Forecasting financial volatility of the Athens Stock Exchange daily returns: An application of the asymmetric normal mixture GARCH model

by

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Abstract

In this paper we model the return volatility of stocks traded in the Athens Stock Exchange using alternative GARCH models. We employ daily data for the period January 1998 to November 2008 allowing us to capture possible positive and negative effects that may be due to either contagion or idiosyncratic sources. The econometric analysis is based on the estimation of a class of five GARCH models under alternative assumptions with respect to the error distribution. The main findings of our analysis are: First, based on a battery of diagnostic tests it is shown that the normal mixture asymmetric GARCH (NM-AGARCH) models perform better in modeling the volatility of stock returns. Second, it is shown that with the use of the Kupie's tests for in-sample and out-of-sample forecasting performance the evidence is mixed since the choice of the appropriate volatility model depends on the trading position under consideration. Third, at the 99% confidence interval the NM-AGARCH model with skewed student-t distribution outperforms all other competing models both for in-sample and out-of-sample forecasting performance. This increase in predictive performance for higher confidence intervals of the NM-AGARCH model with skewed student-t distribution makes this specification consistent with the requirements of the Basel II Agreement.

Keywords: volatility, risk management, GARCH, asymmetric normal mixture GARCH, Kupiec test

JEL Codes: C52, C32, G13

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1. Introduction

During the past two decades, volatility in financial markets and their models and forecasts have attracted growing attention by academic researchers, policy makers and practitioners. The recent financial turmoil has brought on the surface once again the need for appropriate estimation of the volatility of stocks, derivatives and other financial instruments as well as the implementation and use of models for evaluating the trading positions, capital adequacy and Value-at-Risk measures according to the Basle II agreement. Over the years several issues have been raised regarding the development of theoretical models of asset returns volatility and their applications to real world problems.

First, modelling volatility is a key element in the pricing of derivative securities. An investor's choice of portfolio implies the maximization of his expected return subject to a risk constraint, or the minimization of his risk under an expected return constraint. A good forecast of an asset's price volatility provides an initial point for the assessment of investment risk. For example, the application of the Black and Scholes valuation model requires the knowledge of the volatility of the underlying asset in order to price an option. In fact, volatility is the only parameter that needs to be estimated and we could argue that in option markets, investors trade volatility. Second, volatility measurement is an important issue for policy makers, portfolio managers and financial market participants because it can be used as a measurement of risk, providing an important input for portfolio management, option pricing and market regulation (Poon and Granger, 2003 and Badescu *et al*. 2008). Third, a related issue, which has become very clear during the current financial crisis, is that greater volatility in the financial markets raises important questions about the stability of global financial system and its consequences on the real economy. Finally, with respect to the issue of modelling volatility of stock returns and the forecasting performance of alternative models the time-varying property of forecast confidence intervals calls for more sophisticated models which are at the same time tractable and they are able to capture higher conditional moments and provide more accurate forecasts at the 99% confidence interval.

The enormous growth of trading activity that was observed during the recent years in both the developed but mainly in the emerging markets has led the financial regulators and supervisory committees to seek well justified methods to quantify the risk.[1] The importance of financial risk management has significantly increased since the mid-1970s, which saw both the collapse of the fixed exchange rate system and two oil price crises. These major events led to considerable volatility in the capital markets, which together with the emergence of the derivatives market, increased trading volumes and technological advances, led to increasing concerns about the effective measurement and management of financial risk. This need was further reinforced by a number of financial crises that took place in the 1980s and 1990s such as the worldwide stock markets collapse in 1987, the Mexican crisis in 1995, the Asian and Russian financial crises in 1997-1998 as well as the Orange County default and the Barings Bank and Long Term Capital Management bankruptcy cases.

The increased financial uncertainty led to a rise of the likelihood for financial institutions to suffer substantial losses as a result of their exposure to unpredictable market changes as it has been made evident once again during the current financial crisis that already led several major financial institutions to bankruptcy in the U.S. and the U.K. These events have made investors become more cautious in their

[1]For a detailed analysis see the Basel Committee on Banking Supervision's (1996a, b), Duffie and Pan (1997), Jorion (2000), Alexander (2005) and Drzik (2005) provide a comprehensive overview of value at risk measures.
[2] See also Bank for International Settlements (1988, 1999a,b,c, 2001).

investment decisions, while it has also led to an increased need for more careful study of price volatility in stock markets.

A stylized fact that is well documented in the literature is that stock returns for mature and emerging stock markets behave as martingale processes with leptokurtic distributions and conditionally heteroskedastic errors. Furthermore, these data exhibit volatility across time and the unconditional variance is constant even though the conditional variance during some periods is unusually large. Therefore, it is argued that estimation methods that use conditional variances are more appropriate for this type of data, as the heteroscedasticity in the disturbances biases the test statistics, leading to incorrect inferences. In the presence of heteroscedasticity, the estimators themselves are no longer efficient and hence for the purpose of forecasting return series, more accurate intervals can be obtained by modeling volatility of returns.

Following the seminal work by Engle (1982) a voluminous literature has emerged that extensively analysed the volatility pattern of all major stock markets as well as many of the emerging stock markets of the world by applying the ARCH and GARCH class of models as well as numerous variants of these specifications. Overall, it has been shown that these methodologies have done well in describing the time-dependent heteroskedasticity present in stock returns data. Alexander and Lazar (2003, 2004, 2005 and 2006) have recently proposed a class of GARCH(1,1) models with normal mixture conditional densities having flexible individual variance processes and time-varying conditional higher moments in order to model volatility. An extension of these models is the asymmetric normal mixture GARCH model. The importance of using (asymmetric) normal mixture GARCH process is based on the fact that it can capture tails in the financial time series more accurately a property which is particularly useful for modelling return volatility in the emerging financial markets where asymmetric high volatility is observed during financial shocks. The emerging markets are subject to internal or external shocks observed due to short run capital flows, thin trading and instability. Moreover, the normal mixture GARCH models are similar to Markov switching models (Hamilton and Susmel, 1994) in capturing the effects of sudden shifts in the DGP of the stochastic evolution of the financial variables in the emerging markets but they are easier to estimate (Alexander and Lazar, 2005 and 2006).

This paper considers the modelling of the stock returns volatility in the Athens Stock Exchange a capital market which has been characterized by substantial volatility during the last decade. Although this market has recently grown in size and has been upgraded to the mature market status it still exhibits features that are found in the emerging markets. The existence of a rational bubble in the period 1998-2000 has dominated its behaviour since then and it has been recently affected substantially by the financial crisis which is still unfolding and has recorded a loss of 52% in capital value since the beginning of 2008. This loss in capital value has been mainly caused by the capital flight initiated by foreign institutional investors and hedge funds during the current financial, banking and credit crisis which in such an uncertain environment prefer to take positions in safer stock markets and currencies. This negative trend has been also reflected in the recent increase of the spread of the Greek 10-year bond spread by 252 base points compared to the German bund. It is evident therefore that it is of crucial importance to use alternative models in order to assess their performance in modelling the stock returns volatility and evaluate their forecasting accuracy especially for the case of emerging markets.[2]

[2]During the last decade a number of studies have examined several issues with respect to the workings of the ASE. Thus, Alexakis and Xanthakis (1995), Dockery and Kavoussanos (1996, 2001), Fountas

In this paper we adopt Alexander and Lazar (2004, 2005) methodology and we compare five main GARCH models with alternative probability density functions for the error term in order to capture time-varying conditional skewness and kurtosis for the daily stock returns of the Athens Stock Exchange. Specifically, we conduct our analysis by using five GARCH models including the normal mixture GARCH models with three alternative normality distributions, namely normal distribution, Student's-*t* distribution and skewed Student's-*t* distribution.

There are a number of important findings that stem from our analysis. First, using daily data for the period January 1998 to November 2008 for FTSE20 of the Athens Stock Exchange we show that the Normal Mixture AGARCH model with skewed student-t distribution performs better over a set of competing GARCH type models under different error distributions, based on several criteria which include the maximum likelihood, the Newey (1985) moment specification tests, the unconditional density test and the autocorrelation function. Second, based on Kupiec in-sample and out-of-sample forecasting performance we found that the NM-AGARCH with skewed student-t distribution has the best in-sample and out-of-sample performance. Third, the choice of the appropriate specification to model stock returns volatility for the ASE may depend partially on the chosen confidence level and on the trading position. Fourth, the predictive performance of the NMAGARCH model is superior at the 99% confidence interval which is in accordance with the requirements of Basel II Accord with respect to the use of models that capture accurately the volatility of stock

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and Segredakis (2002) have examined the efficient market hypothesis and seasonal effects that the stock returns exhibit, while Philippas (1998) examine the fitness of various market models, when trying to price the shares of the market. Furthermore, Alexakis and Petrakis, (1991), Apergis and Eleptheriou (2001) Leledakis *et al*., (2002), Siourounis, (2002) and Diamandis et al. (2007) provided evidence that the stock returns of ASE can be described by ARCH and GARCH processes .

returns and provide appropriate VaR measures.

The remainder of the paper is organized as follows. Section 2 presents a discussion on recent developments on modelling volatility. In section 3 we present the econometric methodology. Section 4 reports the data and the preliminary diagnostic statistics. In section 5 we present and discuss the empirical results while section 6 presents our summary and concluding remarks.

2. Volatility modelling: A review of the literature

The seminal works of Mandelbrot (1963) and Fama (1965) found that the empirical distribution of price changes of financial assets is leptokurtic when compared to the normal distribution, thus rejecting the assumption of normality. Furthermore, Mandelbrot (1967) and Fielitz (1971) provide evidence rejecting the assumptions of homoskedasticity and independence over time.

In order to account for these 'peculiarities' Engle (1982) developed the autoregressive conditional heteroskedastic (ARCH) methodology which allows for the modelling of the time-varying volatility of the financial assets. This methodology was later generalized by Bollerslev (1986) who proposed the generalized ARCH (GARCH) methodology. The works provided the framework for modeling the dynamics of the volatility process of returns of financial assets. The advantage of these models is that they are easy to estimate and in addition this framework allows us to conduct diagnostic tests. Several variations of these models have appeared along with numerous empirical applications in the financial markets in the last decade (see Bollerslev *et al.*, 1992 and Bera and Higgins, 1993 for an extensive literature review).

However, it was observed that the GARCH(1,1) specification could only capture some of the skewness and leptokurtosis commonly found in financial data.[3] This weakness of the original GARCH(1,1) model led subsequently to the development of a large number of extensions in order to provide a better description of the data. The main strand of this literature focused on the introduction of alternative distributional functions for the error term in order to provide a better description of the data and to account for the conditional excess kurtosis in the data. Thus, Bollerslev (1987) replaced the assumption of normality with Student's-t whereas Nelson (1990) introduced the GED distribution. However, it was also shown that even these specifications were unable to fully capture the observed non-normalities in both conditional and unconditional returns in daily or higher frequency data. Furthermore, Nelson (1991) and Glosten *et al.* (1993) introduced asymmetry in the GARCH specification by incorporating leverage effects whereas Forsberg and Bollerslev (2002) assume skewed innovation densities for the normal inverse Gaussian distribution. Finally, Christoffersen *et al.* (2006) assume skewed innovation densities for the inverse Gaussian density, Lanne and Saikkonen (2007) for the z-distribution and Aas and Haff (2006) for a Generalized Hyperbolic skewed Student's-t distribution.[4]

Recently Haas *et al.* (2004) and Alexander and Lazar (2003, 2004, 2005, 2006) have introduced a general class of GARCH models where errors have a normal mixture conditional distribution with GARCH variance components. An important advantage of the Normal Mixture GARCH (NM-GARCH) models is

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[3]Riskmetrics, 1996 developed by Morgan Stanley provides a comprehensive technical tool on calculating VaR measures based on GARCH-type of models. See also Lambert and Laurent (2001) for a detailed discussion of the use of GARCH models within this framework.

[4]The literature on modeling volatility of stock returns includes additional models such as the Asymmetric Power ARCH, Integrated GARCH, Fractionally Integrated GARCH, Fractionally Integrated Exponential GARCH, Fractionally Integrated Asymmetric Power ARCH and Hyperbolic GARCH. For a comprehensive analysis of alternative model of returns volatility (see Bera and Higgins, 1993; Bollerslev *et al*., 1992; Bollerslev *et al*., 1994; Lambert and Laurent, 2001;Engle 2002; Tsay, 2002; Alexander 2005).

that besides assuming a skewed leptokurtic conditional density they also take into consideration time variation in the conditional skewness and kurtosis. Furthermore, Haas *et al*. (2004) and Alexander and Lazar (2004, 2005) have shown that these models provide a superior fit to physical conditional densities as compared to other GARCH specifications. This is due to the fact that standard asymmetric GARCH models have only a single volatility component; the leverage effect. Alexander and Lazar (2005) discuss and show the superiority of the NM-GARCH models over several GARCH models by explicitly taking into account more than one volatility component. They argue that the incorporation in the GARCH models two asymmetric variance components provide us with a much better understanding of the behaviour of stock market returns.

Alexander and Lazar (2004, 2005) extended the NM-GARCH(1,1) model to introduce two distinct sources of skewness, one 'persistent' and one 'dynamic'. Persistent asymmetry arises when the conditional density is a mixture of normal density components having different means. This type of asymmetry is the result of obtaining different expected returns under different market conditions. Dynamic asymmetry arises from the existence of 'short-term' asymmetries due to the leverage effect. Furthermore, the asymmetric NM-GARCH model can accommodate different states of time-varying volatility including a leverage effect in each variance component. The main advantage of this approach compared to other models is that even without a risk premium, the asymmetric normal mixture GARCH implies a volatility skew that is significant and exhibits persistence although at a diminishing rate. Alexander and Lazar (2004, 2005) used this model in order to examine the determinants of the index skew for the stock markets of France, Germany, UK, Japan and US. Alexander and Lazar (2006) also applies a set of fifteen GARCH models using alternative density function for the case of three bilateral exchange rates, namely sterling-dollar, euro-dollar and yen-dollar. In both applications Alexander and Lazar (2004, 2005, 2006) found that with the use of a battery of diagnostic tests the asymmetric normal mixture GARCH(1,1) model is superior in accounting for leptokurtosis in these two sets of financial data.

3. Econometric Methodology

Given that the theoretical specifications of the large variety of GARCH models which have been developed over the last twenty years have repeatedly discussed and employed in numerous applications we restrict our discussion to the presentation of the Asymmetric Normal Mixture GARCH model (NM-AGARCH) following Alexander and Lazar (2004, 2005).

The asymmetric normal mixture GARCH model has one equation for the mean and K conditional variance components representing different market conditions. To simplify matters the conditional mean equation is written as $y_t = \varepsilon_t$ assuming that there are no explanatory variables since these can be estimated separately. The error term ε_t is assumed to have a conditional normal mixture density with zero mean, which is a probability weighted average of K normal density functions with different means and variances given as follows:

$$\varepsilon \mid I_{t-1} \sim NM(p_1,....p_k, \mu_1,......, \mu_k, \sigma_{1t}^2,....\sigma_{kt}^2), \quad \sum_{i=1}^{k} p_i = 1, \ \sum_{i=1}^{k} p_i \mu_i = 0 \qquad (1)$$

Therefore the conditional density of the error term is derived as

$$\eta(\varepsilon_t) = \sum_{i=1}^{K} p_i \phi_i(\varepsilon_t) \quad (2)$$

where ϕ_i represents normal density functions with different constant means μ_i and different time varying variances σ_{it}^2 for $i = 1,...,K$.

The conditional variance behaviour is described by K variance components - and these could possibly characterize different market circumstances. These variances are assumed to follow any GARCH process but for the present analysis we follow Alexander (2004, 2005) and we assume there are two alternative asymmetric specifications.

(i) NM-GARCH which the symmetric normal mixture GARCH(1,1) studied Alexander and Lazar (2004) for which:

$$\sigma_{it}^2 = \alpha_0 + \alpha_i \varepsilon_{t-1}^2 + \beta_i \sigma_{it-1}^2, \quad \text{for } i = 1,...,K \quad (3)$$

(ii) NM-AGARCH which is the asymmetric normal mixture GARCH model due to Engle and Ng, (1993) for which:

$$\sigma_{it}^2 = \alpha_0 + \alpha_i (\varepsilon_{t-1}^2 - \lambda_i)^2 + \beta_i \sigma_{it-1}^2, \quad \text{for } i = 1,...,K \quad (4)$$

(iii) NM-GJR GARCH due to Glosten *et al.* (1993) for which:

$$\sigma_{it}^2 = \alpha_0 + \alpha_i \varepsilon_{t-1}^2 + -\lambda_i d_{t-1}^- \varepsilon_{t-1}^2 + \beta_i \sigma_{it-1}^2, \quad \text{for } i = 1,...,K \quad (5)$$

where $d_t^- = 1$ if $\varepsilon_t < 0$ and 0 otherwise.

In all cases, the overall conditional variance is

$$\sigma_t^2 = \sum_{i=1}^{K} p_i \sigma_{it}^2 + \sum_{i=1}^{K} p_i \mu_i^2 \tag{6}$$

For $K > 1$, the existence of second, third and fourth moments are guaranteed by imposing less restricted conditions than in the single component ($K = 1$) models. Thus, Alexander and Lazar (2005) show that is not required that $\alpha_i + \beta_i < 1$ to hold in every case whereas Haas *et al*. (2004) have also found that $\alpha > 1$ could occur on the second and higher variance components.

Based on these results Alexander and Lazar (2004, 2005) argue that a particular set of conditions for the non-negativity of variance and the finiteness of third moment is required which is given as follows:

$$0 < p_i < 1, i = 1, \ldots.., K, \sum_{i=1}^{K-1} p_i < 1, 0 < \alpha_i, 0 \leq \beta_i < 1 \tag{7}$$

Therefore, for the case of the NM-GARCH model we should satisfy:

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i \omega_i}{(1-\beta_i)} > 0, n = \sum_{i=1}^{K} \frac{p_i (1-\alpha_i - \beta_i)}{(1-\beta_i)} > 0$$

$$\text{and } \omega_i + \alpha_i \frac{m}{n} > 0 \tag{8}$$

the required conditions for the NM-AGARCH model are given by:

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i(\omega_i + \alpha_i \lambda_i^2)}{(1-\beta_i)} > 0, n = \sum_{i=1}^{K} \frac{p_i(1-\alpha_i - \beta_i)}{(1-\beta_i)} > 0$$

$$\text{and } \omega_i + \alpha_i(\frac{m}{n} + \lambda_i^2) > 0 \quad (9)$$

and finally for the NM-GRJ GARCH model we should have:

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i \omega_i}{(1-\beta_i)} > 0, n = \sum_{i=1}^{K} \frac{p_i(1-\alpha_i - 0.5\lambda - \beta_i)}{(1-\beta_i)} > 0$$

$$\omega_i + (\alpha_i + 0.5\lambda)\frac{m}{n} > 0 \quad (10)$$

As we already explained in the previous section Alexander and Lazar (2004, 2005) argue that there are two distinct sources of asymmetry in the model; (a) *Persistent* asymmetry and *Dynamic* asymmetry. The former arises in both symmetric and asymmetric normal mixture GARCH models and is due to the fact that when the conditional returns density is a mixture of normal density components it has different means. The latter appears only in the two asymmetric models and depends on the λ_i parameters in the component variance processes. These components capture time-varying short-term asymmetries arising from the leverage effect. The sign of the coefficient λ_i determines the leverage effect. A positive sign implies the standard result in stock markets that "bad news" has a greater effect than “good news” and this corresponds to a negative unexpected return. An important result of the combined effect of these two types of skewness in the physical conditional returns density is that we are allowed to draw more information about the shape of stock index skews than those obtained from the estimation of standard GARCH models and therefore we are able to provide a more accurate analysis of volatility of financial returns.

A final qualification of the normal mixture GARCH specifications is that can be considered as a restricted form of the Markov switching GARCH model where the transition probabilities are independent of the past state. These models are considerably easier to estimate than the class of Markov Switching GARCH (MS-GARCH) models, since normal mixture models have a straightforward relationship between the regimes and the transition probabilities are not historical state-dependent (Alexander and Lazar, 2003, 2004, 2005 and 2006).

4. Data and preliminary results

In this paper we model the volatility of stock returns traded in the Athens Stock Exchange (ASE). We use daily data for the period January 2, 1998 to November 30, 2008. The price data for the ASE is the closings price quotations of the FTSE/ASE20, a joint venture between FTSE and the ASE which is a capitalisation weighted index, consisting of the top 20 companies by market capitalisation (mainly banking sector and telecommunications) and it has been drawn from DATASTREAM. The sample consists of 2930 observations. The estimation process is run using 10 years of data (1998-2007) while the remaining 5 year (252*5 days) is used for out-of-sample forecasting.

Table 1(A) reports the results from unit root and stationarity tests for the CSE general stock price index and its first difference in order to obtain a clear picture of the stochastic properties of the series. Specifically, in order to test for the presence of a unit root in the level of the series we apply a set of unit root tests developed by Elliott *et al*. (1996) and Elliott (1999) as well as by Ng and Perron (2001). These tests modify conventional ADF and Philips-Perron unit root tests in order to derive tests that have better size and power. The use of these recently developed tests lead to firmer conclusions with respect to the integration properties of the stock price series since rejections of the null hypothesis of nonstationarity will not be attributed to size distortions, whereas nonrejections is not the outcome of a low probability of rejecting a false null hypothesis. The null hypothesis for the Elliot *et al*. (1996) GLS augmented Dickey-Fuller test (DF-GLS$_u$) and Ng and Perron (2001) GLS version of the modified Phillips-Perron (1988) tests (MZ_a^{GLS} and MZ_t^{GLS}) is that of a unit root against the alternative that the initial observation is drawn from its unconditional distribution. These tests use the GLS-detrending technique proposed by Elliott *et al*. (1996) and extended by Elliott (1999), to maximize power, and a modified selection criterion to select the lag truncation parameter in order to minimize size distortion. In the GLS procedure of Elliot *et al*. (1996), the standard unit root tests (without trend) are applied after the series are first detrended under the local alternative $\rho = 1 + \alpha / T$. This methodology resulted to a substantial increase in power for the DF-GLS$_u$ test deriving power functions that lie just under the asymptotic power envelope. Ng and Perron (2001) find similar gains for the MZ_a^{GLS} and MZ_t^{GLS} tests and they have also derived a modified version of the AIC criterion (MIC) that give rise to substantial size improvements over alternative selection rules such as BIC. Finally, we apply the Kwiatkowski *et al*. (1992) KPSS test for the null hypothesis of level or trend stationarity against the alternative of non-stationarity and these additional results will provide robust inference. The overall evidence for this set of tests is that the FTSE20 price index is nonstationary while its first difference is a stationary process.

Provided that the stock price index is a nonstationary variable we only consider the first differences of the stock price index:

$$\Delta p_t = 100 * (p_t - p_{t-1})$$

which corresponds to the approximate percentage nominal return on the stock price series obtained from time t to $t-1$.

We also calculate several descriptive statistics for monthly percentage changes in the stock prices which are given in Table 1(B). We clearly observe that the mean-return over the whole sample period is positive, on average about 0.6%, the return series are positively skewed whereas the large returns (either positive or negative) lead to a large degree of kurtosis. Furthermore, the Lung-Box Q^2 statistic for all returns series is statistically significant, providing evidence of strong second-moment dependencies (conditional heteroskedasticity) in the distribution of the stock price changes. The evidence based on the third and fourth moment of the returns FTSE20 of the ASE implies that the assumption of normality is rejected and this finding is in line with the stylised fact that financial returns exhibit leptokurtosis and volatility clustering that has been document in the finance literature. Figure 1 provides further insights on the non-normality issue based on the density graphs and the QQ-plots against the normal distribution for the daily returns and it is shown that their distribution exhibit fat tails. Furthermore, the QQ-plots imply that there is an asymmetry in the fat tails. An additional result of these graphical expositions show that the return series exhibit volatility clustering, which means that there are periods of large absolute changes tend to cluster together followed by periods of relatively small absolute changes.

5. Empirical Results

5.1. Model estimates

Given these salient features of the daily returns for the FTSE/ASE20 we now move to estimate the conditional variance parameters independently on the residuals e_t from the $AR(p)$ conditional mean equation with $p=3$ based on the Akaike

Information Criterion. We then obtain the optimal parameters values by maximizing the following likelihood function as suggested by Alexander and Lazal (2004, 2005)[5]:

$$L(\theta \mid e) = \sum_{i=1}^{T} \ln[\eta(e_t)] \tag{11}$$

We constructed and estimated fifteen GARCH(1,1) models for the return volatility of the ASE/FTSE20 price index under different distributional assumptions and we then evaluated their predictive performance[6]. These model specifications are summarized as follows:

A. Models with normally distributed errors

(1) GARCH

(2) GJR

(3) FIGARCH

(4) HYGARCH

(5) NM-AGARCH

B. Models with symmetric Student's t distributed errors

(1) GARCH

(2) GJR

(3) FIGARCH

(4) HYGARCH

(5) NM-AGARCH

C. Models with skewed Student's t distributed errors

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[5] Alexander and Lazal (2004, 2005) provide the updating formula for the grid search.

[6] All computations were performed with G@RCH 5.0 procedure on Ox package (see Laurent and Peters, 2002; Laurent 2007).We also use the Ox programming language for the estimation of the Asymmetric Normal Mixture GARCH (see Alexander and Lazar, 2005 and 2006; Doornik, 2007) and the parameters are estimated using the Quasi Maximum likelihood method (Bollerslev and Wooldrige, 1992) and the BFGS Quasi-Newton optimization algorithm.

(1) GARCH

(2) GJR

(3) FIGARCH

(4) HYGARCH

(5) NM-AGARCH

Four model selection criteria are used to assess the appropriateness of the fitted models to the returns volatility of the Athens stock market: (a) The maximum likelihood; (b) The Newey (1985) *moment specification tests*. Under this statistical criterion we test for normality in the standardized residuals, checking the first four moments and for zero autocorrelations in the powers, using a Wald test. The test statistics for the moments are distributed as $\chi^2(1)$ and the cumulative test is distributed as $\chi^2(20)$. The results shown in Tables 2-4 refer to the number of tests (out of 20) that are rejected at 1% critical level; (c) The unconditional density test, which tests the histogram fit between the model simulated data and the original data. The model selection criterion is based on the modified Kolmogorov-Smirnov statistic, and (d) The Autocorrelation Function (ACF) test, which captures the dynamic properties of the model squared returns. Following Alexander and Lazar (2004) we apply the Mean Square Error criterion to assess the fit of the theoretical autocorrelation functions with the empirical autocorrelations.

The estimations for the fifteen models are reported in Tables 2-4.[7] The reported estimated coefficients and calculated statistics are very informative and provide evidence in favour of the NM-AGARCH model for modelling the volatility of stock returns of the ASE. First, we note that the Ljung-Box Q^2-statistic is not

[7]The parameters are estimated by MLE. Numbers in parentheses are t-statistics with (*) and (**) denote statistical significance at the 5% and 1% critical level respectively.

statistically significant implying that all models under consideration are successful in taking into account the conditional heteroskedasticity exhibited by the data. Second based on the estimated likelihood it is shown that the NM-AGARCH model with non-zero means in the components is preferable. Third, we find that the application of the moment specification tests leads to the conclusion that the basic models do not capture the higher moments. Furthermore, for the rest of the models we observe that the moment tests do not distinguish well between the models. We find that some models pass all the tests whereas some others have several rejections but we observe that the NM-GARCH model performs well based on this criterion. Fourth, the unconditional density test clearly shows that the NM-AGARCH with all three sources of asymmetry is preferred against all the other models. Finally, looking into the evidence provided by the ACF test we again find evidence in favour of the NM-AGARCH models.

Table 2 and 3 reports the results for the models with Student's-t distribution and we note that the parameters (v) are statistically significant for all the GARCH models and therefore we conclude that the returns series are fat tailed. Furthermore, when we examine the skewed Student's-t distribution, the asymmetric parameters (ξ) are negative and statistically significant for all GARCH models. This is an indication that the density distribution of the FTSE/ASE20 is skewed to the left. Estimated long memory parameter d for the FIGARCH model (Chung, 1999) and the hyperbolic parameter $Ln(\alpha)$ for the HYGARCH model are also found to be statistically significant and these results are shown in Table 3.

Table 4 reports the estimates of the parameters ω, α, β_1 as well as the normal mixture γ parameter for the Asymmetric Normal Mixture models and we observe that these are statistically significant for all three cases. Additionally, the

Student's-t and skewed Student's-t parameters, v – Student-t, ξ- Skewness and v -Skewness are also found to be statistically significant. Thus, the overall results show that NM-AGARCH models perform better in modeling the volatility of stock returns of the Athens Stock Exchange.

5.2. Forecasting performance evaluation

The next step of the present analysis involves the evaluation of forecasting performance of the competing models. This task is accomplished with the application of the backtesting procedure provided by the Kupiec (1995) test and out-of-sample VaR evaluation criteria.[8]

Table 5 reports several statistics for the evaluation and comparison of model forecast error. Specifically we consider the Root Mean Squared Error (RMSE), the Mean Squared Error (MSE), the Akaike information criterion and the Nyblom test for parameter stability (Nyblom, 1994). The evidence leads to the conclusion that the RMSE and/or the MSE may not be adequate backtesting tests because they do not take into consideration tail probability and overshooting effects. This finding is further confirmed in Figure 2 as RMSE obtains its maximum value at a point where the AIC is not at maximum for NMGARCH models. Moreover, the application of the Nyblom test statistic shows that the parameters of each model do not exhibit instability in recursive estimations.

We next move to examine whether the NM-AGARCH model provides better VaR estimates and forecasting performance than all other competing models. To this end we move on to provide in-sample VaR computations and this is accomplished by computing the one-step-ahead VaR for all models. This procedure is equivalent to

[8] For a detailed analysis of selection and evaluation criteria for VaR models see Andersen and Bollerslev (1998) Christoffersen, (1998), Sarma *et al*. (2003) and Bams *et al*. (2005).

backtesting the model on the estimation sample. We test all models with a VaR level of significance, (α), that takes values from 1% and 5% and we then evaluate their performance by calculating the failure rate for the returns series y_t. The failure rate is defined as the number of times returns exceed the forecasted VaR. Following Giot and Laurent (2003) we define a failure rate f_l for the long trading positions, which is equal to the percentage of negative returns smaller than one-step-ahead VaR for long positions. In a similar manner, we define f_s as the failure rate for short positions as the percentage of positive returns larger than the one-step-ahead VaR for short position.[9]

In Kupiec's test, we define f as the ratio of the number of observations exceeding Var(x) to the number of total observation (T) and pre-specified VaR level as a (Tang and Shieh, 2006). The statistic of Kupiec LR test is given by Eq. (12) (Kupiec, 1995). Under the null hypothesis Kupiec (1995) developed a likelihood ratio statistic (LR) distributed as chi-square distribution which is given as follows:

$$LR = 2\left\{\log[f^x(1-f)^{T-x}] - \log[\alpha^x(1-\alpha)^{T-x}]\right\} \qquad (12)$$

where $f = N/T$ is the failure rate, $\hat{f}$ is the empirical (estimated) failure rate, N is the number of days over a period T that a violation has occurred. Giot and Laurent (2003) suggest that the computation of the empirical failure rate defines a sequence of yes/no, under this testable hypothesis.

The VaRs of α quantile for long and short trading position are computed as in Equation 13, 14 and 15 for normal, Student-t and skewed Student-t respectively (Tang and Shieh, 2006).

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[9] When the VaR model is correctly specified then the failure rate should be equal to the pre-specified VaR level.

$$VaR_{long} = \hat{\mu}_t - z_{\alpha}\hat{\sigma}_t, \qquad VaR_{short} = \hat{\mu}_t + z_{\alpha}\hat{\sigma}_t \tag{13}$$

$$VaR_{long} = \hat{\mu}_t - st_{\alpha,\nu}\hat{\sigma}_t \qquad VaR_{short} = \hat{\mu}_t + st_{\alpha,\nu}\hat{\sigma}_t \tag{14}$$

$$VaR_{long} = \hat{\mu}_t - skst_{\alpha,\nu,\xi}\hat{\sigma}_t \qquad VaR_{short} = \hat{\mu}_t + skst_{\alpha,\nu,\xi}\hat{\sigma}_t \tag{15}$$

where z_{α}, $st_{\alpha,\nu}$ and $skst_{\alpha,\nu,\xi}$ are left or right tail quantile at $\alpha\%$ for normal, Student-t and skewed Student-t distributions respectively.

Table 6 reports the corresponding p-values for all competing VaR models given that the number of days for the in-sample-forecasting is taken to be 15 and two alternative significance levels $\alpha = 0.01$ and $\alpha = 0.05$. Figure 3 provides plots of Kupiec's test results. The main finding from this analysis is that the NM-AGARCH with student-t distribution for short position and the HYGARCH(1,d,1) with skewed student-t distribution and GJR with skewed student-t distribution for long position perform better for $\alpha = 0.05$ whereas NM-AGARCH with skewed student-t distribution for short position and NM-AGARCH with skewed student-t distribution and FIGARCH(1,d,1) with skewed student-t distribution for long position perform better for $\alpha = 0.01$. Therefore, we argue that these findings show that at the 95% confidence interval the evidence is mixed but as we move to the 99% confidence interval the NM-AGARCH with skewed student-t distribution outperforms all other competing models based on Kupiec in-sample forecasting.

Given that the in-sample forecasting estimates VaR are only based on knowing the past performance, we further investigate the performance of the competing models by conducting an out-of-sample forecasting analysis. For the out-of-sample forecast evaluation we use 252*5 days forecast sample in order to provide one-step-ahead prediction. Table 7 and Figure 4 provide a summary of out-of-sample VaR forecasts for long and short trading positions. The results of Kupiec out-of-sample forecasting test show that the NM-AGARCH with skewed student-t distribution and FIGARCH(1,d,1) with skewed student-t distribution for short position and HYGARCH(1,d,1) with student-t and GJR with skewed student-t distribution perform better for $\alpha = 0.05$ whereas NM-AGARCH with skewed student-t distribution and GJR with skewed student-t distribution and GJR with student-t distribution for short position and NM-AGARCH with skewed student-t distribution and NM-AGARCH with student-t distribution for long position perform better for $a = 0.01$. The overall empirical evidence based on Kupiec in-sample and out-of-sample forecasting performance suggests that the NM-AGARCH with skewed student-t distribution has the best in-sample and out-of-sample performance. However, we must note that the choice of the appropriate specification to model stock returns volatility for the ASE also depends on the chosen confidence level and on the trading position since in some cases other model specifications could possibly provide better in-sample and out-of-sample performance.

Our results can be directly linked with the provisions made by Basel II Agreement regarding the model accuracy for estimating VaR measures and modelling volatility. The Basel II Accord which came in force on November 2007 requires the use of a volatility model, which is statistically significant at 99% confidence level, in order to provide an accurate measurement of the VaR under extreme cases.[10] Figure 5 provides a comparison of out-of-sample performance for GARCH and NM-AGARCH with Normal and skewed student-t distribution. It is shown that the NM-AGARCH

[10] For a detailed analysis see the Basel Committee on Banking Supervision's (1996a, b).

captures fat-tailed behavior of the data much better than GARCH model and more important NM-AGARCH model with skewed student-t distribution has better predictive performance for the 99% confidence interval for the case of the Athens Stock Exchange. Our results are in line with those reported in Alexander and Lazar (2004, 2005) for the case of four major stock markets, namely CAC40, DAX30, FTSE100 and NIKKEI225, in Alexander and Lazar (2003, 2006) for the exchange rates of pound sterling, yen and euro vis-à-vis the US dollar and in Badescu *et al*. (2008) for the case of option valuation using the S&P index option.[11] Therefore, we provide evidence that this recently developed class of models is an important tool for portfolio managers, institutional investors and hedge funds in order to model stock returns volatility and VaR measures in a more accurate way, which is much needed especially during the current turbulent period in the global financial markets.

5. Summary and concluding remarks

The recent global credit and financial turmoil which has led to bankruptcy several major financial and banking institutions and to huge losses in market capitalization both the mature and emerging stock markets around the world makes the need for the development and use of more sophisticated techniques for modeling volatility and measuring VaR. This need is further reinforced by the provisions of the Basel II Accord which was put in force in November 2007 and which it underlines the importance for using accurate volatility models that provides statistically significant results at the 99% confidence interval.

This paper considers the modeling of the stock returns volatility in the Athens Stock Exchange a capital market which has been characterized by substantial volatility

[11] Cifter and Ozun (2007) obtained similar but less clear-cut results for the case of an emerging market, the Istanbul Stock Exchange.

during the last decade. During the current financial crisis it has lost 52% of its capital value since the beginning of 2008. To this end we use daily data for the period January 2, 1998 to November 30, 2008 a period that includes the existence of a rational bubble in 1999-2000 as well as the current financial crisis. We employ a general class of GARCH models where the error term follows a normal mixture distribution which introduces a new type of asymmetry and has the advantage of capturing the observed time variation in higher conditional moments and at the same time they are tractable.

Our results show that the Normal Mixture AGARCH model with skewed student-t distribution performs better over a set of competing GARCH type models under different error distributions, based on several criteria which include the maximum likelihood, the Newey (1985) moment specification tests, the unconditional density test and the autocorrelation function. Furthermore, based on Kupiec in-sample and out-of-sample forecasting performance we found that the NM-AGARCH with skewed student-t distribution has the best in-sample and out-of-sample performance. In addition, we note that the choice of the appropriate specification to model stock returns volatility for the ASE also depends to some extent on the chosen confidence level and on the trading position since in some cases other model specifications like FIGARCH, GJR and HYGARCH could possibly provide better in-sample and out-of-sample performance. An important finding of this study is that the predictive performance of the NMAGARCH model is superior at the 99% confidence interval which is in accordance with the provisions made by Basel II Accord and it provides support to the need for portfolio managers, institutional investors and hedge funds who are active in both developed and emerging markets to employ advanced models in order to model accurately the volatility of stock returns and provide appropriate VaR measures.

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Table 1. (A) Unit root and stationarity tests

Variable			Statistic			
	t_μ	t_τ	MZ_a^{GLS}	MZ_t^{GLS}	η_μ	η_τ
p	-0.78 [3]	-0.42 [3]	-0.29 [2]	-0.36 [2]	2.019*	0.822*
Δp	-13.71* [3]	-14.03* [3]	-29.42* [3]	-18.16* [3]	0.212	0.113

Notes: p and Δp are the prices and returns, respectively.

- The DF-GLS$_u$ is due to Elliot et al. (1996) and Elliott (1999) is a test with an unconditional alternative hypothesis. The standard Dickey-Fuller tests are detrended (with constant or constant and trend). The critical values for the DF-GLS$_u$ test at the 5% significance level are:-2.73 (with constant, t_μ) and -3.17 (with constant and trend, t_τ), respectively (Elliott,1999).
- MZ_a and MZ_t are the Ng and Perron (2001) GLS versions of the Phillips-Perron tests. The critical values at 5% significance level are: -8.10 and -1.98 (with constant), respectively (Ng and Perron, 2001, Table 1).
- η_μ and η_τ are the KPSS test statistics for level and trend stationarity respectively (Kwiatkowski *et al*. 1992). For the computation of theses statistics a Newey and West (1994) robust kernel estimate of the "long-run" variance is used. The kernel estimator is constructed using a quadratic spectral kernel with VAR(l) pre-whitening and automatic data-dependent bandwidth selection [see, Newey and West, 1994 for details]. The 5% critical values for level and trend stationarity are 0461 and 0.148 respectively, and they are taken from Sephton (1995, Table 2).

Numbers in brackets denotes the lag structure to ensure absence of serial correlation. (*) indicates significance at the 95% confidence level.

(B) Descriptive Statistics – Daily Data

	Mean	Standard Deviation	m_3	m_4	JB	$Q(24)$	$Q^2(24)$
p_t	5.96	0.27	2.15*	0.49	1229.8*	639.63*	1509.2*
Δp_t	0.008	0.11	6.69*	166.2*	6.3x10^6	1287.2*	1880.0*

Notes: The average return is expressed in terms of $x10^3$; m_3 and m_4 are the coefficients of skewness and kurtosis of the standardized residuals respectively; JB is the statistic for the null of normality; $Q(24)$ and $Q^2(24)$ are the Ljung-Box test statistics for up to 24th-order serial correlation in the Δp_t and Δp_t^2 series, respectively. (*) denotes statistical significance at the 5 percent critical level.

Table 2. Estimation Results from GARCH(1,1) and GJR(1,1)

	GARCH	**GARCH-** t	**GARCH-*Skew***	**GJR**	**GJR-** t	**GJR-*Skew***
ω	0.04936* (5.904)	0.0706* (4.018)	0.0708* (4.016)	0.04801* (5.068)	0.0737* (4.208)	0.0734* (4.198)
α	0.1163* (13.03)	0.1241* (7.867)	0.1253* (7.884)	0.0821* (8.971)	0.0906* (5.767)	0.0914* (5.780)
β_1	0.8732* (100.4)	0.8571* (53.74)	0.8564* (53.60)	0.8725* (87.2)	0.8508* (41.18)	0.8511* (61.23)
$\nu - Student - t$		6.488* (7.97)			6.703* (7.721)	
$\xi - Skewness$			-0.0399 (-2.123)			-0.0341 (-1.5196)
$\nu - Skewness$			6.4786* (7.999)			6.688* (7.722)
$\gamma_1 - GJR$				0.0769* (6.492)	0.0847* (3.827)	0.0829* (3.706)
Volatility	0.0217201	0.0194261	0.01966775	0.0102672	0.0112142	0.0112976
L	6999.8504	7057.4518	7058.1301	7013.0935	7065.7830	7065.9273
AIC	-5.51367	-5.55828	-5.55802	-5.52332	-5.56405	-5.56338
$Q^2(10)$	19.9797 [0.0104]	18.9134 [0.0813]	18.2616 [0.0193]	15.6626 [0.0475]	13.8074 [0.0869]	13.8983 [0.0844]
Moment tests 1%	4	2	3	1	0	0
Density	1.7981	1.6834	1.3925	0.9754	1.2357	1.1209
ACF	0.4209	0.3515	0.2618	1.0983	0.8767	0.6789

Notes: Parameters were estimated by MLE. L is the Likelihood function. AIC is the Akaike Information Criterion for model selection. $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 for serial correlation on the squared series. Moment is the Newey (1985) moment specification tests. Density is the unconditional density test. ACF is the autocorrelation function. Numbers in parenthesis are t-statistics. Numbers in brackets are p- values. (*) denotes statistical significance at the 5% critical level.

Table 3. Estimation Results from FIGARCH(1,d,1) and HYGARCH(1,d,1)

	FIGARCH-Chung	**FIGARCH Chung-** t	**FIGARCH Chung-Skew**	**HYGARH**	**HYGARH-** t	**HYGARCH-*Skew***
ω	4.0714* (3.411)	4.9295* (2.142)	5.154* (2.149)	0.3072* (3.550)	0.3510* (2.770)	0.3469* (2.764)
α	0.2687* (3.648)	0.2523* (2.179)	0.2519* (2.024)	-0.2678 (-1.768)	-0.1719 (-0.7704)	-0.1617 (-0.731)
β_1	0.5143* (6.993)	0.4260* (3.437)	0.4213* (3.477)	-0.1149 (-0.667)	-0.0054 (-0.208)	0.0072 (0.0278)
$\nu - Student - t$		6.7040* (7.568)			7.1127* (7.307)	
$\xi - Skewness$			-0.0274 (-1.001)			-0.0272 (-1.007)
$\nu - Skewness$			6.6815* (7.592)			7.1011* (7.312)
d FIGARCH	0.3974* (10.45)	0.4008* (7.963)	0.4044* (7.757)	0.1579* (2.935)	0.2122* (2.316)	0.2138* (2.356)
HYGARCH Ln(α)				0.4901* (2.221)	0.2840 (1.189)	0.2749 (1.187)
Volatility	0.0298293	0.0244554	0.0251226	0.0047133	0.005460	0.00548113
L	7028.8083	7074.1989	7074.7354	7025.7463	7071.8861	7072.4357
AIC	-5.5265	-5.57068	-5.57032	-5.5325	-5.56807	-5.56772
$Q^2(10)$	4.28644 [0.8304]	4.29997 [0.8291]	4.38645 [0.8207]	2.44310 [0.9643]	3.11411 [0.9270]	3.18131 [0.9225]
Moment tests 1%	2	2	0	0	1	2
Density	0.8523	0.7727	0.6982	0.9192	0.7888	0.6798
ACF	1.2398	0.8788	0.5978	0.4523	0.6209	0.2309

Notes: Parameters were estimated by MLE. L is the Likelihood function. AIC is the Akaike Information Criterion for model selection. $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 for serial correlation on the squared series. Moment is the Newey (1985) moment specification tests. Density is the unconditional density test. ACF is the autocorrelation function. Numbers in parenthesis are t-statistics. Numbers in brackets are p- values. (*) denotes statistical significance at the 5% critical level.

Table 4: Estimation Results from NORMAL MIXTURE-AGARCH(1,1)

	NM-AGARCH	**NM-AGARCH-** t	**NM-AGARCH–Skew**
ω	0.04167* (9.008)	0.065347* (3.573)	0.065465* (3.592)
α	0.120339* (12.81)	0.13874* (8.103)	0.13077* (8.090)
β_1	0.86762* (97.92)	0.847743* (52.95)	0.848117* (52.95)
$\nu - Student - t$		6.75367* (7.727)	
$\xi - Skewness$			-0.03020 (-1.3760)
$\nu - Skewness$			6.742793* (7.725)
$\gamma - Normal\ \ Mixture$	0.003736* (6.302)	0.003704* (3.927)	0.00364* (3.825)
Volatility	0.0220358	0.0197378	0.0198021
L	7013.9332	7066.3075	7086.3826
AIC	-5.52398	-5.56447	-5.56374
$Q^2(10)$	15.3885 [0.0520]	13.0074 [0.1116]	13.0034 [0.1117]
Moments tests 1%	1	0	0
Density	0.7654	0.6245	0.5898
ACF	0.1345	0.1009	0.0625

Notes: Parameters were estimated by MLE. L is the Likelihood function. AIC is the Akaike Information Criterion for model selection. $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 for serial correlation on the squared series. Moment is the Newey (1985) moment specification tests. Density is the unconditional density test. ACF is the autocorrelation function. Numbers in parenthesis are t-statistics. Numbers in brackets are p- values. (*) denotes statistical significance at the 5% critical level.

Table 5: Forecast Evaluation Measures

Method	MSE	RMSE	AIC	$Q^2(10)$	$N-test$
GARCH-*Normal*	1.158e-007	0.0003400	-5.455077	18.1348 [0.0202384]	2.76533
GARCH- t	1.158e-007	0.0003403	-5.500143	15.8671 [0.0443219]	3.23907
GARCH-*Skew*	1.159e-007	0.0003404	-5.499449	15.7861 [0.045546]	3.76229
GJR-*Normal*	1.136e-007	0.0003370	-5.462809	14.8664 [0.0617958]	2.6402
GJR-*t*	1.135e-007	0.0003369	-5.504390	12.5601 [0.127909]	3.22445
GJR-*Skew*	1.135e-007	0.0003369	-5.503519	12.5712 [0127481]	3.77487
FIGARCH-*Normal*	1.139e-007	0.0003375	-5.479032	4.28309 [0.830722]	1.73392
FIGARCH- t	1.140e-007	0.0003376	-5.513748	4.03299 [0.854135]	2.07801
FIGARCH-*Skew*	1.140e-007	0.0003377	-5.512977	4.06854 [0.850887]	2.6452
HYGARCH-*Normal*	1.137e-007	0.0003372	-5.476837	2.9250 [0.938976]	1.62957
HYGARCH- t	1.141e-007	0.0003377	-5.511258	3.19047 [0.921842]	2.3441
HYGARCH-*Skew*	1.142e-007	0.0003378	-5.510484	3.20433 [0.920888]	2.84299
NMAGARCH-*Normal*	1.474e-007	0.0003839	-5.462561	15.3885 [0.0520165]	2.37792
NMAGARCH- t	1.474e-007	0.0003840	-5.504262	13.0074 [0.111595]	3.05126
NMAGARCH-*Skew*	1.475e-007	0.0003838	-5.503387	13.0034 [0.111733]	3.63598

Notes: We report 1 day ahead out-of-sample forecasting based on 252 days evaluation. MSE denotes Mean Squared Error. RMSE denotes Root Mean Squared Error. AIC is the Akaike Information Criterion for model selection. $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 for serial correlation on the squared standardized residuals. $N-test$ is the Nymblom (1994) test for parameter stability.

Table 6: In Sample Forecasting Kupiec Test

In Sample Forecasting 95% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
GARCH-*Normal*	0.95114	0.07027	0.79094	0.047281	0.40190	0.52611
GARCH- t	0.94720	0.41096	0.52148	0.051615	0.13804	0.71024
GARCH-*Skew*	0.94602	0.82571	0.36352	0.048463	0.12741	0.72113
GJR-*Normal*	0.95193	0.20164	0.65340	0.044917	1.4271	0.23224
GJR- t	0.94720	0.41096	0.52148	0.051615	0.13804	0.71024
GJR-*Skew*	0.94720	0.41096	0.52148	**0.051675**	0.67213	0.41231
FIGARCH- *Normal*	0.95548	1.6614	0.19742	0.044129	1.9142	0.16650
FIGARCH- t	0.95154	0.12741	0.72113	0.045311	1.2112	0.27110
FIGARCH- *Skew*	0.95075	0.03087	0.86229	0.044917	1.4271	0.23224
HYGARCH- *Normal*	0.95232	0.29309	0.58825	0.044917	1.4271	0.23224
HYGARCH- t	0.94838	0.13804	0.71024	0.048069	0.20164	0.65640
HYGARCH- Skew	0.94563	0.99497	0.31853	**0.051675**	0.29309	0.58825
NMAGARCH- *Normal*	0.95035	0.00673	0.93460	0.044523	1.6614	0.19742
NMAGARCH- t	**0.95602**	0.82571	0.36352	0.048069	0.20164	0.65640
NMAGARCH-Skew	0.94563	0.99497	0.31853	0.047675	0.29309	0.58825

In Sample Forecasting 99% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	p **-value**	**Failure Rate**	**Kupiec LR**	p **-value**
GARCH-*Normal*	0.98700	2.1112	0.14623	0.013002	2.1112	0.14623
GARCH- t	0.99094	0.23276	0.62948	0.010244	0.015177	0.90195
GARCH-*Skew*	0.99094	0.23276	0.62948	0.010244	0.015177	0.90195
GJR-*Normal*	0.98700	2.1112	0.14623	0.013396	2.6727	0.10208
GJR- t	0.99094	0.23276	0.62948	0.010244	0.015177	0.90195
GJR-*Skew*	0.98936	0.10232	0.74907	0.009850	1.0057756	0.93942
FIGARCH- *Normal*	0.98779	1.1742	0.27854	0.013396	2.6727	0.10208
FIGARCH- t	0.99212	1.2423	0.26502	0.009456	0.077191	0.78114
FIGARCH- *Skew*	0.99133	0.47611	0.49019	**0.015366**	0.23276	0.62948
HYGARCH- *Normal*	0.98779	1.1742	0.27854	0.010266	6.3428	0.011786
HYGARCH- t	0.99133	0.47611	0.49019	0.011032	0.26434	0.60715
HYGARCH- *Skew*	0.99054	0.07719	0.78114	0.010244	0.015177	0.90195
NMAGARCH-*Normal*	0.98700	2.1112	0.14623	0.013396	2.6727	0.10208
NMAGARCH- t	0.98936	4.64e-0.4	0.98907	0.010638	0.10232	0.74907
NMAGARCH-*Skew*	**0.99332**	0.10232	0.74907	**0.015366**	0.0057756	0.93942

Notes: Number of in-sample forecaring is 15 days.

TABLE 7: Out-of- Sample Forecasting Kupiec Test

Out-of- Sample Forecasting 95% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
GARCH-*Normal*	0.97540	20.881	0.0000	0.018254	34.971	0.0000
GARCH-*t*	0.97063	13.177	0.000283	0.02222	25.601	0.0000
GARCH-*Skew*	0.97460	19.442	0.000001	0.024603	20.881	0.0000
GJR-*Normal*	0.96508	6.7128	0.009574	0.026984	16.756	0.0000
GJR-*t*	0.96746	9.1778	0.002445	0.021429	27.318	0.0000
GJR-*Skew*	0.97063	13.177	0.000284	**0.042857**	22.385	0.0000
FIGARCH- *Normal*	0.96508	6.7128	0.009572	0.026190	18.068	0.0000
FIGARCH-*t*	0.95952	2.5664	0.10916	0.029365	13.177	0.00028
FIGARCH- *Skew*	**0.96984**	0.01679	0.89689	0.030952	11.071	0.00088
HYGARCH- *Normal*	0.96508	6.7128	0.00957	0.23810	22.385	0.0000
HYGARCH-*t*	0.95873	2.1441	0.14312	0.028571	1.4192	0.23353
HYGARCH-*Skew*	0.95079	0.01679	0.89689	**0.042857**	14.312	0.00001
NMAGARCH-*Normal*	0.97302	16.756	0.0000	0.16667	39.315	0.0000
NMAGARCH-*t*	0.96746	9.1778	0.002449	0.020635	29.110	0.0000
NMAGARCH-*Skew*	**0.96984**	12.097	0.000505	0.024603	20.881	0.0000

Out-of- Sample Forecasting 99% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
GARCH-*Normal*	0.99286	1.1539	0.28274	0.0055556	2.9961	0.08347
GARCH- t	0.99383	8.0799	0.00448	0.0015873	13.928	0.000189
GARCH-*Skew*	0.99383	8.0799	0.00448	0.0039683	6.0036	0.014277
GJR-*Normal*	0.98333	4.7114	0.29964	0.01111	0.15167	0.69695
GJR- t	**0.99683**	8.0799	0.00448	0.0015873	13.928	0.000189
GJR-*Skew*	**0.99683**	8.0799	0.00448	0.003175	8.0799	0.00448
FIGARCH- *Normal*	0.99048	0.29325	0.86403	0.008730	0.21442	0.64333
FIGARCH- t	0.99444	2.9961	0.08347	0.0047619	4.3316	0.037411
FIGARCH- *Skew*	0.99444	2.9961	0.08347	0.0047619	4.3316	0.037411
HYGARCH- *Normal*	0.99206	0.58318	0.44507	0.0095238	0.029325	0.86403
HYGARCH- t	0.99524	4.3316	0.03741	0.0039683	6.0036	0.014277
HYGARCH- *Skew*	0.99524	4.3316	0.03741	0.005556	2.9961	0.083466
NMAGARCH-*Normal*	0.99286	1.1539	0.28274	0.01111	0.15167	0.69695
NMAGARCH- t	0.99603	6.0036	0.01427	**0.0113810**	10.663	0.001093
NMAGARCH-*Skew*	**0.99683**	8.0799	0.00447	**0.0113810**	10.663	0.010929

Note: Number of forecasts: 252*5 days and 1 day ahead

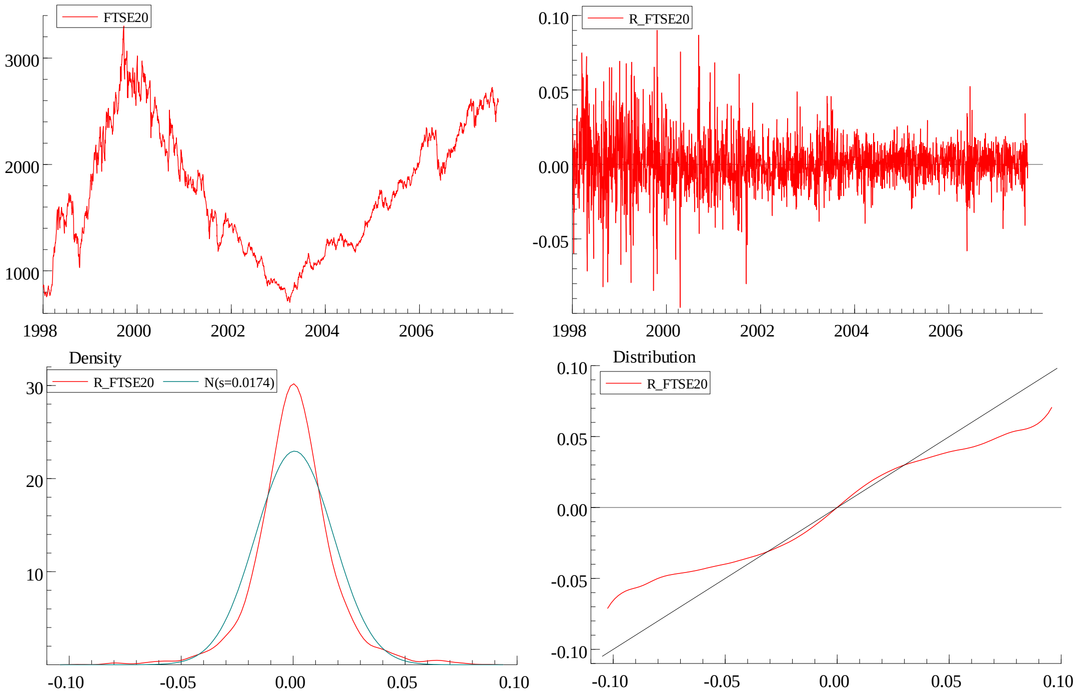

Figure 1: FTSE20/ASE stock index in level, daily returns, daily returns density (versus normal) and QQ-plot against the normal distribution. The time period is 02/01/1998 -10/05/2007.

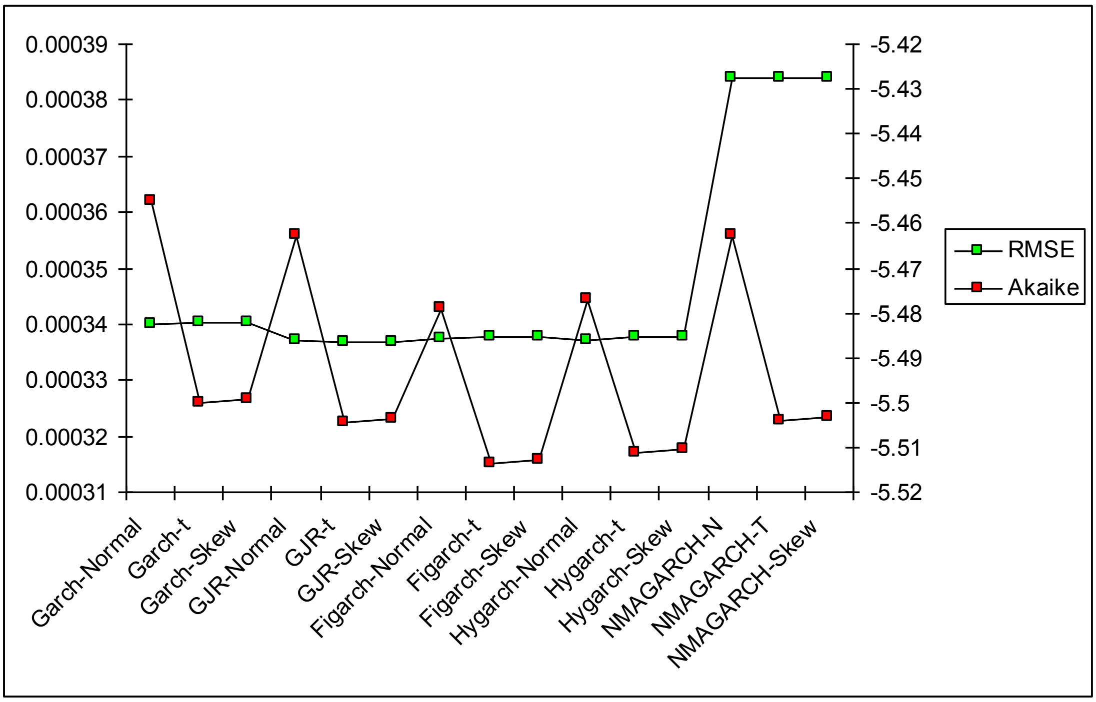

Figure 2: RMSE and Akaike Values

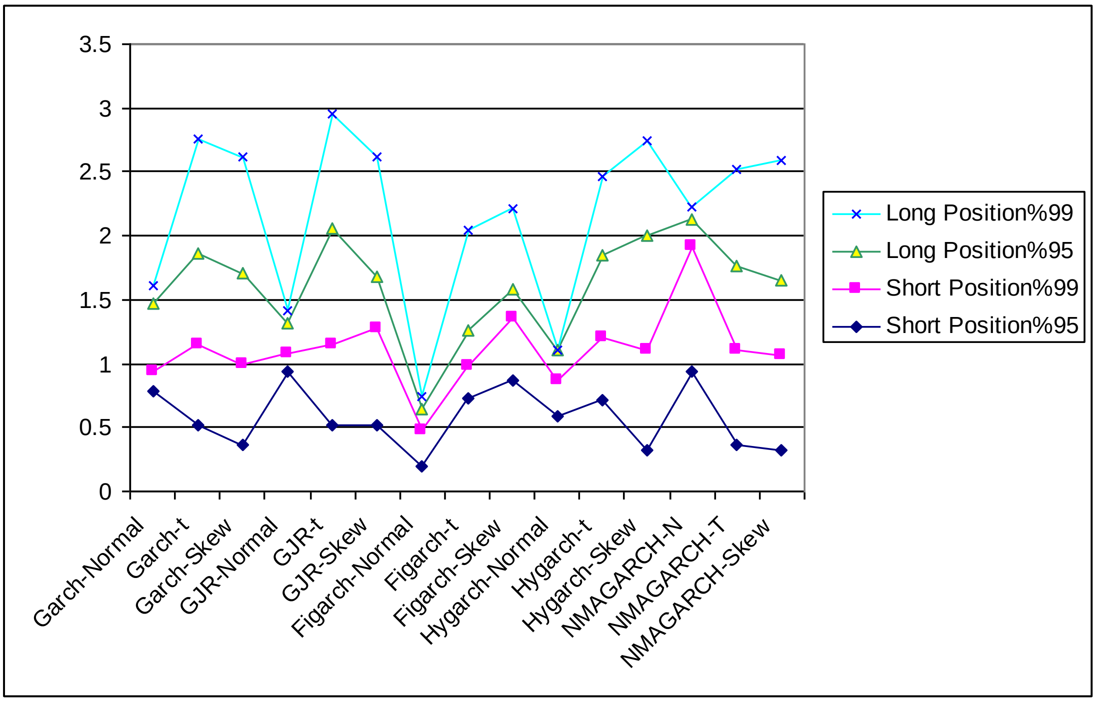

Figure 3. In-sample Kupiec test p-value

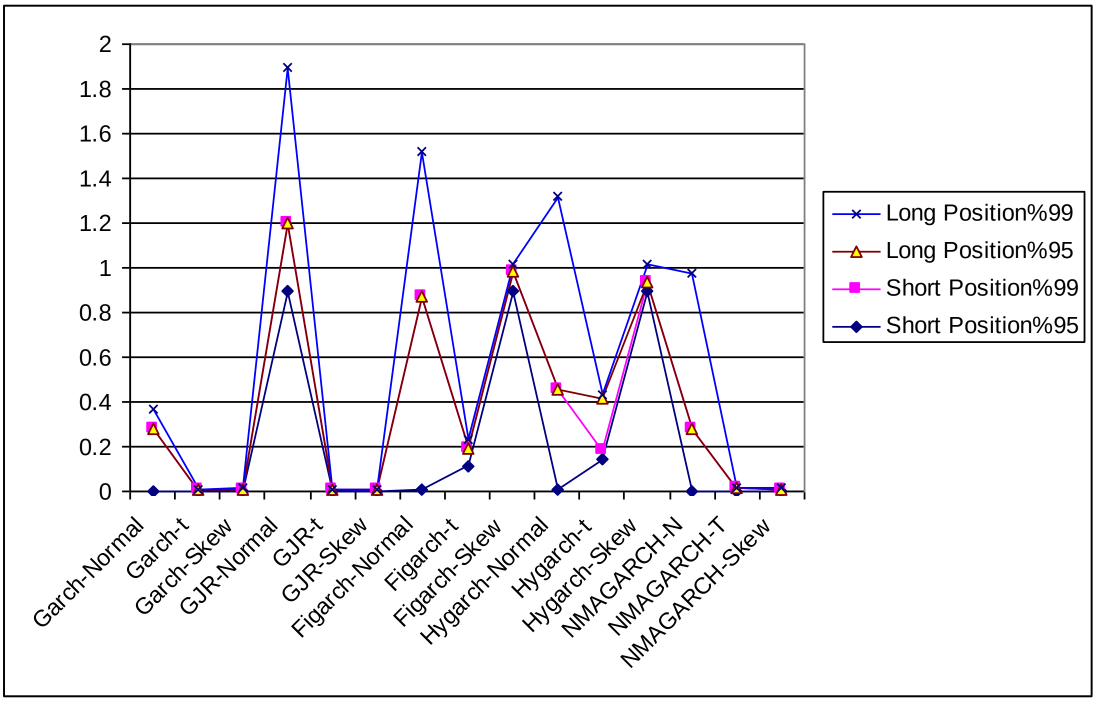

Figure 4. Out-of-Sample Kupiec Test p-value

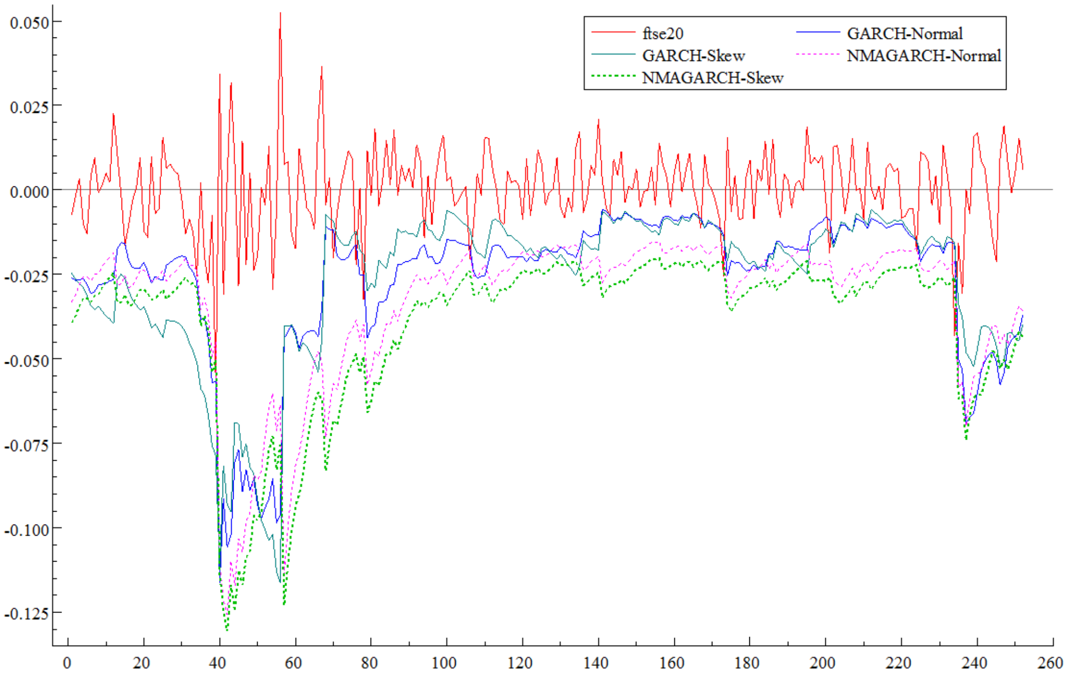

Figure 5. Out-of-Sample Forecasting (Last 252 days)