ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

“GREEN FINANCE”

Giourga Sofia

sofiagiou9@gmail.com

Supervisor: Topaloglou Nikolas

Athens University of Economics and Business

Department of International and European Economic Studies

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Chapter 1: Introduction

Given the nature of modern society is widely recognized that the development of technology, the urbanization and the lack of sensitivity of individuals have caused unprecedented ecological problems. Some characteristic examples are the greenhouse effect, the hole of the ozone layer and the air pollution. It is essential to prevent the aforementioned in order to make our lives better and to conserve our planet. Nowadays, more and more people, governments and enterprises are aware of environmental problems and are aiming to protect the environment through new green projects. Green financing is booming and more and more issuers have started issuing green bonds. Although, a plethora of states, businesses and investors keep paying more attention to their own private profits without taking into consideration the problems that our planet is facing today. The main question that this thesis is coming to answer is if it is in the interest of an investor to invest most of its available capital in green bonds. Will this investor receive higher returns than he would have if he had only invested in regular bonds? Which will be his optimal portfolio?

So, the answers of the above main questions will be given later, but before we get there it is important to refer to some important meanings related to Green Finance and Green Bonds in general. The first part of this thesis concerns the theoretical part. The Chapter 2 is about to understand the whole notion of Green Finance. According to the authors, there is not a specific definition, but in general with the term Green Finance we refer to investments that are taking into consideration the environmental protection and sustainability. Green Finance aims to improve the financial sector by contributing to the tackling of the climate change and by applying new, green projects. The reference to the corporate environment responsibility and to the criteria that a business should meet in order to be characterized as Green is extremely important.

In addition, the Chapter 3 is about to understand the Green Bonds and the bond market. Green Bonds are those that finance only environmentally friendly projects, such as alternative energy projects, green buildings etc. In order to become familiar with the Green Bonds it is important to make a reference at the whole notion of the bonds and their categories. A bond must follow some specific standards, the Green Bond Principles in order to be characterized as Green Bond. In 2007, the European Investment Bank was the first issuer, which was followed by the World Bank. Nowadays, more and more countries and enterprises are issuing Green Bonds with USA, China and France as the protagonists.

The last Chapter of the theoretical part, the Chapter 4 is about the risk measures and their mathematical formulations. Investors are facing the problem of optimal portfolio selection and Markowitz was the first who tried to solve this problem. Investors aim to make the right financial decision by choosing in our case the best optimal portfolio, the one with Regular Bonds or the one with Green and Regular Bonds. More specifically, they allocate their available capital to specific assets in order to gain the maximum possible return with the least possible loss. So, it is essential to be able to measure the risk of their investment. There are a lot of different risk measures (variance, value at risk, conditional value at risk etc.) which are analyzed in detail in this chapter.

The second- empirical part of this thesis is about to answer the main questions and to present some interesting results. With the usage of historical data- returns of Regular and Green Bonds we made some statistic and dynamic tests. With the usage of the GAMS system, the statistic tests gave us the efficient frontiers of the portfolios of the Regular Bonds and of the Green and Regular Bonds. In order to answer the question "Does it worth to invest and to build a portfolio with not only regular, but with green bonds too?", we calculated the Sharpe Ratios of both cases and we compared them. The dynamic tests- Backtesting experiments gave us the real returns of the minimum variance portfolio for the last 2 years in both cases. Apart from this, was very interesting to examine how the investors act and in which bonds they choose to invest more, depending on their aversion to risk.

Chapter 2: The meaning of green finance

2.1 The meaning of entrepreneurship

The literature supports the view that the term entrepreneurship has many different meanings, as it is particularly difficult to define. Although, in general, entrepreneurship is the process by which people, capital, facilities, effort and knowledge about a specific object are combined in order to create a business which did not exist before or in order to expand an existing business. This process is characterized by innovation, social responsibility and respect for the environment in order to become or remain not only sustainable but also profitable (William B. Gartner, 1990). In addition, Business clusters, i.e., groups of companies usually located close to each other that operate with common goals and interact with each other, are often observed.

The [1]European Commission states that entrepreneurship is the thought and the process of creating and developing economic activities based on the combination of risk-taking (financial risks) and creativity-innovations within a new or existing economic organization. Through this process competition and growth are promoted, productivity is increased, jobs are created and, in some cases, exports are increased too (European Commission, 2004).

The following forms of entrepreneurship are worth mentioning:

- Social Entrepreneurship: It has as its main goal to highlight and solve important and complex social problems such as environmental destruction, unemployment, poverty, etc. The difference between entrepreneurship and social entrepreneurship lies in the fact that the second focuses more on the social prosperity and not only on the creation of economic wealth. (Abu-Saifan, 2012).

- Green entrepreneurship: Aims at the need to protect the environment through business actions. Actions that focus on sustainability and "on protecting and preserving the natural environment'". It is a new type of entrepreneurship that seems to have brought about significant changes in the way with which businesses operate (Lotfi, M.; Yousefi, A.; Jafari, S., 2018).

2.2 Green finance

According to the literature, we cannot give a specific definition of Green Finance due to the fact that ''many publications do not try to define the term and also the definitions that are proposed vary significantly'' (Lindenberg, Nannette, 2014).

Although, by using the term green finance, in general, we refer to financial investments that are made on the basis of environmental protection and sustainability. These

[1] European Commission: Institution of the European Union for the protection of the Community interests of the Member States. It was founded in 1951 with headquarters in Brussels.

financial investments (or green investments) are made in new projects and innovations that support a better tomorrow for our planet. The purpose of the Green Finance is to improve the financial system by making it to include the problem of the climate change, of the greenhouse effect, of the water and air pollution and etc., in its financial risks (Stefano Spinaci, 2021). A big part of the Green Finance stresses the need to tackle climate change and to reduce greenhouse emissions, but not only that. In Figure 1 it is observed that green investments include investments not only in climate change adaptation (blue bubbles), but also include investments in other significant sections such as recycling, biodiversity protection, etc.(green bubbles).

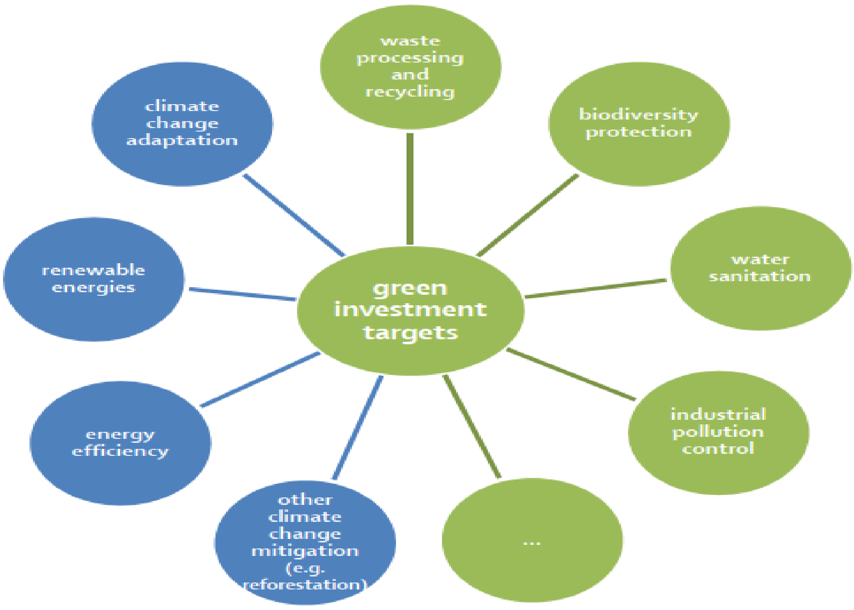

Figure 1*/ Source: Lindenberg, Nannette, Definition of Green Finance (April 15, 2014)*

The Green Finance consists of (Lindenberg, Nannette, 2014) the financing of public and private green investments, of public policies and of components of the financial system that adopt and encourage some of the following:

- Research and development of innovative technologies
- Energy saving programs
- Water saving programs
- Replacing some environmentally harmful products with friendly ones, such as paper instead of glass or plastic
- Usage of renewable energy sources
- Recycling of old appliances and placing bins in all stores
- Green transport of the merchandise
- Battery recycling

Equally important is the reference to another definition of Green Finance. “For the banking sector, green finance is defined as financial products and services, under the consideration of environmental factors throughout the lending decision making, ex-post monitoring and risk management processes, provided to promote environmentally responsible investments and stimulate low-carbon technologies, projects, industries and businesses" (Price Waterhouse Coopers Consultants (PWC), 2013).

Green entrepreneurs are the ones who deal with businesses based on the principle of sustainability and emphasize on green values. They took into consideration both financial and ethical criteria and are possessed by moral barriers and ecological conscience. Essentially, their purpose in investing in green entrepreneurship is not only their private profit, but also the creation of a better society. This is obvious when they not only contribute to the creation of investments in a small segment of the market, but also when they try hard to expand gradually their investments so that the sustainable transformation of an entire industry can take place. Green entrepreneurs strive to be constantly up to date with the technology and innovations and some are considered as "social activists who promote and conduct activities which generate social and economic values on a regional level" (Lutz E. Schlange, 2006).

A reference is made, with the above, to the so-called Green Economy which aims at the development of awareness for the protection of the environment. It is considered as a system of economic activities that refers to the production, consumption and distribution of goods and services in a way that leads to an increase in the quality of life of people. The most important goal of the Green Economy is to protect the environment for the future generations (UNEP, 2011).

2.3 Corporate environmental responsibility

In the context of adapting corporate social responsibility standards, and in particular the environmental responsibility that aims to promote sustainable development, governments in recent years have included a number of environmental criteria in their legislation. "More than 170 countries and several hundred non-governmental and inter-governmental organizations (like the European Union and its Member States, the US, Japan and other non-EU OECD countries, Russia and other countries in transition to a market economy, oil-exporting developing countries of the Organization of Petroleum Exporting, the developing countries bound together in the Alliance of Small Island States and the majority of developing countries including China and India)" collaborated on an innovative project, the foundations of which were laid in 1997 with the Kyoto Protocol (Sebastian Oberthür, Hermann E. Ott, 1999). This protocol came into force in 2005 and had as its goal to reduce greenhouse gas emissions and tackle the climate change. Governments that adopt the Kyoto protocol have forced their businesses, through the legislation, to adopt environmentally friendly tactics, which leads to reduction of the carbon dioxide emissions in order to achieve a better future. But each country was and still is committed to achieving different percentages of greenhouse gas emissions depending on their different culture and abilities.

The sensitivity to environmental issues, which is observed not only in the states but also in the citizens themselves, (Gyöngyi Kovács, 2008) was the impetus for a plethora of businesses to focus on eco-friendly practices and on creating innovations. Innovations with environmental and sustainable direction combined with means that reduce the production costs. In this case businesses increase their subjective value by recognizing their environmental responsibility and by clearly defining the standards they want to follow and adopt. More specifically, many companies undertake to embrace a "green" corporate social responsibility program by agreeing to the following actions in order to comply with the legislation, to reduce their environmental impact and to contribute to a better society (Klaus North, 1997):

- Environmental performance and control: The qualitative and quantitative data related to the energy consumed by the company, the emissions and the control of the waste and resources are recorded, in order to observe the improvement or the deterioration of these quantities after the adoption of environmentally friendly practices.

- Employee Practices: The implementation of an integrated environmental policy should be adopted by all company staff. The company is the one that undertakes the education and the training of the employees in environmental issues and developments. It is very important for a company to have a well-trained staff at all levels who can keep up with the technology and innovations.
- Green Supplies: A self-respecting company that wants to meet the demands of its consumers and is committed to adhering to the principles of sustainability must work with suppliers who embrace the same principles and ensure that the goods and services produced by them are environmentally friendly.

- Green Products: We are talking about companies that are committed to the quality of the ingredients of their products and to the way they are produced. This commitment concerns the environmental character of the products, as well as the possibility of recycling them and their wrappers.

- Corporate Environmental Policy: It includes the principles that the company wants to represent and the precise goals and practices that intends to follow in order to reduce its environmental impact. “The vital elements in any Corporate Environmental Policy are the environmental audits, publication of a statement of environmental policy to inform shareholders and the public, quality management procedure, group board commitment to the protection of the environment, etc.” (Pamela Shimell, 1991).

2.4 Practices that characterize a business as Green

Businesses are grouped into those who are not interested in environmental issues and have no ecological conscience and moreover pollute the environment without hesitation and ignoring the promotion of sustainable development and to those who are environmentally conscious. The first category, ignores the laws of their country and choose to pay any fines imposed on them. On the contrary, the companies that carry out ecological operations with systems and new technologies can prevent environmental pollution and improve the quality of life. Many of them are characterized by sensitivity and respect for the environment and follow ecological practices beyond the logic of compliance with the regulations. They follow these practices because they feel they owe it to our planet and to the future generations.

As cited in section 2.3, many companies choose to adopt a "green" corporate social responsibility program by operating environmental performance and control, training properly their staff, cooperating with green suppliers, producing green products and following corporate environmental policy. These practices characterize a business as Green. More specifically, green companies tend to reduce the pollution they cause to the environment with the proper preparation of their operation, with the proper conduct of the required transport and with the proper management of both their resources and their waste. Significant are the following examples as listed in Table 1, in order to point out some of the actions that a green business should follow and to make clear and explain the use of the term “proper”:

Waste	Recycling
	Energy production from waste
	Reuse- Upgrade Waste into a value adding co-product
Energy	Reduce energy waste
	Renewable energy sources
	Reduction of CO_2 emissions
	Reducing greenhouse gas[2] emissions
Transfers	Supplies supervision
	Digital communication
	Supervision of goods’ transport
Raw Materials and Resources	Recycling
	Raw materials from renewable energy sources
	Management of all types of materials
	Water stock management

Table 1: *Points of interest for reducing environmental impact by companies.*

In addition, there are some initiatives that companies with a positive ecological sign (the so-called green companies) can implement, in combination with the aforementioned.

[2] Greenhouse gases: Some gases such as ozone, methane, carbon dioxide and nitrous oxide absorb and emit energy radiation within the thermal infrared area, causing the greenhouse effect.

Firstly, more and more companies use the “free gifts of nature” and generate harmful effects, like waste. These companies in order to contribute to the waste reduction should be part of an Industrial Symbiosis. Industrial symbiosis “engages traditionally separate industries in a collective approach to competitive advantage involving physical exchange of materials, energy, water, and or by-products” (Samuel Bautista-Lazo, Tim Short, 2013). This relationship provides not only economic, but also ecological advantages. Economic advantages from the point of view that businesses spend less on useful materials and environmental advantages from the point of view that a huge amount of waste does not end up in the landfills and are therefore reused.

On the other hand, companies in order to avoid the generation of waste should adopt the “Fit thinking” (Samuel Bautista-Lazo et al., 2013). This is an advanced idea that concerns the production and the product’s characteristics. In short, the products must meet the expectations of customers (and of course follow the environmental standards) both in quality and cost in order to be purchased and used by them. There are many cases of products that are either used for a short period of time by consumers or are not even bought due to the high cost. This results in the creation of waste or in the fastest creation of waste that could have been avoided. In addition, companies should be open to innovations and new technologies, because these will be the future methods that will contribute to the depletion, and why not to the zero generation of waste by achieving alchemic transformations of them.

An additional example of an action that green companies can implement is the adoption of a transition plan from depletable to renewable resources. From the previous years until today, it is found that most companies, and not only, use uncontrollably the resources that they consider to be abundant. We may not know exactly when they will run out, but the only sure thing is that resources such as oil, coal, gas and generally all carbon-based resources, nuclear energy, etc. will cease to exist. Or there is a possibility that they will be used by the elites, as their cost will be very high. Apart from that, however, such resources leave significant residues in both the atmosphere and the water, resulting in the destruction of our natural environment. Therefore, it is obvious that there is a need to turn to renewable energy sources which will not emit greenhouse gases, which are harmful to the atmosphere. Renewable energy comes in many different forms (Tom Tietenberg, Lynne Lewis, 2009) such as hydroelectric energy, wind energy, photovoltaic systems, solar energy, marine energy, biomass fuels and geothermal energy. It is certain that in many cases a combination of the above sources of energy will be needed and in the beginning their relative cost may be high. However, their use will lead to future social, environmental and economic benefits which have a higher and longer-term value for both present and future generations.

Apart from these, green companies can work with non-governmental organizations dealing with environmental issues. These organizations are able to help businesses to develop further their ideas and innovations, as well as instill the value of sustainable entrepreneurship in all members of the business. They can work together to create annual awareness campaigns for employees, future entrepreneurs, as well as suppliers, on environmental protection and sustainability issues. Making targeted moves with the cooperation of competent persons to promote green views based on entrepreneurship, more and more emerging entrepreneurs are awakened. “Developing appropriate management behaviors which will enable leaders to develop and operate sustainable organizations in this new environment is at the heart of debates about the nature and character of corporate responsibility” (Patricia Hind, Andrew Wilson, Gilbert Lenssen, 2009). At the same time, it becomes clear to consumers themselves the real interest of green businesses in promoting the interests of society and environment and not only their financial. Designing such campaigns is therefore very important if we take into account that improving the sustainability of a company’s actions is not an easy task to execute and some will agree that changing the whole mindset of a business is even harder.

2.5 The reality of small and medium enterprises

For most companies, green entrepreneurship is a growth opportunity that lays the groundwork for a new modern global economy. There are more and more green companies with large capitals who can keep up with the new technologies and with the current environmental situation. These companies aim for a better tomorrow, but also aim to meet the needs and the standards of the consumers, investors and shareholders. Of course, on the other hand there are some companies who believe that the environmental protection, expressed through green entrepreneurship, is necessary, but it does not bring immediate financial benefits due to the high cost of the change and adoption of new environmentally friendly equipment and technologies.

Given the current state of the global economy, it is clear that all companies are aware of the environmental reality. It is understood that as green entrepreneurship evolves, significant changes are taking place in the global economy. The economic sector of the countries will be affected, which means that it must be harmonized with the new conditions. However, some companies are afraid that with the development of a new green business project, they will take a risk without any guarantee that what they do will succeed. Small and medium-sized enterprises belong to this category of enterprises, which wonder if a new project is likely to pay off and if they should make a start. Some researchers believe that small and medium companies are going to take, in this case, a big risk due to the fact that they do not have the same large capital, the expertise and resources as other companies, like the multinationals (Christopher J. Newell, Walter B. Moore, 2010).

But if we study in depth the reality of small and medium enterprises, we will find that this is not always the case (Heledd Jenkins, 2006). These companies see the importance to develop a green business spirit for the following reasons:

- To harmonize with the legislation of their country
- To receive rewards and funding from European programs

- To be able to compete, to some extent, with larger companies
- To attract more consumers, aware of environmental issues

European funding programs, such as the NSRF, obviously can help businesses evolve and keep pace with current developments by adopting environmentally friendly equipment and technologies. Technological developments in modern society have allowed polluting companies to protect the environment at minimal cost by taking advantage of new detection systems, sensors and telecommunications systems. In this way, the amount of money lost in possible fines is reduced, but at the same time the future benefits of society and of the businesses increase. An added advantage is that small and medium-sized enterprises can adapt more quickly and with less difficulty to new developments due to the smaller number and volume of their equipment. "SMEs can be very adaptive, swiftly adjusting their trading capacities according to changing market opportunities. This flexibility means that they can respond quickly to changing circumstances" (Heledd Jenkins, 2006).

Apart from these, small and medium companies are able to take advantage of the movements of large companies. Movements such as the big days of information and awakening of citizens for the protection of the environment and the importance of turning to environmentally friendly products. As a result, consumers will turn to the companies that produce such products, regardless of their size. In addition, they can take advantage of large companies' negotiations with suppliers. These negotiations are essentially about suppliers' compliance with legal regulations on environmental protection. In this way, small and medium-sized businesses save time and money and gain the ability to become competitive, without having put much effort and a lot of money.

Chapter 3. The meaning of green bonds

3.1 Bonds

A bond means a debt security with a specific duration, such as a month, a year or a decade. The issuer of the bond wishes to raise funds and undertakes to "pay a certain amount of money at regular intervals in the future as interest" (the so-called coupons) or a face value at maturity (Ilias Tzavalis, Athanasios Petralias, 2009). A bond includes all the information and details about the duration and the payments that the lender should know. The main features, therefore, that it is important for a modern investor to know, in order to be able to easily evaluate and choose any type of bond in the world markets, are the issuer of each bond, its nominal value, the coupons, the frequency of payment of each coupon, the expiration date, the currency, as well as the creditworthiness of each issuer (Choudhry Moorad, 2004).

Bonds, according to their issuer, are divided into 4 important categories:

1. Sovereign bonds (such as Greek government bonds): Nowadays, the governments are issuing more and more bonds and actually they “replaced bank borrowing with bond issuance” (Stijn Claessens, Daniela Klingebiel, Sergio L. Schmukler, 2007). The sovereign bond market is the largest bond market and it is observed that in developed countries with developed financial systems the demand for government bonds is particularly high.

2. Municipal bonds (such as bonds issued by each US state): “are represent a promise by local government units, the issuers, to repay to lenders an amount of money borrowed according to a fixed schedule” (Temel Judy Wesalo, 2001). These bonds are used for financing school and higher education infrastructure, streets, transportation facilities, hospitals, electric power generating facilities, water tunnels, resource recovery plants etc.

3. Supranational bonds: these are bonds issued, mainly, by international organizations such as the World Bank and the European Investment Bank (Choudhry Moorad, 2004). These international organizations have issued bonds in all major countries' currencies and have the best risk assessment rating (AAA).

4. Corporate bonds (such as bonds issued by OTE): It is about bonds issued by companies whether they are listed on the stock market or not, in order to raise finance for new projects or to cover debts. There is also the financial sector bond, which is about a type of corporate bond issued by a financial institution (OECD, green bonds, 2015). Nowadays, corporate bond market is thriving considering that in the US “the corporate bond market is massive, with more than 40000 corporate bond issues outstanding and a principal amount of more than $8.3 trillion” (Maureen O’ Hara, Yihui Wang, Xing (Alex) Zhou, 2018).

On the other hand, bonds can be divided, depending on some special characteristics such as the interest rate, the duration, the method of repayment, etc., into the following categories (Ilias Tzavalis, Athanasios Petralias, 2009):

- Fixed Rate Bonds: “are the most common type of bond” and they have a fixed issue interest rate until the maturity of the bond. They regularly give coupons to the holders as well as the face value at the expiration.

- Floating rate notes: these bonds do not have fixed coupons and if the issue rate increases, the coupons that the holder will receive will be more. Based on the configuration of the interest rate index, the coupon is configured.

- Zero-coupon bonds: do not give coupons to the holders, but they simply pay the nominal value at maturity. These bonds usually have a maturity of three months to one or two years.

- Perpetual bonds: These are bonds that do not have an expiration date and holders receive coupons at regular intervals indefinitely. The issuer is not required to "redeem them or pay the so-called face value".

- Callable/ Putable bonds: are bonds according to which the issuer can buy them back at a pre-agreed price. Respectively, the owner has the right to sell them at a predetermined price in the future but before their expiration date. If the government or a company issues the bonds there is an initial period in which the issuer cannot buy them back.

- Convertible bonds: these are corporate bonds only. The holder has the right to exchange them with other bonds or shares of the same company, at predetermined prices and dates. All the convertible bonds are callable bonds, too.

3.2 Green bonds

Many authors cite that, the term Green or Climate bonds refers to bonds whose proceeds “are committed to finance environmental and climate friendly projects, such as renewable energy, green buildings, resource conservation”, bioenergy, low cardon transports (Josué Banga, 2019), pollution prevention and control or alternative energy projects. The World Bank defines a green bond as “a debt security that is issued to raise capital specifically to support climate-related environmental projects” (World Bank, 2015). These bonds are issued by central governments, banks or companies with a specific duration and the issuers must indicate that the proceeds from the bonds are used for green projects. It is obvious that the difference between the conventional bonds and the green bonds is that the proceeds from the latter will be allocated exclusively to environmentally friendly projects.

However, in order for a bond to be named and characterized as green, the aforementioned alone are not enough. It must follow the Green Bond Principles (GBP's), which are "a set of voluntary standards established by industry participants including major banks and non-profit organizations" (Linh Pham, 2016) and provide integrity in the Green Bond Market. The GBP have four core components (International Capital Market Association, 2018):

1. Use of the Proceeds: the proceeds of a green bond must finance green projects which should be explicitly described in the legal documentation. Indicative categories of green projects, as presented, are the renewable energy, energy efficiency, pollution prevention and control, environmentally sustainable management of living natural resources and land use, terrestrial and aquatic biodiversity conversation, clean transportation, sustainable water, climate change adaptation, eco-efficient and/or circular economy adapted products and green buildings.

2. Process for Project Evaluation and Selection: The issuer have to inform the investors about its purposes, "the environmental sustainability objectives", which green project from the above will follow, how he will execute it and about the potential environmental and social risks which are associated with the specific project.

3. Management of Proceeds: "The GBP encourage a high level of transparency and recommend that an issuer's management of proceeds be supplemented by the use of an auditor, or other third party, to verify the internal tracking method and the allocation of funds from the Green Bond proceeds".

4. Reporting on actual use of funds: all the issuers, without exceptions, must keep up to date and inform the investors about the use of the funds. This can be done with an annual report which includes the descriptions of the projects, what actions have taken so far, the expected impact of the projects and what actions will be taken next until the project is completed.

3.3 Advantages and disadvantages of green bonds

An investor in order to build his optimal portfolio and decide to include green bonds will consider the pros and cons. In Chapter 5, an extensive analysis will be made with specific data in order to determine whether it benefits the investor to include green bonds in his portfolio or not.

In general, however, it is obvious that green bonds are a means of achieving ecological and environmentally friendly projects. Investments are necessary in the modern era and the investor achieves not only profits but also a better future. In addition, in many countries, the ability of banks to provide long-term green loans is limited due to their large liabilities. Businesses that only have access to short-term bank credit often face risks of refinancing and of failure for long-term green projects. If banks and businesses can issue green bonds in the medium- and long-term for this kind of projects, the restrictions on long-term green financing can be mitigated. By issuing long-term bonds, issuers have a safeguard that for a certain period of time they will have the necessary capital to achieve their environmentally friendly goals. Another advantage of the green bonds is that issuers' reputation is improving. Through the green bond market, the issuer highlights its commitment to improving the environment and striving for sustainability. Issuing a green bond is an effective way to develop and implement a credible sustainability strategy and the general public, by investing in such bonds, will contribute to a range of tangible environmental projects.

On the other hand, there are some disadvantages regarding the green bonds. Constant information and monitoring of green projects are required, as well as the avoidance of accidents. Green bonds are closely linked to these projects and any delay in completing the project could lead to a loss of issuer credibility. It is also important to avoid mistakes and accidents, because the further burden and pollution of the environment, leads not only to the destruction of the environment, but also to a loss of issuer credibility, too. Another important disadvantage that is often mentioned is the "green washing risk". "Green washing risk" is defined as the case in which the issuer issues green bonds without supporting the green character of the project for which funding is requested. He issues these bonds in order to attract investors and increase his capital with supposedly environmentally friendly goals. Flammer (2018) expresses concerns about money laundering in the green bond market due to the absence of strict public regulations.

3.4 Historical background of green bonds

The green bond was first launched by the European Investment Bank (EIB)[3] in 2007 (Stephen Kim Park, 2018), which was followed by the World Bank[4] and more specifically by the department "International Bank for Reconstruction and Development" (IBRD), in 2008. The World Bank issued a Green Bond, the so called "climate awareness bond" in order to finance climate change mitigation project (OECD, Green Bonds, 2015). At first, there was no interest in green bonds as environmental projects were considered unprofitable in comparison with other investments. Nevertheless, in recent years there is a big increase in their issue, with the average annual growth rate at approximately 95%. This significant increase is because of the growing awareness of traditional investors about the benefits of green investments and the impact of climate change on their financial assets and their environment. An

[3] European Investment Bank: The bank was founded in 1958 with the entry into force of the Treaty of Rome. It aimed to finance projects needed to achieve the EU's goals. Today, "it is the biggest multilateral financial institution in the world and one of the largest providers of climate finance".
[4] World Bank: An international financial institution that provides technical and financial assistance to developing countries and aimed at reducing poverty. It was founded in 1944 with headquarters in Washington.

important factor, was actually the fact that in 2013, the Bank of America[5] and Vasakronan[6] created the first corporate green bond. The wider bond market began to react after the sale of the first green 1 billion bond, which sold within an hour of being issued by the IFC in March 2013.

The entrance of the corporate green bonds in the green bond market changes dramatically the whole market. In 2013 with the Vasakronan and in 2014 with the Toyota's green bonds for electric vehicles and hybrids. Also, significant was the issue of the Apple's first green bond, in the amount of $1.5 billion, "the largest issuance to date by a U.S. corporation", in 2016 (Stephen Kim Park, 2018).

In 2020 the Covid-19 pandemic played an important role in bond issuance, as there was a slowdown of the green bonds' issuance compared to 2019. Both companies and governments decided to issue "pandemic bonds" in order to "mitigate the effects of the coronavirus as well as to pave the road to recovery" (Climate Bonds Initiative, Green Bond Market Summary, H1, 2020).

In Figure 2 are presented the green bond milestones, the remarkable development of the green bonds market. Green bonds are now issued by companies, governments and banks and the demand for them is very high. Most of the publishers come from Europe, China or North America, with most of the projects funded in developing countries.

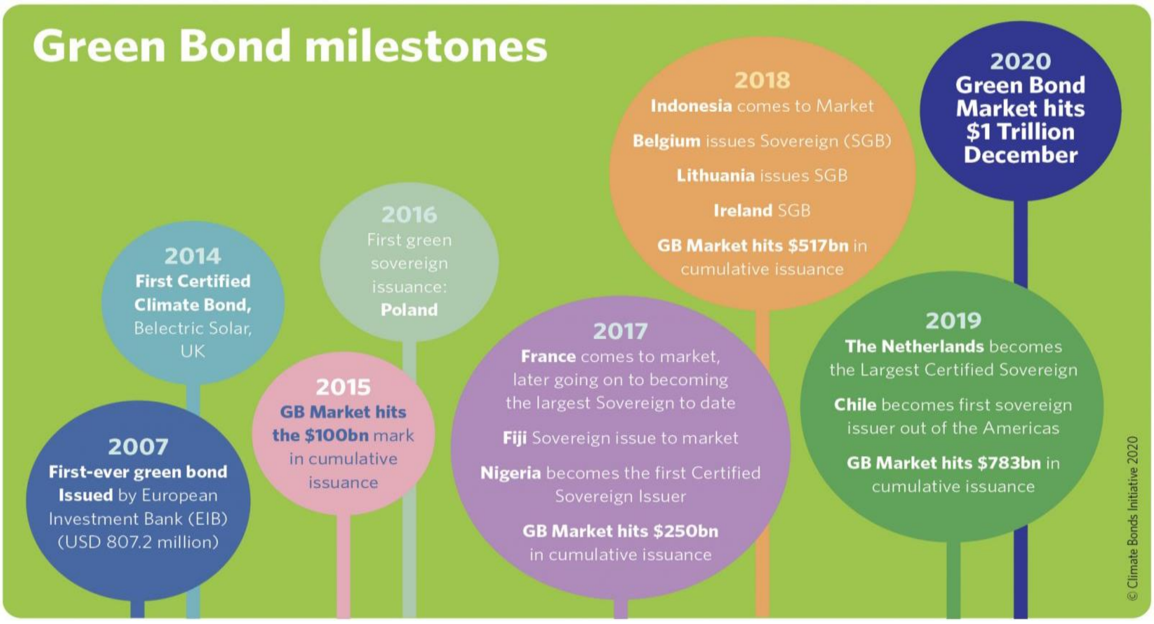

Figure 2*/ Source: Climate Bonds Initiative, Explaining Green Bonds, 2020, https://www.climatebonds.net/market/explaining-green-bonds*

[5] Bank of America: American Multinational Bank for Investment and Financial Services based in San Francisco and established in 1998.

[6] Vasakronan: Swedish Real Estate Agency founded in 1993. Formerly state property, but now belongs to the first, second, third and fourth Swedish pension office.

3.5 Countries issuing green bonds

During the last years, the companies are increasingly investing in green bonds, which are intended to boost investments related to the environment and climate change. Green bonds finance projects that benefit the planet and are issued by governments, companies or financial institutions. The main condition is that they are used for actions related to the management of renewable energy sources, transport, waste management, green buildings and the conservation of biodiversity. The USA, China and France are the top three countries that make the most of the financial instruments to reduce their environmental footprint. The largest issuer of green bonds for the USA is the Federal National Mortgage Association (FNMA)[7], commonly known as "Fannie Mae", for China, the main issuers are Banks[8], while for France the main title is the Government Bond. "Together they accounted for 44% of global issuance in 2019" (Climate Bonds Initiative, Green Bond Summary, H1, 2020).

France issued its first green bond in 2017 and since then, together with China and the USA, they issue green bonds of many millions or even billions of dollars every year. Indicatively, in 2018, 52% of the profits from the issuance of green bonds were invested in energy projects, 13% in buildings, 11% in the transport sector, 5% in water supply programs and a 4% was invested in waste management programs (Monica Filkova, CFA, C. Frandon-Martinez and A. Giorg, 2019). In 2019, the USA issued green bonds that reached 51.3 billion dollars. Green bonds issued by China in the same year reached the 31.3 billion dollars, while those of France, exceeded the 30 billion dollars (30.1 billion). The investments of the profits from the issuance of green bonds for the year 2019 per sector, are as follows (Almeida, M., 2020):

- 32 % in renewable energy sources
- 30% in green buildings
- 20% in the domain of transportation
- 9% in the water sector
- 3% in the agricultural domain

Top 3 Countries	Amount Issued	Number of Issuers	2018-19 change (amount)
USA	USD51.3bn	105	+44%
China	USD31.3bn	79	+1%
France	USD30.1bn	19	+113%

Table 2: *Top 3 countries at the green bond market in 2019.*

[7] Federal National Mortgage: It was founded in 1938 and since 1968 it is a Public Service.
[8] In 2018, the top China issuer was the Industrial Bank (USD9.6bn) and in 2019, the top China issuer was the bank ICBC, which is the largest bank in China and worldwide.

The year 2020 is considered as a difficult and different year due to the Covid-19 pandemic. It is characterized by high mobility of green bonds in H1[9] 2020, as worldwide their issuance reached almost USD250bn issued versus USD341bn for the full year of 2019(Almeida, M., Mok, L., Tukiainen, K., 2020). The Climate Bond Initiative (CBI) said that "if we did not go through the period of the COVID-19 pandemic, which has hit the markets worldwide, this amount would be much higher". Therefore, due to the coronavirus, the market's composition was very different this year as shown in the Diagram 1. A different category of bonds, the so-called pandemic bonds, was created this year, with China leading the way in February 2020.

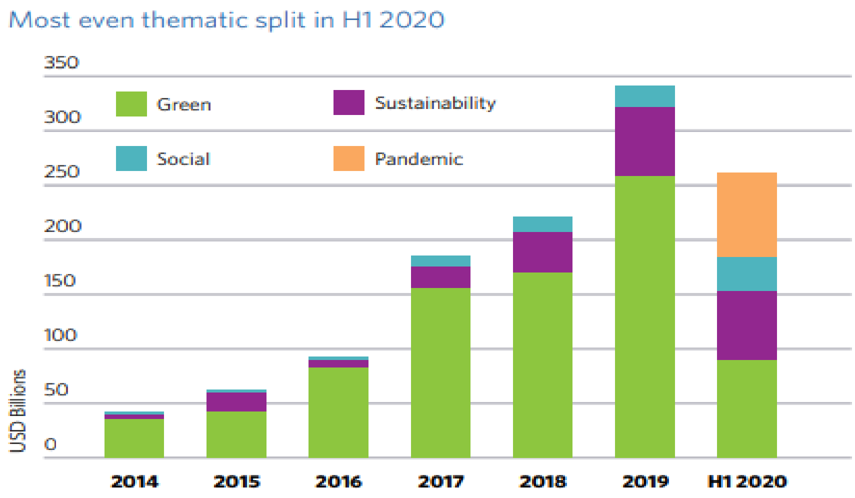

Diagram 1/ Source: Almeida, M., Mok, L., Tukiainen, K., 2020

The reins in the green bond issue category for this year remained in the "hands" of the USA, followed by France and in the third place was Netherlands. China, by "having spent relatively more on pandemic-related investments (at the expense of others, such as green)" and being the first country to be hit by the virus, was in the 7th place.

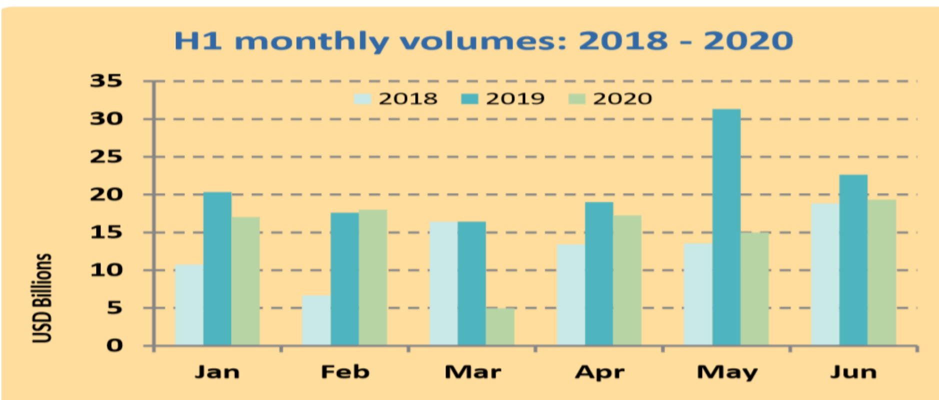

Diagram 2*: Comparison between year/ Source: Climate Bonds Initiative, August 2020, https://www.climatebonds.net/files/reports/h1_2020_highlights_final.pdf*

[9] H1: refers to the first half of a year.

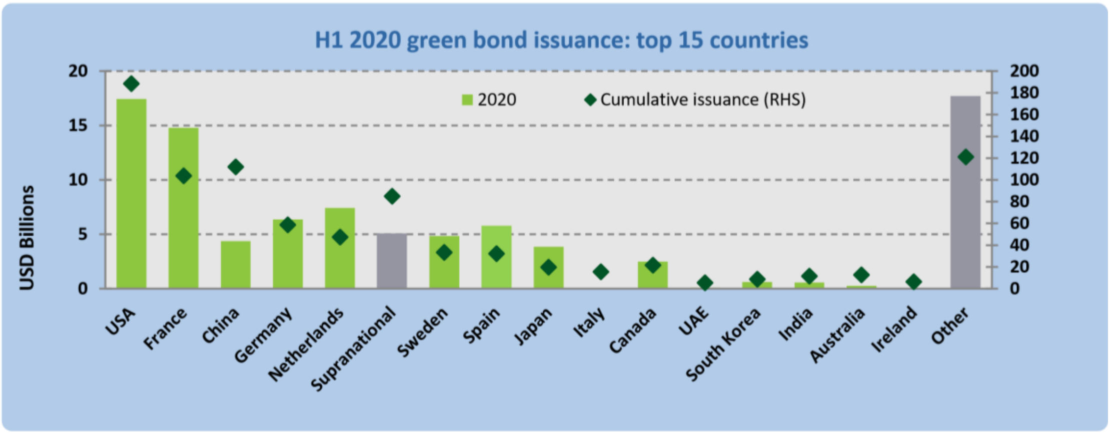

Diagram 3*: The first positions in the green bond market in 2020/ Source: Climate Bonds Initiative, August 2020, https://www.climatebonds.net/files/reports/h1_2020_highlights_final.pdf*

Chapter 4. Risk measures and mathematical formulations

The problem of optimal portfolio selections, the process of selecting the "best" portfolio from a set of assets, is the cornerstone of the modern financial theory. It has piqued the interest of many researchers due to the fact that the appropriate portfolio of assets in which to invest is a difficult and multifactorial process. In 1952, Markowitz “provided the foundation for modern portfolio theory as a mathematical problem” when he tried to answer the question “Which portfolio is the best?” (Hannes Marling and Sara Emanuelsson, 2012).

The issue of portfolio optimization is the most representative problem in the financial sector faced by investors. In fact, with portfolio optimization, investors seek to make the right financial decision by allocating their available capital to specific assets, in order to achieve the maximum possible return with the least possible loss, given the financial risks. Investors want portfolios that have the minimum risk and give the maximum expected return. In order to gain that it is important to understand and quantify the risk and determine their risk tolerance. It is therefore important to present and analyze the financial risks and their mathematical formulations. As shown below, there are a lot of different approaches of measuring portfolio risk, from which some of them are usually combined.

4.1 Variance

The term mean variance first appeared by Markowitz in 1952, when he published the article “Portfolio Selection”. He was later awarded the Nobel Prize in Economics for his work in the Modern Portfolio Theory, in 1990. Below is an introduction to this theory, which “helps investors determine the biggest reward at a given level of risk or the least risk at a given level of return” (Investopedia). The investors, who are usually risk-averse, aim to build a portfolio with the lowest possible risk and at the same time with the highest return. But, in general, high investment profits are associated with high risk and low profits are associated with low risk.

The term portfolio refers to the collection of the assets, for example stocks, bonds, currencies or commodities, which are owned by an entity. The Markowitz theory focuses on two parameters the expected return and risk. As he cited in his article, “we illustrate geometrically relations between beliefs and choice of portfolio according to the expected returns—variance of returns rule”.

In general, the return Rt at time t of a portfolio is:

$$R_t = \frac{\mathrm{T_t}}{\mathrm{T_{t-1}}} - 1$$

and it is calculated with the total value Tt of the assets of the portfolio divided by the total value at an earlier time t – 1. It actually reflects the percentage change of the total value from time (t-1) to time (t) (Hannes Marling and Sara Emanuelsson, 2012).

More specifically, we consider a portfolio with N assets and which one has a return r_i, expected return μ_i, variance σ_i and X_i which is the percentage of the amount invested in asset i (weight). The mathematical representation of the Markowitz model is given below:

(1) $R = \sum_{i=1}^{N} X_i r_i$: Return of the portfolio
(2) $\mu = E(R) = \sum_{i=1}^{N} X_i \mu_i$: Expected return of the portfolio
(3) $\sum_{i=1}^{N} X_i = 1$: where $X_i > 0$

The variance's (/standard deviation's) type as a measure of risk, according to Markowitz is:

$$\sigma^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{i,j} X_i X_j$$

where X_i is the percentage of the amount invested in assets i (i=1, 2, …, N), X_j is the percentage of the amount invested in assets j (j=1, 2, …, N) and $\sigma_{i,j} = E[(r_i - \mu_i)(r_j - \mu_j)]$ is the covariance between r_i and r_j (between the assets i and j). The variance shows the risk of the investment, and in general with the term "risk" we are referring to the deviations of the actual return from the expected one. A low variance is equal to a low risk.

So according to the above, it is obvious that investors, in order to find a solution to their optimization problem, aim to maximize the portfolio's return (r_i) given the portfolio's risk (volatility- variance) or they aim to minimize the portfolio's risk (-variance of return) given the minimum desired expected return (a fixed return $\bar{r}$) (Vaclavik Milan, Jablosky Josef, 2012), as shown below:

1st case:

$$\max \sum_{i,j=1}^{N} w_i r_i$$

$$s.t. \quad \sigma^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

$$\sum_{i=1}^{N} w_i = 1, \sum_{j=1}^{N} w_j = 1$$

2nd case:

$$\min \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

$$s.t. \quad \bar{r} = \sum_{i,j=1}^{N} w_i \bar{r}_i$$

$$\sum_{i=1}^{N} w_i = 1, \sum_{j=1}^{N} w_j = 1$$

It is easier to solve the second problem, because investors will have to solve a non-linear function, but with linear constraints. In general, according to Markowitz (1952), investors in trying to minimize, to make the variance small they should not only invest in many different assets (diversification[10]), but also, they should invest in securities with low covariances among themselves, which means that the assets should not be selected individually.

As we already point out, portfolio optimization, according to Markowitz, is based on the assumption that the investor aims for high returns and a minimum level of risk. However, this Markowitz optimization model has some difficulties which make its implementation in the real market not applicable in practice (Yongma Moon and Tao Yao, 2011). And this is happening due to the fact that:

- the optimization model is a "quadratic programming problem" and it is more difficult to solve and apply in relation to "linear problems"
- the mean- variance model is not reliable when we have non- normal distribution.
- the size of the covariances table is very large and difficult to estimate
- the model fails in times of crisis, as estimates are based on previous time periods
- the original model was designed only for stocks, but nowadays a portfolio consists of a plethora of different assets, such as stocks, bonds, currencies or commodities

Because of the aforementioned, a lot of researchers have developed a lot of different and improved models in order to solve the Markowitz's problems, which are presented below.

4.2 Value at Risk (VaR)

Value at Risk is a widely used risk measure, basically by industries, that quantifies the risk, i.e., the potential loss of money, of an investment- of a given portfolio for a certain

[10] By diversification we mean that the portfolio as a whole will have less risk compared to the risk that each asset would have separately.

period of time. Value at Risk is an alternative model, which "give us risk measures that are valid in the face of more general distributions" (Kevin Dowd, 2007).

Value-at-Risk (VAR) was widely used in the 1990s to measure market risk in trading portfolios. However, this method is dated back to the past, in 1922, when the need to apply it arises due to the capital requirements imposed on member companies of the New York Stock Exchange. The roots of the VAR method can also be traced to portfolio theory, for which, since 1945, an initial theoretical approach has been attempted. The origins of the portfolio theory can be traced back to non-mathematical discussions about the portfolio creation. The benefits of the portfolio diversification were approached in detail by authors such as Hardy (1923) and Hicks (1935), while Leavens (1945) made an example of a quantitative approach, that can be argued to be the first VAR application in the literature. To be precise, he approached in detail the behavior of a portfolio of ten bonds, for a defined period of time (Glyn A. Holton, 2002).

Value at Risk is about the tail (the worst p% of the outcomes) of the distribution and is defined as the maximum allowable loss, given a level of confidence "α*100%". Having the α*100% level of confidence, we set the probability "$p = 1 - \alpha$" and the VaR of the portfolio is equal to "$VaR = -q_p$".

- q_p : is the point on the x-axis (as shown in the Diagram 4) that cuts off the top α*100% observations from the (bottom 1-α of) tail observations.
- confidence level α*100%: means that there is a*100% chance of a return better than the VaR
- α∈ [0,1]

Investors are trying to measure and be accurate in their calculations regarding their potential losses from an investment. With the Value at Risk are able to measure the maximum possible loss with a given level of confidence and with a given probability in a time period. Usually, the confidence level is equal to 95% or 99% and the fixed probability is either 5% or 1%.

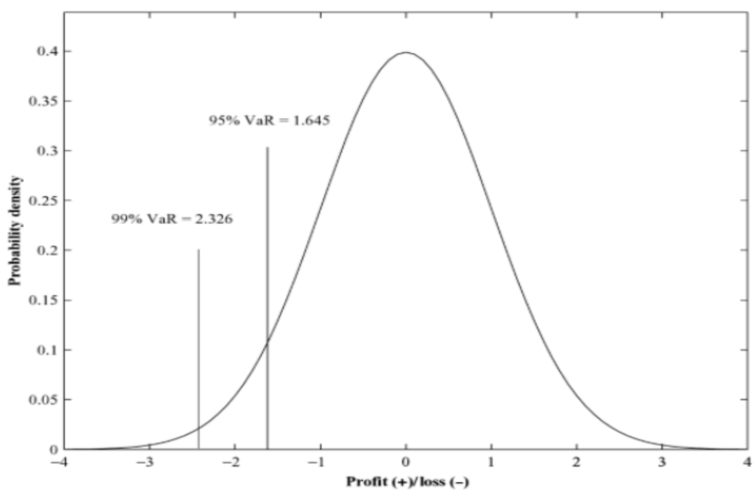

Diagram 4*/ Source: Kevin Dowd, 2007*

Using the illustrated diagram as an example, we have a Profit and Loss (P/L) probability density function over chosen holding period and it has fatter tails than a normal distribution. VaR, in this case, with 95% confidence is 1.645 and with 99% confidence is 2.326. By this, we mean that the worst outcome at these levels of confidence will be a loss of 1.645 and 2.326, respectively. In both cases, the Value at Risk "is given by the negative of the point on the x-axis that cuts off the top 95% (or 99%) of P/L observations from the bottom 5% (or 1%) of tail observations".

VaR's simplicity and the relative ease of being intuitively perceived make it one of the most widespread risk measures in the fields of insurance, portfolio management, etc. Its use, however, as a measure of risk is also associated with some problems-drawbacks. The most important of these are (Angulo, Javier A, 2009):

- Value at Risk may show the maximum allowable loss (given a confidence level), but it does not take into consideration the case of a distribution with long tail. As a result, using VaR as a risk measure may not take into account cases in which the investor has big loses.
- Value at Risk does not satisfy the theoretical property of Sub- Additivity. This theoretical property shows that the risk of a portfolio of shares X, Y, for example, is less than or equal to the risk of the individual shares.

  Sub- Additivity: $\varphi(\tilde{X}+\tilde{Y}) \leq \varphi(\tilde{X})+\varphi(\tilde{Y})$
  - $\tilde{X}, \tilde{Y}$: random variables, the returns of shares X, Y
  - φ: the function- risk measure of our choice

  Because VaR does not meet this requirement, it does not take into account the case of portfolio diversification. The careful addition of additional shares (or assets in general) to the investor's portfolio does not reduce the non-systematic risk of the portfolio.

In order to address the above disadvantages of VaR, an alternative measure-extension of VaR, the Conditional Value at Risk, has been proposed by the literature.

4.3 Conditional Value at Risk (C-VaR)

Conditional Value at Risk, introduced by Rockafellar and Uryasev (1999), is the expected value of returns beyond the Value at Risk. As a risk assessment measure "quantifies the amount of tail risk an investment portfolio has". It gives us a more complete picture of the potential losses in relation to VaR, due to the fact that C-Var is taking into consideration the "extreme" losses in the tail of the distribution, "the worst-case loss scenarios". In addition, it fulfills the property of Sub- Additivity and as a result allows the diversification of the portfolio.

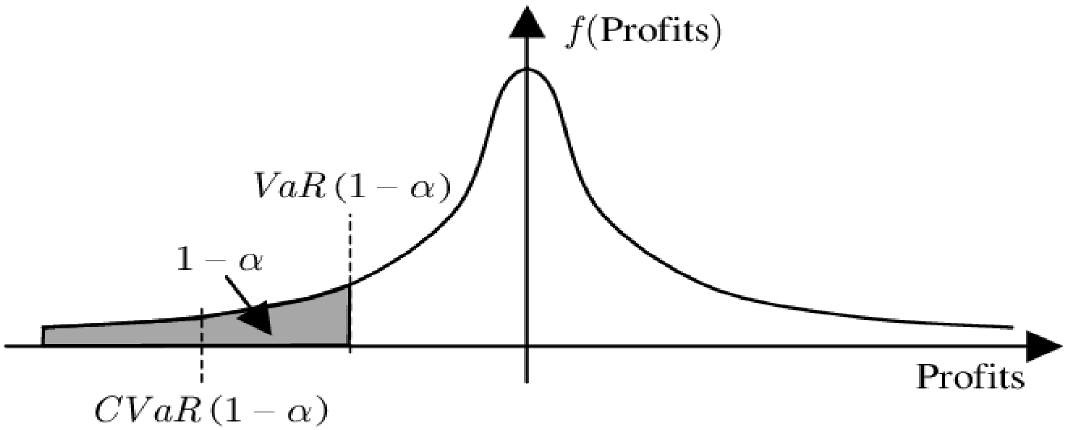

Diagram5/ *Source: https://www.researchgate.net/figure/VaR-and-CVaR-illustration_fig4_226393412*

The mathematical model of CVaR, for continuous distributions, when the VaR is known, is:

$$CVaR(x,a) = E[R(x,\tilde{r})|R(x,\tilde{r}) \leq VaR(x,\alpha)]$$

where:

- x: the investment rates on each asset
- α: confidence level
- $\tilde{r}$: the return of each asset

The above relation shows that the CVaR measures the expected value of the lowest returns of the portfolio X (in the tail area), provided that these returns are less than or equal to VaR. Although, this mathematical relation is not applicable for discrete distributions and because of this there is another definition of the CVaR for general distributions.

$$CVaR(x,\alpha) = z - \frac{\sum_{s\in\Omega} p_s y_s^+}{1-\alpha}$$

where:

- z: Value at Risk [= VaR (x, α)]
- s: each scenario
- p_s: the probability of each scenario
- y_s^+: auxiliary variable which gets a positive value when the current portfolio return is below the Value at Risk and is equal to zero when the return of the portfolio exceeds the VaR.

As we have already said CVaR gives information about the (1-a) * 100% of the distribution, i.e. the tail and makes it suitable in cases that we have left skewed distributions, distributions with long left tail. For the investors at the left tail is the risk where he can lose a lot of money and he wants to minimize it and is obvious that portfolios will low C-Var will have low VaR as well. In order to achieve this, because

CVaR is essentially an expected return (it gives us the average value of the returns below VaR), we will maximize the following relationship taking into account some constraints:

$$
\begin{array}{lll}
\text{Maximize} & z - \frac{1}{1-\alpha} \sum_{s=1}^{S} p_s y_s^+ & \\
\text{s.t.} & \mathbf{x} \in X, \quad z \in \Re & \\
& \mathbf{x}^\top \bar{\mathbf{r}} \geq \mu & \\
& y_s^+ \geq z - \mathbf{x}^\top \mathbf{r}_s & s = 1, 2, \ldots, S \\
& y_s^+ \geq 0 & s = 1, 2, \ldots, S
\end{array}
$$

where:

- $x^T \bar{r}$: the expected return of the portfolio
- μ: expected portfolio's target return
- $x^T r_s$: the return of the portfolio for each scenario s

In chapter 5, as we will see below, this risk measure is used in order to find the optimal portfolio from a series of bonds and will create the efficient frontier as a result of the above optimization problem. In this way, the investor will be able to manage the risk and to know which assets he should use in order to build his portfolio with low risk and with the maximum possible return.

Summing up the risk measure CVaR is a coherent and linear risk measure and in some cases is preferable to VaR as it gives us information about what extreme losses in the tail of the distribution. VaR fails to deal with distributions which are not normal and is not a coherent risk measure, in contrast to CVaR. “CVaR maintains consistency with VaR by yielding the same results in the limited settings where VaR computations are tractable, i.e., for normal distributions” (R. Tyrrell Rockafellar, Stanislav Uryasev, 2002). However, both of them are widely known, used by many sectors of the economy and have aroused the interest of many researchers.

4.4 Mean Absolute Deviation

As already mentioned, the Markowitz model cannot cope with big problems, because the size of the table of covariances of the real markets is very large and difficult to estimate. For this reason, in 1991, Konno and Yamazaki proposed an alternative model, the Mean-Absolute Deviation Model, in order to solve optimization problems using a "linear programming" model (Z. Qin, M. Wen and C. Gu, 2011). This leads to a faster solution of optimization problems compared to the Markowitz model which leads to the solution of a quadratic model. This linear portfolio optimization model of Konno and Yamazaki is shaped by considering a different risk measure, the absolute deviation of the rate of a portfolio’s return instead of the variance. Although, “the authors showed the fact that the mean-absolute deviation model gave essentially the same results as the mean-variance model if all the returns are normally distributed”.

More specifically, in the Mean-Absolute Deviation Model, risk is defined as the expected absolute “distance” (-deviation) of a portfolio’s return from its expected value:

$$MAD(x) = E[|R(x,\tilde{r}) - R(x,\bar{r})|]^{11}$$

where:

- x: the investment rates on each asset
- $\tilde{r}$: the return of each asset

When we have scenarios- a discrete distribution of the asset's return, then the type of the MAD is:

$$MAD(x) = \sum_{s=1}^{S} Prob_s \,|x^T r_s - x^T \bar{r}|$$

In order to optimize the MAD, to minimize the distance, we have to solve the following linear program:

$$\begin{array}{lll} \text{Minimize} & \sum_{s=1}^{S} p_s y_s & \\ \text{s.t.} & \mathbf{x} \in X & \\ & \mathbf{x}^\top \bar{\mathbf{r}} \geq \mu & \\ & y_s \geq \mathbf{x}^\top (\mathbf{r}_s - \bar{\mathbf{r}}) & s = 1, 2, \ldots, S \\ & y_s \geq \mathbf{x}^\top (\bar{\mathbf{r}} - \mathbf{r}_s) & s = 1, 2, \ldots, S \\ & y_s \geq 0 & s = 1, 2, \ldots, S \end{array}$$

where:

- p_s: the probability of each scenario
- y_s: a variable that helps us to "remove" the absolute of the MAD relation and based on the above constraints we conclude that the variable (the distance from the expected value) will always be positive
- $x^T \bar{r}$: the expected return of the portfolio
- $x^T r_s$: the return of the portfolio for each scenario s

In this way, we are creating the efficient frontier of the portfolio, as is the case with CVaR. The investor is able to manage the risk and to choose the assets and the amount that he will invest in each asset in order to build his optimal portfolio. As we already said, MAD is a preferable risk measure compared to the Markowitz model from the point of view that linear programming problems can be solved much faster than the corresponding quadratic programming problems. Also, the linear programming formulation is more suitable to handle problems associated with real transaction environments. However, the aforementioned advantages of the MAD model "are not widely appreciated in the financial engineering community".

4.5 Tracking Models

According to the tracking error model, investors are trying to build their optimal portfolio according to a benchmark portfolio. They want to be as close as possible to

[11] $MAD(x) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right]$

this specific benchmark portfolio, and in this way, they will be able to evaluate the risk and manage their investment. There are many different definitions of tracking error and different tracking error models. "Rudolf et al. (1998) propose a comparison between four different tracking error linear models in a static framework. Roll (1992) proposes a mean-variance analysis of tracking error […]", etc. (Diana Barro, Elio Canestrelli, 2008).

In general, the Tracking Error is the difference between the returns of a portfolio that the investor receives and the return of the benchmark portfolio, i.e., the return which he expected to get. So, in order for the investor to have the desired result he must create a portfolio whose returns will be close to those of the target (/of the benchmark portfolio). If the tracking error is high that means that the investor should replace some of the portfolio's assets with others. For the solution of the tracking error optimization problem, an investor must solve the following:

$$Maximize \sum_{s=1}^{S} p_s\, R(x, r_s)$$

$$\text{s.t.} \qquad x \in X$$

$$R(x, r_s) \geq R(x, \bar{r}) - \varepsilon R(x, \bar{r}), \quad s = 1, 2, 3, \ldots, S$$

The same model, which is defined with respect to the portfolio mean $R(x, \bar{r})$, can be defined also with respect to a target random variable $\tilde{g}$[12]:

$$Maximize \sum_{s=1}^{S} p_s\, R(x, r_s)$$

$$\text{s.t.} \qquad x \in X$$

$$R(x, r_s) \geq g_s - \varepsilon R(x, \bar{r}), \quad s = 1, 2, 3, \ldots, S$$

where:

- ε: a user specified parameter
- g_s: the return of the benchmark portfolio for each scenario s

In this model, "instead of measuring risk using a linear function of the downside deviation, we impose an infinite penalty for any deviations that are more than a user-specified parameter $\varepsilon R(x, \bar{r})$ below the mean" (Zenios, S.A., 2007). The risk we are interested in is the downside risk and deviations which are greater than the mean or "within $\varepsilon R(x, \bar{r})$ below the mean" are not taken into account in the risk measure.

The parameter ε, which is defined by the user, helps him to create his optimal portfolio. It is very important to set the appropriate ε in order to achieve the benchmark portfolio. And this is the result of the choice of a small ε, which leads to closer tracking. "The

[12] If we set $\tilde{g}$ equal to a fixed value the target random variable is the value of the portfolio we want to achieve.

term “tracking” indicates that the portfolio value stays within a margin $-\varepsilon R(x, \bar{r})$ of the target”.

4.6 Put/ Call Efficient Frontiers

The Put/Call Efficient Frontiers Model has the main idea of building a portfolio that trades off the portfolio’s downside risk[13] against the portfolio’s upside- regard[14] given a random target (a benchmark portfolio). “The upside potential has identical payoffs to a call option on the future portfolio value relative to the target” (Zenios, S.A., 2007). The price of the target is the strike price of the call option. It is obvious that when the portfolio value is above the target, the upside potential is the payoff of a call option that it is in-the-money. The investor’s payoff is the difference of the portfolio’s performance with the benchmark. On the other hand, when the portfolio value does not exceed the target, the upside potential is zero[15]. “Similarly, the downside payoffs are identical to those of a short position in a put option on the future portfolio value relative to the target”.

In other words, portfolios which are considered as put/call efficient are those that achieve the higher call value (expected upside) for a given put value (expected downside). So, it is essential to measure the portfolio’s performance above and below the target. The following auxiliary variables help us with the aforementioned due to the fact that they measure the deviations of the portfolio return from the random target $\tilde{g}$:

- $\tilde{y}_+ = \max\,[0, R(x,\tilde{r}) - \tilde{g}]$: measures the positive deviations, the upside potential of the portfolio
- $\tilde{y}_- = \max\,[\tilde{g} - R(x,\tilde{r}), 0]$: measures the negative deviations, the downside risk

As we already said, this model aims to produce a portfolio that gains the higher call value for a given put value. More specifically, it aims to produce a portfolio with the highest return given the level of the risk and vice versa. In order to trace the efficient frontier, we formulate the following linear program:

$$
\begin{array}{ll}
\text{Maximize} & \sum_{s=1}^{S} p_s y_+^s \\
\text{s.t.} & \sum_{s=1}^{S} p_s y_-^s \le \omega \\
 & y_+^s - y_-^s - [R(\mathbf{x}, r^s) - g^s] = 0, \quad \forall s \in \Omega \\
 & y_+^s, y_-^s \ge 0 \quad \forall s \in \Omega.
\end{array}
$$

where:

- ω = the allowable downside, the risk
- Ω={1, 2, …, N} index set of scenarios

[13] We have risk when portfolio values are below the target.

[14] We have reward when portfolio value exceeds the target. When the portfolio value is equal to the target the portfolio has no risk and no reward.

[15] In this case, we could say that the call option is out-of-the-money.

By solving this optimization problem, we get the portfolio whose returns are above the benchmark with the minimum possible risk. The portfolios that are under the efficient frontier are no desirable, as they do not give sufficient return for the level of risk. The portfolios to the right of efficient frontiers are not optimal because they have a higher level of risk for a specific return.

4.7 Expected Utility Maximization

The previous optimization models "optimized a prespecified measure of risk against a prespecified measure of reward to obtain a set of efficient portfolios". The investors in these cases, are able to choose one of the efficient portfolios. The difference of the Expected Utility Maximization with the previous models is that this one allows the investor to set his own criteria regarding the risk and the returns under conditions of uncertainty. This difference seems small but it is quite significant as the investor's choices and decisions play a very important role and must be taken into account.

The Expected Utility Maximization Model gives the opportunity to an investor to select a specific and unique portfolio that optimizes his preferences as expressed through a utility function.

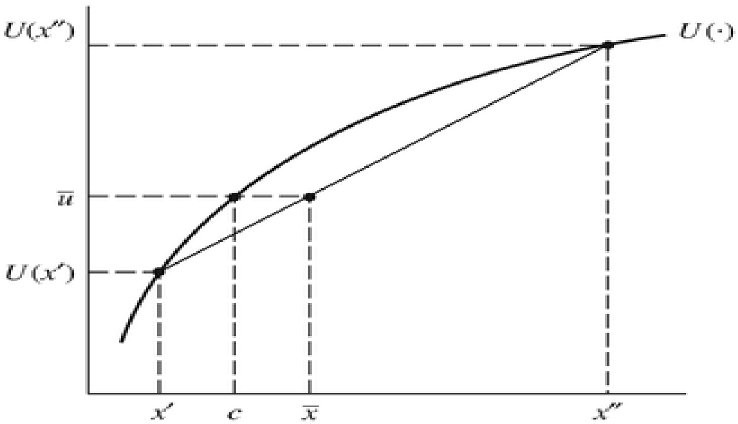

Diagram 6: *Expected Utility function of a risk-averse investor. / Source: Machina M.J. (2018), Expected Utility Hypothesis.*

The decision maker will choose from all the portfolios, the one with the highest expected utility and in order to be able to define and maximize the model, it must be assumed that decisions are made rationally. The mathematical formulation of the model[16] is the following:

[16] For the definition of the Expected Utility Maximization Model, we are using portfolio return, which return is a linear function.

$$
\begin{array}{ll}
\text{Maximize} & \sum_{s=1}^{S} p_s U(R(\mathbf{x}, r^s)) \\
\text{s.t.} & \mathbf{x} \in X \\
 & R(\mathbf{x}, r^s) = \sum_{i=1}^{n} x_i r_i^s \\
 & \sum_{i=1}^{n} x_i = 1
\end{array}
$$

where:

- x: asset allocation vector in percentages of total wealth

The result of the above maximization model allows the investor to choose the portfolio which satisfies a set of constraints “and furthermore satisfies the preference relation over all other feasible portfolios”.

Chapter 5. Empirical Application

In this section we implement portfolio optimization, the process of selecting the "best" portfolio from a set of portfolios based on a goal. It is the cornerstone of modern financial theory, as it is very attractive in the field of selection under uncertainty. By portfolio management-optimization, investors seek to make the appropriate financial decision by allocating their available capital into specific assets, in order to achieve the maximum possible return with the least possible loss, given their risk aversion. In this thesis, we applied the portfolio optimization problem in two cases. The first case concerns the regular bonds (RB) and second case concerns the combination of green and regular bonds (GRB). The assets from which an investor can choose for his portfolio are 6 bond indices for the first case and 11 bond indices for the second case (the same 6 regular bond indices from the first case and other 5 green bond indices).

In order to find all the effective portfolios, we conduct statistic tests. With these tests we are able to construct the efficient frontiers of the RB portfolios and of the GRB portfolios. In this case, we are able to find out how a portfolio's returns vary based on different risk levels. When for a given level of risk, we have the highest expected return or equivalent when for a given level of return, we have the minimum risk, then we say that we have an efficient portfolio. The efficient frontier is a good instrument for all the investors in order to choose their optimal portfolio because it gives useful information for risk averse, risk lovers and risk neutral investors. A risk averse investor, for example, will choose a portfolio which is as close as to the minimum variance portfolio (MVP), which is the right edge (if we have the CVaR risk measure) or the left edge (if we have the VaR risk measure) of the efficient frontier.

Subsequently, we conduct dynamic tests known as Backtesting Experiments. "Backtesting is the general method for seeing how well a strategy would have done ex-post". "Backtesting assesses the viability of a strategy by discovering how it would play out using historical data" (James Chen, 2021). The historical data are reliable and useful, because experience has shown that a portfolio with a good and high performance in the past will usually perform in the same way in the future. It gives us very strong results about the performance of an investment when we will implement it in real time. In our case, we conduct the backtesting experiments in order to assess the performance of the optimal portfolios (MVP model) with RB and with RGB for the last 2 years (105 weeks) and in this way we are able to find out their future performance and compare them. Also, we conduct Backtesting Experiments for 3 models, as we will see below, in order to compare the choices of the different investors (risk averse, risk lovers, risk neutral).

In order to conduct both the statistic and dynamic tests we use the GAMS[17] system and a specific measure of risk. In our empirical application we are using the CVaR Model, which takes into consideration the extreme losses in the tail of the distribution and is better than the VaR.

[17] It is a modeling system for mathematical optimizations. By using the GAMS, it is easy to solve a linear, a non-linear or a mixed optimization problem.

5.1 Data Description

As we already said, we examine 2 cases which are about regular bonds and a combination of green and regular bonds. More specifically, for the RB we have 6 indices from which an investor can build his optimal portfolio:

1. S&P 500 Composite Index
2. S&P IG Corporate Bond Index
3. S&P US HY Corporate Bond Index
4. ICE BofA US Corporate Index
5. FTSE USBIG Corporate Index
6. Bloomberg Barclays US Corporate Index

For the GRB we have 11 indices in total (the 6 aforementioned plus 5 green bond indices):

1. Solactive Green Bond Index
2. S&P Green Bond Index
3. S&P Green Bond Select USD Total Return Index
4. Bloomberg Barclays MSCI Global Green Bond Index
5. ICE BofA Green Bond Index

Over time, historical data were collected for all these 11 indices from the year 2014. In our analysis, we use weekly historical data from 05/12/2014 until 29/05/2020 in order to calculate the returns of each asset of every week of our specific period. By using these returns we made two tables, one for the RB and one for the GRB.

The data that we have to define at the GAMS system in order to conduct our analysis is a table with the returns, how many assets we have (6 RB indices in the first case and 6 RB + 5 GB indices in the second case), how many scenarios we have (n=286 for the efficient frontier, n=181 for the Backtesting Experiments), the probability that each scenario has (Prob(n)=1/286 or Prob(n)=1/181), the target returns and the statistical percentage we want (a= 0.95).

5.2 Methodology

In this part we are going to analyze step by step the whole methodology that we followed in order to conduct the statistic and dynamic tests. First of all, we used, as we already said, the CVaR mathematical model for dealing with the optimization problems at GAMS. After the definition of the data, we define the variables:

- C-var
- z: Value at Risk
- f_return(n): portfolio return under scenario n
- tot_return: expected return of the portfolio
- y(n): loss shortfall beyond VaR
- hold(class): units of asset i purchased under scenario

and then the equations:

- OdjDefCVaR: Objective function definition for CVaR maximization
- Tar_Return: Equation defining a minimum target over the expected return
- FinalReturn: Equation defining the final portfolio return under scenario n
- EXP_Return: Equation defining the portfolio expected return constraint
- init_balance: Asset balance constraint
- VaRCon: Equation defining the Var Deviation constraint

5.2.1 Efficient Frontiers

As mentioned above, we want to find all the efficient portfolios in both cases, the portfolios that give the highest return for a given level of risk and to compare them. In order to achieve that, we have to conduct statistic tests to find the efficient frontiers. For the case with the RB, we put in the GAMS system the table with the RB returns of the 6 indices for the last 5 years, 286 observations in total. For the case with the GRB, we put in GAMS system the table with the GRB returns of the 11 indices, again, for the last 5 years.

The steps we follow in both cases are the same. Firstly, we maximize the CVaR model in GAMS system and in this way, we find the portfolio with the lowest risk, the minimum variance portfolio (MVP) and we get its expected return. This portfolio is the right-bottom edge of the curve of the efficient frontier. Subsequently, in order to find the left-upper edge of the efficient frontier curve, we maximize the total returns in GAMS and again we are getting its expected return. This portfolio contains only one asset (A) and has the highest expected return. After these 2 steps, we have the two edges of the efficient frontier curve and 2 expected returns, the minimum (from the MVP) and the maximum expected return (A). In order to find the intermediate points and finally form the efficient frontiers we are separating the space between the returns in 8 equal spaces and in this way, we are getting 8 more different expected returns (μ_i):

$$d = \frac{\mu_A - \mu_{MVP}}{9}$$

$$\mu_i = \mu_{MVP} + i * d$$

Then, we are maximizing the CVaR model 8 times, by using each time one of the 8 expected returns we calculated, as the portfolios' target return. Thus, we found the combinations of expected return- risk and we are able to form the efficient frontiers.

5.2.2 Backtesting Experiments

At this point of the empirical analysis, we carry out dynamic tests and more specifically we carry out 4 Backtesting Experiments. The first two are about the RB and the GRB and our goal is to find the final performance of the portfolios in each case and to compare them. The whole procedure for these two is the same and we are solving the Minimum Risk model in GAMS system. At first, it is important to adjust our historical data to the optimization problems. As already mentioned, we took the 11 indices and found their returns from the year 2014 until the year 2020. So, for the Backtesting

Experiments, we created from these returns 105 include files (for each case, 210 in total) in which the time horizon changes each time. More specifically, we started with the include file 1, which consists of data from the 12/12/2014 until the 25/05/2018, then we use the include file 2 which consists of the data from the 19/12/2014 until 01/06/2018 and so on. Therefore, all the 105 (210 in total) include files consist of 181 weekly returns for each index.

After solving the Minimum Risk Model through GAMS for both cases, we get the percentages of the total available capital invested in each index, the optimal weights for every week for the last 2 years. Finally, in order to get the real returns of each portfolio, we multiply the optimal weights by the returns of the specific week we want to examine. For example, the weights from the include file 1 (12/12/2014- 25/05/2018) multiplied by the returns of the next week, the 1/06/2018. We repeat the whole procedure for all the weeks of the last two years and in this way, we find 105 Real Returns for the RB and 105 Real Returns for the GRB and we can evaluate the final performance of the portfolios.

The final Backtesting Experiments that we carry out, has as a goal to find the real returns for different degrees of risk aversion. In a few words, we want to understand how the investors react in real time and to compare all of them (risk averse, risk lovers, risk premium). In this part, we use the previous include files of the RB and of the GRB and we solve 3 different models through GAMS in both cases. The 1st model is the maximization of the CVaR, which give as a result the optimal weights of the risk averse investors. The 2nd model is the maximization of the total returns in order to get the optimal weights of the risk lover investors and the 3rd model is again the maximization of the CVaR but we are taking into consideration the target returns and we are getting the optimal weights of the risk neutral investors. Finally, we multiply all the optimal weights with the returns of the weeks we want to examine (the last 2 years, the last 105 weeks) and we get the real returns.

5.3 Results

At this section, we are presenting the results of the efficient frontiers and of the Backtesting Experiments for the cases of Regular Bonds and of the combination of the Green and Regular Bods.

5.3.1 Efficient Frontiers

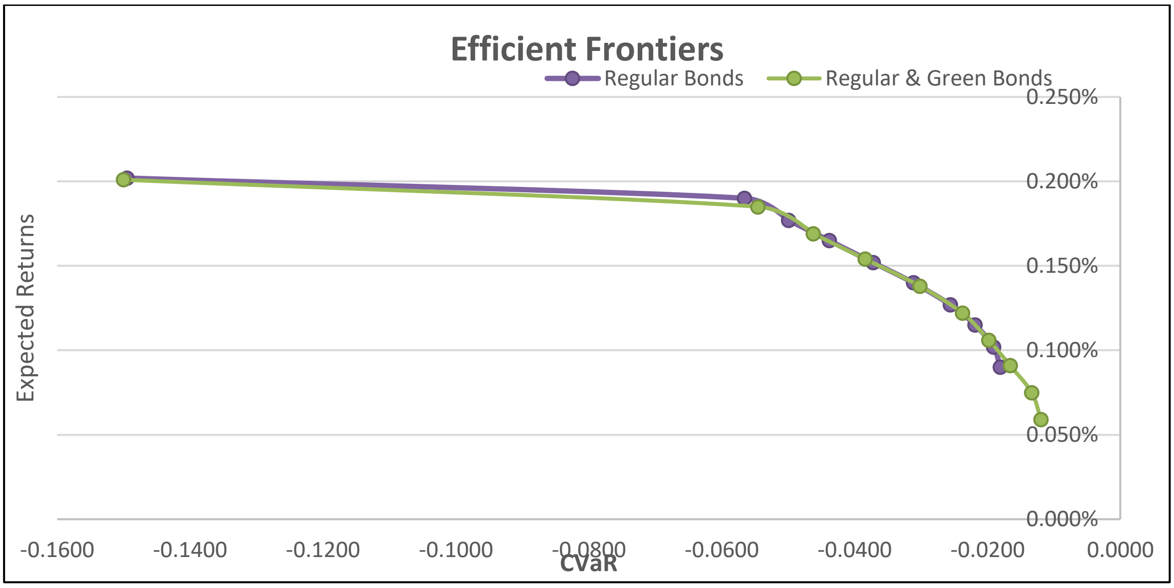

Diagram 7

REGULAR BONDS		
	CVaR	Expected Returns
μ_{MVP}	-0.0181	0.090%
μ_1	-0.0191	0.102%
μ_2	-0.0218	0.115%
μ_3	-0.0256	0.127%
μ_4	-0.0311	0.140%
μ_5	-0.0372	0.152%
μ_6	-0.0438	0.165%
μ_8	-0.0499	0.177%
μ_9	-0.0565	0.190%
μ_A	-0.1495	0.202%

***Table 3**: Data for the efficient frontier of the Regula bonds.*

GREEN AND REGULAR BONDS		
	CVaR	Expected Returns
μ_{MVP}	-0.0119	0.059%
μ_1	-0.0133	0.075%
μ_2	-0.0166	0.091%

μ_3	-0.0198	0.106%
μ_4	-0.0237	0.122%
μ_5	-0.0301	0.138%
μ_6	-0.0384	0.154%
μ_8	-0.0462	0.169%
μ_9	-0.0545	0.185%
μ_A	-0.1500	0.201%

Table 4: *Data for the efficient frontier of the Green and Regular Bonds.*

The results that we get from the statistic tests with GAMS are presented in the Table 3 and Table 4. Graphically, the set of portfolios that present the highest profit for a given level of risk is dispersed in the border of an efficient frontier. To find this set, as we mentioned, portfolio optimization models are used. So, in the Diagram 7 we have the efficient frontiers of the two cases, which resulted from the connection of the effective portfolios that we found, 10 points-portfolios in total.

As we can see from our data the minimum expected return of the RB is 0.090% and the minimum risk (CVaR) is -0.0181. But in the case with the GRB the minimum expected return is equal to 0.059% while the minimum risk is -0.0119. Even if we did not have the data tables, it is obvious from the Diagram 7 that the minimum variance portfolio with the Green Bonds is much better than the portfolio with the Regular Bonds due to the fact that it has lower risk and the most investors will prefer it. The RB MVP consists of bonds of the S&P IG Corporate Bond Index and of the S&PUS HY Corporate Bond Index, while the GRB MVP consists of bonds of the S&P Green Bond Index, ICE BofA Green Bond Index and of S&P 500 Composite Index. It is obvious that the 2nd Minimum Variance portfolio consists mainly of green bonds.

In addition, the maximum return that an investor can receive from the RB is 0.202%, but in the case of the GRB the maximum return is 0.201%, something a little less. Due to the low levels of risk, the efficient portfolios of the GRB are giving lower expected returns compared with the RB, which have a higher risk and they also have higher returns. In other words, this means that there is a positive correlation between the CVaR and the Expected Returns.

5.3.2 Backtesting Experiments

Now, we are presenting the results of the Backtesting Experiments on Tables 5 and 6. For the sake of brevity, we present the results of the first 10 weeks and of the last 10 weeks.

REGULAR BONDS

Weeks	Optimal Weights S1	Optimal Weights S2	Optimal Weights S3	Optimal Weights S4	Optimal Weights S5	Optimal Weights S6	Real Returns	Cumulative Returns
1/6/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	0.0496%	1.0005
8/6/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	-0.0707%	0.9998
15/6/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	0.2105%	1.0019
22/6/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	-0.1985%	0.9999
29/6/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	0.0111%	1.0000
6/7/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	0.2783%	1.0028
13/7/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	0.4335%	1.0072
20/7/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	-0.1458%	1.0057
27/7/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	0.1205%	1.0069
3/8/2018	0.0000	0.6522	0.3478	0.0000	0.0000	0.0000	0.2800%	1.0097
27/3/2020	0.0000	.7222	0.2778	0.0000	0.0000	0.0000	4.2461%	1.0411
3/4/2020	0.0000	.7222	0.2778	0.0000	0.0000	0.0000	0.5662%	1.0470
10/4/2020	0.0000	.7222	0.2778	0.0000	0.0000	0.0000	3.2458%	1.0810
17/4/2020	0.0000	.7222	0.2778	0.0000	0.0000	0.0000	2.2006%	1.1048
24/4/2020	0.0000	.7222	0.2778	0.0000	0.0000	0.0000	-0.2610%	1.1019
1/5/2020	0.0000	.7222	0.2778	0.0000	0.0000	0.0000	0.0316%	1.1023
8/5/2020	0.00000	0.9091	0.0909	0.0000	0.0000	0.0000	-0.7824%	1.0937
15/5/2020	0.0000	0.9000	0.1000	0.0000	0.0000	0.0000	0.3242%	1.0972
22/5/2020	0.0000	0.9000	0.1000	0.0000	0.0000	0.0000	1.4227%	1.1128
29/5/2020	0.0000	0.9000	0.1000	0.0000	0.0000	0.0000	0.7224%	1.1208

Table 5[18].

GREEN AND REGULAR BONDS							
Weeks	Optimal Weights S1	Optimal Weights S2	[…]	Optimal Weights S10	Optimal Weights S11	Real Returns	Cumulative Returns
1/6/2018	0.0000	0.0009		0.0000	0.0000	0.0657%	1.0007

[18] S1= S&P 500 Composite Index, S2= S&P IG Corporate Bond Index, S3= S&P US HY Corporate Bond Index, S4= ICE BofA US Corporate Index, S5= FTSE USBIG Corporate Index, S6= Bloomberg Barclays US Corporate Index.

8/6/2018	0.0000	0.0009		0.0000	0.0000	-0.1871%	0.9988
15/6/2018	0.0000	0.0009		0.0000	0.0000	0.4088%	1.0029
22/6/2018	0.0000	0.0009		0.0000	0.0000	-0.0647%	1.0035
29/6/2018	0.0000	0.0009		0.0000	0.0000	0.0410%	1.0039
6/7/2018	0.0000	0.0009		0.0000	0.0000	0.1845%	1.0058
13/7/2018	0.0000	0.0009		0.0000	0.0000	0.2857%	1.0087
20/7/2018	0.0000	0.0009		0.0000	0.0000	-0.1180%	1.0075
27/7/2018	0.0000	0.0009		0.0000	0.0000	0.0451%	1.0079
3/8/2018	0.0000	0.0009		0.0000	0.0000	0.0618%	1.0085
27/3/2020	0.0000	0.0000		0.0000	0.0000	0.9118%	1.0312
3/4/2020	0.0000	0.0000		0.0000	0.0000	0.0646%	1.0318
17/4/2020	0.0000	0.0000		0.0000	0.0000	1.1187%	1.1048
24/4/2020	0.0000	0.0000		0.0000	0.0000	0.2010%	1.0494
1/5/2020	0.0000	0.0000		0.0000	0.0000	0.7820%	1.0576
8/5/2020	0.00000	0.0000		0.0000	0.0000	-0.4300%	1.0531
15/5/2020	0.0000	0.0000		0.0000	0.0000	-0.1340%	1.0517
22/5/2020	0.0000	0.0000		0.0000	0.0000	0.3766%	1.0556
29/5/2020	0.0000	0.0000		0.0000	0.0000	0.4667%	1.0606

Table 6[19].

From these dynamic tests, after the solution of the MVP model in GAMS for every week for the last 2 years, we received the optimal weights, the percentages of the total available capital of an investor, that he should invest in each bond index. As we already mentioned in 5.2.2. we repeated the process 105 times in total for each case in order to find, at the end, the real returns of the portfolios. The real returns are the result of the multiplication of the optimal weights with the returns-historical returns of the bonds. After this procedure, we calculate the cumulative returns and then we are able to present the Backtesting Results in the form of a diagram (Diagram 8).

[19] From 27/3/2020 to 29/5/2020, we have only the optimal weight S5=1.000. S1= Solactive Green Bond Index, S2= S&P Green Bond Index, S3= S&P Green Bond Select USD Total Return Index S4= Bloomberg Barclays MSCI Global Green Bond Index, S5= ICE BofA Green Bond Index.

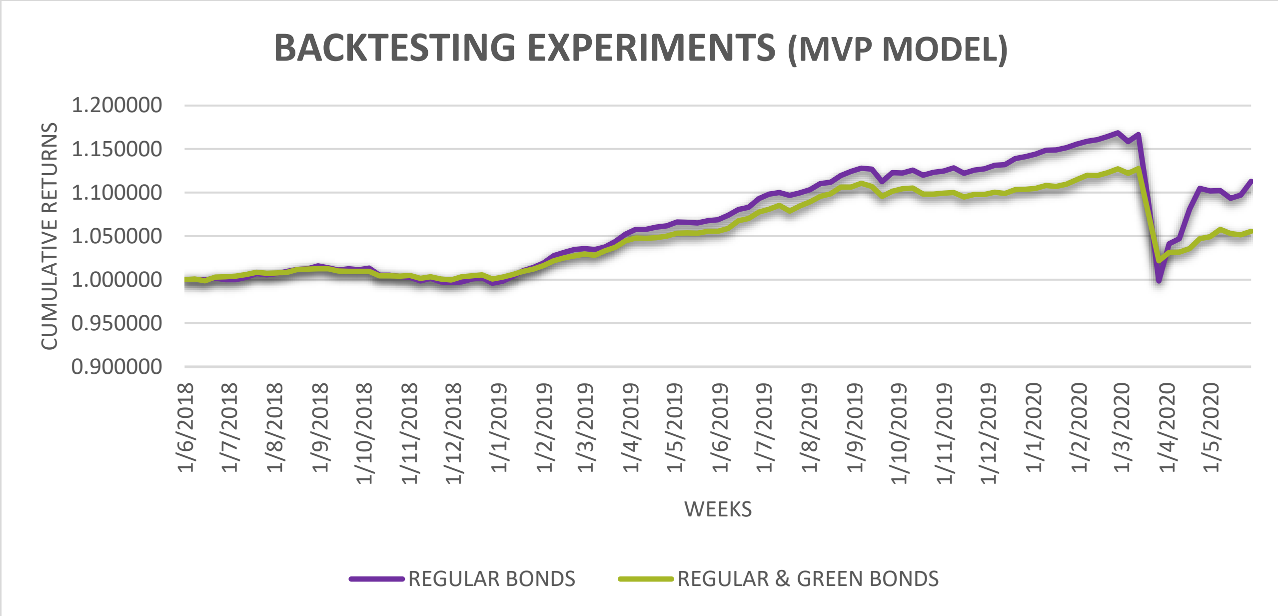

Diagram 8

As shown in the Diagram 8, we have plotted both the movement of the cumulative returns of the optimal portfolio with regular bonds (purple line) and the movement of the real returns of the optimal portfolio with green bonds (green line). At first glance, it is not easy to answer the question "whether the performance of one portfolio is better than the performance of the other". Nevertheless, we observe in both relatively large fluctuations of their cumulative returns, mainly the last weeks and in the case of regular bonds these fluctuations are little bit bigger. More specifically, on 13/03/2020 the cumulative return of the portfolio with RB is 1.0871 and on 27/03/2020, 2 weeks after, this return decreased 4.23%, while the same weeks the cumulative return of the portfolio with GB was 1.0749 on 13/03/2020 and it decreased 4.06%. As we can see, at the last weeks of our analysis the cumulative returns regain an upward course.

One could say that these fluctuations are the result of the covid pandemic in the financial sector. Unfortunately, especially at the beginning of the pandemic, there was an uncertainty that greatly affected the decisions of investors. Although, it is worth mentioning that over time, S&P 500 Composite Index, S&P IG Corporate Bond Index and S&P US HY Corporate Bond Index at the case with the RB and ICE BofA Green Bond Index and S&P US HY Corporate Bond Index at the case with the GRB are the ones we find most often in efficient portfolios compared to the rest, according to the MVP model.

So, in order to answer the important question "whether the performance of one portfolio is better than the performance of the other" we have to find the Sharpe Ratio of both cases and to compare them. The Sharpe Ratio is a commonly used measure of portfolio performance and in general it helps the investor to understand the returns of his investment taking into consideration its risk. It actually describes the returns of holding a risky asset in contrast of holding an asset with zero risk. The general type is the following (William F. Sharpe, 1994):

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

where:

- R_p= the return of a portfolio
- R_f= a risk-free rate[20]
- σ_p= standard deviation of the portfolio

In our analysis, we took as a risk-free rate the "US T-Bill Sec Market 3 Month (D) – Middle Rate", R_f= 0.0341% and we calculated the 2 Sharpe Rations. For the Minimum Variance Portfolio with Regular Bonds the Sharpe Ratio is equal to 0.0659 and the Sharpe Ratio of the Minimum Variance Portfolio with Green and Regular Bonds is equal to 0.0327. In general, a great Sharpe Ratio means an attractive and a good portfolio. So, in our analysis according to the Sharpe Ratio the performance of the MV Portfolio with the RB is little better that the MV Portfolio with the GRB, even if their difference is not that big.

In the last part, we conduct two more Backtesting Experiments, one with the RB and one with the GRB, in order to compare the decisions of the risk averse- risk lovers and risk neutral investors. For these dynamic tests, as already mentioned, we are solving 3 different models. The first model is about the risk averse investor (we maximize CVaR), the second model is about the risk lover investor (we maximize total returns) and the last model is about the risk neutral investor (we maximize CVaR and take into consideration the target returns). After the solution of these optimization problems with GAMS, we take again the optimal weights. These optimal weights concern the percentage of a total capital that an investor wish to allocate to each bond, based on his risk aversion. A risk lover investor wishes to allocate his capital to efficient portfolios that are close to the left edge of the efficient frontier, to the portfolio with the highest expected return in combination to a high risk. Exactly the opposite is desired by a risk averse investor, while a risk neutral is indifferent to risk levels.

We conduct the whole procedure 105 times (210 in total) for the case with RB and for the case with GRB for the last 2 years. In order to get the cumulative returns and compare the investors with the usage of a diagram, we multiply the optimal weights with the historical returns.

[20] Risk-free Rate: The return of an investment with zero risk.

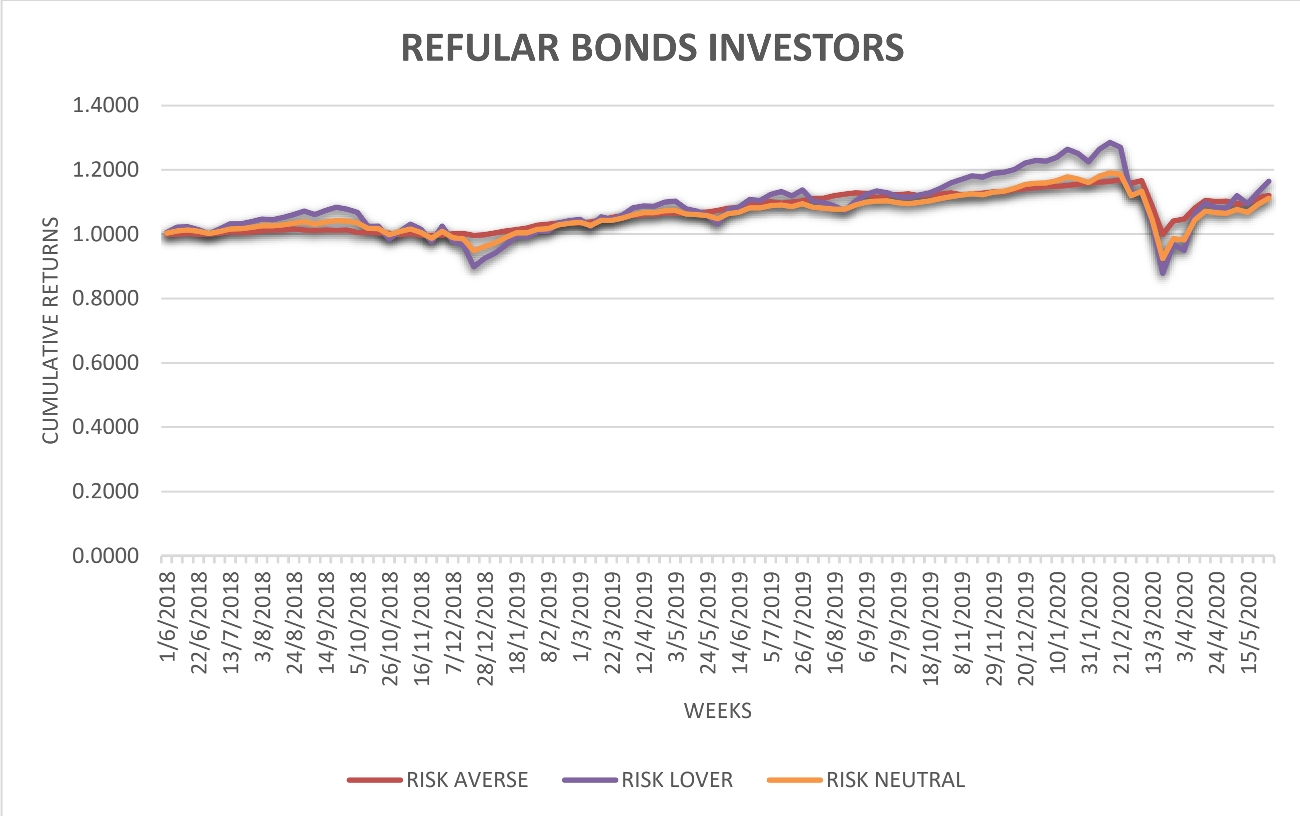

Diagram 9

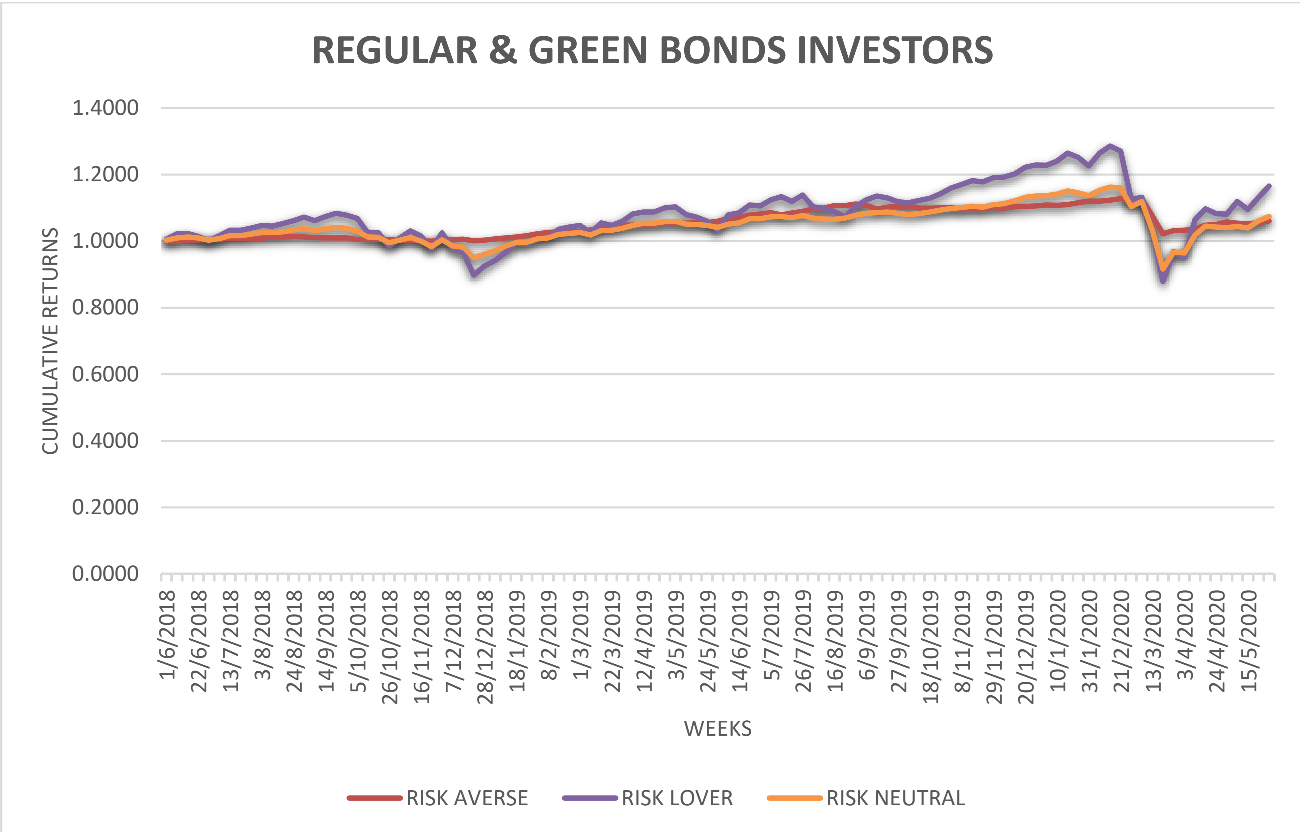

Diagram 10

At the Diagrams 9 and 10, we present the cumulative returns of the investors for the case with the regular bonds and for the case with the green and regular bonds respectively. It is obvious, that there are some fluctuations and especially these are more obvious at the last weeks of our analysis. In the case of the GRB we have smaller fluctuations over time in comparison to the case of the RB, if we exclude the risk neutral investors. More specifically, on 06/03/2020 the cumulative return of a risk averse investor’s portfolio of GRB is 1.1276 and decreased after two weeks, on 20/3/2020, 9.37%, while the cumulative return of a risk averse investor’s portfolio of RB decreased 14,4% the same time. The cumulative return of a risk lover’s portfolio of RB and of GRB is 1.1316 on 06/03/2020 and decreased 22.38% and 22.37% respectively. Last, but not least, the cumulative return of a risk neutral investor’s portfolio of GRB is 1.1185 on 06/03/2020 and decreased 18.12% on 20/03/2020 in contrast to the cumulative return of a risk neutral investor’s portfolio of RB, which decreased just 7.96%.

Based on our data and on these Diagrams, reasonably conclude that the risk lover investor has a higher volatility compared to the others. The risk averse investor, who takes safe decisions in some words, does not have a lot of fluctuations. Of course, there will always be moments of uncertainty and times when historical data will not be very helpful, but most of the times he will probably not face many shocks. The risk neutral investor’s line follows the course of the two lines in general, starting with a stability but also showing large fluctuations in the last weeks of our analysis.

Where do these investors choose to invest their available capital? They prefer to allocate their initial budget, basically, in the following indices, which are the ones that we find most often in the efficient portfolios of their choice:

- Risk Averse investors: they prefer to allocate their initial budget, basically, in the bonds of S&P IG Corporate Bond Index and of S&P US HY Corporate Bond Index (RB) and in the bonds of ICE BofA Green Bond Index and of S&P US HY Corporate Bond Index (GRB).
- Risk Lover investors: they prefer to allocate their initial budget in the bonds of S&P 500 Composite Index in both cases (RB and GRB).
- Risk Neutral investors: they prefer to allocate their initial budget in the bonds of S&P 500 Composite Index and of S&P US HY Corporate Bond Index in both cases (RB and GRB).

So, it is obvious that risk averse investors choose to diversify their portfolio by investing not only in regular, but in green bonds too. Despite the benefits we described in previous chapters of going green and protecting and improving the environment, we see that investors are still opting for regular bonds. Below we present Diagram 11 with the cumulative returns of all investors and in both cases with regular and green and regular bonds.

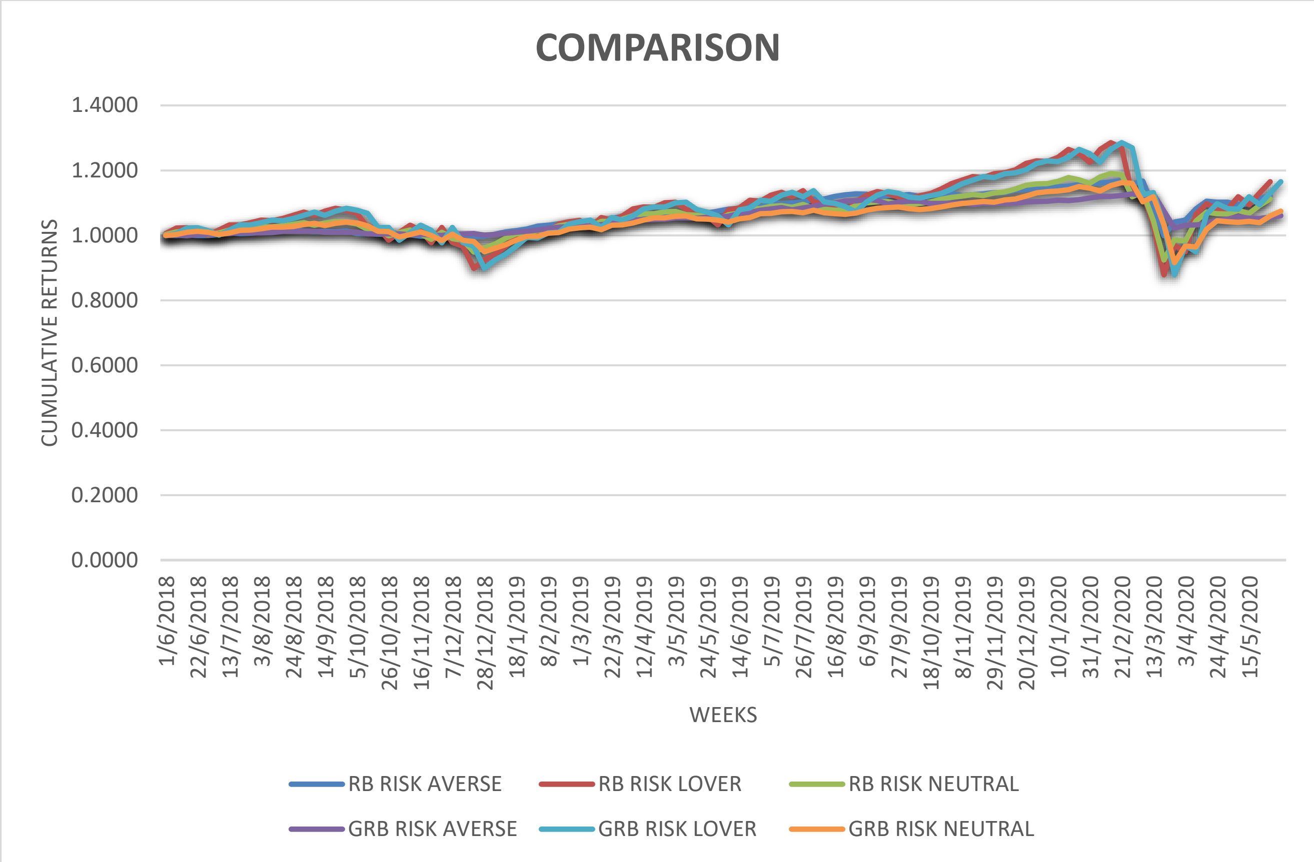

Diagram 11

Therefore, in our analysis of effective frontiers it is clear that many of the efficient portfolios and mainly the MVP which contains green bonds is better than this with regular bonds. And this, because the first ones have a much smaller risk. Also, based on the first 2 Backtesting experiments, we see that portfolios with green bonds have smaller fluctuations compared to the others in our analysis. On the other hand, according to the Sharpe Ratio the performance of the portfolio with RB is a little better than the performance of the portfolio with GRB and therefore investors will choose it easier. Although our literature shows us huge increases in bond issuance and increasing investor interest in them (in GB). In fact, in the case of investors, it is observed that although they choose green bonds in their portfolios, the regular ones are predominant, with the sole exception of the portfolios of risk averse investors.

Chapter 6. Conclusion

At the first, theoretical part of this thesis, we presented some important and interesting definitions. “What is Green Finance?”, “What are Green Bonds?”, “Who are the main issuers of Green Bonds?”. All these were answered in Chapters 2 and 3. Our planet is facing day by day the climate change, the air and water pollution, the greenhouse effect and other serious issues. Large and small businesses, states, workers and non-workers, investors etc. begin to raise awareness of these environmental challenges, but of course not all of them. As we mentioned, Green Finance and the issuing of Green Bonds is booming the last years. Investors are faced with the dilemma of investing in regular bonds or in a combination of green and regular bonds. In these cases, most of them are taking into consideration more their own profit and in order to choose their optimal portfolio they have to be able to measure the risk of their investment. In chapter 4, we made an analysis of the different risk measures and, also, of their mathematical formulations.

At the empirical part of our thesis, we are able to answer the question “Which portfolio is best for an investor to choose? Which portfolio’s performance is better? The portfolios with the regular bonds or the portfolios with the green and regular bonds?”. With the help of the Efficient Frontiers, Backtesting Experiments and of the Sharpe Ration the answer was that an investor will may choose to invest to a portfolio with regular bonds. The difference of the Sharpe Ratios was not that great and in our analysis of the efficient frontiers we see that the minimum variance portfolio with the GRB is much better. Apart from this, the last Backtesting experiments showed that only the risk averse investors choose, basically, green bonds for their optimal portfolio.

To conclude, I have to mention that it is very essential to protect and prevent our environment from future disasters. All of us should try to contribute as much as possible for a less damaged planet, for a better environment and living conditions. Investments should focus more on green projects and states should examine the situation more carefully. In the beginning this will not be easy and more specifically for the investors who largely determine the market. Investors should choose to go Green, even if they will not see big profits quickly, but in the course of time they will benefit. They will have contributed in their own way to the protection of the environment, to the protection of their future, as well and when these projects become more profitable, their profits will also increase. So, in short, all the factors that determine the bond market, as well as those that burden the environment should consider whether they want to remain selfish or to contribute for a better tomorrow.

7. References

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