

# Output Dynamics and Private Sector Borrowing during Sovereign Debt Crises

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# Disclaimer

- The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, its management

# Motivation

- Empirics
  - Private sector external borrowing: severe decline in default/post-default restructuring vs. mild decline in preemptive restructuring
  - Output dynamics: severe decline in default/post-default restructuring vs. mild decline in preemptive restructuring
- Theory
  - Private sector external borrowing: Theory on default/post-default restructuring (Kaas, Mellert and Scholl 2020) vs. No on preemptive restructuring
  - Output dynamics: Theory on default/post-default restructuring (Mendoza and Yue 2012) vs. No on preemptive restructuring
- Question – How to fill a gap between theory and data?

# What We Do in This Paper

- Empirics
  - Data on private sector borrowing from sovereigns' chair creditors
  - New stylized facts
- Theory
  - Sovereign debt model with preemptive and post-default restructurings, private sector borrowing and endogenous output
  - Role of creditor chairs lending to both sovereign government and private sector
  - Role of sovereign default/restructuring on private sector borrowing and output dynamics
- Quantitative analysis
  - Replication of five stylized facts

# Data: Debt Restructurings and Creditor Chairs

- Sovereign debt restructurings – Asonuma and Trebesch (2016)
  - 197 sovereign debt restructurings with private creditors in 1975–2020
    - Post-default restructurings: 116 episodes
    - Preemptive restructurings: 81 episodes
- Creditor chairs in sovereign debt restructurings – Asonuma and Joo (2020)
  - 197 sovereign debt restructurings with private creditors in 1975–2020
    - Creditor chairs (institutions, nationality)

# Data: Private Sector Borrowing

- BIS bank cross-border positions
  - Geographic locations (country-level)
  - 75 sovereign debtors experiencing debt restructurings.
- Private sector borrowing (stock and flow of liabilities)
  - From sovereign's creditor chair countries
  - From rest of the world

# Data: Private Sector Borrowing

Table 1: Private Sector Liability from Sovereigns' Creditor Chair Countries

A. Panel dataset in 1977–2021

	Total	Mean	Median	Std.Dev
<b>Country coverage</b>	71			
<b>Country-creditor chair pairs (episodes)</b>	116			
Flow (Percent of GDP)	5,054	0.8%	0.1%	3.8%
Stock (Percent of GDP)	5,014	12.3%	6.9%	16.1%
Stock (Percent of total liabilities)	5,350	28.0%	25.7%	23.5%

# Stylized Facts

- **Stylized Fact 1:** Private sector borrows from sovereigns' creditor chair countries prior to, during, and after sovereign debt restructurings.

Table 1: Private Sector Liability from Sovereigns' Creditor Chair Countries

## B. Sovereign Debt Restructurings in 1975–2020

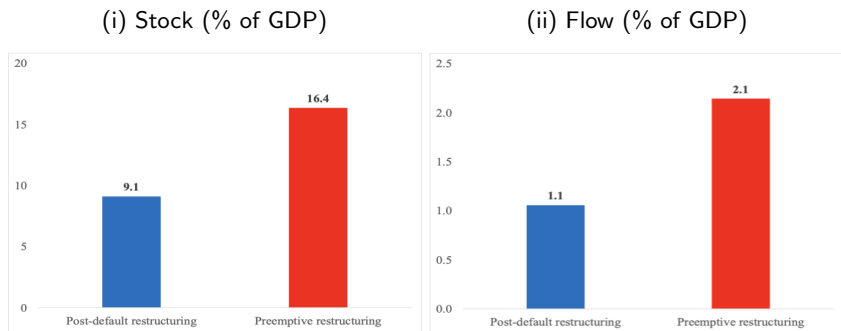
	Observation	Mean	Observation	Mean	Observation	Mean
<b>Restructuring Episodes</b>			197			
<b>Pre-Restructuring Duration</b>			3.2			
	Pre-restructuring periods		Restructuring periods		Post-restructuring periods	
Flow (percent of GDP) <sup>1/</sup>	111	1.4	111	0.8	111	0.8
Stock (percent of GDP) <sup>1/</sup>	111	12.0	111	14.5	111	15.7
Stock (percent of total liabilities) <sup>1/</sup>	121	33.0	121	31.8	121	31.9



# Stylized Facts

- **Stylized Fact 2:** Countries which experience a preemptive (post-default) restructuring had high (low) private sector borrowing from sovereigns' creditor countries.

Figure: Private Sector Liability to Creditor Countries in Pre-restructuring Period

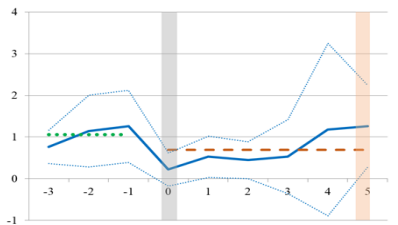


# Stylized Facts

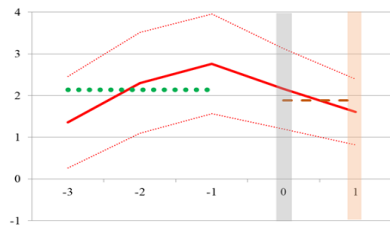
- **Stylized Fact 3:** Private sector borrowing from creditor chair countries declines sharply and recovers gradually in post-default restructurings.
- **Stylized Fact 4:** Private sector borrowing from creditor chair countries declines mildly in preemptive restructurings.

Figure: Private Sector Liability (Flow) to Creditor Chair Countries (% of GDP)

(i) Post-default restructurings



(ii) Preemptive restructurings



— Post-Default Restructurings  
— Average - Restructuring  
— Lower 90% Confidence Interval  
— Upper 90% Confidence Interval  
••• Average - Pre-restructuring

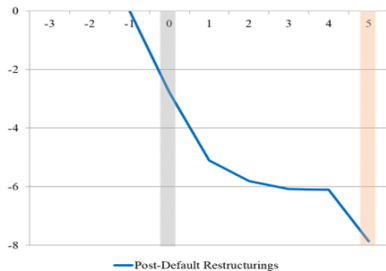
— Preemptive Restructurings  
— Average - Restructuring  
— Lower 90% Confidence Interval  
— Upper 90% Confidence Interval  
••• Average - Pre-restructuring

# Stylized Facts

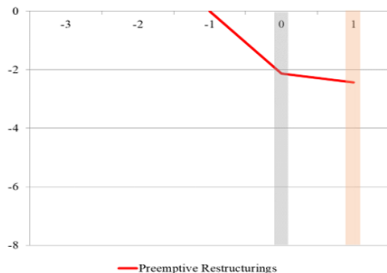
- **Stylized Fact 5:** GDP declines severely in post-default restructurings while mildly in preemptive restructurings.

Figure: Cumulative change in GDP from pre-restructuring year (%)

(i) Post-default restructurings



(ii) Preemptive restructurings



# Stylized Facts

- *Stylized Fact 1*: Private sector also borrows from sovereigns' creditor chair countries prior to and during restructurings.
- *Stylized Fact 2*: Countries which experience a preemptive (post-default) restructuring had high (low) private sector borrowing from sovereigns' creditor chair countries.
- *Stylized Fact 3*: Private sector borrowing from creditor chair countries declines sharply in post-default restructurings.
- *Stylized Fact 4*: Private sector borrowing from creditor chair countries declines mildly in preemptive restructurings.
- *Stylized Fact 5*: GDP declines severely in post-default restructurings while mildly in preemptive restructurings.

# Main Questions

- Why output dynamics differ between preemptive and post-default restructurings?
- Why private sector borrowing differs between preemptive and post-default restructurings?

# Literature Review

- Private sector borrowing and sovereign borrowing
  - Bocola (2016), Arellano, Bai and Bocola, (2024), Aguilar, Amador and Gopinath (2009), Artes and Hale (2008), Kaas, Mellert, and Scholl (2020)
  - Ours: Different patterns of private sector external borrowing
- Output decline in sovereign default
  - Mendoza and Yue (2012), Sosa-Padilla (2017), Gordon and Guerron-Quintana (2018)
  - Ours: Output decline due to private sector borrowing
- Different types of sovereign defaults/restructurings
  - Arellano et al. (2023), Hatchondo et al. (2014), Asonuma and Trebesch (2016),
  - Ours: Different pattern of output and private sector borrowing

# Main Theoretical Findings (Preliminary)

- Preemptive restructurings
  - Ex ante choice (prior to current TFP realization)
  - High likelihood of settlement and high expected recovery rates
  - Foreign creditor's net worth decreases mildly
  - Moderate decline in private sector borrowing and moderate decline in output
  - Quick settlement (no delays)
- Defaults / Post-default restructurings
  - Ex post choice (after current TFP realization)
  - Low likelihood of settlement (delays) and low expected recovery rates
  - Foreign creditor's net worth decreases sharply
  - Severe decline in private sector borrowing and severe decline in output
  - Long delays in settlement

# Model: General Features

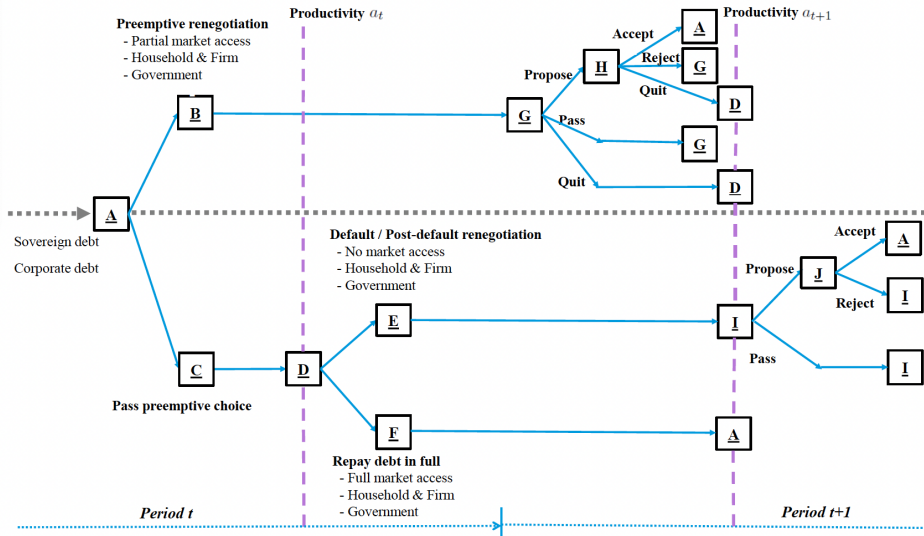
- Sovereign debt in a dynamic small open economy model:
  - Endogenous ex ante choice of preemptive option and passing it
  - Endogenous ex post sovereign's choice of default and repayment
  - Endogenous choice of settlement and delays conditional on preemptive option and default
  - Endogenous lending choice of foreign creditor to sovereign government and private sector
  - Endogenous ex post firm's choice of default and repayment
  - Endogenous production with labor and two types of intermediate goods



## Model: General Features

- A risk averse sovereign debtor, a household, final goods and intermediate goods firms, and a risk-averse foreign creditor
- A stochastic TFP shock  $a_t$
- Credit record  $h_t$  : indicating status of sovereign market access
- Sovereign and firm borrowing from the foreign creditor
- Corporate bonds to finance intermediate goods
- Incomplete capital market: one-period zero-coupon bonds
- One-side commitment
- Two types of debt renegotiations:
  - Preemptive - multi-round before TFP realization
  - Post-default - multi-round after TFP realization

# Timing of the Model



# Model: Household's Problem

- Household maximization problem

$$\max_{c_t, L_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$$

$$s.t. \quad c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t \quad (1)$$

- Optimality condition of household

$$\frac{u_l(c_t, L_t)}{u_c(c_t, L_t)} = w_t \quad (2)$$

# Model: Final Goods Producer's Problem

- Final goods producer's production function

$$y_t = a_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L} (\bar{k})^{\alpha_K} \quad (3)$$

$$M(m_t^d, m_t^*) = [\lambda(m_t^d)^\mu + (1 - \lambda)(m_t^*)^\mu]^{\frac{1}{\mu}} \quad (4)$$

- Final goods producer's profit maximization problem:

$$\begin{aligned} \max_{m_t^d, m_t^*, L_t^f} \pi_t^f(b_t, b_t^f, 0, a_t) &= a_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L} (\bar{k})^{\alpha_K} - w_t L_t^f - p_t^m m_t^d - p_t^* m_t^* - b_t^f \\ \text{s.t. } q^f(b_{t+1}, b_{t+1}^f, h_t, a_t) b_{t+1}^f &= p_t^* m_t^* \end{aligned} \quad (5)$$

- $\bar{k}$  is numeraire (Mendoza and Yue 2012)
- Issue foreign corporate bonds  $b_{t+1}^f$  to finance foreign intermediate goods

# Model: Final Goods Producer's Problem

- “Capacity to repay” default on corporate bonds
  - Repay when  $\pi_t^f(b_t, b_t^f, h_t, a_t) \geq 0$
  - Default when  $\pi_t^f(b_t, b_t^f, h_t, a_t) < 0$
- Corporate (final goods producer) default choice

$$CD(b_t, b_t^f, h_t) = \{a_t \in A : \pi_t^f(b_t, b_t^f, h_t, a_t) < 0\}$$

# Model: Final Goods Producer's Problem

- Optimality conditions

$$\alpha_M a_t (\bar{k})^{\alpha_k} (M(m_t^d, m_t^*))^{\alpha_M - \mu} (L_t^f)^{\alpha_L} (1 - \lambda) (m_t^*)^{\mu - 1} = p_t^* \quad (6)$$

$$\alpha_M a_t (\bar{k})^{\alpha_k} (M(m_t^d, m_t^*))^{\alpha_M - \mu} (L_t^f)^{\alpha_L} \lambda (m_t^*)^{\mu - 1} = p_t^m \quad (7)$$

$$\alpha_L a_t (\bar{k})^{\alpha_k} (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L - 1} = w_t \quad (8)$$

## Model: Intermediate Goods Producer's Problem

- Intermediate goods producer's production function

$$m_t^d = A(L_t^m)^\gamma \quad (9)$$

- Intermediate goods producer's profit maximization problem:

$$\max_{L_t^m} \pi_t^m = A(L_t^m)^\gamma - w_t L_t^m \quad (10)$$

- The optimality condition

$$\gamma A(L_t^m)^{\gamma-1} = w_t \quad (11)$$

# Equilibrium in Factor Markets and Production

$$\alpha_M a_t (\bar{k})^{\alpha_k} (M(m_t^d, m_t^*))^{\alpha_M - \mu} (L_t^f)^{\alpha_L} (1 - \lambda) (m_t^*)^{\mu - 1} = p_t^* \quad (6)$$

$$\alpha_M a_t (\bar{k})^{\alpha_k} (M(m_t^d, m_t^*))^{\alpha_M - \mu} (L_t^f)^{\alpha_L} \lambda (m_t^*)^{\mu - 1} = p_t^m \quad (7)$$

$$\alpha_L a_t (\bar{k})^{\alpha_k} (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L - 1} = w_t \quad (8)$$

$$\gamma A (L_t^m)^{\gamma - 1} = w_t \quad (11)$$

$$L_t^m + L_t^f = L_t \quad (12)$$

$$A (L_t^m)^\gamma = m_t^d \quad (9)$$



# Model: Sovereign's Problem - Ex Ante

- Ex ante value of sovereign

$$V^{EXANTE}(b_t, b_t^f, 0, a_{t-1}) = \max[V^{PRE}(b_t, b_t^f, 0, a_{t-1}), V^{NON-PRE}(b_t, b_t^f, 0, a_{t-1})] \quad (13)$$

- Ex ante value of taking a preemptive restructuring

$$V^{PRE}(b_t, b_t^f, 0, a_{t-1}) = \max_{c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t, b_{t+1}^f} \int_A [u(c_t, L_t) + \beta V(b_t, b_{t+1}^f, 1, a_t)] d\mu(a_t | a_{t-1})$$

$$\text{s.t. } c_t = a_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L} (\bar{k})^{\alpha_K}$$

$$q^f(b_{t+1}, b_{t+1}^f, 0, a_t) b_{t+1}^f = p_t^* m_t^*$$

$$\pi^f(b_t, b_t^f, 0, a_t) \geq 0$$

$$L_t^m + L_t^f = L_t$$

$$A(L_t^m)^\gamma = m_t^d \quad (14)$$

# Model: Sovereign's Problem - Ex Ante

- Ex ante value of sovereign

$$V^{NON-PRE}(b_t, b_t^f, 0, a_{t-1}) = \int_A V(b_t, b_t^f, 0, a_t) d\mu(a_t | a_{t-1}) \quad (15)$$

- Sovereign's preemptive restructuring choice

$$PRE(b_t, b_t^f, 0) = \{a_{t-1} \in A : V^{PRE}(b_t, b_t^f, 0, a_{t-1}) \geq V^{NON-PRE}(b_t, b_t^f, 0, a_{t-1})\} \quad (16)$$

# Model: Sovereign's Problem - Ex Post

- **Ex post** value of sovereign

$$V(b_t, b_t^f, 0, a_t) = \max[V^R(b_t, b_t^f, 0, a_t), V^D(b_t, b_t^f, 0, a_t)] \quad (17)$$

- **Ex post** value of repayment

$$V^R(b_t, b_t^f, 0, a_t) = \max_{c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t, b_{t+1}^f} u(c_t, L_t) + \beta \int_A V(b_{t+1}, b_{t+1}^f, 0, a_{t+1}) d\mu(a_{t+1}|a_t)$$

$$\text{s.t. } c_t = a_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L} (\bar{k})^{\alpha_K}$$

$$q^f(b_{t+1}, b_{t+1}^f, 0, a_t) b_{t+1}^f = p_t^* m_t^*$$

$$\pi^f(b_t, b_t^f, 0, a_t) \geq 0$$

$$L_t^m + L_t^f = L_t$$

$$A(L_t^m)^\gamma = m_t^d \quad (18)$$

# Model: Sovereign's Problem - Ex Post

- **Ex post** value of default/post-default restructuring:

$$V^D(b_t, b_t^f, 0, a_t) = \max_{c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t, b_{t+1}^f} u(c_t, L_t) + \beta \int_A V((1+r^*)b_t, b_{t+1}^f, 2, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (19)$$

$$\text{s.t. } c_t = a_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L} (\bar{k})^{\alpha_K}$$

$$q^f(b_{t+1}, b_{t+1}^f, 0, a_t) b_{t+1}^f = p_t^* m_t^*$$

$$\pi^f(b_t, b_t^f, 0, a_t) \geq 0$$

$$L_t^m + L_t^f = L_t$$

$$A(L_t^m)^\gamma = m_t^d \quad (20)$$

- Sovereign default/post-default restructuring choice

$$D(b_t, b_t^f, 0) = \{a_t \in A : V^D(b_t, b_t^f, 0, a_t) \geq V^R(b_t, b_t^f, 0, a_t)\}$$

# Model: Foreign creditor - Ex Ante

- Ex ante value of foreign creditor

$$V^{*,EXANTE}(b_t, b_t^f, 0, a_{t-1}) = \mathbf{1}_{PRE} V^{*,PRE}(b_t, b_t^f, 0, a_{t-1}) + (1 - \mathbf{1}_{PRE}) V^{*,NON-PRE}(b_t, b_t^f, 0, a_{t-1}) \quad (22)$$

- Ex ante value of taking a preemptive restructuring

$$V^{*,PRE}(b_t, b_t^f, 0, a_{t-1}) = \max_{c_t^*, b_{t+1}^{*f}} \int_A [v(c_t^*) + \beta V^*(b_t, b_{t+1}^{*f}, 1, a_t)] d\mu(a_t | a_{t-1})$$
$$c_t^* - q^f(b_{t+1}, b_{t+1}^{*f}, 0, a_{t-1}) b_{t+1}^{*f} = \bar{W}^* - b_t^{*f} \quad (23)$$

- Prices of corporate bonds

$$q^f(b_{t+1}, b_{t+1}^f, 0, a_{t-1}) = \int_A \beta^* \frac{v'(c_{t+1}^*)}{v'(c_t^*)} [\mathbf{1}_{Non-Cor-Def}] d\mu(a_t | a_{t-1}) \quad (24)$$

- Ex ante value of passing preemptive restructuring

$$V^{*,Non-PRE}(b_t, b_t^f, 0, a_{t-1}) = \int_A V^*(b_t, b_t^f, 1, a_t) d\mu(a_t | a_{t-1}) \quad (25)$$

# Model: Foreign creditor - Ex Post

- **Ex post** value of foreign creditor

$$V^*(b_t, b_t^f, 0, a_t) = \mathbf{1}_{Non-Def} V^{*R}(b_t, b_t^f, 0, a_t) + (1 - \mathbf{1}_{Non-Def}) V^{*D}(b_t, b_t^f, 0, a_t) \quad (26)$$

- **Ex post** value when the sovereign repays

$$V^{*,R}(b_t, b_t^f, 0, a_t) = \max_{c_t^*, b_{t+1}^{*f}, b_{t+1}^{*f}} v(c_t^*) + \beta^* \int_A V^*(b_{t+1}, b_{t+1}^f, 0, a_{t+1}) d\mu(a_{t+1}|a_t)$$

s.t.

$$c_t^* - q(b_{t+1}, b_{t+1}^{*f}, 0, a_t) b_{t+1} + q^f(b_{t+1}, b_{t+1}^{*f}, 0, a_t) b_{t+1}^{*f} = \bar{W}^* - b_t - b_t^{*f} \quad (27)$$

- Prices of sovereign bonds

$$q(b_{t+1}, b_{t+1}^f, 0, a_t) = \int_A \beta^* \frac{v'(c_{t+1}^*)}{v'(c_t^*)} \left[ \mathbf{1}_{Non-Def} \left( \mathbf{1}_{Non-Pre} + (1 - \mathbf{1}_{Non-Pre}) \gamma(b_{t+1}, b_{t+1}^f, a_{t+1}) \right) + (1 - \mathbf{1}_{Non-Def}) \alpha(b_{t+1}, b_{t+1}^f, a_{t+1}) \right] d\mu(a_{t+1}|a_t) \quad (28)$$

## Model: Foreign creditor - Ex Post

- Prices of corporate bonds

$$q^f(b_{t+1}, b_{t+1}^f, 0, a_t) = \int_A \beta^* \frac{v'(c_{t+1}^*)}{v'(c_t^*)} [\mathbf{1}_{Non-Cor\_Def}] d\mu(a_{t+1}|a_t) \quad (24')$$

- **Ex post** value when the sovereign default

$$V^{*D}(b_t, b_t^f, 0, a_t) = \max_{c_t^*, b_{t+1}^{*f}} v(c_t^*) + \beta^* \int_A V^*((1+r^*)b_t, b_{t+1}^f, 2, a_{t+1}) d\mu(a_{t+1}|a_t)$$
$$s.t. \quad c_t^* - q^f(b_{t+1}, b_{t+1}^f, 0, a_t) b_{t+1}^{*f} = \bar{W}^* - b_t^{*f} \quad (29)$$

# Model: Renegotiation Problem

- Preemptive vs. post-default renegotiations
  - Symmetric in bargaining game and power
  - Timing: **Prior to** vs. **after** TFP realization
  - Sovereign's outside options: **Non-preemptive option** vs. **permanent autarky**
  - Creditors' outside options: **Ex ante expected return** vs. **zero recovery rates**
- Strategies of the proposer  $i$  and the other party  $j$  (for  $i, j = B, L$ ) depending on state, current offer and types of debt renegotiations:

- Post-default renegotiations

$$\theta_i = \{1 \text{ (propose)}\} \quad \& \quad \theta_j = \{1 \text{ (accept)}\}$$

$$\theta_i = \{0 \text{ (pass)}\} \quad \& \quad \theta_j = \{0 \text{ (reject)}\}$$

- Preemptive renegotiations

$$\theta_i = \{1 \text{ (propose)}\} \quad \& \quad \theta_j = \{1 \text{ (accept)}\}$$

$$\theta_i = \{0 \text{ (pass)}\} \quad \& \quad \theta_j = \{0 \text{ (reject)}\}$$

$$\theta_i = \{-1 \text{ (quit)}\} \quad \& \quad \theta_j = \{-1 \text{ (quit)}\}$$



# Model: Preemptive Debt Renegotiation

- Case when the borrower B is the proposer
- If B proposes and the proposal is accepted,

$$V^{PRO}(b_t, b_t^f, 1, a_{t-1}) = \max_{T_t} \int_A [u(c_t, L_t) + \beta \int_A V(b_{t+1}, b_{t+1}^f, 0, a_t)] d\mu(a_t | a_{t-1}) \quad (26)$$

s.t. (8) (9) (10) and

$$T_t = \delta_t^B b_t \quad (16a)$$

$$V^{PRO}(b_t, b_t^f, 1, a_{t-1}) \geq V^{NON-PRE}(b_t, b_t^f, 0, a_{t-1}) \quad (27)$$

$$V^{*,ACT}(b_t, b_t^f, 1, a_{t-1}) = \max_{c_t^*, b_{t+1}^{*f}} \int_A [v(c_t^*) + \beta V^*(b_{t+1}, b_{t+1}^{*f}, 0, a_t)] d\mu(a_t | a_{t-1})$$

$$\text{s.t. } c_t^* - q(b_{t+1}, b_{t+1}^f, 1, a_t) b_{t+1} - q^f(b_{t+1}, b_{t+1}^f, 1, a_t) b_{t+1}^{*f} = \bar{W}^* - \delta_t^B b_t - b_t^{*f} \quad (28)$$

$$V^{*,ACT}(b_t, b_t^f, 1, a_{t-1}) \geq V^{*Non-PRE}(b_t, b_t^f, 0, a_{t-1}) \quad (29)$$

# Model: Preemptive Debt Renegotiation

- If B passes,

$$V^{PASS}(b_t, b_t^f, 1, a_{t-1}) = \max_{g_t, k_{t+1}^g, T_t} \int_A [u(c_t, L_t) + \beta \int_A \Psi(b_t, b_{t+1}^f, 1, a_t)] d\mu(a_t | a_{t-1}) \quad (30)$$

s.t. (8) (9) (10) and

$$V^{PASS}(b_t, b_t^f, 1, a_{t-1}) \geq V^{NON-PRE}(b_t, b_t^f, 0, a_{t-1}) \quad (27a)$$

$$V^{*,REJ}(b_t, b_t^f, 1, a_{t-1}) = \max_{c_t^*, b_{t+1}^{*f}} \int_A [v(c_t^*) + \beta V^*(b_t, b_{t+1}^{*f}, 1, a_t)] d\mu(a_t | a_{t-1})$$

$$\text{s.t. } c_t^* - q^f(b_{t+1}, b_{t+1}^f, 1, a_t) b_{t+1}^{*f} = \bar{W}^* - b_t^f \quad (31)$$

$$V^{*,REJ}(b_t, b_t^f, 1, a_{t-1}) \geq V^{*Non-PRE}(b_t, b_t^f, 0, a_{t-1}) \quad (29a)$$

- If B quits,

$$V^{QUIT}(b_t, b_t^f, 1, a_{t-1}) = V^{NON-PRE}(b_t, b_t^f, 0, a_{t-1}) \quad (33)$$

$$V^{*,QUIT}(b_t, b_t^f, 1, a_{t-1}) = V^{*,NON-PRE}(b_t, b_t^f, 0, a_{t-1}) \quad (34)$$

# Model: Preemptive Debt Renegotiation

- Equilibrium

$$\begin{aligned} \delta_t^{B*} &= \operatorname{argmax} V^{PRO}(b_t, b_t^f, 1, a_{t-1}) \\ \text{s.t. } V^{PRO}(b_t, b_t^f, 1, a_{t-1}) &\geq V^{PASS}(b_t, b_t^f, 1, a_{t-1}) \\ V^{*ACT}(b_t, b_t^f, a_{t-1}) &\geq V^{*REJ}(b_t, b_t^f, a_{t-1}) \end{aligned} \quad (35)$$

- If both parties reach an agreement,

$$\begin{aligned} \Psi^B(b_t, b_t^f, 1, a_{t-1}) &= V^{PRO}(b_t, b_t^f, 1, a_{t-1}) \\ \Psi^{B*}(b_t, b_t^f, 1, a_{t-1}) &= V^{*ACT}(b_t, b_t^f, 1, a_{t-1}) \end{aligned} \quad (36)$$

- Otherwise,

$$\begin{aligned} \Psi^B(b_t, b_t^f, 1, a_{t-1}) &= V^{PASS}(b_t, b_t^f, 1, a_{t-1}) \\ \Psi^{B*}(b_t, b_t^f, 1, a_{t-1}) &= V^{*REJ}(b_t, b_t^f, 1, a_{t-1}) \end{aligned} \quad (36a)$$

or

$$\begin{aligned} \Psi^B(b_t, b_t^f, 1, a_{t-1}) &= V^{QUIT}(b_t, b_t^f, 1, a_{t-1}) \\ \Psi^{B*}(b_t, b_t^f, 1, a_{t-1}) &= V^{*REJ-PRE}(b_t, b_t^f, 1, a_{t-1}) \end{aligned} \quad (36b)$$

# Model: Preemptive Debt Renegotiation

- Settlement set for preemptive renegotiation

$$R^B(b_t, b_t^f, 1) = \left\{ a_{t-1} \in A : \begin{array}{l} V^{PRO}(b_t, b_t^f, 1, a_{t-1}) \geq V^{PASS}(b_t, b_t^f, 1, a_{t-1}) \\ V^{*ACT}(b_t, b_t^f, 1, a_{t-1}) \geq V^{*REJ}(b_t, b_t^f, 1, a_{t-1}) \end{array} \right\} \quad (37)$$

# Quantitative Analysis

- TFP process -AR(1) process:

$$\log(a_t) = \rho \log(a_{t-1}) + \epsilon_t,$$

- Household utility function - CRRA:

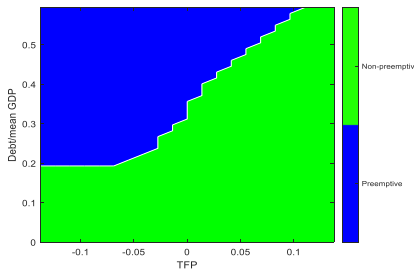
$$u(c_t, l_t) = \frac{(c_t - \frac{l_t^{1+\psi}}{1+\psi})^{1-\sigma}}{1-\sigma},$$

Parameter	Value	Source
Int. goods share in gross output of final goods	$\alpha_m = 0.43$	Argentina's national accounts
Capital share in gross output of final goods	$\alpha_k = 0.17$	Standard capital share in GDP (0.3)
Labor share in gross output of final goods	$\alpha_L = 0.4$	Standard labor share in GDP (0.7)
Labor share in GDP of int. goods	$\gamma = 0.7$	Standard labor share in GDP (0.7)
Coefficient of relative risk aversion	$\sigma = 2$	Standard RBC value
Risk-free interest rate	$r^* = 0.01$	Standard RBC value
Bargaining power	$\theta = 0.975$	Asonuma and Joo (2020) -Argentina 2001-05
Curvature parameter of labor supply	$\omega = 1.455$	Frisch wage elasticity (2.2)
Armington weight of domestic inputs	$\lambda = 0.62$	Regression estimate
Armington curvature parameter	$\mu = 0.65$	Regression estimate
Dixit-Stiglitz curvature parameter	$\nu = 0.59$	Gopinath and Neiman (2010)
Autocorrelation of TFP shocks	$\rho_\epsilon = 0.95$	Estimated
Standard deviation of TFP innovations	$\sigma_\epsilon = 0.017$	Estimated
Intermediate goods TFP coefficient	$A = 0.31$	Estimated
Subjective discount factor	$\beta = 0.88$	Estimated
TFP semi-elasticity of exogenous capital flows	$\xi = -0.67$	Estimated

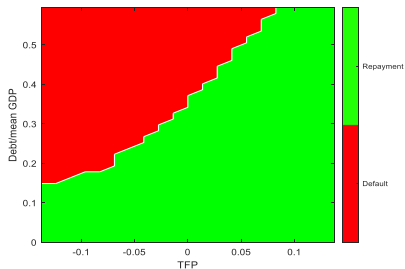
# Quantitative Analysis - Steady-state Distribution

- Debtor's choice between preemptive and non-preemptive and between repayment and default
  - Preemptive - when debt is high
  - Default - when debt is high and current TFP is low

(a) Choice for Preemptive Restructuring  
(ex-ante: lagged TFP)



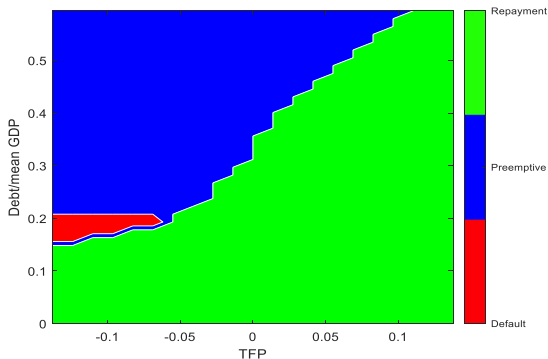
(b) Choice for Default and Repayment  
(ex post: current TFP)



# Quantitative Analysis - Steady-state Distribution

- Debtor's choice among preemptive, default and repayment
  - Replication of Asonuma and Trebesch (2016)

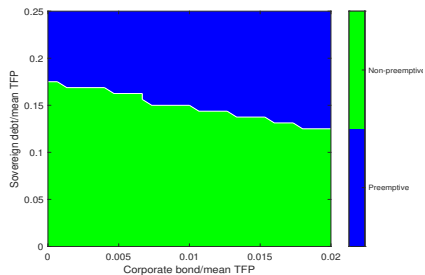
(c) Choice for Preemptive Choice, Default and Repayment



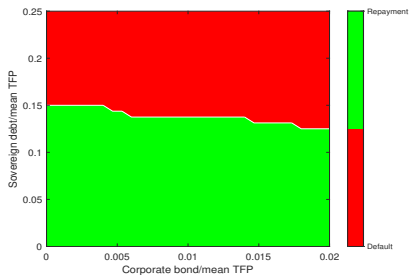
# Quantitative Analysis - Steady-state Distribution

- Debtor's choice between preemptive and non-preemptive and between repayment and default

(a) Choice for Preemptive Restructuring  
(ex-ante: mean lagged TFP)



(b) Choice for Default and Repayment  
(ex post: mean current TFP)

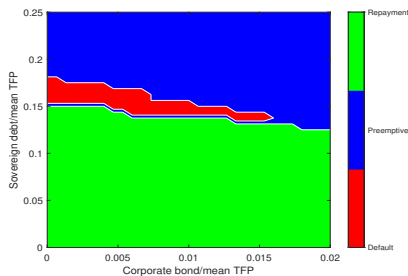




# Quantitative Analysis - Steady-state Distribution

- Debtor's choice between preemptive and non-preemptive and between repayment and default

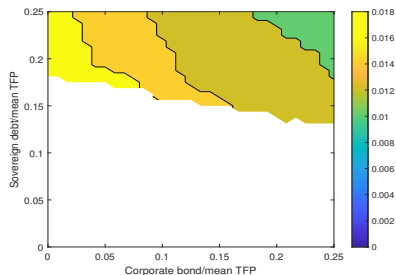
(b) Choice for Default and Repayment  
(ex post: mean current TFP)



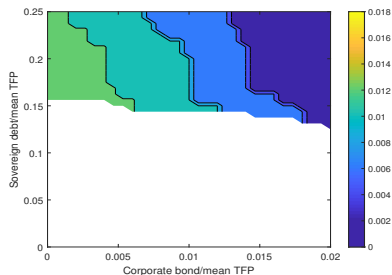
# Quantitative Analysis - Steady-state Distribution

- Private sector borrowing choice

(a) Corporate borrowing under preemptive



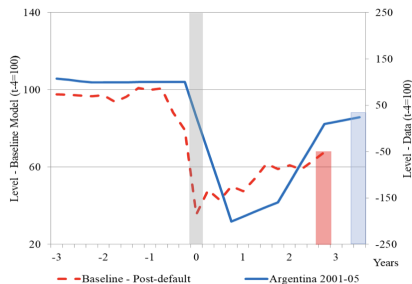
(b) Corporate borrowing under default



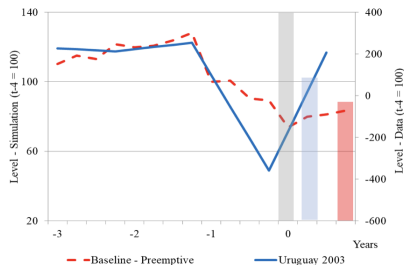
# Quantitative Analysis

- Private sector borrowing dynamics

(a) Post-default – Argentina



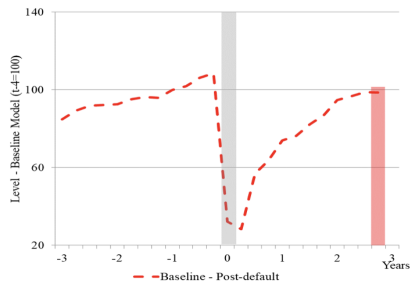
(b) Preemptive – Uruguay



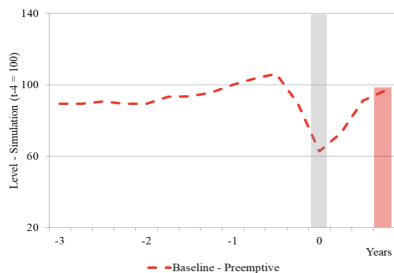
# Quantitative Analysis

- Output dynamics

(a) Post-default – Argentina



(b) Preemptive – Uruguay



# Conclusion

- New data and stylized facts on private sector borrowing, output dynamics during sovereign debt restructurings.
- New theoretical explanation on private sector borrowing and output dynamics during sovereign debt restructurings
  - Role of sovereign default/restructuring on private sector borrowing and output dynamics
- Quantitative analysis of model rationalizes the stylized facts

# Model: Final Goods Producer's Problem under Corporate Defaults

- Final goods producer's production function under corporate default

$$y_t^{CD} = a_t(M^{f,D}(m_t^d))^{\alpha_M}(L_t^f)^{\alpha_L}(\bar{k})^{\alpha_K} \quad (3')$$

$$M^{CD}(m_t^d) = [\lambda(m_t^d)^\mu]^{\frac{1}{\mu}} \quad (4')$$

- Final goods producer's production function under corporate default

$$\max_{m_t^d, m_t^*, L_t^f} \pi_t^{CD}(b_t, b_t^f, 0, a_t) = a_t(M^{f,D}(m_t^d))^{\alpha_M}(L_t^f)^{\alpha_L}(\bar{k})^{\alpha_K} - w_t L_t^f - p_t^m m_t^d$$

# Model: Sovereign's Problem - Ex Post

- Ex post value of sovereign

$$V(b_t, b_t^f, 0, a_t) = \max[V^R(b_t, b_t^f, 0, a_t), V^{D,CD}(b_t, b_t^f, 0, a_t)] \quad (17')$$

- Ex post value of repayment

$$V^R(b_t, b_t^f, 0, a_t) = \max_{c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t, b_{t+1}^f, b_{t+1}} u(c_t, L_t) + \beta \int_A V(b_{t+1}, b_{t+1}^f, 0, a_{t+1}) d\mu(a_{t+1}|a_t)$$

$$\text{s.t. } c_t = a_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L} (\bar{k})^{\alpha_K}$$

$$q^f(b_{t+1}, b_{t+1}^f, 0, a_t) b_{t+1}^f = p_t^* m_t^*$$

$$L_t^m + L_t^f = L_t$$

$$A(L_t^m)^\gamma = m_t^d \quad (18)$$

# Model: Sovereign's Problem - Ex Post

- **Ex post** value of default/post-default restructuring:

$$V^D(b_t, b_t^f, 0, a_t) = \max_{c_t, m_t^d, L_t^f, L_t^m, L_t} u(c_t, L_t) + \beta \int_A V((1+r^*)b_t, b_{t+1}^f, 2, a_{t+1}) d\mu(a_{t+1}|a_t)$$

$$\text{s.t. } c_t = a_t (M^{f,D}(m_t^d))^{\alpha_M} (L_t^f)^{\alpha_L} (\bar{k})^{\alpha_K}$$

$$L_t^m + L_t^f = L_t$$

$$A(L_t^m)^\gamma = m_t^d \tag{20'}$$



## Model: Post-default Renegotiation

- Case when the borrower B is the proposer
- If B proposes and the proposal is accepted,

$$V^{PRO}(b_t, b_t^f, 2, a_t) = \max_{T_t} u(c_t, l_t) + \beta \int_A V(b_{t+1}, b_t^f, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (38)$$

s.t. (8b), (9b), (10b) and

$$T_t = \alpha_t^B b_t \quad (8b)$$

$$V^{*ACT}(b_t, b_t^f, 2, a_t) = -\alpha_t^B b_t \quad (39)$$

## Model: Post-default Renegotiation (cont.)

- If B passes,

$$\begin{aligned} V^{PASS}(b_t, b_t^f, 2, a_t) &= \max_{T_t} u(c_t, l_t) \\ &+ \beta \int_A V((1+r^*)b_t, b_t^f, 2, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (40) \\ &s.t. (8b), (9b), (10b) \end{aligned}$$

$$V^{*REJ}(b_t, b_t^f, 2, a_t) = \frac{1}{1+r^*} \int_A \Gamma^*((1+r^*)b_t, b_t^f, 2, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (41)$$

## Model: Post-default Renegotiation (cont.)

- Equilibrium

$$\begin{aligned} \alpha_t^{B*} &= \operatorname{argmax} V^{PRO}(b_t, b_t^f, 2, a_t) \\ \text{s.t. } V^{PRO}(b_t, b_t^f, 2, a_t) &\geq V^{PASS}(b_t, b_t^f, 2, a_t) \\ V^{*ACT}(b_t, b_t^f, 2, a_t) &\geq V^{*REJ}(b_t, b_t^f, 2, a_t) \end{aligned} \quad (42)$$

- If both parties reach an agreement,

$$\begin{aligned} \Gamma^B(b_t, b_t^f, 2, a_t) &= V^{PRO}(b_t, b_t^f, 2, a_t) \\ \Gamma^{B*}(b_t, b_t^f, 2, a_t) &= V^{*ACT}(b_t, b_t^f, 2, a_t) \end{aligned} \quad (43)$$

- Otherwise,

$$\begin{aligned} \Gamma^B(b_t, b_t^f, 2, a_t) &= V^{PASS}(b_t, b_t^f, 2, a_t) \\ \Gamma^{B*}(b_t, b_t^f, 2, a_t) &= V^{*REJ}(b_t, b_t^f, 2, a_t) \end{aligned} \quad (43a)$$

- Settlement set for post-default renegotiation

$$R^B(b_t, b_t^f, 2) = \left\{ a_t \in A : \begin{aligned} &V^{PRO}(b_t, b_t^f, 2, a_t) \geq V^{PASS}(b_t, b_t^f, 2, a_t) \\ &V^{*ACT}(b_t, b_t^f, 2, a_t) \geq V^{*REJ}(b_t, b_t^f, 2, a_t) \end{aligned} \right\} \quad (44)$$