A Microfounded Approach to Currency Substitution and Government Policy*

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Abstract

This paper develops a search-theoretic dynamic dual-currency model featuring endogenous currency substitution as a function of jointly-determined fiscal and monetary policy. Benevolent governments, unable to commit to future policies, weigh distortion-smoothing and time-consistency, where steady-state public expenditures, public debt, labor taxation, and inflation are determined using the notion of a Markov-perfect equilibrium. Currency substitution arises endogenously following cross-country differences in fundamentals, characterizing the relationship between payment patterns, fiscal, and monetary policy. An extension incorporating 'de jure' dollarization eliminates time-consistency concerns and reduces the objective of the government to distortion-smoothing exclusively.

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1 Introduction

The usage of alternative media of exchange in transactions has been of interest to economists for centuries, dating back to the influential work of Jevons (1875) and Menger (1892). Early on it was well understood that aside from regulations, payment patterns are an outcome of individual decisions made by economic agents and policies imposed by official authorities. Recent advances in monetary search theory, surveyed in Lagos et al. (2017), provide microfoundations for said strategies by tying the individuals' choices to preferences and technologies. As a result, not only does a medium of exchange arise endogenously if markets are subject to a distinct set of frictions, but further can agents' optimal portfolio choices and payment strategies be tied to the innate characteristics of each asset. Not surprisingly, in equilibrium, a widely recognizable object proves to be superior at facilitating trade than a more opaque alternative, unless the expected rate of return of the latter warrants its authentication.[1]

The distinct microfoundations make monetary search theory well-suited to study endogenous currency substitution. Recent examples include Lester et al. (2012) and Zhang (2014). To date, however, these advances almost exclusively restrict their attention to monetary policy.[2] Fiscal policy considerations, specifically the trade-off between distortion-smoothing and time-consistency following Barro (1979) and Lucas and Stokey (1983), remain predominantly absent.[3] This paper aims to close this gap by answering a set of questions. First and foremost, under what circumstances does currency substitution arise endogenously? Is currency substitution purely a monetary phenomenon or can it originate from fiscal imbalances? And if so, can it aggravate fiscal imbalances? What are its implications for government financing – domestic and abroad? And lastly, what is the role of commitment, i.e., do policy prescriptions differ from the Ramsey literature if the government is unable to commit to future policies?

To answer these questions, I embed the work of Martin (2011) – explicitly modeling fiscal and monetary policy under limited commitment – into a dynamic two-country dual-currency framework featuring endogenous currency substitution along the lines of Lester et al. (2012). Using the notion of a Markov-perfect equilibrium, optimal policy is determined following the primal approach. Governments, unable to commit to policies beyond the current term, finance public expenditures with distortionary labor taxation, public debt, and seigniorage revenue. They do so by weighing distortion-smoothing and time-consistency. In other words, government policy results from the

[1] Prominent examples go back to the uncertainty regarding the weight and purity of coins and gemstones in ancient times. More recent examples include the counterfeiting of paper currencies and information frictions regarding the quality of exotic financial assets used in transactions. See e.g. Brunner and Meltzer (1971), Alchian (1977), Williamson and Wright (1994), Banerjee and Maskin (1996), Kim (1996), and Berentsen and Rocheteau (2004) for studies on the relationship between authentication, recognizability, and liquidity.

[2] Earlier theoretical work relating currency substitution to inflation differentials includes Calvo and Rodriguez (1977), Girton and Roper (1981), Chen and Tsaur (1983), Chen et al. (1989), Végh (1989), Guidotti and Rodriguez (1992), Chang (1994), Savastano (1996), Uribe (1997), Engineer (2000), Martin (2006). An overview of the early empirical literature is provided by Calvo and Végh (1992).

[3] A notable exception is Sims (2001) studying the fiscal consequences of dollarization in the case of Mexico by introducing the fiscal theory of the price level into a Barro (1979) tax-smoothing model. Explicit foundations of money, and thus the transmission channel of monetary policy to payment patterns, however, remain absent.

interaction of monetary policy and public debt, where cross-country heterogeneities in the preference for public goods yield international differences in allocations. Anticipating future policies, private citizens accumulate portfolios of domestic and foreign currency alongside illiquid nominal government bonds in a Walrasian market to subsequently trade them for consumption in a local decentralized market. While local currency is always accepted locally, the acceptance of foreign currency hinges upon the trading partner's ex-ante investment into an authentication technology.[4] As a result, the internationalization of currencies is an outcome of both, decisions made by private citizens and policies introduced by official authorities, and does not rely on restrictions imposed exogenously. Furthermore, it originates from cross-country differences in fundamentals rather than ad-hoc assumptions on inflation differentials.

The presented framework provides a rich set of new insights. To set the stage, let us start with the key properties of the Markov-perfect monetary equilibrium. Given the governments' inability to commit to policies beyond the current term, if there are expected distortions in the future, a government has an incentive to smooth distortions over time, and thus will not implement socially-efficient allocations in the current period. As a consequence, due to a positive relationship between inflation and public debt, the Friedman rule is not sustainable, and thus, independent of currency substitution, steady states are constrained-efficient featuring positive net nominal liabilities, labor taxation, and inflation. Furthermore, in the steady state, the allocations are robust to commitment frictions, and thus endowing the government with commitment power would not change long-run allocations.[5]

To assess the effects of currency substitution on government policy, a distinction between an interior solution and the limiting cases in which either none or all producers accept foreign currency in transactions is necessary. If local producers exclusively accept local currency, i.e., if there is no currency substitution, then domestic and foreign policies are fully independent. Once a subset of local producers accepts foreign currency, given global market clearing conditions, domestic and foreign prices become intertwined. A strategic complementarity between the producers' authentication decision and the consumers' portfolio choice imposes a constraint on the domestic government's ability to generate seigniorage revenue, and thus to condition monetary policy on public debt choices. In the steady state, the government trades off the incentive to smooth distortions inter-temporally against the negative effects on the demand for its own currency. The existence of the interior solution hinges upon the costliness of authentication, while persistence is determined by the required frequency of the investment. In the limiting case where domestic cur-

[4] Implicitly, this modeling choice assumes that foreign currency can be counterfeited at no cost and is thus rejected in transactions unless verified. Relaxing this assumption, i.e., assuming counterfeiting of foreign currency is costly, would result in foreign currency trading with a positive probability even in the absence of costly verification. See Rocheteau (2008), Li and Rocheteau (2010), Lester et al. (2012), Li et al. (2012), and Gomis-Porqueras et al. (2017) for a discussion of costly counterfeiting equilibria.

[5] The monetary policy prescription suggesting a positive nominal interest rate is in line with Aruoba and Chugh (2010) embedding a Ramsey problem into the Lagos and Wright (2005) structure. A detailed discussion comparing the result to the optimality of the Friedman rule prescribed by e.g. Chari et al. (1991) is provided in Section 4.1. The corresponding Ramsey problem is solved in the Appendix.

rency ceases to be valued, the domestic government loses its ability to finance expenditures using seigniorage revenue and debt denominated in the domestic currency, and thus has to rely solely on revenue generated via labor taxation. Said limiting case motivates government transaction policies to anchor the local demand for domestic currency, as previously formalized by Aiyagari and Wallace (1997) and Li and Wright (1998). Furthermore, it warrants a discussion contrasting the presented economy to an economy in which currency substitution is 'de jure' imposed.

Next to the policy implications for the domestic government, the model further provides important insights into the foreign country's best response to domestic strategies. If a country successfully manages to have its currency accepted internationally, national welfare unambiguously increases due to the additional seigniorage revenue imported from abroad. The increased global demand relaxes the government's budget constraint, thus affecting foreign fiscal and monetary policy. In the steady state, the foreign government trades off the incentive to push distortions into the future against the cost of losing the international status of its currency, and thus the ability to import seigniorage revenue from abroad.[6]

To compare the allocations of agent-driven currency substitution to the case in which currency substitution is imposed by official authorities, an extension studies 'de jure' dollarization. If a government abandons its currency to officially adopt the foreign currency as the legal tender, it gives up its ability to tailor monetary policy to the aggregate state and thus relies exclusively on revenue generated via taxation and debt. As a result, time-consistency concerns disappear, reducing the government's objective to inter-temporal distortion-smoothing exclusively. In general, it does so if the increased trade volume warrants giving up its monetary independence. Since agent-driven currency substitution imposes a constraint on the government's ability to inflate away nominal liabilities, said trade-off weakens. In the limiting case in which domestic agents stop valuing domestic currency altogether, 'de jure' dollarization proves to be a viable option, as it allows the government to regain the ability to smooth distortions inter-temporally by issuing public debt denominated in the foreign currency.

The rest of the paper is organized as follows: Section 1.1 reviews the related literature. Section 2 characterizes the economy and determines the monetary equilibrium. Government policies and steady states are determined in Section 3 and 4, respectively. A discussion relating the testable implications of the theory to empirical observations is provided in Section 5. Lastly, Section 6 concludes.

1.1 Related Literature

The theory draws from two strands of literature: government policy under limited commitment and dual-currency systems. Here I discuss the theoretical literature closest to my work. Empirical

[6] A similar result, trading off of the temptation to inflate against the negative demand effects, has been identified by Li and Matsui (2009) and Zhang (2014) in monetary models.

evidence follows in Section 5.

Starting with the literature on government policy, the paper builds on the influential work of Barro (1979) and Lucas and Stokey (1983) highlighting the importance of government debt as a buffer to smooth distortions over time. Their approach, however, relies on the assumption that governments can commit to future policies, causing time-inconsistency and making the analysis ill-suited as a positive theory, despite valuable normative insights.[7] Among the first to relax the commitment assumption were Ortigueira (2006) and Klein et al. (2008), characterizing Markov-perfect equilibria in frameworks of optimal taxation. Jointly determined fiscal and monetary policy, on the other hand, were studied by Díaz-Giménez et al. (2008), Martin (2009, 2011, 2013), and Niemann (2011), where Martin (2011, 2013) are the papers closest to mine. In a Lagos and Wright (2005) monetary model with competitive markets, Martin (2011) studies the determination of government policy under limited commitment, extended to incorporate financial intermediation and trading frictions in Martin (2013). This paper embeds these insights into a two-country dual-currency model to analyze the effects of currency substitution on domestic and foreign monetary and fiscal policy.

By analyzing co-existing media of exchange, the paper further draws from the literature on dual-currency systems. For brevity, I restrict the attention to the advances featuring two-country dual-currency models focusing on transactional motives.[8] Building on Kiyotaki and Wright (1993), the first search-theoretic two-country dual-currency framework was provided by Matsuyama et al. (1993), later on extended by Zhou (1997) to allow for currency exchange. The indivisibility of both goods and money, however, precluded a discussion of prices and exchange rates, motivating the work by Trejos and Wright (1995, 1996) and Camera and Winkler (2003), featuring divisible goods alongside indivisible fiat money. The first two-country dual-currency model with both divisible goods and fiat money was provided by Head and Shi (2003), extending the large household model by Shi (1999). Unlike the paper at hand, however, both currencies are perfectly recognizable and the determinacy of the exchange rate hinges on search frictions. The paper closest to mine, featuring perfectly-divisible currencies, is Zhang (2014). Building on Lester et al. (2012) to incorporate costly authentication, Zhang (2014) studies the strategic interaction between monetary authorities under currency substitution in line with Li and Matsui (2009) and Liu and Shi (2010). However, the framework exclusively features monetary policy and thus remains silent about the inter-temporal trade-offs between distortion-smoothing and time-consistency arising from fiscal considerations – a channel present in the paper at hand.

[7] If the government has commitment power, conditional on an initial debt choice, any debt level can be supported in the long run, yielding counterfactually smooth nominal interest rates and taxes. Removing the government's commitment power resolves said issue.

[8] Single country frameworks with competing media of exchange are vast. See Trejos and Wright (1996) and Craig and Waller (2000) for a survey of the earliest search-theoretic dual-currency frameworks. Outside of monetary search theory, the paper further embodies insights by Kindleberger (1967) and Krugman (1984). Dual-currency cash-in-advance models are surveyed in Obstfeld and Rogoff (1996). Transactional motives aside, recent studies analyzing financial liability dollarization to hedge exchange rate risks, distinguishing between firms and households, are provided by Bocola and Lorenzoni (2020) and Montamat (2020), respectively.

2 The Economy

2.1 Environment

Consider a two-country dual-currency variant of the monetary framework proposed by Lagos and Wright (2005). Time is discrete, continues forever, and each period is divided into two sequentially opening markets: a decentralized market (DM) and a centralized market (CM), as visualized in Figure 1. In each market, a perishable good is produced and consumed. While exchange in the CM occurs in a competitive Arrow-Debreu fashion, in the DM agents meet bilaterally at random and bargain over the terms of trade.

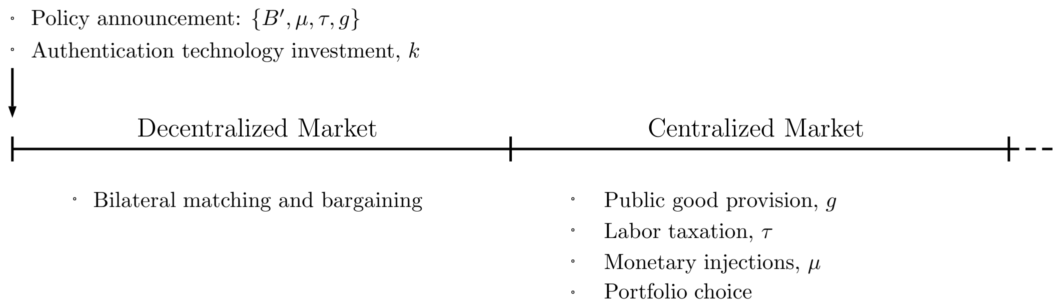

Figure 1: Timing of events in a representative period

There are two countries, domestic and foreign $j \in \{d, f\}$, each populated by a unit measure of infinitely-lived agents evenly divided between $n_j = 1/2$ consumers and $1 - n_j$ producers denoted by the superscripts $i \in \{c, p\}$. All agents are immobile, preventing them from trading in the other country's DM.[9] Exchange in the CM, on the other hand, is unrestricted.

In the DM, consumers and producers meet bilaterally at random with probability one. Consumers want to consume a local search good $x \in \mathbb{R}_+$ but cannot produce, while producers can produce but do not want to consume. The utility of consumption, $u(x)$, is strictly increasing and concave with $u_x > 0 > u_{xx}$, $u(0) = 0$, $u_x(0) = \infty$, and $u_x(\infty) = 0$. The DM cost function, $-x$, in turn, is linear, where efficiency occurs for $\hat{x} \in (0, \infty)$ solving $u_x(\hat{x}) = 1$. In the CM, all agents consume and produce a numéraire good $c \in \mathbb{R}_+$. The utility of consumption, $U(c)$, is strictly increasing and concave with $U_c > 0 > U_{cc}$, $U(0) = 0$, $U_c(0) = \infty$, and $U_c(\infty) = 0$. The production technology is linear in hours worked, where the disutility of labor is given by $-ah$ with h hours worked and $a > 0$. Let the socially-efficient consumption $\hat{c} \in (0, \infty)$ solve $U_c(\hat{c}) = a$.

Apart from consumers and producers, there is a benevolent government in each country supplying a local public good $g \in \mathbb{R}_+$ in the CM, where g is transformed one-to-one from the numéraire good. Agents draw utility $\eta v(g)$ of consuming the public good, where $v(g)$ is strictly increasing and concave with $v_g > 0 > v_{gg}$, $v(0) = 0$, $v_g(0) = \infty$, and $v_g(\infty) = 0$, and the parameter $\eta > 0$

[9] See e.g. Zhang (2014) for a discussion of an open-economy model with mobile consumers and immobile producers.

represents the country's citizens' preference for the public good. Let the first-best public good provision, $\hat{g} \in (0, \infty)$, be such that $\eta v_g(\hat{g}) = a$. The discount factor across periods is $\beta = (1+r)^{-1}$, where $r > 0$ is the rate of time preference, characterizing the consumers' and producers' expected lifetime utilities, $\mathbb{E}\sum_{t=0}^{\infty}\beta^t[u(x_t)+U(c_t)+\eta v(g_t)-ah_t]$ and $\mathbb{E}\sum_{t=0}^{\infty}\beta^t[-x_t+U(c_t)+\eta v(g_t)-ah_t]$, where preferences are additively separable for tractability.

To finance public expenditures g, a government may use three financing channels: (i) distortionary labor taxation τ in the CM, (ii) print perfectly divisible domestic currency at rate μ (seigniorage revenue), and (iii) issue perfectly divisible one-period nominal bonds, b, redeemable in the country's currency in period $t+1$. Policies are announced at the beginning of each period before the DM opens. A government can commit to policies within a period but not future policies. Furthermore, it remains inactive in the DM and only actively participates in the CM. Going forward, we consider a Markov-perfect equilibrium in which policies are a function of fundamentals only - see e.g. Maskin and Tirole (2001). In doing so, the monetary authority accommodates the decisions of the fiscal authority, allowing us to look at a consolidated budget constraint for each country, $j \in \{d, f\}$:

$$\phi_j(1+B_j) + g_j = \tau_j h_j + \phi_j(1+\mu_j)(1+q_j B_j'). \tag{1}$$

All nominal variables – except for bond prices – are normalized by the aggregate currency stock, A_{m_j}. Thus, today's aggregate currency supply is equal to $A_{m_j}/A_{m_j} = 1$, and tomorrow's is $A'_{m_j}/A_{m_j} = 1+\mu_j$, where 'primes' denote variables evaluated in the following period. Today's aggregate bonds-to-money ratio is $B_j = A_{b_j}/A_{m_j}$ with A_{b_j} denoting today's aggregate bond supply, q_j is the issuance price for one nominal bond that pays out one unit of the country's currency to the bearer in the subsequent CM, and $1/\phi_j$ is the normalized price of the numéraire good. Lastly, the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, is $e \equiv \phi_f/\phi_d$.[10] The above budget constraint restricts a government's policy choices to $(B_j', \mu_j, \tau_j, g_j)$, taking as given the current bonds-to-money ratio, B_j, inherited from its past self, and the assumption that agents respond optimally to current and (anticipated) future policy, as reflected in prices and the labor supply. Lastly, note that the stock of bonds and currency does not need to grow at the same rate, i.e., the bonds-to-money ratio does not need to be constant over time and a government is free to choose it every period, provided it satisfies (1) in a monetary equilibrium.

Agents are anonymous, lack commitment, and there is no enforcement technology in place. Further, bilateral matches are dissolved at the end of each DM, precluding credit arrangements and asking for an instantaneous settlement. Hence, for gains from trade to be realized in the DM, a medium of exchange is essential.[11] Since all goods are perishable and governments do not participate in the DM (and thus cannot provide verification or intermediation services), the only

[10] Note that since market clearing in the CM implies that the law of one price holds, consumers and producers can trade currencies at the market-clearing exchange rate allowing the CM to function as a competitive foreign exchange market.

[11] An in-depth discussion is provided by Kocherlakota (1998), Wallace (2001), and Shi (2006).

available forms of money are domestic and foreign currency.[12] While domestic currency is accepted with probability one by domestic producers, the acceptance of foreign currency is conditional on a domestic producer's ex-ante investment in an authentication technology at a flow cost $k \geq 0$. At the beginning of the DM, before matches are formed, each producer draws an idiosyncratic cost k, where k is distributed according to the density function $f(k)$ over the interval $k \in [\underline{k}, \bar{k}]$ with $f(k) \geq 0$ and $\int_{\underline{k}}^{\bar{k}} f(k)dk = 1$.[13] Let $\alpha \in [0,1]$ denote the share of a country's producers accepting both domestic and foreign currency, and $1-\alpha$ the share of a country's producers accepting domestic currency exclusively.[14]

2.2 Value Functions

Domestic and foreign agents enter the CM with normalized nominal wealth $\omega \equiv \phi_d(m_d + b_d) + \phi_f(m_f + b_f)$ and $\breve{\omega} \equiv \phi_d(\breve{m}_d + \breve{b}_d) + \phi_f(\breve{m}_f + \breve{b}_f)$, respectively, consisting of individual money and bond balances, (m_j, b_j) and $(\breve{m}_j, \breve{b}_j)$. The CM and DM value functions of domestic (foreign) agents are denoted by $W_d^i(\omega)$ and $V_d^i(\omega)$ ($W_f^i(\breve{\omega})$ and $V_f^i(\breve{\omega})$), respectively. Going forward, all value functions are presented for the domestic agents only but can be mirrored to study the decisions of foreign agents.

A domestic agent, $i \in \{c, p\}$, entering the CM with normalized nominal wealth ω solves:

$$W_d^i(\omega) = \max_{c_d, h_d, m'_j, b'_j} U(c_d) + \eta_d v(g_d) - a_d h_d + \beta V_d^i(m'_d, m'_f, b'_d, b'_f), \tag{2}$$

$$\text{s.t. } (1-\tau_d)h_d + \omega = c_d + \phi_d(1+\mu_d)(m'_d + q_d b'_d) + \phi_f(1+\mu_f)(m'_f + q_f b'_f), \tag{3}$$

where:

$$V_d^c(m_d, m_f, b_d, b_f) = \alpha_d\big[u(x_{d,2}) + W_d^c(\omega - d_d - d_f)\big] + (1-\alpha_d)\big[u(x_{d,1}) + W_d^c(\omega - d_d)\big], \tag{4}$$

represents the consumers' continuation value of entering the DM with a *normalized* portfolio (m_d, m_f, b_d, b_f), $x_{d,1}(d_d)$ the quantity of domestic search goods financed with domestic currency only, $x_{d,2}(d_d, d_f)$ the quantity of domestic search goods financed with both domestic and foreign currency, and $d_j \in [0, m_j]$ the amount of currency transferred. Agents choose their labor supply, consumption, and portfolio to maximize their discounted lifetime utility subject to the budget constraint (3). Note that the composition of the nominal wealth when entering the CM is irrelevant since bonds are redeemed for fiat money at par. The composition of the nominal wealth carried

[12] Since bonds are book entries only, bonds cannot be used as a medium of exchange without verification. See Aruoba and Chugh (2010), Berentsen and Waller (2011), and Martin (2011) for that matter.

[13] Aside from authentication, there is a variety of costs associated with accepting new media of exchange (see Lotz and Rocheteau (2002) for an overview). Examples include time spent learning how to use the new means of payment, keeping up with exchange rates, and the costs of installing and maintaining new technologies. Heterogeneity of these costs across producers can occur for different degrees of sophistication.

[14] Once a consumer and a producer are matched, it is common knowledge whether the producer invested, and thus foreign currency is rejected unless the authentication cost was incurred. This formalization is in line with the work by Chang (1994), Engineer (2000), Martin (2006), Lester et al. (2012), and Zhang (2014).

into the DM, on the other hand, matters since only domestic currency, m_d, and, conditional on the producer's investment, foreign currency, m_f, can be used to settle transactions. Since $W_d^c(\omega)$ is linear in current wealth, i.e., $W^c_{d,m_j} = W^c_{d,b_j} = a_d\phi_j(1-\tau_d)^{-1}$, a consumer's future portfolio, (m'_d, m'_f, b'_d, b'_f), is independent of current financial wealth, ω. Substituting (3) and (4) into (2) and updating reduces the consumer's portfolio choice to:

$$\begin{aligned}\max_{m'_d,m'_f,b'_d,b'_f} \quad & - \quad a_d(1-\tau_d)^{-1}\sum_{j=d,f}\left[m'_j[\phi_j(1+\mu_j)/\beta-\phi'_j]+b'_j[\phi_j q_j(1+\mu_j)/\beta-\phi'_j]\right] \\ & + \quad \alpha'_d\Delta^c_{d,2}(m'_d,m'_f)+(1-\alpha'_d)\Delta^c_{d,1}(m'_d),\end{aligned} \tag{5}$$

where $m'_j[\phi_j(1+\mu_j)/\beta-\phi'_j]$ represents the opportunity cost of carrying currency across a period, $b'_j[\phi_j q_j(1+\mu_j)/\beta-\phi'_j]$ the cost of holding nominal bonds, and $\Delta^c_{d,1}(m'_d)$ ($\Delta^c_{d,2}(m'_d,m'_f)$) the consumer's surplus in a single (dual) currency match determined via bilateral bargaining in Section 2.3. Note from (5) that all consumers in a country leave the CM with the same money and bond balances, (m_d, m_f, b_d, b_f).[15] Furthermore, if $V^c_{d,b'_j} < V^c_{d,m'_j}$, then bonds are only held if agents are compensated for their illiquidity, i.e., if $q_j < 1$.[16] Lastly, since producers have no use for currency in the DM, it is evident from (5) that positive balances are only held if the rate of return on currency and bonds is positive, i.e., if $\phi_j(1+\mu_j)/\beta < \phi'_j$ and $\phi_j q_j(1+\mu_j)/\beta < \phi'_j$.

2.3 Bargaining Game

For producers to invest in the costly authentication technology, the pricing mechanism in the DM must allow them to extract a fraction of the match surplus. The terms of trade in the domestic and the foreign DM, $(x_{d,1}, x_{d,2}, d_d, d_f)$ and $(x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f)$, are determined according to Kalai's (1977) proportional bargaining rule and depend on the consumers' currency balances, (m_d, m_f) and $(\breve{m}_d, \breve{m}_f)$, and the producers' acceptance strategies. Given the linearity of the CM value function (2), the bargaining problem in the domestic DM solves:

$$(x_{d,1},x_{d,2},d_d,d_f)\in\arg\max_{x_{d,1},x_{d,2},d_d,d_f}\mathbb{I}_d u(x_{d,2})+(1-\mathbb{I}_d)u(x_{d,1})-\frac{a_d}{(1-\tau_d)}[\phi_d d_d+\mathbb{I}_d\phi_f d_f], \tag{6}$$

$$\begin{aligned}\text{s.t. } (1-\theta)&\left[\mathbb{I}_d u(x_{d,2})+(1-\mathbb{I}_d)u(x_{d,1})-\frac{a_d}{(1-\tau_d)}[\phi_d d_d+\mathbb{I}_d\phi_f d_f]\right] \\ &=\theta\left[-\mathbb{I}_d x_{d,2}-(1-\mathbb{I}_d)x_{d,1}+\frac{a_d}{(1-\tau_d)}[\phi_d d_d+\mathbb{I}_d\phi_f d_f]\right],\end{aligned} \tag{7}$$

$$\text{s.t. } d_d\in[0,m_d], d_f\in[0,m_f], \tag{8}$$

[15] If the domestic and the foreign currency end up being perfect substitutes, consumers in a country could strictly speaking hold different portfolios, but they will have the same real value.

[16] The marginal values of entering the DM with money and bonds, $\beta V^c_{d,m'_j} = a_d(1+\mu_j)\phi_j(1-\tau_d)^{-1}$ and $\beta V^c_{d,b'_j} = a_d(1+\mu_j)\phi_j q_j(1-\tau_d)^{-1}$, follow from the first-order conditions of (2), where $V^c_{d,b'_j} < V^c_{d,m'_j}$ if and only if $q_j < 1$.

where $\theta \in (0,1)$ denotes the consumer's bargaining power, $(x_{d,1}, x_{d,2})$ the number of consumption goods acquired, (d_d, d_f) the amount of currency transferred, and $\mathbb{I}_d \in \{0,1\}$ is an indicator function equal to one if the domestic consumer is in a dual currency match and zero otherwise. Eq. (7) ensures that the total surplus is split proportionally between the consumer and the producer, and (8) represents the domestic consumer's feasibility constraints. Solving (7) for $a_d(1-\tau_d)^{-1}[\phi_d d_d + \mathbb{I}_d \phi_f d_f]$ and plugging into (6) and (8) reduces the bargaining problem in the domestic DM to:

$$(x_{d,1}, x_{d,2}) \in \arg\max_{x_{d,1}, x_{d,2}} \ \mathbb{I}_d \theta[u(x_{d,2}) - x_{d,2}] + (1-\mathbb{I}_d)\theta[u(x_{d,1}) - x_{d,1}], \tag{9}$$

$$\text{s.t. } \mathbb{I}_d \Phi(x_{d,2}) + (1-\mathbb{I}_d)\Phi(x_{d,1}) \leq a_d(1-\tau_d)^{-1}[\phi_d m_d + \mathbb{I}_d \phi_f m_f], \tag{10}$$

with $\Phi(x) \equiv (1-\theta)u(x) + \theta x$. The terms of trade in the foreign DM, $(x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f)$, arise from an analogous problem solving:

$$(x_{f,1}, x_{f,2}) \in \arg\max_{x_{f,1}, x_{f,2}} \ \mathbb{I}_f \theta[u(x_{f,2}) - x_{f,2}] + (1-\mathbb{I}_f)\theta[u(x_{f,1}) - x_{f,1}], \tag{11}$$

$$\text{s.t. } \mathbb{I}_f \Phi(x_{f,2}) + (1-\mathbb{I}_f)\Phi(x_{f,1}) \leq a_f(1-\tau_f)^{-1}[\phi_f \breve{m}_f + \mathbb{I}_f \phi_d \breve{m}_d]. \tag{12}$$

Definition 1. *An equilibrium of the bargaining game in the domestic and the foreign DM is a set of strategies, $(x_{d,1}, x_{d,2}, d_d, d_f)$ and $(x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f)$, such that the terms of trade, $(x_{d,1}, x_{d,2}, d_d, d_f)$ and $(x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f)$, are a solution to the bargaining problems (9)-(10) and (11)-(12), respectively.*

Let us distinguish between single currency matches, $\mathbb{I}_j = 0$, and dual currency matches, $\mathbb{I}_j = 1$, starting with $\mathbb{I}_j = 0$. If (10) and (12) do not bind, then gains from trade are maximized and consumption is at the first-best level, $x_{d,1} = x_{f,1} = \hat{x}$, suggesting $a_d(1-\tau_d)^{-1}\phi_d d_d = a_f(1-\tau_f)^{-1}\phi_f \breve{d}_f = (1-\theta)u(\hat{x}) + \theta\hat{x}$. If (10) and (12) bind, then $x_{d,1} < \hat{x}$ and $x_{f,1} < \hat{x}$ solve $\Phi(x_{d,1}) = a_d(1-\tau_d)^{-1}\phi_d m_d$ and $\Phi(x_{f,1}) = a_f(1-\tau_f)^{-1}\phi_f \breve{m}_f$, respectively. Vice versa, if the consumer is in a dual currency match, i.e., if $\mathbb{I}_j = 1$, then $x_{d,2} = x_{f,2} = \hat{x}$ solve $a_d(1-\tau_d)^{-1}[\phi_d d_d + \phi_f d_f] = a_f(1-\tau_f)^{-1}[\phi_d \breve{d}_d + \phi_f \breve{d}_f] = (1-\theta)u(\hat{x}) + \theta\hat{x}$ if (10) and (12) do not bind, and $\Phi(x_{d,2}) = a_d(1-\tau_d)^{-1}[\phi_d m_d + \phi_f m_f]$ and $\Phi(x_{f,2}) = a_f(1-\tau_f)^{-1}[\phi_d \breve{m}_d + \phi_f \breve{m}_f]$ with $x_{d,2} < \hat{x}$ and $x_{f,2} < \hat{x}$ otherwise.

2.4 Foreign Currency Acceptance Decision

Having determined the agents' portfolio choice in the CM and the terms of trade in the DM, let us now turn to the producers' decision to invest in the authentication technology and accept foreign currency. Given the consumers' portfolio choice, (5), consider the situation in which all domestic consumers enter the DM with the same normalized portfolio (m_d, m_f, b_d, b_f). A producer invests if:

$$\Pi_d(\alpha_d) \equiv (1-\theta)\big[u(x_{d,2}(\alpha_d)) - x_{d,2}(\alpha_d) - [u(x_{d,1}(\alpha_d)) - x_{d,1}(\alpha_d)]\big] \geq k_d, \tag{13}$$

i.e., if the producer's surplus of accepting foreign currency, $\Pi_d(\alpha_d)$, exceeds the investment cost, k_d. Since the DM allocations, $(x_{d,1}, x_{d,2})$, solve the consumer's portfolio choice in (5) for a given α_d, both $x_{d,1}(\alpha_d)$ and $x_{d,2}(\alpha_d)$ are a function of the producers' acceptance strategies. For a given $(x_{d,1}, x_{d,2})$, contrarily, the share of producers accepting foreign currency, α_d, satisfies one of the following configurations:

$$\alpha_d \begin{cases} = 0 & \text{if} \quad \Pi_d < \underline{k} \\ \in (0,1) & \text{if} \quad \Pi_d \in [\underline{k}, \bar{k}) \\ = 1 & \text{if} \quad \Pi_d \geq \bar{k}, \end{cases} \tag{14}$$

where a producer invests with an arbitrary probability $\Lambda \in [0,1]$ under indifference, i.e., if k_d satisfies $\Pi_d(\alpha_d) = k_d$.[17] In words, if the cost of the investment exceeds the benefits for all producers, no producer invests, and thus foreign currency is not accepted domestically. Vice versa, if the producers' benefit exceeds the cost for all producers, then all producers invest and foreign currency is accepted with probability one. Note that if $\alpha_d = 1$ and $x_{f,1} > x_{d,1}$, then the demand for domestic currency is zero, and thus the domestic currency ceases to be valued in equilibrium. An interior solution with $\alpha_d \in (0,1)$, i.e., an equilibrium in which both currencies are valued, is guaranteed for an intermediate range where producers drawing a low cost invest and producers drawing a high cost do not, providing a natural anchor for the domestic currency.[18]

Given the properties of α_d, multiple equilibria can exist due to a strategic complementarity between the producers' investment decision and the consumers' demand for currency, reflected in the prices ϕ_d and ϕ_f. Intuitively, if α_d is low, then the foreign currency is fairly illiquid in the domestic DM, and thus its price ϕ_f is (relatively) low due to the low domestic demand. As a result, few producers find it worthwhile to invest. As α_d increases, the foreign currency becomes more liquid domestically, causing consumers to reallocate their portfolios towards the foreign currency in the CM. As a result, ϕ_f (ϕ_d) increases (decreases), increasing $\Pi_d(\alpha_d)$. Therefore, an increase in α_d increases the producers' incentive to invest, i.e., $\partial \Pi_d / \partial \alpha_d > 0$.

It is important to point out that theoretically one could envision an environment with a myriad of foreign currencies, m_j. In equilibrium, assuming each currency needs to be authenticated separately, a producer would choose to authenticate the pair, (m_j, k_j), yielding the highest gains from trade. For that reason, this paper abstains from explicitly modeling a vector of alternative media of exchange, and restricts attention to a dual currency system instead.[19]

[17] Note that for $\theta = 1$, i.e., if the terms of trade in the DM are characterized by consumers' take-it-or-leave-it offers, then, unless $\bar{k} = 0$, foreign currency will never be accepted in equilibrium.

[18] The distribution of costs, $f(k)$, with $k \in [\underline{k}, \bar{k}]$ guarantees the stability of the interior solution, $\alpha \in (0,1)$. If the cost was degenerate, i.e., if $k = \kappa$ for all producers in a country, then $\alpha \in (0,1)$ iff $\Pi = \kappa$, and thus for an arbitrarily small deviation $\kappa + \epsilon$, $\alpha = 1$ ($\alpha = 0$) if $\epsilon < 0$ ($\epsilon > 0$). This is consistent with information costs à la Lester et al. (2012), the Zhu and Wallace (2007) trading mechanism, and Aiyagari et al. (1996) government agents. The latter, i.e., government agents prohibited from accepting foreign currency in transactions, can be formalized with a positive share of producers consistently drawing a large k_d satisfying $k_d > \Pi_d$.

[19] Arguably, in the presence of unexpected shocks and policy changes, there could arise an insurance motive to authenticate multiple currencies. Albeit an important question, this paper abstains from such shocks and thus leaves the analysis incorporating a vector of currencies for future research.

Lastly, note that by assuming a flow cost, dollarization is reversible. In contrast, if k_d constitutes a one-time payment in the first period, then, provided $\Pi_d \geq \underline{k}$ in $t = 0$, the domestic economy experiences *hysteresis* and remains dollarized with $\alpha_d \in (0, 1]$ in all future periods, even if $\Pi_d < \underline{k}$ in $t = 1, ..., \infty$. This alternative formalization confirms the persistence of dollarization presented by Uribe (1997) in a cash-in-advance model with increasing returns to scale, as well as the stylized model by Guidotti and Rodriguez (1992).

2.5 Monetary Equilibrium

In foresight of the Markov-perfect equilibrium, let us determine the CM prices of the domestic and the foreign currency, ϕ_d and ϕ_f, the growth rates of the currency supply, μ_d and μ_f, the bond prices, q_d and q_f, and the labor tax rates, τ_d and τ_f. Definition 2 defines the monetary equilibrium. Let (m_d, m_f, b_d, b_f) and $(\breve{m}_d, \breve{m}_f, \breve{b}_d, \breve{b}_f)$ denote a domestic and a foreign consumer's portfolio, respectively, $(x_{d,1}, x_{d,2}, d_d, d_f)$ and $(x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f)$ their corresponding terms of trade in the DM, $\{\alpha_d, \alpha_f\}$ the domestic and foreign producers' acceptance strategies, and $n_d m_j + n_f \breve{m}_j = 1$ and $n_d b_j + n_f \breve{b}_j = B_j$ the normalized market clearing conditions ensuring that the global supply of currency and bonds equals the global demand.

Definition 2. *A monetary equilibrium is a list of portfolios* $\{(m_d, m_f, b_d, b_f), (\breve{m}_d, \breve{m}_f, \breve{b}_d, \breve{b}_f)\}$, *quantities traded* $\{(x_{d,1}, x_{d,2}, d_d, d_f), (x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f)\}$, *producers' strategies* $\{\alpha_d, \alpha_f\}$, *and market clearing conditions such that:*

(i) $\{(x_{d,1}, x_{d,2}, d_d, d_f), (x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f)\}$ *solve the bargaining problems in* (9)-(10) *and* (11)-(12);

(ii) $\{\alpha_d, \alpha_f\}$ *solves the producers' acceptance decision in* (13) *satisfying* (14);

(iii) $\{(m_d, m_f, b_d, b_f), (\breve{m}_d, \breve{m}_f, \breve{b}_d, \breve{b}_f)\}$ *solves the consumers' portfolio choice in* (5); *and*

(iv) market clearing satisfies $n_d m_j + n_f \breve{m}_j = A_{m_j}/A_{m_j} = 1$ *and* $n_d b_j + n_f \breve{b}_j = A_{b_j}/A_{m_j} = B_j$.

Lemma 1. *For* $j = \{d, f\}$ *a monetary equilibrium is spanned by a triple* $\{\Phi(x_{j,1}),\ \Phi(x_{j,2}),\ \Omega(x_{j,1}, x_{j,2})\}$ *in* (15)-(20) *with the equilibrium prices and policy variables:*

$$\tau_j = 1 - \frac{a_j}{U_c(c_j)}, \tag{15}$$

$$\phi_j = n_j \frac{\Phi(x_{j,1})}{U_c(c_j)} + n_{-j} \frac{\Phi(x_{-j,2}) - \Phi(x_{-j,1})}{U_c(c_{-j})}, \tag{16}$$

$$\mu_j = \beta \frac{\phi_j'}{\phi_j} \left[1 + \Omega(x_{j,1}', x_{j,2}')\right] - 1, \tag{17}$$

$$q_j = \left[1 + \Omega(x_{j,1}', x_{j,2}')\right]^{-1}, \tag{18}$$

where:

$$\Phi \equiv (1-\theta)u(x_j) + \theta x_j, \tag{19}$$

and:

$$\Omega \equiv \theta\left[\frac{\alpha_j[u_x(x_{j,2})-1]}{(1-\theta)u_x(x_{j,2})+\theta} + \frac{(1-\alpha_j)[u_x(x_{j,1})-1]}{(1-\theta)u_x(x_{j,1})+\theta}\right], \tag{20}$$

are both continuously differentiable.

Lemma 1 summarizes the equilibrium prices and policy variables. The proof is delegated to Appendix A. Starting with the labor tax rates, τ_d and τ_f, (15) characterizes the negative relationship between taxation and consumption of the numéraire good in the CM. Eq. (16) captures the positive relationship between the equilibrium price of the domestic (foreign) currency, ϕ_d (ϕ_f), and the global demand. Moving on to the growth rate of the currency supply in (17) and the equilibrium bond prices in (18), μ_j and q_j depend on the liquidity premium, $\Omega \geq 0$. A few observations distinguishing between different currency regimes are in order. Precisely, we distinguish between (i) local circulation, (ii) international circulation of one currency, and (iii) international circulation of both currencies.

Let us start with case (i) in which local producers only accept local currency, i.e., $(\alpha_d, \alpha_f) = (0, 0)$. In said regime, the domestic and foreign DM allocations are fully independent, and thus local government policy does not affect foreign allocations and vice versa. Since local consumers have no demand for the other country's currency, i.e., $m_f = \breve{m}_d = 0$, market clearing satisfies $n_d m_d = 1$ and $n_f \breve{m}_f = 1$. Currency is costless to carry across periods, i.e., $\mu_j = \beta - 1$ (the Friedman rule), if the local economy is in a first-best steady state, i.e., if $x_{j,1} = x'_{j,1} = \hat{x}$ suggesting $V^c_{j,m_j} = W^c_{j,m_j}$. Furthermore, $V^c_{j,m'_j} = V^c_{j,b'_j}$ suggests $q_j = 1$ since currency does not incorporate a liquidity premium, and thus consumers do not need to be compensated for holding illiquid bonds. If the nominal interest rate is positive, i.e., if $\mu_j > \beta - 1$, on the other hand, then the consumers' feasibility constraint binds, suggesting a positive liquidity premium $\Omega(x'_{j,1}) > 0$, and thus $V^c_{j,m_j} > W^c_{j,m_j}$ given $x'_{j,1} < \hat{x}$. Consequentially, $q_j < 1$.[20] Note that given (13) the regime with $(\alpha_d, \alpha_f) = (0, 0)$ can exist even if $\mu_d \neq \mu_f$ as long as $\Pi_j < \underline{k}$, i.e., as long as the authentication cost exceeds the benefit of accepting the other country's currency for all producers.

Consider now case (ii) in which one currency circulates internationally while the other currency only circulates locally, i.e., either $(\alpha_d, \alpha_f) = ((0, 1], 0)$ or $(\alpha_d, \alpha_f) = (0, (0, 1])$. Focusing on the former, if foreign currency is accepted domestically, then domestic consumers hold portfolios consisting of both currencies while foreign consumers only hold foreign currency, yielding the market clearing conditions $n_d m_d = 1$ and $n_d m_f + n_f \breve{m}_f = 1$.[21] Existence of $(\alpha_d, \alpha_f) = ((0, 1], 0)$ requires $\Pi_d \geq \underline{k}$ and $\Pi_f < \underline{k}$. Given the domestic demand for foreign currency, domestic and

[20] Since the liquidity premium $\Omega \geq 0$, it holds that q_j cannot exceed one. Hence, both domestic and foreign nominal interest rates are guaranteed to be non-negative in equilibrium.

[21] The latter case with $(\alpha_d, \alpha_f) = (0, (0, 1])$ has similar properties and therefore I abstain from a separate discussion.

	$\partial\phi_d/\partial\cdot$	$\partial\phi_f/\partial\cdot$	$\partial e/\partial\cdot$	$\partial q_d/\partial\cdot$	$\partial q_f/\partial\cdot$
μ_d	-	+	+	-	+
τ_d	-	-	+	-	-

Table 1: Steady-state comparative statics for $\alpha_d \in (0,1)$ and $\alpha_f = 0$ with $x_{j,1} \in (0,\hat{x})$ and $x_{j,2} \in (0,\hat{x})$.

foreign prices become intertwined yielding important comparative statics, summarized in Table 1. The derivation is delegated to Appendix B. Given (15)-(18), the policy variables are restricted to the growth rate of the money supply and the labor tax rate. To determine the comparative statics with respect to public debt, B_d, one needs to understand the effects of inter-temporal distortion-smoothing on domestic and foreign allocations. The government's problem in Section 3 identifies the relevant trade-offs with the steady-state allocations characterized in Section 4.

First and foremost, an increase in μ_d depreciates the value of the domestic currency, ϕ_d. Due to general equilibrium effects, the demand for domestic currency falls and domestic consumers substitute into foreign currency, causing its price, ϕ_f, and the exchange rate, $e \equiv \phi_f/\phi_d$, to increase. Along the same lines, the price of domestic (foreign) bonds, q_d (q_f), decreases (increases) following an increase in μ_d. Turning to the effects of taxation, an increase in the domestic labor tax rate reduces the domestic consumers' demand for currency, and thus the prices ϕ_d and ϕ_f. Since domestic currency is only held locally while foreign currency is held globally, the reduction in the price of the domestic currency exceeds the reduction in the price of the foreign currency, causing the exchange rate to increase.

Let us now turn to case (iii) with two international currencies, i.e., $(\alpha_d, \alpha_f) = ((0,1],(0,1])$. Existence of this case requires $\Pi_j \geq \underline{k}$ in both countries, and thus $\underline{k} = 0$. Hence, unless authentication is free for some producers in both countries, there cannot exist an equilibrium with two international currencies. Note that if $\bar{k} = 0$ in both countries, i.e., if authentication is free for all producers, then $(\alpha_d, \alpha_f) = (1,1)$ is unique, and thus domestic and foreign currency are perfect substitutes. The latter case yields exchange-rate indeterminacy, as discussed by Kareken and Wallace (1981) in an overlapping generations model. Intuitively, suppose $\mu_d = \mu_f = \mu$ in the steady state such that $x_{j,2}$ solves $\mu = \beta[1+\Omega] - 1$ with $\Omega = \theta[u_x(x_{j,2}) - 1][(1-\theta)u_x(x_{j,2}) + \theta]^{-1}$. In said scenario, albeit domestic and foreign consumers may hold different portfolios, (m_d, m_f) and $(\breve{m}_d, \breve{m}_f)$, satisfying the market clearing conditions (iv) in Definition 2, consumption $(x_{d,2}, x_{f,2})$ is uniquely pinned down. As a result, there exist more unknowns, (ϕ_d, ϕ_f, e), than equations, rendering the exchange rate indeterminate.[22]

[22]Aside dual-currency systems, there is a large body of work assessing the exchange rate indeterminacy addressed by Kareken and Wallace (1981). While Aiyagari and Wallace (1997), Li and Wright (1998), Curtis and Waller (2000), Li (2002), Lotz and Rocheteau (2002), Waller and Curtis (2003), and Lotz (2004) rely on legal restrictions, Zhu and Wallace (2007) break the exchange-rate indeterminacy in an environment with co-existing fiat money and higher-return assets. For a model with counterfeitable currencies see Gomis-Porqueras et al. (2017).

3 Government Policy

Having determined the monetary equilibrium, we now proceed to study optimal fiscal and monetary policy. Recall that a government cannot commit to future policy choices and announces current policy, $\{B'_j, \mu_j, \tau_j, g_j\}$, at the beginning of each period after observing the current domestic and foreign bonds-to-money ratios, B_d and B_f. Following Martin (2013), I use the notion of the Markov-perfect equilibrium, where policy functions depend on fundamentals only. Using the primal approach all prices and policy variables (except for debt) are replaced with first-order conditions, reducing the government's choice variables to future debt and current allocations, $\{B'_j, x_{j,1}, c_j, g_j\}$. Since $k_j \in [\underline{k}, \bar{k}]$ with $\underline{k} > 0$ in both countries implies either $(\alpha_d, \alpha_f) = ([0,1], 0)$ or $(\alpha_d, \alpha_f) = (0, [0,1])$, the main analysis focuses on the problem of the domestic government with $\alpha_d \in [0,1]$ and $\alpha_f = 0$. Implications for the foreign government are introduced when warranted.

3.1 Government Budget Constraint

Plugging the aggregate resource constraint, $c_d + g_d = h_d$, and prices and policy variables, τ_d, ϕ_d, μ_d, and q_d with $x_{f,2} = x_{f,1}$ (given $\alpha_f = 0$) in (15)-(18) into (1) yields the updated budget constraint:[23]

$$
\begin{aligned}
\varepsilon(B_d, B'_d, x_{d,1}, x'_{d,1}, x'_{d,2}, c_d, g_d) \;&\equiv\; [U_c(c_d) - a_d]c_d - a_d g_d - n_d \Phi(x_{d,1})[1 + B_d] \\
&+\; \beta n_d \Phi(x'_{d,1})[1 + \Omega(x'_{d,1}, x'_{d,2}) + B'_d] = 0,
\end{aligned} \tag{21}
$$

with $\Phi(x_{d,1})$ and $\Omega(x_{d,1}, x_{d,2})$ in (19) and (20), where $x'_{d,1}$ and $x'_{d,2}$ represent the DM good allocations the current government anticipates the future government and citizens will implement as a function of the inherited debt, B'_d and B'_f. Due to limited commitment, the government today takes these allocations as given.

To examine the effects of changes in the allocations on the government's budget constraint it is helpful to analyze the partial derivatives of (21): $\varepsilon_{B_d} = -n_d \Phi(x_{d,1})$, $\varepsilon_{B'_d} = \beta n_d \Phi(x'_{d,1})$, $\varepsilon_{x_{d,1}} = -n_d \Phi_x(x_{d,1})[1 + B_d]$, $\varepsilon_{x'_{d,1}} = \beta n_d [\Phi_x(x'_{d,1})[1 + \Omega(x'_{d,1}, x'_{d,2}) + B'_d] + \Phi(x'_{d,1}) \Omega_{x'_{d,1}}(x'_{d,1}, x'_{d,2})]$, $\varepsilon_{x'_{d,2}} = \beta n_d \Phi(x'_{d,1}) \Omega_{x'_{d,2}}(x'_{d,1}, x'_{d,2})$, $\varepsilon_{c_d} = U_c(c_d) - a_d + U_{cc}(c_d) c_d$, and $\varepsilon_{g_d} = -a_d$.

Considering first $\varepsilon_{B'_d} > 0$, an increase in tomorrow's debt level B'_d reduces the need for taxation, and thus relaxes the government's budget constraint today. The additional financial burden resulting from an increase in B'_d, however, tightens tomorrow's budget constraint, i.e., $\varepsilon'_{B_d} < 0$. Furthermore, it affects tomorrow's monetary policy, as reflected in $\varepsilon_{x'_{d,1}}$, and thus, given (17) and (18), today's demand for money and bonds, i.e., today's budget constraint. Whether the domestic government has an incentive to increase or decrease future debt will depend on the sign of $\varepsilon_{x'_{d,1}}$, and thus, implicitly, the domestic producers' acceptance strategies. In that context note that given the partial derivative $\Omega_{x'_{d,2}} \leq 0$, a decrease in future foreign inflation tightens the domestic

[23] While individual labor effort varies with the agent's role in DM (consumer or producer), private and public consumption, c_d and g_d, is the same for all agents, yielding the aggregate CM resource constraint.

government's budget constraint today, $\varepsilon_{x'_{d,2}} \leq 0$, due to the reduced demand for domestic currency.

The partial derivative $\varepsilon_{x_{d,1}}$ captures the effect of the current domestic debt level on current domestic monetary policy. If net nominal liabilities are positive, i.e., if $1 + B_d > 0$, then $\varepsilon_{x_{d,1}} < 0$, and thus the government has an incentive to reduce $x_{d,1}$ (i.e., increase the growth rate of the money supply μ_d) to reduce the real value of its financial burden, and thus relax the budget constraint. If $1 + B_d < 0$, on the other hand, then an increase in inflation tightens the budget constraint.

Lastly, note from $\varepsilon_{g_d} < 0$ that an increase in public expenditures tightens the budget constraint while, given $\varepsilon_{c_d} < 0$ in equilibrium, an increase in labor taxation relaxes the budget constraint.[24]

3.2 The Problem of the Government

The current domestic government chooses $\{B'_d, x_{d,1}, c_g, g_d\}$ to maximize the domestic consumers' and producers' joint period utility, $\mathcal{Z}_d(B_d, B_f) = n_d V_d^c + (1 - n_d)V_d^p$, taking as given the current domestic and foreign debt level, B_d and B_f, the agents' best response to the policy announcement, $\{(x_{d,1}, x_{d,2}, d_d, d_f), (x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f), (\alpha_d, \alpha_f), (m_d, m_f, b_d, b_f), (\breve{m}_d, \breve{m}_f, \breve{b}_d, \breve{b}_f)\}$, and the anticipation that future agents implement the future allocations $x'_{d,1} \equiv \mathcal{X}'_{d,1}(B'_d, B'_f)$ and $x'_{d,2} \equiv \mathcal{X}'_{d,2}(B'_d, B'_f)$ as a function of the debts, B'_d and B'_f, chosen today. Let $\Gamma \in [-1, \bar{B}]$ be the set of possible debt levels, where $\bar{B}$ is large enough such that neither the upper nor the lower bound of Γ constrain government behavior. The latter is shown in Proposition 1.[25] The problem of the domestic government is:

$$\begin{aligned} \max_{B'_d, x_{d,1}, c_d, g_d} \mathcal{Z}_d(B_d, B_f) &= \alpha_d[n_d u(x_{d,2}) - (1 - n_d)x_{d,2}] + (1 - \alpha_d)[n_d u(x_{d,1}) - (1 - n_d)x_{d,1}] \\ &+ U(c_d) + \eta_d v(g_d) - a_d(c_d + g_d) - \alpha_d(1 - n_d)\mathbb{E}(k_d \mid k_d \leq \Pi_d(\alpha_d)) \\ &+ \beta \mathcal{Z}_d(B'_d, B'_f), \end{aligned} \tag{22}$$

$$\text{s.t. } \varepsilon(B_d, B'_d, x_{d,1}, \mathcal{X}'_{d,1}(B'_d, B'_f), \mathcal{X}'_{d,2}(B'_d, B'_f), c_d, g_d) = 0, \tag{23}$$

$$\text{s.t. } n_d \Phi(x_{d,1})\Omega(x_{d,1}, x_{d,2}) \geq 0, \tag{24}$$

where $\mathbb{E}(k_d \mid k_d \leq \Pi_d(\alpha_d)) \equiv \int_{\underline{k}}^{\Pi_d(\alpha_d)} k_d f(k_d) dk_d / \int_{\underline{k}}^{\Pi_d(\alpha_d)} f(k_d) dk_d$ denotes the expected cost of the producers investing into the authentication technology, (23) satisfies the government's period budget constraint in (21), and (24) represents the feasibility constraint $V^c_{d,m_d} - W^c_{d,m_d} \geq 0$, implying $x_{d,1} \leq \hat{x}$ (i.e., ruling out over-consumption in the DM) with $\Phi(x_{d,1})$ and $\Omega(x_{d,1}, x_{d,2})$ defined in (19) and (20). Definition 3 defines the Markov-perfect monetary equilibrium.

[24] To assure uniqueness of the steady state with positive net nominal liabilities assume the following regularity condition holds: $U_{cc}c - (U_c - a)(1 + U_{ccc}c/U_{cc}) < 0$ for $c \in (0, \hat{c}]$, implying c and g are moving in the same direction. The functional form $U(c) = c^{1-\rho}/(1-\rho)$ with $\rho > 0$ satisfies this requirement.

[25] Enlarging the set to allow for strictly negative net nominal liabilities, i.e., $B_j < -1$, yields discontinuities in the debt function, $\mathcal{B}'_j(B_j)$. The imposed set $\Gamma \in [-1, \bar{B}]$ ensures that the Markov-perfect monetary equilibrium is well-behaved and the steady state is unique (as shown in the proof of Proposition 2 and 3). For a discussion of the aforementioned discontinuities see the single-currency competitive-market version in Martin (2011) featuring $\Gamma \in [B^L, B^H]$ with $-\infty < B^L < B^H < \infty$.

Definition 3. *A Markov-perfect monetary equilibrium is a set of functions* $\{\mathcal{B}'_d, \mathcal{X}_d, \mathcal{C}_d, \mathcal{G}_d, \mathcal{Z}_d\}$ *such that for all* $B_j \in \Gamma$*:*

(i) $\{\mathcal{B}'_d(B_d, B_f), \mathcal{X}_{d,1}(B_d, B_f), \mathcal{C}_d(B_d, B_f), \mathcal{G}_d(B_d, B_f)\} \in \arg\max_{B'_d, x_{d,1}, c_d, g_d} \alpha_d[n_d u(x_{d,2}) - (1-n_d)x_{d,2}] + (1-\alpha_d)[n_d u(x_{d,1}) - (1-n_d)x_{d,1}] + U(c_d) + \eta_d v(g_d) - a_d(c_d + g_d) - \alpha_d(1-n_d)\mathbb{E}(k_d \mid k_d \leq \Pi_d(\alpha_d)) + \beta \mathcal{Z}_d(B'_d, B'_f)$ *subject to* $\varepsilon(B_d, B'_d, x_{d,1}, \mathcal{X}'_{d,1}(B'_d, B'_f), \mathcal{X}'_{d,2}(B'_d, B'_f), c_d, g_d) = 0$ *in (23), and* $V^c_{d,m_d} - W^c_{d,m_d} \geq 0$ *in (24);*

(ii) $\mathcal{Z}_d(B_d, B_f) = \alpha_d[n_d u(\mathcal{X}_{d,2}(B_d, B_f)) - (1-n_d)\mathcal{X}_{d,2}(B_d, B_f)] + (1-\alpha_d)[n_d u(\mathcal{X}_{d,1}(B_d, B_f)) - (1-n_d)\mathcal{X}_{d,1}(B_d, B_f)] + U(\mathcal{C}_d(B_d, B_f)) + \eta_d v(\mathcal{G}_d(B_d, B_f)) - a_d[\mathcal{C}_d(B_d, B_f) + \mathcal{G}_d(B_d, B_f)] - \alpha_d(1-n_d)\mathbb{E}(k_d \mid k_d \leq \Pi_d(\alpha_d)) + \beta \mathcal{Z}_d(\mathcal{B}'_d(B_d, B_f), \mathcal{B}'_f(B_d, B_f))$*; and*

(iii) the agents' optimal choices, $\{(x_{d,1}, x_{d,2}, d_d, d_f), (x_{f,1}, x_{f,2}, \breve{d}_d, \breve{d}_f), (\alpha_d, \alpha_f), (m_d, m_f, b_d, b_f), (\breve{m}_d, \breve{m}_f, \breve{b}_d, \breve{b}_f)\}$*, satisfy Definition 2.*

Solving (22) subject to (23) and (24) with the respective Lagrange multipliers, λ and ξ, yields the first order conditions:

$$(\lambda - \lambda')\varepsilon_{B'_d} + \lambda\left[\varepsilon_{x'_{d,1}} \frac{\partial \mathcal{X}'_{d,1}}{\partial B'_d} + \varepsilon_{x'_{d,2}} \frac{\partial \mathcal{X}'_{d,2}}{\partial B'_d}\right] = 0, \tag{25}$$

$$n_d \Omega_d \Phi_{x_{d,1}}(x_{d,1})/\theta + \lambda \varepsilon_{x_{d,1}} + \xi n_d\left[\Phi_{x_{d,1}}(x_{d,1})\Omega_d + \Phi(x_{d,1})\Omega_{d,x_{d,1}}\right] = 0, \tag{26}$$

$$U_c(c_d) - a_d + \lambda \varepsilon_{c_d} = 0, \tag{27}$$

$$\eta_d v_g(g_d) - a_d + \lambda \varepsilon_{g_d} = 0, \tag{28}$$

with $\Omega_d \equiv \Omega(x_{d,1}, x_{d,2})$ in (20), where, given (21) $\mathcal{Z}_{d,B'_d} = \lambda' \varepsilon'_{B_d}$ and $\varepsilon_{B'_d} = -\beta \varepsilon'_{B_d}$ are used in (25), and given (10) $\partial x_{d,2}/\partial x_{d,1} = \Phi_x(x_{d,1})/\Phi_x(x_{d,2})$ is used in (26). Considering (25), the first term on the left-hand side, $(\lambda - \lambda')\varepsilon_{B'_d}$, represents the trade-off between current and future distortions, where distortions are perfectly smoothed if $\lambda = \lambda'$. The second term, i.e., the derivatives $\partial \mathcal{X}'_{d,1}/\partial B'_d$ and $\partial \mathcal{X}'_{d,2}/\partial B'_d$, emerge from the government's inability to commit and captures how the domestic debt choice, B'_d, affects the current domestic budget constraint through its effect on future strategies. In equilibrium, government policy results from the interaction of monetary policy and government debt. The tax rate is determined residually to satisfy the government budget constraint. Before proceeding to the determination of government policy, Proposition 1 characterizes some key properties of the Markov-perfect monetary equilibrium.

Proposition 1. *Assume* $\alpha_d \in [0, 1)$*. For all* $B_j \in \Gamma$*, a Markov-perfect monetary equilibrium features: (i) positive labor taxation* $c_d < \hat{c}$*; (ii) constrained-efficient public good provision* $g_d < \hat{g}$*; (iii) positive net nominal liabilities* $\mathcal{B}'_d(B_d, B_f) > -1$*; and (iv) a positive nominal interest rate,* $q_d < 1$*, and thus* $x_{d,1} < \hat{x}$*.*

Proof in Appendix C. Let us start with (i) and (ii) implying $\lambda > 0$ for all $\Gamma \in [-1, \bar{B}]$. Given (25)-(28) it is understood that if there are expected distortions in the future, the government has an incentive to smooth distortions over time, and thus will not implement first-best allocations in the current period. To eliminate future distortions, and hence guarantee $\{\hat{x}, \hat{c}, \hat{g}\}$ solving $u_x(\hat{x}) = 1$, $U_c(\hat{c}) = a_d$, and $\eta_d v_g(\hat{g}) = a_d$, respectively, it needs to run the Friedman rule ($\mu_d = \beta - 1$), collect zero taxes ($\tau_d = 0$), and provide the first-best level of public expenditure in every period.[26] Plugging $\{\hat{x}, \hat{c}, \hat{g}\}$ into the government budget constraint in (21), the first-best steady-state debt level solves $\hat{B} = -1 - a_d\hat{g}/(1-\beta)n_d\Phi(\hat{x})$, suggesting negative net nominal obligations. In other words, to eliminate future distortions and maintain a first-best steady state, the government would need to start with sufficient claims on the private sector. This option, however, is outside of Γ. Thus, for $B_d \in \Gamma$, to pay for first-best public good provision $\hat{g}$ and maintain $\{\hat{x}, \hat{c}\}$ the government would need to continuously roll over positive net nominal liabilities. Put differently, to guarantee $\mathcal{X}_{d,1}(B_d, B_f) = \hat{x}$ for all $B_d \geq -1$ (implying $\lambda = \lambda'$ in (25)), it needs to run a Ponzi-scheme of ever-increasing debt, where $\mathcal{B}'_d(B_d) = a_d\hat{g}/\beta n_d\Phi(\hat{x}) + (1 + B_d)/\beta - 1 > B_d$ solves (21). Since agents are better off not holding domestic bonds in this scenario, maintaining the first-best allocation $\{\hat{x}, \hat{c}, \hat{g}\}$ is not sustainable in the long run, and thus, given (27) and (28), for all $B_j \in \Gamma$ the equilibrium is characterized by positive labor taxation and public good provision below the efficient level, i.e., $c_d < \hat{c}$ and $g_d < \hat{g}$.

Moving on to (iii) and (iv), the results show that for $B_j \in \Gamma$ the government always carries over strictly positive net nominal liabilities (bonds and money), $\mathcal{B}'_d(B_d, B_f) > -1$, and never runs the Friedman rule, as implied by $q_d < 1$, and thus $x_{d,1} < \hat{x}$, suggesting a positive nominal interest rate, $i_d = 1/q_d - 1 > 0$, in the steady state. This result originates from the fact that there exists a positive relationship between domestic inflation and domestic debt, $\partial x_{d,1}/\partial B_d < 0$, implying that the government cannot increase welfare by increasing public debt. Thus, even though the first-best allocation in the DM is technically attainable through monetary trade, due to the government's incentive to smooth distortions over time, it is never implemented in equilibrium. In this context, an important result to highlight is that given the fact that $x_{d,1} = \hat{x}$ is feasible, the non-negativity constraint, $V^c_{d,m_d} - W^c_{d,m_d} \geq 0$, does not bind, and thus $\xi = 0$ for all $B_j \in \Gamma$.[27]

3.3 Determination of Government Policy

The determination of government policy hinges on the *intra-temporal* and *inter-temporal* trade-offs introduced by the interaction between monetary policy and government debt.

Starting with equation (26), note from the government budget constraint in (21) that the partial derivative $\varepsilon_{x_{d,1}} < 0$ for all $B_d > -1$. Hence, if net nominal liabilities are positive, then

[26] In that context, note that if $x_{d,1} = \hat{x}$, and thus $x_{d,2} = \hat{x}$, it holds that $\Pi_d = 0$ in (13), and thus, provided $\underline{k} > 0$, domestic producers have no incentive to accept foreign currency in transactions. In other words, $\alpha_d = 0$ for $\{\hat{x}, \hat{c}, \hat{g}\}$.

[27] In equilibrium it holds that $x_{d,1} = \hat{x}$ for $B_d = -1$ and $x_{d,1} < \hat{x}$ for $B_d > -1$. Note, however, that this is contingent on the fact that the pricing mechanism supports efficiency. Under the Nash bargaining solution, this result is not guaranteed.

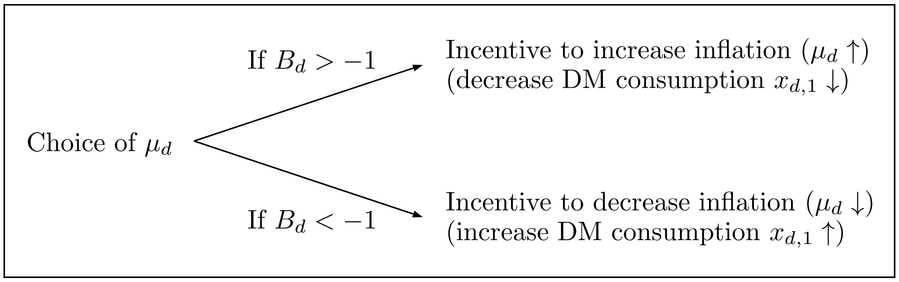

Figure 2: Intra-temporal trade-off

an increase in $x_{d,1}$ tightens the budget constraint, and thus, despite the distortionary effects on the DM allocation, the government has an incentive to inflate away its nominal liabilities to relax the budget constraint. In other words, for $B_d > -1$, given $x_{d,1} < \hat{x}$ and thus $\xi = 0$ (see proof in Appendix C), an increase in public debt implies a decrease in DM consumption, $\partial x_{d,1}/\partial B_d < 0$, and thus $\partial x_{d,2}/\partial B_d \leq 0$ (holding with equality if domestic DM consumption is exclusively financed with foreign currency). This is the channel through which current debt affects current inflation, characterizing the *intra-temporal* trade-off between monetary policy and public debt (as visualized in Figure 2).

Consider now the first-order condition (25). The government chooses to increase future debt, B'_d, and push distortions to the future, $\lambda < \lambda'$, if it relaxes the budget constraint, i.e., if the second term on the left-hand side of (25) is positive. On the contrary, the domestic government decreases B'_d, and thus reduces distortions in the future, $\lambda > \lambda'$, if the second term on the left-hand side of (25) is negative, i.e., if increasing B'_d tightens the budget constraint. To determine which effect dominates, since $\lambda > 0$, $\partial \mathcal{X}'_{d,1}/\partial B'_d < 0$, $\varepsilon_{x'_{d,2}} \leq 0$ (holding with equality if $\alpha'_d = 0$), and $\partial \mathcal{X}'_{d,2}/\partial B'_d \leq 0$ (holding with equality if $\alpha'_d = 1$) for all $B_j \in \Gamma$, we need to determine the sign of $\varepsilon_{x'_{d,1}}$. From the monetary equilibrium we know that $V_{d,m_d} = n_d \Phi(x_{d,1})[1 + \Omega(x_{d,1}, x_{d,2})]$ and $V_{d,b_d} = n_d \Phi(x_{d,1})$, and thus $\varepsilon_{x'_{d,1}} = \beta \frac{\partial V'^c_{d,m_d}}{\partial x'_{d,1}} + \beta \frac{\partial V'^c_{d,b_d}}{\partial x'_{d,1}} B'_d$, representing the marginal value of (normalized) nominal balances. If $\frac{\partial V'^c_{d,m_d}}{\partial \mathcal{X}'_{d,1}} \frac{\partial \mathcal{X}'_{d,1}}{\partial B'_d} > 0$, then an increase in B'_d relaxes the budget constraint. In the opposite case, the government decreases B'_d. In any case, note that $\frac{\partial V'^c_{d,b_d}}{\partial x'_{d,1}} > 0$, and thus $\frac{\partial V'^c_{d,b_d}}{\partial \mathcal{X}'_{d,1}} \frac{\partial \mathcal{X}'_{d,1}}{\partial B'_d} < 0$. Hence, a higher B'_d, and thus higher future inflation, decreases the future value of domestic bonds, and therefore their demand. Thus, to buy bonds, agents will ask for a lower price q_d (i.e. a higher return), which (provided $B'_d > 0$) tightens the budget constraint. This is the *inter-temporal* trade-off between monetary policy and public debt capturing the degree of distortion-smoothing over time. To summarize: A higher B'_d, and thus higher future domestic inflation has two effects: (a) it increases the future marginal value of the domestic currency since a consumer would have preferred to arrive with an additional unit in the DM. Thus, the current demand for domestic currency increases, which relaxes the government's budget constraint. (b)

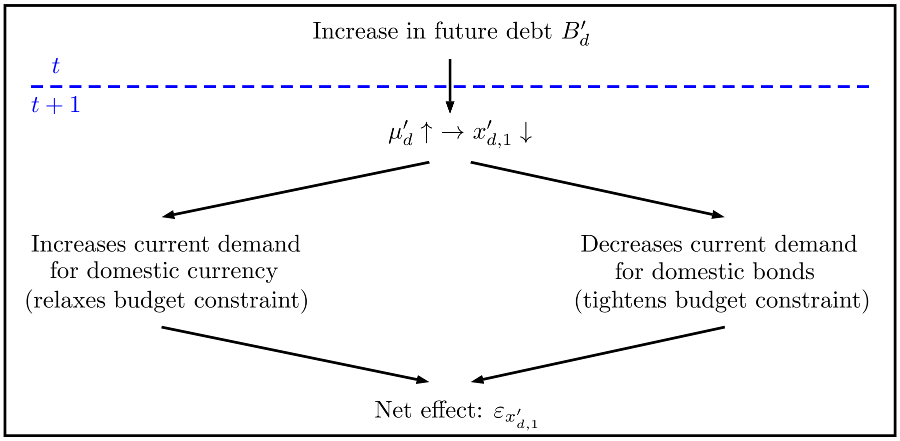

Figure 3: Inter-temporal trade-off

An increase in future inflation (due to higher debt today) reduces the current demand for domestic bonds. Thus, the interest paid on debt increases, which tightens the government budget constraint. Figure 3 visualizes the inter-temporal trade-off.

4 Steady States

Having characterized the government's problem, we now study the steady states. In a first step, I characterize a benchmark steady state in which domestic currency substitution is prohibited, and thus $\alpha_d = 0$ for all combinations of μ_d and μ_f. Once established, we proceed to the equilibrium steady state with $\alpha_d \in (0,1]$ if $\Pi_d \geq \underline{k}$.

4.1 Benchmark Steady State: Prohibited Dollarization

Assume payment in foreign currency is prohibited by law, and thus transactions in the domestic economy are required to be settled with domestic currency exclusively. In other words, $\alpha_d = 0$ for all μ_d and μ_f. To determine the benchmark steady state, $\{B^N, x^N, c^N, g^N\}$, we start with the first-order condition (25). In a steady state, following Barro (1979), distortions are perfectly smoothed over time, i.e., $\lambda = \lambda'$. Hence, given $\alpha_d = 0$, (25) reduces to $\lambda \varepsilon_{x'_{d,1}} \partial \mathcal{X}'_{d,1}/\partial B'_d = 0$. Given $\lambda > 0$ and $\partial \mathcal{X}'_{d,1}/\partial B'_d < 0$ for all $B_d \geq -1$, it holds that $\varepsilon_{x'_{d,1}} = 0$. Solving for the steady-state public debt level B^N yields:

$$B^N = -1 - \frac{\Phi(x^N)\Omega_x(x^N) + \Phi_x(x^N)\Omega(x^N)}{\Phi_x(x^N)}, \tag{29}$$

with $1 + B^N > 0$ if $\Phi(x^N)\Omega_x(x^N) + \Phi_x(x^N)\Omega(x^N) < 0$, where $\Phi(x^N)$ and $\Omega(x^N)$ are given in (19) and (20) for $\alpha_d = 0$. The term $\varepsilon_{x'_{d,1}} = 0$ implies that the increased demand for domestic currency and the decreased demand for domestic bonds, triggered by the anticipated policy response to an increase in domestic debt, are exactly offset in the benchmark steady state. In other words, the time-consistency problem driving the change in public debt is canceled out.

From Proposition 1 we know that a first-best steady state with $x_{d,1} = x'_{d,1} = \hat{x}$ is not sustainable and therefore $\Omega(x^N) > 0$, suggesting $\xi = 0$. Furthermore, $\alpha_d = 0$ reduces $n_d \Omega_d \Phi_{x_{d,1}}(x_{d,1})/\theta$ in (26) to $n_d[u_x(x^N) - 1]$. Solving (28) for $\lambda = \eta_d v_g(g^N)/a_d - 1$ and plugging into (26), (27), and the government budget constraint (21), yields the steady-state system of equations, evaluated at B^N:

$$a_d[u_x(x^N) - 1] - [\eta_d v_g(g^N) - a_d]\Phi_x(x^N)[1 + B^N] = 0, \tag{30}$$

$$[\eta_d v_g(g^N) - a_d]U_{cc}(c^N)c^N + \eta_d v_g(g^N)[U_c(c^N) - a_d] = 0, \tag{31}$$

$$[U_c(c^N) - a_d]c^N - a_d g^N + \beta n_d \Phi(x^N)\Omega(x^N) - (1 - \beta)n_d \Phi(x^N)[1 + B^N] = 0. \tag{32}$$

Note that, given $\alpha_d = 0$, the domestic allocations are fully independent of foreign policy, i.e., $\{B^N, x^N, c^N, g^N\}$ solving the steady-state system of equations, (29)-(32), is the same for all $B_f \in \Gamma$.

Proposition 2. *For $(\alpha_d, \alpha_f) = (0, 0)$, the domestic benchmark steady-state allocations, $\Upsilon^N \equiv \{B^N, x^N, c^N, g^N\}$, solve (29)-(32), where for all $B_d \in \Gamma$ it holds that:*

(i) $\mathcal{B}'_d(B^N) = B^N$ and $\mathcal{B}'_d(B_d) \to B^N$ for $v_g(g_d) \to \psi$ with $\eta_d \psi > a_d$;

(ii) given an initial debt level B^N, a government with and without commitment both implement Υ^N in every period; and

(iii) steady-state allocations represent the citizens' preferences for public goods, η_d, with $\partial B^N/\partial \eta_d > 0$, $\partial x^N/\partial \eta_d < 0$, $\partial c^N/\partial \eta_d > 0$, and $\partial g^N/\partial \eta_d > 0$.

Proposition 2 proceeds in three steps with the corresponding proof in Appendix D. The first result, (i), builds on the fact that the time-consistency problem is eliminated at B^N. Given that result, the domestic government has no incentive to substitute distortions inter-temporally, and hence maintains a constant debt level, B^N, in the long run. In other words, the steady-state B^N is unique. Furthermore, (i) shows that for CRRA preferences, $v(g_d) = \psi g_d^{1-\gamma}/(1 - \gamma)$ with $\gamma > 0$, if $\gamma \to 0$, then the public debt level converges to B^N for all $B_d \in \Gamma$ in the long run, i.e., $\mathcal{B}'_d(B_d) \to B^N$.

Drawing from (i), in line with Martin (2011), the second result (ii) shows that the constrained-efficient steady state is robust to commitment frictions. In other words, endowing the domestic government at B^N with commitment power would not change the long-run equilibrium allocations. The corresponding Ramsey problem to verify this property is provided in Appendix D. Furthermore, despite the fact that the government inflates away nominal liabilities, i.e., $x^N < \hat{x}$, there is no time-consistency problem at B^N, and thus, unlike proposed by Alvarez et al. (2004) under commitment, time-consistency does not strictly require monetary policy at the Friedman rule. The result is in line with Aruoba and Chugh (2010) embedding a Ramsey (commitment) problem into the Lagos-Wright structure. Analogous to the paper at hand, their monetary policy prescription contrasts the suggested optimality of the Friedman rule in Chari et al. (1991). The discrepancy, however, follows from standard Ramsey theory, according to which all final goods should be taxed to some degree in an optimal tax system. In Chari et al. (1991) the optimality of the Friedman rule relies on the fact that cash and credit goods are taxed uniformly in the spirit of Atkinson and Stiglitz (1980). Since a deviation from the Friedman rule would imply that cash goods are taxed more than credit goods, this cannot be optimal, rendering the Friedman rule the optimal policy. The preference structure of Lagos and Wright (2005) differs in that regard as labor taxation only applies to the CM good. Thus, to satisfy the requirements of an optimal tax system, taxation of the DM good requires a positive nominal interest rate.[28]

Lastly, the third result (iii) characterizes the effects of an increase in the preference parameter η_d on steady-state allocations. From (21) and (28) we know that an increase in the marginal value of public goods, $\eta_d v_g$, tightens the government budget constraint. Thus, to maximize domestic welfare, the government adjusts long-run fiscal and monetary policy. Precisely, to smooth distortions over time, it increases public debt B^N, followed by an increase in the growth rate of the currency supply, μ^N to, on one hand, finance the additional expenditures with seigniorage revenue and, on the other hand, alleviate the financial burden of the heightened nominal debt. The proportional labor tax rate, τ^N, is determined residually to maximize tax revenue τh, where given $dc/dg > 0$, it holds that aggregate CM output increases, $\partial h/\partial \eta_d > 0$.

4.2 Equilibrium Steady State

To assess the effects of currency substitution on long-run allocations, let us now study the steady state without legal restrictions and thus, following (14), $\alpha_d \in (0,1]$ if $\Pi_d \geq \underline{k}$. To determine the domestic steady-state allocations $\Upsilon^D = \{B^D, x_1^D, x_2^D, c^D, g^D\}$ we revisit (25). Decomposing $\frac{\partial \mathcal{X}'_{d,2}}{\partial B'_d}$ into $\frac{\partial \mathcal{X}'_{d,2}}{\partial \mathcal{X}'_{d,1}} \frac{\partial \mathcal{X}'_{d,1}}{\partial B'_d}$ with $\frac{\partial \mathcal{X}_{d,2}}{\partial \mathcal{X}_{d,1}} = \frac{\Phi_{x_{d,1}}(x_{d,1})}{\Phi_{x_{d,2}}(x_{d,2})}$ from (10) and plugging in $\lambda = \lambda'$, the steady-state debt solves:

$$B^D = -1 - \frac{\Phi(x_1^D)\Omega_{x_1^D}(x_1^D, x_2^D) + \Phi_{x_1^D}(x_1^D)\Omega(x_1^D, x_2^D)}{\Phi_{x_1^D}(x_1^D)} - \frac{\Phi(x_1^D)\Omega_{x_2^D}(x_1^D, x_2^D)}{\Phi_{x_2^D}(x_2^D)}. \tag{33}$$

A few observations, distinguishing between the different currency regimes, are in order. Given $\lambda > 0$ and $\partial \mathcal{X}'_{d,1}/\partial B'_d < 0$, equation (25) suggests $\varepsilon_{x'_{d,1}} + \varepsilon_{x'_{d,2}} \frac{\Phi_x(x'_{d,1})}{\Phi_x(x'_{d,2})} = 0$ in the steady state. If $\alpha_d = 0$, then $\varepsilon_{x'_{d,2}} = 0$, and thus $B^D = B^N$ in (29). Consider now $\alpha_d \in (0,1)$. If domestic producers accept foreign currency, then, given $\varepsilon_{x'_{d,2}} < 0$ for $\alpha_d \in (0,1)$, it holds that the marginal value of domestic nominal balances $\varepsilon_{x'_{d,1}} > 0$, and thus increasing B'_d tightens the domestic government's budget constraint. In the steady state, this is offset by the incentive to increase B'_d due to the reduced demand for the domestic currency, i.e., $\varepsilon_{x'_{d,2}} \equiv \partial \beta V'^c_{d,m_d}/\partial x'_{d,2} < 0$. Lastly, if $\alpha_d = 1$, then

[28] See Aruoba and Chugh (2010) for a detailed comparison of the two literatures and an in-depth discussion of alternative tax schemes.

there is no demand for the domestic currency, $\phi_d = 0$, and thus $B^D = -1$.

Since $x_{j,1} = x'_{j,1} = \hat{x}$ is not sustainable in the long-run, it holds that $\Omega(x_1^D, x_2^D) > 0$, and thus $\xi = 0$. Solving (28) for $\lambda = \eta_d v_g(g^D)/a_d - 1$ and plugging into (26), (27), and (21), respectively, yields the steady-state system of equations, evaluated at B^D:

$$a_d\Omega(x_1^D, x_2^D)/\theta - [\eta_d v_g(g^D) - a_d][1 + B^D] = 0, \tag{34}$$

$$[\eta_d v_g(g^D) - a_d]U_{cc}(c^D)c^D + \eta_d v_g(g^D)[U_c(c^D) - a_d] = 0, \tag{35}$$

$$[U_c(c^D) - a_d]c^D - a_d g^D + \beta n_d \Phi(x_1^D)\Omega(x_1^D, x_2^D) - (1-\beta)n_d\Phi(x_1^D)[1 + B^D] = 0. \tag{36}$$

Proposition 3. *Suppose $\alpha_f = 0$ with the foreign steady-state allocations $\Upsilon^F = \{B^F, x_1^F, c^F, g^F\}$. There exist the threshold values $\eta_d = \{\underline{\eta}, \bar{\eta}\}$ with $\underline{\eta} < \bar{\eta}$ such that:*

$$\alpha_d \begin{cases} = 0 & \text{if } \eta_d < \underline{\eta} \\ \in (0,1) & \text{if } \eta_d \in [\underline{\eta}, \bar{\eta}) \\ = 1 & \text{if } \eta_d \geq \bar{\eta}, \end{cases} \tag{37}$$

with the domestic steady-state allocations, $\Upsilon^D \equiv \{B^D, x_1^D, x_2^D, c^D, g^D\}$, solving (10) and (33)-(36).

Proof in Appendix E.[29] For a given Υ^F and the correspondence $x_{d,2}(x_{d,1})$ in (10), the system of equations (33)-(36) yields Υ^D, where steady-state public debt and inflation increase in the marginal value of public goods, $\eta_d v_g$. The intuition behind this result follows the standard tax-smoothing argument. Unlike in the benchmark steady state, however, the reduction in the domestic currency's purchasing power may trigger currency substitution. Revisiting the producers' authentication decision in (14), in combination with the mapping $\eta_d \to x_1^D$ defined in (33)-(36), for a given Υ^F, there emerge the two threshold values, $\eta_d = \{\underline{\eta}, \bar{\eta}\}$ solving $\Pi_d = \underline{k}$ and $\Pi_d = \bar{k}$ in (13), respectively. Three cases are considered: (i) $\alpha_d = 0$, (ii) $\alpha_d \in (0,1)$, and (iii) $\alpha_d = 1$.

Let us start with case (i). If $\eta_d < \underline{\eta}$, then the surplus of accepting foreign currency in the domestic DM does not warrant the investment into the authentication technology, and thus (33)-(36) collapse to (29)-(32). Hence, if $\alpha_d = 0$, then the domestic and foreign steady-state allocations, Υ^D and Υ^F, and thus the domestic and foreign government policies, are fully independent. Allocations can still differ across countries due to heterogeneities in fundamentals.

Consider now case (ii). For an intermediate range of $\eta_d \in [\underline{\eta}, \bar{\eta})$, investing in the authentication technology is worthwhile for a subset of producers, yielding the interior solution $\alpha_d \in (0,1)$. As a consequence, domestic and foreign allocations become intertwined. The increased demand for foreign currency and the reduced demand for domestic currency impose a constraint on the

[29] The proof includes the verification that the properties (i) - (iii) in Proposition 2 hold for $\Upsilon^D \equiv \{B^D, x_1^D, x_2^D, c^D, g^D\}$. However, for brevity, I refrain from a repeated discussion in the main text.

domestic government's ability to generate seigniorage revenue.[30] To compensate for that constraint, the domestic government adjusts fiscal policy, where in the steady state, it trades off the incentive to smooth distortions inter-temporally against the negative effects on the demand for its own currency, thus confirming the intuition in the earlier work of Rojas-Suárez (1992) suggesting that currency substitution may aggravate fiscal imbalances.

Lastly, (iii) considers the case where $\eta_d \geq \bar{\eta}$, and thus $\alpha_d = 1$. If the domestic currency is not valued by domestic agents, as suggested by $x_{d,2} - x_{d,1} > 0$, and thus $\phi_d = 0$ given $\bar{k} > 0$, then neither domestic nor foreign agents hold assets denominated in the domestic currency, reducing the choice variables in the domestic government's objective function, (22), to (c_d, g_d), while $x_{d,2}$ is determined by the foreign government exclusively. As a consequence, the domestic government loses its ability to finance expenditures using seigniorage revenue and debt, and thus has to rely solely on revenue generated via labor taxation. This result provides an intuitive motivation for government transaction policies along the lines of Aiyagari and Wallace (1997) and Li and Wright (1998). By restricting a subset of producers to exclusively accept the domestic currency, which can be accomplished by increasing $\bar{k} \to \infty$ for a positive share of producers in the framework at hand, an interior solution with $\alpha_d \in (0, 1)$ is guaranteed.[31] Alternatively, to regain the ability to smooth distortions over time, the government could issue public debt denominated in the foreign currency. Section 4.3 discusses this alternative.

Proposition 4. *Suppose $(\alpha_d, \alpha_f) = ([0, 1], 0)$ with the domestic and foreign steady-state allocations Υ^D and Υ^F, respectively. Then, an increase in μ_d yields $\partial \mathcal{Z}_f / \partial \mu_d \mid_{\Upsilon^F} > 0$ for $\alpha_d \in (0, 1)$, and $\partial \mathcal{Z}_f / \partial \mu_d \mid_{\Upsilon^F} = 0$ for $\alpha_d = 0$, where $\mathcal{Z}_f$ denotes the foreign government's period utility.*

Proposition 4 discusses the effects of domestic inflation on the foreign government's period welfare:

$$\mathcal{Z}_f(B_d, B_f) = n_f u(x_{f,1}) - (1 - n_f)x_{f,1} + U(c_f) + \eta_f v(g_f) - a_f(c_f + g_f).$$

If $\alpha_d = 0$, then there is no domestic demand for foreign currency, and thus domestic and foreign policies are fully independent. As a result, an increase in domestic inflation does not affect foreign welfare. If $\alpha_d \in (0, 1)$, then, for a given Υ^F, an increase in domestic inflation increases ϕ_f due to the heightened domestic demand for foreign currency, and thus foreign welfare, $\mathcal{Z}_f$. The proof follows directly from the comparative static $\partial \phi_f / \partial \mu_d > 0$ in Table 1, suggesting $\partial x_1^F / \partial \mu_d > 0$ for a given μ_f. Hence, having its currency circulate internationally allows the foreign government to import seigniorage revenue from abroad which unambiguously leads to higher steady-state consumption.

To determine the foreign government's best response to domestic currency substitution, let

[30] This constraint originates from general equilibrium effects, as an increase in domestic inflation reduces (increases) the demand for domestic (foreign) currency, increasing the acceptability of foreign currency, α_d.

[31] Alternatively, the government could anchor the domestic currency by paying wages, enforcing tax payments, or paying interest on deposits denominated in the domestic currency. An explicit formalization of these policies, however, goes beyond the scope of this paper and is left for future research.

us revisit the government budget constraint in (1). Plugging in the equilibrium prices and policy variables, τ_f, ϕ_f, μ_f, and q_f in (15)-(18) yields the updated foreign government budget constraint $\varepsilon_f(B_f, B'_f, c_d, c_f, g_f, x_{f,1}, x'_{f,1}, x_{d,1}, x_{d,2}, x'_{d,1}, x'_{d,2}) = 0$, where $x'_{f,1} \equiv \mathcal{X}'_{f,1}(B'_d, B'_f)$, $x'_{d,1} \equiv \mathcal{X}'_{d,1}(B'_d, B'_f)$, and $x'_{d,2} \equiv \mathcal{X}'_{d,2}(B'_d, B'_f)$ represent the allocations the foreign government anticipates future citizens and governments will implement as a response to today's policy decisions. Thus, it takes those allocations as given. Note that the foreign budget constraint is a function of domestic allocations due to the global market clearing condition $n_d m_f + n_f \breve{m}_f = 1$, and thus $\phi_f(c_f, c_d, x_{f,1}, x_{d,1}, x_{d,2})$ in (16) for $\alpha_d > 0$. The explicit characterization of $\varepsilon_f(\cdot)$ and the partial derivatives are provided in Appendix F. The foreign government solves:

$$
\begin{aligned}
\max_{B'_f, x_{f,1}, c_f, g_f} \mathcal{Z}_f(B_d, B_f) &= n_f u(x_{f,1}) - (1-n_f)x_{f,1} + U(c_f) + \eta_f v(g_f) - a_f(c_f + g_f) \\
&+ \beta \mathcal{Z}_f(B'_d, B'_f),
\end{aligned}
$$

$$
\text{s.t. } \varepsilon_f(B_f, B'_f, c_d, c_f, g_f, x_{f,1}, x'_{f,1}, x_{d,1}, x_{d,2}, x'_{d,1}, x'_{d,2}) = 0, \tag{38}
$$

$$
\text{s.t. } V^c_{f,\breve{m}_f} - W^c_{f,\breve{m}_f} \geq 0, \tag{39}
$$

where (39) represents the feasibility constraint $U_{c_f} \phi_f \Omega(x_{f,1}) \geq 0$ ensuring $x_{f,1} \leq \hat{x}$ analog to (24). Using the Lagrange multipliers λ and ξ the first-order conditions are:

$$
(\lambda - \lambda')\varepsilon_{f,B'_f} + \lambda \left[\varepsilon_{f,x'_{f,1}} \frac{\partial \mathcal{X}'_{f,1}}{\partial B'_f} + \varepsilon_{f,x'_{d,1}} \frac{\partial \mathcal{X}'_{d,1}}{\partial B'_f} + \varepsilon_{f,x'_{d,2}} \frac{\partial \mathcal{X}'_{d,2}}{\partial B'_f} \right] = 0, \tag{40}
$$

$$
n_f[u_x(x_{f,1}) - 1] + \lambda \varepsilon_{f,x_{f,1}} + \xi[n_f \Phi_x(x_{f,1})\Omega(x_{f,1}) + U_c(c_f)\phi_f \Omega_x(x_{f,1})] = 0, \tag{41}
$$

$$
U_c(c_f) - a_f + \lambda \varepsilon_{f,c_f} + \xi n_d [U_{cc}(c_f)/U_c(c_d)][\Phi(x_{d,2}) - \Phi(x_{d,1})]\Omega(x_{f,1}) = 0, \tag{42}
$$

$$
\eta_f v_g(g_f) - a_f + \lambda \varepsilon_{f,g_f} = 0, \tag{43}
$$

where $\lambda > 0$, $\xi = 0$, and $\partial \mathcal{X}'_{f,1}/\partial B'_f < 0$ characterize the Markov-perfect monetary equilibrium.[32] Let us distinguish between the different currency regimes. First and foremost, note that if $\alpha_d = 0$, then the foreign government budget constraint is a function of foreign allocations exclusively, and thus (40)-(43) are equivalent to (25)-(28) (with $\alpha_d = 0$), where, given $\partial \mathcal{X}'_{f,1}/\partial B'_f < 0$, $\varepsilon_{f,x'_{f,1}} = 0$ solves for the foreign steady-state debt (analog to $\varepsilon_{x'_{d,1}} = 0$ in the domestic government's problem). In other words, if domestic agents do not accept foreign currency, then the foreign steady state corresponds to the benchmark steady state $\{B^N, x^N, c^N, g^N\}$ in (29)-(32). Consider now the case with $\alpha_d \in (0,1)$. If there is domestic demand for foreign currency, then this affects the foreign government's public debt policy, as reflected in the last two terms inside the brackets of (40). From the government budget constraint we know that for $B_f \in \Gamma$, it holds that $\varepsilon_{f,x'_{d,1}} < 0$ and $\varepsilon_{f,x'_{d,2}} > 0$. Since $\partial \mathcal{X}'_{f,1}/\partial B'_f < 0$ holds for all $B_f \in \Gamma$, given general equilibrium effects, it is implied

[32]These properties can be easily verified using the proof of Proposition 1 in Appendix C.

that an increase in foreign inflation increases (decreases) $x_{d,1}$ ($x_{d,2}$), and hence $\partial \mathcal{X}'_{d,1}/\partial B'_f > 0$ and $\partial \mathcal{X}'_{d,2}/\partial B'_f < 0$, thus rendering $\varepsilon_{f,x'_{f,1}} < 0$ in the steady state (to satisfy (40) for $\lambda = \lambda'$). In other words, in the steady state, given $\varepsilon_{f,x'_{f,1}} < 0$, the foreign government has a strict incentive to increase public debt, B'_f, and push distortions into the future. This effect, however, is offset by the domestic demand for foreign currency, providing the foreign government with an incentive to decrease B'_f to maintain the international status of its currency, and thus to retain the ability to import seigniorage revenue from abroad. Lastly, if $\alpha_d = 1$, then domestic currency is not valued anymore, and thus $\varepsilon_{f,x'_{d,1}} = 0$. Taking as given domestic allocations, the foreign steady-state allocations, $\Upsilon^F = \{B^F, x_1^F, c^F, g^F\}$, solve the system of equations (40)-(43) and the foreign budget constraint (38) with $\lambda = \lambda' > 0$ and $\xi = 0$.

4.3 De Jure Dollarization

The presented framework emphasizes how the economic agents' decisions affect payment patterns. Nonetheless, it is equipped to examine the implications for government policy if currency substitution is imposed exogenously. Suppose the domestic government abandons its currency and officially adopts the foreign currency as its legal tender. By giving up its monetary independence, it 'de facto' eliminates its ability to tailor monetary policy to the aggregate state, and thus relies exclusively on revenue generated by taxation and debt denominated in the foreign currency.[33] To reconcile the effects of de jure dollarization in the framework at hand, let us revisit the government budget constraint in (1). The domestic government's choice variables reduce to $\{B_d, \tau_d, g_d\}$, yielding the normalized domestic budget constraint:

$$\tau_d h_d - g_d + \phi_f(1+\mu_f) q_f B'_d - \phi_f B_d = 0, \tag{44}$$

with τ_d, ϕ_f, μ_f, and q_f given in (15)-(18), where the latter three, and thus domestic DM consumption, x_d, are taken as given by the domestic government. Solving:

$$\max_{B'_d, c_d, g_d} \mathcal{Z}_d(B_d, B_f) = n_d u(x_d) - (1-n_d)x_d + U(c_d) + \eta_d v(g_d) - a_d(c_d + g_d) + \beta \mathcal{Z}_d(B'_d, B'_f),$$

subject to (44), the first-order condition with respect to B'_d yields $\lambda - \lambda' = 0$, and thus $B'_d = B_d$. In words, if the domestic country officially adopts the foreign currency as its legal tender, then time-consistency concerns disappear, i.e., $\partial x_d/\partial B_d = 0$, reducing the objective of the government to inter-temporal distortion-smoothing exclusively. As a consequence of giving up the ability to inflate away nominal liabilities in the future, every debt level is a steady state under 'de jure' dollarization. Conditional on the choices of τ_d and g_d, the government is free to choose any

[33] Albeit identified as a cost in the context of this model, Barro and Gordon (1983) argue that losing monetary independence can be beneficial if the dollarized country is prone to an inflation-bias problem as dollarization curbs the government's incentive to overstimulate the economy and excessively monetize deficits and debts. A similar logic has been revisited by Chari et al. (2020), analyzing the effects of heterogeneities in *temptation* shocks – shocks that provide incentives to generate surprise inflation – on the incentives to form currency unions.

debt level B_d satisfying the budget constraint (44), where B_d remains constant for a fixed foreign inflation rate, μ_f.[34]

Transition costs aside, a government has an incentive to officially adopt the foreign currency as its legal tender if the increased trade volume warrants giving up the country's monetary independence.[35] When compared to an economy with endogenous currency substitution, i.e., $\alpha_d \in [0, 1)$, this trade-off weakens, as the domestic agents' option to deviate into foreign currency already imposes a constraint on the domestic government's ability to generate seigniorage revenue. Since in the limiting case with $\alpha_d = 1$ the government has to rely on taxation exclusively, by 'de jure' dollarizing, it regains B'_d as a choice variable, and thus the ability to smooth distortions inter-temporally by issuing public debt denominated in the foreign currency.

Lastly, the presented framework provides an intuitive setting to revisit an early result by Friedman (1953) and Mundell (1961), according to whom a currency union tends to be less costly for countries with similar aggregate shocks. To confirm this intuition through the lens of the model, assume the foreign country is subject to an idiosyncratic expenditure shock (e.g. a war). To smooth distortions over time, it increases public debt, and thus inflation. Since the domestic government is unable to adjust monetary policy, this represents a negative externality on the domestic citizens. The less correlated the business cycles of the two countries, the higher the cost of giving up the ability to condition monetary policy on the aggregate state.

5 Historical Evidence

> *["In the post-WW1 European hyperinflation] the lack of a stable domestic means of payment was a serious inconvenience in trade and production, and foreign currencies therefore came to be desired not merely as a store of value but actually as a means of domestic payment. ... Thus, in advanced inflation, Gresham's law was reversed; good money tended to drive out bad ..."*
>
> League of Nations (1946, p.48)

This section revisits the testable implications of the theory and reconciles empirical facts documented in the existing literature and data on currency substitution. First and foremost, the study emphasizes the information frictions associated with less familiar media of exchange in transactions. The fundamental idea dates back to the influential work by Jevons (1875) and Menger (1892), later revisited by Alchian (1977), stressing that once there exist differences in the degrees of knowledge, it is the costliness of information that induces the use of widely recognized

[34] Note that similar implications apply to an exchange-rate peg or inflation targeting, as both cases restrict the monetary authority's ability to adjust the growth rate of the currency supply in response to changes in fiscal policy.

[35] A list of the pros and cons is provided by Alesina and Barro (2001). According to their results, a country is more likely to abandon its currency for an anchor currency if it has: (a) a history of high inflation, (b) a large international trade volume with the anchor country, (c) business cycles that covary with the anchor country, and (d) stable real exchange rates with respect to the anchor country.

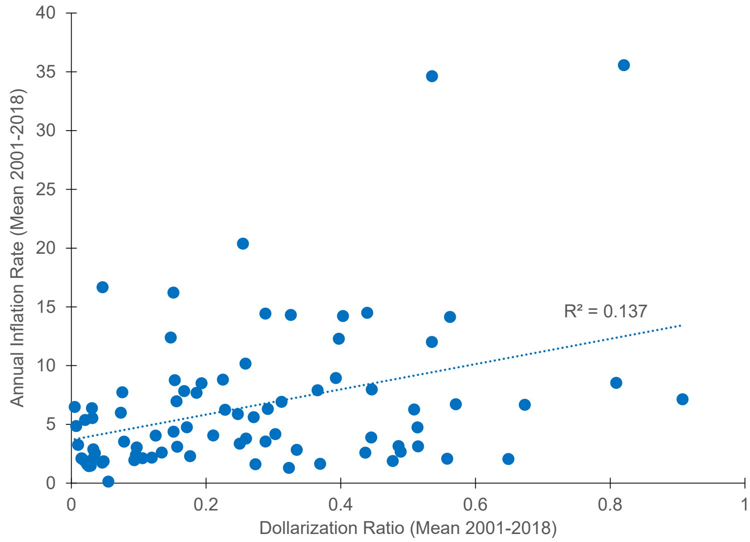

Figure 4: Arithmetic mean (2001-2018) of the 'annual headline consumer price index (CPI) inflation' and 'dollarization ratios'. Sources: Yeyati (2021) and Ha et al. (2021).

media of exchange. This paper builds on these early ideas by incorporating information acquisition about foreign currencies. Furthermore, by encompassing the recent advances in monetary search theory, it establishes an explicit transmission channel of government policy to payment patterns.

The positive relationship between domestic inflation and currency substitution, documented in the extensive literature on dollarization summarized in Calvo and Végh (1992), finds proof in historic data. Figure 4 plots the statistically significant correlation between the arithmetic mean of the annual domestic inflation rate and the share of foreign-denominated assets in domestic deposits (labeled 'dollarization ratio') for 118 countries over the period 2001-2018.[36] The data used is provided by Yeyati (2021) and Ha et al. (2021) and excludes *de jure* dollarized economies.[37] To obtain reliable results, offshore financial centers, whose dollarization may be due to extranational factors, are excluded.[38] Furthermore, observations pre-2001 are omitted to exclude episodes of hyperinflation in Latin America, Africa, and Eurasia.

By explicitly incorporating the interaction of fiscal and monetary policy, the analysis goes beyond the relationship between inflation and currency substitution and yields important new insights for government financing. Identifying said effects requires explicitly modeling the inter-temporal

[36]Since obtaining reliable data on the public's holdings of foreign currency outside of the banking system presents a challenge, to proxy for the use of foreign currency in domestic transactions, the literature relies on the share of foreign denominated assets in domestic deposits. Admittedly, this approach implicitly assumes a degree of financial integration – a premise that could be challenged for some developing economies.

[37]The dataset on 'dollarization ratios' by Yeyati (2021) assembles data reported in various central bank bulletins, IMF Article IV Staff Reports, and previous empirical work by De Nicoló et al. (2005), Arteta (2002), and Bennett et al. (1999) dating back to 1970 for some countries.

[38]The following countries are defined as offshore financial centers by the IMF: Andorra, Anguilla, Antigua and Barbuda, Aruba, Bahamas, Bahrain, Barbados, Belize, Bermuda, British Virgin Islands, Cayman Islands, Cook Islands, Costa Rica, Cyprus, Dominica, Gibraltar, Grenada, Guernsey, Hong Kong, Ireland, Isle of Man, Jersey, Lebanon, Liechtenstein, Luxembourg, Macau, Malta, Marshall Islands, Mauritius, Monaco, Montserrat, Nauru, Netherlands Antilles, Niue, Panama, Palau, Samoa, Seychelles, Singapore, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Switzerland, Turks and Caicos Islands, and Vanuatu.

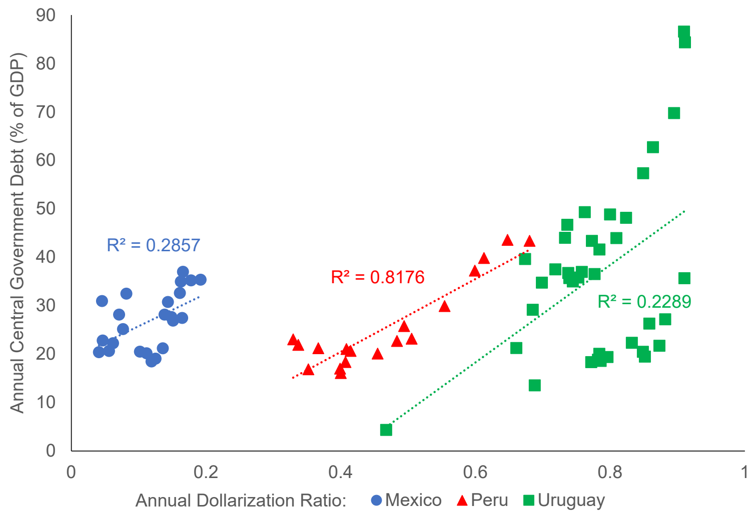

Figure 5: Annual 'dollarization ratios' and 'central government debt (percent of GDP)'. Sources: Yeyati (2021) and Mauro et al. (2015).

trade-off between distortion-smoothing and time-consistency. Currency substitution aside, the results confirm the early insights of Easterly and Schmidt-Hebbel (1991) according to whom inflation originates from fiscal imbalances.[39] More precisely, the incentive to inflate away nominal liabilities necessitated by persistent fiscal deficits. However, unlike in a single currency environment, once competing media of exchange can coexist, expansionary monetary policy may further aggravate fiscal imbalances by giving rise to endogenous currency substitution, thus imposing a constraint on the government's ability to generate seigniorage revenues. The root cause lies in the 'reversed Gresham's Law' referred to by the League of Nations (1946) in the quote above. To maintain a balanced budget, for a given level of public expenditures, the reduction in seigniorage revenue further necessitates borrowing, suggesting a positive relationship between currency substitution and public debt. Recent experiences in Latin America confirm this relationship. Figure 5 plots the correlation between the annual dollarization ratio and the annual central government debt to GDP ratio for Mexico (1991-2018), Peru (2002-2018), and Uruguay (1981-2019). The observations are in line with the empirical work of Rojas-Suárez (1992) studying currency substitution in Peru, as well as Sims (2001) studying the case of Mexico by introducing the fiscal theory of the price level into a Barro (1979) tax-smoothing model. The paper at hand complements these studies by providing a microfounded framework explicitly characterizing the transmission channels at work.

6 Conclusion

This paper presents a search-theoretic dynamic dual-currency model featuring endogenous currency substitution as a function of jointly-determined fiscal and monetary policy. Benevolent

[39] A discussion of the relationship between fiscal imbalances and inflation in episodes of extreme (hyper-) inflation is provided by Dornbusch et al. (1990).

governments, unable to commit to policies beyond the current term, weigh distortion-smoothing and time-consistency, where steady-state public expenditures, public debt, labor taxation, and inflation are determined using the notion of a Markov-perfect equilibrium. Currency substitution arises as an endogenous response to cross-country differences in fundamentals.

Due to general equilibrium effects, changes in payment patterns yield important implications for government financing. While domestic and foreign policies are fully independent in the absence of currency substitution, due to global market clearing conditions, policies become intertwined once local agents settle transactions in foreign media of exchange. The reduced demand for local currency imposes a constraint on the local government's ability to generate seigniorage revenue. In the limiting case where local currency is not valued in equilibrium, taxation remains as the only revenue source. The foreign government benefits from having its currency accepted internationally, as it allows it to import seigniorage revenue from abroad.

Having established the effects of agent-driven currency substitution, the paper further studies an extension featuring 'de jure' dollarization, i.e., currency substitution imposed by official authorities. By abandoning its monetary independence, a country's policy objectives are reduced to inter-temporal distortion-smoothing exclusively. Since time-consistency is no longer a concern, the government is free to implement any public debt level, provided it satisfies the government's inter-temporal budget constraint.

The results presented in the paper provide important insights into the effects of endogenous currency substitution on government financing. While presented in the context of domestic and foreign currency, it is worthwhile to note that the constraints currency substitution (or the possibility of it) imposes on domestic government financing are applicable to a wider range of alternative media of exchange (e.g. crypto-currencies or financial assets). Thus, this paper intends to be a guiding reference for future studies on the relationship between fiscal policy, monetary policy, and payment patterns.

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Appendix: Derivations and Proofs

A. Proof: Lemma 1

Let us start with the labor tax rate, τ_j. Substituting (3) solved for h into (2) and taking the derivative with respect to c_j yields the equilibrium tax rates τ_d and τ_f in (15).

Next, we determine the CM prices of domestic and foreign currency, ϕ_d and ϕ_f. Using (10) and (12) with $\mathbb{I}_j = 0$ yields:

$$\phi_d m_d a_d (1-\tau_d)^{-1} = (1-\theta)u(x_{d,1}) + \theta x_{d,1}, \tag{A.1}$$

$$\phi_f \breve{m}_f a_f (1-\tau_f)^{-1} = (1-\theta)u(x_{f,1}) + \theta x_{f,1}. \tag{A.2}$$

Plugging τ_j in (15), (A.1), and (A.2) into (10) and (12) with $\mathbb{I}_j = 1$ using the market clearing conditions $n_d m_d + n_f \breve{m}_d = 1$ and $n_d m_f + n_f \breve{m}_f = 1$ yields the equilibrium prices ϕ_d and ϕ_f in (16).

Lastly, to determine the growth rates of the domestic and the foreign currency, μ_d and μ_f, as well as the issuance prices of domestic and foreign bonds, q_d, and q_f, we revisit the domestic and foreign consumers' portfolio choice problems in (5):

$$\begin{aligned} \max_{m'_d, m'_f, b'_d, b'_f} \quad & - \quad a_d(1-\tau_d)^{-1} \sum_{j=d,f} \big[m'_j[\phi_j(1+\mu_j)/\beta - \phi'_j] + b'_j[\phi_j q_j(1+\mu_j)/\beta - \phi'_j]\big] \\ & + \quad \theta\big[\alpha'_d[u(x'_{d,2}) - x'_{d,2}] + (1-\alpha'_d)[u(x'_{d,1}) - x'_{d,1}]\big], \end{aligned} \tag{A.3}$$

and:

$$\begin{aligned} \max_{\breve{m}'_d, \breve{m}'_f, \breve{b}'_d, \breve{b}'_f} \quad & - \quad a_f(1-\tau_f)^{-1} \sum_{j=d,f} \big[\breve{m}'_j[\phi_j(1+\mu_j)/\beta - \phi'_j] + \breve{b}'_j[\phi_j q_j(1+\mu_j)/\beta - \phi'_j]\big] \\ & + \quad \theta\big[\alpha'_f[u(x'_{f,2}) - x'_{f,2}] + (1-\alpha'_f)[u(x'_{f,1}) - x'_{f,1}]\big], \end{aligned} \tag{A.4}$$

respectively. Taking the first-order conditions of (A.3) and (A.4) and plugging in the tax rates, τ_d and τ_f, in (15) as well as the derivatives $\partial x'_{d,1}/\partial m'_d = U_c(c_d)\phi'_d/[(1-\theta)u_x(x'_{d,1})+\theta]$, $\partial x'_{d,2}/\partial m'_d = U_c(c_d)\phi'_d/[(1-\theta)u_x(x'_{d,2})+\theta]$, $\partial x'_{d,2}/\partial m'_f = U_c(c_d)\phi'_f/[(1-\theta)u_x(x'_{d,2})+\theta]$, $\partial x'_{f,1}/\partial \breve{m}'_f = U_c(c_f)\phi'_f/[(1-\theta)u_x(x'_{f,1}) + \theta]$, $\partial x'_{f,2}/\partial \breve{m}'_f = U_c(c_f)\phi'_f/[(1-\theta)u_x(x'_{f,2}) + \theta]$, and $\partial x'_{f,2}/\partial \breve{m}'_d = U_c(c_f)\phi'_d/[(1-\theta)u_x(x'_{f,2}) + \theta]$ obtained from (10) and (12) yields the equilibrium growth rates and bond issuance prices μ_d, μ_f, q_d, and q_f in (17)-(18). ■

B. Derivation: Comparative Statics

This section determines the comparative statics for $\alpha_d \in (0,1)$ and $\alpha_f = 0$ presented in Table 1. Let us start with the comparative statics with respect to the growth rate of the domestic currency supply, μ_d, taking as given the tax rate, τ_d. Given $x_{j,1} = x'_{j,1}$ and $x_{j,2} = x'_{j,2}$, and thus $\phi_j = \phi'_j$ in a steady state, the growth rates of the currency supply reduce to $\mu_j = \beta[1 + \Omega(x_{j,1}, x_{j,2})] - 1$ with Ω defined in (20). Implicit differentiation yields the inverse derivatives $\partial x_{d,1}/\partial \mu_d = [\beta \Omega_{x_{d,1}}]^{-1} < 0$

and $\partial x_{d,2}/\partial \mu_d = [\beta \Omega_{x_{d,2}}]^{-1} < 0$. Using $\phi_d = n_d \frac{\Phi(x_{d,1})}{U_c(c_d)}$ and $\phi_f = n_f \frac{\Phi(x_{f,1})}{U_c(c_f)} + n_d \frac{\Phi(x_{d,2}) - \Phi(x_{d,1})}{U_c(c_d)}$ given $\alpha_d \in (0,1)$ and $\alpha_f = 0$, and taking the derivative with respect to μ_d yields:

$$\frac{\partial \phi_d}{\partial \mu_d} = n_d \frac{\partial \Phi(x_{d,1})}{\partial x_{d,1}} \frac{\partial x_{d,1}}{\partial \mu_d} U_c^{-1}(c_d) < 0, \tag{B.1}$$

$$\frac{\partial \phi_f}{\partial \mu_d} = n_f \frac{\partial \Phi(x_{f,1})}{\partial x_{f,1}} \frac{\partial x_{f,1}}{\partial \mu_d} U_c^{-1}(c_f) + n_d \left[\frac{\partial \Phi(x_{d,2})}{\partial x_{d,2}} \frac{\partial x_{d,2}}{\partial \mu_d} - \frac{\partial \Phi(x_{d,1})}{\partial x_{d,1}} \frac{\partial x_{d,1}}{\partial \mu_d} \right] U_c^{-1}(c_d) > 0, \tag{B.2}$$

given $\partial \mu_d / \partial x_{f,1} = 0$ from $\mu_d = \beta[1 + \Omega(x_{d,1}, x_{d,2})] - 1$, and consequentially $\partial e/\partial \mu_d > 0$ with $e = \phi_f/\phi_d$. Furthermore, since $\partial \Omega(x_{d,1}, x_{d,2})/\partial \mu_d > 0$ and $\partial \Omega(x_{f,1})/\partial \mu_d < 0$ given $\partial \phi_f/\partial \mu_d > 0$, it holds that $\partial q_d/\partial \mu_d < 0$ and $\partial q_f/\partial \mu_d > 0$. The effects of a change in the domestic tax rate, τ_d, taking as given μ_d, are:

$$\frac{\partial \phi_d}{\partial \tau_d} = -n_d \frac{U_{cc}(c_d)\Phi(x_{d,1})}{U_c^2(c_d)} \frac{\partial c_d}{\partial \tau_d} < 0,$$

$$\frac{\partial \phi_f}{\partial \tau_d} = -n_d \frac{U_{cc}(c_d)[\Phi(x_{d,2}) - \Phi(x_{d,1})]}{U_c^2(c_d)} \frac{\partial c_d}{\partial \tau_d} < 0,$$

given $\partial c_d/\partial \tau_d < 0$ in (15). Further, it holds that:

$$\frac{\partial e}{\partial \tau_d} = \frac{\Phi(x_{f,1})}{\Phi(x_{d,1})} \frac{U_{cc}(c_d)}{U_c(c_f)} \frac{\partial c_d}{\partial \tau_d} > 0.$$

Lastly, since $\partial \phi_d/\partial \tau_d < 0$ and $\partial \phi_f/\partial \tau_d < 0$, it holds that $\partial q_d/\partial \tau_d < 0$ and $\partial q_f/\partial \tau_d < 0$.

C. Proof: Proposition 1

The proof proceeds in two steps. In the first step, I prove the results (i) and (ii), followed by the proof of (iii) and (iv) in the second step.

Proof of (i) and (ii): The proof of (i) and (ii), i.e., the proof that $\lambda > 0$ for all $\Gamma \in [-1, \bar{B}]$ given (27) and (28), starts with an auxiliary lemma:

Lemma 2. *$\lambda = 0$ if and only if $\{x_{d,1}, c_d, g_d\} = \{\hat{x}, \hat{c}, \hat{g}\}$ in all periods.*

Proof of Lemma 2: Suppose $\lambda = 0$. From equation (26), we get $x_{d,1} = \hat{x}$, and thus, given $\Phi(\hat{x})\Omega_{x_{d,1}}(\hat{x}) < 0$ and $\alpha_d = 0$, $\Omega(\hat{x})\Phi_x(\hat{x})/\theta = u_x(\hat{x}) - 1 = \xi = 0$ is the only solution. Plugging $\lambda = 0$ into (27) and (28) suggests $c_d = \hat{c}$ and $g_d = \hat{g}$. From (25), we get $\lambda' = 0$, and thus $x'_{d,1} = \hat{x}$, $c'_d = \hat{c}$, and $g'_d = \hat{g}$. Assuming now that $\{\hat{x}, \hat{c}, \hat{g}\}$ in all periods, then (26)-(28) all imply $\lambda = \lambda' = 0$, suggesting $\mu_d = \beta - 1$, $\tau_d = 0$, $\phi_d = n_d\Phi(\hat{x})/a_d$, $q_d = 1$, and $\hat{B} = -1 - a_d\hat{g}/(1-\beta)n_d\Phi(\hat{x})$. ■

Proof by contradiction. Suppose $\lambda = 0$ for some $B_d \in \Gamma$, then (26)-(28) imply $x_{d,1} = \hat{x}$, $c_d = \hat{c}$, and $g_d = \hat{g}$, yielding the $\mathcal{B}'_d(B_d) = a_d\hat{g}/\beta n_d\Phi(\hat{x}) + (1 + B_d)/\beta - 1 > B_d$. Given Lemma (2), $\lambda' = 0$, and thus $\{\hat{x}, \hat{c}, \hat{g}\}$ is also implemented in all future periods. To check whether said first-best steady state is sustainable by continuously rolling over debt, what remains to be shown is whether agents are willing to hold domestic bonds in the first place. In the first-best steady state outlined above the domestic government's value function (22) corresponds to:

$$\hat{\mathcal{Z}}_d = (1-\beta)^{-1}[n_d u(\hat{x}) - (1-n_d)\hat{x} + U(\hat{c}) + \eta_d v(\hat{g}) - a_d(\hat{c}+\hat{g})],$$

given $\alpha_d = 0$ for $x_{d,1} = \hat{x}$. Using the CM budget constraint (3), assuming a consumer spends all her money in the DM, and thus arrives with $m_d = 0$ in the CM, if a consumer leaves the CM with $m'_d = 1$ and $b'_d = \hat{B}$ she works $\hat{h} = \hat{c} + \hat{g} + \frac{n_d \Phi(\hat{x})}{a_d}$, suggesting the following equilibrium payoff in the CM:[40]

$$\hat{W}^c_d = U(\hat{c}) + \eta_d v(\hat{g}) - a_d(\hat{c}+\hat{g}) - n_d\Phi(\hat{x}) + \beta\hat{\mathcal{Z}}_d.$$

Assume now a consumer deviates from the above strategy and instead arrives in the DM with $m'_d = 1$ and $b'_d = 0$, and thus she sells all her bonds to save on work in the CM. In said situation, in future periods, given (3), the consumer works $h_{dev} = \hat{c} - (1-\beta)\frac{n_d\Phi(\hat{x})}{a_d}$. In the current period of deviation, the consumer's labor input is $h_{dev} = \hat{c} + (\beta - B_d)\frac{n_d\Phi(\hat{x})}{a_d}$, suggesting a CM payoff of:

$$\hat{W}^c_{d,dev} = U(\hat{c}) + \eta_d v(\hat{g}) - a_d\hat{c} - (\beta - B_d)n_d\Phi(\hat{x}) + \beta\hat{\mathcal{Z}}_{d,dev},$$

with the continuation value:

$$\hat{\mathcal{Z}}_{d,dev} = (1-\beta)^{-1}[n_d u(\hat{x}) - (1-n_d)\hat{x} + U(\hat{c}) + \eta_d v(\hat{g}) - a_d\hat{c} + (1-\beta)n_d\Phi(\hat{x})],$$

given the future labor input, $h_{dev} = \hat{c} - (1-\beta)\frac{n_d\Phi(\hat{x})}{a_d}$. After some rearrangement, one can show that if $B_d > \hat{B}$ the deviation is profitable, i.e., $\hat{W}^c_{d,dev} > \hat{W}^c_d$ for $B_d > \hat{B} \equiv -1 - a_d\hat{g}/(1-\beta)n_d\Phi(\hat{x})$. Since $\hat{B} < -1$, this deviation is profitable for all $B_d \in \Gamma$, contradicting the conjecture that $\lambda = 0$ for some $B_d \in \Gamma$. Hence, continuously rolling-over debt to maintain a first-best steady state is not sustainable in the long run. ■

Proof of (iii) and (iv): The proof of (iii) and (iv) starts with two auxiliary lemmata.

Lemma 3. *$\xi = 0$ for all $B_j \in \Gamma$.*

Proof of Lemma 3: The proof follows from (i) and (ii). Given $\lambda > 0$, and thus $\lambda\varepsilon_{x_{d,1}} \equiv -\lambda n_d \Phi_{x_{d,1}}(x_{d,1})[1+B_d] \leq 0$ in (26) for $B_d \in \Gamma$, it needs to hold that $\Omega_d\Phi_{x_{d,1}}(x_{d,1})/\theta + \xi[\Phi_{x_{d,1}}(x_{d,1})\Omega_d + \Phi(x_{d,1})\Omega_{d,x_{d,1}}] \geq 0$. Suppose $\Omega_d\Phi_{x_{d,1}}(x_{d,1})/\theta + \xi[\Phi_{x_{d,1}}(x_{d,1})\Omega_d + \Phi(x_{d,1})\Omega_{d,x_{d,1}}] = 0$, and thus $B_d = -1$ for (26) to hold with equality, the only solution is $x_{d,1} = \hat{x}$ and $\xi = 0$. Suppose now that $\Omega_d\Phi_{x_{d,1}}(x_{d,1})/\theta + \xi[\Phi_{x_{d,1}}(x_{d,1})\Omega_d + \Phi(x_{d,1})\Omega_{d,x_{d,1}}] > 0$, and thus $B_d > -1$, then given $x_{d,1} < \hat{x}$ from (i), the only solution is $\xi = 0$. ■

Lemma 4. *$\partial x_{d,1}/\partial B_d < 0$ for all $\Gamma \in [-1, \bar{B}]$.*

Proof of Lemma 4: Proof by contradiction. Suppose $\partial x_{d,1}/\partial B_d \geq 0$ (and thus $\partial x_{d,2}/\partial B_d \geq 0$)

[40]Note that for the proof the domestic agents' foreign bond holdings, b_f, can be ignored without loss of generality.

for some $\Gamma \in [-1, \bar{B}]$. Then, given $\xi = 0$, totally differentiating (26) yields:

$$\left[\Omega_{d,x_{d,1}} \frac{\partial x_{d,1}}{\partial B_d} + \Omega_{d,x_{d,2}} \frac{\partial x_{d,2}}{\partial B_d}\right] \Phi_{x_{d,1}}(x_{d,1})/\theta + [\Omega_d/\theta - \lambda[1+B_d]]\Phi_{x_{d,1}x_{d,1}}(x_{d,1}) \frac{\partial x_{d,1}}{\partial B_d}$$
$$= \lambda \Phi_{x_{d,1}}(x_{d,1}) + \frac{\partial \lambda}{\partial B_d} \Phi_{x_{d,1}}(x_{d,1})[1+B_d],$$

where the left hand side is weakly negative given $\Omega_d/\theta - \lambda[1+B_d] = 0$ from (26) with $\xi = 0$. Thus, given $\lambda > 0$, $\lambda \Phi_{x_{d,1}}(x_{d,1}) + \partial\lambda/\partial B_d \Phi_{x_{d,1}}(x_{d,1})[1+B_d] < 0$ cannot be satisfied for $B_d = -1$, and suggests $\partial\lambda/\partial B_d < 0$ for $B_d > -1$. Thus, given (28), $\partial\lambda/\partial B_d < 0$ implies $\partial g_d/\partial B_d > 0$, and consequentially $\partial c_d/\partial B_d > 0$. The latter, i.e., the fact that g_d and c_d move in the same direction, $dc_d/dg_d > 0$, can be shown using the Implicit Function Theorem. Combining (27) and (28) gives $F(c_d, g_d) = \eta_d v_g(g_d)[U_c(c_d) - a_d] + [\eta_d v_g(g_d) - a_d]U_{cc}(c_d)c_d = 0$, where $dc_d/dg_d = -F_g/F_c$ with $F_g = \eta_d v_{gg}(g_d)[U_c(c_d) - a_d + U_{cc}(c_d)c_d] > 0$ (given $\lambda > 0$ suggesting $U_c(c_d) - a_d + U_{cc}(c_d)c_d < 0$ in (27)) and $F_c = \eta_d v_g(g_d)U_{cc}(c_d) + (\eta_d v_g(g_d) - a_d)[U_{ccc}(c_d)c_d + U_{cc}(c_d)]$, which can be rewritten to $F_c = \eta_d v_g(g_d)[U_{cc}(c_d)c_d - [U_c(c_d) - a_d][1 + U_{ccc}(c_d)c_d/U_{cc}(c_d)]]/c_d < 0$. In words, if $\partial x_{d,1}/\partial B_d \geq 0$ was true, then the government could increase welfare by increasing public debt, $\partial x_{d,1}/\partial B_d \geq 0$, $\partial g_d/\partial B_d > 0$, and $\partial c_d/\partial B_d > 0$. A contradiction with the envelope condition $\mathcal{Z}_{d,B_d} = \lambda \varepsilon_{B_d} < 0$. ∎

Proof by contradiction. Suppose future debt $\mathcal{B}'_d(B_d, B_f) = -1$ for some $B_d \in \Gamma$, then (25) reduces to:

$$(\lambda - \lambda')\Phi(x'_{d,1}) + \lambda\left[\left[\Phi_{x'_{d,1}}(x'_{d,1})\Omega'_d + \Phi(x'_{d,1})\Omega'_{d,x'_{d,1}}\right]\frac{\partial \mathcal{X}'_{d,1}}{\partial B'_d} + \Phi(x'_{d,1})\Omega'_{d,x'_{d,2}}\frac{\partial \mathcal{X}'_{d,2}}{\partial B'_d}\right] = 0,$$

where the second term on the left-hand side is positive for $\Phi_{x'_{d,1}}(x'_{d,1})\Omega'_d + \Phi(x'_{d,1})\Omega'_{d,x'_{d,1}} < 0$, and thus, given $\Phi(x'_{d,1}) > 0$, suggesting $\lambda - \lambda' < 0$, which holds regardless of $x'_{d,1} = \hat{x}$ or $x'_{d,1} < \hat{x}$, ruling out $B_d = -1$. Thus, $\mathcal{B}'_d(-1) > -1$. Suppose now $B_d > -1$. Then, evaluating the government budget constraint (21) today and tomorrow yields:

$$[U_c(c_d) - a_d]c_d - a_d g_d - n_d\Phi(x_{d,1})[1 + B_d] = 0$$
$$[U_c(c'_d) - a_d]c'_d - a_d g'_d + \beta n_d \Phi(x''_{d,1})[1 + \Omega(x''_{d,1}, x''_{d,2}) + B''_d] = 0,$$

where $B''_d \equiv \mathcal{B}''_d(-1)$, $x''_{d,1} = \mathcal{X}''_{d,1}(B''_d, B''_f)$ and $x''_{d,2} = \mathcal{X}''_{d,2}(B''_d, B''_f)$. Considering the first equation, given $B_d > -1$ and $n_d\Phi(x_{d,1}) > 0$, we know that $[U_c(c_d) - a_d]c_d - a_d g_d > 0$. Furthermore, given $B''_d > -1$, $\Phi(x_{d,1}) > 0$, and $\Omega(x_{d,1}, x_{d,2}) > 0$, it holds that $[U_c(c'_d) - a_d]c'_d - a_d g'_d < 0$ in the second equation. Since c_d and g_d move in the same direction, i.e., $dc_d/dg_d > 0$, $[U_c(c_d) - a_d]c_d - a_d g_d > 0$ and $[U_c(c'_d) - a_d]c'_d - a_d g'_d < 0$ imply that $c < c'$ and $g < g'$. Given (28), this implies $\lambda - \lambda' > 0$ which contradicts $\lambda - \lambda' < 0$ above. Thus, $\mathcal{B}'_d(B_d) > -1$ for all $B_d \in \Gamma$. $q_d < 1$ follows immediately since $B_d > -1$ implies $x_{d,1} < \hat{x}$. ∎

D. Proof: Proposition 2

The proof proceeds in two steps. In part (i) I determine $\mathcal{B}'_d(B^N) = B^N$ and $\mathcal{B}'_d(B_d) \to B^N$ for $v_g \to \psi$ with $\eta_d\psi > a_d$, followed by a characterization of the Ramsey problem in part (ii).

Part (i): Starting with $\mathcal{B}'_d(B^N) = B^N$, the uniqueness of B^N follows from the uniqueness of the solution to the steady-state system of equations, (29)-(32), evaluated at B^N. For a given $\{x^N, c^N\}$, (29) yields a unique solution for B^N, while (32) yields a unique solution for g^N. To verify whether (30) and (31) yield unique solutions for x^N and c^N for any given $\{B^N, g^N\}$, we need to show that (30) is strictly decreasing in x^N and (31) is strictly decreasing in c^N. Rearranging (30) to $a_d[u_x(x^N) - 1]/\Phi_x(x^N) - [\eta_d v_g(g^N) - a_d][1 + B^N] = 0$ and taking the derivative with respect to x^N yields $a_d u_{xx}(x^N)/[(1-\theta)u_x(x^N) + \theta]^2 < 0$. Further, taking the derivative of (31) with respect to c^N yields $[\eta_d v_g(g^N) - a_d][U_{cc}(c^N) + U_{ccc}(c^N)c^N] + \eta_d v_g(g^N)U_{cc}(c^N) < 0$ (as shown in part (iv) of Appendix C). Thus B^N is unique.

Let us now determine $\mathcal{B}'_d(B_d) \to B^N$ for $v_g \to \psi$ with $\eta_d\psi > a_d$. Consider (25) with $\lambda = \lambda'$ and $\lambda = \eta_d v_g/a_d - 1$, yielding $[\eta_d v_g - a_d]\varepsilon_{x'_{d,1}}\partial\mathcal{X}'_{d,1}/\partial B'_d = 0$. From Proposition 1 we know that $\partial\mathcal{X}'_{d,1}/\partial B'_d < 0$ and $g_d < \hat{g}$ for $B_d \geq -1$, thus implying $\varepsilon_{x'_{d,1}} = 0$. If $v_g \to \psi$ with $\eta_d\psi > a_d$, the aforementioned expression becomes a function of B'_d and $\mathcal{X}'_{d,1}(B'_d)$ exclusively, i.e., $[\eta_d\psi - a_d]\varepsilon_{x'_{d,1}}\partial\mathcal{X}'_{d,1}/\partial B'_d = 0$, and thus the solution B'_d is the same for all $B_d \in \Gamma$, i.e., $\mathcal{B}'_d(B_d)$ is a constant. Given the uniqueness of B^N, i.e., $\mathcal{B}'_d(B^N) = B^N$, it holds that $\mathcal{B}'_d(B_d) = B^N$ for all $B_d \in \Gamma$. What remains to be verified is whether $\partial\mathcal{X}_{d,1}/\partial B_d < 0$ holds in said limiting case. To do so, use (26), rearranged to $[u_x(x_{d,1}) - 1]/\Phi_x(x_{d,1}) = [\eta_d\psi - a_d][1 + B_d]/a_d$. Differentiating with respect to B_d yields:

$$\frac{u_{xx}(x_{d,1})}{[(1-\theta)u_x(x_{d,1}) + \theta]^2}\frac{\partial x_{d,1}}{\partial B_d} = \frac{\eta_d\psi - a_d}{a_d} > 0,$$

and thus $\partial x_{d,1}/\partial B_d < 0$ given $u_{xx}(x_{d,1})/[(1-\theta)u_x(x_{d,1}) + \theta]^2 < 0$. ■

Part (ii): To formulate the problem of a government with commitment, we set up a standard Ramsey problem. Following Chari et al. (1991), we reduce the government budget constraints into a single implementability constraint. Using the domestic government budget constraint in (1) with the aggregate resource constraint $h = c + g$, multiplying by β^t for every period, and summing up over all periods, $t = 0, ..., \infty$, gives:

$$\sum_{t=0}^{\infty}\beta^t\Big[\tau_t c_t - (1-\tau_t)g_t + \phi_t\big[(1+\mu_t)(1+q_t B_{t+1}) - (1+B_t)\big]\Big] = 0. \tag{D.1}$$

For brevity, since the analysis focuses on the domestic government, the subscript d is omitted. Using $\tau_t = 1 - a/U_{c,t}$ in (15) and the transversality condition $\lim_{T\to\infty}\beta^T\phi_T(1+\mu_T)(1+q_T B_{T+1}) = 0$, equation (D.1) simplifies to:

$$\begin{aligned}&\textstyle\sum_{t=0}^{\infty}\beta^t\big[(U_{c,t}-a)c_t-ag_t\big]-U_{c,0}\phi_0(1+B_0)+\\&\textstyle\sum_{t=1}^{\infty}\beta^{t-1}\big[U_{c,t-1}\phi_{t-1}(1+\mu_{t-1})(1+q_{t-1}B_t)-\beta U_{c,t}\phi_t(1+B_t)\big]=0,\end{aligned}$$

which can be rearranged to:

$$\begin{aligned}&\textstyle\sum_{t=0}^{\infty}\beta^t\big[(U_{c,t}-a)c_t-ag_t\big]-U_{c,0}\phi_0(1+B_0)+\\&\textstyle\sum_{t=1}^{\infty}\beta^{t-1}\Big[U_{c,t-1}\phi_{t-1}(1+\mu_{t-1})-\beta U_{c,t}\phi_t+\big[U_{c,t-1}\phi_{t-1}(1+\mu_{t-1})q_{t-1}-\beta U_{c,t}\phi_t\big]B_t\Big]=0.\end{aligned}\tag{D.2}$$

Using the equilibrium conditions (16), (17), and (18), the term (D.2) reduces to:

$$\sum_{t=0}^{\infty}\beta^t\big[(U_{c,t}-a)c_t-ag_t+n\Phi(x_t)\Omega(x_t)\big]-n\Phi(x_0)\Omega(x_0)-n\Phi(x_0)(1+B_0)=0,\tag{D.3}$$

which characterizes our implementability condition. Hence, given $B_0>-1$, the problem of a government with commitment is to solve:

$$\max_{x_t,c_t,g_t}\quad\sum_{t=0}^{\infty}\beta^t\big[nu(x_t)-(1-n)x_t+U(c_t)+\eta v(g_t)-a(c_t+g_t)\big],$$

subject to (D.3). Using the Lagrange multiplier λ, the first-order conditions are:

$$u_{x_0}-1-\lambda(1+B_0)\Phi_{x_0}=0\quad\text{for }\ t=0,\tag{D.4}$$
$$u_{x_t}-1+\lambda[\Phi_{x_t}\Omega_t+\Phi_t\Omega_{x_t}]=0\quad\text{for all }\ t\geq 1,\tag{D.5}$$
$$U_{c_t}-a+\lambda[U_{c_tc_t}c_t+U_{c_t}-a]=0\quad\text{for all }\ t\geq 0,\tag{D.6}$$
$$\eta v_{g_t}-a-\lambda a=0\quad\text{for all }\ t\geq 0.\tag{D.7}$$

From the first-order condition (D.6) and (D.7) one can see that c_t and g_t are constant for all $t\geq 0$, while the first-order condition (D.5) shows that x_t is constant for all $t\geq 1$, but may be different in $t=0$. Given that, one can rewrite the implementability condition as:

$$(U_c-a)c-ag+\beta n\Phi(x_1)\Omega(x_1)-(1-\beta)n\Phi(x_0)(1+B_0)=0.\tag{D.8}$$

Plugging the above equation (D.8) into the updated government budget constraint (21) yields:

$$B_t=\frac{\Phi(x_0)(1+B_0)}{\Phi(x_1)}-1,$$

for all $t\geq 1$, suggesting that debt remains constant after the initial period. Solving (D.7) after λ and plugging into (D.4)-(D.6) and (21) gives the updated equilibrium conditions solving the allocation $\{x_0, x_1, c, g\}$:

$$
\begin{aligned}
a(u_{x_0} - 1) - (\eta v_g - a)(1 + B_0)\Phi_{x_0} &= 0,\\
a(u_{x_1} - 1) + (\eta v_g - a)[\Phi_{x_1}\Omega_1 + \Phi_1\Omega_{x_1}] &= 0,\\
(\eta v_g - a)U_{cc}c + \eta v_g(U_c - a) &= 0,\\
(U_c - a)c - ag + \beta n\Phi(x_1)\Omega(x_1) - (1 - \beta)n\Phi(x_0)(1 + B_0) &= 0.
\end{aligned}
$$

Suppose the initial debt level is equivalent to the steady-state debt level, i.e., $B_0 = B^N$ in (29). Then the above FOCs are equivalent to the FOCs in the equilibrium steady state without commitment, (30)-(32). Thus, given $B_0 = B^N$ in (29), the solution to the Ramsey problem of a government with commitment is equivalent to the equilibrium steady state without commitment, i.e., $\{B_t, x_t, c_t, g_t\} = \{B^N, x^N, c^N, g^N\}$ solving (29)-(32) in Proposition 2, for all $t \geq 0$. ■

E. Proof: Proposition 3

The proof proceeds in three steps. Part (i) shows that for a given foreign steady-state Υ^F, the domestic steady-state allocations, $\Upsilon^D \equiv \{B^D, x_1^D, x_2^D, c^D, g^D\}$, solve (33)-(36), where for all $B_d \in \Gamma$ and $\alpha_d \in [0, 1)$ it holds that (a) $\mathcal{B}'_d(B^D, B^F) = B^D$ and (b) $\mathcal{B}'_d(B_d, B^F) \to B^D$ for $v_g(g_d) \to \psi$ with $\eta_d\psi > a_d$. Part (ii) confirms (37). Lastly, part (iii) shows that at B^D, the equilibrium steady state, Υ^D, is robust to commitment frictions.

Part (i): The proof proceeds in two steps, (a) and (b). Assume $\alpha_d \in [0, 1)$. Starting with (a), i.e., $\mathcal{B}'_d(B^D, B^F) = B^D$, the uniqueness of B^D follows from the uniqueness of the solution to the steady-state system of equations, (33)-(36), evaluated at B^D. For a given Υ^F, and thus a given $\{x_1^D, x_2^D(x_1^D), c^D\}$, where the correspondence $x_2^D(x_1^D)$ is uniquely pinned down in (10), (33) yields a unique solution for B^D and (36) yields a unique solution for g^D. Vice versa, for a given Υ^F and a given $\{B^D, g^D\}$, we need to show that (34) is strictly decreasing in x_1^D and (35) is strictly decreasing in c^D. Taking the derivative of (34) with respect to x_1^D yields $a_d\Omega_{d,x_1^D}/\theta < 0$. Further, taking the derivative of (35) with respect to c^D yields $[\eta_d v_g(g^D) - a_d][U_{cc}(c^D) + U_{ccc}(c^D)c^D] + \eta_d v_g(g^D)U_{cc}(c^D) < 0$ (as proven in part (iv) of Appendix C). Thus B^D is unique.

Moving on to (b), to show that $\mathcal{B}'_d(B_d, B^F) \to B^D$, i.e., B_d converges to B^D in the long run for $v_g(g_d) \to \psi$ with $\eta_d\psi > a_d$, consider (25) with $\lambda = \lambda'$ and $\lambda = \eta_d v_g/a_d - 1$, yielding $[\eta_d v_g - a_d][\varepsilon_{x'_{d,1}} + \varepsilon_{x'_{d,2}}\Phi_x(x'_{d,1})/\Phi_x(x'_{d,2})]\partial\mathcal{X}'_{d,1}/\partial B'_d = 0$. Given $\partial\mathcal{X}'_{d,1}/\partial B'_d < 0$ and $g_d < \hat{g}$ for $B_d \in \Gamma$, it holds that $\varepsilon_{x'_{d,1}} + \varepsilon_{x'_{d,2}}\Phi_x(x'_{d,1})/\Phi_x(x'_{d,2}) = 0$, where for a given Υ^F, $\varepsilon_{x'_{d,1}} + \varepsilon_{x'_{d,2}}\Phi_x(x'_{d,1})/\Phi_x(x'_{d,2}) = 0$ is a function of B'_d, $\mathcal{X}'_{d,1}(B'_d, B^F)$, and $\mathcal{X}'_{d,2}(B'_d, B^F)$ (with $\mathcal{X}'_{d,2}(\mathcal{X}'_{d,1})$ uniquely pinned down for a given Υ^F), and thus so is $[\eta_d v_g - a_d][\varepsilon_{x'_{d,1}} + \varepsilon_{x'_{d,2}}\Phi_x(x'_{d,1})/\Phi_x(x'_{d,2})]\partial\mathcal{X}'_{d,1}/\partial B'_d = 0$ if $v_g(g_d) \to \psi$ with $\eta_d\psi > a_d$. In other words, for a given Υ^F, B'_d is the same for all $B_d \in \Gamma$, i.e., $\mathcal{B}'_d(B_d, B^F)$ is a constant. Given the uniqueness of B^D, i.e., $\mathcal{B}'_d(B^D, B^F) = B^D$, it holds that $\mathcal{B}'_d(B_d, B^F) = B^D$ for all $B_d \in \Gamma$. Lastly, to verify that $\partial\mathcal{X}_{d,1}/\partial B_d < 0$ holds in said limiting case, combine (26),

$\lambda = \eta_d v_g/a_d - 1$, and $\xi = 0$ to $\Omega_d/\theta = [\eta_d\psi - a_d][1 + B_d]/a_d$ and differentiate with respect to B_d to get:

$$\left[\Omega_{d,x_{d,1}} \frac{\partial \mathcal{X}_{d,1}}{\partial B_d} + \Omega_{d,x_{d,2}} \frac{\partial \mathcal{X}_{d,2}}{\partial B_d}\right]/\theta = \frac{\eta_d \psi - a_d}{a_d} > 0,$$

and thus $\partial \mathcal{X}_{d,1}/\partial B_d < 0$ given $\Omega_{d,x_{d,1}} < 0$ and $\Omega_{d,x_{d,2}} \leq 0$.

Part (ii): As shown in (i), for a given Υ^F and the correspondence $x_{d,2}(x_{d,1})$ in (10), the steady-state system of equations (33)-(36) solves $\Upsilon^D = \{B^D, x_1^D, x_2^D, c^D, g^D\}$, and thus the mapping $g^D \to x_1^D$. Given (14) and incorporating the solutions to the consumer's portfolio choice in (5), for a given foreign inflation rate μ_f, an increase in domestic inflation μ_d (driven by an increase in public expenditures g_d) increases a domestic producers' benefits of accepting foreign currency Π_d, i.e., $\partial\Pi_d/\partial\mu_d|_{\mu_f} > 0$. Thus, for a given foreign steady-state Υ^F, there exist the threshold domestic public expenditures $g^D = \{\underline{g}, \bar{g}\}$, and hence $\eta_d = \{\underline{\eta}, \bar{\eta}\}$, satisfying $\Pi_d = \underline{k}$ and $\Pi_d = \bar{k}$, respectively, characterizing α_d in (37).

Part (iii): To verify robustness to commitment frictions at B^D, plug (16), (17), and (18) into (D.2) to get the implementability condition:

$$\sum_{t=0}^{\infty} \beta^t \big[(U_{c,t} - a)c_t - ag_t + n\Phi(x_{1,t})\Omega(x_{1,t}, x_{2,t})\big] - n\Phi(x_{1,0})\Omega(x_{1,0}, x_{2,0}) - n\Phi(x_{1,0})(1 + B_0) = 0. \quad \text{(E.1)}$$

Solving the government's objective function:

$$\max_{x_{1,t}, c_t, g_t} \quad \sum_{t=0}^{\infty} \beta^t \big[\alpha_t[nu(x_{2,t}) - (1-n)x_{2,t}] + (1 - \alpha_t)[nu(x_{1,t}) - (1-n)x_{1,t}] \\ + U(c_t) + \eta v(g_t) - a(c_t + g_t) - \alpha_t(1 - n)\mathbb{E}(k_t \mid k_t \leq \Pi_t)\big],$$

subject to (E.1) with the Lagrange multiplier λ yields the first-order conditions:

$$\Omega(x_{1,0}, x_{2,0})/\theta - \lambda(1 + B_0) = 0 \quad \text{for } t = 0, \quad \text{(E.2)}$$
$$\Omega(x_{1,t}, x_{2,t})\Phi_{x_{1,t}}(x_{1,t})/\theta + \lambda[\Phi_{x_{1,t}}\Omega_t + \Phi_t\Omega_{x_{1,t}}] = 0 \quad \text{for all } t \geq 1, \quad \text{(E.3)}$$
$$U_{c_t} - a + \lambda[U_{c_t c_t} c_t + U_{c_t} - a] = 0 \quad \text{for all } t \geq 0, \quad \text{(E.4)}$$
$$\eta v_{g_t} - a - \lambda a = 0 \quad \text{for all } t \geq 0. \quad \text{(E.5)}$$

From the first-order condition (E.4) and (E.5) one can see that c_t and g_t are constant for all $t \geq 0$, while the first-order condition (E.3) shows that $x_{1,t}$ and $x_{2,t}$ are constant for all $t \geq 1$, but may be different in $t = 0$. Given that, one can rewrite the implementability condition as:

$$(U_c - a)c - ag + \beta n\Phi(x_{1,t})\Omega(x_{1,t}, x_{2,t}) - (1 - \beta)n\Phi(x_{1,0})(1 + B_0) = 0.$$

Plugging the above equation into the updated government budget constraint (21) yields:

$$B_t = \frac{\Phi(x_{1,0})(1+B_0)}{\Phi(x_{1,1})} - 1,$$

for all $t \geq 1$, suggesting that debt remains constant after the initial period. Solving (E.5) after λ and plugging into (E.2)-(E.4) and (21) gives the updated equilibrium conditions:

$$\begin{aligned}
a\Omega(x_{1,0}, x_{2,0})/\theta - [\eta v_g - a](1+B_0) &= 0, \\
a\Omega(x_{1,1}, x_{2,1})\Phi_{x_{1,1}}(x_{1,1})/\theta + [\eta v_g - a][\Phi_{x_{1,1}}\Omega_1 + \Phi_1 \Omega_{x_{1,1}}] &= 0, \\
[\eta v_g - a]U_{cc}c + \eta v_g[U_c - a] &= 0, \\
(U_c - a)c - ag + \beta n \Phi(x_{1,1})\Omega(x_{1,1}, x_{2,1}) - (1-\beta)n\Phi(x_{1,0})(1+B_0) &= 0.
\end{aligned}$$

Suppose the initial debt level is equivalent to the steady-state debt level, i.e., $B_0 = B^D$ in (33). Then the above FOCs are equivalent to the FOCs in the equilibrium steady state without commitment, (34)-(36). ■

F. Derivation: Foreign Government Budget Constraint

Plugging τ_f, ϕ_f, μ_f, and q_f in (15)-(18), into (1) and rearranging yields:

$$\begin{aligned}
\varepsilon_f(\cdot) \;\; \equiv \;\; & [U_c(c_f) - a_f]c_f - a_f g_f - \left[n_f\Phi(x_{f,1}) + n_d\tfrac{U_c(c_f)}{U_c(c_d)}[\Phi(x_{d,2}) - \Phi(x_{d,1})]\right][1+B_f] \\
+ \;\; & \beta\left[n_f\Phi(x'_{f,1}) + n_d\tfrac{U_c(c_f)}{U_c(c_d)}[\Phi(x'_{d,2}) - \Phi(x'_{d,1})]\right][1+\Omega(x'_{f,1}) + B'_f] = 0,
\end{aligned}$$

in (38) with the partial derivatives:

$$\begin{aligned}
\varepsilon_{f,B_f} &= -n_f\Phi(x_{f,1}) - n_d\tfrac{U_c(c_f)}{U_c(c_d)}[\Phi(x_{d,2}) - \Phi(x_{d,1})], \\
\varepsilon_{f,B'_f} &= \beta n_f\Phi(x'_{f,1}) + \beta n_d\tfrac{U_c(c_f)}{U_c(c_d)}[\Phi(x'_{d,2}) - \Phi(x'_{d,1})], \\
\varepsilon_{f,c_f} &= U_c(c_f) - a_f + U_{cc}(c_f)c_f \\
&- n_d\tfrac{U_{cc}(c_f)}{U_c(c_d)}\Big[[\Phi(x_{d,2}) - \Phi(x_{d,1})][1+B_f] - \beta[\Phi(x'_{d,2}) - \Phi(x'_{d,1})][1+\Omega(x'_{f,1}) + B'_f]\Big], \\
\varepsilon_{f,c_d} &= n_d\tfrac{U_{cc}(c_d)U_c(c_f)}{U_c^2(c_d)}\Big[[\Phi(x_{d,2}) - \Phi(x_{d,1})][1+B_f] - \beta[\Phi(x'_{d,2}) - \Phi(x'_{d,1})][1+\Omega(x'_{f,1}) + B'_f]\Big], \\
\varepsilon_{f,g_f} &= -a_f, \\
\varepsilon_{f,x_{f,1}} &= -n_f\Phi_x(x_{f,1})[1+B_f], \\
\varepsilon_{f,x'_{f,1}} &= \beta n_f\Phi_x(x'_{f,1})[1+\Omega(x'_{f,1}) + B'_f] + \beta\Big[n_f\Phi(x'_{f,1}) + n_d\tfrac{U_c(c_f)}{U_c(c_d)}[\Phi(x'_{d,2}) - \Phi(x'_{d,1})]\Big]\Omega_x(x'_{f,1}), \\
\varepsilon_{f,x_{d,1}} &= n_d\tfrac{U_c(c_f)}{U_c(c_d)}\Phi_x(x_{d,1})[1+B_f], \\
\varepsilon_{f,x_{d,2}} &= -n_d\tfrac{U_c(c_f)}{U_c(c_d)}\Phi_x(x_{d,2})[1+B_f], \\
\varepsilon_{f,x'_{d,1}} &= -\beta n_d\tfrac{U_c(c_f)}{U_c(c_d)}\Phi_x(x'_{d,1})[1+\Omega(x'_{f,1}) + B'_f], \\
\varepsilon_{f,x'_{d,2}} &= \beta n_d\tfrac{U_c(c_f)}{U_c(c_d)}\Phi_x(x'_{d,2})[1+\Omega(x'_{f,1}) + B'_f].
\end{aligned}$$