

# THE CONUNDRUM OF RECOVERY POLICIES: GROWTH OR JOBS?

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**Abstract:** This paper adopts a Neo-Schumpeterian approach to macroeconomics, by proposing a model which includes fully-endogenous growth, involuntary search-based unemployment, and financial frictions. The model analyzes the effects of several recovery policies used by governments to fight unemployment or/and enhance growth. Employment protection legislation reduces growth and unemployment. Policies that reduce the cost of job vacancies decrease unemployment and raise growth. Industrial policies in the form of production subsidies to young small firms, production taxes to adult large firms, and R&D subsidies increase growth and unemployment. Policies that reduce financial frictions accelerate growth but exert an ambiguous effect on unemployment.

*Keywords:* fully- endogenous growth, Schumpeterian unemployment, financial frictions, recovery policies, vacancy creation.

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# 1 Introduction

The recent great recession has sparked the debate of how governments create jobs (Blinder, 2009) and, more generally, how they identify and implement effective recovery policies. Caballero (2010, p. 96) states that crises appear to be inevitable and unpredictable. As a result, he suggests macroeconomists emphasize crafting appropriate recovery policy responses.

In this paper, we analyze the effects of recovery policies used by governments aiming at fighting unemployment and/or enhance economic growth. What are the effects of recovery policy instruments on involuntary unemployment and endogenous growth? Which policies unequivocally stimulate the rate of growth and reduce unemployment? Which policies generate tradeoffs between job-creation and growth, highlighting the conundrum of recovery? Are there any policy combinations leading to lower unemployment and higher growth rates?

We address these questions by using a dynamic, general-equilibrium model of Schumpeterian growth, search unemployment, and financial frictions with the following main features.<sup>1</sup> First, growth is endogenously driven by deliberate innovation efforts of entrepreneurial firms. Innovators discover production techniques that lower costs. The arrival of innovations is governed by a stochastic Poisson process. It generates fully-endogenous growth of output and total factor productivity (TFP). An innovator enjoys temporary monopoly profits that fuel investments in R&D.

Second, innovators encounter labor market frictions. They must engage in a stochastic search process to find, organize, and train workers prior to starting production at full capacity. The matching process requires the creation, maintenance, and management of costly job vacancies. Firms optimize the amount of vacancies based on profit-maximization considerations. Matching takes place between blocks of vacant positions and workers. As in the case of innovations, the arrival of job-matches is also governed by a stochastic Poisson process. The endogenous arrival of new technologies together with labor market frictions gives rise to involuntary, search-based unemployment of the Diamond-Mortensen-Pissarides (DMP) type.

Third, firms undertake Rent-Protection Activities (RPAs) to discourage the innovation efforts of potential competitors, with a view to prolonging monopoly tenure and delaying the emergence of a new technology leader. We assume that innovation depends directly on R&D investment and inversely on RPAs. The latter are financed by retained earnings and include expenditures on patent enforcement, trade

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<sup>1</sup> The term “Schumpeterian growth” refers to endogenous growth generated through the process of creative destruction, as described by Schumpeter (1934).

secrets, lobbying, and securing property rights etc.<sup>2</sup> In this model, RPAs have two key features: RPAs remove the counterfactual scale-effects property from the model resulting in fully-endogenous growth;<sup>3</sup> RPAs may dilute, even reverse, the expansionary impact of job-creating policies on growth. In other words, policies that reduce unemployment and channel resources into investment activities may hamper economic growth by stimulating RPAs more than R&D investment. As a result, RPAs are a driving force behind the recovery conundrum: jobless growth or stagnant growth with job-creation.

The paper does not model the nature and causes of financial frictions, e.g., informational asymmetries in financial markets, credit constraints, housing-market bubbles, fiscal and monetary policies, etc. Instead, to simplify the analysis and clarify the intuition of results, we assume the existence of an exogenous aggregate systemic risk that raises the probability of default for all firms. Specifically, we identify the impact of the U.S. 2007-2009 financial crisis by substantially increasing the systemic risk.

The model's equilibrium is unique, and entails the simultaneous presence of involuntary, search-based unemployment as well as Schumpeterian growth. The expected life of a firm is finite, and consists of four distinct, consecutive stages. The length of each stage is stochastic and endogenous. In the R&D phase, firm size is indeterminate, i.e., each firm is infinitesimally small. Upon discovering a new process innovation, a firm becomes a young technology leader, captures an exogenous and small share of the market, and enters the vacancy-creation process. It advertises new positions, interviews prospective workers, develops distribution systems, trains and organizes workers and suppliers. This process is stochastic and upon completion, the firm expands production and enters adult stage. The adult firm immediately captures the whole market. It is then targeted by potential innovators, and engages in RPAs to delay the emergence of a new technology leader. Lastly, the firm enters its old stage, during which it becomes a technology follower competing against a young technology leader. As an old firm, it still captures a large part of the market. It does not however engage in RPAs and will eventually be replaced by a new technology leader.

The model generates two types of industries, referred to as A and B industries. Type A industries consist of adult firms that serve the whole market and engage in RPAs. They are targeted by prospective in-

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<sup>2</sup> Costly vacancy creation, stochastic block (as opposed to individual) matching between firms and job-applicants, and inclusion of RPAs are three central features that differentiate our work from earlier Schumpeterian models of growth and unemployment, such as those of Aghion and Howitt (1994), and Şener (2000, 2001). For detailed empirical evidence on RPAs and theoretical applications, see among others Dinopoulos and Syropoulos (2007), Şener (2008), and Grieben and Şener (2009).

<sup>3</sup> The removal of scale effects further distinguishes our paper from the seminal study by Aghion and Howitt (1994). Ha and Howitt (2007), Madsen (2007, 2008), Ang and Madsen (2011), among others, argue that fully-endogenous growth theory is more empirically relevant than semi-endogenous growth theory. For arguments in favor of semi-endogenous growth theory, see Jones (2005).

novators. Type B industries consist of young and old firms. In a B industry a young technology leader tries to replace an old technology follower by creating more jobs through costly vacancies and stochastic matching. In other words, small, young firms create jobs in our model, whereas large, old firms destroy jobs.<sup>4</sup>

Albeit there exist frictions in the labor market, the model does not have transitional dynamics.<sup>5</sup> The absence of transitional dynamics is driven by two assumptions: perfect foresight in matching; and matching between one firm and many workers (a block) as opposed to matching between one firm and one worker (one-to-one matching). The absence of transitional dynamics suggests a short-run interpretation of our key results. We highlight this interpretation by assuming a fixed population level. Unless appropriate corrective policies are implemented, the absence of transitional dynamics implies that a financial crisis may have permanent adverse effects on employment and growth.

In our model, policies affect employment levels by impacting the rate of job destruction and vacancy creation.<sup>6</sup> We analyze the effects of six policies: four job-creation policies consisting of production subsidies targeting either young or adult technology leaders; taxes on old technology followers firing workers (employment protection); and subsidies targeting vacancies for young technology leaders; a pro-growth investment-related policy subsidizing firms engaged in R&D; and lastly, policies that reduce the systemic risk of default such as interest rate subsidies.

Our model generates intriguing results. A number of policies set up the *conundrum* of recovery. They reveal a positive relationship between growth and unemployment: jobless growth or stagnant growth with job creation. We find that production subsidies for adult firms and employment protection legislation imply a trade-off between growth and job-creation. These policies reduce the rates of unemployment and growth. Similarly, production subsidies for young firms and R&D subsidies also imply the same trade-off between growth and job-creation. They increase growth and unemployment.

We identify two types of recovery policies that unequivocally stimulate the rate of growth and reduce the rate of unemployment: policies that reduce the systemic risk in slow-growth economies; and subsidies

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<sup>4</sup> This modeling feature is consistent with Haltiwanger et al. (2010). They argue that firm age is more important than firm size in the process of job creation and destruction.

<sup>5</sup> The absence of transitional dynamics is a common property of Schumpeterian growth models. See, for example, Grossman and Helpman (1991, chapter 4), and Dinopoulos and Syropoulos (2007). We conjecture that the introduction of one-to-one matching between workers and firms, human and/or physical capital accumulation would generate transitional dynamics.

<sup>6</sup> Our model generalizes the related work of Mortensen (2005) who assumes costless vacancy creation. This assumption implies that firms do not choose optimally the number of job vacancies. As a result, Mortensen's model does not allow policies to affect employment through a vacancy-creation process.

that reduce the cost of vacancy creation *ex-ante* (i.e., prior successful matching) rather than *ex-post* (i.e., after successful matching). These policies reveal no trade-offs between growth and employment.

Simulation analysis suggests that policy combinations can lead to a higher rate of growth and a lower unemployment rate. For example, the appropriate combination of production subsidies targeting adult and young firms, or a combination of an R&D subsidy and a production subsidy targeting adult firms can stimulate growth and reduce unemployment.

The remainder of the paper is organized as follows. Section 2 describes the elements of the model. Section 3 derives the equilibrium conditions formally and illustrates the equilibrium graphically. Section 4 addresses the comparative-static effects of several investment and labor-market policies. Section 5 offers concluding remarks. Algebraic derivations are relegated to various appendices.

## 2 The Model

Our model is a first in adopting a *Neo-Schumpeterian* approach to macroeconomics in analyzing the effects of recovery policies. It combines fully-endogenous Schumpeterian growth and Schumpeterian unemployment. Within the class of endogenous growth models, it is the first to introduce both labor-market and financial frictions.

Our model differs from the real business cycle (RBC) models in two important aspects. First, instead of relying on neoclassical growth theory featuring exogenous long-run TFP growth, our model generates fully-endogenous growth. Second, instead of generating voluntary unemployment through a leisure-work tradeoff as does the RBC approach, our model generates involuntary state-of-the-art equilibrium search unemployment of the type advanced by the DMP literature.

The Neo-Schumpeterian approach to macroeconomics also differs from the so called “periphery” approaches, using Caballero’s (2010) terminology. In contrast to these approaches that highlight the role of informational frictions but rely on a partial-equilibrium framework, we adopt the assumption of rational expectations and use a dynamic general-equilibrium framework.

Our model borrows its elements of growth from Dinopoulos and Syropoulos (2007) which omits unemployment considerations and financial frictions that constitute the main concerns of our paper. In addition, there exist three main differences between the approach of the present paper to modeling search unemployment and the standard DMP literature. First, while the DMP literature relies on the neoclassical growth model and exogenous idiosyncratic shocks to generate labor turnover and unemployment, our model employs an endogenous job-destruction mechanism linked to endogenous technological change.

Second, instead of gradual matches between one worker and one firm, we consider a stepwise matching process: an innovator immediately captures a small portion of the market, and then undertakes another step that involves block matching to drive out the incumbent firm. Thus, in contrast to the studies of Aghion and Howitt (1994) and Mortensen (2005), in our model the matching rate itself contributes to the endogenous job-destruction process. Third, unlike Mortensen (2005) or any other macro-labor study we are aware of, our paper combines a block-matching feature with costly vacancy creation. This combination renders unnecessary any bargaining between a worker and a firm. As a result, we are able to maintain the assumption of perfectly competitive labor markets, in accordance to endogenous growth theory.<sup>7</sup>

## 2.1 Consumers

The economy consists of a continuum of identical and infinitely-lived households whose measure is set equal to one. The size of each household is denoted by  $N$  and remains constant over time. Given the unit measure of households, the size of aggregate population also equals  $N$ .<sup>8</sup> Each household member inelastically supplies one unit of labor per period of time. The representative household maximizes the infinite horizon utility

$$H = \int_0^{\infty} e^{-\rho t} \log h(t) dt, \quad (1)$$

where  $\rho > 0$  is the subjective discount rate. The subutility function  $\log h(t)$  is defined as

$$\log h(t) \equiv \int_0^1 \log y(\omega, t) d\omega, \quad (2)$$

where  $y(\omega, t)$  is the per-capita demand for goods manufactured in industry  $\omega$  at time  $t$ . The economy consists of a continuum of structurally identical industries indexed by  $\omega \in [0, 1]$ . Household optimization can be viewed as a two-stage problem. The first stage is a static optimization problem where each household allocates consumption expenditure to maximize  $h(t)$  for any given product prices. Since goods enter the subutility function in a symmetric fashion, each household spreads its per-capita consumption expenditure  $c(t)$  evenly across all available goods. Thus, demand for each good equals

$$Y(\omega, t) = c(t)N/P(\omega, t), \quad (3)$$

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<sup>7</sup> In Mortensen (2005), the bargaining solution between each firm and each worker substitutes the firm's choice of profit-maximizing vacancies. With block matching, the individual applicant has no bargaining power. This feature is realistic because most unemployed workers are not organized in labor unions and coordination among job applicants does not occur in practice.

<sup>8</sup> Allowing for positive population growth leaves the key results intact.

where  $Y(\omega, t) = y(\omega, t)N$ , and  $P(\omega, t)$  is the market price of the purchased goods in industry  $\omega$  at time  $t$ . From now, for notational simplicity, we drop the time index  $t$  where appropriate.

The second stage involves a dynamic optimization problem in which each household chooses the evolution of  $c$  over time. Substituting (2) into (1) and using  $Y$  from (3), one can simplify the household's dynamic problem to maximizing  $\int_0^\infty e^{-\rho t} \log c \, dt$  subject to the budget constraint  $\dot{A} = W + (r - \chi)A - cN$ , where  $A$  denotes the asset holdings of each household, and  $W$  is household expected wage income. Variable  $r$  is the rate of return obtained from a completely diversified asset portfolio. This portfolio allows investors to avoid idiosyncratic firm-level risk, although they are still subject to systemic aggregate risk. This is captured by the risk premium parameter  $\chi \geq 0$  implying a depreciation of household assets by  $\chi$  percent. That is, in normal times we assume that  $\chi = 0$ . During a financial crisis, households expect a default on  $\chi$  percent of all financial investments, without ex-ante being able to identify risky investments.<sup>9</sup> This implies that the systemic-risk-adjusted rate of return on a fully diversified (idiosyncratic-risk-free) portfolio is  $r - \chi$ . The solution of the dynamic optimization problem provides the Keynes-Ramsey rule, amended by the risk premium,<sup>10</sup>

$$\dot{c}/c = r - \chi - \rho. \tag{4}$$

Because the labor supply and the wage rate are constant in the steady state, equation (4) implies a constant per-capita consumption expenditure measured in units of labor<sup>11</sup> and  $r = \rho + \chi$  in equilibrium. Hence, the return to a fully-diversified portfolio  $r$  must compensate for the systemic risk. In addition, because the systemic risk augments the subjective discount rate, an interest rate subsidy has the same economic impact as a systemic-risk reduction.

## 2.2 Job Creation and Destruction

Labor is the only factor of production. The labor force consists of low-skilled and high-skilled workers.

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<sup>9</sup> We include this risk premium as a convenient shortcut to capture the perceived risk of financial intermediaries going bankrupt, without explicitly modeling such institutions and money. This modeling approach does not require that default on investments actually happens – the mere expectation of it suffices to produce a positive risk premium, reflecting mistrust that Akerlof and Shiller (2009) have identified as one of the main causes of economic depressions.

<sup>10</sup> Each household consists of a large number of members who engage in income transfers such that each member enjoys the same level of consumption regardless of individual earnings. This implies the absence of effective uncertainty in individuals' income and consumption emanating from idiosyncratic firm-level risk. Bayer and Wälde (2011) offer a modified version of (4) which takes into account individual income uncertainty.

<sup>11</sup> Nevertheless, the aggregate price index  $P_{AGG}$  declines over time whenever innovation takes place as will be shown later. As a result, real per-capita consumption measured in units of final output grows over time.

The proportion of the former is given as  $1 - s$  and that of the latter is given as  $s \in (0, 1)$ . Low-skilled workers can be employed in manufacturing only, whereas high-skilled workers can be employed in either R&D or RPAs.<sup>12</sup> We assume that high-skilled workers can find employment instantly without going through a job-matching process. Hence only low-skilled workers are subject to turnover and face the prospect of unemployment.<sup>13</sup>

Consider next the hiring process of an innovator. In each industry, production technology improves through the stochastic arrival of process innovations. We assume that a young technology leader (an entrant) can immediately hire a small number of unskilled workers without engaging in costly search. As a result, it captures an exogenous fraction  $\phi \in (0, 1)$  of the market and forces the incumbent to lay off a corresponding number of workers.<sup>14</sup> To capture the remaining fraction  $1 - \phi$  of the market, an entrant must expand capacity and therefore engage in costly search by posting vacant positions. While the entrant is searching, the incumbent continues to supply a fraction  $1 - \phi$  of the market.<sup>15</sup> When the entrant completes the hiring process, which occurs with endogenous instantaneous probability  $q$ , the incumbent exits the market and all of its remaining workers join the unemployment pool. Further innovation in the industry triggers again the above job creative-destruction cycle. Hence, at any point in time, young firms create and maintain job vacancies and unemployed workers search for and fill the available job vacancies.

### 2.3 *Industry Structure*

The assumptions that all industries are structurally identical and that only adult firms are targeted by challengers engaged in R&D imply that, at each point in time, there are two possible industry configurations which we refer to as A and B industries. In A industries, there is an adult technology leader serving the

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<sup>12</sup> By allowing resource mobility between R&D and RPAs, we endogenize the intensity of R&D activity and capture an essential feature of endogenous growth theory. This labor assignment is similar to Dinopoulos and Syropoulos (2007) and also Grieben and Şener (2009). In these papers labor mobility between R&D and manufacturing is assumed, while the portion of labor devoted to RPAs is kept fixed. In contrast, here the portion of labor allocated to manufacturing is fixed but workers face unemployment in this sector.

<sup>13</sup> This is a commonly used assumption in the literature. See, among others, the dynamic growth settings of Şener (2001, 2006) and the static model of Davis (1998). This assumption captures in a simple way the well-established unemployment differential between high-skilled and low-skilled workers (see e.g. Nickell and Bell, 1995 and 1996, for descriptive evidence on seven major OECD countries). Moreover, because vacancy creation is costly, the assumption of costly high-skilled labor matching would create a conflict with the assumption of free entry in R&D activities.

<sup>14</sup> One can view this feature as follows. The technology leader instantaneously employs a share  $\phi$  of workers employed by the existing incumbent monopolist. Switching to the technology leader makes sense for these workers because they escape the impending unemployment risk.

<sup>15</sup> This step-wise replacement mechanism follows the spirit of Dinopoulos and Waldo (2005, pp. 141-142) where a successful product innovator instantaneously captures a small share of the market followed by a gradual switch of consumers from the previous-generation product to the state-of-the-art quality product.



entire market and entrepreneur firms that invest in R&D to discover the next process innovation. At the same time, each adult firm engages in RPAs to protect its monopoly profits by retarding the innovation effort of challengers. The reader can think of industries A as “growth-oriented” industries because they are targeted by future innovators. In B industries, there is a technology follower in its old phase serving a fraction  $1 - \phi$  of the market, and a young technology leader with the state-of-the-art production process, serving a fraction  $\phi$  of the market and thereby exerting partial monopoly power. At the same time, the new technology leader is searching to hire workers and drive the old technology follower completely out of business. One can think of B industries as “employment-oriented” industries: each young firm invests in vacancy maintenance and hiring of new workers aiming at expanding capacity and employment.

Let  $n_A$  and  $n_B = 1 - n_A$  represent the fraction (measure) of A and B industries, respectively. Let also  $I(\omega) = I$  denote the intensity of the Poisson process that governs the arrival of innovations in each industry. An A industry switches to a B industry with instantaneous probability  $I dt$ . Hence, the expected flow of industries from A into B is  $n_A I dt$ . When a young firm successfully completes its hiring process, a B industry switches to an A industry. The probability of this event is  $q dt$  and hence the expected flow of industries from B into A is  $(1 - n_A) q dt$ . Consequently, the net flow into the A industries is  $dn_A = (1 - n_A) q dt - n_A I dt$ , which implies

$$\dot{n}_A = q(1 - n_A) - I n_A. \quad (5)$$

## 2.4 Product Markets

Manufacturing of final consumer goods uses low-skilled labor only according to a constant returns to scale production function  $Y_i = \lambda^{m_i} Z_i$ , where  $Y_i$  is the output of firm  $i$ ,  $\lambda > 1$  is a parameter capturing the size of each process innovation, integer  $m_i$  is the number of process innovations which have occurred until the time of production, and  $Z_i$  is the number of low-skilled workers employed. In other words, the term  $\lambda^{m_i}$  captures the total factor productivity (TFP) component of production.

Let  $\lambda^{m(\omega)}$  represent the state-of-the-art productivity level in industry  $\omega$ . Consider an adult firm in an A industry that has access to the state-of-the-art  $m^{\text{th}}$  technology and has completed the hiring process. For this firm, the marginal (and average) cost of manufacturing one unit of final goods is  $w_L / \lambda^{m(\omega)}$ , where  $w_L$  is the wage rate of low-skilled labor. Hence  $1 / \lambda^{m(\omega)}$  measures the amount of low-skilled labor required per unit of output in industry  $\omega$ .

The adult firm competes against a follower with access to technology one step down the technology ladder, i.e. the  $[m(\omega) - 1]^{\text{th}}$  technology, and a unit cost of  $w_L/\lambda^{m(\omega)-1}$ . These firms compete in a Bertrand fashion: the technology leader uses its cost advantage to engage in limit pricing and capture the entire market. In equilibrium, the adult firm in an A industry charges a price  $P_a(\omega) = w_L/\lambda^{m(\omega)-1}$  and incurs a unit cost  $w_L(1 - \sigma_a)/\lambda^{m(\omega)}$ , where  $0 < \sigma_a < 1$  ( $\sigma_a < 0$ ) is the adult firm's production subsidy (tax) rate.<sup>16</sup> The adult firm captures the entire market demand  $cN/P_a(\omega)$ . Thus, in an A industry, an adult firm earns a flow of monopoly profits

$$\pi_a = \frac{cN}{w_L/\lambda^{m-1}} \left[ \frac{w_L}{\lambda^{m-1}} - \frac{w_L}{\lambda^m} (1 - \sigma_a) \right] = \frac{cN [\lambda - (1 - \sigma_a)]}{\lambda}. \quad (6)$$

The demand for low-skilled labor engaged in manufacturing equals<sup>17</sup>

$$Z \equiv \frac{cN}{\lambda w_L}. \quad (7)$$

Hence, the incumbent's profit flow and labor demand are independent of  $m$ , the number of cumulative innovations used for production at time  $t$ , but depend on  $\lambda$ , the size of process innovations.

While an adult technology leader earns monopoly profits, it simultaneously invests in RPAs employing high-skilled labor at a wage rate of  $w_H$ . The cost of producing  $X$  units of RPAs is  $w_H \gamma X$ , where  $\gamma$  is the unit-labor requirement of such activities. Hence, the profit flow net of rent protection costs earned by an adult firm is given by

$$\pi_a^{\text{net}} = \pi_a - w_H \gamma X. \quad (8)$$

Consider now a typical B industry where there are two producing firms: a low-cost young firm with state-of-the-art technology  $m(\omega)^{\text{th}}$  that serves a portion  $\phi$  of the market; and a high-cost old firm with  $[m(\omega) - 1]^{\text{th}}$  technology supplying the remaining portion  $1 - \phi$  of the market. The profit flow of a young firm is equal to  $\phi \pi_y$ . In order to determine  $\pi_y$ , note that in B industries, a young firm having access to

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<sup>16</sup> Specifically, the low-cost adult firm can charge price  $P_a(\omega) = [w_L/\lambda^{m(\omega)-1}] - \varepsilon$ , where  $\varepsilon \rightarrow 0$  is infinitesimally positive. The high-cost firm can charge a price as low as its marginal cost  $w_L/\lambda^{m(\omega)-1}$ ; however, this price does not generate positive demand and forces the high-cost firm to exit the market. We assume that followers (previous technology leaders) retain the capacity to produce using their own technology and rehiring their old workers without going through costly worker search again. Thus, they face zero capacity maintenance costs, impose a constant threat to enter the market, and force low-cost producers to engage in limit pricing.

<sup>17</sup> Labor demand is given by output produced  $cN/P_a(\omega)$  times the unit-labor requirement  $1/\lambda^{m(\omega)}$ .

$m(\omega)^{\text{th}}$  technology competes in a Bertrand fashion with a follower having access to the  $[m(\omega) - 1]^{\text{th}}$  technology. In equilibrium, a young firm charges limit price  $P_y(\omega) = w_L / \lambda^{m(\omega)-1}$ , faces market demand  $\phi cN / P_y(\omega)$ , and incurs unit-cost  $\frac{w_L}{\lambda^{m(\omega)}}(1 - \sigma_y)$ , where  $0 < \sigma_y < 1$  ( $\sigma_y < 0$ ) is the young firm's production subsidy (tax) rate. The follower exits the market. The typical young firm does not invest in RPAs since its technology is not (yet) targeted by entrepreneurs. Thus, in a B industry, the profit flow earned by a young firm is given by

$$\phi\pi_y = \phi \frac{cN}{w_L / \lambda^m} \left[ \frac{w_L}{\lambda^m} - \frac{w_L}{\lambda^{m+1}}(1 - \sigma_y) \right] = \frac{\phi cN [\lambda - (1 - \sigma_y)]}{\lambda}. \quad (9)$$

Note that  $\pi_a = \pi_y$  for  $\sigma_y = \sigma_a$ .

In B industries, each old firm with the  $[m(\omega) - 1]^{\text{th}}$  technology can still retain its profit flow in a portion  $1 - \phi$  of the market due to labor market frictions. In this segment of the market, an old firm competes in a Bertrand fashion against another firm with access to the  $[m(\omega) - 2]^{\text{th}}$  technology. An old firm in a B industry charges a price equal to the marginal cost of the rival firm  $P_o(\omega) = w_L / \lambda^{m(\omega)-2}$  and incurs a unit cost  $w_L / \lambda^{m(\omega)-1}$ . Thus, an old firm in a B industry earns a profit flow  $(1 - \phi)\pi_a$ .

## 2.5 Job Vacancies and Matching

In B industries, young technology leaders hold vacancies in order to attract workers. Let  $V_S$  represent the market valuation of a successfully-matched vacancy, i.e., the expected discounted value of profits per worker employed. Let  $V_i$  denote all vacancies created by a young firm  $i$ . Let us also denote with  $\alpha$  the flow cost of holding a vacancy, which can be interpreted as a fixed recruitment cost that the firm incurs regardless of whether a job is filled.<sup>18</sup> Let  $q$  denote the probability that all vacant positions of a firm are matched. In other words,  $q$  is the probability that a young firm in a B industry becomes an adult firm serving an entire A industry. Young firm  $i$  chooses vacancies  $V_i$  to maximize  $qV_S V_i - \alpha V_i$ . The first term is the expected return from posting  $V_i$  vacancies and the second term is the cost of holding those vacancies. The firm takes the matching rate  $q$  and the marginal return from vacancy holding  $V_S$  as given. Maximizing the above expression with respect to  $V_i$  yields the first-order condition  $qV_S = \alpha$ .

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<sup>18</sup> Pissarides (1985) interprets vacancy costs as (fixed) opportunity costs of machines (capital) required for new job openings. We model vacancy maintenance costs as fixed costs following the standard search unemployment literature.

How is  $V_S$  determined? Successful matching implies a change in the valuation of a young firm that is given by  $V_a - V_y > 0$ , where  $V_a$  and  $V_y$  represent the valuation of an adult firm and a young firm, respectively. Dividing  $V_a - V_y$  by the amount of jobs held by an adult firm  $J_a$  yields  $V_S = (V_a - V_y)/J_a$ . In equilibrium, the amount of available jobs (demand for labor) must equal the amount of vacancies held by a young firm, that is,  $J_a = V_i = (1 - \phi)Z$ . All vacancies are subject to the same matching rate (i.e., there is block matching). A young firm does not find profitable to maintain more vacancies than the number of workers it will employ as an adult firm.<sup>19</sup> Substituting  $V_S$  and  $J_a$  into the first order condition  $qV_S = \alpha$  yields the following vacancy creation (VC) condition

$$q \frac{V_a - V_y}{(1 - \phi)Z} = \alpha \quad \mathbf{VC}, \quad (10)$$

where the LHS is the firm-specific expected benefit from holding a vacancy, and the RHS is the cost of maintaining a vacancy.<sup>20</sup>

Next, we establish a link between the firm-specific vacancy matching rate  $q$  and aggregate labor market conditions. Let  $V \equiv \sum V_i$  represent the level of economy-wide vacancies and  $U$  the level of economy-wide unemployment. The arrival of successful job matches is governed by a stochastic process whose intensity is given by the matching function  $M(U, V)$ . We assume that the matching function is concave, homogeneous of degree one and increasing in both arguments in accordance to the DMP literature.<sup>21</sup>

Let  $\theta \equiv V/U$  denote the number of vacancies per unemployed worker capturing labor-market tightness. Dividing  $M(U, V)$  by  $V$  yields the matching (hiring) rate of young firms  $q(\theta) = M(U/V, 1) = M(1/\theta, 1)$ . Similarly, dividing  $M(U, V)$  by  $U$  yields the job-finding rate of unemployed workers  $p(\theta) = M(1, V/U) = M(1, \theta)$ . Note that  $q(\theta)$  and  $p(\theta)$  are stochastic Poisson arrival rates, unlike the deterministic rates in

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<sup>19</sup> If a young firm opens more vacancies than the number of workers employed by an adult firm, then the return to holding an extra vacancy drops down to zero. Specifically,  $V_S > 0$  for  $V_i \in [0, (1 - \phi)Z]$ , whereas  $V_S = 0$  for  $V_i > (1 - \phi)Z$ . Note also that although the first-order condition  $qV_S = \alpha$  leaves firm-level vacancies indeterminate, it must hold for a finite level of vacancies.

<sup>20</sup> Equation (10) implies a knife-edge equilibrium condition which can be justified by an adjustment process linked to changes in matching rate  $q$ . Consider, for example, an increase in the marginal return of vacancy creation  $(V_a - V_y)/[(1 - \phi)Z]$ . This encourages young firms to offer more vacancies. For any aggregate unemployment rate, the excess supply of vacancies makes it more difficult for young firms to attract workers. Thus the firm-specific matching rate  $q$  declines to restore equilibrium.

<sup>21</sup> The matching function has the following additional properties:  $M(0, V) = M(U, 0) = 0$ ,  $\lim_{x \rightarrow 0} \partial M / \partial x = +\infty$ , and  $\lim_{x \rightarrow \infty} \partial M / \partial x = 0$ ,  $x \in \{U, V\}$ .

Aghion and Howitt (1994).<sup>22</sup> Observe that  $\partial q(\theta)/\partial \theta < 0$ , that is, as vacancies per unemployed worker increase, it becomes more difficult for firms to fill their vacant positions. Observe also that  $\partial p(\theta)/\partial \theta > 0$ , that is, as vacancies per unemployed worker increase, unemployed workers can find jobs more easily. The transition rates  $p(\theta)$  and  $q(\theta)$  satisfy  $p(\theta)U = q(\theta)V = M(U, V)$ . This matching scheme implies that vacant positions are matched at the rate of  $q(\theta)$  because  $V$  and  $U$  can be interpreted as firm-specific vacancies and job-applicants, respectively.

## 2.6 Innovation

Entrepreneurial firms engage in sequential and stochastic R&D races targeting A industries to discover next-generation process innovations and replace adult incumbent firms. The latter engage in RPAs in order to retard innovation efforts of challengers. The intensity of the Poisson process that governs the arrival of innovations for firm  $j$  is given by

$$I_j = R_j/D \quad \text{with} \quad D = \delta X, \quad (11)$$

where  $R_j$  represents R&D services of R&D lab  $j$ , and  $D$  measures the difficulty of conducting R&D. We model R&D difficulty  $D$  as a flow variable, where  $X$  is the level of RPAs undertaken by an incumbent adult firm, and parameter  $\delta$  is the efficiency of RPAs.

We assume that Poisson arrival rates are independently distributed across firms, industries, and time. Therefore, the industry-wide Poisson arrival rate equals

$$I = \sum_j I_j = \frac{R}{D} \quad \text{with} \quad R = \sum_j R_j. \quad (12)$$

## 2.7 Financial Market

There exists a stock market that channels household savings to firms engaged in R&D. Retained earnings finance RPAs and vacancy maintenance. During a typical R&D race, a firm issues a flow of shares to pay wages of R&D researchers. If a firm wins an R&D race, then it distributes the flow of profits to its stockholders as dividends; if the firm does not win an R&D race, its stockholders receive nothing. The existence of a continuum of industries and the assumption that Poisson arrival rates are independent across firms and over time imply that investors can fully diversify firm-specific idiosyncratic risk by holding an appropriate portfolio. The return to this stock portfolio is deterministic and equals the market interest rate

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<sup>22</sup> See Pissarides (2000, chapter 6) and Aghion and Howitt (1998, section 4.5) for alternative stochastic matching models, where only a fraction of contacts between workers and open vacancies lead to successful matches.

minus the systemic risk of default,  $r - \chi$ . At each instant in time, there exist distinct stocks issued by R&D labs, young, adult, and old firms. These stocks are traded freely among investors. The absence of profitable arbitrage in the stock market relates the expected equity returns to the effective interest rate of a riskless asset. In what follows we derive the no-arbitrage condition for each of the four stocks.

Let  $V_R$  denote the value of a firm engaged in R&D to discover the state-of-the-art process innovation. The no-arbitrage condition implies that the expected return to any stock issued by an R&D lab must equal the return generated by a fully diversified (idiosyncratic-risk-free) portfolio of equal size. In other words, the expected return of investing  $V_R$  in an R&D lab must equal  $(r - \chi)V_R$ . The expected income from investing  $V_R$  in an R&D lab is calculated as follows. Over a time interval  $dt$ , an R&D lab innovates with probability  $I_j dt$ , becomes a young firm, and realizes a valuation gain  $V_y - V_R$ . This firm incurs R&D costs  $w_H(1 - \sigma_R)\beta R_j$ , where  $0 < \sigma_R < 1$  ( $\sigma_R < 0$ ) is an R&D subsidy (tax) rate, and  $\beta > 0$  is the unit-labor requirement of R&D. With probability  $(1 - I_j dt)$ , however, success does not materialize, and stockholders absorb capital loss  $dV_R = \dot{V}_R dt$ . The presence of non-diversifiable risk implies that a firm engaged in R&D defaults with probability  $\chi dt$ , and its stockholders absorb capital loss  $V_R$ . Adding these components of equity return, we may write the no-arbitrage condition for an R&D lab as

$$I_j dt (V_y - V_R) - w_H (1 - \sigma_R) \beta R_j dt + (1 - I_j dt) \dot{V}_R dt - \chi V_R dt = (r - \chi) V_R dt . \quad (13)$$

It is apparent from (13) that capital loss generated by the systemic risk cancels out and does not affect the valuation of R&D firms. Free-entry in R&D activities drives firm value to zero, i.e.,  $V_R = \dot{V}_R = 0$ . Taking limits as  $dt \rightarrow 0$  and using (11) yields the following R&D free-entry condition<sup>23</sup>

$$V_y = \beta \delta w_H (1 - \sigma_R) X . \quad (14)$$

Consider now the stock market valuation of a young firm in a B industry. This firm serves a fraction  $\phi$  of the market by employing  $\phi Z$  units of labor and realizes profit flow  $\phi \pi_y$ . At the same time it maintains  $V_i = (1 - \phi)Z$  vacant positions aiming at expanding its capacity and capturing the entire market. As mentioned earlier, each vacancy costs  $\alpha > 0$  to maintain per unit of time. Thus, over time interval  $dt$  total costs of holding vacancies are  $\alpha(1 - \phi)Z dt$ . By incurring these costs, a young firm succeeds to complete the hiring process with instantaneous probability  $q(\theta) dt$ . This firm becomes an adult firm serving the entire market,

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<sup>23</sup> An alternative derivation of R&D free-entry condition is as follows. Consider firm  $j$  that is engaged in R&D. During the time interval  $dt$ , this firm incurs with certainty a cost of  $w_H(1 - \sigma_R)\beta R_j dt$  which corresponds to the subsidized wage bill of employing  $\beta R_j$  researchers. The expected benefit of R&D investment is  $V_y I_j dt$ . Setting the expected benefit equal to the cost of R&D, and using (11), yields (14).

and moves from industry B to A. Its stockholders realize a capital *gain*  $V_a - V_y > 0$ . With probability  $1 - q(\theta)dt$ , no matching occurs. In this case stockholders realize a change in valuation  $dV_y = \dot{V}_y dt$ . Finally, the firm defaults with instantaneous probability  $\chi dt$ , and its stockholders absorb capital loss  $V_y$ . In the absence of stock-market arbitrage opportunities, the expected return generated by investing an amount  $V_y$  in stocks issued by a young firm must equal the return of a fully diversified portfolio of equal size  $(r - \chi)V_y$ . Collecting terms, we may write the no-arbitrage condition for a young firm as

$$\phi\pi_y dt + q(\theta)(V_a - V_y)dt - \alpha(1 - \phi)Zdt + [1 - q(\theta)dt]\dot{V}_y dt - \chi V_y dt = (r - \chi)V_y dt. \quad (15)$$

Taking limits as  $dt \rightarrow 0$  yields the following expression for the stock market value of a young firm

$$V_y = \frac{\phi\pi_y + q(\theta)V_a - \alpha(1 - \phi)Z}{q(\theta) + r - \dot{V}_y/V_y}. \quad (16)$$

Next, consider the stock-market valuation of an adult firm. Over a small time interval  $dt$ , its stockholders receive dividends equal to the net profit flow  $\pi_a^{net} dt = (\pi_a - w_H \gamma X)dt$ . With instantaneous probability  $I dt$ , further process innovation occurs, and the adult monopolist becomes an old firm in a B industry with valuation  $V_o$ . In this case, stockholders of an adult firm absorb capital *loss*  $V_a - V_o > 0$ . In addition, the firm must lay off  $\phi Z$  manufacturing workers incurring a firing cost  $f > 0$  per worker.<sup>24</sup> With probability  $(1 - I dt)$ , no further innovation occurs in the industry. In this case stockholders realize a capital gain  $dV_a = \dot{V}_a dt$ . Finally, with instantaneous probability  $\chi dt$ , the firm defaults and its stockholders absorb capital loss  $V_a$ . Collecting terms, we may write the no-arbitrage condition of an adult firm as

$$(\pi_a - w_H \gamma X)dt - I(V_a - V_o + f\phi Z)dt + (1 - I dt)\dot{V}_a dt - \chi V_a dt = (r - \chi)V_a dt. \quad (17)$$

Taking limits as  $dt \rightarrow 0$  yields the following expression for the stock market value of an adult firm

$$V_a = \frac{I(V_o - f\phi Z) + \pi_a - w_H \gamma X}{I + r - \dot{V}_a/V_a}. \quad (18)$$

Finally, consider the stock market valuation of an old firm. This firm is a technology follower and serves  $1 - \phi$  fraction of a B-industry market. In a time interval  $dt$ , stockholders of an old firm receive  $(1 - \phi)\pi_a dt$  as dividend payments. With probability  $q(\theta)dt$ , a young technology leader drives an old firm out of

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<sup>24</sup> Parameter  $f$  can be interpreted as an employment termination or layoff tax imposed by the government on firms that lay off workers. This parameter captures the effects of employment protection legislation.

the market. In this event, the stockholders of an old firm absorb capital loss  $V_o$ . In addition, the old firm must lay off the remaining  $(1 - \phi)Z$  workers and must incur firing cost  $f > 0$  per worker. With probability  $1 - q(\theta)dt$ , no matching occurs in the industry. In this event, stockholders realize capital gain  $dV_o = \dot{V}_o dt$ . Finally, with probability  $\chi dt$ , the old firm defaults and the stockholders absorb capital loss  $V_o$ . Collecting terms, we may write the no-arbitrage condition of an old firm as

$$(1 - \phi)\pi_a dt - q(\theta)[V_o + f(1 - \phi)Z]dt + [1 - q(\theta)dt]\dot{V}_o dt - \chi V_o dt = (r - \chi)V_o dt. \quad (19)$$

Taking limits as  $dt \rightarrow 0$  yields the following expression for the stock market value of an old firm

$$V_o = \frac{(1 - \phi)[\pi_a - fq(\theta)Z]}{q(\theta) + r - \dot{V}_o/V_o}. \quad (20)$$

## 2.8 Rent Protection Activities

Adult firms serving A industries, face the threat of innovation and undertake rent protection activities (RPAs), denoted by  $X$ , aiming to prolong the expected duration of temporary monopoly profits by delaying the success of challengers. Adult firms optimally choose  $X$  at each point in time to maximize expected discounted profits, as stated in LHS of (17). This maximization yields the following *RPA condition*:<sup>25</sup>

$$w_H \gamma X = I(V_a - V_o + f\phi Z) \quad \mathbf{RPA}. \quad (21)$$

The LHS of (21) equals RPAs expenditure and increases with the threat of innovation  $I$  and the capital loss associated with successful innovation  $V_a - V_o + f\phi Z$ .

## 2.9 Labor Markets

At each instant in time, each low-skilled worker can either be employed or unemployed. In B industries, when a young firm expands capacity, an old firm exits the market and fires its low-skilled workers. The fraction of industries that experience this type of labor turnover is  $q(\theta)(1 - n_A)$ . In each B industry the number of workers employed by an old firm is  $(1 - \phi)Z$ . As a result, the flow of workers into the unemployment pool during time period  $dt$  equals  $q(\theta)(1 - n_A)(1 - \phi)Zdt$ .<sup>26</sup> The flow of workers out of unem-

<sup>25</sup> The RPA condition is derived as follows. Use (12) and (11), note that  $I(X) = R/(\delta X)$ , and set the derivative of the LHS of (17) with respect to  $X$  to zero. Using  $dI(X)/dX = -I/X < 0$  and taking limits as  $dt \rightarrow 0$  yields (21).

<sup>26</sup> There are additional flows into and out of unemployment that cancel out each other. In A industries, with instantaneous probability  $I$ , an entrepreneur successfully innovates, the incumbent monopolist loses a fraction  $\phi$  of the market, and lays off  $\phi Z$  workers. This event creates an inflow  $I n_A \phi Z$  into the unemployment pool. However, this is matched by instantaneous hiring of the same number of workers by successful entrepreneurs in all A industries.



ployment during time period  $dt$  is driven by successful job finding of unemployed workers, which is given by  $p(\theta)Udt$ . As a result, the equation of motion for the level of unemployment  $U$  is given by

$$\dot{U} = q(\theta)(1 - n_A)(1 - \phi)Z - p(\theta)U. \quad (22)$$

At each point in time, young technology leaders in B industries maintain vacant positions to hire workers. The fraction of B industries is equal to  $1 - n_A$ . The number of vacant positions in each industry is equal to labor demand  $V_i = (1 - \phi)Z$ . Thus, the economy-wide vacancy rate, defined as vacancies per low-skilled worker  $v \equiv \sum V_i / (1 - s)N = V / [(1 - s)N]$ , equals

$$v = \frac{(1 - n_A)(1 - \phi)Z}{(1 - s)N}. \quad (23)$$

There is a labor market for low-skilled workers and a separate one for high-skilled workers. In each market, the supply of employed workers must equal the demand for labor. As a result, the labor market clearing conditions for low-skilled and high-skilled workers may be written as

$$(1 - u)(1 - s)N = Z[n_A + (1 - n_A)\phi + (1 - n_A)(1 - \phi)] = Z, \quad (24)$$

$$sN = n_A(\gamma X + \beta R), \quad (25)$$

where  $u \equiv U / [(1 - s)N]$  is the unemployment rate of low-skilled workers.

Substituting  $Z$  from (24) into (22), and using the definitions of  $u$  and  $v$ , provides the following equation of motion for the rate of unemployment  $u$

$$\dot{u} = q(\theta)(1 - n_A)(1 - \phi)(1 - u) - p(\theta)u = 0, \quad (26)$$

where  $q(\theta)(1 - n_A)(1 - \phi)$  is the economy-wide *job-destruction rate*, and  $p(\theta)u$  is the economy-wide *job-finding rate*. Equation (26) states that  $\dot{u} = 0$ . This result is obtained by using (23) to substitute  $V/Z$  for  $(1 - n_A)(1 - \phi)$ ,  $V = v(1 - s)N$ , (24) to substitute for  $Z$ , and identity  $p(\theta)u \equiv q(\theta)v$  from the matching function. Equation (26) holds both in and out of steady-state equilibrium and implies that unemployment does not exhibit transitional dynamics. The absence of transitional dynamics is driven by two assumptions: perfect

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<sup>27</sup> Note that the equation of motion for vacancies  $V$  is the same as (22). In a fraction  $n_A$  of industries, with probability  $I$  young firms create  $V_i$  vacancies. The total matching at each point in time is given by  $pU = qV$ . It follows that  $\dot{V} = In_A V_i - qV$ . In the steady-state equilibrium,  $\dot{V} = 0$  and therefore  $V_i = qV / (In_A)$ . Substituting into this expression the stock of vacancies  $V$  from equation (23) and taking into account  $(1 - n_A) / n_A = I / q$  from in (5) with  $\dot{n}_A = 0$ , provides  $V_i = (1 - \phi)Z$ , and thus  $\dot{V} = In_A(1 - \phi)Z - pU$ . Using  $In_A = (1 - n_A)q$  yields the RHS of (22).

foresight in matching; and matching between one firm and many workers (a block) as opposed to matching between one firm and one worker (one-to-one matching).

### 3 Steady-State Equilibrium

We establish that the equilibrium is unique and does not exhibit transitional dynamics as in Dinopoulos and Syropoulos (2007). Appendices A, B and D provide algebraic details. We choose low-skilled labor as the numéraire by setting  $w_L \equiv 1$ .<sup>28</sup> At the steady-state equilibrium per-capita consumption expenditure  $c$  is constant over time. It then follows from equation (4) that the market interest rate is  $r = \rho + \chi$ .<sup>29</sup> In contrast, the arrival of process innovations generates positive endogenous growth of TFP, output, per-capita consumption expenditure measured in units of output, and consumer utility. The arrival of innovations generates deflation as the aggregate price level of final goods falls at constant rate  $\dot{P}_{AGG}/P_{AGG} = -n_A \Pi \log \lambda$  (please see Appendix C for details).

The equilibrium is characterized by the following system of three equations in three unknowns: the matching rate  $q(\theta)$ ; the rate of innovation  $I$  and the rate of unemployment  $u$  (please see Appendix A for details). The steady-state values of these variables are constant over time.

$$\frac{q}{(1-\phi)} \left[ \frac{\lambda-1+\sigma_a}{\rho+\chi} - \underbrace{\frac{2\phi(\lambda-1+\sigma_y)}{B(1-\sigma_R)(\rho+\chi)^2}}_{v_a/Z} - \underbrace{\frac{\phi(\lambda-1+\sigma_y)}{\rho+\chi}}_{v_y/Z} \right] = \alpha \quad \mathbf{VC}, \quad (27)$$

$$I = \frac{\frac{\phi(\lambda-1+\sigma_y)}{\rho+\chi}}{B(1-\sigma_R) \left[ \frac{\lambda-1+\sigma_a}{\rho+\chi} - \underbrace{\frac{2\phi(\lambda-1+\sigma_y)}{B(1-\sigma_R)(\rho+\chi)^2}}_{v_a/Z} - \underbrace{\frac{(1-\phi)(\lambda-1-fq)}{\rho+\chi+q}}_{v_o/Z} + f\phi \right]} \quad \mathbf{RP}, \quad (28)$$

$$\frac{qI}{I+q} (1-u)(1-\phi) = pu \quad \mathbf{CD}. \quad (29)$$

<sup>28</sup> Alternatively, one could normalize per capita consumption expenditure by setting  $c \equiv 1$ . This normalization provides explicit determination of low-skilled wage rate  $w_L$ . Equation (7) implies  $Z \equiv cN/(\lambda w_L) = N/(\lambda w_L)$ . Combining this result and (24) generates  $w_L = 1/[\lambda(1-u)(1-s)]$ . This approach, however, complicates the presentation of steady-state equilibrium without providing additional insights.

<sup>29</sup> In addition, the absence of population growth implies that the following endogenous variables remain constant over time: rate of vacancies  $v$ , R&D investment  $R$ , level of rent protection activities  $X$ , profit flows, stock market values of firms, allocation of labor across activities, and the wage of high-skilled labor  $w_H$ .

The **vacancy-creation (VC) condition** (27) is the reduced form of (10) and expresses the equilibrium matching rate  $q(\theta)$  as a function of parameters. The matching rate is a monotonically decreasing function of labor-market tightness, that is  $\partial q(\theta)/\partial \theta < 0$ . Let us denote with  $\theta \equiv \mu(q)$  the inverse function that determines the market tightness measure  $\theta$  as a declining function of the matching rate  $q$ . Therefore (27) pins down  $q$  and (unique)  $\theta = v/u$ , from which the job-finding rate of workers  $p(\theta)$  is determined.

The **relative-profitability (RP) condition** (28) expresses the rate of innovation  $I$  as a function of matching rate  $q(\theta)$  and parameters. Let us first consider the RHS. Term  $1/[B(1 - \sigma_R)]$  is the cost of R&D relative to RPAs. The numerator is the market value of a young firm per unit of output  $V_y/Z = \phi(\lambda - 1 + \sigma_y)(\rho + \chi)^{-1}$ . The term in square brackets captures the expected return to RPAs, measured by the difference in per-output market value between an adult firm and an old firm  $(V_a - V_o - f\phi)/Z$ . As a result, the RHS of the RP condition is proportional to  $V_y/(V_a - V_o + f\phi Z)$  which is the expected return of R&D relative to RPAs, i.e., the relative profitability of R&D. Observe that the RHS of (28) is monotonically decreasing in  $q$ . As a result, once the matching rate  $q$  is determined, equation (28) pins down the equilibrium innovation rate  $I$ .

The **creative destruction (CD) condition** (29) combines (26) with equation (5) with  $\dot{n}_A = 0$  imposed. The LHS of (29) corresponds to the rate of labor flow into unemployment, and the RHS represents the rate of labor flow out of unemployment. With  $q$ ,  $p$  and  $I$  determined, CD condition (29) and the expression of  $\theta = v/u$  from the VC equation (27) determine simultaneously the equilibrium levels of  $u$  and  $v$ .<sup>30</sup>

The creative destruction condition corresponds to a general-equilibrium version of the Beveridge curve, which plays a prominent role in DMP models of search unemployment.<sup>31</sup> Specifically, one version of the Beveridge curve is obtained by assuming that the job-separation function  $\Psi(I, \theta) \equiv (1 - \phi)q(\theta)I/[I + q(\theta)] = \bar{\psi}$  is an exogenous parameter. One can then express equation (29) as  $u = \bar{\psi}/[\bar{\psi} + p(v/u)]$  which

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<sup>30</sup> With  $I$ ,  $q$ ,  $\theta$ , and  $u$  determined, the remaining endogenous variables can be obtained in a standard recursive fashion. The level of  $n_A$  follows from imposing  $\dot{n}_A = 0$  in (5). Substituting  $R = I\delta X$ , from (11) and (12), into (25) yields  $X$ . Substituting  $u$  into (24) gives  $Z$  and thereby  $c$ ,  $\pi_a$  and  $\pi_y$  can be recovered from (6) and (9). With  $\pi_y$  and  $X$  determined,  $w_H$  is derived from (A.2).

<sup>31</sup> In line with empirical evidence, the present model generates a Beveridge curve which is unambiguously downward sloping in  $(u, v)$  space. Consider an increase in  $v$ , holding  $u$  constant. Restoring the CD condition requires a lower  $u$  because of two reasons. First, a higher  $v/u$  ratio raises the job-finding rate  $p(\theta)$  and reduces  $u$ . Second, a higher  $v/u$  ratio decreases the matching rate of young firms  $q(\theta)$  and hence the replacement rate of old firms, again reducing  $u$ . The unambiguously downward sloping Beveridge curve is an improvement over the DMP literature according to which the slope of this curve is ambiguous (see e.g. Mortensen and Pissarides 1994, p. 403, Pissarides 2000, p. 47, or Caballero 2007, pp. 127-128).

is the standard representation of the Beveridge curve. The present model complements and generalizes the DMP theory of unemployment by recognizing explicitly and taking into account the dependence of the job-separation rate on labor market tightness, and on virtually all general-equilibrium parameters – including the systemic risk – that affect the rates of innovation and growth. Changes in these parameters shift the standard representation of the Beveridge curve.<sup>32</sup>

We next derive an expression for the rate of growth. The endogenous arrival of process innovations generates growth in instantaneous consumer utility. The latter captures the appropriately weighted consumption index and corresponds to real per-capita income. It is possible to obtain the following expression for instantaneous utility<sup>33</sup>

$$\log h(t) = \log(c/\lambda) - (1 - n_A)(1 - \phi)\log \lambda + (\log \lambda)n_A It. \quad (30)$$

The first term captures the effect of per capita output expressed in units of low-skilled labor  $c/\lambda$  under the assumption that all firms charge the same price as young firms. The second term reflects a price adjustment based on the fact that old firms charge a higher price than young firms,  $P_o > P_y$ . This price prevails in a share  $1 - \phi$  of each B industry. These industries account for fraction  $1 - n_A$  of all industries. If there are no B industries, i.e.,  $1 - n_A = 0$ , or no old firms, i.e.,  $1 - \phi = 0$ , the second term is zero. The third term captures the standard dynamic effect caused by the arrival of process innovations: every time an innovation occurs in an industry, the instantaneous utility jumps up by  $\log \lambda$ ; and during the period from time zero to  $t$  the expected number of innovations occurring in each of  $n_A$  growth-oriented industries is  $It$ . Differentiating (30) with respect to time yields the growth rate of instantaneous utility

$$g = n_A \log \lambda = \frac{Iq \log(\lambda)}{I + q}. \quad \mathbf{GR} \quad (31)$$

The growth rate is proportional to the rate of innovation  $I$ , the measure of growth-oriented industries  $n_A$ , and parameter  $\log(\lambda)$  capturing the impact of innovation size.

We now illustrate the equilibrium graphically in  $(q, I)$  space. We assume a Cobb-Douglas matching function:  $M(U, V) = V^\eta U^{1-\eta}$ , which implies  $q = \theta^{-(1-\eta)}$ ,  $\theta = \mu(q) = q^{\frac{1}{1-\eta}}$ , and an elasticity of market tightness  $\theta$  with respect to the matching rate  $q$  denoted by  $\varepsilon = -(\partial \mu / \partial q)q / \mu = 1/(1 - \eta) > 1$ . The VC condition

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<sup>32</sup> Recent shifts in the Beveridge curve have been identified and discussed by prominent economists, e.g., Diamond (2011), Mortensen (2011), and Pissarides (2011). Our model offers a Neo-Schumpeterian perspective capturing general equilibrium shifts in the Beveridge curve.

<sup>33</sup> Appendix C provides detailed calculations yielding (30).

(27), which determines the equilibrium value of  $q$  as a function of the model's parameters, is shown in Figure 1 by vertical line VC. The RP condition (28), which establishes an inverse relationship between  $I$  and  $q$  is shown by the downward sloping curve RP. The intuition behind the shape of curve RP is as follows. A higher matching rate  $q$  reduces the valuation of old firms  $V_o/Z$  by increasing their replacement rate and expected firing costs. Thus, for an adult firm the incentives to avoid replacement and engage in RPAs become stronger. The profitability of RPAs relative to R&D increases, and this leads to a lower innovation rate  $I$ .

To show the unemployment and growth rates in  $(q, I)$  space, we utilize the CD condition (29) and GR equation (31). The corresponding graphs are illustrated with the iso-unemployment (UU) and iso-growth (GG) curves. Consider first the UU curve. Totally differentiating (29) for a given  $u$  using  $q(\theta)\theta = p(\theta)$  and  $\theta \equiv \mu(q)$  yields

$$\left. \frac{dI}{dq} \right|_{u=\bar{u}} = -\frac{I}{q} \left( \varepsilon - 1 + \varepsilon \frac{I}{q} \right) < 0, \quad (32)$$

where  $\varepsilon > 1$  under a Cobb-Douglas matching function. In this case, the typical UU curve in  $(q, I)$  space is downward sloping and convex to the origin. Moving away from the origin in  $(q, I)$  space implies higher unemployment rates. The intuition behind the negative slope of a UU curve is as follows. A higher matching rate  $q$  increases aggregate job destruction rate  $qI/(I+q)$ . Higher  $q$  also implies a lower vacancy-unemployment ratio  $\theta$  and hence a lower job-finding rate  $p$ . Both effects raise the unemployment rate  $u$ . Along a UU curve the unemployment rate must remain constant. This requires a reduction in the innovation rate  $I$  reducing the labor flow into unemployment by lowering the mass of industries subject to replacement  $1 - n_A = I/(I+q)$ .

Consider next the iso-growth GG curve. Totally differentiating equation (31) for a given  $g$  yields

$$dI/dq \Big|_{g=\bar{g}} = -(I/q)^2. \quad (33)$$

A typical GG curve is convex to the origin and downward sloping. The intuition behind the negative slope is as follows. A higher  $q$  increases growth by raising the fraction of growth-oriented industries  $n_A$ . Along a GG curve the rate of growth must be constant. This property requires a reduction in the innovation rate  $I$ . Moving away from the origin in  $(q, I)$  space implies higher growth rates. Direct comparison between (32) and (33) establishes the following property. If  $\varepsilon > 1$ , a condition which holds under a Cobb-Douglas matching function, then the UU curve is steeper than the GG curve for any pair of  $(q, I)$ .

**Insert Figure 1 (Steady-state equilibrium) here**

Figure 1 illustrates the iso-unemployment UU and iso-growth GG curves passing through the initial equilibrium levels of  $u$  and  $g$ . These curves divide the  $(q, I)$  space into four quadrants. It follows from (29) and (31) that the GG curve depends only on  $\lambda$ , and that the UU curve depends only on technological and policy-invariant parameters  $\phi$  and  $\varepsilon$ . The rest of parameters, including policy-related ones that change the initial equilibrium by shifting the RP and the VC curves, neither affect the position nor the shape of iso-unemployment and iso-growth contours. This leads to the following result.

**Proposition 1:** *An economic policy that shifts the initial equilibrium point  $E$  in Figure 1 to a new equilibrium point located in:*

- *quadrant I generates higher growth  $g$  and lower unemployment rate  $u$ ;*
- *quadrant II generates higher growth  $g$  and higher unemployment  $u$ ;*
- *quadrant III generates lower growth  $g$  and higher unemployment rate  $u$ ;*
- *quadrant IV generates lower growth  $g$  and lower unemployment  $u$ .*

The four quadrants in Figure 1 illustrate clearly the *conundrum* associated with recovery policies or economic shocks. Quadrants II and IV show the area where growth and unemployment are positively correlated. A move from the initial equilibrium to any point in quadrant II generates jobless growth; whereas a move to any point in quadrant IV results in stagnant growth with job creation. One of the main insights of the neo-Schumpeterian approach to macroeconomics is that higher growth may *not* necessarily come with lower unemployment and vice versa. Quadrants I and III establish a negative relationship between unemployment and growth as one would find in a conventional textbook model of macroeconomics. A move to a final equilibrium point located in quadrant I generates an economic recovery with higher output growth and lower unemployment; whereas a move to quadrant III generates a recession with lower growth and higher unemployment.

The model also captures the possibility of a recession followed by a jobless recovery. For example, consider a shock (e.g., captured by an increase in systemic risk) which moves the initial equilibrium to quadrant III. The absence of transitional dynamics implies that the economy jumps instantaneously to a lower rate of output growth and a higher rate of unemployment. The instantaneous drop in employment implies a downward jump of output which will show up as negative GDP growth when measured in discrete time intervals. After the initial jump, the level of output increases gradually at the rate of lower but still positive TFP growth, while the rate of unemployment remains unchanged. Eventually, the level of output reaches and surpasses its initial level without any change in the rate of unemployment generating a jobless recovery.

## 4 Economic Policies

We begin our analysis by considering policies shifting only the RP curve or only the VC curve. Then we analyze other policies shifting both curves. To resolve ambiguities, we assume a Cobb Douglas matching function and perform numerical simulations using benchmark parameters from the U.S..<sup>34</sup> Table 1 shows the numerical implementation of our model for the benchmark case, as well as numerical results that correspond to the policy analysis in subsections 4.1 – 4.4. We use Figure 1 to illustrate the “conundrum of recovery policies” and the growth and unemployment effects of various policies. In Figure 1, we draw the RP curve such that it lies between the UU and GG curves. This turns out to be the empirically relevant case which is consistent with numerical simulations.

**Insert Table 1 (Numerical analysis) here.**

### 4.1 *Employment Protection*

Most advanced countries implement some form of employment protection legislation, which imposes restrictions on worker dismissals that may take the form of cumbersome administrative procedures increasing firings costs.<sup>35</sup> An increase in employment-separation (firing) costs  $f > 0$  shifts the RP curve down in Figure 1 without affecting the VC curve. As a result, the rate of innovation  $I$  declines but the matching rate  $q$  and  $v/u$  ratio remain the same as before the policy change. The equilibrium point moves to quadrant IV generating lower unemployment and lower growth rates.

What is the intuition behind this result? Higher firing costs  $f$  exert no influence on the vacancy creation incentives as captured by (27). This leaves  $q$  and  $v/u$  unaffected. An increase in  $f$ , however, leads to a fall in the relative R&D-RPA profitability as captured by the RHS of (28) and thus reduces  $I$ . The mechanism works primarily through RPA incentives. First, there is a direct effect of higher  $f$  on RPA incentives as captured by the last term in (21). With the increase in  $f$ , the firing costs that adult firms incur rise. Thus, adult firms have more to lose if further innovation occurs. This strengthens their incentives to invest in RPAs. Second, a higher  $f$  reduces the value of old firms  $V_o$  by increasing their expected firing costs. This implies that adult firms have even more at stake to lose if further innovation occurs. This strengthens their incentives to engage in RPAs. Both effects clearly imply a fall in the relative R&D-RPA profitability and a lower rate of innovation  $I$ .

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<sup>34</sup> In general, the benchmark parameters are in line with recent growth literature employing numerical simulations. Notes under Table 1 provide details on the choice of benchmark parameters.

<sup>35</sup> Skedinger (2010) offers a comprehensive study of this legislation and its effects.

The intuition behind the effect on unemployment is straightforward. A lower innovation rate  $I$ , caused by higher firing costs, decreases the fraction of B industries  $1 - n_A = I/(I + q)$  that are subject to replacement. The reduction in the fraction of B industries lowers the aggregate job-destruction rate  $(1 - \phi)qI/(I + q)$  and the unemployment rate  $u$ . The growth effect is also clear. With  $q$  unchanged, the lower rate of innovation  $I$  unambiguously reduces  $g$ .

In short, changes in employment protection policies, captured by  $f$ , imply a tradeoff between growth and job creation illustrating the conundrum of recovery.

**Proposition 2:** *A higher employment-termination cost  $f > 0$  has no effect on matching rate  $q$  but reduces the rates of unemployment  $u$ , innovation  $I$ , and growth  $g$ .*

## 4.2 Vacancy Costs

Such costs may consist of advertising vacant positions, maintaining a human resources department, reimbursing outlays of applicants (e.g. travel and lodging costs), etc.<sup>36</sup> Please consider Figure 1. A decrease in vacancy cost parameter  $\alpha$  shifts the VC curve left without affecting the RP curve. The innovation rate  $I$  increases, the matching rate  $q$  and the  $v/u$  ratio decline. Under the empirically relevant case, where the RP curve lies between the UU and GG curves, the economy moves to quadrant I increasing growth and reducing unemployment.

What is the intuition behind these beneficial effects? A lower vacancy cost  $\alpha$  strengthens the vacancy creation incentives and leads to more vacancies per unemployed worker  $\theta \equiv v/u$ . The higher  $\theta$  makes it more difficult for young firms to match their positions, thus the matching rate  $q$  decreases. Formally, the fall in  $q$  and the rise in  $\theta$  follow from (27) and  $\theta \equiv \mu(q)$ , respectively. A lower matching rate  $q$  increases the valuation of old firms  $V_o$  because their likelihood to be replaced and expected firing costs decline. As a result, adult firms face weaker incentives to defend their positions through RPAs and reduce their level  $X$ . Lower R&D difficulty implies a higher innovation rate  $I$ .

Unemployment is affected through several channels. First, when young firms face a lower matching rate  $q$ , the rate at which old firms are replaced decreases and the fraction of industries subject to replacement  $(1 - n_A)$  increases. The net effect points to a fall in the aggregate job-destruction rate  $(1 - \phi)qI/(I + q)$ , and thus the unemployment rate  $u$  declines. Second, when the incentives to create vacancies and hence

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<sup>36</sup> A decrease in  $\alpha$  may become operational by defining marginal vacancy creation costs  $\alpha \equiv \hat{\alpha}(1 - \sigma_v)$ , and considering an increase in the vacancy creation and maintenance subsidy rate  $\sigma_v$ . See Hagedorn and Manovskii (2008, pp. 1698-99) and the references therein for empirical evidence on search costs associated with vacancies.



the  $\theta \equiv v/u$  ratio increases, the job finding rate of unemployed workers  $p$  increases and thus  $u$  declines. Third, with faster innovation  $I$ , the fraction of B industries  $1 - n_A$  increases and therefore  $u$  increases. In the empirically relevant case where the RP curve is between the UU and GG curves, the first two effects dominate the third one, causing a decline in the unemployment rate  $u$ .

Growth is affected through two channels: higher innovation rate  $I$  promotes growth, whereas the reduction in the fraction of growth-oriented industries  $n_A$  (driven by lower  $q$  and higher  $I$ ) reduces the rate of growth. The former effect dominates increasing the equilibrium rate of growth  $g$ .

The aforementioned analysis leads to

**Proposition 3:** *Policies that reduce the costs of vacancies  $\alpha$  reduce the matching rate  $q$ . For the empirically relevant case where the RP curve is between the UU and GG curves, the economy moves to quadrant I, attaining a higher rate of growth  $g$ , a higher rate of innovation  $I$ , and a lower rate of unemployment  $u$ .*

### 4.3 Production and R&D Subsidies

#### a. Production Subsidies Targeting Small Young Firms

An increase in production subsidy for young, small and scalable (job-creating) firms  $\sigma_y$  reduces the marginal profitability of vacancy creation by increasing the market valuation of young firms  $V_y$  and reducing the market value of adult firms  $V_a$ , as indicated by the LHS of (27). The VC curve shifts to the right, leading to a higher matching rate  $q$  and lower  $\theta \equiv v/u$  ratio.<sup>37</sup> An increase in  $\sigma_y$  raises also the returns to R&D relative to RPAs, captured by the RHS of (28). For a given matching rate  $q$ , higher R&D profitability stimulates innovative activity  $I$  and hence the RP curve also shifts to the right. In Figure 1, with both RP and VC curves shifting to the right, the economy can potentially move to quadrant II or III. Clearly,  $q$  increases but the change in  $I$  is indeterminate. Numerical simulations show that the rate of innovation  $I$  increases.

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<sup>37</sup> The mechanism behind this result is as follows. An increase in  $\sigma_y$  raises the profit flows of young firms  $\pi_y$  in (9), and thus their valuation  $V_y$  in (16). This also implies higher rewards from innovation as captured by the  $V_y$  term in (14). Maintaining the R&D free-entry condition (14) requires an increase in R&D costs, which are proportional to  $w_H \gamma X$ . This, in turn, translates into higher RPA expenditure for adult monopolists in A industries and thus their valuation  $V_a$  as captured in (18) decreases. With the marginal gains from vacancy creation  $V_a - V_y$  decreasing, each young firm in B industries creates fewer vacancies. In other words, the market tightness  $\theta \equiv v/u$  decreases and  $q(\theta)$  rises, an adjustment which restores the zero-profit condition in vacancy creation (10).

In regards to economic intuition, note that several competing forces affect the rate of innovation. A larger  $\sigma_y$  directly increases the valuation of young firms  $V_y$ , strengthening the entrepreneurs' incentives to engage in R&D. However, there exist indirect effects that work through a higher  $q$  and a lower  $V_a$ . A higher matching rate  $q$  reduces the valuation of old firms  $V_o$  and encourages adult firms to increase RPAs. This indirect effect reduces the relative R&D-RPA profitability and exerts downward pressure on  $I$ . A lower valuation of adult firms  $V_a$  works in the opposite direction by reducing their RPA incentives. Numerical simulations show that the direct effect dominates increasing the equilibrium rate of innovation  $I$ .

A higher subsidy to young firms raises the rate of unemployment through three complementary channels. First, a higher matching rate  $q$  implies that old firms get replaced at a faster rate and the fraction of employment-oriented industries  $1 - n_A$  decreases. The net effect of these changes is a higher aggregate job-destruction rate  $(1 - \phi)qI/(I + q)$ . Second, when vacancy-creation incentives decline and young firms create fewer openings, the job finding rate of workers  $p$  decreases. Third, when the rate of innovation  $I$  increases, the mass of industries subject to turnover  $1 - n_A$  also increases. All three effects raise the unemployment rate  $u$ . With regards to growth, we observe two effects working in the same direction. The faster innovation rate  $I$  and the increase in the fraction of growth-oriented industries  $n_A$  driven by higher  $q$  (despite the mitigating effect of higher  $I$ ) both work to accelerate the growth rate  $g$ .

In short, a subsidy to small young firms creates a tradeoff between jobs and growth and hence a recovery-policy conundrum. It accelerates the rates of innovation and growth by increasing the relative profitability of R&D. It also reduces profitability from vacancy creation by encouraging firms to remain small and young. Thus a subsidy to young firms decreases the job-finding rate of workers and increases the replacement rate of old firms as more firms prefer to stay young and small. These two effects along with the rise in the mass of industries subject to turnover lead to more unemployment.

### **b. Production Subsidies Targeting Large Adult Firms**

An increase in  $\sigma_a$  raises the profitability of vacancy creation by raising the valuation of adult firms  $V_a$ , as captured by the LHS of (27). The VC curve shifts left leading to a lower matching rate  $q$  and higher  $\theta \equiv v/u$  ratio. An increase in  $\sigma_a$  also reduces the relative profitability of R&D as indicated by the RHS of (28). For a given  $q$ , this hinders innovative activity  $I$  and hence the RP curve also shifts left. In Figure 1, with both RP and VC curves shifting left, the economy can potentially move to quadrant I or IV. Clearly,  $q$  decreases but the change in  $I$  is indeterminate. Numerical simulations show that the equilibrium rate of innovation  $I$  decreases.

What is the intuition? A higher subsidy to adult firms  $\sigma_a$  directly increases their valuation  $V_a$  by increasing their profit flows. Why should this encourage the matching effort of young firms? The reason is that young firms make their vacancy-creation decisions by taking into account the increase in market value upon successful matching. Subsidies to adult firms raise their market value and strengthen the incentives of young firms to engage in vacancy creation in order to become adult firms. Young firms open up more vacancies generating a higher  $v/u$  ratio, and a lower matching rate  $q$ .

What changes the innovation rate despite the fact that there is no direct or indirect effect of this subsidy on the valuation of young firms  $V_y$ ? The mechanism again works through RPA incentives and relative R&D profitability. Specifically, when adult firms are subsidized and attain higher valuations, they now have higher incentives to defend their position through RPAs. We should note that there is a mitigating factor due the lower matching rate  $q$ , which increases the valuation of old firms  $V_o$  and reduces the RPA incentives of adult firms. The net effect is a reduction in the relative profitability of R&D and a decline in the equilibrium rate of innovation  $I$ .

We briefly summarize three distinct effects of a higher  $\sigma_a$  on the rate of unemployment. First, a lower matching rate  $q$  generates a lower aggregate job destruction rate  $(1 - \phi)qI/(I + q)$ . Second, when young firms create relatively more openings they increase the job-finding rate of workers  $p$ . Third, a lower rate of innovation leads to a lower fraction of industries subject to turnover  $1 - n_A$ . All three effects reduce unemployment. With regards to growth, we may identify two effects working in the same direction. A lower rate of innovation  $I$  and a smaller fraction of growth-oriented industries  $n_A$  driven by lower  $q$  (despite the mitigating effect of lower  $I$ ) both work to reduce the growth rate  $g$ .

In short, a policy of subsidizing large adult firms implies a tradeoff between growth and jobs. It belongs to the conundrum of recovery policies. It raises the returns from vacancy creation by rewarding successful job-matching more generously. This helps with fighting unemployment. It has no direct effect on R&D incentives (as measured by the direct rewards from successful R&D  $V_y$ ) but it motivates adult firms to defend their positions more rigorously. This reduces relative profitability of R&D and retards innovation. The lower innovation rate also helps to reduce unemployment by slowing the rate of industry turnover.

### **c. R&D Subsidies**

An increase in R&D subsidy rate  $\sigma_r > 0$  reduces the market valuation of adult firms  $V_a$  in (27) and (28).<sup>38</sup>

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<sup>38</sup> An increase in the tax rate on RPAs could be incorporated by replacing  $B$  in (28) with  $\hat{B} \equiv B/(1 + \sigma_x)$  and consid-

As a result, this growth-oriented investment policy generates the same qualitative effects as those triggered by an increase in production subsidy targeting young firms  $\sigma_y$ . It increases the rates of growth and innovation by reducing the costs of the latter, and raises the rate of unemployment by leading to a higher labor turnover. It thus creates a recovery-policy conundrum as well. The analysis of production and R&D subsidies leads to

**Proposition 4:** *The following policies raise the matching rate  $q$ , and (based on numerical simulations) increase the rates of unemployment  $u$ , growth  $g$  and innovation  $I$ :*

- i) *an increase in young firms' production subsidy rate  $\sigma_y > 0$ ;*
- ii) *a decrease in adult firms' production subsidy rate  $\sigma_a > 0$ ;*
- iii) *an increase in R&D subsidy rate  $\sigma_R > 0$ .*

Even though single subsidy policies belong to the conundrum of recovery, numerical simulations show that combinations of such policies can lead to a recovery with higher growth and lower unemployment. For example increasing  $\sigma_a$  from zero to 0.1 and  $\sigma_y$  from zero to 0.2 reduces the rate of unemployment from 8.15% to 6.75% and increases the rate of growth from 0.62% to 0.66%. In general, we find that setting  $\sigma_y$  higher than  $\sigma_a$  can lead to lower unemployment and higher growth. Alternatively, a combination of an R&D subsidy  $\sigma_R$  and a subsidy to adult firms  $\sigma_a$  can generate lower unemployment and higher growth.

#### 4.4 *Financial Frictions*

Hall (2010, pp. 12-13) points out that a key element of the U.S. 2007-2009 financial crisis was the dramatic increase in credit spreads (difference between private borrowing rates and long-term treasury bond rates), reflecting *inter alia* the expected default of many financial assets. Hall (2010, Figure 4) reports that the spread between the Baa corporate bond rate and the 20-year treasury bond rate, which is a reasonable measure of the systemic risk  $\chi$  in our model, increased gradually from about 1.5 percentage points in January, 2007, to about 5 percentage points in January, 2009. Since February 2009, the credit spread showed a gradual decline reflecting among other factors the impact of expansionary monetary and fiscal policies. As a result, the evidence from the initial period 2007-2008 of the great recession is more appropriate to study the effects of financial frictions. In this paper, we do not seek the reasons behind these events, but

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ering an increase in  $\sigma_\chi > 0$ . This policy is isomorphic to an increase in an R&D subsidy  $\sigma_R$  because it reduces the relative profitability of RPAs.

rather analyze the effects of an upward once-and-for-all jump in the risk premium  $\chi$  on employment, vacancy creation, and growth.

Turning to the model, we start our analysis with the following observations. The VC condition (27) delivers a dynamic general-equilibrium channel that relates the systemic risk parameter  $\chi$  to labor-market frictions. It fills a gap in the DMP model which is based on the assumption of perfect financial markets.<sup>39</sup> Changes in the systemic risk parameter work through changes in the effective discount rate which equals the market interest rate  $r = \rho + \chi$ . For example, in the model a two percent increase in the systemic risk can be fully neutralized by a two percent subsidy in the market interest rate. Motivated by the U.S. 2007-2009 financial crisis, we focus on the effects of a higher systemic risk. The analysis is readily applicable to monetary policies that reduce the effective discount rate such as interest rate and credit subsidies. These policies deliver opposite general equilibrium effects compared to the effects generated by an increase in systemic risk.

As said, the direct effect of a higher systemic risk  $\chi$  is to raise the effective discount rate for both young and adult firms and reduce their valuations  $V_y$  and  $V_a$ . For adult firms, there is also an additional effect associated with RPAs. A larger  $\chi$  reduces what is at stake for the adult firms and thereby lowers their RPA expenditure. The fall in  $V_a$  turns out to be larger than the fall in  $V_y$  if and only if  $\phi < \phi^c \equiv B(\chi + \rho)/4$ , a condition which is satisfied for a wide range of parameters consistent with an interior equilibrium.<sup>40</sup> Thus we conclude that a higher  $\chi$  reduces  $V_a - V_y$  and hence decreases the incentives to engage in vacancy creation. This shifts the VC curve to the right, resulting in a lower labor-market tightness  $\theta = v/u$  and a higher matching rate  $q$ .<sup>41</sup>

Turning to the RP condition, we note that what matters is the change in the R&D incentives *relative* to RPA incentives, i.e., the  $V_y/(V_a - V_o + f\phi Z)$  ratio, which corresponds to the RHS of (28). All stock market valuations decline as a result of an increase in the risk premium. A higher  $\chi$  reduces  $V_y$  and  $V_o$  by increasing their effective discount factor,  $\rho + \chi$  and  $\rho + \chi + q$ , respectively, as shown in (27) and (28). A higher

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<sup>39</sup> Concluding his Nobel Prize lecture, Pissarides (2011, pp. 1103-04) identifies this research gap by stating that “The financial crisis of 2008 has thrown open the question of the interaction between capital and labor markets. Equilibrium matching models are built on the assumption of perfect capital markets. The implied arbitrage equations under perfect foresight and unlimited borrowing and lending are used to calculate a value for jobs and workers. These are good starting assumptions, and they have yielded important results. But future work needs to explore other assumptions about capital markets and integrate the financial sector with the labor market”.

<sup>40</sup> This condition follows from imposing  $\partial(V_a - V_y)/\partial\chi < 0$  using (27) and setting subsidy rates to zero to simplify.

<sup>41</sup> These effects are consistent with labor-market evidence witnessed during the U.S. 2007-2009 financial crisis. Pissarides (2011, Figure 7, p.1101) illustrates that, during the period of January 2008 and April 2009, the US unemployment rate increased and the vacancy rate declined as the US moved along its Beveridge curve.

$\chi$  reduces the valuation of adult firms  $V_a$  if and only if  $\phi < \phi^{CC} \equiv 1/[(4/B(\chi + \rho)) + 1]$ , a condition which also holds for a wide range of parameters consistent with an interior equilibrium.<sup>42</sup> Numerical simulations show that R&D incentives  $V_y$  fall by more than RPA incentives  $V_a - V_o + f\phi Z$ . For a given  $q$  this leads to a decline in the equilibrium innovation rate  $I$ , and a leftward shift of the RP curve.

The intuition behind these changes is as follows. An increase in risk premium  $\chi$  exerts a proportionally larger impact on the effective discount rate of the young successful innovators  $\rho + \chi$ . This is because the effective discount rates applying to adult firms and old firms depend on additional factors which dilute the impact of  $\chi$ . For adult firms, these factors are the partial replacement threat due to innovation and the RPA costs [please see second component of term  $V_a/Z$  in (27)]. For old firms, these factors are the full replacement threat due to matching by young firms  $q$  and expected firing costs incurred by old firms  $f\phi Z$  [please see term  $V_o/Z$  in (28)]. In summary, with VC shifting to the right and RP shifting to the left due to a higher systemic risk, the innovation rate  $I$  declines. In Figure 1, the economy can potentially move to quadrant III or IV. The growth rate  $g$  is subject to two forces. The lower  $I$  decreases  $g$  and the higher  $q$  increases it. Numerical simulations show that the change in innovation rate  $I$  dominates the change in matching rate  $q$  decreasing the equilibrium growth rate  $g$ .<sup>43</sup>

Next, we consider the effects of a higher systemic risk on the rate of unemployment. A higher  $q$  increases the aggregate job-destruction rate  $(1 - \phi)qI/(I + q)$ , raising unemployment. A lower tightness  $\theta = v/u$  decreases the job-finding rate of workers  $p$ , also raising unemployment. The only effect that reduces unemployment is related to the decline in innovation rate  $I$ . The latter effect reduces the fraction of B industries subject to labor turnover  $1 - n_A$  and thus decreases unemployment.

Numerical simulations show that the impact of the risk premium  $\chi$  on the unemployment rate  $u$  is complex and depends on the level of growth. Specifically, at sufficiently low initial innovation and growth rates, an increase in  $\chi$  raises the unemployment rate. In other words, when growth is low, the reduction in vacancy creation incentives is larger relative to the reduction in relative R&D profitability. The effects of higher  $q$  and lower  $p$  dominate over the effect of the lower  $I$  leading to a net increase in unemployment. Whereas at sufficiently high innovation and growth rates, an increase in  $\chi$  reduces the unemployment rate. This is because when growth is high, the reduction in vacancy creation incentives is now smaller compared to the fall in relative R&D profitability.

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<sup>42</sup> This condition follows from imposing  $\partial V_a/\partial \chi < 0$  from (27) and setting subsidy rates to zero to simplify.

<sup>43</sup> Our finding that an increase in systemic risk reduces economic growth is empirically supported by Adrian et al. (2010).

We thus conclude that a financial crisis generated by a once-and-for-all increase in the systemic risk is more detrimental to low-growth economies by reducing growth and increasing the rate of unemployment. Figure 2 illustrates the growth and unemployment effects of continuously rising values of  $\rho + \chi$  for alternative values of the innovation-size parameter  $\lambda$ , which is related positively to the rate of growth: as the systemic risk rises, the rate of growth declines monotonically but the rate of unemployment first declines (for high growth rates) and then rises (for low growth rates).<sup>44</sup>

**Insert: Figure 2 (Growth and unemployment effects of rising  $\rho + \chi \in [0.035, 0.095]$ ) here.**

The following proposition summarizes the main aforementioned results.

**Proposition 5:** *Numerical simulations show that an increase in the systemic risk of default  $\chi$*

- i) *raises the matching rate  $q$ , and reduces the rates of innovation  $I$  and growth  $g$ ;*
- ii) *increases (reduces) the unemployment rate  $u$  at low (high) initial growth rates.*

## 5 Concluding Remarks

This paper is a first to adopt a neo-Schumpeterian macroeconomic approach to financial frictions, growth and jobs. It develops a model that highlights the general-equilibrium nexus among financial frictions, unemployment and fully-endogenous Schumpeterian growth. Financial frictions are modeled as an exogenous systemic risk that augments the market rate of interest faced by consumers and firms. Unemployment is modeled according to the DMP theory. Fully-endogenous growth stems from the market interaction between profit-maximizing R&D efforts of entrepreneurs and rent protection activities (RPAs) of adult firms that wish to protect the flow of temporary profits. RPAs deliver a scale-free growth environment, but they set up the conundrum of recovery: policies that reduce the rate of unemployment may reduce the rate of growth by increasing the profitability of RPAs relative to R&D investment, and by shifting resources away from firms engaged in R&D.

The model delivers a steady-state equilibrium which is unique and does not exhibit transitional dynamics. It also generates a version of the Beveridge curve that allows us to trace the general-equilibrium ef-

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<sup>44</sup> Based on Figure 2, our model implies that shocks to  $\lambda$  generate a negative correlation between TFP growth  $g$  and unemployment rate  $u$ . This is consistent with the empirical evidence as shown by Postel-Vinay (2002, Figure 1 on p. 740). Our result contrasts with Aghion and Howitt (1994, p. 488) who find a positive relationship between  $g$  and  $u$  in their endogenous growth setting. We think that there is a good case for considering shocks to  $\lambda$  as the source of TFP growth shocks since other policy/technology parameters (affecting  $g$  through the innovation rate  $I$ ) seem more stable over time. Moreover, shocks to  $\lambda$  can capture not only technology shocks common across countries but also input price shocks that affect profit margins.

fects of financial frictions and recovery policies on unemployment and vacancies. We use the model to analyze the effects of five recovery policies aiming at reducing unemployment or/and accelerating economic growth. Table 2 provides a summary of analytical and numerical results.

### **Insert: Table 2 (Summary of results)**

Growth-stimulating policies, such as R&D subsidies or production subsidies targeting young firms searching for workers, have trade-offs. They indeed boost growth but also raise unemployment. In contrast, trade-offs between growth and employment disappear with certain policies. For example, subsidizing the costs of vacancy creation directly (i.e., by reducing vacancy creation costs during the search process) results in higher growth *and* higher employment. In addition, simulation analysis suggests that the policy maker can find combinations of policies to reduce unemployment and increase growth. These policies include subsidizing young and adult firms at different rates, or combining an R&D subsidy with a production subsidy targeting adult firms.

We also investigate the impact of financial frictions by analyzing an increase in the systemic risk of default. We find that an increase in the risk premium decreases the rates of innovation and growth, while the unemployment effect is ambiguous. According to numerical analysis, at sufficiently low growth rates an increase in the risk premium raises the rate of unemployment; whereas it reduces the rate of unemployment at sufficiently high growth rates. As a result, in slowly growing economies, a rise in financial frictions measured by a higher systemic risk may have particularly severe economic adverse effects. Interest rate policies that reduce the effective discount rate reverse the effects of a higher systemic risk.

We are the first to admit that these novel results are suggestive rather than conclusive, because they depend on reasonable but undoubtedly somewhat restrictive assumptions and in some cases on numerical simulations. For instance, financial frictions are modeled as an exogenous systemic risk of default; the model omits human and physical capital accumulation; the scale effect property is removed in a particular way. The model assumes that subsidies are financed by lump sum taxes and abstracts from public finance issues stemming from government budget deficits. Relaxing these assumptions leads to feasible and welcome generalizations and extensions of our model.

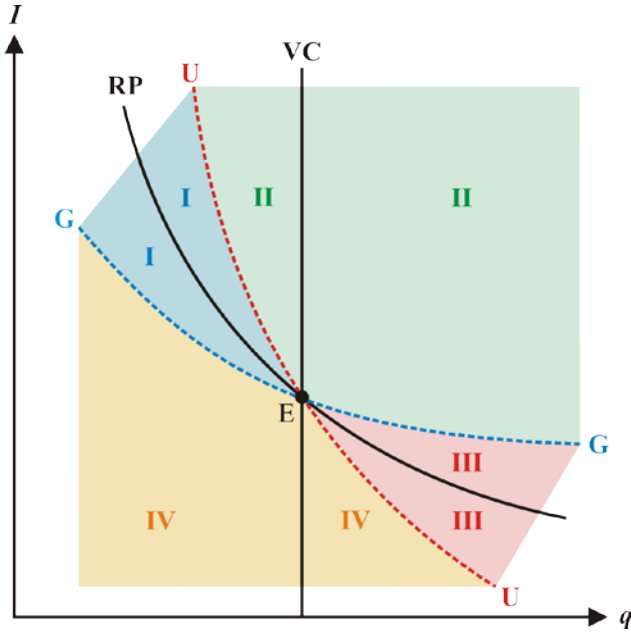
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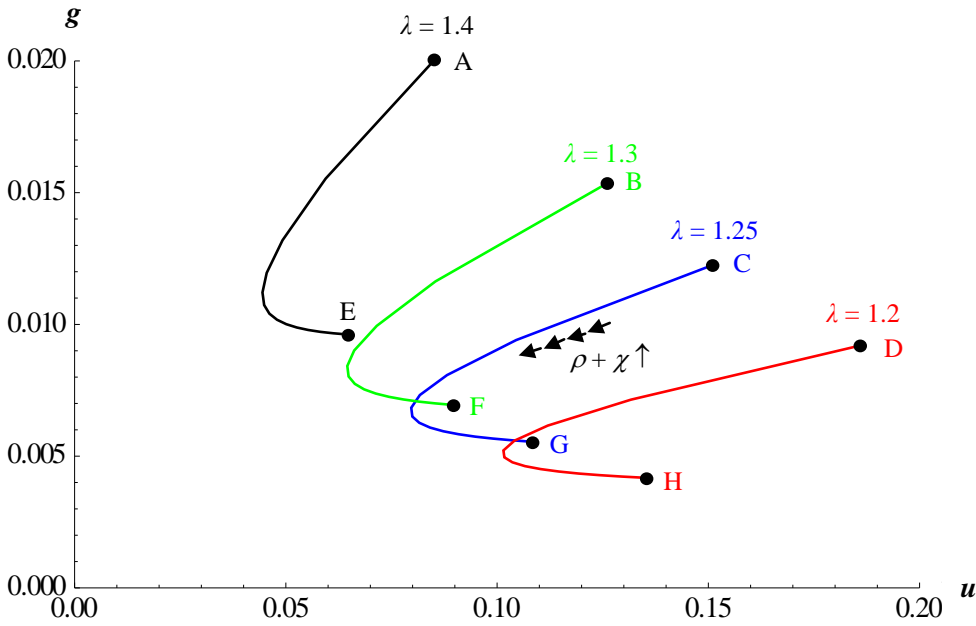


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**Figure 1: Steady-State Equilibrium**



**Figure 2: Growth and unemployment effects of rising  $\rho + \chi \in [0.035, 0.095]$**

Notes: In this graph, each line is drawn for a given  $\lambda$  and identifies the equilibrium combinations  $(u, g)$  that result when  $\rho + \chi$  changes, while all other parameters take their benchmark values from Table 1. Moving down on each line corresponds to higher levels of  $\rho + \chi$ . At points A, B, C and D, respectively,  $\rho + \chi$  takes its lowest value 0.035. At points E, F, G and H, respectively,  $\rho + \chi$  takes its highest value 0.095. For all cases of  $\lambda$  considered here, the critical  $\rho + \chi$  that makes the impact on unemployment switch is in the 0.05-0.06 range.

**Table 1: Numerical analysis**

Benchmark parameters:  $\sigma_y = 0$ ,  $\sigma_a = 0$ ,  $f = 0$ ,  $\lambda = 1.25$ ,  $\rho + \chi = 0.06$ ,  $\alpha = 0.2$ ,  $\phi = 0.01$ ,  $N = 1$ ,  $s = 0.01$ ,  $A = 0.13$ ,  $\eta = 0.6$ ,  $\sigma_R = 0$ ,  $B \equiv \beta\delta/\gamma = 1$ ,  $\gamma = 1$ ,  $w_L = 1$

Endogenous variables	Benchmark solution	$\sigma_y = 0.05$	$\sigma_a = 0.05$	$f = 0.05$	$\rho + \chi = 0.07$	$\alpha = 0.18$
Innovation rate $I$	0.04589	0.06735	0.02838	0.04454	0.04027	0.05210
A industry share $n_A$	0.61194	0.54544	0.66147	0.61902	0.66154	0.55559
Matching rate $q$	0.07237	0.08082	0.05547	0.07237	0.07872	0.06513
Job finding rate $p$	0.31301	0.26522	0.46640	0.31301	0.27590	0.36660
Vacancy rate $v$	0.35284	0.39575	0.32229	0.34692	0.30583	0.40807
Unemployment rate $u$	0.08157	0.12059	0.03833	0.08021	0.08726	0.07250
High-skilled wage rate $w_H$	2.42472	2.53428	2.69847	2.45325	2.22091	2.23641
Consumption expenditure $c$	1.13655	1.08827	1.19006	1.13825	1.12952	1.14779
RPA level $X$	0.01562	0.01718	0.01470	0.01546	0.01453	0.01711
Firm value $V_a$	2.52567	2.17654	3.43796	2.52944	2.30514	2.55064
Firm value $V_y$	0.03789	0.04353	0.03966	0.03794	0.03227	0.03826
Firm value $V_o$	1.70012	1.53021	2.44875	1.67801	1.50381	1.81622
Stock market value $V^{TOT}$	2.19886	1.87599	3.09493	2.19881	2.02894	2.21636
Utility growth rate $g = n_A \lambda \log \lambda$	0.00627	0.00820	0.00419	0.00615	0.00594	0.00646

**Notes:** Here we provide the main results of a Mathematica<sup>®</sup> Appendix, which is available upon request and also on the authors' websites. The stock market value  $V^{TOT}$  is defined as  $V^{TOT} \equiv n_A V_a + (1 - n_A)[\phi V_y + (1 - \phi)V_o]$ . We chose the size of innovations,  $\lambda = 1.25$ , so as to be consistent with the gross markup (the ratio of the price to the marginal cost) enjoyed by innovators. According to the literature, the value of the markup is between 1.05 and 1.4 (see Basu 1996 and Norrbin 1993). The subjective discount rate  $\rho$  is set at 0.06 to capture a real interest rate of 6 percent. This value is in the range suggested by Mehra and Prescott (1985) as the average real return on the U.S. stock market during the past century (7 percent) and the value of 3 percent used by Dinopoulos and Segerstrom (1999). Jones and Williams (2000, p. 73) argue in favor of using such relatively high real interest rates rather than risk-free rates on treasury bills of around 1%. The matching function takes the Cobb Douglas form as in Blanchard and Diamond (1989) with  $M(U, V) \equiv AV^\eta U^{1-\eta}$  where  $\eta = 0.6$ , such that  $q = A(1/\theta)^{0.4}$  and  $p = A\theta^{0.6}$ . The benchmark value for the vacancy-creation costs  $\alpha = 0.2$  is very close to the value 0.213 used by Shimer (2005). We chose other benchmark parameters  $\phi = 0.01$ ,  $A = 0.13$ ,  $B \equiv \beta\delta/\gamma = 1$  and  $\gamma = 1$  with the objective to generate reasonable values for various endogenous variables. Specifically, these parameters generate a growth rate close to  $g = 0.5\%$  as suggested by Denison (1985), and an unemployment rate around 8 percent. We note that a low  $\phi$  corresponds to a high degree of labor market frictions faced by successful innovators. It also implies *ceteris-paribus* a low R&D reward. Initial low profits earned by young successful firms are consistent with the notion of "crossing the chasm" in studies of how high-tech markets evolve over time, e.g., see Moore (2002). The proportion of high-skilled workers  $s$  is set at 0.01 to generate a wage differential  $w_H/w_L \equiv w_H$  that is significantly greater than 1. Our definition of "high-skilled" workers is very narrow, because it comprises only those working in R&D and RPAs.  $N = 1$  and  $w_L = 1$  are convenient normalizations. Finally, setting  $\sigma_a = \sigma_y = \sigma_R = f = \chi = 0$  serves as a useful, distortion-free reference case.

**Table 2: Summary of results**

Policy change	Effects in Figure 1			Analytical results					Simulation results				
	VC	RP	Quadrant move	$q$	$I$	$g$	$u$	$n_A$	$q$	$I$	$g$	$u$	$n_A$
$\sigma_a \uparrow$	left	left	I, IV	$\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\sigma_y \uparrow$	right	right	II, III	$\uparrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$f \uparrow$	none	left	IV	$-$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$-$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\alpha \downarrow$	left	none	I	$\downarrow$	$\uparrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\chi \uparrow$	right iff $\phi < \phi^C$	left/ right	III, IV	$\uparrow$ iff $\phi < \phi^C$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow\downarrow$	$\uparrow$

**Notes:** An increase in R&D subsidy rate  $\sigma_R > 0$  or RPA tax rate  $\sigma_X > 0$  yields the same qualitative effects as an increase in  $\sigma_y$ . Interest rate policies reducing the effective discount rate correspond to a decline in  $\chi$ .

## Appendix A: Derivation of Equilibrium Equations

The VC condition (27) is derived as follows. Start with substituting  $r = \rho + \chi$  and  $\dot{V}_y/V_y = 0$  in (16). Next, substituting  $\alpha(1 - \phi)Z = q(\theta)(V_a - V_y)$  from (10) and  $\pi_y$  from (9) into the resulting expression provides the following equation for  $V_y$

$$V_y = Z \left[ \frac{\phi(\lambda - 1 + \sigma_y)}{\rho + \chi} \right]. \quad (\text{A.1})$$

Combining (A.1) with (14) yields the following free-entry in R&D condition

$$w_H(1 - \sigma_r)\beta\delta X = Z \left[ \frac{\phi(\lambda - 1 + \sigma_y)}{\rho + \chi} \right] \quad \text{FE.}^{45} \quad (\text{A.2})$$

Substituting (21) into (17), using expressions  $r = \rho + \chi$  and  $\dot{V}_a/V_a = 0$  and taking limits as  $dt \rightarrow 0$  yields

$$V_a = \frac{\pi_a - 2w_H\gamma X}{\rho + \chi}. \quad (\text{A.3})$$

Solve (A.2) for  $w_H X$  and substitute the resulting expression and  $\pi_a$  from (6) into (A.3) to obtain

$$V_a = Z \left[ \frac{\lambda - 1 + \sigma_a}{\rho + \chi} - \frac{2\phi(\lambda - 1 + \sigma_y)}{B(1 - \sigma_r)(\rho + \chi)^2} \right], \quad (\text{A.4})$$

where  $Z = cN/\lambda$  from (7) and choice of numéraire, i.e.  $w_L \equiv 1$ ; and  $B \equiv \beta\delta/\gamma$  is the resource requirement of R&D relative to RPAs. Substitute  $V_y$  from (A.1) and  $V_a$  from (A.4) into the vacancy-creation condition (10), and observe that  $Z$  cancels out. This yields the VC condition (27) in the main text.

The RP condition (28) is derived as follows. Substituting  $r = \rho + \chi$ ,  $\dot{V}_o/V_o = 0$ ,  $\pi_a$  from (6) into (20), and noting  $Z = cN/\lambda$  yields

$$V_o = Z \left[ \frac{(1 - \phi)(\lambda - 1 - fq)}{\rho + \chi + q} \right]. \quad (\text{A.5})$$

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<sup>45</sup> Equations (A.1) and (A.2) illustrate the necessity of the assumption that an innovator captures a small fraction of the market immediately (i.e.,  $\phi > 0$ ). Where  $\phi = 0$ , the reward to R&D vanishes, i.e.,  $V_y = 0$  and there is no Schumpeterian growth and labor turnover.

Dividing (A.2) by (21) using  $V_y$  from (A.1),  $V_a$  from (A.4), and  $V_o$  from (A.5), yields the **RP** condition (28) in the main text.

Finally, the **creative-destruction (CD)** condition (29) is derived as follows. Setting  $\dot{n}_A = 0$  in (5) yields  $n_A = q(\theta)/[I + q(\theta)]$ . Next, substitute this expression into (26) to obtain equation (29) in the main text.

## ***Appendix B: Existence and Uniqueness of the Equilibrium***

Solving (27) for  $q$  and simplifying implies that for  $q > 0$ , the following parametric restriction must hold

$$q = \frac{\alpha(\rho + \chi)}{\lambda - 1 + \frac{\sigma_a}{1 - \phi} - \frac{\phi}{1 - \phi} \left[ \frac{2(\lambda - 1 + \sigma_y)}{(\rho + \chi)(1 - \sigma_R)B} + \sigma_y \right]} > 0. \quad (\text{B.1})$$

Because  $q$  is strictly declining in  $\theta \equiv v/u$ , condition (B.1) guarantees the existence and uniqueness of  $\theta$ , and hence of  $q(\theta)$  and  $p(\theta)$ . As a result to have unique  $I > 0$ , the denominator of (28) must be positive, which gives us our second parametric restriction

$$\frac{\lambda - 1 + \sigma_a}{\rho + \chi} - \frac{2\phi(\lambda - 1 + \sigma_y)}{B(1 - \sigma_R)(\rho + \chi)^2} - \frac{(1 - \phi)(\lambda - 1 - fq)}{\rho + \chi + q} + f\phi > 0, \quad (\text{B.2})$$

where  $q$  is given by (B.1). Conditions (B.1) and (B.2) must jointly hold for a unique interior equilibrium. Our numerical simulations show that these restrictions indeed hold for a wide range of empirically relevant parameters.<sup>46</sup> The existence of unique  $u \in (0, 1)$  then follows from solving (29) for  $u$  yielding

$$u = \left[ 1 + \frac{p(I + q)}{qI(1 - \phi)} \right]^{-1}.$$

## ***Appendix C: Growth Rates***

### **a. Growth Rate of Instantaneous Utility**

We obtain the growth rate of instantaneous utility  $h(t)$  as follows. Substituting  $y(\omega, t) = c(t)/P(\omega, t)$  from (3) into (2); using  $P_a(\omega, t) = P_y(\omega, t) = w_L/(\lambda^{m(\omega)-1})$  for a fraction  $n_A + (1 - n_A)(1 - \phi)$  of industries;  $P_o(\omega, t) = w_L/\lambda^{m(\omega)-2}$  for a fraction  $(1 - n_A)\phi$  of industries,  $w_L \equiv 1$ ; and taking into account that only a fraction  $n_A$  of

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<sup>46</sup> The simulations use *Mathematica* version 8. The source programs are available upon request and also on the authors' websites.

industries are targeted for innovation at each point in time, provides

$$\begin{aligned}
\log h(t) &= \int_{n_A} \log \left[ \frac{c(t)}{P_a(t)} \right] d\omega + \int_{(1-n_A)(1-\phi)} \log \left[ \frac{c(t)}{P_o(t)} \right] d\omega + \int_{(1-n_A)\phi} \log \left[ \frac{c(t)}{P_y(t)} \right] d\omega \\
&= \int_{n_A} \log \left[ \frac{c(t)\lambda^{m(\omega,t)-1}}{w_L} \right] d\omega + \int_{(1-n_A)(1-\phi)} \log \left[ \frac{c(t)\lambda^{m(\omega,t)-2}}{w_L} \right] d\omega + \int_{(1-n_A)\phi} \log \left[ \frac{c(t)\lambda^{m(\omega,t)-1}}{w_L} \right] d\omega \\
&= n_A \log(c) + (1-n_A)(1-\phi)\log(c) + (1-n_A)\phi\log(c) \\
&\quad + \int_{n_A} \log \left[ \lambda^{m(\omega,t)-1} \right] d\omega + \int_{(1-n_A)(1-\phi)} \log \left[ \lambda^{m(\omega,t)-1} \lambda^{-1} \right] d\omega + \int_{(1-n_A)\phi} \log \left[ \lambda^{m(\omega,t)-1} \right] d\omega \\
&= \log(c) + \int_0^1 \log \lambda^{m(\omega,t)-1} d\omega + \int_{(1-n_A)(1-\phi)} \log \lambda^{-1} d\omega \\
&= \log(c) - \log \lambda - (1-n_A)(1-\phi)\log \lambda + \int_0^1 \log \lambda^{m(\omega,t)} d\omega \\
&= \log(c/\lambda) - (1-n_A)(1-\phi)\log \lambda + (\log \lambda)n_A I t.
\end{aligned}$$

The last line, which is equation (30) in the main text, uses the property  $\int_0^1 \log \lambda^{m(\omega,t)} d\omega = (\log \lambda) I_{AGG} t$  of stochastic Poisson processes (see Grossman and Helpman 1991, p. 97). We note that  $I_{AGG} = I n_A$  captures the expected aggregate innovation rate. Every time a labor saving innovation takes place in a fraction  $n_A$  of the industries, a potential for a price decline by  $\lambda$  materializes and the value of the integral term  $\int \log \lambda^{m(\omega,t)} d\omega$  increases by  $\log \lambda$ . This is the dynamic component of welfare due to technological progress. In Appendix E, we provide a formal welfare analysis.

The static component is the logarithm of the quantities consumed of goods summed over all industries, and it is given by  $\log(c/\lambda) - (1-n_A)(1-\phi)\log \lambda$ . The first term is standard, whereas the second term accounts for different prices charged by old and young firms: old firms charge a price  $1/\lambda^{m(\omega)-2}$  in a share  $1-\phi$  of B industries, whose fraction is  $1-n_A$ . This exceeds the price charged by young firms,  $1/\lambda^{m(\omega)-1}$ .

Differentiating expression  $\log h(t)$  derived above with respect to time and taking into account  $\dot{n}_A = \dot{c} = 0$ , which hold in the steady-state equilibrium, yields the growth rate of instantaneous utility

$$\dot{h}/h \equiv g = I n_A \log(\lambda) = I q \log(\lambda) / (I + q).$$

## b. Growth Rate of Aggregate Output

The economy-wide output level at time  $t$  is calculated as follows. In each of the  $n_A$  growth-oriented industries, an adult firm produces  $\lambda^{m-1} Z$  units of output. In the remaining  $1-n_A$  employment-oriented indus-



tries, a young firm produces  $\phi\lambda^{m-1}Z$  units of output and an old firm produces  $(1-\phi)\lambda^{m-2}Z$  units of output. Hence aggregate output is given by

$$Y(t) = \int_{n_A} Z \lambda^{m(\omega,t)-1} d\omega + \int_{(1-n_A)(1-\phi)} Z \lambda^{m(\omega,t)-2} d\omega + \int_{(1-n_A)\phi} Z \lambda^{m(\omega,t)-1} d\omega.$$

The Poisson process governing the arrival of innovations implies that the expected number of innovations  $E[m(\omega)]$  in industry  $\omega$  at time  $t$  equals  $I(\omega)n_A t$ . At each point in time only a fraction  $n_A$  of all industries are targeted for innovation; thus,  $I(\omega)n_A$  is the expected innovation success rate in industry  $\omega$ . The assumption of structurally identical industries implies that in the steady-state equilibrium we have  $E[m(\omega)] = m$  and thus:

$$Y(t) = n_A \lambda^{m-1} Z + (1-n_A) \lambda^{m-1} Z [\phi + (1-\phi)\lambda^{-1}] = \Psi \lambda^{m-1} Z,$$

where  $\Psi \equiv n_A + (1-n_A)[\phi + (1-\phi)\lambda^{-1}]$  is constant over time at the steady-state equilibrium. Taking logs and differentiating with respect to time, and using  $E[m] = n_A I t$ , gives the following output growth rate:

$$\log Y(t) = \log \Psi + (m-1) \log \lambda + \log Z = \log \Psi + (n_A I t - 1) \log \lambda + \log Z \Rightarrow \dot{Y}/Y = n_A I \log \lambda.$$

### c. Growth Rate of Prices

The growth rate of prices is derived as follows. We start with the price growth rate of a typical industry. In the present model, firm level prices remain the same during each phase and follow a stepwise process with each jump caused by a switch to firm status. At the industry level, however, process innovations generate downward price adjustments. Consider first an industry  $\omega$  that is currently registered as an A industry, where  $P^A = w_L/\lambda^{m(\omega)-1}$ . Using  $E[m(\omega)] = I(\omega)n_A t$  and  $w_L \equiv 1$ , one can calculate the growth rate of expected prices in this industry as

$$\begin{aligned} \log P^A(\omega) &= \log(1) - \log[\lambda^{m(\omega)-1}] = -[m(\omega) - 1] \log \lambda = -[I(\omega)n_A t - 1] \log \lambda \\ \Rightarrow \frac{\dot{P}^A}{P^A} &= -I(\omega)n_A \log \lambda. \end{aligned}$$

To determine the growth rate in expected goods prices in the  $\phi$  and  $1-\phi$  segments of any industry currently registered as B industry, first note that  $P^{B,\phi} = w_L/\lambda^{m(\omega)-1}$  and  $P^{B,1-\phi} = w_L/\lambda^{m(\omega)-2}$ , and then use derivations analogous to the case of an A industry analyzed above to obtain

$$\frac{\dot{P}^{B,\phi}}{P^{B,\phi}} = \frac{\dot{P}^{B,1-\phi}}{P^{B,1-\phi}} = -I(\omega)n_A \log \lambda.$$

Thus, we conclude that in any industry  $\omega$ , regardless of which type of industry it is currently registered as,

the rate of decline in expected price level is  $I(\omega)n_A \log \lambda$ . With structural symmetry across a continuum of industries, the growth rate in the aggregate price level is deterministic and given by  $\dot{P}_{AGG}/P_{AGG} = -n_A I \log \lambda$ . It follows that per-capita consumption measured in units of output (i.e.,  $c/P_{AGG}$ ) grows at the rate  $n_A I \log \lambda$ .

### ***Appendix D: Absence of Transitional Dynamics***

The discussion following equation (26) establishes that the rate of unemployment  $u$  does not exhibit transitional dynamics. It then follows from (24) that there cannot be transitional dynamics for  $Z$  either. Since we normalized  $w_L \equiv 1$ , (7) implies that there are no transitional dynamics for  $c$  as well. Hence equation (4) implies that  $r$  is always equal to  $\rho + \chi$  in and out of steady-state equilibrium. Equations (6) and (9) then imply that there are no transitional dynamics for  $\pi_a$  and  $\pi_y$ , respectively. Furthermore, since there are no transitional dynamics for  $u$ , and given  $\partial q(\theta)/\partial \theta < 0$  from the matching function which pins down a unique  $\theta = (v/u)$ , there cannot be transitional dynamics for  $v$  either. Since there are no transitional dynamics for  $Z$ , it then follows from (23) that  $n_A$  does also not display transitional dynamics. Next, since there are no transitional dynamics for  $u$  and  $v$ , the same applies to  $q(\theta)$  and  $p(\theta)$ . Using the results obtained so far, it follows from (20) that there are no transitional dynamics for  $V_o$ .

Since  $n_A = q(\theta)/[I + q(\theta)]$ , absence of transitional dynamics for  $n_A$  and  $q(\theta)$  implies absence of transitional dynamics for  $I$ . Given no transitional dynamics for  $q(\theta)$  and  $Z$ , it follows from (10) that there are no transitional dynamics for  $V_a - V_y$ . Using this fact in (16) together with the results obtained so far, it follows that there are no transitional dynamics for  $V_y$ , and hence for  $V_a$  as well. According to (14) or (21), it follows that there are no transitional dynamics for  $w_H X$ . Using this as well as the absence of transitional dynamics for  $n_A$  and  $I$ , it follows from (25) that there are no transitional dynamics for  $X$ , given  $R = ID = I\delta X$ . Finally, this result together with the previous argument implies that there cannot be transitional dynamics for  $w_H$  as well.

### ***Appendix E: Welfare Analysis***

Substituting expression (30) into the intertemporal utility function (1), taking into account the absence of transitional dynamics, and evaluating the resulting integral yields the following expression for the household discounted utility:

$$H = \frac{1}{\rho} \left[ \frac{n_A I \log(\lambda)}{\rho} + \log\left(\frac{c}{\lambda}\right) - (1 - n_A)(1 - \phi) \log(\lambda) \right].$$

Note that  $g = n_A \Pi \log(\lambda)$  and  $c/\lambda = (1 - u)(1 - s)$ , which follows from (24) and (7). Thus, policies that increase  $g$  (dynamic effect) and reduce unemployment  $u$  (a static effect) are good candidates to raise welfare. Nevertheless, we also need to consider the change in  $1 - n_A$ , which captures the presence of old firms in B industries charging higher prices (a static effect).

The welfare effects of recovery policies are ambiguous and depend on parameter values. It is apparent from the welfare expression that policies generating a recovery conundrum yield ambiguous welfare effects. However, even policies that increase growth and reduced unemployment may generate ambiguous welfare effects. For example, consider first a reduction in vacancy costs  $\alpha$  that increases  $g$  and reduces  $u$ . Both effects increase welfare. However, a lower  $\alpha$  reduces  $q$  and increases  $I$  leading to an increase in  $1 - n_A = I/(I + q)$  and a decrease in welfare. As a result, the overall welfare effect of lower vacancy costs is ambiguous. Another recovery policy to consider is a reduction in risk premium  $\chi$  in a low-growth economy. A lower  $\chi$  raises welfare by increasing  $g$  and reducing  $u$ . But again a lower  $\chi$  increases  $1 - n_A$  by reducing  $q$  and increasing  $I$ . This effect reduces welfare. Extensive numerical simulations show that policy welfare effects depend on the parameters of the model.