Unemployment Persistence, Inflation and Monetary Policy, in a Dynamic Stochastic Model of the "Natural Rate".

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Abstract

This paper analyzes monetary policy in the context of a small dynamic stochastic general equilibrium model of the "natural rate" characterized by endogenous unemployment persistence. The model is based on one period nominal wage contracts and wage setting by labor market "insiders". Product and financial markets are assumed perfectly competitive. The persistence of unemployment results in persistent inflation as, under the Taylor rule, the central bank responds systematically to persistent deviations of unemployment from its natural rate. Inflation persists, even though prices are fully flexible. The analysis provides an equilibrium interpretation of the persistence or real and nominal aggregates in the United States.

Keywords: unemployment persistence, inflation, monetary policy
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One of the main characteristics of business cycles is the relatively high degree of persistence of fluctuations of both real and nominal variables around their steady state values. Modern dynamic macroeconomic theory aims to explain business cycles in terms of relatively simple dynamic general equilibrium models subjected to exogenous stochastic shocks.

Persistence is explained either in terms of the persistence of nominal and real shocks, or in terms of propagation mechanisms such as consumption and investment dynamics, or, in imperfectly competitive models, the dynamics of wage and price setting.

Models of the "new" neoclassical synthesis that has emerged since the 1980s differ according to the distortions they introduce relative to the basic competitive dynamic stochastic model, the nature of the shocks assumed, and the propagation mechanisms that they imply.[1]

This paper analyzes the persistence of real and nominal aggregates in a dynamic stochastic general equilbrium model of the "natural" rate, characterized by endogenous unemployment persistence. Unemployment persists because of distortions that arise primarily in the labor market.

It is shown that the degree of unemployment persistence is translated into persistence of deviations of all other real variables from their "natural" rates. Thus, deviations of all real variables around their "natural" rates display the same degree of persistence as unemployment. It is also shown that in the presence of a contingent monetary policy rule, such as the Taylor rule, inflation and nominal interest rates are also characterized by the same degree of persistence as unemployment and other real variables. This is because monetary policy responds to both deviations of inflation from the target of the central bank and persistent deviations of unemployment from its "natural" rate.

These predictions of the model are consistent with the evidence for aggregate fluctuations in the United States, which we present in the first section of the paper. This evidence suggests that one cannot reject the hypothesis that real variables such as the unemployment rate and real output, and nominal variables, such as inflation and nominal interest rates, display the same degree of persistence. Thus, our model offers an equilibrium interpretation

[1] The term "new" neoclassical synthesis has been coined by Goodfriend and King [1997] to describe approaches that rely on dynamic stochastic general equilibrium (DSGE) models based on explicit dynamic microeconomic foundations. It encompasses both "new" classical, or, real business cycle models, based on Kydland and Prescott [1982] and "new" keynesian models such as those collected in Mankiw and Romer [1991].

of these, relatively neglected, stylized facts.

The main distinguishing characteristic of the model put forward in this paper is a dynamic "insider outsider" version of the "Phillips Curve", which accounts for endogenous persistence of unemployment following nominal and real shocks. This model of the "Phillips curve" differs from the typical "new keynesian" model of aggregate fluctuations, in that the propagation mechanism of shocks is not the staggered setting of prices and wages, but the gradual adjustment of employment due to the behavior of "insiders" in the labor market. The model thus provides for richer endogenous dynamics compared to models of staggered wage and price setting, which have been shown to underestimate the degree of persistence of nominal and real variables.[2]

The model combines and extends two strands of the literature.

First, the Gray-Fischer-Taylor model of predetermined nominal wages, according to which nominal wages are set periodically and remain fixed between periods. Because shocks to inflation are not known when nominal wages are set, unanticipated inflation reduces real wages and causes employment to increase along a downward sloping labor demand curve.[3]

The second strand of the literature we embody in our model is the insider-outsider theory of wage determination of Lindbeck and Snower [1986], Blanchard and Summers [1986] and Gottfries [1992]. According to this approach, there is an asymmetry in the wage setting process between "insiders", who already have jobs, and "outsiders" who are seeking employment. "Outsiders" are disenfranchised from the labor market, and wages are set by "insiders", who seek to maximize the expected real wage consistent with their own employment.

The dynamic "Phillips Curve" that we derive provides an alternative formulation to the "new keynesian Phillips Curve", and this model provides an alternative source of unemployment persistence, compared to the "new keynesian" models which are based on imperfect competition and staggered price and wage contracts.

The propagation mechanism that causes unanticipated nominal and real shocks to produce persistent deviations of unemployment from its "natural" rate is the gradual adjustment of employment to shocks, due to the market power of labor market insiders, and not the staggered adjustment of wages

[2]Insert footnote on the evidence here.

[3]In the Gray [1976] and Fischer [1977b] model, the one we utilize, nominal wages are fixed at the beginning of each period, whereas Fischer [1977a] and Taylor [1979] present models in which they are fixed in the beginning of alternate periods in a staggered fashion.

and prices. Nominal wages are fixed for only one period in our model, and are renegotiated every period, while prices are assumed fully flexible. Hence, because of the renegotiation of wages in every period, nominal wage stickiness would not be able to account for the persistent effects of nominal shocks in the absence of the gradual adjustment of employment in this model.

The distortions that matter for the fluctuations of unemployment and other real variables around their "natural" rates in this model are distortions in the labor market. They arise because of one period nominal wage contracts, and the market power of "insiders" in the wage determination process. The product market is assumed competitive, although it would be straightforward to introduce product market imperfections as well.[4]

On the demand side we assume that aggregate consumption and money demand are determined by a representative household, which maximizes its intertemporal utility, and which can borrow and lend freely in a competitive financial market, at the market interest rate. Money enters the utility function of the representative household, and the demand for real money balances is proportional to consumption, and inversely related to the nominal interest rate. The Euler equation for consumption determines the evolution of private consumption and aggregate demand. The preferences of the representative household for consumption and real money balances are subject to persistent stochastic shocks, which shift both the Euler equation for consumption and the demand for money function.

Product market equilibrium is achieved through adjustments of the real interest rate, which is the relative price which adjusts in order to equate aggregate demand with aggregate supply. Thus, the equilibrium real interest rate depends on both demand and supply shocks.

The demand for real money balances turns out to be proportional to real output and inversely related to the nominal interest rate. If the central bank follows an interest rate rule, as we assume in this paper, the money supply adjusts endogenously to equilibrate the money market. If the central bank follows a money supply rule, nominal interest rates would be determined endogenously by the equilibrium condition in the money market.

We solve the model under the assumption that the central bank follows a feedback Taylor [1993] rule, adjusting nominal interest rates in response

[4]In fact, such labor market distortions were the main focus of Keynes [1936]. Alogoskoufis and Giannoulakis [2017], contains an analysis of an extension of this model, with the addition of additional distortions, such as imperfect competition in the product market and staggered pricing. The results are of a similar nature.

to changes in the "natural" real interest rate, deviations of inflation from a fixed inflation target, and deviations of unemployment from its "natural" rate. In addition we assume that the interest rate rule is subject to a white noise monetary policy shock. This monetary policy shock captures potential errors in the implementation of monetary policy.[5]

We demonstrate that under such a Taylor rule, the only shocks that are not completely neutralized by monetary policy are productivity shocks and shocks to monetary policy. Fluctuations of deviations of unemployment and output from their "natural" rates display persistence and are driven by these two types of disturbances.

Since productivity shocks are supply shocks, their real effects can only partially be offset through unanticipated inflation, as they imply a tradeoff between deviations of inflation from target, and unemployment from its "natural" rate. This is not the case for aggregate demand shocks, which, with the exception of monetary policy errors, can be fully neutralized by monetary policy through appropriate changes in the nominal interest rate. It is for this reason that the only shocks which cause fluctuations in deviations of unemployment, output and other real variables from their "natural" rates are productivity and monetary policy shocks.

[5]The analysis of monetary policy usually focuses on policy rules that seek to stabilize inflation around a low inflation target and unemployment around its "natural" rate, even if the "natural" rate itself is inefficiently high. As demonstrated by Kydland and Prescott [1977] and Barro and Gordon [1983], if the central bank seeks to use monetary policy to reduce unemployment below its "natural" rate, the outcome is an upwards bias in equilibrium inflation, as the inflationary expectations of labor market participants rise to ensure that the central bank has no incentive to systematically try to raise inflation above inflationary expectations. Delegating monetary policy to an independent central banker who does not seek to reduce unemployment below its "natural" rate, as first suggested by Rogoff [1985], can address this inflation bias problem, and still allow central banks to seek to stabilize deviations of unemployment from its "natural" rate in response to unanticipated shocks. The most widely discussed and analyzed monetary policy rule since the mid-1990s is the Taylor [1993] rule. The Taylor rule, which is a generalization of the celebrated Wicksell [1898] rule, has been shown to be a fairly close description of the monetary policy rule followed by the Federal Reserve Board and central banks in the other main industrial economies. It has also been extensively adopted and analyzed in the context of "new keynesian" business cycle models based on staggered pricing. See Taylor [1999] for how the Taylor rule describes the monetary policy of the Federal Reserve Board. See also Clarida et al. [1999] for the properties of the Taylor rule in "new keynesian" models with staggered price setting. More recent analyses and surveys can be found in, among others, Gali [2008], Gali [2011a], ,Gali [2011b], Taylor and Williams [2011] and Woodford [2003], Woodford [2011].

Because of the endogenous persistence of deviations of unemployment from its "natural" rate, under a Taylor rule, the equilibrium inflation rate also displays the same degree of persistence around the target of the monetary authorities as unemployment. The persistence of inflation arises from the fact that the central bank adjusts nominal interest rate in response to deviations of unemployment from its "natural" rate. This is anticipated by wage setters, who condition their inflationary expectations on past deviations of unemployment from its "natural" rate, and therefore neutralize the attempts of the monetary authorities to smooth these deviations. Thus, under the Taylor policy rule, the persistence in the fluctuations of the inflation rate does not affect unemployment. It is only the unanticipated part of monetary policy and the inflation rate that can affect unemployment in this model of the "natural" rate.

One could interpret the persistence of inflation under a Taylor rule as arising from the lack of central bank anti-inflationary credibility. If the central bank seeks to use monetary policy in order to smooth deviations of unemployment from its "natural" rate, and there is endogenous persistence in these deviations due to the behavior of wage setters, there will be persistence in inflationary expectations and actual inflation as well. Thus, the persistence of inflation in the presence of endogenous unemployment persistence arises for the same reasons as the inflationary bias in the Kydland and Prescott [1977] and Barro and Gordon [1983] models, when the central bank systematically seeks to reduce unemployment below its "natural" rate. Since the employment objectives of wage setters and the central bank differ under the Taylor rule, the only way for wage setters to ensure that the central bank will follow the expected policy, is to raise their expectations of inflation to the level which will ensure that the central bank has no further incentive to deviate from the expected policy. It is exactly this mechanism, which is responsible for the persistence of inflation when there is endogenous unemployment persistence, as in this model. Under a Taylor rule, there is persistence of deviations of inflation from the central bank target, without any impact on the persistence of deviations of unemployment from its "natural" rate.

The lack of credibility that results in inflation persistence can be addressed in one of two ways.

One would be to modify the monetary policy rule in order to make inflation the sole objective of monetary policy. This solution, which amounts to abandoning the Taylor rule in favour of a Fisher [1919] rule of complete stabilization of inflation, would result in non persistent inflation, as expected inflation will always be equal to the central bank target. However, although this response is optimal in the presence of monetary policy shocks, this will forego the stabilizing role of monetary policy following productivity shocks, and would result in a suboptimally high variance of the persistent deviations of unemployment from its "natural" rate.

The second type of solution, would be to maintain the Taylor rule, but modify it in order to make nominal interest rates respond more to deviations of inflation from target, relative to deviations of unemployment from its "natural" rate. This policy, by appropriate choice of parameters, could result in an optimal tradeoff between the variance of inflation around the target of the central bank and the variance of unemployment around its "natural" rate, in the presence of both monetary and real shocks.

The rest of the paper is as follows: In section 1 we present evidence from the United States with regard to the degree of persistence of real variables, such as the unemployment rate and real output, and nominal variables, such as inflation and nominal interest rates. All variables display positive persistence of deviations from their "natural" rates, and one cannot reject the hypothesis that they display a common degree of persistence. In section 2 we present our basic dynamic model of the "Phillips curve", based on the distinction between "insiders" and "outsiders" in the labor market. In section 3 we derive the evolution of aggregate consumption and money demand, from the behavior of a representative household with access to a competitive financial market. In section 4 we analyze how the real interest rate adjusts to bring about equilibrium between aggregate demand and aggregate supply in the product market. In section 5 we solve the model under the assumption that the central bank follows a Taylor [1993] rule, and derive our main new result, linking the persistence of unemployment to that of inflation. In section 6 we show that the persistence of inflation is the same as the persistence of unemployment even under an "optimal" monetary policy rule. We also discuss the optimal derivation of the parameters of the Taylor rule in the presence of unemployment persistence, and demonstrate that, if the central bank cares sufficiently about inflation, unemployment persistence calls for a higher optimal response to deviations of inflation from target relative to deviations of unemployment from its "natural" rate. The last section sums up our conclusions.

1 The Persistence of Aggregate Fluctuations in the United States

One of the main characteristics of business cycles is the relatively high degree of persistence of fluctuations of both real and nominal variables around their steady state, or "natural rate" values. In this section we document the main characteristics of US business cycles with regard to the degree of persistence of such fluctuations for both real variables, such as the unemployment rate and real output, and nominal variables, such as inflation and nominal interest rates.

Single equation estimates of autoregressive (AR) processes for unemployment, u, the log of real GDP y, the inflation rate π and short term nominal interest rates i are presented in Table 1. The variables are defined as deviations from their steady state values or "natural" rates. The steady state or "natural" rates of unemployment, output, and the nominal interest are approximated by Hodrik and Prescott [1997] filters. Steady state inflation is approximated by a constant.

A number of interesting conclusions can be derived from the estimates in Table 1.

First, all variables appear to be 2nd order autoregressive processes. In order to summarize the degree of persistence of these variables, we also report the sum of the estimated parameters on the two lags of the dependent variables, with their appropriate standard errors. This is the parameter called "persistence" in Table 1.

Second, neither deviations of unemployment, output and the nominal interest rate from their Hodrik Prescott "natural" rate, nor the inflation rate appear to be characterized by a unit root. The relevant Augmented Dickey Fuller (ADF) statistics do not indicate the presence of a unit root at conventional levels of significance for any of the variables. Thus, one cannot reject the hypothesis that these variables are stationary.

Third, the degree of persistence, i.e the sum of the two autoregressive parameters, is positive and statistically significant for all variables and subperiods. In all cases, the degree of persistence is statistically significant even at the 1% significance level. Thus, on the basis of these estimates, there does not seem to be evidence of significant differences in the degree of persistence of deviations of real and nominal variables. Unemployment, output, inflation and nominal interest rates appear to be characterized by a degree of persistence of a similar order of magnitude. We shall return to this point below.

In order to investigate the dynamic interconnections between these variables, we present In Table 2 both unrestricted and restricted vector autoregressions (VARs) of the same four variables. The restricted VARs imply that none of the variables Granger causes any of the others, as all variables only depend on their own past values, and not the past values of the other variables. The estimates of the unrestricted VAR are in the (a) columns, and the estimates of the restricted VAR the (b) columns.

The $\chi^2(6)$ Granger causality statistics suggest that none of the four variables in the VAR is Granger caused by the other three variables. The critical value of $\chi^2(6)$ at 5% is equal to 12.592, and at 1% equal to 16.812. The statistics reported in the final row of Table 2 are all below these critical values. In addition, the Wald test for the joint hypothesis that none of the four variables in the unrestricted VAR is Granger caused by the other three variables gives a $\chi^2(24)$ statistic of 32.309, with critical values of 36.415 at 5% and 42.980 at 1%. Hence, a restricted VAR, as in the (b) columns, in which each variable is Granger caused only by its own lagged values, cannot be rejected by this evidence, and appears to be an adequate statistical representation of the data.

Our final question is whether the degree of persistence differs between the four variables. From the estimates of the restricted VAR, we can test the hypothesis that the sum of the coefficients of the lagged variables is the same for all four variables in the restricted VAR, as well as the hypothesis that each of the two coefficients of the lagged variables is the same for each equation. The Wald statistic for the hypothesis that the sum of the coefficients of the lagged variables, i.e the degree of persistence, is the same in all equations in the restricted VAR is equal to 6.801. This is asumptotically distributed as $\chi^2(3)$, with critical values equal to 7.815 at the 5% level, and 11.345 at the 1% level. Thus, the hypothesis that all variables, real and nominal, display the same degree of persistence cannot be rejected at conventional significance levels. A more powerful hypothesis, that both coefficients on the two lagged variables are the same across equations, cannot be rejected either. The relevant Wald statistic, which is asymptotically distributed as $\chi^2(6)$ is equal to 9.883. The critical values are equal to 12.592 at the 5% level, and 16.812 at the 1% level. Thus, the hypothesis that all variables, real and nominal, have the same lag structure, cannot be rejected either.

When the restricted VAR is estimated under the additional restriction that all equations have the same lag structure and, therefore, the same degree of persistence, the coefficient on the first lag is estimated at 0.822, with an asymptotic standard error of 0.042, and the coefficient on the second lag is estimated at -0.344, with an asymptotic standard error of 0.042. The degree of persistence, which is the sum of the two, is estimated at 0.479, with an asymptotic standard error of 0.037.

To summarize, the evidence presented for the US economy from 1892 to 2014 suggests that real variables such as deviations of unemployment and output from their "natural" rates, and nominal variables, such as inflation and deviations of nominal interest rates from their "natural" rate, follow stationary univariate stochastic processes and are not Granger caused by variables other than their own lagged values. Furthermore, these stochastic processes seem to be characterized by identical coefficients, which result in the same degree of persistence for all variables. This degree of persistence is estimated at 0.479, or about 50%.

In what follows, we suggest a dynamic stochastic general equilibrium model of the "natural" rate which can account for these empirical facts.

2 "Insiders vs Outsiders" in a Dynamic Model of Wage Setting

Consider an economy in which output is produced by a continuum of competitive firms, indexed by i, where $i \in [0, 1]$.

The production function of firm i is given by,

$$Y(i)_t = A_t L(i)_t^{1-\alpha} \tag{1}$$

where $Y(i)$ is output of firm i, A is exogenous productivity, and $L(i)$ is employment by firm i. t is a discrete time index, where $t = 0, 1, ...$

Employment is determined by firms, who maximize profits, by equating the marginal product of labor to the real wage. Thus, employment is determined by the marginal productivity condition,

$$(1-\alpha) A_t L(i)_t^{-\alpha} = \frac{W(i)_t}{P_t} \tag{2}$$

where $W(i)$ is the nominal wage of firm i, and P is the price for the product of firm i. Since the product market is assumed to be competitive, all firms face the same price, and $P(i) = P$ for all firms.

In log-linear form, (1) and (2) can be written as,

$$y(i)_t = a_t + (1-\alpha)l(i)_t \tag{3}$$

$$l(i)_t = \bar{l} - \frac{1}{\alpha}(w(i)_t - p_t - a_t) \tag{4}$$

where $\bar{l} = \frac{\ln(1-\alpha)}{\alpha}$.

Lowercase letters denote the logarithms of the corresponding uppercase variables. (3) determines output as a positive function of employment, and (4) determines employment as a negative function of the deviation of real wages from productivity.

2.1 Wage Setting and Employment

Nominal wages are set by "insiders" in each firm at the beginning of each period, before variables, such as current productivity and the current price level are known. Nominal wages remain constant for one period, and they are renegotiated at the beginning of the following period. Thus, this model is characterized by nominal wage stickiness of the Gray [1976], Fischer [1977b] variety. Employment is determined ex post by the firm, given the contract wage, the price level and productivity.

Following Blanchard and Summers [1986], we assume that the number of "insiders", who at the beginning of each period determine the contract wage, consists of an exogenous number of "core insiders", and those employed by the firm in the previous period. Their key objective is to set the maximum nominal wage which, given their rational expectations about the price level and productivity, will minimize deviations of expected employment from the target number of "insiders". This target is a weighted average of all those who were employed in period $t-1$, and the exogenous number of "core" employees of each firm. Thus, this model is characterized by a state dependent pool of insiders, as in Blanchard and Summers [1986]. The employment target in period t is determined by,

$$\bar{n}(i)_t = \delta l(i)_{t-1} + (1-\delta)\bar{n}(i) \tag{5}$$

where $l(i)_{t-1}$ is the number of those who were actually employed in the previous period, and $\bar{n}(i)$ is the logarithm of the number of "core" employees of firm i, assumed exogenous. δ is the weight of those recently employed relative to "core" employees, in the employment target of insiders. This formulation is the one proposed by Blanchard and Summers [1986].

The expectations on the basis of which wages are set depend on information available until the end of period $t-1$, but not on information about prices and productivity in period t. On the basis of the above, we assume that the objective of wage setters is to choose the path of the maximum wages that would minimize deviations of the expected employment path, from the expected path of the employment target of current "insiders".

This can be modeled as a maximin problem. Insiders are assumed to choose the expected employment path that minimizes deviations from their target, and select the maximum wage path that satisfies their optimal employment path subject to the labor demand curve. Thus, the problem can be formalized as choosing the path of current and expected future wages which minimizes the following quadratic intertemporal loss function,

$$\min E_{t-1} \sum\nolimits_{s=0}^{\infty} \beta^s \frac{1}{2} \left(l(i)_{t+s} - \bar{n}(i)_{t+s} \right)^2 \tag{6}$$

subject to the sequence of labor demand equations (4) and employment targets $\bar{n}(i)_t$, as defined in (5). $\beta = 1/(1+\rho) < 1$ is the discount factor, with ρ being the pure rate of time preference. As can be seen from (6), "outsiders", i.e the unemployed, have no influence on the wage setting process.

We shall assume that the total number of "core" employees in the economy is always strictly smaller than the labor force. This assumption ensures that the "natural rate" of unemployment is strictly positive. We thus assume that,

$$\int_{i=0}^{1} \bar{n}(i) di = \bar{n} < n \tag{7}$$

where n is the log of the labor force.

From the first order conditions for a minimum of (6), wages are set at the maximum level which ensures that expected employment by each firm satisfies,

$$E_{t-1} l(i)_t = \frac{\beta\delta}{1+\beta\delta^2} E_{t-1} l(i)_{t+1} + \frac{\delta}{1+\beta\delta^2} l(i)_{t-1} + \frac{(1-\beta\delta)(1-\delta)}{1+\beta\delta^2} \bar{n}(i) \tag{8}$$

The implied contract wage can be derived by using the labor demand (marginal productivity) condition (4) to substitute for employment in (8).

Integrating over i, *expected aggregate employment* must then satisfy,

$$E_{t-1}l_t = \frac{\beta\delta}{1+\beta\delta^2}E_{t-1}l_{t+1} + \frac{\delta}{1+\beta\delta^2}l_{t-1} + \frac{(1-\beta\delta)(1-\delta)}{1+\beta\delta^2}\bar{n} \tag{9}$$

(9) is the same as (8) without the i index.

Wage contracts that satisfy (9) encompass Gray-Fischer wage contracts and Blanchard-Summers wage contracts as special cases.

With Gray-Fischer contracts, $\delta = 0$, as past employment does not exert any separate influence on the wage setting process. Only "core" employees would matter in Gray-Fischer type contracts. Setting $\delta = 0$ in (6), nominal wages in Gray-Fischer contracts would be set at the maximum level which ensures that,

$$E_{t-1}l_t = \bar{n}$$

On the other hand, with Blanchard-Summers contracts, there is no consideration of the effects of current contracts on expected employment beyond period t. This is equivalent to setting $\beta = 0$ in (9), i.e with myopic behavior. Setting $\beta = 0$ in (9) implies that nominal wages would be set in order to ensure that,

$$E_{t-1}l_t = \delta l_{t-1} + (1-\delta)\bar{n}$$

This is identical to equation (3.2) in Blanchard and Summers [1986]. Nominal wages with Blanchard-Summers contracts would be set at the maximum level which ensures that expected employment equals a weighted average of "core" employees, and those recently employed, without consideration for the effects on future employment.

In our more general dynamic model, wages are set at the maximum level which ensures that expected employment in period t is given by (9), which also depends on expected employment in period $t+1$. This is because expected employment at t will affect the number of insiders who will negotiate wages for period $t+1$. Thus, in our model, labor market "insiders" are forward looking, in that they set nominal wages in order to achieve an employment target which depends on "core" employees, those previously employed, but also on those expected to be employed in the future, as expected future employment will affect the future number of "insiders" and thus future wage setting behavior.

As a result, this dynamic model is more general than the Gray-Fischer model and slightly more general than the Blanchard-Summers model.

2.2 Wage Determination, Unemployment Persistence and the Phillips Curve

Subtracting (9) from the log of the labor force n, after some rearrangement, we get,

$$E_{t-1}u_t = \frac{\beta\delta}{1+\beta\delta^2}E_{t-1}u_{t+1} + \frac{\delta}{1+\beta\delta^2}u_{t-1} + \frac{(1-\beta\delta)(1-\delta)}{1+\beta\delta^2}u^N \tag{10}$$

where, $u_t \simeq n - l_t$ is th current unemployment rate, and $u^N \simeq n - \bar{n} > 0$ is the "natural rate" of unemployment. The "natural rate" of unemployment in this model is defined in terms of the difference between the labor force and the number of core "empoyees". This is the equilibrium rate towards which the economy would converge in the absence of shocks.

To solve (10) for expected unemployment, define the operator F, as,

$$F^s u_t = E_{t-1}u_{t+s} \tag{11}$$

We can then rewrite (10) as,

$$\left((1+\beta\delta^2)F^0 - \beta\delta F - \delta F^{-1}\right)u_t = (1-\beta\delta)(1-\delta)u^N \tag{12}$$

(12) can be rearranged as,

$$-\beta\delta F^{-1}\left(F^2 - \frac{1+\beta\delta^2}{\beta\delta}F + \frac{1}{\beta}\right)u_t = (1-\beta\delta)(1-\delta)u^N \tag{13}$$

It is straightforward to show that if $0 < \beta < 1$ and $0 < \delta < 1$, the characteristic equation of the quadratic in the forward shift operator (in brackets) has two distinct real roots, which lie on either side of unity. The two roots satisfy,

$$\lambda_1 + \lambda_2 = \frac{1+\beta\delta^2}{\beta\delta}, \lambda_1\lambda_2 = \frac{1}{\beta} \tag{14}$$

Using (14) we can rewrite (13), as,

$$(F-\lambda_1)(F-\lambda_2)u_t = -\frac{(1-\beta\delta)(1-\delta)}{\beta\delta}u^N \tag{15}$$

Assuming λ_1 is the smaller root, we can solve (15) as,

$$E_{t-1}u_t = \lambda_1 u_{t-1} + (1-\lambda_1)u^N \tag{16}$$

(16), which is the rational expectations solution of (10), determines the path of expected unemployment implied by the wage setting behavior of "insiders".

It is straightforward to show that λ_1, the coefficient that determines the persistence of expected unemployment, is equal to δ, the relative weight of recent employees in the wage setting process. From (14), which defines the two roots, it follows that since $\lambda_2 = 1/\beta\lambda_1$, it follows that,

$$\lambda_1 + \frac{1}{\beta\lambda_1} = \frac{1+\beta\delta^2}{\beta\delta} = \delta + \frac{1}{\beta\delta} \tag{17}$$

Thus, the degree of persistence of unemployment λ_1 is equal to the weight of recent employees relative to "core" employees in the wage setting process δ, exactly as suggested by Blanchard and Summers [1986].

Actual unemployment, is determined from the employment decisions of firms, after information about prices, productivity and other shocks has been revealed.

Integrating the labour demand function (4) over the number of firms i, aggregate employment is given by,

$$l_t = \bar{l} - \frac{1}{\alpha}(w_t - p_t - a_t) \tag{18}$$

Subtracting the aggregate employment equation (18) from the log of the labor force n, actual unemployment is determined by,

$$u_t = n - \bar{l} + \frac{1}{\alpha}(w_t - p_t - a_t) \tag{19}$$

Taking expectations on the basis of information available at the end of period $t-1$, the wage is set in order to make expected unemployment equal to the expression in (16), which defines the rate of unemployment consistent with the wage setting behavior of "insiders".

Thus, from (19), the wage is thus set in order to satisfy,

$$w_t = E_{t-1}p_t + E_{t-1}a_t + \alpha \left(E_{t-1}u_t - n + \bar{l}\right) \tag{20}$$

where $E_{t-1}u_t$ is determined by (16).

Substituting (20) for the nominal wage in (19), the unemployment rate evolves according to,

$$u_t = E_{t-1}u_t - \frac{1}{\alpha}(p_t - E_{t-1}p_t + a_t - E_{t-1}a_t) \tag{21}$$

Substituting (16) in (21), taking into account that $\lambda_1 = \delta$, gives us the solution for the unemployment rate.

$$u_t = \delta u_{t-1} + (1-\delta)u^N - \frac{1}{\alpha}(p_t - E_{t-1}p_t + a_t - E_{t-1}a_t) \tag{22}$$

From (22), the unemployment rate is equal to the expected unemployment rate, as determined by the behavior of "insiders" in the labor market, and depends negatively on unanticipated shocks to inflation and productivity. Unanticipated shocks to inflation reduce unemployment by a factor which depends on the elasticity of labor demand with respect to the real wage, as unanticipated inflation reduces real wages. Unanticipated shocks to productivity also reduce unemployment, as they reduce the difference between real wages and productivity and increase labor demand.

We can express (22) in terms of inflation, by adding and subtracting the lagged log of the price level in the last parenthesis. Thus, (22) takes the form of a dynamic, expectations augmented "Phillips Curve".

$$u_t = \delta u_{t-1} + (1-\delta)u^N - \frac{1}{\alpha}(p_t - E_{t-1}p_t + a_t - E_{t-1}a_t) \tag{23}$$

From (23), deviations of unemployment from its "natural rate" depend negatively on unanticipated shocks to inflation and productivity, as these cause a discrepancy between real wages and productivity, due to the fact that nominal wages are predetermined. Unanticipated shocks to inflation reduce real wages and induce firms to increase labor demand and employment beyond their "natural rates". Thus, unemployment falls relative to its "natural rate". Unanticipated shocks to productivity, given inflation, cause an increase in productivity relative to real wages, and also cause firms to increase labor demand, employment and output beyond their "natural rates", which reduces unemployment.

It can easily be confirmed from (23) that following a shock to inflation or productivity, unemployment will converge gradually back to its "natural rate", with the speed of adjustment being $(1 - \lambda_1)$ per period. Thus, following shocks to inflation or productivity, deviations of unemployment from its "natural rate" will display persistence.

2.3 The Relation between Output and Unemployment Persistence

The persistence of employment and unemployment, will also be translated into persistent output fluctuations.

Aggregating the firm production functions (3), the aggregate production function can be written as,

$$y_t = a_t + (1 - \alpha) l_t \tag{24}$$

Adding and subtracting $(1 - \alpha)(n - \bar{n})$, the production function can be written as,

$$y_t = y_t^N - (1 - \alpha)(u_t - u^N) \tag{25}$$

where,

$$y_t^N = (1 - \alpha)\bar{n} + a_t \tag{26}$$

is the log of the "natural rate" of output.

(26) is an Okun [1962] type of relation, which suggests that fluctuations of output around its "natural rate" will be negatively related to fluctuations of the unemployment rate around its own "natural rate".

From (25) and (23), deviations of output from its "natural rate" are determined by,

$$y_t - y_t^N = \delta(y_{t-1} - y_{t-1}^N) + \frac{1 - \alpha}{\alpha}(\pi_t - E_{t-1}\pi_t + a_t - E_{t-1}a_t) \tag{27}$$

(27) shows that deviations of output from its "natural" level also display persistence, because of the persistence of employment and unemployment.

(27) is a *dynamic output supply function.* Deviations of output from its "natural rate" depend positively on unanticipated shocks to inflation and

productivity, as these cause a discrepancy between real wages and productivity, due to the fact that nominal wages are predetermined. Unanticipated shocks to inflation reduce real wages and induce firms to increase labor demand, employment and output. Unanticipated shocks to productivity, given inflation, cause an increase in productivity relative to real wages, and also cause firms to increase labor demand, employment and output, beyond their "natural rates". On the other hand, anticipated shocks to productivity increase both output and its "natural rate" by the same proportion.

This concludes the discussion of the labor market and the supply side of the model. We next turn to the determination of aggregate demand.

3 The Determination of Aggregate Consumption and Money Demand

We next turn to the determination of aggregate demand. We assume that the economy consists of a continuum of identical households j, where $j \in [0, 1]$. Each household member wishes to supply one unit of labor inelastically, and unemployment impacts all households in the same manner. The proportion of insiders is assumed to be the same for all households. In addition, the proportion of the unemployed is also assumed to be the same for all households.

The representative household chooses (aggregate) consumption and real money balances in order to maximize,

$$E_t \sum\nolimits_{s=0}^{\infty} \left(\frac{1}{1+\rho} \right)^s \left(\frac{1}{1-\theta} \left(V^C_{t+s} C^{1-\theta}_{t+s} + V^M_{t+s} \left(\frac{M}{P} \right)^{1-\theta}_{t+s} \right) \right) \tag{28}$$

subject to the sequence of expected budget constraints,

$$E_t \left(F_{t+s+1} = (1 + i_{t+s}) \left(F_{t+s} - \frac{i_{t+s}}{1 + i_{t+s}} M_{t+s} + P_{t+s} \left(Y_{t+s} - C_{t+s} - T_{t+s} \right) \right) \right) \tag{29}$$

where $F_t = B_t + M_t$, denotes the financial assets held by the representative household. ρ denotes the pure rate of time preference, θ is the inverse of the elasticity of intertemporal substitution, i the nominal interest rate, B one period nominal bonds, M nominal money balances, Y real non interest income and T real taxes net of transfers. V^C and V^M denote exogenous stochastic shocks in the utility from consumption and real money balances respectively.

From the first order conditions for a maximum,

$$V_t^C C_t^{-\theta} = \lambda_t (1 + i_t) P_t \tag{30}$$

$$V_t^M \left(\frac{M}{P}\right)_t^{-\theta} = \lambda_t i_t P_t \tag{31}$$

$$E_t \lambda_{t+1} = E_t \left(\frac{1+\rho}{1+i_{t+1}}\right) \lambda_t \tag{32}$$

where λ_t is the Lagrange multiplier in period t.

(30)-(32) have the standard interpretations. (30) suggests that at the optimum the household equates the marginal utility of consumption to the value of savings. (31) suggests that the household equates the marginal utility of real money balances to the opportunity cost of money. (32) suggests that at the optimum, the real interest rate, adjusted for the expected increase in the marginal utility of consumption, is equal to the pure rate of time preference.

From (30), (31) and (32), eliminating λ, implies that,

$$\left(\frac{M}{P}\right)_t = C_t \left(\frac{V_t^C}{V_t^M} \frac{i_t}{1+i_t}\right)^{-\frac{1}{\theta}} \tag{33}$$

$$E_t \left(\frac{V_{t+1}^C (C_{t+1})^{-\theta}}{P_{t+1}}\right) = \left(\frac{1+\rho}{1+i_t}\right) \left(\frac{V_t^C (C_t)^{-\theta}}{P_t}\right) \tag{34}$$

(34) is the money demand function, which is proportional to consumption and a negative function of the nominal interest rate, and (35) is the familiar Euler equation for consumption.

Log-linearizing (34) and (35),

$$m_t - p_t = c_t - \frac{1}{\theta} \ln \left(\frac{i_t}{1+i_t}\right) + \frac{1}{\theta} \left(v_t^M - v_t^C\right) \tag{35}$$

$$c_t = E_t c_{t+1} - \frac{1}{\theta} (i_t - E_t \pi_{t+1} - \rho) + \frac{1}{\theta} (v_t^C - E_t v_{t+1}^C) \tag{36}$$

where lowercase letters denote natural logarithms, and $\pi_t = p_t - p_{t-1}$ is the rate of inflation.[6]

We then turn to the determination of equilibrium in the product and money markets.

4 Equilibrium in the Product and Money Markets

Since there is no capital and investment in this model, and no government expenditure, product market equilibrium implies that output is equal to consumption.

$$Y_t = C_t \tag{37}$$

This product market equilibrium condition allows us to substitute output for consumption in the money demand function and the Euler equation for consumption, and derive the "new keynesian" LM and IS curves.

4.1 The "New Keynesian" IS and LM Curves

Substituting (37) in (35) and (36), we get the money and product market equilibrium conditions,

$$m_t - p_t = y_t - \frac{1}{\theta}\ln\left(\frac{i_t}{1+i_t}\right) + \frac{1}{\theta}\left(v_t^M - v_t^C\right) \tag{38}$$

$$y_t = E_t y_{t+1} - \frac{1}{\theta}\left(i_t - E_t\pi_{t+1} - \rho\right) + \frac{1}{\theta}(v_t^C - E_t v_{t+1}^C) \tag{39}$$

(38) is the money market equilibrium condition, the equivalent of the LM Curve in the traditional models of the "neoclassical synthesis", and (39) is the product market equilibrium condition, the equivalent of the IS Curve. (38) and (39) are often referred to as the "new keynesian" LM curve and the "new keynesian" IS curve respectively.

[6] Technically, since the logarithm of the expectation of a product (or ratio) of two random variables is not equal to the sum (or difference) of the expectations of the logarithms of the relevant random variables, (36) must also contain second order terms, depending on the covariance matrix of consumption, inflation and shocks to preferences for consumption and money. Assuming that all exogenous shocks are independent stationary stochastic processes, these second order terms are constant and can be ignored.

4.2 The "Natural" Real Interest Rate and the Current Equilibrium Real Interest Rate

The real interest rate is defined by the Fisher [1896] equation,[7]

$$r_t = i_t - E_t\pi_{t+1} \tag{40}$$

The "natural" real interest rate is determined by the product market equilibrium condition, when output is at its "natural" rate. From (26) and (39), the "natural" real interest rate is given by,

$$r^N{}_t = \rho - \theta\left(a_t - E_t a_{t+1}\right) + \left(v_t^C - E_t v_{t+1}^C\right) \tag{41}$$

The "natural" real interest rate is equal to the pure rate of time preference, but also depends positively on deviations of current shocks to consumption from anticipated future shocks, and negatively on deviations of current productivity shocks from anticipated future shocks. Thus, real shocks, such as productivity shocks, that cause a temporary increase in the "natural" level of output reduce the "natural" real rate of interest, in order to bring about a corresponding increase in consumption and maintain product market equilibrium. On the other hand, real consumption preference shocks that cause a temporary increase in consumption, require an increase in the "natural" real rate of interest, in order to reduce consumption back to the "natural" level of output, and maintain product market equilibrium.

Because of the nominal rigidity of wages for one period, the current equilibrium real interest deviates from its "natural" rate. The current real interest rate is determined by the equation of the output demand function (39) with the output supply function (27). It is thus determined by,

$$r_t = r_t^N - \theta(1-\delta)(y_t - y_t^N)$$

Deviations of the current real interest rate from its "natural" rate depend negatively on deviations of output from its own "natural" rate. Since devia-

[7]To quote from Fisher [1896], "When prices are rising or falling, money is depreciating or appreciating relative to commodities. Our theory would therefore require high or low interest according as prices are rising or falling, provided we assume that the rate of interest in the commodity standard should not vary." (p. 58). The rate of interest in the commodity standard is the real interest rate, and rising or falling prices are expected inflation.The Fisher equation was further elaborated in Fisher [1930], where it was made even clearer that Fisher referred to expected inflation.

tions of output from its "natural" rate tend to persist, deviations of the real interest rate from its "natural" rate will tend to persist as well.

Unanticipated shocks to inflation or productivity, which cause a temporary rise in current output relative to its "natural" rate, will reduce the current real interest rate relative to its "natural" rate. This is the well known "Wicksellian" mechanism, emphasized for the first time by Wicksell [1898].

4.3 Equilibrium Fluctuations with Exogenous Preference and Productivity Shocks

In what follows, we shall assume that the logarithms of the exogenous shocks to preferences and productivity follow stationary $AR(1)$ processes of the form.

$$v_t^C = \eta_C v_{t-1}^C + \varepsilon_t^C \tag{42}$$

$$v_t^M = \eta_M v_{t-1}^M + \varepsilon_t^M \tag{43}$$

$$a_t = \eta_A a_{t-1} + \varepsilon_t^A \tag{44}$$

where the autoregressive parameters satisfy, $0 < \eta_C, \eta_M, \eta_A < 1$, and ε^C, ε^M, ε^A, are white noise processes.

With these assumptions, current employment, unemployment, output (and consumption), real wages and the real interest rate, as functions of the exogenous shocks and shocks to inflation, evolve according to,

$$l_t = \bar{n} + \delta(l_{t-1} - \bar{n}) + \frac{1}{\alpha}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) \tag{45}$$

where, $\bar{n}$, the aggregate number of "core employees", is the "natural rate" of employment.

$$u_t = u^N + \delta(u_{t-1} - u^N) - \frac{1}{\alpha}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) \tag{46}$$

where $u^N = n - \bar{n}$ is the "natural rate" of unemployment.

$$y_t = y_t^N + \delta(y_{t-1} - y_{t-1}^N) + \frac{1-\alpha}{\alpha}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) \tag{47}$$

where $y_t^N = (1-\alpha)\bar{n} + a_t$ is the "natural rate" of output.

$$w_t - p_t = \omega_t^N + \delta\left((w-p)_{t-1} - \omega_{t-1}^N\right) - \left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) \tag{48}$$

where $\omega_t^N = a_t - \alpha(\bar{n} - \bar{l})$ is the "natural rate" of real wages.

$$r_t = r_t^N + \delta(r_{t-1} - r_{t-1}^N) - \frac{\theta(1-\alpha)(1-\delta)}{\alpha}(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A) \tag{49}$$

where $r_t^N = \rho - \theta(1-\eta_A)a_t + (1-\eta_C)v_t^C$, is the "natural" real interest rate. The real interest rate is defined by the Fisher equation (16.32).

The "natural rates" of real variables evolve as functions only of the exogenous real shocks. However, unanticipated inflation, and innovations to productivity, by reducing real wages relative to their "natural rate", cause persistent increases in employment and output above their "natural rates", and persistent reductions in unemployment and the real interest rate, below their "natural rates". The degree of persistence in these deviations is the same for all real variables, and is equal to δ, the weight of recent employees in the wage setting process.

5 Fluctuations of Unemployment and Inflation under a Taylor Rule

Assume that the central bank follows a Taylor rule of the form,

$$i_t = r_t^N + \pi^* + \phi_\pi\left(\pi_t - \pi^*\right) - \phi_u(u_t - u^N) + \varepsilon_t^i \tag{50}$$

where $\phi_\pi, \phi_u > 0$ are policy parameters, and ε_t^i is a white noise monetary policy shock.

According to this rule, the central bank aims for a nominal interest rate which is equal to the "natural" real rate of interest, plus a target inflation rate equal to π^*. If actual inflation is higher than the target π^*, then the central bank raises interest rates in order to reduce inflation towards its target. In addition, if the unemployment rate is higher than its "natural rate", then the central bank reduces nominal interest rates, in order to increase aggregate demand and bring unemployment back to its "natural rate".

We have expressed the Taylor rule in terms of deviations of unemployment and not output from its "natural rate". This does not affect the results, as through the Okun type relation (25), deviations of unemployment from its "natural rate" are a negative linear function of deviations of output from its own "natural rate".

Under the Taylor rule (50), one can use the dynamic "Phillips curve" (46), the Fisher equation (40), and the real interest rate equation (49) to solve for inflation. Once one solves for inflation, one can also determine unanticipated inflation, and the evolution of employment, unemployment, output, real wages and the real interest rate, through equations (45), (46) and (47), (48) and (49).

5.1 The Persistence of Inflation under a Taylor Rule

Substituting (50) in the Fisher equation (40), after using the real interest rate equation (49) and the dynamic Phillips curve (46), we get the following process for inflation,

$$\pi_t = \gamma_1 E_t \pi_{t+1} + \gamma_2 E_{t-1}\pi_t + \gamma_3 \pi_{t-1} + \gamma_4 \pi^* + \gamma_5 \varepsilon_t^a + \gamma_6 \varepsilon_t^i + \gamma_7 \varepsilon_{t-1}^i \tag{51}$$

where,

$$\gamma_1 = \frac{\alpha}{\phi_\pi \alpha + \phi_u + \theta(1-\delta)(1-\alpha) + \delta\alpha}$$

$$\gamma_2 = \frac{\phi_u + \theta(1-\delta)(1-\alpha)}{\phi_\pi \alpha + \phi_u + \theta(1-\delta)(1-\alpha) + \delta\alpha}$$

$$\gamma_3 = \frac{\delta\phi_\pi\alpha}{\phi_\pi \alpha + \phi_u + \theta(1-\lambda_1)(1-\alpha) + \lambda_1\alpha}$$

$$\gamma_4 = \frac{(\phi_\pi - 1)(1-\delta)\alpha}{\phi_\pi \alpha + \phi_u + \theta(1-\delta)(1-\alpha) + \delta\alpha}$$

$$\gamma_5 = -\gamma_2$$

$$\gamma_6 = -\gamma_1$$

$$\gamma_7 = \delta\gamma_1$$

Note that, because of the persistence of unemployment, the inflationary process also displays persistence. It also depends on current expectations about future inflation, through the definition of the real interest rate and on both parameters of the Taylor rule, as unanticipated inflation causes the unemployment rate and the real interest rate to deviate from their "natural rates". Finally, because of the persistence of unemployment, both current and past monetary policy shocks affect the inflationary process. The effects of productivity and monetary policy shocks on inflation also depend on the parameters of the Taylor rule.[8]

In order to solve for inflation, we first take expectations of (51) conditional on information available up to the end of period $t-1$. This yields,

$$E_{t-1}\pi_t = \frac{1}{\phi_1+\delta}E_{t-1}\pi_{t+1} + \frac{\phi_1\delta}{\phi_1+\delta}\pi_{t-1} + \frac{(\phi_1-1)(1-\delta)}{\phi_1+\delta}\pi^* + \frac{\delta}{\phi_1+\delta}\varepsilon^i_{t-1} \quad (52)$$

The process (52) has two roots, δ and ϕ_π , and will be stable if the two roots lie on either side of unity. Since $\delta < 1$, the expected inflation process will be stable if,

$$\phi_\pi > 1 \quad (53)$$

Condition (53), is the *Taylor principle*. It requires that nominal interest rates over-react to deviations of current inflation from target inflation, in order to affect expected real rates. This is a sufficient condition for a stable and determinate process for expected (and actual) inflation.[9]

If (53) is satisfied, then the solution for the expected inflation process (54) is given by,

$$E_{t-1}\pi_t = (1-\delta)\pi^* + \delta\pi_{t-1} + \frac{\delta}{\phi_\pi}\varepsilon^i_{t-1} \quad (54)$$

From (54), it follows that,

$$E_t\pi_{t+1} = (1-\delta)\pi^* + \delta\pi_t + \frac{\delta}{\phi_1}\varepsilon^i_t \quad (55)$$

[8] (51) being the inflationary process from a dynamic stochastic general equilibrium model, in which the policy rule of the monetary authorities is taken into account when agents form their expectations, it does not suffer from the Lucas [1976] critique. Changing the parameters of the policy rule, would also change the parameters of the inflationary process.

[9] Clarida et al. [1999], Woodford [2003], and Gali [2008] among others, contain detailed discussions of the Taylor principle, and its significance for the resolution of the price level and inflation indeterminacy problem which affects non contingent interest rate rules.

Substituting (16.94) and (16.95) in the inflation process (16.91), the rational expectations solution for inflation is given by,

$$\pi_t = (1-\delta)\pi^* + \delta\pi_{t-1} - \psi_1\varepsilon_t^a - \psi_2\varepsilon_t^i + \psi_3\varepsilon_{t-1}^i \tag{56}$$

where,

$$0 < \psi_1 = \frac{\phi_u + \theta(1-\delta)(1-\alpha)}{\phi_\pi\alpha + \phi_u + \theta(1-\delta)(1-\alpha)} < 1$$

$$0 < \psi_2 = \frac{\phi_\pi - \delta}{\phi_\pi}\frac{\alpha}{\phi_\pi\alpha + \phi_u + \theta(1-\delta)(1-\alpha)} < 1$$

$$0 < \psi_3 = \frac{\delta}{\phi_\pi} < 1$$

From (56), the fluctuations of inflation around the target of the monetary authorities π^* are persistent, and depend on the current innovation in productivity and current and past monetary policy shocks, as the central bank is using the short run tradeoff between inflation and unemployment to partly counteract the real effects of such shocks.

Furthermore, the persistence of inflation is the same as the persistence of deviations of unemployment and other real variables, such as output, from their "natural rate" and independent of the parameters of the Taylor rule. This is because of the persistent effects of both nominal and real shocks on deviations of real variables such as the unemployment rate.

From (56), the variance of inflation is given by,

$$Var(\pi_t) = E(\pi_t - \pi^*)^2 = \frac{1}{(1-\delta^2)}\left(\psi_1^2\sigma_a^2 + (\psi_2^2 + \psi_3^2)\sigma_i^2\right) \tag{57}$$

Under a Taylor rule the variance of inflation depends positively on the variances of both real and monetary shocks.

It is straightforward to show that the higher is the response of nominal interest rates to unemployment ϕ_u, the higher is ψ_1, and therefore the higher the impact of the variance of real shocks on the variance of inflation. ψ_2 depends negatively on ϕ_u and, therefore, the higher is the response of nominal interest rates to unemployment ϕ_u, the lower is ψ_2, and therefore the lower is the impact of the variance of monetary shocks on the variance of inflation. On the other hand, the higher is the response of nominal interest rates to inflation, ϕ_π, the lower are ψ_1, ψ_2 and ψ_3, and therefore the lower the impact of the variance of monetary shocks on the variance of inflation.

These properties would apply even in a model without unemployment persistence. Unemployment persistence δ magnifies these effects.

5.2 Fluctuations in Unemployment and Output

The fluctuations of unemployment and output around their "natural rates" are driven by unanticipated inflation and innovations in productivity. From (56), unanticipated inflation is determined by,

$$\pi_t - E_{t-1}\pi_t = -\psi_1\varepsilon_t^a - \psi_2\varepsilon_t^i \tag{58}$$

Substituting (58) in the "dynamic" Phillips curve (46) and the dynamic output supply function (47), deviations of unemployment and output from their "natural rates" are determined by,

$$(u_t - u^N) = \delta(u_{t-1} - u^N) - \frac{1}{\alpha}\left((1-\psi_1)\varepsilon_t^a - \psi_2\varepsilon_t^i\right) \tag{59}$$

$$(y_t - y_t^N) = \delta(y_{t-1} - y_{t-1}^N) + \frac{1-\alpha}{\alpha}\left((1-\psi_1)\varepsilon_t^a - \psi_2\varepsilon_t^i\right) \tag{60}$$

Thus, under the Taylor rule (50), only innovations in productivity and monetary policy shocks induce fluctuations of deviations of unemployment and output from their "natural rates". Anticipated supply and monetary shocks are incorporated in the expectations of wage setters, while other demand shocks, such as shocks to consumption preferences, are fully neutralized by monetary policy, since the nominal interest rate is assumed to fully accommodate changes in the "natural rate" of interest.

However, because of the persistence in deviations of unemployment and output from their "natural rates", the real effects of these shocks are not short lived, as in a model without endogenous persistence, but they display persistence. The higher the persistence of deviations of unemployment from its "natural rate", the higher the persistence of the effects of temporary nominal and real shocks.

Thus, this model can explain the persistence of both inflation and unemployment, without recourse to staggered price and wage setting. In fact nominal wages contracts last only for one period, and prices are fully flexible. The propagation mechanism that results in persistent effects of productivity and monetary shocks is the wage setting behavior of insiders, and its dynamic interaction with the Taylor rule followed by the central bank.

One could also solve for the fluctuations of real wages and the real interest rate, by substituting (57) into (48) and (49). One can confirm that only innovations in productivity and monetary policy shocks induce fluctuations of deviations of real wages and the real interest rate from their "natural rates".

From (59) and (60), the variances of deviations of unemployment and output from their "natural" rates are given by,

$$Var(u_t) = E(u_t - u^N)^2 = \frac{1}{(1-\delta^2)\,\alpha^2}\left((1-\psi_1)^2\sigma_a^2 + \psi_2^2\sigma_i^2\right) \tag{61}$$

$$Var(y_t) = E(y_t - y_t^N)^2 = \frac{(1-\alpha)^2}{(1-\delta^2)\,\alpha^2}\left((1-\psi_1)^2\sigma_a^2 + \psi_2^2\sigma_i^2\right) \tag{62}$$

It is straightforward to show that the higher is the response of nominal interest rates to unemployment ϕ_u, the higher is ψ_1, and therefore the lower the impact of the variance of real shocks on the variance of unemployment and output. ψ_2 depends negatively on ϕ_u and, therefore, the higher is the response of nominal interest rates to unemployment ϕ_u, the lower is ψ_2, and therefore the lower is the impact of the variance of monetary shocks on the variance of unemployment and output. On the other hand, the higher is the response of nominal interest rates to inflation, ϕ_π, the lower are ψ_1 and ψ_2. Hence, the impact of real shocks on unemployment and output is amplified, while the impact of monetary shocks is dampened.

5.3 Inflation Stabilization and the Divine Coincidence

It is important to note that, unlike the bechmark "new keynesian" model with staggered prices and wages, this model is not characterized by the "divine coincidence" of output stabilization when inflation itself is stabilized. Stabilization of inflation around the target inflation rate of the central bank does not automatically lead to output and employment stabilization around their "natural rates". This is because of the labor market distortions implied by the wage setting behavior of insiders.[10]

[10] See Blanchard and Gali [2007] for a discussion of the "divine coincidence". In order to deal with this problem in the benchmark new keynesian model with staggered prices and wages, one has to superimpose ad hoc additional labor market distortions. This is

To see this, suppose that the central bank allows its response to deviations of inflation from its target ϕ_π to become infinite. Then, from the definition of the ψ 's in (56), ψ_1, ψ_2 and ψ_3 would be driven to zero, and neither nominal nor real shocks would affect inflation. Inflation would converge to the target rate of the central bank π^*, and, from (57), the variance of inflation would tend to zero. Inflation would thus be fully stabilized.

However, from (59) and (60), real shocks would continue to affect deviations of unemployment and output from their "natural rates", even if ψ_1 and ψ_2 are driven to zero. As can be seen from (61) and (62) by setting $\psi_1 = \psi_2 = 0$, inflation stabilization does not result in output and unemployment stabilization in this model. Real shocks continue to affect the relationship between real wages and productivity, even when there is no unanticipated inflation.

The variance of deviations of the unemployment rate from its "natural" rate is then given by,

$$Var(u_t) = E(u_t - u^N)^2 = \frac{1}{(1-\delta^2)\,\alpha^2}\sigma_a^2 \tag{63}$$

Thus, the divine coincidence does not hold in this model. In the presence of real shocks, inflation stabilization does not result in unemployment and output stabilization, as there is always a tradeoff between the stabilization of inflation and the stabilization of unemployment around its "natural rate" in the presence of real shocks.

6 The Optimal Taylor Rule

We have shown that in a model with endogenous unemployment persistence, as is the model in this paper, the Taylor rule results in inflation persistence. One can show that inflation persistence would also be associated with optimal monetary policy, if the loss function of the central bank depends on both deviations of inflation from target and deviations of unemployment from its "natural" rate.[11]

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not the case with the model in this paper, as labor market distortions are part and parcel of the model and not an afterthought.

[11] See Alogoskoufis [2017] for a proof of this proposition and proposed solutions.

6.1 Optimal Inflation Policy

In order to derive optimal monetary policy, one has to specify an appropriate social welfare function. Assuming that the optimal steady state inflation rate is equal to π^*, the only other distortion in this model is the fact that unemployment deviates from its "natural" rate. Thus, one can assume that the central bank would seek to minimize an intertemporal loss function that depends on deviations of inflation from its steady state optimal rate π^*, and deviations of unemployment from its "natural" rate u^N. This can be written as,

$$\Lambda_t = E_t \sum\nolimits_{s=0}^{\infty} \beta^s \left(\frac{1}{2}(\pi_{t+s} - \pi^*)^2 + \frac{\zeta}{2}(u_{t+s} - u^N)^2 \right) \tag{64}$$

β is the discount factor, $\beta = 1/(1+\rho)$, where ρ is the pure rate of time preference, and ζ is the relative weight attached by the central bank to deviations of unemployment from its "natural" rate, relative to deviations of inflation from target.[12]

The optimal policy is the one that minimizes (64) subject to the dynamic stochastic expectational Phillips curve (46). We term this policy as the *optimal time consistent contingent policy*, because the central bank has no short run incentive to deviate from this policy, and its choice of inflation will depend on the current state of the economy, summarized in the deviations of the current unemployment rate from its "natural" rate.[13]

Note that under this policy, there is a clash between the objectives of the monetary authorities and the objectives of wage setting insiders regarding unemployment. The central bank seeks to minimize deviations of unemployment from its natural rate, whereas wage setters seek to minimize deviations of unemployment from a weighted average of the "natural" rate and past

[12] In accordance with the conventions of the literature on monetary policy, e.g Barro and Gordon [1983], Rogoff [1985], we treat (64) as a measure of the intertemporal welfare costs of inflation and unemployment. However, we assume that the central bank does not seek to systematically reduce unemployment below its inefficiently high "natural" rate. Thus, we abstract from the systematic inflation bias that would result in the case in which the central bank also sought to use inflation in order to reduce unemployment below its "natural rate", as in Kydland and Prescott [1977] and Barro and Gordon [1983].

[13] Note that the optimal contingent policy is time consistent, in the sense of Kydland and Prescott [1977]. In Alogoskoufis [2017] we also examine time inconsistent policy rules which may result in even lower intertemporal losses.

unemployment. This clash is what accounts for the persistence of inflation under the contingent optimal policy.

From the first order conditions for a minimum of (64) subject to (46), we get,

$$\pi_t = \pi^* + \frac{\zeta}{\alpha}\left(u_t - u^N\right) + \frac{\zeta\beta\delta}{\alpha}E_t\left(u_{t+1} - u^N\right) \tag{65}$$

Using (46) to substitute for current and expected future deviations of the unemployment rate from its natural rate, after some rearrangement, we get,

$$\pi_t = \delta\pi_{t-1} + (1-\delta)\pi^* - \frac{\zeta(1+\beta\delta^2)}{\alpha^2}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^a\right) \tag{66}$$

The rational expectations solution of (66) is given by,

$$\pi_t = \delta\pi_{t-1} + (1-\delta)\pi^* - \frac{\zeta(1+\beta\delta^2)}{\alpha^2 + \zeta(1+\beta\delta^2)}\varepsilon_t^a \tag{67}$$

From (67), deviations of the "optimal", time consistent, contingent inflation rate from the inflation target π^* display the same degree of persistence, as the persistence of deviations of unemployment from its "natural" rate. The reason is that the central bank allows inflation to fluctuate in order to minimize deviations of unemployment from its natural rate. Since these deviations in unemployment display persistence, deviations of inflation from target will also display persistence under the optimal contingent policy.[14]

It is worth noting that the persistence of inflation under the optimal time consistent contingent monetary policy does not affect the persistence of unemployment. The reason is that wage setters can anticipate the persistent part of the inflation process, incorporate it in their expectations when they set nominal wages, and neutralize the effects of persistent inflation on unemployment. Thus, the only element of monetary policy that matters for unemployment is the unanticipated part, which is a function of the current productivity shock.

Under the optimal contingent policy, the variances of inflation and unemployment are given by,

[14]Note from (67) that if deviations of unemployment from its natural rate did not persist, i.e in the case $\delta = 0$, the optimal contingent monetary policy rule would not result in persistent deviations of inflation from target. There would be deviations of inflation from π^* only in response to unanticipated shocks to productivity.

$$Var(\pi_t) = E_t(\pi_t - \pi^*)^2 = \frac{1}{1-\delta^2}\frac{\zeta^2(1+\beta\delta^2)^2}{(\alpha^2+\zeta(1+\beta\delta^2))^2}\sigma_a^2 \tag{68}$$

$$Var(u_t) = E_t(u_t - u^N)^2 = \frac{1}{1-\delta^2}\frac{\alpha^2}{(\alpha^2+\zeta(1+\beta\delta^2))^2}\sigma_a^2 \tag{69}$$

where σ_a^2 is the variance of the innovation in productivity.

From (64), the expected intertemporal welfare loss under the optimal contingent monetary policy rule is thus given by,

$$\Lambda_t^C = \frac{1}{2}\frac{1+\rho}{\rho}\frac{\zeta}{1-\delta^2}\left(\frac{\alpha^2+\zeta(1+\beta\delta^2)^2}{(\alpha^2+\zeta(1+\beta\delta^2))^2}\right)\sigma_a^2 \tag{70}$$

where superscript C denotes the optimal *contingent* inflation policy.

6.2 The Optimal Taylor Rule in the Presence of Nominal Shocks

It is straightforward to deduce from our discussion of the divine coincidence in the previous section, that if nominal shocks, in the form of monetary policy errors, were the only source of uncertainty, then the optimal Taylor rule would imply an infinitely large reaction of nominal interest rates to deviations of inflation from the central bank target. This would stabilize both inflation and unemployment in such a case.

6.3 The Optimal Taylor Rule in the Presence of Real Shocks

Under the Taylor rule, assuming real shocks and no monetary policy errors, the expected intertemporal welfare loss is given by,

$$\Lambda_t^T = \frac{1}{2}\frac{1+\rho}{\rho}\frac{1}{1-\delta^2}\left(\frac{\zeta(1-\psi_1)^2+\alpha^2\psi_1^2}{\alpha^2}\right)\sigma_a^2 \tag{71}$$

where superscript T denotes the Taylor rule policy and ψ_1 is a function of the Taylor rule and other parameters, defined in (56).

Thus, from (70) and (71), the optimal Taylor rule is the one that results in a reaction of inflation to real shocks ψ_1 which satisfies,

$$\frac{\zeta(1-\psi_1)^2+\alpha^2{\psi_1}^2}{\alpha^2}=\frac{\zeta\left(\alpha^2+\zeta(1+\beta\delta^2)^2\right)}{\left(\alpha^2+\zeta(1+\beta\delta^2)\right)^2} \tag{72}$$

Let us denote by ψ_1^* the optimal ψ_1 satisfying (72). ψ_1^* is the solution to the following quadratic equation,

$$\psi_1^{*2}-b\psi_1^*+c \tag{73}$$

where,

$$b=2\frac{\zeta/\alpha^2}{1+\zeta/\alpha^2}$$

$$c=\frac{\zeta}{1+\zeta/\alpha^2}\left(\frac{1}{\alpha^2}-\frac{\alpha^2+\zeta(1+\beta\delta^2)^2}{\left(\alpha^2+\zeta(1+\beta\delta^2)\right)^2}\right)$$

In general there will be two solutions for ψ_1^*, corresponding to the two solutions of the quadratic equation (73). The solutions will take the form,

$$\psi_1^{*1,2}=\frac{1}{2}\left(b\pm\sqrt{b^2-4c}\right) \tag{74}$$

Both solutions will depend on all technological and preference parameters of the model, but also the degree of unemployment persistence δ, as well as ζ, the parameter denoting the relative weight that the central bank attaches to unemployment relative to inflation.

It is straightforward to demonstrate that a higher degree of unemployment persistence δ will result in a rise in one of the solutions for ψ_1^* and a fall in the other. Thus, from (56), it will affect the optimal parameters of the Taylor rule in opposite directions.

If the central bank does not care about unemployment, and $\zeta=0$, then, from (73), the optimal ψ_1 is also equal to zero. From the definition of ψ_1 in (56), it then follows that the response of nominal interest rates to deviations of inflation from target in the Taylor rule should be infinitely large. This would stabilize the inflation rate, but, in the presence of real shocks, result in the largest possible variance of deviations of unemployment from its natural rate, as in (63). Thus, only if the central bank is not concerned with unemployment is the optimal Taylor rule a Fisher [1919] rule of complete inflation stabilization in the case of both nominal and real shocks.

If the central bank cares about both unemployment and inflation, i.e $\zeta > 0$, then, in the presence of real shocks, both ϕ_π and ϕ_u should be positive and finite.

One can calculate the optimal ψ_1's from (73), as functions of ζ. These functions are depicted in Figure 1 assuming that ζ, on the horizontal axis, lies between zero and 1. In the functions depicted in Figure 1, we have assumed that the other structural parameters of the model take the values $\alpha = 1/3$, $\beta = 0.98$, corresponding to a pure rate of time preference $\rho = 0.02$, and that $\delta = 1/2$, which is the average of the estimates of the persistence of unemployment in the main industrial economies, from Alogoskoufis [2017]. In Figure 1 we also display the optimal ψ_1 corresponding to the absence of unemployment persistence, i.e $\delta = 0$.

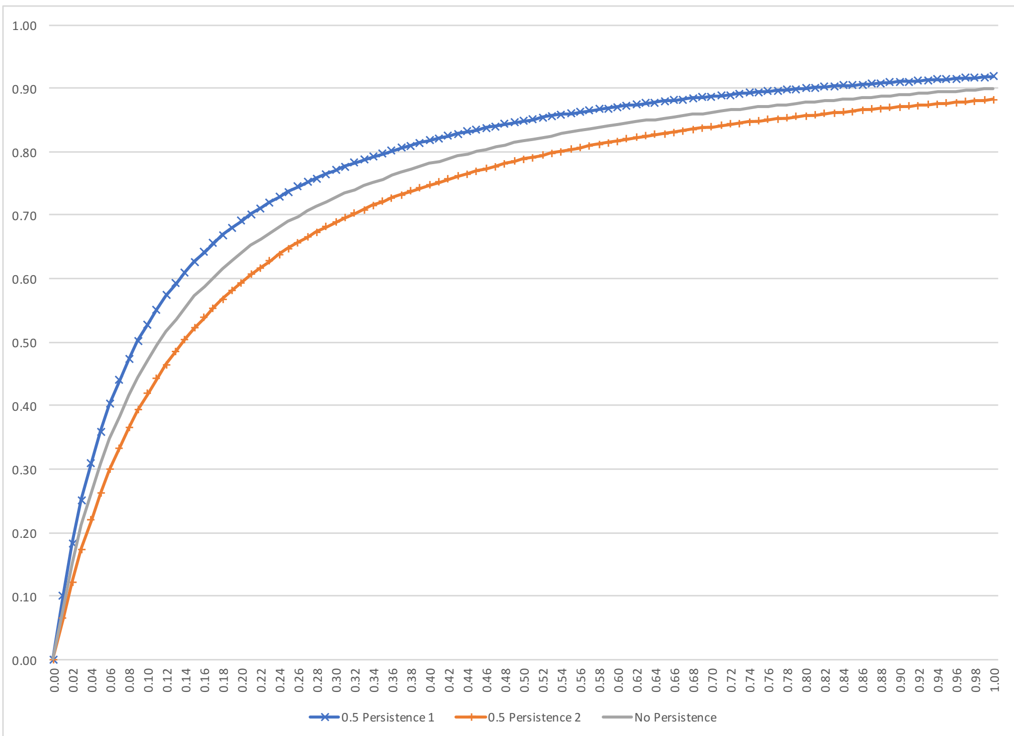

Figure 1: Optimal Response of Inflation to Real Shocks as a Function of the Preferences of the Central Bank for Unemployment Relative to Inflation

From the optimal ψ_1's one can calculate the optimal Taylor rule parameters ϕ_π and ϕ_u, using the definition of ψ_1 in (56). This implies the following relation between the optimal Taylor rule parameters and the optimal ψ_1^*,

$$\phi_\pi^* = \frac{1-\psi_1^*}{\alpha\psi_1^*}\left(\phi_u^* + \theta(1-\delta)(1-\alpha)\right) \tag{75}$$

It is clear from (74) that, given the response of nominal interest rates to unemployment ϕ_u, the relation between the optimal response of nominal interest rates to inflation and the optimal ψ_1 is negative. Thus, the less the central bank cares about unemployment relative to inflation, the higher the optimal response of nominal interest rates to inflation relative to unemployment.

In Figure 2 we present the optimal responses of nominal interest rates to inflation ϕ_π^*, measured on the vertical axis, as functions of the relative weight attached by the central bank to unemployment relative to inflation ζ, measured on the horizontal axis. We have assumed a response to unemployment ϕ_u equal to 0.75, which corresponds to a response to output equal to 0.5, as suggested by Taylor [1993], and an intertemporal elasticity of substitution of consumption $1/\theta = 1$. The other parameters are as in Figure 1. As in Figure 1 we also present the optimal ϕ_π in the absence of unemployment persistence as well.

In the absence of unemployment persistence, a response of nominal interest rates to inflation equal to 1.5, as suggested by Taylor [1993], would be optimal if the central bank attached a weight to unemployment relative to inflation equal to about a quarter (0.24). With unemployment persistence equal to 0.5, the optimal response for this weight would rise from 1.5 to 1.8 if one chooses the higher ϕ_π which is more likely to be higher than unity, and thus satisfy the Taylor principle. Alternatively, if one were to choose the lower value of ϕ_π, this would fall to 1.2.

In conclusion, unemployment persistence calls for a modification of the optimal response on nominal interest rates to inflation, as inflation persists as well.

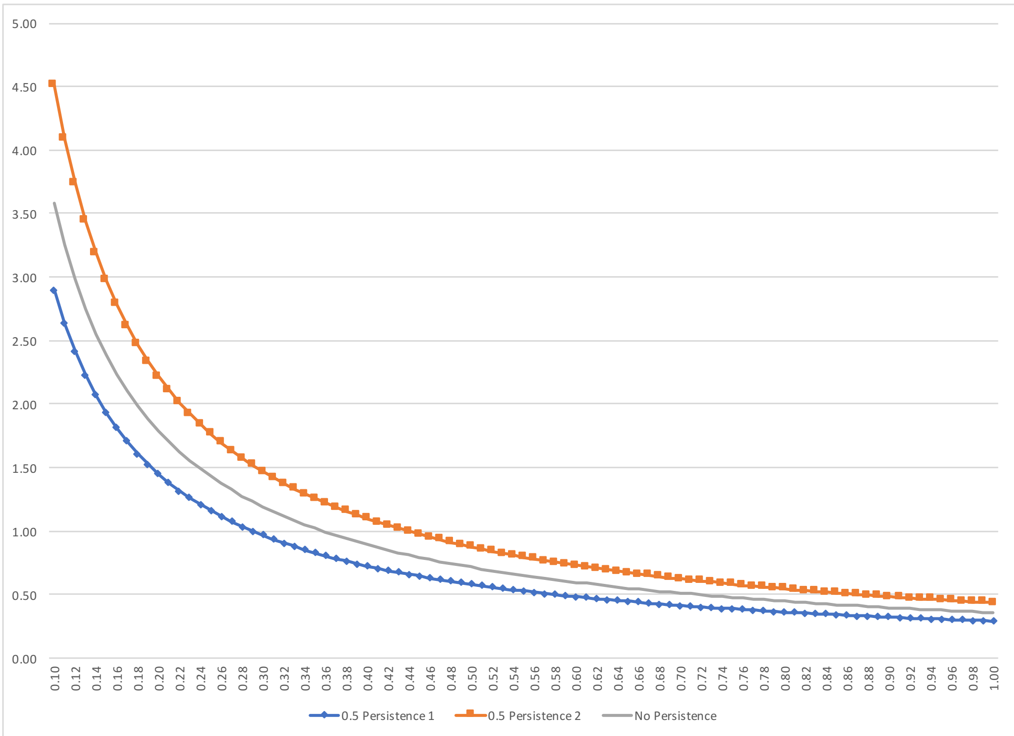

Figure 2: Optimal Response of Nominal Interest Rates to Inflation as a Function of the Aversion of the Central Bank to Unemployment Relative to Inflation

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Data Appendix

The data set used in this study is as follows:

u is the civilian unemployment rate, from *Economic Report of the President* and *Historical Statistics of the United States, From Colonial Times to 1970*. For the pre World War II period, the data used are as proposed by Darby (1976) and Romer (1986).

y is the log of real GDP, again from *Economic Report of the President* and *Historical Statistics of the United States, From Colonial Times to 1970*.

π is the rate of change of the Consumer Price Index, from *Economic Report of the President* and *Historical Statistics of the United States*, From Colonial Times to 1970.

Table 1

The Persistence of Unemployment, Output, Inflation and Nominal Interest Rates in the USA

Annual Data, 1892-2014

OLS Estimates

	$(u\text{-}u^N)_t$		$(y\text{-}y^N)_t$		π_t		$(i\text{-}i^N)_t$
Constant	0.000		0.000		0.012		0.000
	(0.001)		(0.004)		(0.004)		(0.001)
$(u\text{-}u^N)_{t-1}$	0.944	$(y\text{-}y^N)_{t-1}$	0.899	π_{t-1}	0.787	$(i\text{-}i^N)_{t-1}$	0.769
	(0.080)		(0.085)		(0.089)		(0.079)
$(u\text{-}u^N)_{t-2}$	-0.480	$(y\text{-}y^N)_{t-2}$	-0.356	π_{t-2}	-0.212	$(i\text{-}i^N)_{t-2}$	-0.499
	(0.080)		(0.085)		(0.089)		(0.079)
Persistence	0.465		0.543		0.575		0.271
	(0.068)		(0.070)		(0.074)		(0.078)
R^2	0.542		0.511		0.448		0.446
s	0.014		0.039		0.035		0.007
DW	2.032		2.132		1.926		2.107
ADF	-7.861		-6.574		-5.706		-9.332

Note: u^N , y^N and i^N are approximated by a Hodrick-Prescott filter. "Persistence" is the sum of the coefficients of the lagged dependent variables. Standard errors in parentheses. R^2 is the coefficient of determination, *s* the standard error of estimate, DW the Durbin Watson statistic and ADF is the Augmented Dickey-Fuller test.

Table 2

Unrestricted and Restricted VARs for Unemployment, Output, Inflation and Nominal Interest Rates

Annual US Data, 1892-2014

Seemingly Unrelated Regression (SURE) Estimates

	$(u\text{-}u^N)_t$		$(y\text{-}y^N)_t$		π_t		$(i\text{-}i^N)_t$	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
Constant	0.001	0.000	0.001	0.000	0.010	0.012	0.000	0.000
	(0.001)	(0.001)	(0.004)	(0.003)	(0.004)	(0.004)	(0.001)	(0.001)
$(u\text{-}u^N)_{t-1}$	0.897	0.848	-0.647		0.162		-0.004	
	(0.137)	(0.061)	(0.396)		(0.356)		(0.078)	
$(u\text{-}u^N)_{t-2}$	-0.559	-0.363	0.717		0.620		-0.034	
	(0.139)	(0.061)	(0.401)		(0.360)		(0.079)	
$(y\text{-}y^N)_{t-1}$	-0.043		0.761	0.800	0.108		0.000	
	(0.047)		(0.135)	(0.068)	(0.121)		(0.026)	
$(y\text{-}y^N)_{t-2}$	-0.017		-0.172	-0.288	0.048		-0.007	
	(0.047)		(0.135)	(0.068)	(0.121)		(0.027)	
π_{t-1}	0.038		-0.056		0.734	0.808	0.004	
	(0.037)		(0.106)		(0.095)	(0.083)	(0.021)	
π_{t-2}	-0.057		0.076		-0.091	-0.240	0.001	
	(0.036)		(0.105)		(0.094)	(0.083)	(0.021)	
$(i\text{-}i^N)_{t-1}$	0.304		-1.156		-0.105		0.757	0.811
	(0.144)		(0.415)		(0.373)		(0.082)	(0.076)
$(i\text{-}i^N)_{t-2}$	-0.023		0.647		-0.359		-0.501	-0.493
	(0.147)		(0.425)		(0.382)		(0.084)	(0.076)
R^2	0.578	0.533	0.551	0.505	0.500	0.447	0.449	0.444
s	0.013	0.014	0.038	0.039	0.035	0.035	0.008	0.007
$\chi^2(6)$ test		9.850		10.204		11.854		0.554

Note: u^N , y^N and i^N are proxied by a Hodrick-Prescott filter. R^2 is the coefficient of determination, and s the standard error of estimate. $\chi^2(6)$ test refers to the Granger causality test for the exclusion of the other lagged variables in each equation, which is asymptotically distributed as $\chi^2(6)$.

Table 3

Restricted VARs of Unemployment, Output, Inflation and Nominal Interest Rates

Annual US Data, 1892-2014

Seemingly Unrelated Regression (SURE) Estimates

	$(u\text{-}u^N)_t$		$(y\text{-}y^N)_t$		π_t		$(i\text{-}i^N)_t$	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
Constant	0.000	0.000	0.000		0.012		0.000	
	(0.001)	(0.001)	(0.003)		(0.004)		(0.001)	
$(u\text{-}u^N)_{t-1}$	0.848	0.822						
	(0.061)	(0.042)						
$(u\text{-}u^N)_{t-2}$	-0.363	-0.344						
	(0.061)	(0.042)						
$(y\text{-}y^N)_{t-1}$			0.800	0.822				
			(0.068)	(0.042)				
$(y\text{-}y^N)_{t-2}$			-0.288	-0.344				
			(0.068)	(0.042)				
π_{t-1}					0.808	0.822		
					(0.083)	(0.042)		
π_{t-2}					-0.240	-0.344		
					(0.083)	(0.042)		
$(i\text{-}i^N)_{t-1}$							0.811	0.822
							(0.076)	(0.042)
$(i\text{-}i^N)_{t-2}$							-0.493	-0.344
							(0.076)	(0.042)
Persistence	0.485	0.479	0.512	0.479	0.568	0.479	0.318	0.479
	(0.054)	(0.037)	(0.058)	(0.037)	(0.069)	(0.037)	(0.075)	(0.037)
R^2	0.533	0.530	0.505	0.505	0.447	0.435	0.444	0.411
s	0.014	0.014	0.039	0.039	0.035	0.036	0.007	0.008
χ^2 test	9.850		10.204		11.854		0.554	

Note: u^N, y^N and i^N are proxied by a Hodrick-Prescott filter. "Persistence" is the sum of the coefficients of the lagged dependent variables. R^2 is the coefficient of determination, and s the standard error of estimate. χ^2 test refers to the Granger causality test for the exclusion of the other lagged variables, which is asymptotically distributed as $\chi^2(6)$.