Value-at-Risk for long and short trading positions: Evidence from developed and emerging equity markets

by

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May 31, 2010

Abstract

The financial crisis of 2007-2009 has questioned the provisions of Basel II agreement on capital adequacy requirements and the appropriateness of VaR measurement. This paper reconsiders the use of Value-at-Risk as a measure for potential risk of economic losses in financial markets by estimating VaR for daily stock returns with the application of various parametric univariate models that belong to the class of ARCH models which are based on the skewed Student distribution. We used daily data for three groups of stock market indices, namely Developed, Southeast Asia and Latin America. The data covered the period 1987-2009. We conducted our analysis with the adoption of the methodology suggested by Giot and Laurent (2003). Therefore, we estimated an APARCH model based on the skewed Student distribution to fully take into account the fat left and right tails of the returns distribution. The main finding of our analysis is that the skewed Student APARCH improves considerably the forecasts of one-day-ahead VaR for long and short trading positions. Additionally, we evaluate the performance of each model with the calculation of Kupiec's (1995) Likelihood Ratio test on the empirical failure test. Moreover, for the case of the skewed Student APARCH model we computed the expected shortfall and the average multiple of tail event to risk measure. These two measures helped us to further assess the information we obtained from the estimation of the empirical failure rates.

Keywords: Value-at-Risk, risk management, APARCH models, skewed Student distribution

JEL Classification: C53, G21; G28

*We have benefited from comments by seminar participants at the Athens University of Economics and Business, the University of Crete and Philips College. Kouretas acknowledges financial support from a Marie Curie Transfer of Knowledge Fellowship of the European Community's Sixth Framework Programme under contract number MTKD-CT-014288, as well as from the Research Committee of the University of Crete under research grants #2016, #2030 and #2257. We also thank without implicating, Richard Baillie, Dimitris Georgoutsos, Dimitris Moschos, Lucio Sarno and Elias Tzavalis for numerous valuable comments and discussions on an earlier draft.

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1. Introduction

During the recent years the importance of effective risk management has become extremely crucial. This was the outcome of several significant factors. First, the enormous growth of trading activity that has been taking place in the stock markets, especially those of the emerging economies which, however, led to an increase in financial uncertainty and increased volatility in the stock returns. Indeed, during the period 1992-2008 an enormous inflow of portfolio funds to the emerging markets of Central and Eastern Europe, Southeast Asia and Latin America was recorded and this capital inflow was due to the fact that over this period the mature markets have reached their limitations with respect to profit opportunities leading portfolio managers and institutional investors to look for new opportunities in these new markets. Second, the financial disasters that took place in the 1990s led to bankruptcy well-known financial institutions. These events have put great emphasis for the development and adoption of accurate measures of market risk by financial institutions. Financial regulators and supervisory committee of banks have favoured quantitative risk techniques which can be used for the evaluation of the potential loss that financial institutions can suffer. Furthermore, given that the nature of these risks changes over time effective risk management measures must be responsive to news such as other forecasts as well as to be easy understood even in complicated cases.

This need was further reinforced by a number of financial crises that took place in the 1980s, 1990s and the 2000s such as the worldwide stock markets collapse in 1987, the Mexican crisis in 1995, the Asian and Russian financial crises in 1997-1998, the Orange County default, the Barings Bank, the dot.com bubble and Long Term Capital Management bankruptcy cases as well as the financial crisis of 2007-2009 which led several banks to bankruptcy worldwide with Lehman Brothers being the most notable case. Such financial uncertainty has increased the likelihood of financial institutions to suffer substantial losses as a result of their exposure to unpredictable market changes. These events have made investors to become more cautious in their investment decisions while it has also led for the increased need for a more careful study of price volatility in stock markets. Indeed, recently we observed an intensive research from academics, financial institutions and regulators of the banking and financial sectors to better understanding the operation of capital markets and to develop sophisticated models to analyze market risk.

Basle I and II Agreements has been the main vehicle globally for the set up of the regulatory framework on financial markets following the dramatic events in financial markets in the late 1980s and early 1990s. Basle I was introduced in late 1980s and it was based on risk classification of assets with the main purpose to force banks to provide sufficient capital adequacy against these assets based on their respective risks. However, it turned out that this attempt to impose capital ratios for banks had adverse effects since Basle I put a low risk weight on loans by banks to other financial institutions. In this framework banks were given an incentive to transfer risky assets off their balance sheets. Regulation arbitrage was further incurred since Basle I made possible for banks to treat assets that were insured as government securities with zero risk a feature that was fully exploited by the banks and led to the huge increase of the market for CDS.

In an attempt to remedy some of the problems created since the implementation of Basle I Agreement, Basle II was introduced in the 1990s and it was put in full implementation in 2007. A central feature of the modified Basle II Accord was to allow banks to develop and use their own internal risk management models conditional upon that these models were tested under extreme circumstances and properly 'backtested" and "stress tested". Value-at-Risk has become the standard tool used by financial analysts to measure market risk. VaR is defined as a certain amount lost on a portfolio of financial assets with a given probability over a fixed number of days. The confidence level represents 'extreme market conditions' with a probability that is usually taken to be 99% or 95%. This implies that in only 1% (5%) of the cases will lose more than the reported VaR of a specific portfolio. VaR has become a very popular tool among financial analysts which is widely used because of its simplicity. Essentially the VaR provides a single number that represents market risk and therefore it is easily understood.[1]

During the last decade several approaches in estimating the profit and losses distribution of portfolio returns have been developed, and a substantial literature of empirical applications have emerged which provided an overall support for the use of VaR as the appropriate measure of market risk. A number of these models have focused on the computation of the VaR on the left tail of the distribution which corresponds to the negative returns. This implies that it is assumed that portfolio managers or traders have long trading positions, which means that they bought an asset at a given price and they are concerned with the case that the price of this asset falls resulting in losses. More recent approaches dealt with modeling VaR for portfolios that includes both long and short positions. Therefore, they considered the modeling and calculation of VaR for portfolio managers who have taken either a long position (bought an asset) or a short position (sold an asset). As it is well known, in the former case the risk of a loss occurs when the price of the traded asset falls, while in the later case the trader will incur a loss when the asset price increases.[2] Hence, in

[1] See also Bank for International Settlements (1988, 1999a,b,c, 2001).
[2] Bodie *et al*. (2009) provide a comprehensive analysis of trading strategies.

the first case we model the left tail of the distribution of returns and in the second case we model the right tail of the distribution.

Furthermore, given the stylized fact that the distribution of asset returns is non-symmetric, Giot and Laurent (2003) using daily data for FTSE100, NASDAQ and NIKKEI225 have shown that models which rely on a symmetric density distribution for the error term underperform with respect to skewed density models when the left and right tails of the distribution of returns must be modeled. Therefore, in such a case the VaR for portfolio managers or traders who hold both long and short positions cannot be accurately modeled by the application of the standard normal and Student distributions. Giot and Laurent (2003) also showed that similar problems arise when we try to model the distribution with the asymmetric GARCH models which assume that an asymmetry exists between the conditional variance and the lagged squared error term, (see also El Babsiri and Zakoian, 2001). So and Yu (2006), Tang and Shieh (2006), McMillan and Speight (2007) and McMillan and Kambouridis (2009) provided recent evidence on the performance of alternative VaR models for a large number of stock as well as exchange rate markets. They confirmed prior evidence that models which take into consideration the asymmetric effects and long memory features of the data perform better than the specifications which model the conditional variance errors to be normally distributed.

The financial crisis of 2007-2009 has raised questions regarding the usefulness of the regulatory framework underlined by the Basle I and II agreements and it also questioned the appropriateness of the VaR as the measure to capture extreme cases like the ones the banking sector and global financial markets experienced over this turbulent period (See for example, Brunnermeier, 2009; De Grauwe (2009) and Welfens, (2009). De Grauwe (2009) argued that the Basle approach to stabilize the banking system has an implicit assumption that financial markets are efficient. Market efficiency implies that returns are normally distributed. However, it has by now documented in the finance literature that asset returns are not normally distributed but they have distributions with fat tails. Therefore, De Grauwe (2009) argued that the Basle Accords have failed to provide stability in the banking sector since the risks linked with universal banks are tails risks associated with bubbles and crises. Fat tails are linked to the occurrence of bubbles and crises and this implies that models based on normally distributed errors substantially underestimate the probability of large shocks. These consequences of the financial crisis of 2007-2009 increased the need for a closer look in modeling the volatility of returns of asset markets and more importantly to model VaR for portfolios on long and short positions which are mainly constructed from stocks which are traded in both the mature and emerging markets. Thus, we focus on the joint behaviour of VaR models for long and short trading positions.

The aim of this paper is to reconsider the evidence on the forecasting ability of four competing models in estimating VaR of stock market indices. We conducted our study to portfolios for long and short positions on daily stock indexes of 21 stock market indices of developed and emerging markets using data that extends from 1980 to the end of 2009 in order to take into consideration any effects due to the financial crisis of 2007-2009. In order to account for possible asymmetries in the behavior of stock returns we applied the univariate Asymmetric Power ARCH (APARCH) model introduced by Ding *et al*. (1993) which allows to model and calculate the VaR for portfolios defined on long position (long VaR) and short position (short VaR). The in-sample and out-of sample performance of this model was compared with those of the standard parametric Riskmetrics and normal and Student APARCH models.

Following Giot and Laurent (2003), So and Yu (2006), Tang and Shieh (2006), McMillan and Speight (2007) and McMillan and Kambouridis (2009) we examined the performance of these competing models at the 1% and 5% tails

The main finding of our analysis is that the skewed Student APARCH improves considerably the forecasts of one-day-ahead VaR for long and short trading positions. Additionally, we evaluated the performance of each model with the calculation of Kupiec's (1995) Likelihood Ratio test on the empirical failure test. Moreover, for the case of the skewed Student APARCH model we computed the expected shortfall and the average multiple of tail event to risk measure. These two measures helped us to further assess the information we obtained from the estimation of the empirical failure rates.

The remainder of the paper is organized as follows. Section 2 describes the basic concept of VaR and presents the alternative VaR models for modeling financial return series. In section 3 we report our empirical results and finally section 4 provides our concluding remarks.

2. Value-at-Risk and VaR models

We begin with a brief description of the VaR concept. VaR is the standard measure of market risk that provides the financial institutions with the information on the minimum amount that it is expected to lose with a small probability α over a given horizon κ (which is usually taken to be 1 day). Thus, assuming that the VaR model is correct, a 1-day vaR at $\alpha = 5\%$ of 10 million Euros could tell us that one out of 20 days we could expect to occur a loss exceeding 10 million Euros. Therefore, VaR is defined as the maximum loss over a given time horizon at a given confidence level. Coupled with this, the VaR provides information about the required minimum amount of capital required to cover all but a small pre-specified proportion of expected losses. As we already discussed Basle II Agreement requires that financial institutions develop their own internal risk management models to assess their market risk. Then, the true test for these models is provided by "backtesting" which amounts to a comparison of actual trading losses with the estimated VaR and recording the number of exceptions (failures). Given, these considerations we define the k-day VaR on day t is given by:

$$P(P_{t-k} - P_t \leq VaR(t,k,\alpha)) = 1-\alpha$$

where P is the stock price on day t.

Assuming a specific distribution of the stock return, we calculate the VaR in terms of a given percentile of this distribution (Jorion, 2000, Alexander 1003, 2005). Therefore, if we define q_α as the α percentile of the given distribution of returns, then VaR can be expressed as:

$$VaR(t,k,a) = (1-e^{q_\alpha})P_{t-k}$$

In obtaining good VaR measures it is crucial therefore to produce accurate forecasts of the percentiles q_α. This in turn requires the adoption of alternative distributional specifications of asset returns that allows for various hypotheses on conditional volatility.

This section draws heavily on Giot and Laurent (2003) and we provide a description of the four models used in the analysis. The starting point is the definition of the conditional mean and variance of the disturbance term which is relevant for all alternative VaR specifications. Therefore, we consider a series of daily returns, y_t, with $t = 1...T$. In order to take into account the serial correlation that daily returns exhibit as it is well known we fit an $AR(n)$ model on the y_t series:

$$\Phi(L)(y_t - \mu) = \varepsilon_t \qquad (1)$$

where $\Phi(L) = 1 - \phi_1 L - .....\phi_n L^n$ is defined as an AR lag polynomial of order n. Thus, the conditional mean of y_t, i.e. μ_t, is equal to $\mu + \sum_{j=1}^{n} \phi_j (y_{t-j} - \mu)$. The crucial issue in VaR modeling is the specification that the conditional variance takes. As we have already mentioned in the present paper we considered four models with corresponding conditional variance specification, namely, Riskemetrics, Normal APARCH, Student APARCH and skewed Student APARCH.[3] The performance of each model was based on how well it can predict long VaR trading positions (i.e. to model large negative returns) while with respect to the right tail of the distribution of returns the predictive performance of of short VaR is evaluated by its ability to model large positive returns.

2.1. Riskmetrics

J.P. Morgan's Riskmetrics (1996) model combines an econometric model with the assumption of conditional normality for the returns series. Specifically, this model rely on the specification of the variance equation of the portfolio returns and the assumption that the standardized errors are i.i.d.. In this model the autoregressive parameter is pre-specified at given value λ whereas the coefficient of ε_{t-1}^2 equals to $1 - \lambda$. For the case of daily data, $\lambda = 0.94$ and we then obtain:

[3] Jorion (2000) and Alexander (2003) provide a complete analysis of the VaR methodology and alternative estimation methodologies

$$\varepsilon_t = \sigma_t z_t \tag{2}$$

where the standardized error z_t is i.i.d $N(0,1)$ and the variance σ^2 is defined as:

$$\sigma_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2 \tag{3}$$

Then the one-step-ahead VaR forecast computed in $t-1$ for the case of long positions is calculated by $\mu_t + z_a\sigma_t$, and for the short position is calculated by $\mu_t + z_{1-a}\sigma_t$, with α chosen to be a standard level of significance.[4] Since $z_\alpha = -z_{1-\alpha}$ the forecasted long and short VaR will be equal.

2.2. *Normal* APARCH

The normal APARCH developed by Ding *et al*. (1993) is an extension of the GARCH model, (Bollerslev; 1986). The advantage of this class of models is its flexibility since it includes a large number of alternative GARCH specifications. The APARCH (1,1) model is given by the following expression:

$$\sigma^2 = \omega + \alpha_1(|\varepsilon_{t-1}| - \alpha_n\varepsilon_{t-1})^\delta + \beta_1\sigma_{t-1}^\delta \tag{4}$$

where $\omega, \alpha_1, \alpha_n, \beta_1$ and δ are parameters to be estimated in addition to μ_t and σ_t. The term $\alpha_n(-1 < \alpha_n < 1)$, represents the leverage effect, while the coefficient $\delta(\delta > 0)$ is a Box-Cox transformation of σ_t.[5] He and Terasvista (1999a,b) provide a thorough analysis of the properties of the APARCH model.

[4] We note that when calculating the VaR the conditional mean and variance are computed with the replacement of the unknown parameters in equation (1) with their MLE estimates.

[5] Black (1976), French *et al*. (1987) and Pagan and Schwert (1990) among others suggest that the leverage effect means that a positive (negative) value of α_n implies that the past negative (positive) shocks have a deeper impact on current conditional volatility than past positive shocks.

The one-step-ahead VaR forecast for the normal APARCH is computed with the same way as for the Riskmetrics model with the only difference that the conditional variance is given by equation (4).[6]

2.3. Student APARCH

It has been well documented in the finance literature that that models which rely on the assumption that the distribution of returns follows the normal one fail to take into account the fat tails of the distribution of results leads to the underestimation of the VaR. This underestimation can be corrected by allowing alternative distributions of the errors such as the Gaussian, *Student's-t* and Generalized Error Distribution. The adoption of the Student APARCH (ST APARCH) is a potential solution to the problem. The specification of errors is given by:

$$\varepsilon_t = \sigma_t z_t \qquad (5)$$

where z_t is i.i.d. $t(0,1,\upsilon)$ and σ_t is defined as in equation (4).

The one-step-ahead VaR for long and short positions is given by $\mu_t + st_{\alpha,\upsilon}\sigma_t$ and $\mu_t + st_{1-\alpha,\upsilon}\sigma_t$, with α chosen to be a standard level of significance.[7]

2.4. Skewed Student APARCH

Fernandez and Steel (1998) have extended the student distribution with the addition of a skewness parameter to take into consideration the problems of skewness and kurtosis detected in financial databases. This has led to the development of the

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[6] As before σ_t is evaluated at its MLE.

[7] As in the case of the normal of distribution, since $st_\alpha = -st_{1-\alpha}$ the forecasted long and short VaR will be equal.

skewed student APARCH. However, their approach has the disadvantage that this proposed skewness parameter is expressed in terms of the mode and the dispersion. To avoid this deficiency Lambert and Laurent (2001) have re-expressed the skewed student density in terms of the mean and the variance by a re-parameterization of the density so that the innovation process has zero mean and unt variance.[8]

We draw mainly on Giot and Laurent (2003) and we provide a discussion of the statistical properties of the skewed Student APARCH model based on the approach suggested by Lambert and Laurent (2001).

The innovation process z_t is distributed according to the standardized skewed Student distribution if:

$$f(z \mid \xi, \upsilon) = \begin{cases} \dfrac{2}{\xi + \dfrac{1}{\xi} sg[sz + m) \mid \upsilon]} \text{ if } z < -\dfrac{m}{s} \\ \dfrac{2}{\xi + \dfrac{1}{\xi} sg[(sz + m) / \xi \mid \upsilon]} \text{ if z} \geq -\dfrac{m}{s} \end{cases} \tag{6}$$

where $g(. \mid \upsilon)$ is the symmetric (unit variance) Student density and ξ is the asymmetry coefficient. In addition, m and s^2 are respectively the mean and the variance of the non-standardized skewed Student:

$$m = \frac{\Gamma(\frac{\upsilon - 1}{2})\sqrt{\upsilon - 2}}{\sqrt{\pi}\Gamma(\frac{\upsilon}{2})}(\xi - \frac{1}{\xi}) \tag{7}$$

[8] Hansen (1994) argues that this is necessary otherwise we are unable to discriminate between the fluctuations occurred in the mean and variance from the fluctuations occurred in the shape of the conditional density.

and

$$s^2 = (\xi^2 + \frac{1}{\xi^2} - 1) - m^2 \tag{8}$$

where the density function $f(z_t \mid 1/\xi, \upsilon)$ is the opposite of $f(z_t \mid \xi, \upsilon)$ with respect to the zero mean. Thus, the sign of $\log(\xi)$ gives an indication of the direction of skeweness, i.e. the skewness factor (m_3) is positive (negative), and the probability density function is skewed to the right (left), if $\log(\xi) > 0 (< 0)$.

Moreover, Lambert and Laurent (2000) show that the quantile function of $skst^*_{\alpha,\upsilon,\xi}$ of a non standardized skewed Student density is:

$$skst^*_{\alpha,\upsilon,\xi} = \begin{cases} \frac{1}{\xi} st_{a,u}[\frac{\alpha}{2}(1+\xi^2) \text{ if } \alpha < \frac{1}{1+\xi^2} \\ -\xi st_{\alpha,\upsilon}[\frac{1-\alpha}{2}(1+\xi^{-2})] \text{ if } \alpha \geq \frac{1}{1+\xi^2} \end{cases} \tag{9}$$

where $sk_{\alpha,\upsilon}$ is the quantile function of the (unit variance) Student-*t* density. Then we obtain the quantile function of the standardized skewed Student distribution as follows:

$$skst_{\alpha,\upsilon,\xi} = \frac{skst_{\alpha,\upsilon,\xi} - m}{s}$$

Following Ding *et al*. (1993), if it exists, a stationary solution of equation (4) is given by:

$$E(\sigma_t^\delta) = \frac{\omega}{1 - \alpha_1 E(|z| - \alpha_n z)^\delta - \beta_1} \tag{10}$$

which is a function of the density of z. Such a solution exists if

$$V = \alpha_1 E(|z| - \alpha_n z)^{\delta} + \beta_1 < 1$$

Ding et al. (1993) derived the expression for $E(|z| - \alpha_n z)^{\delta}$ for the Gaussian case. We can also show that for the standardized skewed Student distribution is given as follows:

$$E(|z| - \gamma z)^{\delta} = \{\xi^{-(1+\delta)}(1+\gamma)^{\delta} + \xi^{1+\delta}(1-\gamma)^{\delta}\}\frac{\Gamma(\frac{\delta+1}{2})\Gamma(\frac{\upsilon-\delta}{2})(\upsilon-2)^{\frac{1+\delta}{2}}}{(\xi+\frac{1}{\xi})\sqrt{(\upsilon-2)\pi}\,\Gamma(\frac{\upsilon}{2})} \quad (11)$$

For the skewed Student APARCH model, the VaR for long and short positions is given by $\mu_t + skst_{\alpha,\upsilon,\xi}\sigma_t$ and $\mu_t + skst_{1-\alpha,\upsilon,\xi}, \sigma_t$. $skst_{\alpha,\upsilon,\xi}$ $(skst_{1-\alpha,\upsilon,\xi})$ is the left(right) quantile of the skewed Student distribution at level of significance $\alpha\%$ $(1-\alpha\%)$ with υ degrees of freedom whereas ξ is the asymmetry coefficient. If $\log(\xi)$ is smaller than zero $(or\ \xi < 1)$ then $|skst_{\alpha,\upsilon,\xi}| > |skst_{1-\alpha,\upsilon,\xi}|$ and in this case the VaR for long trading positions will be larger (for the same conditional variance) than the VaR for the short position. When $\log(\xi)$ is positive the opposite situation arises.

3. Empirical results

We applied alternative VaR model specifications on daily returns. The data set referred to three groups of stock market indices, namely developed, Latin America and Asia/Pacific. Specifically, the following stock market indices data were used:

Australia (ALL ORDINATES, AOI); France (CAC40); Germany (DAX30); Greece (FTSE20); Japan (NIKKEI225); Spain (MADRID General, SMSI); UK (FTSE100); U.S.A (S&P 500); Argentina (MERVAL, MERV); Brazil (BOVESPA, BVSP); Mexico (IPC); China (SHANGHAI Composite Index, SSG); Hong Kong (HANG SENG Index, HSI); India (BOMBAY, BSE30); Indonesia (JAKARTA Composite, JSX); Malaysia (Kuala Lumpur Composite Index, KLSE); Philippines (PSI); Singapore (STRAIT TIMES Industrial Index, STII); South Korea (SEOUL Composite, KOSPI); Sri Lanca (COLOMBO Index Average, CSE) and Taiwan (Taiwan Weighted Stock Index, WEIGHTED). The starting year of the series under consideration varies and it was based on availability. The time span of the data sets is presented in Table 1. All data sets end on December 31, 2009. The complete data set was obtained from DATASTREAM. Given the recent financial crisis of 2007-2009 it is of crucial importance to investigate the performance of the VaR measures of market risk for the case of stocks traded in developed as well as in emerging markets. In order to implement our analysis we constructed historical portfolios for each case and we chose a specification of the functional form of the distribution of returns. We successively considered the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH. The estimation process was conducted for the full sample whereas we used the last 5 years (each year is taken to have 252 trading days) to conduct the out-of-sample forecasting evaluation.

The daily returns are computed as 100 times the difference of the log of the prices, i.e. $y_t = 100[\ln(p_t) - \ln(p_{t-1})]$. Table 1 reports descriptive statistics for the returns series. As it was expected higher returns were recorded for the emerging markets compared to those of the developed markets. Moreover, according to the estimated standard deviations it was clear that the developed markets were the less risky with the exception of Greece which although is considered a mature market it exhibited return volatility compared to that of the emerging markets. We also observed that all 21 stock return series display similar statistical properties with respect to skewness and kurtosis. Thus, the return series were skewed (either negatively or positively) whereas the large returns (either positive or negative) lead to a large degree of kurtosis. Furthermore, The Lung-Box Q^2 statistics for all returns series were statistically significant, providing evidence of strong second-moment dependencies (conditional heteroskedasticity) in the distribution of the stock price changes.

Given these salient features of the daily returns for all three groups of stock market returns we now move to perform the VaR analysis based on the four chosen volatility models and we computed and fully characterized the corresponding VaR results for long and short trading positions. Table 2 reports the results for the (approximate maximum likelihood) estimation of the skewed Student APARCH model on all 21 daily return series.[9] The calculated Ljung-Box Q^2-statistic was not significant for all stock returns and this implied that the skewed Student APARCH model was successful in taking into account the conditional heteroskedasticity exhibited by the data. Furthermore, it was shown that the autoregressive coefficient in the volatility specification β_1 takes values between 0.62 to 0.93 suggesting that there were substantial memory effects. The coefficient α_n was found to be positive and statistically significant for all series, indicating the existence of a leverage effect for negative returns in the conditional variance specification. The next important result concerns the value of $\log(\xi)$, which is negative in all twenty one cases and this

[9] All computations were performed with G@RCH 6.0. procedure on Ox package (see also Laurent and Peters, 2002, 2009). We used the method of maximum likelihood with a Gaussian density and the BGFS algorithm in order to estimate all parameters of the four VaR models.

finding implies that we were correct in incorporating the asymmetry element in the Student distribution in order to model the distribution of returns in an appropriate way. The final significant result reported in Table 2 refers to the value of δ which takes values from 1.12 and 1.78 statistically significant from 2. In summary the above results indicated that the skewed Student APARCH model takes into consideration the feature of a negative leverage effect (conditional asymmetry) for the conditional variance and it is also consistent with the fact that an asymmetric distribution for the error term (unconditional asymmetry) exists.

We next moved to examine whether the skewed Student APARCH model provided better VaR estimates and forecasting performance than the other three models, Riskemetrics, Normal APARCH and Student APARCH. To this end we provided in-sample VaR computations and this was accomplished by computing the one-step-ahead VaR for all models. This procedure is equivalent to backtesting the model on the estimation sample. We tested all models with a VaR level of significance, (α), that takes values from 0.25% to 5% and we then evaluated their performance by calculating the failure rate for the returns series y_t. The failure rate is defined as the number of times returns exceed the forecasted VaR. Following Giot and Laurent (2003) we defined a failure rate f_l for the long trading positions, which is equal to the percentage of negative returns smaller than one-step-ahead VaR for long positions. In a similar manner, we defined f_s as the failure rate for short positions as the percentage of positive returns larger than the one-step-ahead VaR for short position.[10]

[10] When the VaR model is correctly specified then the failure rate should be equal to the pre-specified VaR level.

To evaluate the in-sample forecasting ability of the alternative VaR measures we employed the unconditional backtesting criterion developed by Kupiec (1995). This criterion tests the hypothesis that the proportion of violations (failures) is equal to the expected one.[11] Under the null hypothesis Kupiec (1995) developed a likelihood ratio statistic given as follows:

$$LR_{uc} = 2\ln[1-\hat{f}]^{T-N} fN] - 2\ln[(1-f)^{T-N} f^{N}\} \sim \chi_1^2 \tag{12}$$

where $f = N/T$ is the failure rate, $\hat{f}$ is the empirical (estimated) failure rate, N is the number of days over a period T that a violation has occurred. Giot and Laurent (2003) suggested that the computation of the empirical failure rate defines a sequence of yes/no, under this testable hypothesis. Table 3(a)-(d) reports the corresponding p-values for the four VaR models and for given significance levels.[12] Based on the results of the in-sample VaR computations we can arrive at the following conclusions: For the long position with $\alpha = 1\%$ VaR models based on the normal distribution (i.e. Riskmetrics and normal APARCH model) performed a particularly poor job in modelling large positive and negative returns, indicating that they generated biased VaR estimates. The symmetric Student APARCH model provided substantially better results compared to those of the normal model. Finally, the skewed Student APARCH model provided the best performance of all VaR models for both negative and positive returns. In the relatively less extreme case of a 5% VaR we observed that Riskemetrics still had a difficult job in modelling large positive and negative returns

[11] A violation is defined as the case where the predicted VaR is unable to cover the realized loss (or to foresee the realized profit)

[12] To save space we report only the results for 1% and 5%. However the results for the other values of α are consistent with those for 1% and 5%. These results are available upon request.

for the developed and Latin America stock markets but did a fairly good job in most of the Asia/Pacific markets. The normal APARCH has partially improved its performance since it provides accurate VaR estimates for all developed and Latin America markets although it fails to do so for most of the markets of the Asia/Pacific region. The symmetric Student APARCH performed well for most of the stock markets except for the cases of Japan and South Korea whereas the skewed Student APARCH performed exceptionally well.

In the short position with $\alpha = 1\%$ we observed that the Riskmetrics model provided accurate VaR estimates for all the developed markets except the US and for the Latin America markets but not for most of the Asia/Pacific markets. An interesting finding was that the normal APARCH performed better than the symmetric Student APARCH model but was again the skewed Student APARCH that performed better in terms of the insignificance of the Kupiec test. Looking at the 5% VaR results we observe that the Riskemetrics model performed well since except for Australia and India all Kupiec statistics were insignificant, and in fact much better than either the normal APARCH or the symmetric Student APARCH. Again, the best performance was given by the skewed Student APARCH. This apparent asymmetry in the performance of the Riskemetrics model between the long and short positions may be the result of skewed distribution of returns it may also be due to the volatility asymmetry effect in response to good and bad news.

The picture that emerges from Table 4(a)-(b) further reinforced the superiority of the skewed Student APARCH model over the alternative specifications. Indeed, this specification performed correctly in 100% of all cases for the negative returns (long position) and with only few exceptions for the positive returns (short position). Moreover, we noted that the skewed Student APARCH performed better than the student APARCH since it corrected a number of deficiencies that the latter model has inherited as a result of its conservatism.

Engle and Manganelli (2004) argued that the Kupiec test is an unconditional test of VaR accuracy which given the substantial time-variation within volatility may lead to incorrect conclusions and therefore conditional accuracy of the VaR estimates must be considered. Therefore, in a correctly specified VaR model we should not only examine whether the exceptions occur at the specified rate (i.e. 1% or 5%) but we also need to examine whether these exceptions are not serially correlated. To this end Engle and Manganelli (2004) proposed the following Dynamic Quantile test statistic (DQ) which takes this issue into consideration. Engle and Manganelli (2004) define the following sequence:

$$Hit_k = I(r_k < -VaR_k) - a$$

Thus, this sequence takes the value (1 — *α)* in the case that the stock returns, r_k are less than the VaR quantile and the value (*α*) otherwise, with the expected value of Hit_k equal to zero. Within this setting the Kupiec test is a sub-case since although it tests that this sequence will be the correct fraction of exceptions it does not test the null hypothesis that this sequence is uncorrelated with past information and have a mean value of zero, which implies that the hits will not be autocorrelated. To test the null hypothesis of no serial correlation in the hit sequence Hit_k Engle and Manganelli (2004) proposed to run a regression on five lags (days) and the current VaR estimate. The DQ test statistic is then computed as: $DQ = \hat{\beta}' X' X \hat{\beta} / \alpha(1-\alpha)$ where X is the vector of explanatory variables and $\hat{\beta}$ the OLS estimates. The DQ test is distributed as a x^2 with degrees of freedom equal to the number of parameters.

Tables 5(a)-(d) provide the evidence from the application of the DQ statistic for the in-sample performance of the four competing models. For the case of the long position and at both 1% and 5% VaR the Riskmetrics model performed the worst across all markets, and the normal APARCH and the Symmetric Student APARCH provided almost equivalent VaR accuracy as with the ones resulted from the Kupiec test. The superiority of the skewed Student APARCH was confirmed again with the use of the most restricted DQ test. Examining the case for the short position we notably observed that the Riskmetrics model performed much better than the normal APARCH and the Symmetric Student APARCH. Again, this asymmetric result may be due to skewed distribution of returns and/or volatility asymmetry in response to good and bad news. Finally, the Skewed Student APARCH performed better than its competing models.

We further assessed the performance of the four models by computing the out-of-sample VaR forecasts. This is considered as the 'true' test for any VaR model. Out-of-sample evaluation of a specific model requires the estimation of the model for the known data points and then based on the estimated equation we provided forecasts for a specific time horizon. This testing procedure is implemented to provide one-day-ahead VaR forecasts.[13] Following Giot and Laurent (2003) we applied an iterative procedure in which the estimated model for the whole sample was estimated and we then compared the predicted one-day-ahead VaR for both the long and short positions with the actual return. This procedure was repeated for all known observations and every time the estimation sample included one more day and we forecasted the corresponding VaR. These forecasts were saved and they were used for the evaluation

[13] Christoffersen and Diebold (2000) documented that the ARCH-class of models exhibit good volatility forecasting ability for short horizon their performance is poor when it comes to long horizon prediction. Although the latter may be more important for portfolio managers we only provide short run analysis of predictive performance.

of the out-of-sample predictive performance of the models.[14] The iteration procedure ends when, as it is the common practice, we have included the $t-1$ days in the estimation of the model. The predictive performance of the skewed Student APARCH model is then evaluated with the use of the Kupiec (1995) likelihood ratio test as in the in-sample case. However, this time the failure rate was calculated for both the long and short positions by comparing the corresponding forecasted VaR_{t+1} with the observed return y_{t+1}.

The results obtained from the Kupiec test for all return series are given in Table 6(a)-(d). Like Table 3(a)-(d) we report the calculated p-values for alternative level of significance for both long and short trading positions. These findings revealed that Riskemetrics had a very poor out-of-sample forecasting performance in both long and short trading positions making this model an inappropriate specification for financial institutions to rely upon especially during very turbulent periods like the 2007-2009 financial crisis. For the long position the normal APARCH and the Symmetric Student APARCH performed fairly well in out-of-sample prediction VaR accuracy and their results were similar to the in-sample ones although the former missed out Australia, UK and US, China, India and Taiwan and the latter the UK. Turning our attention to the skewed Student APARCH model we concluded that it performed well for out-of-sample VaR prediction for both long and short trading positions and for both 1% and 5% VaR. Furthermore, we noted that the combined (i.e. long and short VaR) success rate was equal to 100% in all three groups of stock returns and this finding further reinforced the suitability of the skewed Student APARCH model in measuring market risk in developed and emerging.

[14] To conduct our out-of-sample forecasting analysis we employ the last five years (1260 obs.) of our sample. We also use a ‘stability window’ of 50 days to update the model parameters.

The results from the application of the DQ test for the out-of-sample performance of the four VaR models are given in Tables 7(a)-(d). The overall evidence was very similar to the one obtained from the Kupiec test. Riskemetrics was again a poor model in predicting VaR accuracy for all twenty one markets. Normal APARCH and Symmetric Student APARCH improved considerably on the performance of the Riskemetrics model but their performance was still not satisfactory in all cases. The skewed Student APARCH model improved on all other specifications for both negative and positive returns and for either 1% or 5% critical level. The model performed correctly in 100% of all cases for the long position and for the short position.[15]

4. Summary and conclusions

During the last decade we have observed a substantial change in the way financial institutions evaluate risk. Faced with increased volatility of stock returns as well as with the heavy losses that banks and securities houses have experienced portfolio managers and supervising committees of financial markets have sought for a continuous improvement to potential measures of market risk. Value at Risk is one of the major tools for measuring market risk on a daily basis and is recommended by the Basel Committee on Banking Supervision and this is documented in the Basel Accord Amendment of 1996.

These models of risk management have become the standard tool for measuring internal risk management as well as for external regulatory purposes. However, most of the recent applications of evaluating VaRs for a wide range of

[15] We have further tested the performance of the four models with the calculation of two additional measures relevant to the VaR analysis: The expected shortfall (Scaillet, 2000) and the average multiple of tail event to risk measure (Hendricks (1996). Both measures provided additional support in favour of the skewed Student APARCH model. To save space the results are available upon request.

markets have mostly applied for the case of negative returns, i.e. for the negative tail of the distribution of returns. The present paper dealt with modeling VaR for portfolios that includes both long and short positions. Therefore, we considered the modeling and calculation of VaR for portfolio managers who have taken either a long position (bought an asset) or a short position (sold an asset).

The financial crisis of 2007-2009 has raised questions regarding the usefulness of the regulatory framework underlined by the Basle I and II agreements and it also questioned the appropriateness of the VaR as the measure to capture extreme cases like the ones the banking sector and global financial markets experienced over this turbulent period since several financial institutions went bankrupt under these extreme conditions.

This paper extends previous works on the forecasting performance of alternative VaR models. Specifically, we focused on the comparison of four alternative models for the estimation of one-step-ahead VaR for long and short trading positions. We have applied a battery of univariate tests on four parametric VaR models namely, RiskMetrics, normal APARCH, Student APARCH and skewed Student APARCH. We employed data that covered the recent financial crisis of 2007-2009 for twenty of stock markets which included both developed and emerging stock markets. Our purpose was twofold. First we seek to examine whether alternative VaR models provided different evidence for the case of developed as opposed to the emerging markets and second. We examined the performance of these models when periods of turbulence are included in the sample. Our overall results lead to the overwhelming conclusion that the skewed Student APARCH model outperforms all other specification modelling VaR for either long or short positions. These findings confirmed earlier evidence which show that models which include an asymmetric parameter perform substantially better than those with normal errors and this asymmetric feature is important in forecasting accuracy of the VaR models.

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Table 1. Descriptive statistics of stock market returns

Index	Starting Date	*n*	Mean	Standard deviation	Skewness	Kurtosis	$Q^2(10)$
Developed							
Australia	3/8/84	6368	0.0297	1.0124	-4.62	108.43	209.558*
France	1/3/90	4946	0.0147	1.4173	-0.03	4.77	2536.12*
Germany	26/11/90	4759	0.0287	1.4737	-0.09	5.00	2331.99*
Greece	2/1/98	2913	0.0176	1.8698	-0.03	0.48	1092.31*
Japan	4/1/84	6332	0.0003	1.4713	-0.22	11.51	2661.15*
Spain	8/1/02	1990	0.0210	1.3669	-0.06	10.33	1374.76*
UK	2/4/84	6442	0.0238	1.1248	0.40	9.13	3762.85*
US	2/1/80	7507	0.0306	1.1492	-1.25	29.00	1149.29*
Asia/Pacific							
China	4/1/00	2522	0.0270	1.6798	-0.08	4.45	332.80*
Hong Kong	31/12/86	5643	0.0374	1.8215	-2.46	56.16	188.06*
India	1/7/97	3026	0.0456	1.7991	-0.10	4.95	570.71*
Indonesia	1/7/97	2873	0.0409	1.8644	-0.13	6.18	523.94*
Malaysia	3/12/93	3906	0.0410	1.5762	0.40	40.24	939.25*
Philippines	2/7/97	3024	0.0004	1.6292	0.34	10.32	1766.14*
Singapore	28/12/87	5434	0.0216	1.3299	-0.12	8.29	1518.25*
South Korea	1/7/97	3018	0.0262	2.1612	-0.17	3.39	951.24*
Sri Lanca	1/7/97	2936	0.0443	1.5116	2.28	123.92	403.62*
Taiwan	2/7/97	3009	-0.006	1.1669	-0.12	2.17	878.26*
Latin America							
Argentina	8/10/96	3210	0.0391	2.2833	-0.25	8.31	1325.21*
Brazil	27/4/93	4066	0.1925	2.5934	0.46	8.68	1070.12*
Mexico	8/11/91	4468	0.0677	1.6746	0.04	5.13	914.37*

Notes: The columns provide the starting date of the data for each market, total number of observations, arithmetic mean, standard deviation, skewness, kurtosis and $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 on the squared series. The end date of the sample for all markets is 31/12/2009.

Table 2. Skewed Student APARCH

Index	ω	α_1	α_n	β_1	δ	$\log(\xi)$	v	V	$Q^2(10)$
Developed									
Australia	0.020*	0.09*	0.31*	0.90*	1.23*	-0.09*	8.38*	0.98	17.81
France	0.019*	0.06*	0.65*	0.93*	1.17*	-0.08*	12.79*	0.99	23.09
Germany	0.018*	0.08*	0.50*	0.92*	1.17*	-0.10*	9.79*	0.99	14.49
Greece	0.054*	0.12*	0.19*	0.86*	1.64*	-0.05*	7.00*	0.99	12.91
Japan	0.022*	0.09*	0.49*	0.91*	1.17*	-0.06*	7.49*	0.98	18.67
Spain	0.015*	0.06*	0.84*	0.93*	1.35*	-0.12*	9.43*	1.00	9.05
UK	0.015*	0.07*	0.34*	0.92*	1.57*	-0.11*	14.56*	0.99	10.11
US	0.010*	0.06*	0.58*	0.94*	1.17*	-0.04*	7.15*	0.99	15.58
Asia/Pacific									
China	0.031*	0.11*	0.21*	0.90*	1.39*	-0.05*	3.86*	0.99	19.42
Hong Kong	0.027*	0.09*	0.40*	0.91*	1.07*	-0.15*	6.06*	0.98	16.81
India	0.084*	0.13*	0.40*	0.85*	1.54*	-0.09*	8.14*	0.98	15.20
Indonesia	0.164*	0.17*	0.23*	0.79*	1.71*	-0.02*	5.06*	0.95	13.41
Malaysia	0.018*	0.14*	0.26*	0.88*	1.22*	-0.04*	5.16*	0.98	12.46
Philippines	0.127*	0.19*	0.17*	0.80*	1.54*	-0.05*	5.32*	0.93	15.30
Singapore	0.036*	0.13*	0.26*	0.86*	1.35*	-0.09*	6.06*	0.97	18.31
South Korea	0.022*	0.08*	0.32*	0.93*	1.35*	-0.08*	7.45*	0.99	10.18
Sri Lanca	0.112*	0.38*	0.08*	0.62*	1.45*	-0.03*	3.92*	0.91	13.26
Taiwan	0.024*	0.07*	0.51*	0.93*	1.12*	-0.02*	8.60*	0.98	10.80
Latin America									
Argentina	0.123*	0.11*	0.27*	0.88*	1.77*	-0.08*	5.22*	0.98	23.08
Brazil	0.073*	0.10*	0.19*	0.89*	1.78*	-0.07*	9.66*	0.99	18.12
Mexico	0.049*	0.10*	0.54*	0.89*	1.37*	-0.12*	8.04*	0.97	13.89

Notes: Estimation results for the validity specification of the Skewed Student APARCH model. $V = \alpha_1 E(|z| - \alpha_n z)^{\delta} + \beta_1 < 1$ and $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 on the squared series. (*) denotes significance at the five percent critical value.

Table 3(a). VaR results for the long position of the 21 stock markets (in-sample): Kupiec Test

$a = 1\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0	0.08	0.77
France	0	0.01	0.36	0.83
Germany	0	0	0.23	0.70
Greece	0	0.02	0.48	0.48
Japan	0	0	0.67	0.03
Spain	0	0	0.05	0.49
UK	0	0	0.06	0.76
US	0	0	0.91	0.28
Asia/Pacific				
China	0	0	0.39	0.08
Hong Kong	0	0	0.74	0.64
India	0	0.02	0.62	0.82
Indonesia	0	0	0.96	0.89
Malaysia	0	0.09	0.86	0.43
Philippines	0	0.06	0.88	0.96
Singapore	0	0.07	0.38	0.38
South Korea	0	0	0.17	0.55
Sri Lanca	0	0.30	0.02	0.02
Taiwan	0	0	0.22	0.22
Latin America				
Argentina	0	0	0.49	0.58
Brazil	0	0.02	0.95	0.16
Mexico	0	0.04	0.52	0.52

Table 3(b). VaR results for the long position of the 21 stock markets (in-sample): Kupiec test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.51	0.46	0.51
France	0	0.22	0.03	0.66
Germany	0	0.89	0.01	0.94
Greece	0.03	0.82	0.53	0.53
Japan	0	0.35	0	0.37
Spain	0.01	0.14	0.03	0.50
UK	0	0.30	0.02	0.73
US	0	0.47	0.13	0.82
Asia/Pacific				
China	0	0.28	0.01	0.10
Hong Kong	0.18	0.06	0.58	0.59
India	0.17	0.20	0.52	0.20
Indonesia	0.26	0.52	0.30	0.30
Malaysia	0.45	0	0.40	0.98
Philippines	0.04	0.17	0.63	0.75
Singapore	0.10	0	0.63	0.63
South Korea	0	0.03	0	0.07
Sri Lanca	0.15	0	0.20	0.31
Taiwan	0.03	0.30	0.08	0.17
Latin America				
Argentina	0.04	0.49	0.01	0.39
Brazil	0	0.89	0.26	0.89
Mexico	0.02	0.56	0.86	0.80

Notes: P -values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the long trading position is equal to α). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH.

Table 3(c). VaR results for the short position of the 21 stock markets (in-sample): Kupiec test

$a = 1\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0.73	0.83	0	0.32
France	0.72	0.21	0.01	0.35
Germany	0.49	0.02	0	0
Greece	0.07	0.16	0.83	0.83
Japan	0.09	0.96	0.04	0.41
Spain	0.36	0.83	0	0.66
UK	0.11	0.02	0	0.17
US	0	0.49	0	0.05
Asia/Pacific				
China	0	0.06	0.02	0.08
Hong Kong	0	0.73	0.01	0.01
India	0.13	0.32	0.03	0.07
Indonesia	0	0.03	0.95	0.81
Malaysia	0	0.01	0.50	0.24
Philippines	0	0.23	0.55	0.55
Singapore	0	0.19	0.14	0.14
South Korea	0.30	0.55	0	0.43
Sri Lanca	0	0.03	0.76	0.90
Taiwan	0.06	0.86	0.08	0.17
Latin America				
Argentina	0.01	0.09	0.06	0.70
Brazil	0.71	0.83	0.11	0.56
Mexico	0.96	0	0.62	0.62

Table 3(d). VaR results for the short position of the 21 stock markets (in-sample): Kupiec test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0	0	0.71
France	0.09	0	0.03	0.83
Germany	0.50	0	0.03	0.23
Greece	0.53	0.36	0.63	0.63
Japan	0.02	0	0.04	0.12
Spain	0.19	0	0	0.57
UK	0.11	0	0	0.81
US	0.90	0.30	0.57	0.06
Asia/Pacific				
China	0.40	0.16	0.99	0.28
Hong Kong	0.18	0.26	0.71	0.54
India	0	0	0	0.26
Indonesia	0.84	0.04	0.52	0.58
Malaysia	0.12	0.29	0.43	0.84
Philippines	0.30	0	0.88	0.69
Singapore	0.86	0	0.50	0.50
South Korea	0.68	0.05	0.20	0.79
Sri Lanca	0.98	0	0.34	0.92
Taiwan	0.53	0.53	0.71	0.90
Latin America				
Argentina	0.65	0.02	0.15	0.96
Brazil	0.65	0.03	0.11	0.55
Mexico	0.14	0.35	0.87	0.82

Notes: P-values for the null hypothesis $f_s = \alpha$ (i.e. failure rate for the short trading position is equal to α). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH.

Table 4(a). VaR results for the long position of the 21 stock markets (in-sample): Kupiec test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	40	40	100
France	0	20	80	100
Germany	0	40	80	100
Greece	0	40	100	100
Japan	0	20	80	100
Spain	0	20	60	100
UK	0	20	60	100
US	0	20	100	100
Asia/Pacific				
China	0	20	80	100
Hong Kong	20	20	100	100
India	20	40	40	100
Indonesia	20	40	100	100
Malaysia	20	0	100	100
Philippines	0	40	100	100
Singapore	20	40	100	100
South Korea	0	0	60	100
Sri Lanca	20	60	80	100
Taiwan	0	40	60	100
Latin America				
Argentina	0	20	60	100
Brazil	0	40	100	100
Mexico	0	40	100	100

Notes: P -values for the null hypothesis $f_s = \alpha$ (i.e. failure rate for the short trading position is equal to α). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH.

Table 4(b). VaR results for the short position of the 21 stock markets (in-sample): Kupiec test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	60	60	0	60
France	100	60	0	100
Germany	80	40	0	60
Greece	40	60	100	100
Japan	20	20	40	100
Spain	100	60	20	100
UK	80	20	40	80
US	20	100	40	60
Asia/Pacific				
China	20	40	60	100
Hong Kong	20	100	20	40
India	60	60	40	100
Indonesia	20	20	100	100
Malaysia	20	40	100	80
Philippines	40	40	100	100
Singapore	40	40	100	100
South Korea	100	80	40	100
Sri Lanca	20	20	100	100
Taiwan	100	100	60	60
Latin America				
Argentina	40	80	80	100
Brazil	100	60	80	100
Mexico	100	40	100	100

Notes: P -values for the null hypothesis $f_s = \alpha$ (i.e. failure rate for the short trading position is equal to α). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH.

Table 5(a). VaR results for the long position of the 21 stock markets (in-sample): Dynamic Quantile test

$a = 1\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.01	0.01	0
France	0	0.12	0.30	0.35
Germany	0	0.03	0.18	0.25
Greece	0	0	0	0
Japan	0	0.18	0.46	0.01
Spain	0	0	0.07	0.01
UK	0	0.02	0.40	0.24
US	0	0	0.22	0.04
Asia/Pacific				
China	0	0.04	0.14	0
Hong Kong	0	0.01	0	0
India	0	0	0.09	0.02
Indonesia	0	0.27	0.83	0.81
Malaysia	0	0.02	0.09	0.02
Philippines	0	0.48	0.84	0.82
Singapore	0	0.24	0.76	0.76
South Korea	0	0.10	0.21	0.94
Sri Lanca	0	0	0.02	0.02
Taiwan	0	0.02	0	0
Latin America				
Argentina	0	0.01	0.74	0.74
Brazil	0	0.21	0.82	0.27
Mexico	0	0.06	0.03	0.03

Table 5(b). VaR results for the long position of the 21 stock markets (in-sample): Dynamic Quantile test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0.01	0.99	0.53	0.30
France	0	0.18	0.06	0.85
Germany	0	0.10	0.40	0.32
Greece	0	0.63	0.11	0.11
Japan	0	0.47	0.03	0.38
Spain	0	0.21	0.06	0.28
UK	0	0.75	0.17	0.61
US	0	0.40	0.10	0.21
Asia/Pacific				
China	0	0.09	0.08	0.31
Hong Kong	0	0	0	0
India	0	0.66	0.94	0.60
Indonesia	0	0.03	0.09	0.10
Malaysia	0	0	0.07	0.22
Philippines	0	0.04	0.07	0.05
Singapore	0	0	0.03	0.03
South Korea	0	0.52	0.09	0.54
Sri Lanca	0	0	0.34	0.32
Taiwan	0.06	0.27	0.21	0.35
Latin America				
Argentina	0	0.84	0.33	0.89
Brazil	0	0.81	0.24	0.59
Mexico	0	0.66	0.53	0.56

Notes: P-values for the null hypothesis $f_s = \alpha$ (i.e. failure rate for the long trading position is equal to α). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH.

Table 5(c). VaR results for the short position of the 21 stock markets (in-sample): Dynamic Quantile test

$a = 1\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0.44	0.57	0	0.70
France	0.28	0	0.02	0
Germany	0.86	0.12	0	0.13
Greece	0.04	0.17	0.80	0.80
Japan	0.47	0	0	0
Spain	0.27	0	0	0.04
UK	0.48	0.16	0	0
US	0.12	0	0	0
Asia/Pacific				
China	0.01	0.03	0.01	0.07
Hong Kong	0.10	0.45	0.06	0.06
India	0.02	0	0	0.34
Indonesia	0.29	0.03	0	0.09
Malaysia	0.18	0.34	0.36	0.32
Philippines	0	0	0.70	0.70
Singapore	0	0.43	0.05	0.05
South Korea	0.69	0	0	0
Sri Lanca	0.51	0.51	0.88	0.96
Taiwan	0.18	0	0	0
Latin America				
Argentina	0.41	0.24	0	0.36
Brazil	0.84	0	0.18	0.68
Mexico	0.87	0.19	0.85	0.85

Table 5(d). VaR results for the short position of the 21 stock markets (in-sample): Dynamic Quantile test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0.01	0	0	0.08
France	0.22	0	0.06	0.28
Germany	0.77	0.03	0.09	0.16
Greece	0.26	0.04	0.06	0.06
Japan	0.06	0	0	0.24
Spain	0.16	0	0	0.26
UK	0.65	0	0	0.13
US	0.42	0.64	0.95	0.62
Asia/Pacific				
China	0.19	0.15	0.21	0.14
Hong Kong	0.23	0.66	0.88	0.84
India	0.60	0.54	0.03	0.13
Indonesia	0.18	0.02	0.48	0.53
Malaysia	0	0.04	0.61	0.58
Philippines	0	0.01	0.56	0.23
Singapore	0.90	0	0.01	0.01
South Korea	0.69	0	0	0
Sri Lanca	0	0	0.01	0.22
Taiwan	0.83	0.76	0.80	0.17
Latin America				
Argentina	0.30	0.24	0.28	0.65
Brazil	0.73	0.10	0.22	0.64
Mexico	0.46	0.62	0.59	0.56

Notes: P-values for the null hypothesis $f_s = \alpha$ (i.e. failure rate for the short trading position is equal to α). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH.

Table 6(a). VaR results for the long position of the 21 stock markets (out-of-sample): Kupiec test

$a = 1\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0	0.05	0.15
France	0	0.69	0.44	0.44
Germany	0	0.05	0.50	0.91
Greece	0	0.91	0.44	0.64
Japan	0	0.05	0.64	0.64
Spain	0	0.12	0.25	0.71
UK	0	0	0	0.23
US	0	0	0.36	0.50
Asia/Pacific				
China	0	0	0.23	0.83
Hong Kong	0	0.50	0.28	0.28
India	0	0	0.15	0.23
Indonesia	0	0.03	0.86	0.86
Malaysia	0	0.05	0.91	0.23
Philippines	0	0.15	0.69	0.91
Singapore	0	0.15	0.91	0.91
South Korea	0	0.23	0.35	0.44
Sri Lanca	0	0.15	0.08	0.10
Taiwan	0	0	0.23	0.15
Latin America				
Argentina	0	0.04	0.35	0.16
Brazil	0	0.91	0.28	0.10
Mexico	0	0.15	0.91	0.35

Table 6(b). VaR results for the long position of the 21 stock markets (out-of-sample): Kupiec test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.03	0	0.05
France	0	0.13	0.20	0.05
Germany	0	0.31	0	0.05
Greece	0	0.43	0.70	0.99
Japan	0	0	0	0.05
Spain	0	0.06	0.02	0.43
UK	0	0.16	0.06	0.44
US	0	0.16	0.05	0.06
Asia/Pacific				
China	0	0.01	0	0.12
Hong Kong	0	0.70	0.02	0.15
India	0	0.70	0.44	0.99
Indonesia	0	0.14	0.51	0.60
Malaysia	0	0.06	0.14	0.18
Philippines	0	0.14	0.89	0.89
Singapore	0	0.60	0.25	0.31
South Korea	0	0.31	0.08	0.16
Sri Lanca	0	0.18	0.69	0.89
Taiwan	0	0.60	0.80	0.70
Latin America				
Argentina	0	0.89	0.60	0.70
Brazil	0	0.70	0.31	0.61
Mexico	0	0.21	0.10	0.10

Notes: P -values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the long trading position is equal to α, top of the table). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH (out-of-sample estimation).

Table 6(c). VaR results for the short position of the 21 stock markets (out-of-sample): Kupiec test

$a = 1\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.50	0.16	0.91
France	0	0.08	0.16	0.25
Germany	0	0.08	0.04	0.28
Greece	0	0.44	0.01	0.09
Japan	0	0.01	0	0.05
Spain	0	0.03	0.06	0.45
UK	0	0.01	0	0.21
US	0	0.08	0	0.11
Asia/Pacific				
China	0	0.02	0.23	0.32
Hong Kong	0	0.44	0.28	0.28
India	0	0.44	0.16	0.28
Indonesia	0	0.16	0.01	0.11
Malaysia	0	0.64	0.16	0.08
Philippines	0	0.64	0.08	0.08
Singapore	0	0.35	0.64	0.64
South Korea	0	0.08	0.01	0.06
Sri Lanca	0	0.01	0.51	0.50
Taiwan	0	0.28	0.08	0.10
Latin America				
Argentina	0	0.50	0	0.09
Brazil	0	0.44	0.16	0.28
Mexico	0	0.86	0.16	0.17

Table 6(d). VaR results for the short position of the 21 stock markets (out-of-sample): Kupiec test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.02	0.35	0.99
France	0	0	0.08	0.08
Germany	0	0.29	0.60	0.44
Greece	0	0.11	0.04	0.07
Japan	0	0.29	0.43	0.89
Spain	0	0.11	0	0.33
UK	0	0.14	0.14	0.79
US	0	0.06	0.18	0.70
Asia/Pacific				
China	0	0.97	0	0.06
Hong Kong	0	0.04	0.29	0.29
India	0	0.18	0.23	0.70
Indonesia	0	0.08	0.23	0.23
Malaysia	0	0.99	0.70	0.89
Philippines	0	0.08	0.37	0.44
Singapore	0	0	0.18	0.14
South Korea	0	0	0.03	0.23
Sri Lanca	0	0.03	0.89	0.60
Taiwan	0	0.03	0.10	0.08
Latin America				
Argentina	0	0.01	0.10	0.43
Brazil	0	0	0	0.09
Mexico	0	0	0.06	0.14

Notes: P-values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the short trading position is equal to α, top of the table). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH (out-of-sample estimation).

Table 7(a). VaR results for the long position of the 21 stock markets (out-of-sample): Dynamic Quantile test

$a = 1\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.09	0.04	0.02
France	0	0.11	0	0.06
Germany	0	0.20	0.14	0.39
Greece	0	0.06	0.15	0.31
Japan	0	0.15	0.15	0.08
Spain	0	0	0.03	0.45
UK	0	0	0	0.07
US	0	0	0	0.11
Asia/Pacific				
China	0	0	0.07	0.99
Hong Kong	0	0.18	0.02	0.08
India	0	0.03	0.02	0.02
Indonesia	0	0.25	0.99	0.99
Malaysia	0	0	0	0.04
Philippines	0	0.80	0.98	0.99
Singapore	0	0.19	0.38	0.38
South Korea	0	0	0	0.05
Sri Lanca	0	0.30	0.58	0.58
Taiwan	0	0.03	0	0.05
Latin America				
Argentina	0	0.24	0.82	0.28
Brazil	0	0.99	0.93	0.07
Mexico	0	0.80	0.99	0.93

Table 7(b). VaR results for the long position of the 21 stock markets (out-of-sample)

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.29	0.07	0.24
France	0	0.63	0.46	0.56
Germany	0	0.78	0.08	0.57
Greece	0	0.31	0.36	0.13
Japan	0	0.23	0.14	0.33
Spain	0	0.12	0.11	0.53
UK	0	0.75	0.71	0.93
US	0	0.04	0.04	0.10
Asia/Pacific				
China	0	0.11	0.02	0.05
Hong Kong	0	0.80	0.04	0.04
India	0	0.73	0.02	0.01
Indonesia	0	0.04	0.26	0.34
Malaysia	0	0	0.02	0.04
Philippines	0	0.73	0.98	0.98
Singapore	0	0.63	0.72	0.85
South Korea	0	0.46	0.30	0.37
Sri Lanca	0	0.53	0.57	0.57
Taiwan	0	0.03	0.57	0.62
Latin America				
Argentina	0	0.04	0.24	0.06
Brazil	0	0.81	0.38	0.86
Mexico	0	0.78	0.59	0.59

Notes: P-values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the long trading position is equal to α, top of the table). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH (out-of-sample estimation).

Table 7(c). VaR results for the short position of the 21 stock markets (out-of-sample): Dynamic Quantile test

$a = 1\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.53	0.81	0.38
France	0	0.58	0	0.82
Germany	0	0.58	0.28	0.93
Greece	0	0.98	0.07	0.10
Japan	0	0	0	0.05
Spain	0	0.01	0.22	0.33
UK	0	0.07	0	0.07
US	0	0	0	0.07
Asia/Pacific				
China	0	0.18	0.77	0.04
Hong Kong	0	0.98	0.93	0.93
India	0	0.06	0	0.02
Indonesia	0	0.02	0	0.09
Malaysia	0	0.99	0.58	0.15
Philippines	0	0.14	0.58	0.58
Singapore	0	0.27	0.99	0.99
South Korea	0	0.59	0.07	0.28
Sri Lanca	0	0.29	0.55	0.55
Taiwan	0	0.03	0	0
Latin America				
Argentina	0	0.55	0	0.13
Brazil	0	0.98	0.82	0.93
Mexico	0	0.99	0.82	0.82

Table 7(d). VaR results for the short position of the 21 stock markets (out-of-sample): Dynamic Quantile test

$a = 5\%$

Index	RM	N-APARCH	ST-APARCH	SKST-APARCH
Developed				
Australia	0	0.29	0.67	0.89
France	0	0.20	0.11	0.46
Germany	0	0.32	0.32	0.62
Greece	0	0.60	0.33	0.27
Japan	0	0.27	0.20	0.47
Spain	0	0.01	0.05	0.21
UK	0	0.47	0.48	0.63
US	0	0.22	0.55	0.61
Asia/Pacific				
China	0	0.05	0.02	0.02
Hong Kong	0	0.04	0.49	0.35
India	0	0.06	0.16	0.39
Indonesia	0	0.14	0.29	0.29
Malaysia	0	0.53	0.49	0.55
Philippines	0	0.52	0.73	0.79
Singapore	0	0	0.08	0.05
South Korea	0	0.05	0.09	0.64
Sri Lanca	0	0.02	0.49	0.41
Taiwan	0	0.25	0.36	0.30
Latin America				
Argentina	0	0.04	0.06	0.32
Brazil	0	0.02	0.06	0.10
Mexico	0	0.02	0.25	0.13

Notes: P-values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the short trading position is equal to α, top of the table). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH (out-of-sample estimation).