Optimal Sovereign Debt under Excusable Default

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Abstract

The present paper belongs to the very rich stream of papers that have sought to calibrate countries' optimal debt-to-GDP ratios. It diverges from that stream in assuming a self-interested government that engages in 'excusable' default where other most other papers have assumed a benevolent government that engages in strategic default. This assumption implies that the government maximizes the utility of its own consumption over a period of time that is the government's expected stay in office, which the government fearing loss of office upon default seeks to extend to the fullest by defaulting 'excusably' when unable to gather the funds necessary for debt service rather than strategically on deeming default a better option than debt service. The calibrated optimal debt ratios appear to be much closer to historically observed ratios than those calibrated under the alternative assumption of a benevolent government: our baseline optimal debt-to-GDP ratio obtained under excusable default is around 85%, where its strategic default counterpart is but 2.7%. We conduct a number of sensitivity analyses which reveal that optimal debt is primarily sensitive to the same determinants as maximum debt, specifically the maximum primary surplus the government can sustain, the mean and volatility of the growth in output, and the interest rate to a lesser extent.

1 Introduction

In the wake of Eaton and Gersovitz's (1981) seminal work on optimal sovereign debt, many papers have attempted to calibrate optimal sovereign debt levels. These papers have delivered numerous valuable insights, extending beyond optimal debt levels to their variation over the business cycle and their relation to macroeconomic variables such as the current account, but it is perhaps not unfair to state that existing papers' calibrated debt levels have failed to reach those levels recently observed in advanced economies. To take but two examples, at the two endpoints of a near-decade of work on that topic, Aguiar and Gopinath (2006) calibrate an optimal debt-to-GDP ratio of 5%, Cohen and Villemot (2013) of 38%.[1] These levels, admittedly calibrated to developing countries such as Argentina and pertaining to foreign rather than total debt, are well short of the 90%-plus ratios now prevailing in a large number of industrialized countries.

The calibration of higher optimal sovereign debt levels than has so far been possible is the purpose of the present paper. It differs from that work in one essential respect: where previous work has assumed that governments default strategically, weighting the costs of debt service against those of default in their decision whether to service their debt or to default, our work assumes that governments engage in what Grossman and Van Huyck (1988) call excusable default. Excusable default has a government default only when the entirety of the resources it can muster, the country's maximum primary surplus and any proceeds from new debt issuance, fail to cover the cost of debt service. Thus, whereas strategic default can be viewed as a matter of will (the government decides to default as the result of a cost-benefit analysis that deems default more attractive than debt service), excusable default can be viewed as a matter of means (the government cannot but default as it lacks the means fully to service its debt). Collard, Habib, and Rochet (2015) have used the concept of excusable default to calibrate maximum sustainable debt levels; we use that same concept to calibrate optimal debt levels. Our calibrated optimal debt levels are much closer to prevailing debt levels than are those calibrated under strategic default: our benchmark case has optimal debt-to-GDP ratio 85%.

The central intuition for our result is simple: optimal debt depends, inter alia, on the cost of debt; lenders can be expected to be much more willing to provide high levels of debt at reasonable interest rates when they expect borrowers to do their utmost to service that debt than when they expect borrowers continuously to trade off the costs and benefits of debt service and default in deciding whether to service the debt. Debt being cheaper under excusable default, optimal debt is higher in that case. Indeed, we find optimal debt

[1]See Cohen and Villemot (2013) and Table 1 therein. As noted by Cohen and Villemot, the ratio reported for Aguiar and Gopinath (2006) is that obtained by Harchondo, Martinez, and Sapriza (2009) using their perhaps more precise numerical technique.

to be close to maximum sustainable debt: the former differs from the latter in optimal debt's concern for future payoffs; these are jeopardized only little through debt-induced default, because the low volatility of growth makes the probability of default very low even at maximum sustainable debt; there is therefore little reason for optimal debt to be much below maximum sustainable debt, which represents the most advantageous trade-off between payment promised and proceeds received.

Underlying the central assumption of excusable default is that of a self-interested government: the government does its utmost to stave off default because it expects to lose power upon default; senior members of a government that has lost power see the end of their political careers.[2] A self-interested government that engages in excusable default therefore maximizes not total consumption but that which accrues to the government and its favored constituents, over a period of time that extends not over an infinite horizon but over the government's expected time in office. This is in contrast to governments that engage in strategic default, which are generally assumed to behave altruistically, maximizing the entire population's consumption over the infinite horizon that spans the successive lifetimes of the country's present and future generations.[3] An implication of the distinction between self-interested and altruistic government is that the former is concerned with total debt, that owed both nationals and foreigners, whereas the latter is concerned only with foreign debt, domestic debt being but a transfer between nationals.

We examine the sensitivity of optimal debt and its associated default probability (PD) to various parameters of interest: the maximum primary surplus (MPS) which, along with the proceeds from new debt issuance, serves to service maturing debt, the fraction of total output that accrues to the self-interested government, the government's risk aversion and its discount factor, the risk-free interest rate, and the mean and volatility of the rate of growth in output. Changes in the MPS, the interest rate, and the mean and volatility of the growth rate have major impacts on optimal debt. For example, optimal debt increase from 40% of GDP to 170% as MPS increases from 2.5 to 10%. These changes appear to be due to changes in maximum sustainable debt (MSD): optimal debt is generally only a few percentage points below maximum debt; large changes in MSD consequently result in large changes in optimal debt. MSD increases in MPS and in the mean growth rate, reflecting the increased availability of resources for debt service; it decreases in the risk-free interest rate and in the volatility of the growth rate, the former result reflecting the increased attractiveness of the risk-free investment opportunity and the latter the decreased attractiveness of its now riskier alternative. In contrast to the large changes in optimal debt, the probability of default is hardly if at all affected by changes in the four

[2]Section 2 further justifes the assumption of excusable default.

[3]Amador (2004), Cuadra and Sapriza (2008), Hatchondo, Martinez, and Sapriza (2009), Amador and Aguiar (2011), and Acharya and Rajna (2013) are exceptions in this regard.

parameters: changes in these parameters are accommodated nearly exclusively through changes in the quantity (level) rather than the quality (risk) of debt. Collard et al. (2015) find a similar result for MSD, which they attribute to their – and our – assumptions of lognormally distributed growth rate and zero recovery in default.

Confirming the central role of maximum debt for optimal debt, there is little change in optimal debt where MSD is left unaffected. Optimal debt decreases by only a few percentage points as the government's take and the discount factor increase: the larger stake in the future these imply leads the government to decrease the probability of default by decreasing indebtedness. Much the same is true of risk-aversion: a more risk-averse government decreases debt in order to decrease the probability of default.

In order to shed further light on our results and on the importance of our assumption of excusable default, we consider the alternative case of an altruistic government that engages in strategic default. We adapt our model to that case, using Aguiar and Gopinath's (2006) 2% loss of output in autarky and Arellano's (2008) 73.4% probability of escaping that state.[4] We obtain optimal debt ratio 2.7%, slightly above Arellano's 1% and below Aguiar and Gopinath 5%, and very much below the 85% value obtained under excusable default.[5] While it is possible to obtain as high a value of optimal debt with strategic default as we have with excusable default, this implies unreasonably high costs of default (49.5% of GDP), or unreasonably low probability of escaping autarky (2.3% per annum). We interpret these results as providing at least partial support for our assumptions of excusable default and self-interested government.

Our benchmark case has probability of default at optimal debt 0.167%, which corresponds to the rather improbable frequency of one default every six centuries. Introducing the possibility of growth disasters (Rietz, 1988; Barro, 2006; Barro and Ursua, 2011) increases the probability of default at optimal debt to 0.971%; it decreases optimal debt to 70%.

The paper proceeds as follows. Section 2 briefly reviews the literature. Section 3 derives the expression for maximum sustainable debt. Sections 4 and 5 derive the Bellman equations for optimal debt under excusable and strategic default, respectively. Section 6 presents the parameter values used for the calibration exercises. Section 7 calibrates optimal debt under excusable default and examines its sensitivity to parameter values. Section 8 calibrates optimal debt under strategic default, analyzes its sensitivity to parameter values, and compares it to optimal debt under excusable default. Section 9 introduces the possibility of growth collapses. Finally, Section 10 concludes.

[4] Arellano's (2008) 28.2% quarterly probability of escaping autarky is our annual 73.4%.

[5] The values for Aguiar and Gopinath (2006) and Arellano (2008) are those reported in Cohen and Villemot (2013, Table 1).

2 Literature review

The extensive literature on sovereign debt is a testimony to the importance of that topic.[6] Our paper is in the line of two strands of work within that literature, the first on optimal sovereign debt and the second on maximum sustainable debt. The work on optimal sovereign debt has quantified, refined, and extended Eaton and Gersovitz's (1981) original insight; it has generally maintained their assumption of strategic default. Collard et al. (2015, pp. 386-387) briefly review various refinements to Eaton and Gersovitz, from Aguiar and Gopinath's (2006) incorporation of a trend into the output process, through Arellano's (2008) asymmetric cost of default, Mendoza and Yue's (2012) endogenous cost of default, Cuadra and Sapriza's (2008) political risk, Yue (2010) and Benjamin and Wright's (2009) renegotiation in default, Hatchondo and Martinez (2009) and Chatterjee and Eyigungor's (2012) debt maturity, and Fink and Scholl's (2015) conditionality, to Cohen and Villemot's (2013) 'prepaid' cost of default.[7] Cohen and Villemot (Table 1) show the calibrated optimal debt ratios.

The work on maximum sustainable debt has received much of its impetus from the recently witnessed explosion in OECD country debt, to levels previously thought unsustainable.[8] Bohn (1998, 2008) has analyzed the requirements for sustainability, which Gosh, Kim, Mendoza, Ostry, and Qureshi (2013) have used to develop of measure of maximum debt and the 'fiscal space' it affords. Tanner (2013) has developed a measure of maximum liability that is more equity- than debt-like. As do Gosh et al., Collard et al. (2015) develop a debt-like measure of maximum sustainable debt. Collard et al.'s measure is perhaps 'less maximum' and 'more sustainable' than Gosh et al.'s, in the sense that any shortfall in growth below that necessary to service maximum debt implies certain default for Gosh et al., whereas it implies more probable but still uncertain default for Collard et al.

Our paper's concern is with optimal debt, which is computed as being some debt level short of maximum sustainable debt. Optimal debt falls short of maximum debt because of government's concern with – its own – future welfare: higher debt implies higher probability of default; default implies foregone utility; a government concerned with future welfare therefore seeks to avoid default by choosing a level of debt lower than maximum debt.

[6] See the surveys by Aguiar and Amador (2014) and Panizza, Sturzenegger, and Zettelmeyer (2009), the monographs by Reinhart and Rogoff (2009) and Sturzenegger and Zettelmeyer (2006), and the references therein.

[7] See also Section 6 in Aguiar and Amador (2014). Hamann (2002) constitutes an early attempt at calibrating optimal sovereign debt levels.

[8] Such work can, however, be traced to Aaron's (1966) early work on constant debt ratios, as well as Reinhart, Rogoff, and Savastano's (2003) later work on 'debt intolerance.'

As noted previously, the assumption of excusable default is central to our analysis. How realistic is it? Very! Levy Yeyati and Panizza (2011) provide strong evidence of governments' reluctance to default: by analogy to Winston Churchill's aphorism that "you can always count on Americans to do the right thing – after they've tried everything else," governments appear to default only after they have tried every possible way of staving default off. While debt service is costly, default is generally even costlier, especially from the point of view of a government that can generally expect to lose power in the aftermath of default (Borensztein and Panizza, 2009; Malone, 2011). Even a less than fully self-interested government may do its utmost to avoid default: Tomz (2007) has argued that creditors are much more lenient towards borrowers for whom default was clearly unavoidable than those who are perceived to have been too quick to default; Bolton and Jeanne (2011) have noted the potential of sovereign default to jeopardize the proper functioning of an entire banking system, in view of government bonds' importance as collateral for bank loans.

3 Maximum sustainable debt

The first step in our analysis consists in estimating maximum sustainable debt. We follow Collard et al. (2015) for that purpose, but simplify their notation and their exposition to make these more intuitive.

Let y_t denote a given country's output in period t, D_t the debt raised by the country's government in that same period, to be repaid in its entirety in the following period $t+1$ (recall that debt is fully amortized every period), B_t the proceeds obtained by the government in period t from raising that debt (we assume all debt is issued in the form of zero-coupon bonds), α the maximum primary surplus (MPS) the country can achieve on a sustainable basis, and r the risk-free interest rate. Expressed as a fraction of period-t output y_t, debt and proceeds can be written $d_t = D_t/y_t$ and $b_t = B_t/y_t$. Let $g_{t+1} \equiv y_{t+1}/y_t$ denote the rate of growth in output between periods t and $t+1$; g_{t+1} is assumed to be i.i.d. lognormally distributed: $\log(g) \sim N(\mu, \sigma^2)$; we denote $F(.)$ and $f(.)$ the cdf and pdf of g, respectively.

We seek maximum sustainable debt (MSD) d_M and maximum sustainable borrowing (MSB) b_M.[9] We start with the latter. If the country were to raise debt $d_t y_t$ in period t, it would default on that debt in the period $t+1$ in which the debt is due when

$$\alpha y_{t+1} + b_M y_{t+1} < d_t y_t. \tag{1}$$

The RHS represents the debt to be repaid in period $t+1$, the LHS the resources available

[9] MSD d_M and MSB b_M will be seen below not to depend on date t.

to the government for that purpose; these are the sum of the MPS the country can achieve, αy_{t+1}, and the maximum proceeds from sustainable new borrowing in period $t+1$, $b_M y_{t+1}$.

Rearranging (1), default occurs when the growth rate g_{t+1} is such that

$$g_{t+1} < \frac{d_t}{\alpha + b_M} \equiv g_{E,t+1}; \tag{2}$$

$g_{E,t+1}$ denotes the critical rate necessary to avoid default. Assuming zero recovery in default, borrowing proceeds $b_t y_t$ corresponding to debt issued $d_t y_t$ equal

$$b_t y_t = \frac{d_t y_t}{1+r} \Pr\left[g_{t+1} \geqslant g_{E,t+1}\right] = \frac{d_t y_t}{1+r}\left[1 - F\left(g_{E,t+1}\right)\right]. \tag{3}$$

Using (2) to write

$$d_t = \left(\alpha + b_M\right) g_{E,t+1} \tag{4}$$

and dividing (3) by y_t, we can write borrowing proceeds as a fraction of output

$$b_t = \frac{\alpha + b_M}{1+r} g_{E,t+1}\left[1 - F\left(g_{E,t+1}\right)\right]. \tag{5}$$

Borrowing proceeds b_t display a 'Laffer Curve' property in the critical rate necessary to avoid default g_{Et+1}. Proceeds are zero when that rate is zero, as only when no debt is raised can there be no default when growth and consequently output are zero ($g_{E,t+1} = 0$ implies $y_{t+1} = 0$). Proceeds are also zero when that rate is infinite, as default occurs with certainty in such case. Proceeds increase and then decrease between these two extremes.[10] Clearly, then, borrowing proceeds are maximized when the critical rate maximizes $g\left[1 - F(g)\right]$; that rate does not depend on t because the cdf $F(.)$ does not. These maximum proceeds are sustainable, for they rely for debt service on future proceeds that are themselves sustainable. Maximum sustainable borrowing b_M therefore is the fixed point

$$b_M = \frac{\alpha + b_M}{1+r} g_M\left[1 - F\left(g_M\right)\right] \tag{6}$$

$$\Rightarrow b_M = \frac{\alpha g_M\left[1 - F\left(g_M\right)\right]}{1 + r - g_M\left[1 - F\left(g_M\right)\right]} \tag{7}$$

where

$$g_M = \arg\max_g \; g\left[1 - F(g)\right] \tag{8}$$

Note that the fixed point property precludes reliance on ever-larger borrowing ratios, $b_t < b_{t+1} < \ldots$; put differently, maximum borrowing is sustainable when it does not rely on a borrowing bubble.

Using (4), we have that MSD d_M equal

$$d_M = \left(\alpha + b_M\right) g_M = \frac{\alpha\left(1+r\right) g_M}{1 + r - g_M\left[1 - F\left(g_M\right)\right]}. \tag{9}$$

We denote PD_M the corresponding probability of default, $PD_M \equiv F\left(g_M\right)$.

[10] Note that $\partial b_t / \partial g_{M,t+1}|_{g_{M,t+1}=0} > 0$.

4 Optimal debt under excusable default

We now turn to the determination of optimal debt, making use of our previous analysis of maximum debt and borrowing for that purpose.

The self-interested government's period-t consumption is $\alpha_u y_t + b_t y_t - d_{t-1} y_{t-1}$, where α_u, $\alpha_u < 1$, denotes the fraction of output that accrues to the government, which further makes use of the entirety of net debt proceeds – new debt proceeds minus debt repayment. Note that the self-interested government does not distinguish between foreign and domestic debt, as both constitute sources of additional funds to the government as well as claims on funds that would otherwise be available to the government; debt under excusable default therefore should be viewed as total debt.

We denote $u(.)$ the utility function that the government maximizes and θ, $\theta < 1$, the (non-discounted) weight the government puts on future periods. Assuming that a self-interested government's horizon coincides with the government's expected time in office, θ might be the government's probability of reelection. Finally, we assume that the senior members of a government that has defaulted never again return to power after default; their political lives end with default, with corresponding payoffs zero.

The value function $V_E(d_{t-1}y_{t-1}, y_t)$ in period t is

$$V_E(d_{t-1}y_{t-1}, y_t) = \max_{d_t} u(\alpha_u y_t + b_t y_t - d_{t-1} y_{t-1}) + \frac{\theta}{1+r} E\left[V_E(d_t y_t, y_{t+1})\right], \tag{10}$$

where the expectation is over period-$t+1$ output. The government chooses debt issuance d_t and corresponding proceeds b_t to maximize the value function; (3) relates d_t and b_t.

Assuming CRRA utility $u(c) = c^{1-\gamma}/(1-\gamma)$, we can rewrite the value function as

$$V_E(d_{t-1}y_{t-1}, y_t) = V_E\left(\frac{d_{t-1}y_{t-1}}{y_t}, 1\right) y_t^{1-\gamma} = v_E(\omega_t)\, y_t^{1-\gamma}, \tag{11}$$

where $\omega_t \equiv d_{t-1}y_{t-1}/y_t$ denotes the stock of debt carried over from period $t-1$ into period t, expressed not as a fraction of period-$t-1$ output as is d_{t-1} but as a fraction of period-t output. This is because it is out of period-t output and not out of period-$t-1$ that the debt $d_{t-1}y_{t-1}$ must be repaid. It is therefore $\omega_t y_t$ and not $d_{t-1}y_{t-1}$ that matters for the determination of default in period t. Intuitively, even very high debt carried over from the previous period can be serviced if growth between that period and the present has been very high. We refer to ω as the realized debt ratio, in contrast to the promised debt ratio d. In a manner analogous to these two ratios, $v_E(.)$ expresses the value function as a fraction of output; it decreases in realized debt ω_t raised in the previous period, to be repaid in the present.[11]

[11] That $v_E'(.) < 0$ is immediate from (12) below.

Using (10) and (11), we can write

$$v_E(\omega_t)\, y_t^{1-\gamma} = \max_{d_t}\; u(\alpha_u + b_t - \omega_t)\, y_t^{1-\gamma} + \frac{\theta}{1+r} E\left[v_E(\omega_{t+1})\, y_{t+1}^{1-\gamma}\right]$$

$$\Leftrightarrow v_E(\omega_t) = \max_{d_t}\; u(\alpha_u + b_t - \omega_t) + \frac{\theta}{1+r} E\left[v_E(\omega_{t+1})\, g_{t+1}^{1-\gamma}\right], \qquad (12)$$

where $\omega_{t+1} \equiv d_t y_t / y_{t-1}$.

Using the lognormality of the (i.i.d.) growth rate, $\log(g) \sim N(\mu, \sigma^2)$, we can write

$$\begin{aligned} 1 - F(g_{E,t+1}) &= \Pr[g \geqslant g_{E,t+1}] \\ &= \Pr[\log(g) \geqslant \log(g_{E,t+1})] \\ &= 1 - \Phi\left(\frac{\log(g_{E,t+1}) - \mu}{\sigma}\right) \\ &= 1 - \Phi(x_{E,t+1}), \end{aligned}$$

where

$$x_{E,t+1} \equiv \frac{\log(g_{E,t+1}) - \mu}{\sigma}$$

and consequently

$$g_{E,t+1} = \exp(\mu + \sigma x_{E,t+1}).$$

We can therefore rewrite (4) and (5) as

$$d_t = (\alpha + b_M)\exp(\mu + \sigma x_{E,t+1}) \qquad (13)$$

and

$$b_t = \frac{\alpha + b_M}{1+r}\exp(\mu + \sigma x_{E,t+1})\left[1 - \Phi(x_{E,t+1})\right], \qquad (14)$$

respectively. By analogy to $x_{M,t+1}$, we define

$$s \equiv \frac{\log(g) - \mu}{\sigma}$$

and consequently

$$g = \exp(\mu + \sigma s). \qquad (15)$$

Substituting (13), (14), and (15) into (12), and changing the control variable from d_t to $x_{E,t+1}$, we can write

$$\begin{aligned} v_E(\omega_t) &= \max_{x_{E,t+1}}\; u\left(\alpha_u + \frac{\alpha + b_M}{1+r}\exp(\mu + \sigma x_{E,t+1})\left[1 - \Phi(x_{E,t+1})\right] - \omega_t\right) \\ &\quad + \frac{\theta}{1+r}\int_{x_{E,t+1}}^{\infty} \left\{ \begin{array}{c} \exp\left[(1-\gamma)(\mu + \sigma s)\right] \times \\ v_E\left((\alpha + b_M)\exp\left[\sigma(x_{E,t+1} - s)\right]\right) \end{array} \right\} d\Phi(s), \end{aligned} \qquad (16)$$

where we have used the growth rate's i.i.d. property to write

$$
\begin{aligned}
E\left[v_E\left(\omega_{t+1}\right) g_{t+1}^{1-\gamma}\right] &= E\left[v_E\left(\frac{d_t}{g_{t+1}}\right) g_{t+1}^{1-\gamma}\right] \\
&= E\left[v_E\left(\frac{d_t}{g}\right) g^{1-\gamma}\right] \\
&= E\left[v_E\left(\frac{\left(\alpha+b_M\right)\exp\left(\mu+\sigma x_{E,t+1}\right)}{\exp\left(\mu+\sigma s\right)}\right)\exp\left[\left(1-\gamma\right)\left(\mu+\sigma s\right)\right]\right] \\
&= \int_{x_{E,t+1}}^{\infty}\left\{\begin{array}{c}\exp\left[\left(1-\gamma\right)\left(\mu+\sigma s\right)\right]\times \\ v_E\left(\begin{array}{c}\left(\alpha+b_M\right)\times \\ \exp\left[\sigma\left(x_{E,t+1}-s\right)\right]\end{array}\right)\end{array}\right\} d\Phi\left(s\right); \qquad (17)
\end{aligned}
$$

the last equality is true by the assumption that payoff remains zero after default, reflecting the government's loss of power following default and its (senior) members' ensuing retirement from politics.

We iterate to find the value function $v_E\left(.\right)$ and solve for optimal debt d_E^* and borrowing b_E^* along the balanced growth path: $d_t = d_{t+1} = d_E^*$ and $b_t = b_{t+1} = b_E^*$ when output grows at the critical rate $g_E^* = \exp\left(\mu+\sigma x_E^*\right)$ where x_E^* solves (16) at $\omega_t = \omega_E^* = \alpha + b_M$.[12] We have

$$d_E^* = \left(\alpha+b_M\right)\exp\left(\mu+\sigma x_E^*\right)$$

and

$$b_E^* = \frac{d_E^*}{1+r}\left[1-\Phi\left(x_E^*\right)\right] = \frac{\alpha+b_M}{1+r}\exp\left(\mu+\sigma x_E^*\right)\left[1-\Phi\left(x_E^*\right)\right].$$

The corresponding probability of default PD_E^* is

$$PD_E^* \equiv \Phi\left(x_E^*\right).$$

5 Optimal debt under strategic default

For comparison purposes, we compute optimal debt in the case of strategic default. We mainly follow Arellano (2008), which we modify slightly in order to exploit the growth rate's i.i.d. property.

We assume that a country that has defaulted strategically is excluded from international financial markets for at least one period. At the end of that period, the country escapes autarky and returns to financial markets with probability λ; with probability $1-\lambda$, the country remains in autarky for one additional period, at the end of which the 'escape'

[12]That $\omega_E^* = \alpha + b_M$ is immediate from

$$\omega_E^* = \frac{d_E^*}{g_E^*} = \frac{\left(\alpha+b_M\right)g_E^*}{g_E^*} = \alpha + b_M.$$

The derivation uses the fact that the realized growth rate equals the critical rate along the balanced growth path.

process repeats itself. While it is in autarky, the country loses a fraction τ of its output. Thus, if the country should be in autarky in period t in which output is y_t, the country's consumption would be $\alpha_u (1-\tau) y_t$; $\alpha_u = 1$ for the altruistic government which we have argued engages in strategic default.[13] If in contrast the country should have access to financial markets in that period, consumption would be

$$\alpha_u y_t + B_t - D_{t-1} = \alpha_u y_t + b_t y_t - d_{t-1} y_{t-1}.$$

Note that debt represents only foreign debt, as domestic debt constitutes neither a source of funds nor a claim on funds for the country's population considered in its entirety.

We denote $V_A(y_t)$ the value function in autarky when output is y_t and $V_S(d_{t-1}y_{t-1}, y_t)$ outside autarky; a country that eschews default and the ensuing autarky must service the debt $d_{t-1}y_{t-1}$ carried over from the previous period. We have

$$V_A(y_t) = u(\alpha_u (1-\tau) y_t) + \frac{\theta}{1+r} E\left[\lambda V_S(0, y_{t+1}) + (1-\lambda) V_A(y_{t+1})\right],$$

where we have assumed that a country that has defaulted repudiates all outstanding debt; $\theta \simeq 1$ for the altruistic government that maximizes the country's population's discounted lifetime utility.

Using utility's CRRA form, $u(c) = c^{1-\gamma}/(1-\gamma)$, we can write

$$\begin{aligned} v_A y_t^{1-\gamma} &= \frac{(\alpha_u (1-\tau) y_t)^{1-\gamma}}{1-\gamma} + \frac{\theta}{1+r} E\left[\lambda v_S(0) y_{t+1}^{1-\gamma} + (1-\lambda) v_A y_{t+1}^{1-\gamma}\right] \\ \Leftrightarrow v_A &= \frac{\alpha_u^{1-\gamma}(1-\tau)^{1-\gamma}}{1-\gamma} + \frac{\theta}{1+r} E\left[\lambda v_S(0) g_{t+1}^{1-\gamma} + (1-\lambda) v_A g_{t+1}^{1-\gamma}\right] \\ \Leftrightarrow v_A &= \frac{\frac{\alpha_u^{1-\gamma}(1-\tau)^{1-\gamma}}{1-\gamma} + \frac{\theta \lambda v_S(0)}{1+r} E\left[g_{t+1}^{1-\gamma}\right]}{1 - \frac{\theta(1-\lambda)}{1+r} E\left[g_{t+1}^{1-\gamma}\right]} \end{aligned} \tag{18}$$

The value function outside autarky is

$$\begin{aligned} V_S(d_{t-1}y_{t-1}, y_t) &= \max\left\{ V_A(y_t), \max_{d_t} \begin{array}{c} u(\alpha_u y_t + b_t y_t - d_{t-1} y_{t-1}) \\ + \frac{\theta}{1+r} E\left[V_S(d_t y_t, y_{t+1})\right] \end{array} \right\} \\ \Leftrightarrow V_S\left(\frac{d_{t-1}y_{t-1}}{y_t}, 1\right) y_t^{1-\gamma} &= \max\left\{ v_A y_t^{1-\gamma}, \max_{d_t} \begin{array}{c} u\left(\alpha_u + b_t - \frac{d_{t-1}y_{t-1}}{y_t}\right) y_t^{1-\gamma} \\ + \frac{\theta}{1+r} E\left[V_S\left(\frac{d_t y_t}{y_{t+1}}, 1\right) y_{t+1}^{1-\gamma}\right] \end{array} \right\} \end{aligned}$$

Recalling that $\omega_t \equiv (d_{t-1}y_{t-1})/y_t$, we can write

$$v_S(\omega_t) = \max\left\{ v_A, \max_{d_t} (\alpha_u + b_t - \omega_t)^{1-\gamma} + \frac{\theta}{1+r} E\left[v_S(\omega_{t+1}) g_{t+1}^{1-\gamma}\right] \right\} \tag{19}$$

As in the case of excusable default, we need to determine the relation between the face value of zero coupon debt raised in period t and due in period $t+1$, $d_t y_t$, and its

[13] We include α_u despite it being equal to one for comparison with the analysis of Section 4; we do likewise for θ below.

corresponding proceeds in period t, $b_t y_t$. For that purpose, we need to determine the range of realized debt ratios for which the government chooses to default in period $t+1$, that is, the range of debt ratios ω_{t+1} such that $v_S(\omega_{t+1}) > v_A$. We define ω_S to be the minimum such ratio, $v_S(\omega_S) = v_A$ (note that ω_S does not depend on t because of the i.i.d. distribution of the growth rate g); we refer to ω_S as maximum feasible debt (MFD). Default occurs over the range of debt ratios $\omega_{t+1} > \omega_S$, that is, over the range of growth rates such that

$$\frac{d_t}{g_{t+1}} > \omega_S$$
$$\Leftrightarrow g_{t+1} < \frac{d_t}{\omega_S} \equiv g_{S,t+1}. \tag{20}$$

We can therefore write

$$d_t = \omega_S g_{S,t+1} \tag{21}$$

and

$$b_t = \frac{d_t}{1+r} \Pr\left[g_{t+1} \geqslant g_{S,t+1}\right] = \frac{\omega_S}{1+r} g_{S,t+1}\left[1 - F\left(g_{S,t+1}\right)\right]. \tag{22}$$

We now make use of the growth rate's (i.i.d.) lognormal distribution, $\log(g) \sim N(\mu, \sigma^2)$, to write

$$E\left[g_{t+1}^{1-\gamma}\right] = E\left[g^{1-\gamma}\right] = \exp\left[(1-\gamma)\left(\mu + (1-\gamma)\sigma^2/2\right)\right]$$

in (18) as well as

$$F\left(g_{S,t+1}\right) = \Phi\left(x_{S,t+1}\right) \tag{23}$$

where

$$x_{S,t+1} = \frac{\log\left(g_{S,t+1}\right) - \mu}{\sigma}$$

and consequently

$$g_{S,t+1} = \exp\left(\mu + \sigma x_{S,t+1}\right). \tag{24}$$

We can therefore rewrite (21) and (22) as

$$d_t = \omega_S \exp\left(\mu + \sigma x_{S,t+1}\right) \tag{25}$$

and

$$b_t = \frac{\omega_S}{1+r} \exp\left(\mu + \sigma x_{S,t+1}\right)\left[1 - \Phi\left(x_{S,t+1}\right)\right]. \tag{26}$$

Substituting (25) and (26) into (19), defining $s \equiv [\log(g) - \mu]/\sigma \sim N(0,1)$ and consequently $g = \exp[\mu + \sigma s]$, and using a similar transformation to that in (17), we can write

$$v_S(\omega_t) = \max\left\{\begin{array}{c} v_A, \max\limits_{x_{S,t+1}} \left(\alpha_u + \frac{\omega_S}{1+r}\exp\left(\mu + \sigma x_{S,t+1}\right)\left[1 - \Phi\left(x_{S,t+1}\right)\right] - \omega_t\right)^{1-\gamma} \\ + \frac{\theta}{1+r}\left[\begin{array}{c} \int_{-\infty}^{x_{S,t+1}} v_A \exp\left[(1-\gamma)(\mu+\sigma s)\right] d\Phi(s) \\ + \int_{x_{S,t+1}}^{\infty} v_S\left(\omega_S \exp\left[\sigma\left(x_{S,t+1} - s\right)\right]\right) \exp\left[(1-\gamma)(\mu+\sigma s)\right] d\Phi(s) \end{array}\right] \end{array}\right\} \tag{27}$$

We iterate to find the value function $v_S(.)$ and solve for optimal debt d_S^* and borrowing b_S^* along the balanced growth path: $d_t = d_{t+1} = d_S^*$ and $b_t = b_{t+1} = b_S^*$ when output grows at the critical rate $g_S^* = \exp(\mu + \sigma x_S^*)$ where x_S^* solves (27) at $\omega_t = \omega_S^* = d_S^*/g_S^*$ and

$$d_S^* = \omega_S \exp(\mu + \sigma x_S^*) .$$

We further have

$$b_S^* = \frac{d_S^*}{1+r} [1 - \Phi(x_S^*)] = \frac{\omega_S}{1+r} \exp(\mu + \sigma x_S^*) [1 - \Phi(x_S^*)]$$

and

$$PD_S^* \equiv \Phi(x_S^*) .$$

6 Parameter values

Given our focus on advanced economies, we use US data over the period 1955-2014 to calibrate the model. We set the real interest rate r to match the return on 1-year Treasury bonds (series GS1 in FRED), net of the GDP deflator (GDPDEF in FRED); $r = 1.85\%$. We set the output process to match the mean and volatility of per-capital output (A939RX0Q048SBEA); $\mu = 1.94\%$ and $\sigma = 2.13\%$. In accordance with IMF (2011) estimates, we set maximum primary surplus equal to 5% of GDP; $\alpha = 0.05$. We set the fraction of output that is of concern to the self-interested government that engages in excusable default $\alpha_u = 0.5$; this is somewhat higher than the ratio of government spending to GDP because the government may be concerned with part of private spending – that by its favored constituents for example – in addition to public spending; we set $\alpha_u = 1$ for the altruistic government that engages in strategic default. We set $\theta = 0.6$ in the case of excusable default and $\theta = 0.968$ in that of strategic default. In the former case, the probability of reelection θ concedes a moderate electoral advantage to the incumbent; in the latter, θ is such that the discount factor $\theta/(1+r)$ equals 0.95, the value set by Arellano (2008).[14] It is also Arellano's value that we choose for the probability of escaping autarky; $\lambda = 0.734$.[15] We follow Aguiar and Gopinath (2006) in setting the loss of output in autarky equal to 2% of GDP; $\tau = 0.02$. Finally, we set CRRA coefficient $\gamma = 0.5$.[16] Table 1 shows the various parameter values, distinguishing between the two cases of excusable and strategic default.

[14] We do not transform Arellano's (2008) quarterly value into its annual equivalent, because that value, 0.814, arguably is too small to be consistent with the assumption of altruistic government.

[15] As noted in Footnote 4, 0.734 is the annual equivalent to Arellano's (2008) quarterly 0.282.

[16] Our CRRA coefficient is bounded above by 1. To see why this is the case, note that the senior members of a government than has engaged in excusable default have zero payoff, as they are assumed never to return to power. A γ larger than 1 would result in the paradoxical situation in which governments would consistently be better off in default, for the zero payoff of default would then be higher than the negative payoff of debt service ($c^{1-\gamma}/(1-\gamma) < 0$ if $c > 0$ and $\gamma > 1$).

7 Calibration: excusable default

7.1 Basic results

Table 2 shows the results calibrated for excusable default. Our main interest is in the results in the first row, which use the parameter values for excusable default ($\alpha_u = 0.5$ and $\theta = 0.6$). Maximum sustainable debt d_M is 85.5%% of GDP, maximum sustainable borrowing b_M is 83.3%. The very small difference between MSD and MSB is due to the low risk-free interest rate, $r = 1.85\%$, and the very low probability of default at MSD, $PD_M = 0.768\%$. This very low probability in turn reflects the low volatility of growth, $\sigma = 2.13\%$, which implies an extremely steep transition from near-zero to near-one probability of default PD as a function of the face value of debt d (see Figure 1). Any face value of debt other than one associated with a very low probability of default would therefore see a collapse in borrowing proceeds. This is confirmed by the Laffer curve in Figure 2, which shows a dramatic decline in borrowing proceeds b past MSD d_M.[17]

The very high MSD, at least in comparison to the values generally obtained under strategic default, reflects the government's high debt service capacity, which under excusable default constitutes the primary determinant of maximum sustainable debt: there can be no excusable default when the government has the capacity to service debt. A government's debt service capacity in turn depends on the maximum primary surplus, the risk-free interest rate, the growth rate, and the ability repeatedly to raise new debt, which in the absence of default effectively serves to make all future primary surpluses available for the repayment even of debt of maturity only one year. As the probability of default is very low at MSD ($PD_M = 0.768\%$), new debt can be counted upon with near certainty, and MSD is raised far above MPS ($d_M = 85.5\% \gg 5\%$ =MPS). We examine the sensitivity of MSD to the maximum primary surplus, the risk-free interest rate, and the growth rate below.

Recall that our model is calibrated to US data. Does US debt at around 100% of GDP not imply that the US should have defaulted already, given the very rapid increase in the probability of default past MSD at 85% (see Figure 1)? That default has not in fact occurred may be attributable to the Federal Reserve's purchase of US public debt: the Fed owns about 15% of US public debt.[18] We assume no such purchases in our model.

Optimal sovereign debt d_E^* equals 84.6% of GDP; it is extremely close to MSD d_M. To understand this result, recall from Section 3 that MSD is the level of debt that maximizes

[17] In order further to highlight the importance of low volatility to the steepness of the transitions in default probability and borrowing proceeds, Figures 1 and 2 show the case $\sigma = 21.3\%$ in addition to $\sigma = 2.13\%$.

[18] That Fed holdings at 15% equal the difference between actual debt at 100% and calibrated MSD at 85% is a – welcome – coincidence. It was not part of the calibration of the model.

borrowing proceeds; it would therefore be chosen by governments concerned only with the current payoff, that is, governments for which $\theta = 0$. As such is not the case, a governments who wishes to avoid jeopardizing future payoffs through debt-induced default chooses a level of debt lower than MSD. This consideration fails to decrease optimal debt markedly below maximal, however, because the probability of default PD_M at MSD is very small already: even a level of debt very close to MSD has very low probability of default. The government consequently chooses optimal debt close to MSD, which represents the most advantageous trade-off between payment promised and proceeds received, the apex of the Laffer curve in Figure 2. The probability of default at optimal debt is extremely low, $PD_E^* = 0.167\%$; it combines with the low risk-free interest rate to make optimal borrowing proceeds b_E^* very close to optimal debt d_E^*, $b_E^* = 82.9\%$ of GDP.

The results in the third row use the parameter values for strategic default ($\alpha_u = 1$ and $\theta = 0.968$).[19] They are of interest mainly in that optimal debt d_E^*, proceeds b_E^*, and probability of default PD_E^* are to be compared with the results in Section 8 calibrated for strategic default, obtained with the same parameter values. Clearly, MSD d_M, MSB b_M, and probability of default PD_M are unaffected by the changes in α_u and θ, as maximum debt is computed independently of any concern for future payoffs.[20] On the other hand, optimal debt d_E^*, proceeds b_E^*, and probability of default PD_E^* all decrease as compared to their earlier values, reflecting the now greater importance attached to future payoffs. This is immediate for θ, perhaps slightly less so for α_u: the concavity of the utility function implies that α_u and borrowing proceeds are strategic substitutes; an increase in α_u therefore decreases optimal debt further away from maximum debt both by decreasing the benefits to be had from increased proceeds in the current period and by increasing the payoff to avoiding default in the next period.[21]

7.2 Sensitivity analysis

Figures 3(a) to 4(c) show the sensitivity of maximum and optimal debt and their corresponding proceeds and associated probabilities of default to the exogenous parameters. Figures 3(a), 4(a), 4(b), and 4(c) confirm the importance of the MPS, the risk-free interest rate, and the mean and volatility of the growth rate to MSD. MSD increases from 40 to 170% of GDP as MPS α increases from 2.5 to 10% of GDP; it increases from 80 to 95% of GDP as the mean growth rate μ increases from 1.5 to 2.5%: a government that generates a higher primary surplus from a faster growing economy has more resources available for debt service; it can therefore borrow more. MSD decreases from 90 to

[19] The results in the second row will be discussed in Section 9.

[20] Formally, equations (7), (8), and (9) involve neither θ nor α_u.

[21] See (12) and note that $v_E(\omega_{t+1})$ in $E\left[v_E(\omega_{t+1})\, g_{t+1}^{1-\gamma}\right]$ is itself an increasing function of α_u.

approximately 75% of GDP as the risk-free interest rate varies over the same interval as μ: a higher opportunity cost of capital decreases lenders' willingness to lend to the government, which consequently can borrow less. MSD decreases from approximately 120 to approximately 75% of GDP as the volatility of the growth rate σ increases from 1.5 to 2.5%, the same interval as μ: the more volatile is growth, the greater the likelihood of low growth realizations that leave the government unable to service its debt, the less the government can borrow.

In line with our discussion above, optimal debt d_E^* closely tracks MSD d_M. This is a consequence of the very low probabilities of default at MSD, never above 1% over the ranges considered; the probabilities of default at optimal debt are lower still, rarely exceeding 0.5%. Note that PD_M is invariant in MPS α, the mean growth rate μ, and the risk-free interest rate r; this is an artifact of growth's lognormal distribution and zero recovery: changes in parameter values that should be accommodated in both the size of debt and its riskiness are accommodated only by the former, leaving the latter unchanged.[22] As expected, PD_M increases in volatility σ, reflecting the higher probability of default associated with more volatile growth. The probability of default PD_E^* follows optimal debt d_E^*, increasing where d_E^* increases (α and very slightly μ) and decreasing where d_E^* decreases (very slightly r and σ). That PD_E^* decreases in volatility σ suggests that the indirect effect of σ on PD_E^* through the decreasing d_E^* dominates its direct effect; this again confirms the centrality of optimal debt's tracking of maximum debt to the analysis of optimal debt. Turning to borrowing proceeds b_M and b_E^*, we note that these follow the same pattern as debt, from which they differ only very little by virtue of the low risk-free interest rate and probabilities of default.

In Figures 3(b), 3(c), and 3(d), maximum debt d_M, proceeds b_M, and probability of default PD_M do not change. This is because the parameters α_u, γ, and θ pertain to a trade-off between present and future payoffs that has no relevance for maximum debt. That trade-off is however central to optimal debt d_E^*, which decreases in all three parameters: as noted in Section 7.1, an increase in α_u or θ increases the relative importance of future payoffs, which are not to be jeopardized by default; an increase in γ increases the desirability of smoothing consumption over time, with future consumption again not to be jeopardized through default. The probability of default PD_E^* is decreased by decreasing optimal debt d_E^*, with attending decrease in optimal proceeds b_E^*. Their diverging paths

[22]Formally, consider g_M and g in (8) and define $x_M \equiv [\log(g_M) - \mu]/\sigma$ and $x \equiv [\log(g) - \mu]/\sigma$. Use the lognormality of $F(.)$ to rewrite (8) as

$$\begin{aligned} x_M &= \arg\max_x \exp(\mu + \sigma x)[1 - \Phi(x)] \\ &= \arg\max_x \exp(\sigma x)[1 - \Phi(x)]. \end{aligned}$$

Clearly, $PD_M = \Phi(x_M)$ depends exclusively on σ.

notwithstanding, maximum and optimal debt and borrowing remain close in value. Again, this is due to the very low probability of default at maximum debt: there is little need for optimal debt d_E^* markedly to deviate from maximum debt d_M for the probability of default at optimal debt PD_E^* to be very small. This can be seen in Figure 3(d) for example, where as θ increases from 0.4 to 1, the difference between d_M and d_E^* increases from zero to somewhat less than 5%, and the probability of default at optimal debt PD_E^* decreases to become effectively nil. This suggests that optimal debt's deviation from the tracking of maximum debt is confined to a very narrow range.

8 Calibration: strategic default

8.1 Basic results

Table 3 shows the results calibrated for strategic default. As in Section 7.1, our main interest is in the results in the first row, which use the parameter values for strategic default ($\alpha_u = 1$ and $\theta = 0.968$). The realized debt ratio beyond which the government defaults, maximum feasible debt MFD, is extremely small, $\omega_S = 2.866\%$ of GDP. This result is a consequence of the growth rate's very low volatility: as noted by Aguiar and Gopinath (2006) and Aguiar and Amador (2012), there is relatively little value to the insurance provided by borrowing when there is little volatility in output; there is therefore relatively little to restrain a government behaving strategically from defaulting; default occurs at low debt ratios. This is especially so under the present parametrization, because the proportional cost of autarky at $\tau = 2\%$ of GDP is low and the probability of escaping autarky at $\lambda = 73.4\%$ is high. Optimal debt $d_S^* = 2.712\%$ of GDP is very close to ω_S, yet its associated probability of default, the probability that growth be less than d_S^*/ω_S, is very low, $PD_S^* = 0.024\%$. Again, this is due to the very low volatility of growth; again, the low probability of default at optimal debt and the low interest rate combine to make optimal borrowing proceeds very close to optimal debt, $b_S^* = 2.663\%$ of GDP. Optimal debt d_S^* is close to MFD ω_S for reasons similar to those discussed in Section 7, namely the desirability of maximum proceeds, mitigated only weakly by concern for future payoffs because of the low probability of default. That both forms of default see a very small difference between maximum (d_M, ω_S) and optimal (d_E^*, d_S^*) debt suggests that the difference between strategic and excusable default pertains not to the desirability of high debt levels but to their feasibility. While part of the difference in optimal debt values must be attributable to the distinction between excusable default's total debt and strategic default's foreign debt, such distinction is very unlikely to account for the entire $d_E^* - d_S^* = 85.5\% - 2.7\% = 82.8\%$ difference: slightly less than half of US public debt is held by non-US investors (Labonte and Nagel, 2015).

The results in the second row use the parameter values for excusable default ($\alpha_u = 0.5$ and $\theta = 0.6$). All calibrated values have the same order of magnitude as the values in the first row, with the exception of the probability of default at optimal debt PD^*_S. This confirms, if there were the need to do so, that the order of magnitude difference between the calibrated values of Section 7 and 8 is due not to different parameter values but to the different assumptions regarding default.

Changes α_u and θ have two effects on debt and borrowing, one direct and the other indirect through the autarky payoff v_A in (18). The direct effect of the decrease in α_u and θ is to decrease the importance attached future payoffs, thereby weakening the restraint on the government to engage in strategic default, possibly leading to a decline in maximum feasible debt and optimal debt and borrowing. The indirect effect through v_A is opposite, as the lower v_A that results from the decrease in α_u and θ strengthens the restraint.[23] The calibrated decrease in debt and borrowing suggests that the direct, restraint-weakening effect dominates. The increase in the probability of default at optimal debt, from $PD^*_S = 0.024\%$ in the first row to $PD^*_S = 0.282\%$ in the second, is consistent with such weakening. We show by way of the sensitivity analysis of Section 8.2 that such interpretation is unwarranted.

8.2 Sensitivity analysis

Figures 7(a) to 8(d) show the sensitivity of maximum feasible debt ω_S, optimal debt d^*_S, optimal borrowing proceeds b^*_S, and the probability of default at optimal debt PD^*_S to the exogenous parameters. Figure 7(a) confirms our interpretation in Section 8.1 of the results in the second row of Table 3. As argued in Section 7.1, an increase in the fraction of output that is of concern to the government α_u increases the importance attached future payoffs; it therefore serves to restrain the government from engaging in strategic default (direct effect), unless offset by the increase of v_A in α_u (indirect effect). The direct effect dominates; it increases both ω_S and d^*_S, the latter closely tracking the former. It also increases b^*_S, but leaves PD^*_S essentially unchanged. This suggests that changes in the extent to which strategic default is restrained sometimes are accommodated along a single margin, the level of debt in the present case, rather than along the two possible margins that are the debt level and the default probability.

Figure 7(b) illustrates a case in which both the direct and indirect effects combine to weaken the restraint on the government to engage in strategic default. A larger probability of escaping autarky decreases the cost of default, thereby weakening the restraint on the government and decreasing feasible and optimal debt and borrowing. This direct effect is compounded by the indirect effect through v_A, as the autarky payoff is shown in Figure 9

[23]Figure 9 shows the variation of the autarky payoff in model parameters.

to increase in λ. Changes in the probability of default at optimal debt PD_S^* are (barely) noticeable only for steep changes in optimal debt d_S^*, over the range $0.1 \leqslant \lambda \leqslant 0.2$: as for α_u, accommodation for the most part proceeds along the debt-level margin.

Much the same phenomenon is at work in Figure 7(c), albeit in the opposite direction: A larger loss of output in autarky increases the cost of default, thereby strengthening the restraint on the government and increasing feasible and optimal debt and borrowing. This direct effect is compounded by the indirect effect through v_A, as the autarky payoff is shown in Figure 9 to decrease in τ. The corresponding increase in PD_S^* is extremely small, confirming yet again the dominance of the debt-level margin over its default-probability counterpart.

An increase in γ increases risk-aversion, thereby decreasing the government's willingness to in engage default, absent an offsetting change in v_A. Debt and borrowing increase, as does the probability of default. The decrease of debt and borrowing in the importance of future payoffs θ suggests that, over the range of values considered at least, the indirect effect of θ through v_A dominates the direct effect: the restraint is weakened despite the greater importance attached future payoffs; debt, borrowing, and the probability of default decrease. The increase in PD_S^* in the second row of Table 3 therefore can be viewed as stemming from the θ-induced increase in v_A.

In Figure 8(b), the greater discounting of future payoffs brought about by an increase in the interest rate r weakens the constraint on the government, absent an offsetting change in v_A. Maximum and optimal debt debt and borrowing decrease; the probability of default is essentially unchanged. In Figure 8(c), the higher mean growth strengthens the restraint on the government, absent an offsetting change in v_A; maximum and optimal debt and borrowing increase, as does the probability of default.

Figure 8(d) is interesting in that it is the only one for which the change in optimal debt and borrowing diverges from that of the probability of default. The essentially unchanged maximum feasible debt suggests that the restraint on the government is unchanged, perhaps because the strengthening direct effect and the weakening indirect effect cancel each other (higher volatility further endangers future payoffs through increased default; it increases the autarky payoff through a higher expected growth rate).[24] Absent a change in maximum debt, optimal debt decreases to account for the now larger possibility of default, but not so much as to prevent an increase in the probability of default.

[24]Recall that $E[g] = \exp\left(\mu + \sigma^2/2\right)$ for g lognormally distributed.

9 Growth collapses

We now add the possibility of growth collapses (Rietz, 1988; Barro, 2006; Barro and Ursua, 2011). Specifically, we assume

$$\log(g) = \mu + u - z;$$

u and v are mutually independent, $u \sim N(0, \sigma^2)$, and

$$v = \begin{cases} z & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases}$$

where z is distributed as

$$f(z) = \begin{cases} \alpha e^{-\alpha(z - z_0)} & \text{if } z \geqslant z_0. \\ 0 & \text{otherwise.} \end{cases}$$

We set $p = 0.01$, $\alpha = 4.5$, $z_0 = -\log(1 - 0.095)$; μ and σ remain as before. There is a 1% probability that there be a growth collapse; collapse, if it should occur, involves a decline in GDP of at least 9.5% and is exponentially distributed with rate 4.5.

The calibrated values are shown in the second row of Table 2. As expected, maximum and optimal debt and borrowing decrease whereas default probabilities increase. The government adjusts along the two available margins to the possibility of a growth collapse that would leave the government with insufficient resources to service its debt; the first margin will be recalled to be the debt level, the second the default probability. The probability of default at optimal debt is now $PD_E^* = 0.971\%$, representing a one in a century frequency of default; optimal debt is $d_E^* = 70\%$, MSD is $d_M = 73.3\%$.

Figure 5 shows the sensitivity of maximum and optimal debt and their corresponding proceeds and associated probabilities of default to the rate of the distribution α, the probability that there be a collapse in growth p, and the minimum decline in output such collapse involves z_0. Panel (a) shows that the rate of the distribution has practically no effect on the values of interest, at least over the range of rates considered; it is only as the distribution of losses becomes very concentrated around the minimum value of 9.5% that the probability of default at optimal debt decreases somewhat. Panel (c) shows that increasing that minimum value beyond 10% also has practically no effect, as does decreasing it with the exception of default probabilities, which decrease as minimum losses decrease. The intuition is that as there is no recovery in default, the decrease in output beyond the level at which default occurs is of no importance to lenders. It is the occurrence rather than the extent of default that matters.

This is confirmed by Panel (b), which shows the variation of the values of interest in the probability p that collapse occurs. There is now a marked decrease in debt and borrowing, and a marked increase in default probabilities as collapse becomes more likely. Lenders react to the greater probability of collapse by requiring a higher implicit interest rate (the ratios d_M/b_M and d_E^*/b_E^* increase from 1.032 to 1.052 and from 1.024 to 1.044, respectively, as p increases from 0.5% to 2.5%; see Figure 6). The government responds along the two margins of quantity and quality, issuing less and riskier debt.

10 Conclusion

We have revisited the issue of optimal sovereign debt, assuming self-interested governments engaging in excusable default where most previous work had assumed benevolent governments engaging in strategic defaults. Our assumption of self-interested government is more in accordance with the Public Choice Theory of government than is the alternative assumption of benevolent government, our assumption of excusable default more in accordance with extensive empirical evidence which documents governments' extreme reluctance to default: governments are no doubt mindful of the loss of power that generally follows default.

Our calibrated optimal debt level (85% of GDP) is well above those obtained under the assumption of strategic default (ranging from 1% to 38%), and much closer to those observed in practice (often exceeding 100%). Lenders more readily lend to governments they expect to do their utmost to avoid default than to governments they fear continuously trade off the costs and benefits of default; governments exploit such readiness to reach optimal debt levels only very slightly below those of maximum sustainable debt (MSD). The very low probability of default at MSD, a consequence of the very low volatility of growth, keeps optimal debt close to MSD, which to a borrowing government represents the most advantageous trade-off between payment promised and proceeds received.

We have found optimal debt to be most sensitive to a country's maximum primary surplus, the mean and volatility of its growth rate, and the interest rate. Incorporating the possibility of growth collapses in our calibration, we have raised the probability of default at optimal debt from an implausibly low 0.167% to a much more realistic 0.971%, corresponding to a one in a century frequency of default for an advanced economy such as the US. The now lower optimal debt at 70% and MSD at 73%, much below prevailing debt levels, suggest the need to incorporate governments' ability to direct central banks' purchases of government debt into our analysis. We leave such extension to future work.

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A Tables

Table 1: Parameter Values

	r (%)	μ (%)	σ (%)	α	α_u	θ	λ	τ	γ
Excusable Default	1.85	1.94	2.13	0.05	0.50	0.60	NA	NA	0.50
Strategic Default	1.85	1.94	2.13	NA	1.00	0.968	0.734	0.02	0.50

Note: NA stands for Not Applicable.

Table 2: Excusable Default

	α_u	θ	d_M (%)	$d_E^\star$ (%)	b_M (%)	$b_E^\star$ (%)	PD_M (%)	$PD_E^\star$ (%)
Baseline	0.50	0.60	85.534	84.610	83.336	82.934	0.768	0.167
Collapse	0.50	0.60	73.318	70.170	70.720	68.225	1.759	0.973
Baseline	1.00	0.968	85.534	81.896	83.336	80.408	0.768	0.000

Note: The subscript M denotes maximum values, the superscript $\star$ denotes optimal values. The Growth Collapse model assumes that the growth process takes the form $g = \mu + \sigma\varepsilon -$, where $z > 0$ with probability p and 0 with probability $1 - p$. When positive, z has pdf $f(z) = \alpha \exp(-\alpha(z - z_0))$ if $z > z_0$, 0 otherwise. In the application, we set $p = 0.01$, $\alpha = 4.5$ and $z_0 = -\log(1 - 0.095)$ implying that only GDP drops of more than 9.5% are considered collapses (see Barro, 2006, and Barro and Ursua, 2014).

Table 3: Strategic Default

α_u	θ	ω_S (%)	$d_S^\star$ (%)	$b_S^\star$ (%)	$PD_S^\star$ (%)
1.00	0.97	2.866	2.712	2.663	0.024
0.50	0.60	2.204	2.119	2.075	0.282

Note: The subscript S stands for 'strategic,' the superscript $\star$ denotes optimal values.

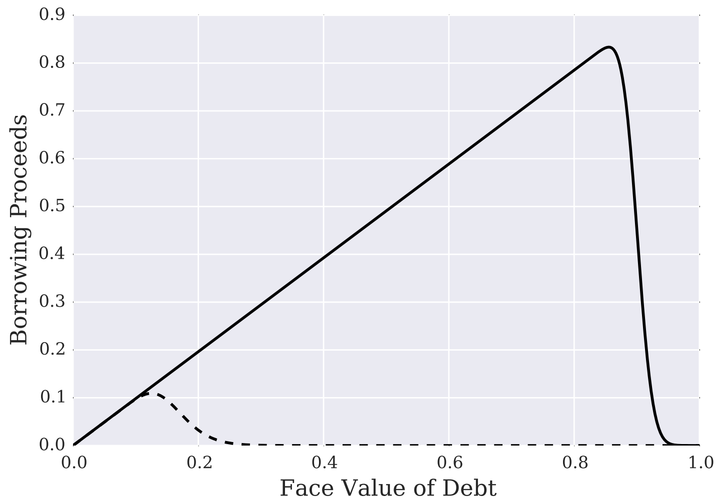
The value of λ needed to generate $d_S^\star = 85\%$ of GDP is 0.023; the corresponding value of τ is 0.495.

B Figures

Figure 1: Probability of Default PD

Benchmark Volatility, 10 × Benchmark Volatility

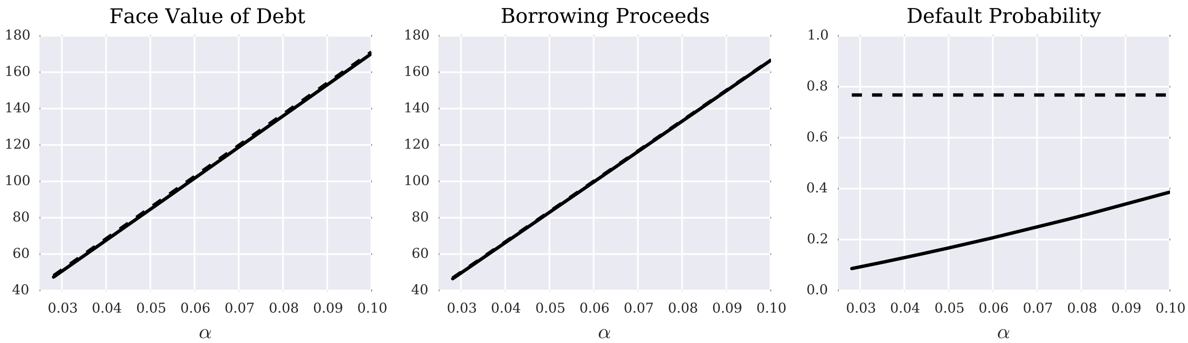
Figure 2: Borrowing proceeds b



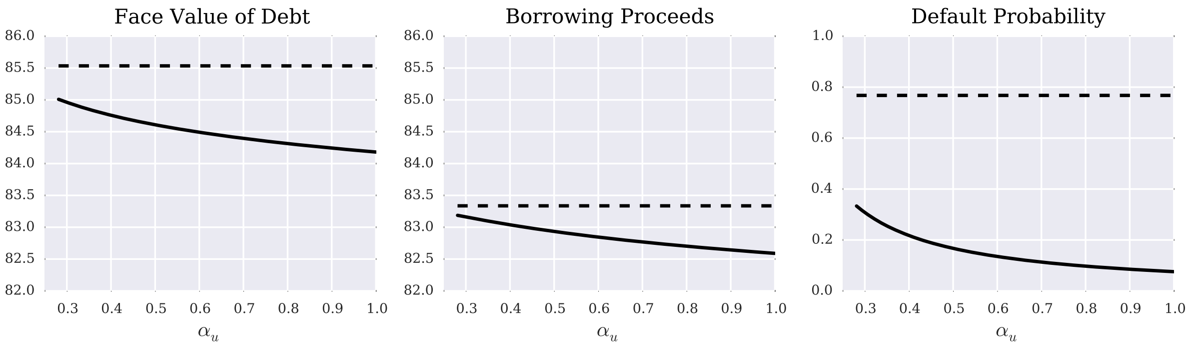
Benchmark Volatility, 10 × Benchmark Volatility

Figure 3: Sensitivity Analysis: Excusable Default Model (I)

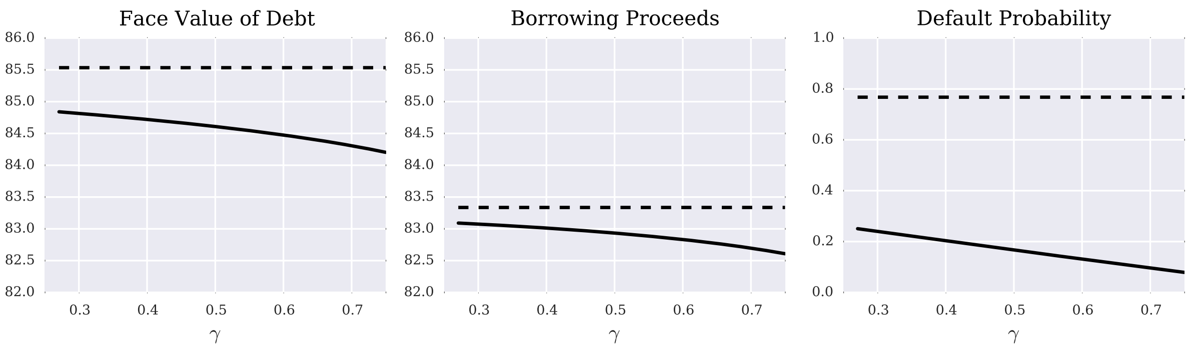
((a)) Variation in α



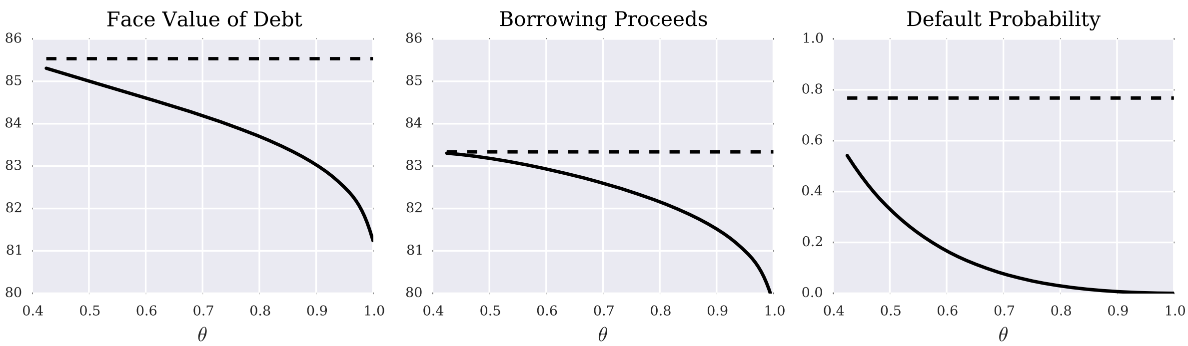
((b)) Variation in α_u



((c)) Variation in γ



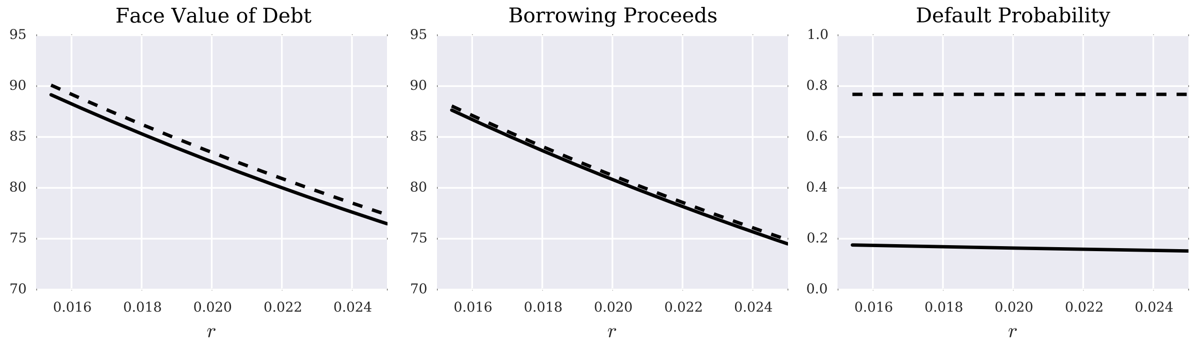
((d)) Variation in θ



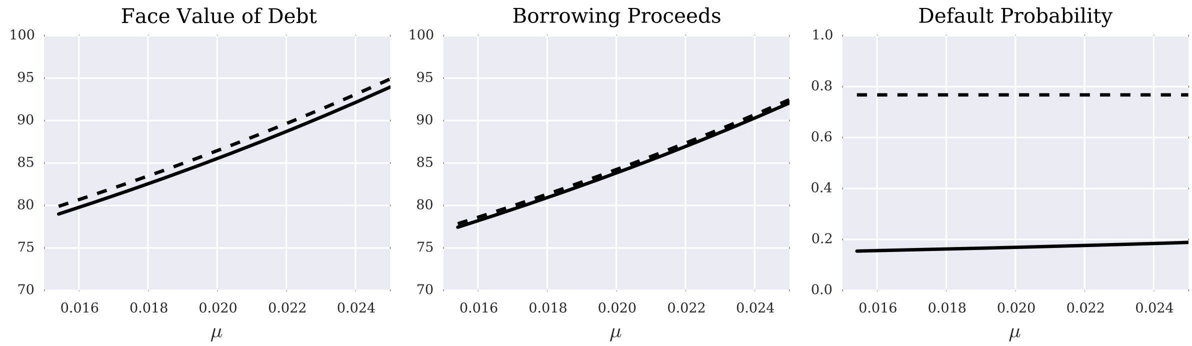
- - - Maximum, —— Optimal

Figure 4: Sensitivity Analysis: Excusable Default Model (II)

((a)) Variation in r



((b)) Variation in μ



((c)) Variation in σ

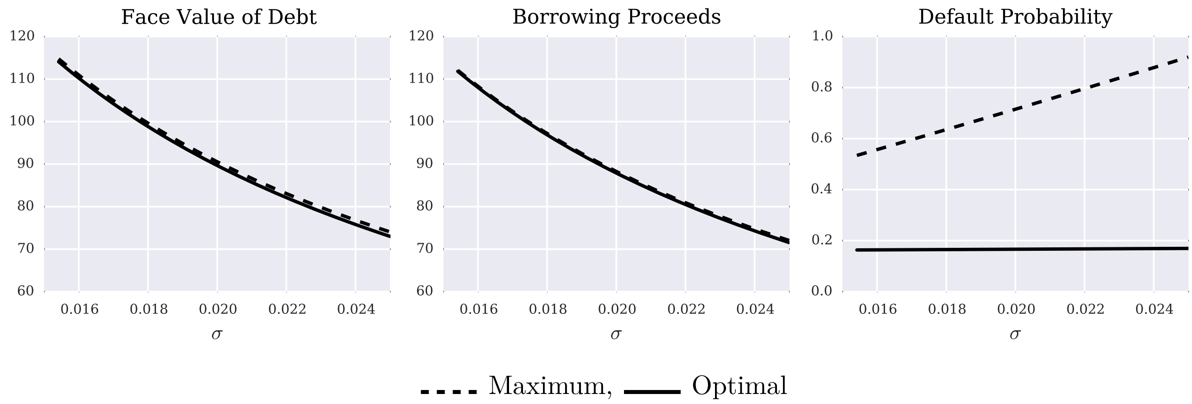
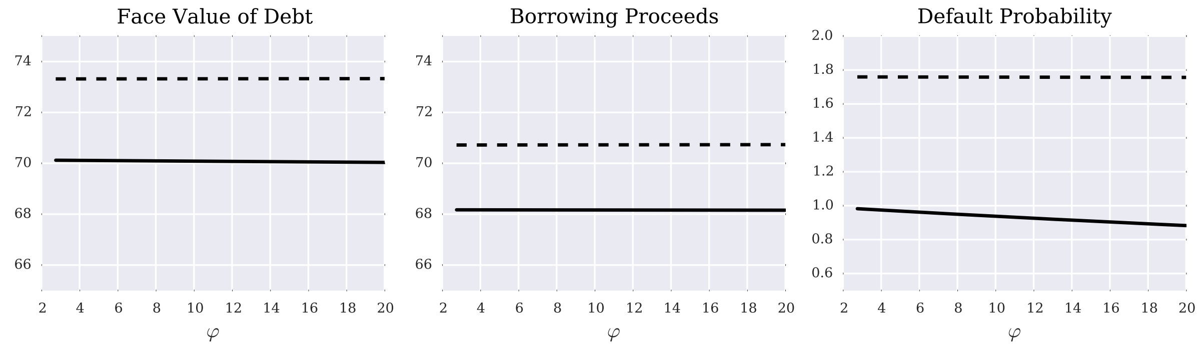
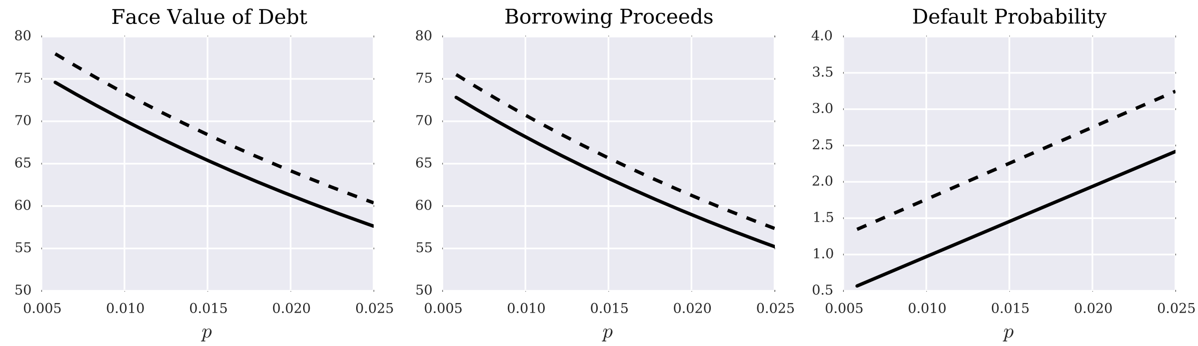


Figure 5: Sensitivity Analysis: Excusable Default Model with Disasters

((a)) Variation in φ



((b)) Variation in p



((c)) Variation in $\overline{\eta}$

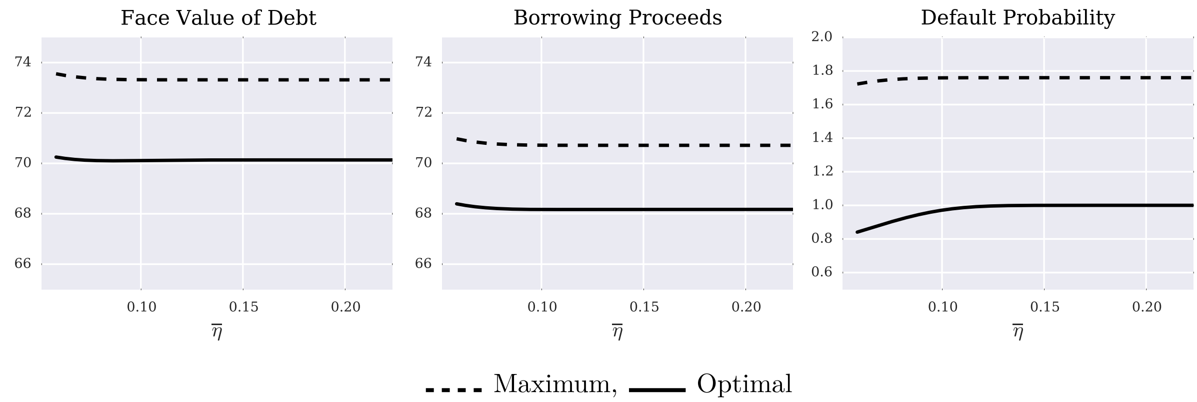


Figure 6: Excusable Default Model with Growth Collapses, Debt/Proceeds Ratio

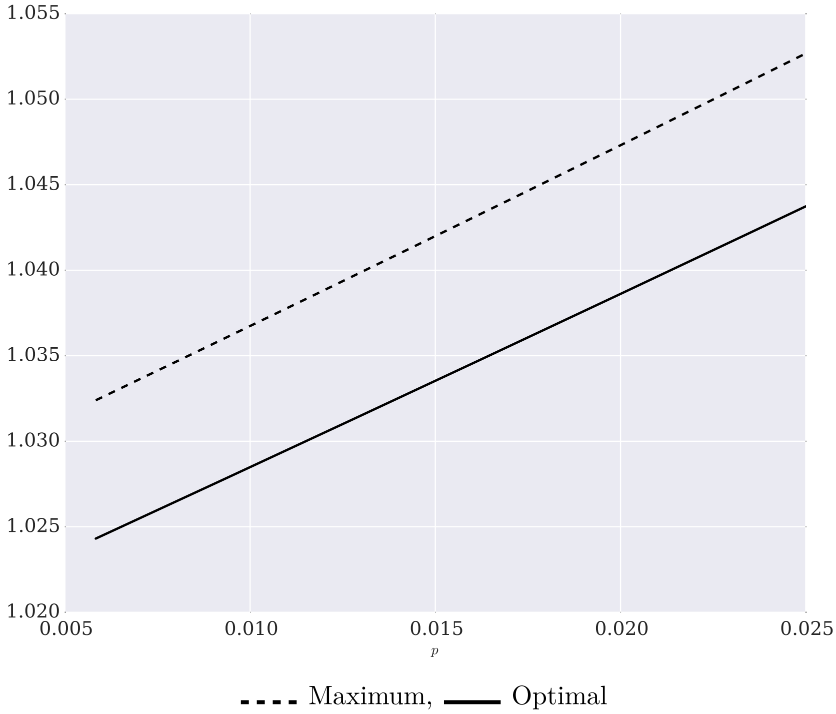
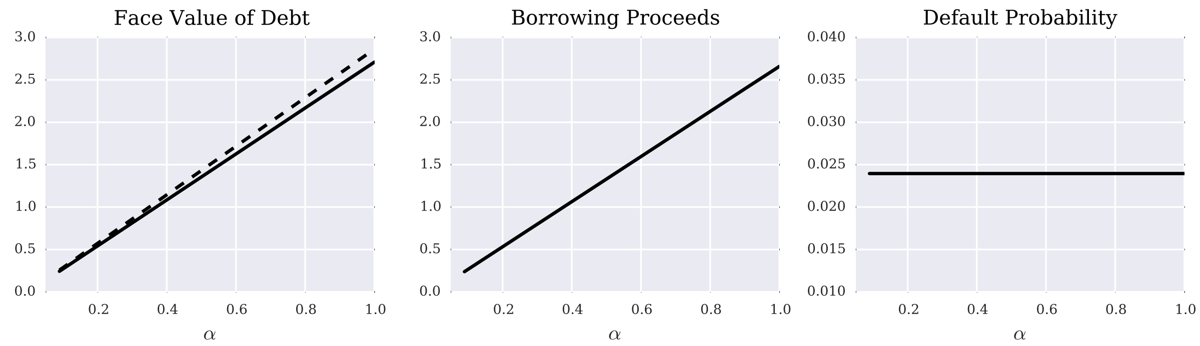
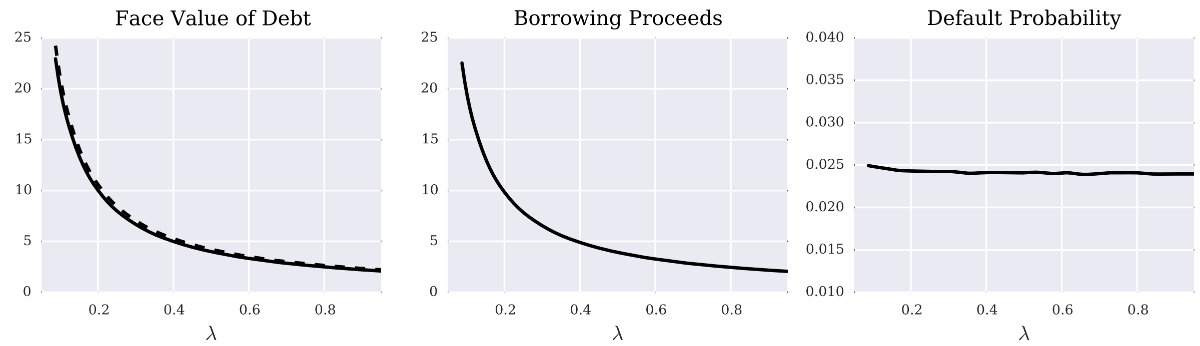


Figure 7: Sensitivity Analysis: Strategic Default Model (I)

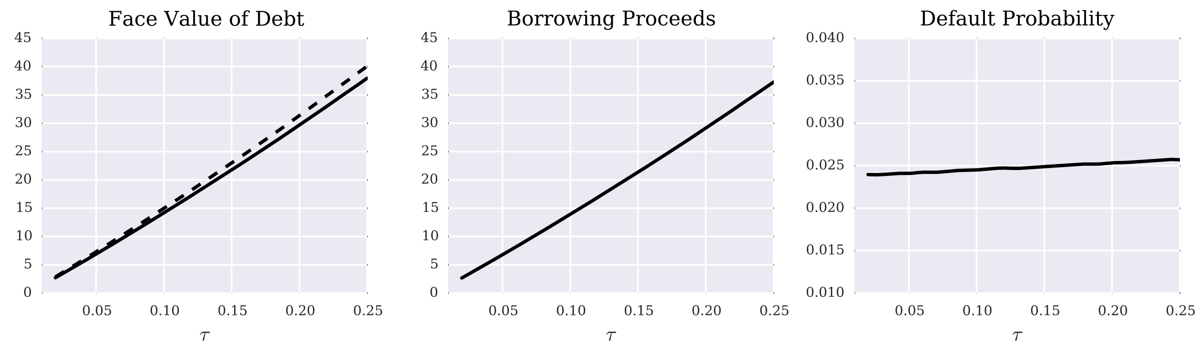
((a)) Variation in α_u



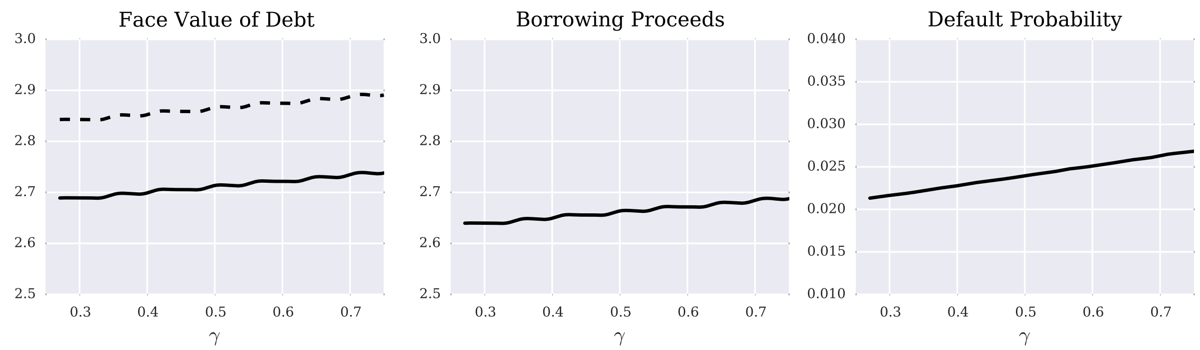
((b)) Variation in λ



((c)) Variation in τ



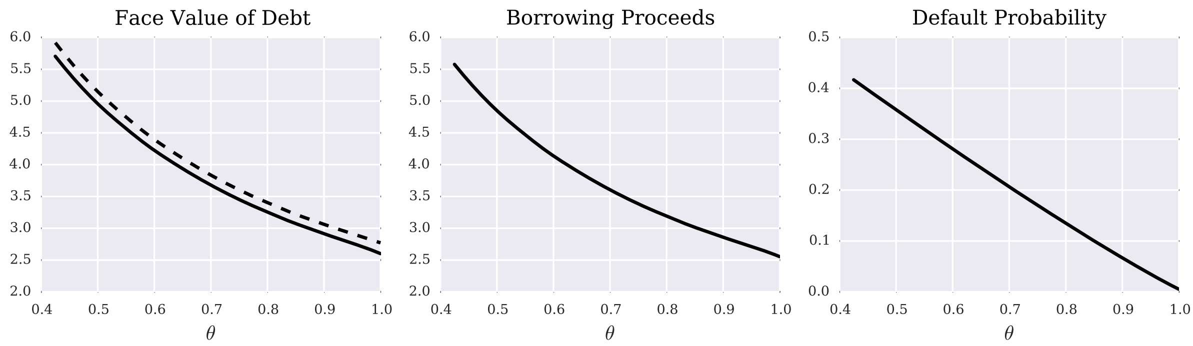
((d)) Variation in γ



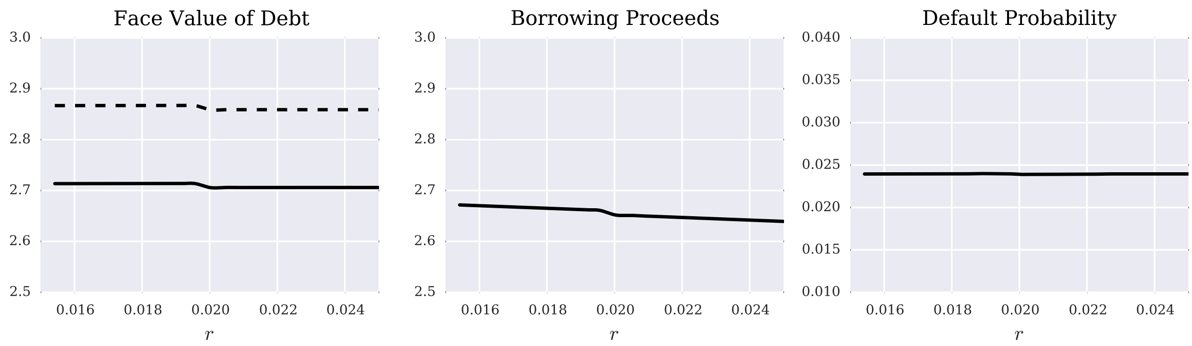
- - - Maximum, —— Optimal

Figure 8: Sensitivity Analysis: Strategic Default Model (II)

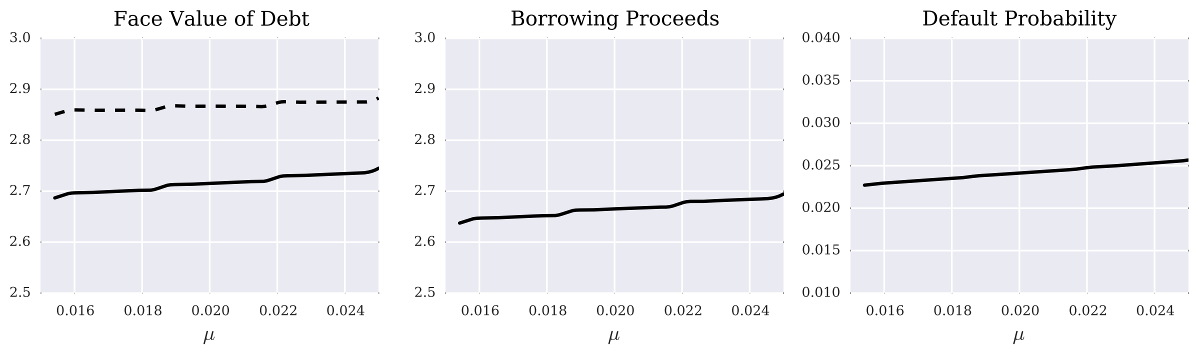
((a)) Variation in θ



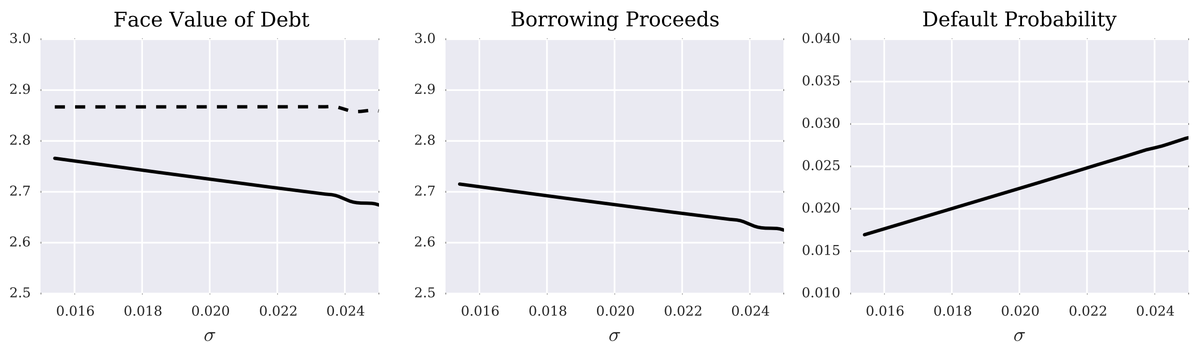
((b)) Variation in r



((c)) Variation in μ



((d)) Variation in σ



- - - Maximum, —— Optimal

Figure 9: Equilibrium Autarky Payoff v_A