# Climate change financial risks: implications for asset pricing and interest rates

September 29, 2021

#### Abstract

In addition to rare macroeconomic disasters (e.g. wars, financial crises and pandemics), climate change poses a threat to financial stability – with extreme climatic events increasing in frequency and intensity and political risks putting pressure on asset valuations. To study the effect of a changing climate on asset prices and interest rates we include both sources of rare disasters in a dynamic CAPM with time-(and stochastically-) varying risk and recursive preferences. A changing climate makes former tail events more frequent and less predictable, increasing the premium of climate risk; interestingly, this change may not be fully reflected on the overall market risk premium that holds both components of risk: macroeconomic and environmental. The same is not true for interest rates, the return on sovereign debt and the participation of climate-risky assets in the market portfolio, that are expected to decline unambiguously as the planet warms.

Keywords: Climate change; Tail events; Time-varying risk; Asset pricing; Interest rates

 $\label{eq:JEL classification: G11, G12, O44, Q51, Q54} JEL \ classification: G11, G12, O44, Q51, Q54$ 

## 1 Introduction

It is by now well understood that climate-related events are increasing in frequency and intensity and that this trend will continue unless there is a radical shift in our emitting behaviour. In the context of the financial system, climate-related risks could affect its stability through losses of big levered financial intermediaries or sudden repricing of asset classes (Brunetti et al. 2021). This is especially relevant if climate change risks are not properly incorporated in asset pricing and risk management methods that usually draw inferences from historical data, when the effects of climate change were much less pronounced, or even absent. This is one reason behind the establishment of the Central Banks and Supervisors NGFS<sup>3</sup> network, and the popular TCFD framework of voluntary climate-related financial disclosures.

Yet although valuation methods that incorporate climate change risks already exist as products from various NGOs and financial data vendors<sup>5</sup> they all remain black boxes that combine complex climate, economic and financial modelling, that is never readily applicable for communication. The purpose of the present paper is to study in a tractable stochastic general equilibrium setting the implications for asset pricing and financial stability of the co-existing and time-varying risk of extreme events of multiple sources – macroeconomic, and those related to climate change. It is also our intention to provide analytical results readily applicable to policy discussions. Our main objective is to explore how these risks can affect the various market measures and in particular risk premia, interest rates and sovereign debt.

There are two main types of climate-related risks relevant for market participants: physical risks and transition risks.<sup>6</sup> Physical risks, associated with physical damages to assets, could be event-driven (droughts, floods, storms, wildfires and crop failures) or chronic, related to long term climate shifts (e.g. sea level rise). The frequency and severity of environmental events can increase as a result of rising global temperatures (IPCC 2018, USGCRP 2018). Transition risks include, among other components,<sup>7</sup> policy risks which emerge from potential introduction of more stringent carbon-pricing policies that can affect the returns of assets related to carbon-intensive technologies or processes.<sup>8</sup> Since investments in the carbon-based economy are mostly

<sup>&</sup>lt;sup>1</sup>See Francis and Vavrus (2012), Cai et al. (2014), Hsiang et al. (2017), Francis (2017) on the frequency and severity of natural disasters.

<sup>&</sup>lt;sup>2</sup>In his path-breaking speech "Breaking the tragedy of the horizon" in 2015, the Governor of the Bank of England, Mark Carney, was the first to highlight the threat of climate change for the stability of the financial system and to identify the risks involved (Carney 2015); see also Battiston et al. (2017), Stolbova et al. (2018).

<sup>&</sup>lt;sup>3</sup>The Network for Central Banks and Supervisors for Greening the Financial System (NGFS) was established in 2017 by eight central banks and supervisors with the purpose of sharing and developing best practices in banking for climate risk management, and mobilizing capital for green and low-carbon investments; it now includes ninety-five members and fifteen observers.

<sup>&</sup>lt;sup>4</sup>The Financial Stability Board released in 2017 the Task Force on Climate-related Financial Disclosures (TCFD), a framework with recommendations on voluntary climate-related disclosures by firms in order to promote more informed investment, credit, and insurance underwriting decisions, reducing thus exposures to climate-related risks.

<sup>&</sup>lt;sup>5</sup>See for example Swiss Sustainable Finance (2019) for an overview of the various data providers and methodologies used.

<sup>&</sup>lt;sup>6</sup>See (Campiglio et al. 2018) for a discussion on the climate-related risk and their relevance for financial markets and central banks.

<sup>&</sup>lt;sup>7</sup>Transition risks also include liability risks, technology risks, market risks, reputation risks.

<sup>&</sup>lt;sup>8</sup>For example, Batten et al. (2016) find contrasting cumulative abnormal returns experienced by a petroleum

irreversible, stringent climate policies are likely to make the operation of carbon-intensive firms unprofitable, and, thereby leave assets stranded. Crucial to individual investors, appropriate pricing of climate-related risks, could lead to more informed, and thus efficient, capital allocation decisions. Of relevance to central banks is the possibility that climate-related damages could result in financial losses that affect the stability of the financial system through the balance sheets of insurers, banks and credit flows. Climate change is also expected to affect monetary policy when this policy is conducted through the nominal interest rate on government bonds by a standard Taylor-type rule. The possibility of declining interest rates due to increasing climate risks (and in turn precautionary savings), especially in an already low-interest environment, raises the incentives for environmental protection also on the central banking level.

The financial literature has come a long way in explaining investors' behavior towards risk, and in particular asset pricing puzzles, by introducing rare disasters like wars, pandemics or strong market corrections – the macroeconomic disasters. This approach suggests that a simple explanation of high risk premia and market volatility is that investors are concerned about equity performance in rare events such as the Great Depression of the 1930s, the two world wars of the 20th century, or the COVID19 pandemic. Since climate-related physical risks can also be regarded as rare disasters – not as rare as world wars, but nevertheless infrequent – it would be natural to consider an analytical dynamic asset pricing framework similar to the one used to price assets under rare macroeconomic disasters for the pricing of climate risks.

To this end we adapt the general equilibrium model with shocks of Ahn and Thompson (1988) to include stochastically time-varying intensities of the Poisson events in the spirit of Wachter (2013), yet of two kinds, macroeconomic and environmental, the latter featuring an increasing trend due to climate change. Ahn and Thompson (1988) extend the (linear) production-based model of Cox et al. (1985a) to include rare disasters captured by a Poisson process which allows for downward jumps in the return of the underlying investments opportunities. The attractive feature of this modelling approach is that it provides a more rigorous general equilibrium representation of uncertain economic activity with endogenous production than the endowment economy setting, popularized by Lucas (1978). In an endowment economy Wachter (2013) introduces a mean-reverting Cox-Ingersoll-Ross (CIR) process (Cox et al. 1985b) for the intensity of the Poisson process and matches the observed post-WWII equity premia and stock market volatility for the US, even for low values of relative risk aversion. To study climate-related risks, we introduce a second Poisson process with stochastic time-varying intensity for climate-related events; these can be either natural disasters or the introduction of a stringent climate policy.<sup>11</sup> We then consider two types of risky production opportunities: a general and

refining company and a wind turbine manufacturer the day after the announcement of the Paris agreement in December 2015.

<sup>&</sup>lt;sup>9</sup>See, e.g., Annicchiarico and Di Dio (2017), Economides and Xepapadeas (2018).

<sup>&</sup>lt;sup>10</sup>See Barro (2006, 2009), Wang and Bidarkota (2009), Gourio (2012), Wachter (2013), Pindyck and Wang (2013), Jin (2014), Tsai and Wachter (2018), Seo and Wachter (2018), Gomes et al. (2018).

<sup>&</sup>lt;sup>11</sup>Since climate-related risks emerge through two major transmission channels – physical disasters and transition risks – it will be natural to approach the problem of pricing climate-related risks by incorporating these type of risks into asset pricing. It should be noted here that transition risks are related to physical risks, since the increasing frequency and intensity of climate-related disasters are expected to accelerate the introduction of more, and more stringent, policies in the transition to a low-carbon economy.

a climate-sensitive one (which we call "brown"), the latter being exposed to transition risks; macroeconomic and climate-related disasters affect both processes, while climate-related policy events affect only the brown.<sup>12</sup> Our proxy for climate change is the change in global average temperature relative to a given time period, i.e., the temperature anomaly. Exogenous temperature paths in this case could be regarded as the ones that correspond to different emissions scenarios as the representative concentration pathways (RCPs) produced by the IPCC (IPCC 2014), that project different GHG concentration pathways up to 2100. We then calibrate the model to historical data and do forward-looking projections for the different market measures based on the RCPs; the portfolio weights get recalculated when we include the transition risk of climate change.

Our contribution is both methodological and empirical. First, we include the aforementioned stochastic intensity for the Poisson process of extreme environmental events that builds on the observed positive relationship between the frequency of extreme climate-related events and temperature anomaly (see Figure 1).<sup>13</sup> Different to Wachter (2013), as climate changes (to the worse), the distribution of the probability of extreme environmental events shifts to higher draws and flattens: events that once constituted the tail of its distribution become more frequent but less predictable, thus raising the importance of climate change risks even in times without disasters. Second, we provide closed form solutions for risk premia (up to an indefinite integral), interest rates and the return on sovereign credit that depend on observables (temperature anomaly) and can be readily used for policy communication under various climate scenarios. Third, our dynamic CAPM naturally draws conclusions about the participation of climate-sensitive assets in the market portfolio. Fourth, using a database of climate-related disasters spanning over hundred years and fourty-two countries we map climate change to GDP drops and provide a link for, both, the intensity and the frequency of extreme events with expected temperature paths relevant for asset pricing and climate risk management.

Using both theory and simulations we show that climate change reduces interest rates and asset valuations. It also entails a positive risk premium which is increasing over time. In addition, our exact solution for the aggregate risk premium exposes a subtle feature of models with time-varying risk of multiple sources, obscured by models that follow the usual Campbell and Shiller (1988) approximation for price-dividend ratios or models with static disaster risk; see Tsai and Wachter (2018). With rising temperatures the increasing risk of environmental events puts a downward pressure on equity prices, which leaves less room for prices to react to the risk of extreme events of either type. As climate changes, the mean and the variance of the distribution of climate risk increase, while those of macroeconomic risk stay unaltered, which increases the relative importance of climate change risk even in times without disasters. This result

<sup>&</sup>lt;sup>12</sup>We follow the definition of Prudential Regulation Authority (2015) and consider as "brown" Tier 1 and 2 assets that are directly exposed to transition risks. Tier 1 assets include coal, oil and gas extraction companies, and conventional utilities; Tier 2 assets include firms that are energy-intensive, e.g. chemicals and mining companies. Together they account for about 30% of global equity and fixed-income investments. These estimates are getting constantly revised through the TCFD framework.

<sup>&</sup>lt;sup>13</sup>Since the link between temperature anomaly and emissions is well established (e.g. Matthews et al. (2009), Hassler et al. (2016), Brock and Xepapadeas (2017)), this establishes the link between emissions and the probability of natural disasters.

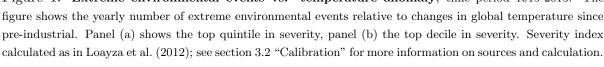
suggests that macroeconomic risk (the major component of the market equity premium) works as a hedging strategy against the risk of climate change, thus losing importance as the planet warms; as a concequence observed equity premia, that combine both sources of extreme events, may be minimally affected. Following increases in the intensity and uncertainty about future catastrophic climate events, precautionary savings reduce interest rates, which has implications for sovereign credit and monetary policy. In addition, apart from pricing climate change for the future, we study the effects of transition risk on portfolio composition; the risk of policy tipping curbs the market price of assets sensitive to climate-policy risk and leads to their stranding.

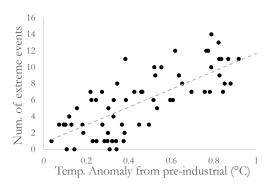
To the best of our knowledge, this is the first paper to incorporate the stochastically time-varying risk of rare disasters related to both macroeconomic events and climate change in a dynamic asset pricing framework with Poisson shocks and to provide exact closed-form solutions. In a long-run risk asset pricing model Bansal et al. (2016) include Poisson shocks due to climate change – but not due to macroeconomic events – and find that global warming carries a positive, and increasing, risk premium and reduces asset valuations. In comparison to us, they find that climate change unambiguously increases risk premia in the market. We show that this result is a limiting case in our model. Brock and Hansen (2018) study the implications of risk, ambiguity and model misspecification on climate economic modeling, while Barnett et al. (2020) incorporate these notions in an endogenous growth model with depletable energy resources and Brownian uncertainty to study asset pricing and the social implications of a changing climate. Bretschger and Vinogradova (2018) and van der Ploeg and van den Bremer (2019) develop macroeconomic models with only climate-related disasters and static disaster risk. Hambel et al. (2020) discuss the implication of various climate uncertainties for portfolio diversification, without stochastic and time-varying risk or closed form solution.

The paper is organized as follows. The next section builds the theoretical framework and formally provides intuition on how climate change risks can affect various market measures such as risk premia, the riskfree rate and sovereign debt. We extend a production-based asset pricing model with Poisson shocks by adding shocks due to climate change with a stochastically time-varying probability that increases in temperature anomaly. Section 3 deals with numerical simulations. We present our methodology, calibration, and discuss our results. Section 4 concludes.

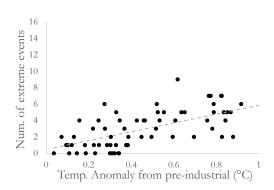
<sup>&</sup>lt;sup>14</sup>Considering uncertain climate change and its catastrophic effects, Weitzman (2013) suggests that emissions abatement can be used as a hedging strategy against macroeconomic risk, which implies a negative climate beta. Our result runs along the same lines but in the opposite direction. A negative climate beta is also assumed in Daniel et al. (2019), who find a declining path for the social cost of carbon as the uncertainty about the effects of carbon emissions is gradually resolved over time. The conditions for whether climate betas are positive or negative are discussed in Dietz et al. (2018).

Figure 1: Extreme environmental events vs. temperature anomaly; time period 1915-2015. The figure shows the yearly number of extreme environmental events relative to changes in global temperature since pre-industrial. Panel (a) shows the top quintile in severity, panel (b) the top decile in severity. Severity index





(a) Top 20% in severity



(b) Top 10% in severity

#### $\mathbf{2}$ The model

#### 2.1Model setup

We build on the general equilibrium model of Cox et al. (1985a) and include jumps in the production possibilities of the economy as in Ahn and Thompson (1988) and time-varying risk as in Wachter (2013). Yet we go one step ahead by including two sources of time-varying risk, i.e., pure macroeconomic disasters and disasters due to climate change, both due to physical and to transition risks.

Time is continuous and denoted by subscript t. Let there be a single physical numeraire good which may be allocated to consumption or investment. The economy has two distinct linear activities for the production of the good: a brown activity (B) which is subject to risk of stringent climate policy and a general activity (G) which differs from B in the sense that is not subject to any climate policy.<sup>15</sup> The transformation of an investment of a vector  $\theta = [Y_G, Y_B]^T$ of amounts of the numeraire good in the two production processes is governed by the following stochastic differential equations:

$$d\theta_{t} = \begin{bmatrix} Y_{Gt} \Big( \mu_{G} dt + \sigma_{G} dW_{Gt} + \sum_{j \in \{M, E\}} (e^{Z_{G}^{j}} - 1) dQ_{t}^{j} \Big) \\ Y_{Bt} \Big( \mu_{B} dt + \sigma_{B} dW_{Bt} + \sum_{j \in \{M, E\}} (e^{Z_{B}^{j}} - 1) dQ_{t}^{j} + (e^{X} - 1) dQ_{t}^{X} \Big) \end{bmatrix}.$$
 (1)

The diffusion term  $\mu_i Y_i dt + \sigma_i Y_i dW_i$  represents the behavior of production process  $i = \{G, B\}$ in normal times (when no disasters take place), such that  $\Delta \log Y_i$  over an interval  $\Delta t$  is normally distributed with mean  $(\mu_i - \sigma_i^2/2)\Delta t$  and variance  $\sigma_i^2\Delta t$ ;  $W_i$  a standard Wiener process repre-

<sup>&</sup>lt;sup>15</sup>Our approach is of course simplistic. A more realistic approach would consider different shades of brown where all assets have a different degree of exposure to climate policy.

senting diffusion risk. Additionally, our model features two types of uncorrelated Poisson shocks that result in economic losses, namely macroeconomic (M) and environmental (E). Macroeconomic shocks are events like wars, economic crises and pandemics, while environmental shocks are severe events related to climate change like huricanes, droughts, floods and wildfires. Each shock  $Q_j$ , with  $j \in \{M, E\}$ , has a time-varying arrival rate  $\lambda^j$ . Brown production possibilities are also exposed to an infrequent policy shock  $Q^X$  that makes production less economically efficient. As the planet deviates from a path consistent with a low-temperature future, the increasing frequency and intensity of climate-related disasters are expected to accelerate the introduction and stringency of policies towards the transition to a low-carbon economy. To capture this correlation, we assume that the Poisson intensity of policy risk is related to the intensity of environmental disasters by  $\lambda^X = \pi \lambda^E$ ,  $\pi \in [0,1]$ .

Processes are denoted by subscripts and shocks by superscripts, i.e.,  $Z_i^j < 0$  denotes the drop in  $\log Y_i$ ,  $i \in \{G, B\}$ , when an event of type  $j \in \{M, E\}$  occurs. For ease of exposition, we assume that each firm operates in the same macroeconomic and natural environment and is subject to the same physical shock  $Z^j < 0$ ; its time-invariant distribution  $z^j$  comes from the data and is independent of all other processes. The above discussion implies that  $Z_G^M = Z_B^M = Z^M$ ,  $Z_G^E = Z_B^E = Z^E$ . When effective, stringent policy acts to further reduce the return on brown production by X < 0, which we assume certain for simplicity.<sup>16</sup> The operator  $\mathbb E$  represents expectation with respect to the underlying distribution.

The system (1) of the available production opportunities specifies the growth of an initial investment when the output of each process is continuously re-invested in the same process, such that  $Y_{it}$  is also cumulative production within time period [0,t] of the *i*-th process. While it does not mean that individuals will indeed re-invest this way, in the presence of stochastic returns to investment there exists a diversification motive that pushes investors to invest in both technologies in equilibrium.

With regards to macroeconomic events  $Q^M$ , we follow Wachter (2013) and assume the following mean-reverting process for the Poisson intensity  $\lambda^M$ :

$$d\lambda_t^M = \kappa^M(\bar{\lambda}^M - \lambda_t^M)dt + \sigma_{\lambda}^M \sqrt{\lambda_t^M} dW_{\lambda t}^M. \tag{2}$$

Variable  $W_{\lambda}^{M}$  is a standard Brownian motion, independent of all other processes. Parameter  $\kappa^{M}$  represents the adjustment speed of the process towards its mean  $\bar{\lambda}^{M}$ ;  $\sigma_{\lambda}^{M}$  is a volatility parameter. The solution to (2) leads to a Gamma stationary distribution for  $\lambda^{M}$ , provided that both  $\kappa^{M}$  and  $\bar{\lambda}^{M}$  are positive, which we will assume. This process has the attractive feature that  $\lambda^{M}$  can never become negative. Moreover, the square root in (2) implies that the resulting stationary distribution is highly right-skewed generating tail events, while at the same time, high realizations of  $\lambda^{M}$  make the process more volatile, and thus even higher realizations more likely, compared to a standard autoregressive process.<sup>17</sup>

<sup>&</sup>lt;sup>16</sup>The policy stringency X could in turn be an increasing function of the intensity of climate damages  $Z^E$ . However, this assumption would not alter the quality of the results, while it would impair the tractability of the model

<sup>&</sup>lt;sup>17</sup>This assumption has an important psychological root: when risk is high, *perceived* disaster probabilities further increase, inducing fear and increasing in turn precautionary savings of risk averse investors.

Climate shocks are triggered by climate change, proxied by temperature anomaly T, i.e., the change in global average temperature relative to a given time period (e.g. for year 2015, T was about 1°C compared to the mean of the pre-industrial period 1850-1900). Natural disasters are increasing in frequency and intensity. Based on observations (Figure 1), we will assume that the probability of natural disasters  $\lambda^E$  is an increasing and linear function of temperature anomaly. However, following the tradition of the time-varying risk literature  $^{19}$ , besides the expected positive relationship between the disaster probability and temperature anomaly, we assume that the arrival rate of natural disasters is rather stochastic. We will therefore assume a process similar to (2) as

$$d\lambda_t^E = \kappa^E (\bar{\lambda}_t^E - \lambda_t^E) dt + \sigma_\lambda^E \sqrt{\lambda_t^E} dW_{\lambda t}^E, \tag{3}$$

with  $\bar{\lambda}_t^E \equiv \tilde{\lambda}^E + \xi T$  and  $\{\tilde{\lambda}^E, \xi\}$  non-negative numbers; variable  $W_{\lambda}^E$  represents a standard Brownian motion, independent of all other processes. Note that, in comparison to (2), although this autoregressive process features mean reversion, the mean itself is time-varying. When temperatures keep rising out of balance natural disasters are becoming more frequent in expectation but less predictable. In equilibrium, the solution of (3) has also a Gamma stationary distribution, where the constant mean and variance are increasing in temperature anomaly.<sup>20</sup>

Finally, the representative agent has the continuous-time analogue of recursive Epstein-Zin preferences, as formulated by Duffie and Epstein (1992a). Accordingly, we use the following recursion to define the utility function U,

$$U_t = \mathbb{E}_t \int_t^\infty f(C_s, U_s) ds, \tag{4}$$

where

$$f(C_t, U_t) = \rho(1 - \gamma)U_t \left(\log C_t - \frac{1}{1 - \gamma}\log((1 - \gamma)U_t)\right). \tag{5}$$

Parameter  $\rho > 0$  is the subjective rate of time preference, and  $\gamma > 0$  measures relative risk aversion. We assume for simplicity that our utility function features a unitary elasticity of intertemporal substitution (EIS). We will conventionally focus on the case of  $\gamma > 1$ , which implies a preference for early resolution of uncertainty.<sup>21</sup>

 $<sup>^{18}\</sup>mathrm{See}$  also Figure 8 in 3.2 "Calibration".

<sup>&</sup>lt;sup>19</sup>E.g. Gourio (2012), Wachter (2013), Seo and Wachter (2018), Gomes et al. (2018).

<sup>&</sup>lt;sup>20</sup>Following Cox et al. (1985b), we can show that given a value of temperature anomaly T, the expected value and variance of  $\lambda^E$  at time s, conditional on its value at time t < s (for s close to t), is given by:  $\mathbb{E}[\lambda_s^E|\lambda_t^E] = \lambda_t^E e^{-\kappa^E(s-t)} + (\tilde{\lambda}^E + \xi T) \left(1 - e^{-\kappa^E(s-t)}\right) \text{ and } \operatorname{Var}[\lambda_s^E|\lambda_t^E] = \lambda_t^E \frac{(\sigma_{\lambda}^E)^2}{\kappa^E} \left(e^{-\kappa^E(s-t)} - e^{-2\kappa^E(s-t)}\right) + (\tilde{\lambda}^E + \xi T) \frac{(\sigma_{\lambda}^E)^2}{2\kappa^E} (1 - e^{-\kappa^E(s-t)})^2$ . The steady-state mean and variance are  $\tilde{\lambda}^E + \xi T$  and  $(\tilde{\lambda}^E + \xi T) \frac{(\sigma_{\lambda}^E)^2}{2\kappa^E}$ , respectively, both increasing in T.

<sup>&</sup>lt;sup>21</sup>A preference for the early resolution of uncertainty has become a standard notion in macroeconomic modeling, and is important to capture concerns about future growth variations, especially those that are persistent as in our model. A prime example of the adoption of such preferences in the macro-finance literature is the seminal work of Bansal and Yaron (2004).

## 2.2 Optimality conditions

We consider a representative investor who maximizes lifetime utility by allocating her wealth, net of consumption, among investments in the risky production opportunities and a riskless asset with an instantaneous rate of return r. Let  $\{n_B, n_G\}$  be the fractions of wealth A invested in the brown and general risky technologies, respectively. Wealth then follows the process:<sup>22</sup>

$$dA_{t} = \left(\sum_{i \in \{G,B\}} n_{it} A_{t}(\mu_{i} - r_{t}) + A_{t} r_{t} - C_{t}\right) dt + \sum_{i \in \{G,B\}} n_{it} A_{t} \sigma_{i} dW_{it}$$

$$+ \sum_{i \in \{G,B\}} n_{it} A_{t} \left(\sum_{j \in \{M,E\}} (e^{Z^{j}} - 1) dQ_{t}^{j}\right) + n_{Bt} A_{t} (e^{X} - 1) dQ^{X}.$$

$$(6)$$

Let  $V(A, \lambda^M, \lambda^E)$  be the value function (maximized utility) in states  $\{\lambda^M, \lambda^E\}$  with wealth A. Using (2), (3), (6), and Itô's Lemma, controls  $v = \{C, n_i\}$ , i.e., optimal consumption expenditure, and portfolio choices must satisfy the following Hamilton-Jacobi-Bellman equation (Duffie and Epstein 1992a):<sup>23</sup>

$$\sup_{v} \{ L^{v}(V(A, \lambda^{M}, \lambda^{E})) + f(C, V(A, \lambda^{M}, \lambda^{E})) \} = 0$$

$$(7)$$

with  $L^{v}(\cdot)$  a differential operator defined as

$$L^{v}(V) = V_{A} \left( \sum_{i \in \{G,B\}} n_{i} A(\mu_{i} - r) + Ar - C) \right) + \frac{1}{2} V_{AA} A^{2} \sigma^{2}$$

$$+ \sum_{j=\{M,E\}} \left( V_{\lambda^{j}} \kappa^{j} (\bar{\lambda}^{j} - \lambda^{j}) + \frac{1}{2} V_{\lambda^{j} \lambda^{j}} (\sigma_{\lambda}^{j})^{2} \lambda^{j} + \lambda^{j} \mathbb{E}_{z^{j}} [\tilde{V}^{j} - V] \right) + \pi \lambda^{E} [\tilde{V}^{X} - V].$$

$$(8)$$

The subscripts of V denote partial derivatives, i.e.,  $V_x = \partial V/\partial x$ . Moreover,

$$\sigma = \sqrt{n_B^2 \sigma_B^2 + n_G^2 \sigma_G^2 + 2n_B n_G \sigma_{GB}},\tag{9}$$

 $\sigma_{GB} \equiv \sigma_{G}\sigma_{B} \text{corr}[dW_{G}, dW_{B}]; \ \tilde{V}^{j} \equiv V(A(1 + \sum_{i \in \{G,B\}} n_{i}(e^{Z^{j}} - 1)), \lambda^{M}, \lambda^{E}), \text{ for } j \in \{M, E\}; \ \tilde{V}^{X} \equiv V(A(1 + n_{B}(e^{X} - 1)), \lambda^{M}, \lambda^{E}), \text{ the value function after the arrival of either disasters (or both together since they are uncorrelated) and policy, and <math>\bar{\lambda}^{E} = \tilde{\lambda}^{E} + \xi T$ . Assuming an interior solution  $(C, n_{i} > 0)$  we get the first order conditions w.r.t to  $C, n_{G}, n_{B}$ :

$$f_C = V_A, \tag{10}$$

<sup>&</sup>lt;sup>22</sup>Let  $\mathcal{B}$  be the part of wealth A held in the riskless asset with an instantaneous rate of return r and denote with  $d\mathcal{R}^A$  the stochastic gross return on wealth such that wealth accumulation follows  $dA = d\mathcal{R}^A - Cdt$ . The gross return to investment reads  $d\mathcal{R}^A = dY_B + dY_G + r\mathcal{B}dt$ . Defining as  $\{n_B, n_G\}$  the fractions of wealth held in the brown and general risky activities the above with (1) leads to (6).

 $<sup>^{23}</sup>$ From now on we will suppress the time subscript t. However, unless otherwise indicated, all variables are time-dependent.

$$r = \mu_G + \frac{V_{AA}A}{V_A} (n_G \sigma_G^2 + n_B \sigma_{GB}) + \sum_{j=\{M,E\}} \lambda^j \mathbb{E}_{z^j} \left[ \frac{\tilde{V}_A^j}{V_A} (e^{Z^j} - 1) \right], \tag{11}$$

$$r = \mu_B + \frac{V_{AA}A}{V_A} (n_B \sigma_B^2 + n_G \sigma_{GB}) + \sum_{j=\{M,E\}} \lambda^j \mathbb{E}_{z^j} \left[ \frac{\tilde{V}_A^j}{V_A} (e^{Z^j} - 1) \right] + \pi \lambda^E \frac{\tilde{V}_A^X}{V_A} (e^X - 1).$$
(12)

Equation (10) is the usual envelope condition for the price of consumption. The system of equations (11) and (12) solves the investor's risky portfolio allocation problem given the riskfree rate, risk and policy. In equilibrium it holds that the riskfree asset is in zero net supply such that  $n_B + n_G = 1$ . Using this for  $n_G$  and equating the right hand sides of (11) and (12) yields a no-arbitrage condition between risky assets; after adjusting for their relative risk, each risky asset should yield the same marginal expected return:

$$\mu_{B} + \frac{V_{AA}A}{V_{A}} n_{B} (\sigma_{B}^{2} - \sigma_{GB}) + \pi \lambda^{E} \frac{\tilde{V}_{A}^{X}}{V_{A}} (e^{X} - 1) =$$

$$\mu_{G} + \frac{V_{AA}A}{V_{A}} (1 - n_{B}) (\sigma_{G}^{2} - \sigma_{GB}).$$
(13)

Equation (13) will be used to calculate the optimal portfolio allocation  $n_B$ . Note that as in Hambel et al. (2020) there are two opposing effects relevant for portfolio composition. On the one hand both assets are needed in general for a diversified portfolio; in particular, from (9) portfolio volatility in times without disasters is a convex quadratic function of  $n_B$ , which takes its minimum value at  $n_{B,min} = \frac{\sigma_G^2 - \sigma_{GB}}{\sigma_G^2 + \sigma_B^2 - 2\sigma_{GB}}$ , with  $\sigma_{GB} = \sigma_G \sigma_B \text{corr}[dW_G, dW_B]$ . The magnitude of this diversification motive against diffusion risk is less pronounced for high values of the correlation coefficient; zero correlation amplifies the diversification motive, which is most pronounced for negative values of the correlation coefficient. On the other hand, since both assets are equally exposed to extreme Poisson events of either type, the investor reacts only to policy risk on climate-sensitive assets, which reduces the diversification motive and lowers  $n_B$ .

#### 2.3 The value function

Apart from performing numerical simulations in the next section, here we aim at deriving closed-form solutions that provide intuition on risk premia, interest rates and sovereign debt under climate change risk. In Appendix A we prove the following proposition.

**Proposition 1** (Value function and SCC) For preferences defined by (4) and (5) the value function that solves (7) reads

$$V(A, \lambda^M, \lambda^E) = \frac{A^{1-\gamma}}{1-\gamma} e^{a+\sum_j b^j \lambda^j},\tag{14}$$

with

$$b^{M} = \frac{\kappa^{M} + \rho}{(\sigma_{\lambda}^{M})^{2}} - \sqrt{\left(\frac{\kappa^{M} + \rho}{(\sigma_{\lambda}^{M})^{2}}\right)^{2} - 2\frac{\mathbb{E}_{z^{M}}\left[e^{(1-\gamma)Z^{M}} - 1\right]}{(\sigma_{\lambda}^{M})^{2}}},$$

$$b^{E} = \frac{\kappa^{E} + \rho}{(\sigma_{\lambda}^{E})^{2}} - \sqrt{\left(\frac{\kappa^{E} + \rho}{(\sigma_{\lambda}^{E})^{2}}\right)^{2} - 2\frac{\mathbb{E}_{z^{E}}\left[e^{(1-\gamma)Z^{E}} - 1\right] + \pi\left[(1 + n_{B}(e^{X} - 1))^{1-\gamma} - 1\right]}{(\sigma_{\lambda}^{E})^{2}}},$$

$$a = (1 - \gamma)(\log \rho - 1) + \frac{1 - \gamma}{\rho}\left(\sum_{i \in \{G, B\}} n_{i}\mu_{i} - \frac{1}{2}\gamma\sigma^{2}\right) + \sum_{j \in \{M, E\}} b^{j} \frac{\kappa^{j}\bar{\lambda}^{j}}{\rho}.$$

$$(15)$$

The propensity to consume out of wealth is constant and equal to  $\rho$ , i.e.,  $C = \rho A$ .  $\square$ 

The fact that the quantities under the root of (15) have to be positive places a joint restriction on the severity of disasters, the risk aversion, the rate of time preference and the volatility of disasters. For  $\gamma > 1$  that we assume,  $b_j > 0$ ,  $j \in \{M, E\}$ , such that draws of higher risk (higher  $\lambda^j$ ) reduce the indirect utility of the risk-averse representative agent – and thus increase the marginal utility, i.e.  $V_{\lambda^j} < 0$  and  $V_{A\lambda^j} > 0$ . Equity premia arise from the co-movement of the marginal utility with the price process of the underlying asset, such that increases in marginal utility should be compensated by higher premia when market prices drop in light of such risk. Moreover, the fact that the propensity to consume out of wealth is constant and equal to  $\rho$  is a feature of the unitary EIS in the utility function and a standard result in the finance literature (Wachter 2013).

## 2.4 The riskfree rate and the return on government debt

Let m denote the state-price density (or pricing kernel) – with the same risk properties as the marginal utility of the risk-averse investor. Long-lived assets with a dividend stream  $D_t dt$  can be priced according to the usual asset pricing equation  $P_t = \mathbb{E}_t \left[ \int_t^\infty \frac{m_s}{m_t} D_s ds \right]$ . In Appendix B we prove the following expressions on the state-price density and the riskfree rate:

**Proposition 2** (State-price density and riskfree rate) Let  $dW_m = [dW_B, dW_G, dW_{\lambda}^M, dW_{\lambda}^E]$ , and  $dQ = [dQ^M, dQ^E, dQ^X]$ . The state-price density has the following dynamics:

$$\frac{dm}{m} = \mu_m dt + \sigma_m dW_m^T + (e^{Z_m} - 1)dQ^T,$$
 (16)

with

$$\mu_m = -r - \sum_{j \in \{M, E\}} \lambda^j \, \mathbb{E}_{z^j} \left[ e^{-\gamma Z^j} - 1 \right] - \pi \lambda^E \left[ (1 + n_B (e^X - 1))^{-\gamma} - 1 \right], \tag{17}$$

$$\sigma_m = [-\gamma n_B \sigma_B, -\gamma (1 - n_B) \sigma_G, b^M \sigma_\lambda^M \sqrt{\lambda^M}, b^E \sigma_\lambda^E \sqrt{\lambda^E}], \tag{18}$$

and

$$e^{Z_m} - 1 = [e^{-\gamma Z^M} - 1, e^{-\gamma Z^E} - 1, (1 + n_B(e^X - 1))^{-\gamma} - 1].$$
(19)

The risk-free rate reads:

$$r = \underbrace{\rho + g - \gamma \sigma^{2}}_{standard \ model} + \underbrace{\sum_{j=\{M,E\}} \lambda^{j} \mathbb{E}_{z^{j}} \left[ e^{-\gamma Z^{j}} (e^{Z^{j}} - 1) \right]}_{macroec. \ \& \ environ. \ risk} + \underbrace{\pi \lambda^{E} \left[ (1 + n_{B}(e^{X} - 1))^{-\gamma} n_{B}(e^{X} - 1) \right]}_{policy \ risk}$$

$$(20)$$

with  $g \equiv \sum_{i \in \{G,B\}} n_i \mu_i - \rho$ , the growth rate of consumption in an economy without risk,  $b_M, b_E > 0$  from (15),  $n_B + n_G = 1$  and  $n_B$  at its optimum level from (13).

Since  $e^{Z_m} - 1 \ge 0$ , according to (16), in the event of a macroeconomic, environmental, or large policy shock marginal utility jumps upwards, increasing investor's required compensation for bearing risk in times of weaker growth prospects. A shift towards a less climate-risky portfolio (lower  $n_B$ ) would reduce m due to the reduction in the exposure of transition risks. In general, higher risk induces precautionary savings, which reduces the riskfree rate; the greater the risk aversion  $\gamma$  the greater is this effect. Volatility aside, the risk-free rate is decreasing in the time-varying disaster probabilities  $\lambda^j$  and in the exposure to these disasters through  $Z^j$ . The effect of the general disaster risk on market fundamentals has been extensively studied in Wachter (2013). Here we concentrate on the effects of economic shocks related to climate change.

A high future temperature path increases the probability of natural disasters  $\lambda^E$  and environmental policy, both reducing the riskfree rate. Conveniently, equation (20) can be readily used to infer possible evolutions of the riskfree rate for various climate scenarios:  $\lambda^E$  follows (3) with its mean and variance increasing as the planet warms, pushing the riskfree rate to the opposite direction. For given distributions  $z^j$  of disaster magnitutes, the distribution of the riskfree rate for each level of temperature anomaly T (through its effect on the distribution of  $\lambda^E$ ) and portfolio composition  $n_B$  (which may, or may not, be set at its optimal value from (13) given (14)) involves so much of complexity as performing a single Monte-Carlo simulation (see section 3.1 "Methodology").

In the expectation that disasters (macroeconomic of environmental) may coinside with at least partial default on government securities, one can use equation (20) to also deduce their effect on sovereign credit risk. Specifically, following Barro (2006) and Tsai and Wachter (2015), we assume that in the event of a disaster  $j \in \{M, E\}$  there will be a default on government liabilities with probability  $q^j$  and when default happens the percentage loss is equal to the percentage decline in consumption; consumption follows (29) in the Appendix. let  $r^L$  denote the instantaneous return on government credit if there were no default (face value), so the observed premium on government debt in samples without disasters is given by

$$r^{L} - r = \sum_{j=\{M,E\}} q^{j} \lambda^{j} \mathbb{E}_{z^{j}} \left[ e^{-\gamma Z^{j}} (1 - e^{Z^{j}}) \right]$$

$$+ \pi q^{E} \lambda^{E} \left[ (1 + n_{B}(e^{X} - 1))^{-\gamma} n_{B} (1 - e^{X}) \right],$$
(21)

while the instantaneous expected return on government debt can be written as

$$r^b \equiv r^L + \sum_{j=\{M,E\}} q^j \lambda^j \, \mathbb{E}_{z^j} \left[ e^{Z^j} - 1 \right] + \pi q^E \lambda^E \left[ n_B(e^X - 1) \right].$$

From the above, the spread of the government bond against the riskfree rate reads

$$r^{b} - r = \sum_{j=\{M,E\}} q^{j} \lambda^{j} \mathbb{E}_{z^{j}} \left[ (e^{-\gamma Z^{j}} - 1)(1 - e^{Z^{j}}) \right]$$

$$+ \pi q^{E} \lambda^{E} \left[ ((1 + n_{B}(e^{X} - 1))^{-\gamma} - 1)n_{B}(1 - e^{X}) \right],$$
(22)

where the first term captures macroeconomic and physical environmental disasters while the second captures the transition risk due to e.g. the abrupt repricing of large climate-sensitive assets in public ownership such as coal mines or energy utilities (Zenios 2021). Both terms have the interpretation of a disaster risk premium for sovereign risk: the percentage change in marginal utility is multiplied by the percentage loss on the government claim. Note that because of the effect of climate change on r, the yield on government debt decreases with increasing temperatures.

Equations (20)-(22) show formally that interest rates and government bond yields are expected to decline as the planet warms due to natural disasters of increasing frequencies and climate policy that responds to deviations from low temperatures, especially in countries with greater exposure to climate-risky assets (high  $n_B$ ). This may be of particular importance in an already low-interest environment because it leaves less room to central banks for effective inflation targeting via the standard Taylor rule. Thus the possibility of a more restrive, low-interest environment due to climate change raises the issue of climate protection for financial stability also on the central banking level.

### 2.5 The market premium of climate change risk

In order to price climate change risks for long-lived assets we follow Abel (1999), Campbell (2003), Wachter (2013) and assume that the aggregate market pays a dividend D, being leveraged consumption, i.e.,  $D = C^{\eta}$ .<sup>24</sup> Equity premia arise from the co-movement of marginal utility of the risk-averse investor with the price of the underlying asset or portfolio, both in normal times and times of disasters. Let R be the expected return on equity; the risk-premium is then calculated by  $R - r = -\sigma_m \sigma_P^T - [\lambda^M, \lambda^E, \lambda^{pol}] \mathbb{E}[(e^{Z_m} - 1) \circ (e^{Z_P} - 1)]^T$ , with  $\sigma_P$  and  $(e^{Z_P} - 1)$  denoting, respectively, the volatility and the expected drop vectors of the corresponding price process for the dividend claim D,  $\lambda^{pol} = \pi \lambda^E$ , the Poisson intensity of the policy shock,  $\sigma_m$  and  $(e^{Z_m} - 1)$  from Proposition 2, and  $\circ$  denoting element-wise multiplication. We prove in Appendix C the following proposition regarding the price of such dividend claim, along with the aggregate risk premium:

**Proposition 3** (Prices and risk premium) Let the aggregate market's dividend be leveraged

<sup>&</sup>lt;sup>24</sup>Consistent with observations (Longstaff and Piazzesi 2004) in the event of a negative shock dividends fall more than consumption when  $\eta > 1$  which we assume.

consumption  $D = C^{\eta}$  with  $\eta > 1$ . The price-dividend ratio for the aggregate market  $G \equiv P/D$  is calculated exactly by:

$$G(\lambda^M, \lambda^E) = \int_0^\infty e^{a_{\eta}(s) + b_{\eta}^M(s)\lambda^M + b_{\eta}^E(s)\lambda^E} ds, \tag{23}$$

with  $a_{\eta}(s), b_{\eta}^{j}(s)$  solutions to the following system of differential equations:

$$a'_{\eta}(s) = \mu_D - g - \rho + (1 - \eta)\gamma\sigma^2 + \sum_{j \in \{M, E\}} \kappa^j \bar{\lambda}^j b^j_{\eta}(s),$$
 (24)

$$(b_{\eta}^{M})'(s) = \frac{1}{2} (\sigma_{\lambda}^{M} b_{\eta}^{M}(s))^{2} + (b^{M} (\sigma_{\lambda}^{M})^{2} - \kappa^{M}) b_{\eta}^{M}(s) + \mathbb{E}_{z^{M}} \left[ e^{(\eta - \gamma)Z^{M}} - e^{(1 - \gamma)Z^{M}} \right],$$

$$(b_{\eta}^{E})'(s) = \frac{1}{2} (\sigma_{\lambda}^{E} b_{\eta}^{E}(s))^{2} + (b^{E} (\sigma_{\lambda}^{E})^{2} - \kappa^{E}) b_{\eta}^{E}(s) + \mathbb{E}_{z^{E}} \left[ e^{(\eta - \gamma)Z^{E}} - e^{(1 - \gamma)Z^{E}} \right]$$

$$+ \pi \left[ (1 + n_{B}(e^{X} - 1))^{\eta - \gamma} - (1 + n_{B}(e^{X} - 1))^{1 - \gamma} \right],$$

$$(25)$$

 $a_{\eta}(0) = b_{\eta}^{M}(0) = b_{\eta}^{E}(0) = 0$ , and  $\mu_{D} = \eta g + \frac{1}{2}\eta(\eta - 1)\sigma^{2}$ . The market's risk premium reads:

$$R - r = \eta \gamma \sigma^{2} - \sum_{j \in \{M, E\}} \overbrace{\lambda^{j} \frac{1}{G} \frac{\partial G}{\partial \lambda^{j}}} b^{j} (\sigma_{\lambda}^{j})^{2} - \sum_{j \in \{M, E\}} \lambda^{j} \mathbb{E}_{z^{j}} \left[ (e^{-\gamma Z^{j}} - 1)(e^{\eta Z^{j}} - 1) \right] - \pi \lambda^{E} \left[ (1 + n_{B}(e^{X} - 1))^{-\gamma} - 1 \right] \left[ (1 + n_{B}(e^{X} - 1))^{\eta} - 1 \right].$$
(26)

Since for s=0 an asset should pay its current dividend, boundary conditions are  $a_{\eta}(0)=b_{\eta}^{M}(0)=b_{\eta}^{E}(0)=0$ . For  $\eta>1$ , both  $a_{\eta}(s)$  and  $b_{\eta}^{j}(s)$  are well defined functions of s such that the infinite integral G converges. The solution to (25) with the previous boundary conditions yields  $b_{\eta}^{j}(s)<0$ ,  $j\in\{M,E\}$ . According to (23), this implies that, ceteris paribus, draws of high disaster risk – macroeconomic, environmental, or policy-related– reduce valuations. Since marginal utility also increases during these times, risk premia should be positive.

The first two terms in the risk premium represent the correlated movement between the pricing-kernel and market prices in times without disasters, while the third term represents the same thing in the event of an economic shock – triggered either from rare macroeconomic disasters such as wars and financial crises, or from natural disasters; the last term captures policy risk, i.e., the policy premium. The first term is the risk premium in the standard CAPM and is negligible for acceptable values of the risk aversion coefficient, while the second term arises from the time variation in disaster risk, and is substantial: the fact that the Poisson intensity of extreme events is itself stochastic is an additional source of risk that should be priced (Wachter 2013). The novelty of our model lies in establishing the link between climate change and the probability of extreme environmental disasters according to (3) and its incorporation in asset pricing. A changing environment makes climate related disasters more frequent and less

predictable; temperature increases shift the distribution of the probability of extreme environmental events to higher draws while it gets flattened at the same time increasing its volatility (see also Figure 7). In line with standard results in the extreme events literature, higher draws of  $\lambda^E$  unambiguously raise the equity premium through the last two terms, the "static" disaster risk terms. However, the second term, due to the time-variation of risk, deserves a closer look.

Of importance is the term  $\epsilon_i \in [-1, 0]$ , defined in (26). Loosely speaking, this term represents the risk "elasticity of valuations", i.e., the variation in the price-dividend ratio in response to variations in macroeconomic and/or climate risk. This term can be decomposed in two parts: the "semi-elasticity of valuations",  $\Delta \log G/\Delta \lambda^j$ , measuring the percentage change in G from a unit increase in  $\lambda^j$ ; and the risk  $\lambda^j$  itself. On the one hand, from (23) and (24), with rising temperatures the increasing expected risk of environmental events  $(\bar{\lambda}^E)$  puts a downward pressure on equity valuations, which leaves less room for prices to react to high draws of risk of either type. This level effect reduces the magnitude of the semi-elasticity of valuations for both types of disasters. On the other hand, as climate changes, the distribution of climate risk shifts to higher draws, while the one of macroeconomic risk stays unaltered, which increases the magnitude of the elasticity of valuations for climate risk through its second part, i.e. the risk itself, and the relative importance of this type of risk even in times without disasters. The equity premium of climate change (whatever multiplies  $\lambda^E$  in (26)) is increasing. Since, however, the risk of rare macroeconomic events makes up the largest part of the equity premium, the magnitude of the aggregate equity risk premium is only minimally affected, while – depending on calibration – it might also decline. This result suggests that, as climate changes, assets that feature relatively lower climate change (but possibly higher macroeconomic) risk work as a hedging strategy against the risk of climate change and should therefore be rewarded with a lower premium.

To build intuition, we show below in Figures 2-4 the semi-elasticity and the elasticitity of valuations, and the two parts of the equity premium (macroeconomic and environmental – without policy shock) as functions of the Poisson intensity of extreme environmental events. In the next section we also calibrate this probability of arrival of extreme environmental events and perform numerical simulations that confirm our claims.

The above subtle relationship between sources of time-varying risk, that works through equity valuations on risk premia, is obscured in models with log-linearization of the price-dividend ratio.<sup>25</sup> To see this, suppose a log-linear approximation of (23) in the state variables in the spirit of Campbell and Shiller (1988), i.e.  $\log G(\lambda^M, \lambda^E) \approx \tilde{a}_{\eta} + \sum_j \tilde{b}^j_{\eta} \lambda^j$ , with  $\tilde{a}_{\eta}, \tilde{b}^j_{\eta}$  scalars and  $\tilde{b}^j_{\eta} < 0$ . The semi-elasticity in this case is just a constant, i.e.  $\Delta \log G/\Delta \lambda^j \approx \tilde{b}^j$ , while the elasticity of valuations in (26) is a linear function of  $\lambda^j$ , i.e.  $\epsilon_j \approx \tilde{b}^j \lambda^j$ . The overall equity premium now reads:

$$R - r \approx \eta \gamma \sigma^2 - \sum_{j \in \{M, E\}} \lambda^j \tilde{b}^j (\sigma_{\lambda}^j)^2 - \sum_{j \in \{M, E\}} \lambda^j \mathbb{E}_{z^j} \left[ (e^{-\gamma Z^j} - 1)(e^{\eta Z^j} - 1) \right] - \pi \lambda^E \left[ (1 + n_B(e^X - 1))^{-\gamma} - 1 \right] \left[ (1 + n_B(e^X - 1))^{\eta} - 1 \right],$$

<sup>&</sup>lt;sup>25</sup>The fact that such models overstate equity premia was noted in (Tsai and Wachter 2018).

which is unambiguously increasing in both  $\lambda^E$  and  $\lambda^M$ . Finally, back in the exact case of equation (26), the overall equity premium increases in  $\lambda^E$  unambiguously when the risk of rare macroeconomic events is governed by a constant Poisson intensity  $\lambda^M$ .<sup>26</sup> We now turn to numerical simulations of our model.

Figure 2: "Semi-elasticity" of valuations  $\frac{1}{G}\frac{\partial G}{\partial \lambda^j}$  as climate changes. Assuming an increasing temperature path, the figures show this term for both types of risk  $j \in \{M, E\}$ , as a function of the probability of extreme environmental events  $\lambda_t^E$ . The solid line shows this term for  $\bar{\lambda}_{2010}^E$ ; the dashed line for  $\bar{\lambda}_{2100}^E$ ; the arrow shows the transition. The probability of extreme macroeconomic events  $\lambda_t^M$  is set at its equilibrium value  $\bar{\lambda}^M$ .

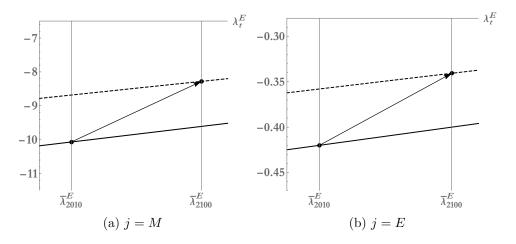
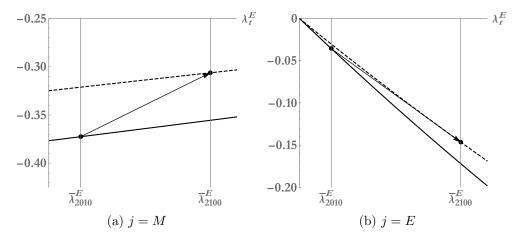
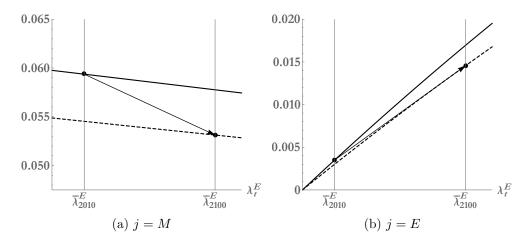


Figure 3: "Elasticity of valuations"  $\epsilon_j = \lambda^j \frac{1}{G} \frac{\partial G}{\partial \lambda^j}$  as climate changes. Assuming an increasing temperature path, the figures show this term for both types of risk  $j \in \{M, E\}$ , as a function of the probability of extreme environmental events  $\lambda_t^E$ . The solid line shows this term for  $\bar{\lambda}_{2010}^E$ ; the dashed line for  $\bar{\lambda}_{2100}^E$ ; the arrow shows the transition. The probability of extreme macroeconomic events  $\lambda_t^M$  is set at its equilibrium value  $\bar{\lambda}^M$ .



<sup>&</sup>lt;sup>26</sup>Whether equity premia increase in the risk of climate change disasters in the exact case of (26) has also to do with the calibration of the severity of these extreme environmental events. Throughout this analysis we assumed that the drop in  $\log Y_i$ ,  $i \in \{G, B\}$ , following an extreme environmental event, draws from a time-invariant distribution which we calibrate to historical data. With the full severity of climate change not having materialized yet, this might be a rather low estimate.

Figure 4: Risk premia as climate changes. Assuming an increasing temperature path, the figures show the  $j = \{M, E\}$  parts of the equity premium as a function of the probability of extreme environmental events  $\lambda_t^E$ . The solid lines are for  $\bar{\lambda}_{2010}^E$ ; the dashed lines for  $\bar{\lambda}_{2100}^E$ ; the arrows show the transition. The probability of extreme macroeconomic events  $\lambda_t^M$  is set at its equilibrium value  $\bar{\lambda}_t^M$ .



## 3 Numerical part

## 3.1 Methodology

A changing climate influences the mean arrival rate of environmental shocks  $\bar{\lambda}^E$ . Our proxy for climate change is the change in global average temperature relative to a given time period, i.e., the temperature anomaly. Exogenous temperature paths in this case could be regarded as the ones that correspond to different emissions scenarios as the Representative Concentration Pathways (RCPs) produced by the IPCC (IPCC 2014), that project different GHG concentration pathways up to 2100. The latest modelling convention links temperature anomaly T to carbon concentration (cumulative emissions CE) within a particular time period, in a rather linear fashion, i.e., according to  $T_t - T_{t_0} \approx \Lambda \times CE_t$ , for  $t > t_0$  and  $t_0$  being the reference year.<sup>27</sup> For example for  $t_0$  corresponding to pre-industrial times (with  $T_{t_0} \approx 0$ ),  $T_{2015}$  was about 1°C. Parameter  $\Lambda$  measures the Transient Climate Response to cumulative carbon Emissions (TCRE) and is estimated to be in the range of  $[0.0008, 0.0024]^{\circ}C/GtC$ ; see Leduc et al. (2016), Matthews et al. (2018). To match the RCP projections (Figure 5) we set  $\Lambda = 0.002^{\circ}C/GtC$  and calculate the various temperature paths based on the corresponding RCP emissions according to the above equation (Figure 6).

<sup>&</sup>lt;sup>27</sup>See Matthews et al. (2009, 2012), Knutti (2013), Knutti and Rogelj (2015), MacDougall et al. (2017), Brock and Xepapadeas (2018), Matthews et al. (2018), Dietz and Venmans (2019).

Figure 5: IPCC projected temperature anomaly paths; source: IPCC (2013)

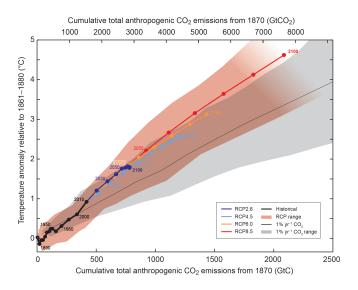
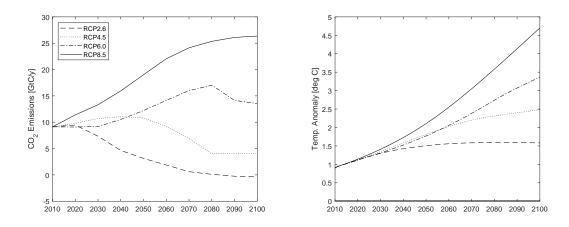
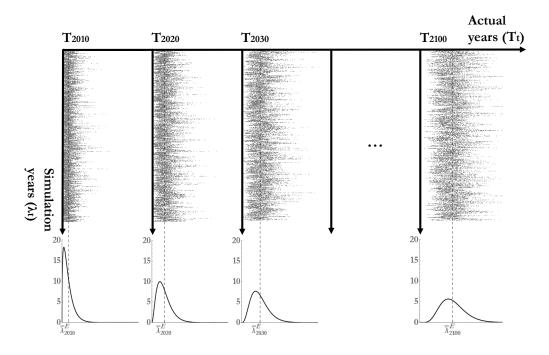


Figure 6: IPCC RCP carbon emissions and (calibrated) temperature anomaly paths; source: (IPCC 2013) (emissions) and own calculation (temperature anomaly).



With regards to simulations of the asset pricing model, the usual methodology assumes time-invariability of the system under study. This allows for a straightforward Monte-Carlo simulation using a large number of random realizations of the relevant stochastic variables. The mean of the process for  $\lambda^E$  changes exogenously with temperature anomaly, such that our model is not time-invariant; see equation (3). We circumvent this issue by assuming that our model is time-invariant for each given level of temperature and simulate the model for each  $\bar{\lambda}_t^E = \bar{\lambda}^E(T_t)$ . We divide our actual time horizon (2010-2100) in decades, and run the model for 100,000 simulation years for each decade. Figure 7 presents our methodology along with the resulting stationary Gamma distribution for  $\lambda^E$  for increasing temperatures, which changes as expected. Finally, we assume throughout that the participation of dirty assets in the portfolio  $n_B$  depends only on the additional policy risk these assets are exposed to (see equation (13)).

Figure 7: Schematic of the simulation methodology.  $\bar{\lambda}^E$  changes with temperature anomaly; for each actual time period (2010-2019, 2020-2029, ...) we keep temperature at its projected mean of the period and run the model for 100,000 simulation years. The figure assumes an increasing temperature path.



## 3.2 Calibration

Table 1: Parameters for the benchmark calibration

All values are in annual terms

Parameters for the stochastic processes	
Average probability of macroeconomic disasters $\lambda^M$	0.0369
Slope of the linear $\bar{\lambda}^E(T)$ curve $\xi$	0.0915
Speed of mean reversion for risk $\kappa^M = \kappa^E$	0.08
Volatility parameter for macroeconomic disasters $\sigma_{\lambda}^{M}$	0.073
Volatility parameter for environmental disasters $\sigma_{\lambda}^{E'}$	0.334
Drift parameter for the general asset $\mu_G$	0.0405
Drift parameter for the brown asset $\mu_B$	0.0395
Volatility parameter for both assets $\sigma_G = \sigma_B$	0.0263
Correlation coefficient $corr(dW_B, dW_G)$	0
Leverage parameter $\eta$	2.36
Probability of policy reaction to extreme climatic events $\pi$	0.6
Utility parameters	
Relative risk aversion $\gamma$	3.5
Intertemporal discount rate $\rho$	0.015

We measure time in years and calibrate the model to match observed climatic and financial historical data. Our initial time period is 2010. Our benchmark is the case without policy risk, i.e. X=0. Table 1 collects the chosen parameter values for the benchmark calibration.

#### 3.2.1 Distributions of macroeconomic and environmental disasters

The percentage decline in per capita consumption features both environmental and macroeconomic shocks that need to be calibrated to the data. Hence, we need to construct a separate dataset for each of the two types of shocks; from these datasets we can then calculate the distribution of percentage drops as well as the average of the Poisson intensities  $\bar{\lambda}^j$ ,  $j \in \{M, E\}$ .

To do so we make use of different data sources as follows. As a first source we extend until 2015 the Barro and Ursúa (2010) dataset, that collects consistent data on GDP per capita for 42 countries for the period 1911-2008.<sup>28</sup> For our purposes this dataset holds the real reported y-o-y changes in GDP per capita, i.e., after accounting of any (negative) growth effects of climate-related events. In order to calculate these growth effects of climate change we act in the following way. We first collect from the international disasters database EM-DAT (2018), all climate-related events for these 42 countries and for the 1915-2015 time period; we consider only events relevant to climate change.<sup>29</sup> We then follow the methodology of Loayza et al. (2012) and calculate the negative growth effects on GDP per capita of extreme environmental events (top 10% in each event category according to the severity index defined in that paper) for each country and each year; from these we keep extreme events that resulted in GDP p.c. drops of more than 1%.30 This is our first dataset including data on environmental damages. To calculate pure macroeconomic damages we add the – absolute value of – environmental damages to growth entries of the extended Barro-Ursúa dataset. This yields the real GDP per capita if no extreme climate-related events had occured. To construct our second dataset containing pure macroeconomic damages we then follow the peak-to-trough methodology for cumulative fractional declines in real GDP per capita as explained in Barro and Ursúa (2008). As in the aforementioned contribution, and in Wachter (2013), we include only peak-to-trough events that resulted in GDP drops more than 10%.<sup>31</sup>

Following Barro and Ursúa (2008) the frequency of large declines in GDP per capita in our pure macroeconomic dataset is calculated  $\bar{\lambda}^M = 0.0369$  while the mean drop size of its time-

<sup>&</sup>lt;sup>28</sup>We use percentage changes in GDP per capita, instead of consumption per capita, as a proxy for damages. Both Barro (2009) and Wachter (2013) find similar results for their CAPMs with rare disasters whether they calibrate to the consumption or GDP data.

<sup>&</sup>lt;sup>29</sup>According to the EM-DAT categories we consider meteorological events (storms/extreme temperatures), hydrological events (floods/avalanches), and climatological events (droughts/wildfires).

<sup>&</sup>lt;sup>30</sup>Loayza et al. (2012) show that extreme climate-related events (top 10%) are always bad for economic growth and calculate the growth elasticities of different event types on different economic sectors: manufacturing; services; agriculture. Using World Bank data we calculate the sectoral shares of GDP for each country and then using these growth elasticities we calculate the country-specific climate-related damages on GDP per capita for each year. The effect of an extreme natural disaster on GDP per capita is robustly negative also in Cavallo et al. (2013) and Felbermayr and Grschl (2014).

<sup>&</sup>lt;sup>31</sup>Using the peak-to-trough methodology for macroeconomic, and not for environmental events, we implicitly make the assumption that macroeconomic events, such as wars or crises, have memory, while climate-related events are memory-less.

invariant distribution is -22.1%. In order to construct the linear relationship  $\bar{\lambda}_t^E = \tilde{\lambda}^E + \xi T_t$  for the time-varying mean of the stochastic process in (3), we divide our sample in ten decades starting from 1915 and calculate  $\bar{\lambda}^E$  for each decade; we use the middle year to indicate a given decade, e.g., 1920 refers to the decade 1916-1925. For the first four decades (i.e. before 1960)  $\bar{\lambda}^E$  stays below 0.004 (which we set as the minimum value for  $\bar{\lambda}^E$  in our calculations), while since then rises sharply with temperature; the fitted line gives  $\tilde{\lambda}^E = 0.0006$  and  $\xi = 0.0915$ . The frequency distribution of climate-related damages has a mean drop size of -1.58%.

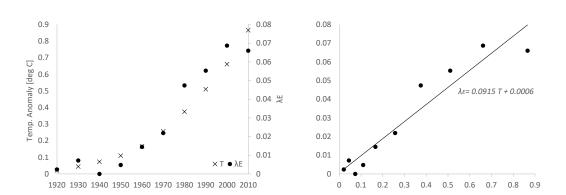


Figure 8: Calibration of the  $\bar{\lambda}_t^E = \tilde{\lambda}^E + \xi T_t$  curve

## 3.2.2 Other parameters

We set the coefficient of relative risk aversion to  $\gamma=3.5$  and the intertemporal discount rate to  $\rho=0.015$ , both widely accepted values in the literature.<sup>32</sup> We follow Wachter (2013) and set the mean reversion parameters of the intensities of disasters to  $\kappa^M=\kappa^E=0.08$ ; this leads to an autocorrelation of the price-dividend ratio of 0.92, its value in the data. The leverage parameter is set to  $\eta=2.36$  to match the observed market equity premium and volatility; this leads to a dividend growth rate in times without disasters of 6% (Wachter (2013) sets  $\eta=2.6$ ).

We define assets with exposure to transition risk as "brown" and follow Prudential Regulation Authority (2015) to set  $n_B = 0.3$  for 2010 in the benchmark; see footnote 12.<sup>33</sup> We assume that in the benchmark the policy risk channel is not active, i.e. X = 0; we subsequently calibrate the policy risk parameter and study the effect of policy risk on the portfolio allocation. In order to calculate the probability  $\pi$  that climate change policy becomes effective after an extreme climate-related event, we use the Grantham-LSE (2018) database that includes all laws and legislations since the 1960s related to climate change, covering 95% of global emissions. In this database there are in total 519 laws for our 42 countries, a quarter of which refers to low carbon transition laws (Nachmany et al. 2017); with 213 severe events (top 10% ever recorded) in our dataset we calculate  $\pi \approx 0.25 \times 519/213 = 0.609$ . With regards to the correlation between

<sup>&</sup>lt;sup>32</sup>Barro (2009) sets  $\gamma = 4$ , while Wachter (2013)  $\gamma = 3$ .

<sup>&</sup>lt;sup>33</sup>In addition, Oestreich and Tsiakas (2015) investigate empirically the effect of EU-ETS on German stock returns in the period 2003-2012. They divide their sample of 65 firms in clean and dirty depending on whether they received free carbon allowances or not; dirty firms occupy about 35% of that sample.

assets in times without disasters, we follow Cochrane et al. (2007) and set  $\operatorname{corr}[dW_G, dW_B] = 0$ . We will therefore examine the case of a strong diversification motive (see section 2.2).

We also set  $\sigma_B = \sigma_G$  and solve (9) for  $n_B = 0.3$  and zero correlation such that  $\sigma = 0.02$  (Wachter 2013), giving  $\sigma_B = \sigma_G = 0.2626$ . The values of  $\mu_B$  and  $\mu_G$  are estimated by solving  $n_B\mu_B + n_G\mu_G - \rho = g = 0.0252$  (Wachter 2013) and (12) again for  $n_B = 0.3$  and X = 0 in the benchmark. Note that the different drift parameters for the two assets are not important for the case without policy risk as it is the aggregate market drift and volatility that matter. Moreover, we need to calibrate the volatility parameters  $\sigma_\lambda^M$  and  $\sigma_\lambda^E$  for processes (2) and (3), respectively. As in Seo and Wachter (2018) volatility parameters are calculated by choosing the discriminant of (15) for both types of disasters to be zero. This yields  $\sigma_\lambda^M = 0.073$  and  $\sigma_\lambda^E = 0.334$  in the benchmark simulation with X = 0.

#### 3.3 Simulation results

Table 2 presents the results of our benchmark simulation in contrast to historical post-WWII US data from Wachter (2013). Our model and its calibration matches observed moments of interest very well: the riskfree rate is 1.4% in comparison with 1.34% in the data, the equity premium generated matches the observed 7.1% p.a., while simulated equity volatility is 18.6% p.a., compared to the observed 17.7% p.a. We turn next to studying the effect of a changing climate on market fundamentals.

Table 2: Moments from simulated vs. historical data.  $R^f$  is the riskfree rate,  $R^e$  the gross return on equity, AR1[P-D] is the first order autocorrelation of the price-dividend ratio and SR the Sharpe ratio. With exception of the SR and AR1[P-D], moments are in percentage terms.

All values are in annual terms		
Moments	Simulation	US Data (1947-2010)
$\overline{\mathbb{E}[R^f]}$	1.40	1.34
$\sigma(R^f)$	2.23	2.66
$\mathbb{E}[R^e - R^f]$	7.07	7.06
$\sigma(R^e)$	18.55	17.72
AR1[P-D]	0.92	0.92
SR	0.38	0.40

All values are in annual terms

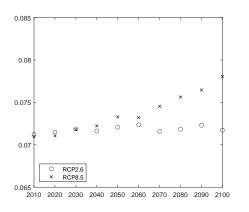
#### 3.3.1 The physical risk of climate change

In this part we explore the pure effects of climate change risk on market fundamentals, with a focus on the equity premium and sovereign debt. Figure 9 presents the effect of the two extreme RCP scenarios (RCP2.6 and RCP8.5) on the risk premium. The risk premium on the aggregate market increases only a little with global warming from its current value of 7.1% to 7.7% p.a. for the worst case scenario (RCP8.5); its change is insignificant in the best case scenario (RCP2.6) where temperature in the end of the century slightly exceed  $1.5^{\circ}C$ . From (26) we can get the part of the equity premium that is solely due to climate change risk. As Figure 10 shows, with our calibration the risk premium of climate change amounts to about 0.5% in 2020; the

remaining part of the aggregate equity premium is mainly due to the risk of rare macroeconomic disasters, and only a very small part is due to the standard CAPM's diffusion risk. The climate risk premium increases to about 1.7% p.a. by the end of the century in the worst case scenario (RCP8.5). What is the reason behind the only slight increase of the aggregate risk premium, although the premium of climate change risk is increasing significantly?

According to our discussion in section 2.5, from (23) and (24), higher temperatures – which increase  $\bar{\lambda}^E$  – affect the way in which valuations react to the different kinds of risk. With our calibration, climate warming reduces valuations (Figure 11) and the magnitude of the risk elasticity of valuations for macroeconomic disasters  $\epsilon^M$  in (26), while it increases the one for environmental  $\epsilon^E$ . Therefore, increasing temperatures, change the relative importance of the two sources of risk in normal times and alongside their contribution to the premium of the aggregate market; see Figure (12). Finally, as expected from our theory, higher temperatures unambiguously decrease interest rates; see figure 13.

Figure 9: The market equity premium (annual terms)



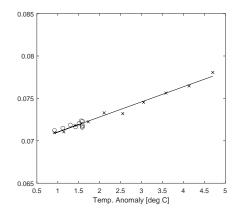
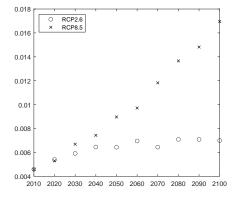


Figure 10: The premium of climate change risk (annual terms)



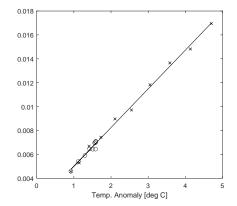


Figure 11: Price-Dividend ratio, G

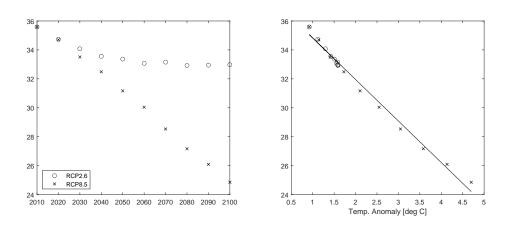


Figure 12: Elasticities of valuation for both sources of risk,  $\epsilon^j$ 

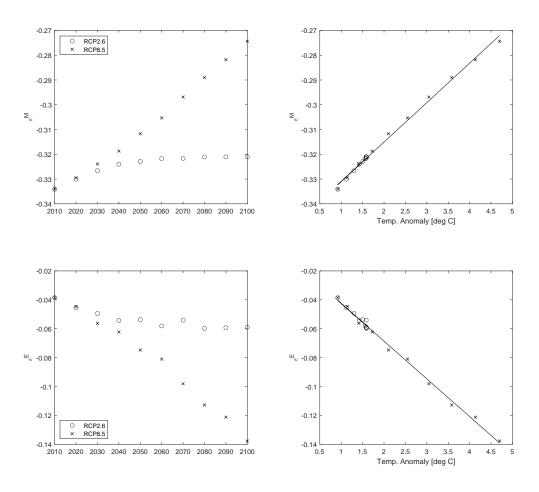
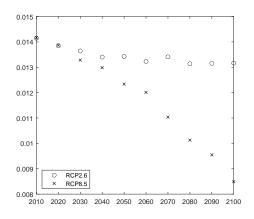
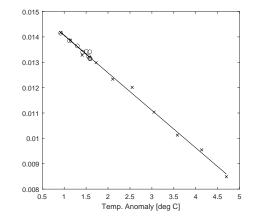


Figure 13: Riskfree rate r (annual terms)





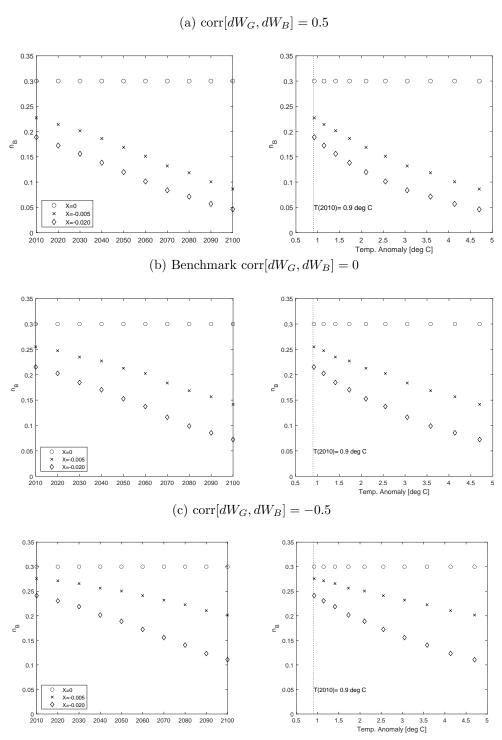
#### 3.3.2 Policy risk and portfolio participation of brown assets

Our benchmark calibration assumes that there is no additional policy risk on brown assets, the share of which we calibrate to  $n_B = 0.3$  by choosing appropriatly the model parameters (see 3.2 "Calibration"). In this paragraph we relax this by assuming the existence of abnormal returns following the announcement of green policies.<sup>34</sup> Ramiah et al. (2013) study the existence of such returns in Australia and document negative mean abnormal returns on the order of -2.8% across 10 industries, including mining, oil, gas and real estate. We measure abnormal returns as the mean difference in actual returns on equity at the time the policy strikes for a carbon-intensive portfolio ( $n_B = 1$ ) whose expected return is evaluated neglecting policy risk.

Using the above we calibrate X = -0.005 that leads to abnormal returns of -2.5%. As the probability of extreme environmental events changes with temperature, investors optimally reallocate their portfolio by choosing  $n_B$  according to (12). Figure 14 presents the simulated portfolio participation of brown assets for the worst IPCC scenario (RCP8.5) for X = 0, X = -0.005 and for X = -0.02, the latter leading to abnormal returns of -6%. To examine the role of the diversification motive of the risk averse investor on portfolio allocation, we also simulate for different values of the correlation coefficient between assets (see section 2.3). First, irrespective of the diversification motive, including the risk of stringent climate policy substantially reduces the participation of brown assets in the market portfolio as temperatures increase. Second, for every level of temperature and policy stringency, a lower correlation between assets increases the diversification motive and leads to a higher share of brown assets in the market portfolio; yet lower than in the benchmark.

<sup>&</sup>lt;sup>34</sup>For specific stocks or portfolios, abnormal returns measure the performance difference on given dates or time periods from expected returns that are calculated by an asset-pricing model.

Figure 14: The effect of policy risk on portfolio participation of brown assets. Temperature follows the worst IPCC scenario (RCP8.5); we use different values for the correlation coefficient of diffusion risk  $\operatorname{corr}[dW_G, dW_B]$  to examine the effect of the diversification motive on portfolio composition.



## 4 Conclusion

In order to price climate change risk for capital markets we develop an asset pricing model with rare events and time-varying probabilities. Such models have been shown to solve several puzzles in the financial literature. In addition to the – already considered – risk of macroe-conomic disasters such as wars, pandemics and financial crises, we include the risk of extreme adverse environmental events related to climate change. Our main methodological contribution lies in establishing the link between carbon concrentrations (temperature anomaly) and the stochastically-varying risk of rare climatic events. Besides the physical risk, which affects the whole market, we also include the transition risk of climate change, i.e., the risk of exposure to stringent environmental policies that lower the returns of carbon-intensive assets.

We confirm the result in the literature that climate change entails a positive and increasing risk premium. We provide exact solutions (both theoretical and numerical) and show, however, that whether this ultimately carries over to the aggregate equity premium depends on the time variation of risk and the severity of environmental events. In our setting with multiple sources of time-varying risk, we expose a subtle issue in models with log-linear approximations of price-dividend ratios or models with static disaster risk, when such models are used for calculating equity premia. With rising temperatures the increasing risk of environmental events puts a downward pressure on equity valuations, which leaves less room for prices to react to the risk of extreme events of either type, macroeconomic or environmental. As climate changes, the distribution of climate risk shifts to higher draws, while that of macroeconomic risk stays unaltered, which increases the relative importance of climate change risk even in times without disasters. Since, however, the risk of rare macroeconomic events makes up the largest part of the equity premium in the data, the magnitude of the aggregate equity risk premium is only minimally affected. We show analytically how log-linearizing the price-dividend ratio – as usually done in asset pricing models – kills this channel and leads to an overstated market equity premium as the risk of climate change increases. In addition, we find that interest rates fall unambiguously in light of a changing climate. This is of particular importance for central banks that use interest rates for inflation targeting and financial stability, especially in an already low-interest environment. Finally, we show that including the transition risk of environmental policies optimally lowers the participation of carbon-intensive assets in the market portfolio, thus leading to their stranding.

## Appendix A - Deriving the value function

The value function  $V(A, \lambda^M, \lambda^E)$  satisfies (7), while, in equilibrium the riskfree asset is in zero net supply, i.e.,  $n_B + n_G = 1$ .

Substitute our conjecture (14) and (5) into (10) to get  $C = \rho A$ . With this and (14) the instantaneous utility reads

$$f(A, \lambda^M, \lambda^E) = \rho A^{1-\gamma} I(\lambda^M, \lambda^E) \left( \log \rho - \frac{\log I(\lambda^M, \lambda^E)}{1-\gamma} \right), \tag{27}$$

with  $I(\lambda^M, \lambda^E) = e^{a + \sum_j b^j \lambda^j}$ . Substitute (27) in the optimized HJB equation (7) to get

$$(1 - \gamma)\rho(\log \rho - 1) - \rho \left( a + \sum_{j} b^{j} \lambda^{j} \right) + (1 - \gamma) \left( \sum_{i} n_{i} \mu_{i} - \frac{\gamma}{2} \sigma^{2} \right)$$

$$+ \sum_{j} b^{j} \kappa^{j} (\bar{\lambda}^{j} - \lambda^{j}) + \frac{1}{2} (b^{j})^{2} (\sigma_{\lambda}^{j})^{2} \lambda^{j} + \lambda^{j} \mathbb{E}_{z^{j}} [e^{(1 - \gamma)Z^{j}} - 1]$$

$$+ \pi \lambda^{E} [(1 + n_{B}(e^{X} - 1))^{1 - \gamma} - 1] = 0,$$

$$(28)$$

Collecting terms in  $\lambda^j$  implies a quadratic equation for each  $b^j$  giving (15) in the main text; the solution with the negative sign in front of the square root is the one with reasonable economic properties (Wachter 2013). Collecting constant terms gives equation for a.

# Appendix B - pricing kernel and riskfree rate

From Itô's Lemma on  $C = \rho A$  using (6) and  $\sum_{i=\{B,G\}} n_i = 1$  in equilibrium, consumption follows:

$$\frac{dC}{C} = gdt + \sum_{i} n_{i}\sigma_{i}dW_{i} + \sum_{j=\{M,E\}} (e^{Z^{j}} - 1)dQ^{j} + n_{B}(e^{X} - 1)dQ^{X},$$
(29)

with  $g = \sum_{i=\{B,G\}} n_i \mu_i - \rho$ . Note that without the stochastic terms, consumption growth is g, i.e. the Keynes-Ramsey rule with EIS=1 in the deterministic environment.

Multiply (11) with  $n_G$  and (12) with  $n_B$ , then add the two and substitute our conjecture (14) and  $n_B + n_G = 1$  to get equation (20) in the main text.

Duffie and Epstein (1992b) and Duffie and Skiadas (1994) show that the state-price density for preferences as given by (4) and (5) in continuous time is given by

$$m_t = \exp\left[\int_0^t f_U(C_s, U_s)ds\right] f_C(C_t, U_t). \tag{30}$$

Itô's Lemma (and employing optimality) implies:

$$\frac{dm}{m} = f_V dt + \frac{df_C}{f_C}. (31)$$

For  $f_C$  from (5) and C following (29), the Poisson jump of m reads  $\tilde{m}/m = \tilde{f}_C/f_C = (\tilde{C}/C)^{-\gamma}$ . Itô's Lemma imply equation (16). It also follows from no-arbitrage:

$$\mu_m = -r - \sum_{j \in \{M, E\}} \lambda^j \, \mathbb{E}_{z^j} \left[ e^{-\gamma Z^j} - 1 \right] - \pi \lambda \left[ (1 + n_B(e^X - 1))^{-\gamma} - 1 \right]. \tag{32}$$

## Appendix C - Pricing climate change risk

Having calculated the state-price density we are now in position to calculate the risk premium for the aggregate equity market. The aggregate market pays a dividend D, being leveraged consumption, i.e.  $D = C^{\eta}$ . From Itô's Lemma it follows directly that

$$\frac{dD}{D} = \mu_D dt + \eta \sum_i n_i \sigma_i dW_i + (e^{Z_D} - 1) dQ^T, \tag{33}$$

where  $\mu_D = \eta g + \frac{1}{2}\eta(\eta - 1)\sigma^2$ ,

$$e^{Z_D} - 1 = [e^{\eta Z^M} - 1, e^{\eta Z^E} - 1, (1 + n_B(e^X - 1))^{\eta} - 1].$$
(34)

We can also show (see Wachter (2013), Seo and Wachter (2018)) that the price for D reads  $P = DG(\lambda^M, \lambda^E)$  with G from (23) in the main text. Itô's Lemma on P = DG using (33) and (23) leads to the process for prices  $dP/P = \mu_P dt + \sigma_P dW_m^T + (e^{Z_P} - 1)dQ^T$ , with  $Z_P = Z_D$  and

$$\sigma_P = \left[ \eta n_B \sigma_B, \eta (1 - n_B) \sigma_G, \frac{1}{G} \frac{\partial G}{\partial \lambda^M} \sigma_\lambda^M \sqrt{\lambda^M}, \frac{1}{G} \frac{\partial G}{\partial \lambda^E} \sigma_\lambda^E \sqrt{\lambda^E} \right]. \tag{35}$$

Variations in  $\lambda^j$ ,  $j \in \{M, E\}$  create variations in G and thus in stock prices, reflected by the second and third term of (35). Equity premia arise from the co-movement of marginal utility of the risk-averse investor with the price of the underlying asset or portfolio, both in normal times and times of disasters. Let R be the expected return on equity; the risk-premium is then calculated by  $R - r = -\sigma_m \sigma_P^T - [\lambda^M, \lambda^E, \lambda^{pol}] \mathbb{E}[(e^{Z_m} - 1) \circ (e^{Z_P} - 1)]^T$ , with  $\sigma_P$  and  $(e^{Z_P} - 1)$  denoting, respectively, the volatility and the expected drop vectors of the corresponding price process,  $\lambda^{pol} = \pi \lambda^E$ , the Poisson intensity of the policy shock,  $\sigma_m$  and  $(e^{Z_m} - 1)$  from Proposition 2, and  $\circ$  denoting element-wise multiplication. Using (18), (19), (34), and (35), we get (26) in the main text. The drift of the price process  $\mu_P$  can be calculated from the definition of the expected return, which comprise the drift, the dividend yield, and the expected drop in prices should an extreme macroeconomic, environmental, or policy event occur.

$$R \equiv \mu_P + \underbrace{D/P}_{G^{-1}} + \sum_{j \in \{M, E\}} \lambda^j \, \mathbb{E}_{z^j} \left[ e^{\eta Z^j} - 1 \right] + \lambda^E \left[ (1 + n_B (e^X - 1))^{\eta} - 1 \right]. \tag{36}$$

## References

- Abel, A. B.: 1999, Risk premia and term premia in general equilibrium, *Journal of Monetary Economics* 43(1), 3-33.
- Ahn, C. M. and Thompson, H. E.: 1988, Jump-diffusion processes and the term structure of interest rates, *The Journal of Finance* **43**(1), 155–174.
- Annicchiarico, B. and Di Dio, F.: 2017, Ghg emissions control and monetary policy, *Environmental and Resource Economics* **67**(4), 823–851.
- Bansal, R., Kiku, D. and Ochoa, M.: 2016, Price of long-run temperature shifts in capital markets, Working paper, National Bureau of Economic Research.
- Bansal, R. and Yaron, A.: 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance* **59**(4), 1481–1509.
- Barnett, M., Brock, W. and Hansen, L. P.: 2020, Pricing Uncertainty Induced by Climate Change, *The Review of Financial Studies* **33**(3), 1024–1066.
- Barro, R. J.: 2006, Rare disasters and asset markets in the twentieth century, *The Quarterly Journal of Economics* **121**(3), 823–866.
- Barro, R. J.: 2009, Rare disasters, asset prices, and welfare costs, *American Economic Review* **99**(1), 243–64.
- Barro, R. J. and Ursúa, J. F.: 2008, Macroeconomic crises since 1870, *Brookings Papers on Economic Activity* **2008**, 255–335.
- Barro, R. J. and Ursúa, J. F.: 2010, Barro-ursua macroeconomic data.

  URL: https://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data
- Batten, S., Sowerbutts, R. and Tanaka, M.: 2016, Let's talk about the weather: the impact of climate change on central banks, *Technical report*, Bank of England.
- Battiston, S., Mandel, A., Monasterolo, I., Schtze, F. and Visentin, G.: 2017, A climate stress-test of the financial system, *Nature Climate Change* 7, 283288.
- Bretschger, L. and Vinogradova, A.: 2018, Best policy response to environmental shocks: Applying a stochastic framework, *Journal of Environmental Economics and Management*.
- Brock, W. A. and Hansen, L. P.: 2018, Climate Change Economics: the Role of Uncertainty and Risk., Wiley, chapter Wrestling with Uncertainty in Climate Economic Models.
- Brock, W. and Xepapadeas, A.: 2017, Climate change policy under polar amplification, *European Economic Review* **94**.
- Brock, W. and Xepapadeas, A.: 2018, Regional climate change policy under positive feedbacks and strategic interactions, *Environmental and Resource Economics*.
- Brunetti, C., Dennis, B., Gates, D., Hancock, D., Ignell, D., Kiser, E. K., Kotta, G., Kovner, A., Rosen, R. J. and Tabor, N. K.: 2021, Climate change and financial stability, FEDS Notes. Washington: Board of Governors of the Federal Reserve.

- Cai, W., Borlace, S., Lengaigne, M., Van Rensch, P., Collins, M., Vecchi, G., Timmermann, A., Santoso, A., McPhaden, M. J., Wu, L. et al.: 2014, Increasing frequency of extreme el niño events due to greenhouse warming, *Nature climate change* 4(2), 111.
- Campbell, J. Y.: 2003, Chapter 13 consumption-based asset pricing, Financial Markets and Asset Pricing, Vol. 1 of Handbook of the Economics of Finance, Elsevier, pp. 803 887.
- Campbell, J. Y. and Shiller, R. J.: 1988, The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors, *The Review of Financial Studies* 1(3), 195–228.
- Campiglio, E., Dafermos, Y., Monnin, P., Ryan-Collins, J. and Guido Schotten, M. T.: 2018, Climate change challenges for central banks and financial regulators, *Nature Climate Change* 8, 462468.
- Carney, M.: 2015, Breaking the tragedy of the horizon climate change and financial stability, Speech in Lloyd's of London.
- Cavallo, E., Galiani, S., Noy, I. and Pantano, J.: 2013, Catastrophic Natural Disasters and Economic Growth, *The Review of Economics and Statistics* **95**(5), 1549–1561.
- Cochrane, J. H., Longstaff, F. A. and Santa-Clara, P.: 2007, Two Trees, *The Review of Financial Studies* **21**(1), 347–385.
- Cox, J. C., Ingersoll, J. E. and Ross, S. A.: 1985a, An intertemporal general equilibrium model of asset prices, *Econometrica* **53**(2), 363–384.
- Cox, J. C., Ingersoll, J. E. and Ross, S. A.: 1985b, A theory of the term structure of interest rates, *Econometrica* **53**(2), 385–407.
- Daniel, K. D., Litterman, R. B. and Wagner, G.: 2019, Declining co2 price paths, *Proceedings of the National Academy of Sciences*.
- Dietz, S., Gollier, C. and Kessler, L.: 2018, The climate beta, *Journal of Environmental Economics and Management* 87, 258 274.
- Dietz, S. and Venmans, F.: 2019, Cumulative carbon emissions and economic policy: In search of general principles, *Journal of Environmental Economics and Management* **96**, 108 129.
- Duffie, D. and Epstein, L. G.: 1992a, Asset pricing with stochastic differential utility, *The Review of Financial Studies* **5**(3), 411–436.
- Duffie, D. and Epstein, L. G.: 1992b, Stochastic differential utility, Econometrica 60(2), 353–394.
- Duffie, D. and Skiadas, C.: 1994, Continuous-time security pricing: A utility gradient approach, *Journal of Mathematical Economics* **23**(2), 107 131.
- Economides, G. and Xepapadeas, A.: 2018, Monetary policy under climate change, *Technical report*, Working Papers 247, Bank of Greece.
- EM-DAT: 2018, The international disasters database. URL: https://www.emdat.be/
- Felbermayr, G. and Grschl, J.: 2014, Naturally negative: The growth effects of natural disasters, *Journal of Development Economics* 111, 92 106.

- Francis, J. A.: 2017, Why are arctic linkages to extreme weather still up in the air?, Bulletin of the American Meteorological Society 98(12), 2551–2557.
- Francis, J. A. and Vavrus, S. J.: 2012, Evidence linking arctic amplification to extreme weather in mid-latitudes, *Geophysical Research Letters* **39**(6).
- Gomes, J. F., Grotteria, M. and Wachter, J. A.: 2018, Cyclical Dispersion in Expected Defaults, *The Review of Financial Studies* **32**(4), 1275–1308.
- Gourio, F.: 2012, Disaster risk and business cycles, American Economic Review 102(6), 2734–2766.
- Grantham-LSE: 2018, Climate change laws of the world database, LSE Grantham Research Institute on Climate Change and the Environment.
- Hambel, C., Kraft, H. and van der Ploeg, F.: 2020, Asset pricing and decarbonization: Diversification versus climate action, *Technical report*, University of Oxford, Department of Economics.
- Hassler, J., Krusell, P. and Smith, A.: 2016, Chapter 24 environmental macroeconomics, Vol. 2 of *Handbook of Macroeconomics*, Elsevier, pp. 1893 2008.
- Hsiang, S., Kopp, R., Jina, A., Rising, J., Delgado, M., Mohan, S., Rasmussen, D. J., Muir-Wood, R., Wilson, P., Oppenheimer, M., Larsen, K. and Houser, T.: 2017, Estimating economic damage from climate change in the United States, *Science* 356(6345), 1362–1369.
- IPCC: 2013, Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change (Summary for Policy Makers), Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.
- IPCC: 2014, Climate Change 2014: Synthesis Report, Contribution to the Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel of Climate Change (Summary for Policy Makers), IPCC, Geneva, Switzerland.
- IPCC: 2018, Global warming of 1.5c. an ipcc special report on the impacts of global warming of 1.5c above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty, *Technical report*, IPCC, Geneva, Switzerland.
- Jin, J.: 2014, Jump-Diffusion Long-Run Risks Models, Variance Risk Premium, and Volatility Dynamics, Review of Finance 19(3), 1223–1279.
- Knutti, R.: 2013, Relationship between global emissions and global temperature rise, *UNFCCC*. URL: https://unfccc.int/sites/default/files/7'knutti.reto.3sed2.pdf
- Knutti, R. and Rogelj, J.: 2015, The legacy of our co2 emissions: a clash of scientific facts, politics and ethics, *Climate Change* **133**(3), 361–373.
- Leduc, M., Damon Matthews, H. and De Ela, R.: 2016, Regional estimates of the transient climate response to cumulative co 2 emissions, *Nature Climate Change* **6**(5), 474–478.
- Loayza, N. V., Olaberra, E., Rigolini, J. and Christiaensen, L.: 2012, Natural disasters and growth: Going beyond the averages, World Development 40(7), 1317 1336.

- Longstaff, F. A. and Piazzesi, M.: 2004, Corporate earnings and the equity premium, *Journal of Financial Economics* **74**(3), 401 421.
- Lucas, R. E.: 1978, Asset prices in an exchange economy, *Econometrica* 46(6), 1429–1445.
- MacDougall, A. H., Swart, N. C. and Knutti, R.: 2017, The uncertainty in the transient climate response to cumulative co2 emissions arising from the uncertainty in physical climate parameters, *Journal of Climate* 30(2), 813–827.
- Matthews, D., Zickfeld, K., Knutti, R. and Allen, M. R.: 2018, Focus on cumulative emissions, global carbon budgets and the implications for climate mitigation targets, *Environmental Research Letters* 13(1).
- Matthews, H. D., Gillett, N., Stott, P. and Zickfield, K.: 2009, The proportionality of global warming to cumulative carbon emissions, *Nature* **459**, 829–833.
- Matthews, H., Solomon, S. and Pierrehumbert, R.: 2012, Cumulative carbon as a policy framework for achieving climate stabilization, *Philosophical Transactions of the Royal Society A: Mathematical*, *Physical and Engineering Sciences* **370**, 4365–4379.
- Nachmany, M., Fankhauser, S., Setzer, J. and Averchenkova, A.: 2017, Global trends in climate change legislation and litigation, *Technical report*, LSE Grantham Research Institute on Climate Change and the Environment.
- Oestreich, A. M. and Tsiakas, I.: 2015, Carbon emissions and stock returns: Evidence from the eu emissions trading scheme, *Journal of Banking & Finance* 58, 294 308.
- Pindyck, R. S. and Wang, N.: 2013, The economic and policy consequences of catastrophes, *American Economic Journal: Economic Policy* **5**(4), 306–339.
- Prudential Regulation Authority: 2015, The impact of climate change on the uk insurance sector, *Technical report*, Bank of England.
- Ramiah, V., Martin, B. and Moosa, I.: 2013, How does the stock market react to the announcement of green policies?, *Journal of Banking & Finance* 37(5), 1747 1758.
- Seo, S. B. and Wachter, J. A.: 2018, Do rare events explain cdx tranche spreads?, *The Journal of Finance* 73(5), 2343–2383.
- Stolbova, V., Monasterolo, I. and Battiston, S.: 2018, A financial macro-network approach to climate policy evaluation, *Ecological Economics* **149**, 239 253.
- Tsai, J. and Wachter, J. A.: 2015, Disaster risk and its implications for asset pricing, *Annual Review of Financial Economics* **7**(1), 219–252.
- Tsai, J. and Wachter, J. A.: 2018, Pricing long-lived securities in dynamic endowment economies, *Journal of Economic Theory* 177, 848 878.
- USGCRP: 2018, Impacts, risks, and adaptation in the united states: Fourth national climate assessment volume ii, *Technical report*, U.S. Global Change Research Program.
- van der Ploeg, R. and van den Bremer, T.: 2019, The risk-adjusted carbon price, *Technical report*, CESifo Working Paper No. 7592.

- Wachter, J. A.: 2013, Can time-varying risk of rare disasters explain aggregate stock market volatility?, *The Journal of Finance* **68**(3), 987–1035.
- Wang, Z. G. and Bidarkota, P. V.: 2009, A Long-Run Risks Model of Asset Pricing with Fat Tails, Review of Finance 14(3), 409–449.
- Weitzman, M. L.: 2013, Tail-hedge discounting and the social cost of carbon, *Journal of Economic Literature* **51**(3), 873–82.
- Zenios, S.: 2021, The risks from climate change to sovereign debt in europe,  $Policy\ Contribution\ 16/2021$ , Bruegel.