A multivariate stochastic volatility model applied to a panel of S&P500 stocks in different industries.

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Abstract

We estimate a multivariate stochastic volatility model for a panel of stock returns for a number of S&P 500 firms from different industries. As in the case of a univariate model we use an efficient importance sampling (EIS) method to estimate the likelihood function of the given multivariate system that we analyze. As opposed to univariate methods where each return is estimated separately for each firm, our results are based on joint estimation that can account for potential common error term interactions based on industry characteristics that cannot be detected by univariate methods. Our results reveal that there are ımportant differences in the industry effects, something that suggests that differential gains to portfolio allocations in the different industries that we examine. There are differences due to idiosyncratic factors and the common industry factors that suggest that each industry requires a separate treatment in arriving at portfolio allocations.

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1 Introduction

Asset volatility has been an area of intense research in finance and financial econometrics in particular in the last three decades. The seminal work of Engle (1982) who introduced the ARCH-type models has been one of the main vehicles to study and model volatility as a conditional moment of interest that needs estimating. in the same way as the conditional mean that defines the typical regression function needs to be estimated. In their generalized (GARCH) form these models assume that conditional variances are functions of past variances and/or past squared returns. At the same time, stochastic Volatility (SV) models, introduced by Taylor (1982, 1994) offer an important alternative to ARCH-type models and are the focus of our paper. Under SV, volatility follows a stochastic process generally independent of the conditional mean process. In GARCH-type models the conditional variance of returns is assumed to be a deterministic function of past returns, whereas in SV models the volatility process is random. The introduction of the additional error term makes the SV model more flexible than the GARCH-type models as well as more directly linked to continuous time models that are frequently used in asset pricing in finance. Hence, the main difference between SV models and GARCH-type models is that the former are conceptually more flexible than the latter in that they allow for random noise in the volatility process.

The simplest benchmark SV model defines volatility as the log of the conditional variance and assumes that it follows an AR (1) process. However, the simple benchmark model is not always able to capture certain key features of the return series data and as such there are a number of proposed extensions of the benchmark SV model that are designed to do that. For example, an important effect which has received attention in the literature is the leverage effect which provides a financial explanation to the observation that volatility tends to rise following a drop in stock prices and it is generally associated with asymmetric volatility responses to increases and decreases in returns, see Cheung and Ng (1993), Campbell and Kyle (1993), Bouchaud, Matacz and Potters (2001) and Ozturk and Richard (2015) and the references therein for some of the more recent literature.

From a computational point of view, the main difference between ARCH-type and SV models is the fact that the latter require a much more intensive computational burden. Since volatility enters non-linearly in SV models, their estimation requires advanced numerical methods, see for example Ghysels, Harvey and Renault (1996), Andersen, Chung and Sorensen (1999) and Ozturk and Richard (2015) and the references therein. In the context of multivariate analysis there has been a number of successful applications of multivariate GARCH models, see Bauwens et al. (2006) for a discussion of such models. The first multivariate stochastic volatility latent factor model in the literature is proposed by Harvey et al. (1994) and then is it extended by Shephard (1996), Pitt and Shephard (1999), Jacquier et al. (1999) and Doz and Renault (2006). In addition to capturing parsimony, multivaritae stochasctic volatility have a direct link with arbitrage pricing theory. Based on the method of inclusion to the mean equation to the mean equationö these models can be classified as additive and multiplicative. The first multivariate factor model proposed by Harvey et al (1994) was additive. In these models returns are decomposed into two additive components. The first whish has a smaller dımension captures the factor which is common to the all assets and the other factor is the idiosyncratic factor which is specific to the asset. These models are then extended by Shephard (1996), Pitt and Shephard (1999) and Aguilar and West (2000). These models have two main drawbacks. The first one is that since homoscedaticity is assumed for the disturbance of return equation when the number of assets in the portfolio is greater than the number of factors some of the portfolios have to have no heteroscedasticity which conflicts with the stylized fact. Furthermore the assumption of the diagonalty in the covariance function of the disturbance term is too strong. Yu and Meyer (2006) included both time varying volatıility and correlations to the multivariate stochastic factor volaility models. Furthermore Philipov and Glickman (2006) also developed a high dimansional factor MSV model where factor covariance matrices are driven by Wishart processes.

On the other hand, in terms of multiplicative models the first model is proposed by Quintana and West (1987) which is also known stochastic discount factor model. This model decomposes return into two multiplicative components, a scalar common factor and a vector of idiosyncratic noise. When compared with the additive models since it has less number of coefficients to be estimated it is computationally more convenient. However since correlation of log volatilities are always assumed to be equal to 1, it is invariant with respect to time. Ray and Tsay (2000) extended this one-factor model to a k-factor model Overall, the theoretical literature on multivariate stochastic volatility (MSV) models has developed significantly over the last few years. see Asai, McAleer and Yu (2006) for an early survey of the multivariate stochastic volatility literature. More recent multivariate models of dynamic stochastic volatility using Bayesian methods include Nakajima and West (2013) and Nakajima (2016)

In this paper we estimate a multivariate SV model and we compute Maximum Likelihood (ML) estimates using Monte Carlo (MC) approximations of the likelihood function produced by Efficient Importance Sampling (EIS). EIS is a backward sequential IS procedure introduced by Richard and Zhang (2007) that can be used for efficient MC evaluation of high-dimensional sequential integrals and is particularly well suited for accurate MC likelihood evaluation of SV models, as shown by Liesenfeld and Richard (2003, 2006) It relies upon a sequence of auxiliary low-dimensional linear regression problems in order to produce global approximation to the likelihood integrals over their full support (as opposed to local approximations obtained e.g. by Taylor Series expansions). Moreover, EIS being generic, once it is implemented for a baseline SV model it can easily be modified to accommodate extensions of that model. Ozturk and Richard (2015) within a univariate framework applied the EIS methodology to estimate a leverage model in six separate sectors, consumer staples, health, industrials, technology, energy and finance. However, estimating an essentially multisectoral system by treating each sector as univariate and separate from each other may conceal and ignore important information about possible interdependence among the different sectors. Ignoring such information when valid may in fact introduce serious misspecification and produce invalid estimates.

In the present paper we estimate a multivariate stochastic volatility model for a panel of stock returns for a number of S&P 500 firms from different industries. Our model is an extension of the dynamic factor model for multivariate count data introduced by Jung et al. (2011). As in the case of a univariate model we use an efficient importance sampling (EIS) method to estimate the likelihood function of the given multivariate system that we analyze. As opposed to univariate methods where each return is estimated separately for each firm, our results are based on joint estimation that can account for potential common error term interactions based on industry characteristics that cannot be detected by univariate methods. Our results reveal that there are important differences in the industry effects, something that suggests that differential gains to portfolio allocations in the different industries that we examine. There are differences due to idiosyncratic factors and the common industry factors that suggest that each industry requires a separate treatment in arriving at portfolio allocations. In particular, when we compare these results with results of the paper by Ozturk and Richard (2015) who employed univariate methods, we can see that our results generally support their results that volatilities of the stocks in the same industry behave similarly but we are also able to show that when an industry factor is taken into consideration we can discern whether this similarity in behavior is specific to the given industry or not.

The paper is organized as follows. In the next section we present the methodology that we adopt in our approach including a presentation of the EIS method. The following section presents the data and the results.

2 Methodology

The model we propose in this paper is an extension of the dynamic factor model for multivariate count data introduced by Jung et al. (2011). The model consists a J-dimensional vector of returns $r_t = (r_{1t}, ..., r_{tj})$ for time $t = 1, \ldots, T$. The dynamic part of the model is developed based on the latent factors. We assume that returns follow the Gaussian normal distributions

$$p(r_{tj}|\theta_{tj}) = \frac{1}{\sqrt{2\pi \exp(\lambda_{tj})}} \exp -\frac{1}{2} \left(\frac{r_{tj}^2}{\exp(\lambda_{tj})} \right), \qquad t = 1, ..., T \quad j = 1, ..., J \tag{1}$$

(define $\lambda' s$).whose standard deviations are a function of the latent random variables. The latent factor θ_t can be expressed as a linear function of P-dimensional vector of latent random factors f_t as:

$$\theta_t = \mu + \Gamma f_t \tag{2}$$

where Γ is a $J \times P$ matrix of factor loadings. We assume that the factors are distributed independently of each other.

Since we estimate a multivariate model for the stocks which belong to the same industry we assume there exists a single common industry-specific factor vector and J stock specific factors vector. In total P which we partition as $P = J+1$. Therefore we also partition the factor loading matrix as $\Gamma = (\Gamma_\tau, \Gamma_\omega)$ where $\Gamma_\tau = (\gamma_j^{\tau_s})$ a $J \times 1$ matrix , and $\Gamma_\omega = diag(\gamma_j^{\omega_j})$ a $J \times J$ diagonal matrix.

Therefore for stock j we can parametrize the variance function θ as

$$\theta_{tj} = \mu_j + \gamma_j^{\tau_j} \tau_{t_j} + \gamma_j^{\omega_j} \omega_{tj} \tag{3}$$

In this setting we assume that the factors follow independent Gaussian AR(1) processes to account for possible serial and cross-correlations. The Gaussian AR(1) processes can be written as:

$$\tau_t | \tau_{t-1} \sim N\left(\kappa^\tau + \delta^\tau \tau_{t-1}, [\nu^\tau]^2\right) \tag{4}$$

$$\omega_{tj} | \omega_{t-1j} \sim N\left(\kappa^{\omega_j} + \delta^{\omega_j} \omega_{t-1j}, [\nu^{\omega_j}]^2\right) \tag{5}$$

We need to assume that $|\delta^\tau| < 1$ and $|\delta^{\omega_j}| < 1$ to assure stationarity. Furthermore to provide the identification of the coefficients we impose the restrictions $\kappa^{\tau_s} = \kappa^{\omega_j} = 0$ for $s = 1, ..., S$ and $j = 1, ..., J$ in order the identify the intercept μ_j's in the variance function. Furthermore, for one arbitrary stock we restrict $\gamma_j^{\omega_j} = 1$.

2.1 EIS Based Inference

The described model in equations 1 through 5 requires the evaluation of likelihood function by integrating the joint density of returns and factors with respect to the TxP, (24924 in our case) latent factors. The likelihood integral needs to be estimated is in the form

$$L(\psi) = \int .... \int \varphi_t(f_t, f_{t-1}; \psi) df_T ... df_1 \tag{6}$$

where ψ represents the parameters in the model and φ_t is the product of the time t densities for r_t given f_t and f_t given f_{t-1}which are given in Equations 1-5. We index the likelihood function based only on the factors since the returns are kept fixed at their observed values. As an initial condition we assume that f_0 is fixed at its unconditional expected value $E(f_t) = 0$. Due to the nonlinearity of the latent factors likelihood function can not be evaluated based on standard integration procedures.

We use the Efficient importance sampling (EIS) Monte Carlo (MC) procedure following Richard and Zhang (2007) for the integration of the likelihood function defined in equation 6. The procedure depends upon a sequence of auxiliary importance sampling densities for f_t given f_{t-1} in the following form:

$$\begin{aligned} m_t(f_t|f_{t-1};a_t) &= \frac{k_t(f_t,f_{t-1};a_t)}{\chi_t(f_{t-1};a_t)}, \quad \text{where} \\ \chi_t(f_{t-1};a_t) &= \int k_t(f_t,f_{t-1};a_t)df_t \end{aligned} \tag{7}$$

for $t=1,....,T$. $\{k_t(f_t,f_{t-1};a_t), a_t \in A_t\}_{t=1}^{T}$ is a preselected class of auxiliary parametric density kernels where $\chi_t(f_{t-1};a_t)$ represents the known analytical integrating factors in f_t given f_{t-1} for the kernel.

Then we can rewrite the likelihood function in (6) in the following form:

$$L(\psi) = \chi_1(f_0;a_1)\int ....\int \prod_{t=1}^{T}\left[\frac{\varphi_t(f_t,f_{t-1};\psi)\chi_{t+1}(f_t;a_{t+1})}{k_t(f_t,f_{t-1};a_t)}\right]\times \prod_{t=1}^{T} m_t(f_t|f_{t-1};a_t)df_T...df_1 \tag{8}$$

with $\chi_{T+1}(\cdot)\equiv 1$.For any value of a_t the final likelihood function in the form

$$L(\psi) = \chi_1(f_0;a_1)\times \frac{1}{N}\sum_{i=1}^{N}\prod_{t=1}^{T}\left[\frac{\varphi_t(\widetilde{f}_t^{(i)},\widetilde{f}_{t-1}^{(i)};\psi)\chi_{t+1}(\widetilde{f}_t^{(i)};a_{t+1})}{k_t(\widetilde{f}_t^{(i)},\widetilde{f}_{t-1}^{(i)};a_t)}\right] \tag{9}$$

where $\left\{\left\{\widetilde{f}_t^{(i)}\right\}_{t=1}^{T}\right\}_{i=1}^{N}$ represents $N-$independent trajectories drawn from the sequence of importance sampling densities $\{m_t(f_t|f_{t-1};a_t)\}_{t=1}^{T}$.

EIS procedure tries to minimize the MC sampling variance of the MC estimate of the likelihood function to select values of $a_t s$. Thus the procedure requires period by period minimization of the variance of $\varphi_t.\chi_{t+1}/k_t$ as function of f_t and f_{t-1} with respect to m_t-distributions. Richard and Zhang (2007) shows that the problem turns out to be solving a back recursive sequence of auxiliary least squares problems in the form:

$$\begin{aligned} (\widehat{c}_t,\widehat{a}_t) = \arg\min_{c_t\in\mathbb{R},a_t\in A_t}\sum_{i=1}^{N}\ln\{&\left[\varphi_t(\widetilde{f}_t^{(i)},\widetilde{f}_{t-1}^{(i)};\psi)\chi_{t+1}(\widetilde{f}_t^{(i)};a_{t+1})\right] \\ &-c_t-\ln k_t(\widetilde{f}_t^{(i)},\widetilde{f}_{t-1}^{(i)};a_t)\}^2 \quad t=1,..,T \end{aligned} \tag{10}$$

where $\left\{\widetilde{f}_t^{(i)}\right\}_{t=1}^{T}$ denotes a trajectory drawn from an initial sequence of auxiliary samplers $\left\{m_t(f_t|f_{t-1};a_t^{(0)})\right\}_{t=1}^{T}$ where $i=1,...,N$ (i.i.d.).

For the initial values of the auxiliary parameters we use Taylor series approximations to $\varphi_t.\chi_{t+1}$. Then we iterate the LS problems by replacing the initial samplers by the previous stage importance samplers. For convergence of these iterations to a fixed-point solution for auxiliary parameters we use common random numbers (CRNs). For the Gaussian EIS Samplers CRNs are $N(0,1)$.

2.2 Implementation of EIS in our model

According to equations 4-5, the conditional distribution of f_t given f_{t-1} can be written as

$$p(f_t|f_{t-1};\psi) = (2\pi)^{-P/2}|H^{-1}|^{-1/2} \times \exp\left\{-\frac{1}{2}\left(f_t - \Delta f_{t-1}\right)' H \left(f_t - \Delta f_{t-1}\right)\right\} \tag{11}$$

where Δ and H are both diagonal where H represents the inverse of the covariance matrix of f_t given f_{t-1}.

Application of sequential EIS to this specific model requires the factor in the model to be parametrized as

$$\varphi_t(f_t, f_{t-1};\psi) = p(f_t|f_{t-1};\psi).\left[\prod_{i=1}^{J} p(r_{tj}|\theta_{tj})\right] \tag{12}$$

where θ_{tj} is a linear function of f_t. When the selected kernel $k_t(f_t, f_{t-1}; a_t)$ is Gaussian in both f_t and f_{t-1} then the integrating constant $\chi_t(f_{t-1}; a_t)$ is itself Gaussian in f_{t-1}. Therefore this means the sole non-Gaussian part $\varphi_t.\chi_{t+1}$ in the likelihood function which is approximated by k_t is the factor of the product of J conditional densities of the return. The Gaussian kernel density is characterized by:

$$k_t(f_t, f_{t-1}; a_t) = p(f_t|f_{t-1};\psi)\zeta_t(f_t; a_t)\chi_{t+}(f_t; a_{t+1}) \tag{13}$$

Where ζ_t is the product of J univariate Gaussian kernels in θ_{tj} which is designed to approximate the product of J conditional densities of the return, $\prod_{i=1}^{J} p(r_{tj}|\theta_{tj})$ and it is parametrized as

$$\zeta_t(f_t; a_t) = \exp\left\{-\frac{1}{2}\left(\theta_t' B_t \theta_t - 2\theta_t' c_t\right)\right\} \tag{14}$$

$B_t = diag(b_{tj})$ represents a $J \times J$ positive-definite diagonal matrix and $c_t = (c_{tj})$ is $J-$dimensional vector. Then the EIS auxiliary parameter a_t is defined as $a_t' = (vech(B_t)', c_t')$. Since $\ln p(f_t|f_{t-1};\psi)$ and $\ln \chi_{t+1}$ appear in both sides of the Equation 10 they cancel out in the derivation and the regression can be estimated as J independent linear least squares regressions of the conditional densities of return, $\left\{\ln p(r_{tj}|\widetilde{\theta}_{tj}^{(i)})\right\}_{i=1}^{N}$ on $\left\{\left(\widetilde{\theta}_{tj}^{(i)}, \left[\widetilde{\theta}_{tj}^{(i)}\right]^2\right)\right\}_{i=1}^{N}$ and intercepts.

Since the density kernels depend upon one period ahead integrating constants we obtain them back-recursively. For the one-period ahead integrating constants which are also a function of f_t we use the following parametrization:

$$\chi_{t+}(f_t; a_{t+1}) = \exp\left\{-\frac{1}{2}\left(f_t' P_t f_t - 2f_t' q_{t+1} + r_{t+1}\right)\right\} \tag{15}$$

where $(P_{t+1}, q_{t+1}, r_{t+1})$ are appropriate functions of the EIS auxiliary parameters a_{t+1} which result from the backward recursive process. Since $\chi_{T+1} \equiv 1$ the initial values of the parameters in the integrating constant are equal to 1. When we write down the Gaussian kernel with respect to the model specific densities and parametrization we obtain

$$k_t(f_t, f_{t-1}; a_t) = (2\pi)^{-P/2} \left|H^{-1}\right|^{-1/2} \times \exp\{-\frac{1}{2}\left[f_t - (d_t + G_t f_{t-1})\right]' M_t \left[f_t - (d_t + G_t f_{t-1})\right] +$$

$$+ \left(\mu' B_t \mu - 2\mu' c_t + f_{t-1}' \Delta' H \Delta f_{t-1} + r_{t+1}\right) - (d_t + G_t f_{t-1})' M_t (d_t + G_t f_{t-1})\} \tag{16}$$

with

$$\begin{aligned} M_t &= \Gamma' B_t \Gamma + H + P_{t+1} \\ d_t &= M_{t-1}' \left[q_{t+1} + \Gamma' \left(c_t - B_t' \mu\right)\right] \\ G_t &= M_{t-1}' H \Delta \end{aligned} \tag{17}$$

The formulation shows that the EIS sampler for $f_t|f_{t-1}$ is given by

$$m_t(f_t|f_{t-1}; a_t) \sim N\left(d_t + G_t f_{t-1}, M_t^{-1}\right) \tag{18}$$

and if we regroup the remaining terms in the kernel, the parameters in the integrating constant are in the form

$$\begin{aligned} P_t &= \Delta' H \Delta - G_t' M_t G_t \\ q_t &= G_t' M_t d_t \\ r_t &= \mu' B_t \mu - 2\mu' c_t + r_{t+1} - d_t' M_t d_t + \ln|H^{-1}| - \ln|M_t^{-1}| \end{aligned} \tag{19}$$

Therefore we can compute the likelihood EIS estimates by following the steps below:

Step 1. Generate $N-$independent $P-$dimensional trajectories from a sequence of initial samplers $\left\{m_t(f_t|f_{t-1}; a_t^{(0)})\right\}$. To obtain the initial samplers we use a second order Taylor Series Approximation in θ_{tj} to $\ln \prod_{i=1}^{N} p(r_{tj}|\theta_{tj})$ around the unconditional expectation of θ_{tj} which is equal to μ_j. In our model these initial values are equal to:

$$B_t^{(0)} = diag\left(\frac{1}{2} r_{tj}^2 \exp(-\mu_j)\right) \text{ and } c_t^{(0)} = -\frac{1}{2}\left(1 + \left(1 + \mu_j\right) r_{tj}^2 \exp(-\mu_j)\right) \tag{20}$$

Then we substitute these values to Equation 17-19 to construct initial sampler

Step 2. We construct $\theta_t = \mu + \Gamma f_t$ by using the the simulated f_t-trajectories in the precious step to solve back recursively the following $J-$independent linear regression functions for each period.

$$\ln p\left(r_{t1}|\widetilde{\theta}_{t1}^{(i)}\right) = cons\tan t + \frac{1}{2}b_{t1}\left[\widetilde{\theta}_{t1}^{(i)}\right]^2 + c_{tJ}\widetilde{\theta}_{t1}^{(i)} + \zeta_{t1}^{(i)} \qquad i = 1, ..., N \quad (21)$$

$$\vdots$$

$$\ln p\left(r_{tJ}|\widetilde{\theta}_{tJ}^{(i)}\right) = cons\tan t + \frac{1}{2}b_{tJ}\left[\widetilde{\theta}_{tJ}^{(i)}\right]^2 + c_{tJ}\widetilde{\theta}_{tJ}^{(i)} + \zeta_{tJ}^{(i)} \qquad i = 1, ..., N \quad (22)$$

where $\zeta_{tj}^{(i)}$ denotes the error term of regression (t, j).

Step 3. The back recursive regression in the previous step provides $\widehat{B}_t = diag(\widehat{b}_{tj})$ and $\widehat{c}_t = (\widehat{c}_{tj})$ which then we use to construct EIS-sampling densities $\{m_t(f_t|f_{t-1};\widehat{a}_t)\}$ back-recursively by replacing the initial values used in *Step 1* by these estimated values.

Step 4. Finally we use these new N-independent trajectories to repeat steps 2 and 3 to compute EIS-MC estimate of the likelihood function until the convergence is achieved.

3 Data

In this paper we use 3 stocks from six different sectors. The data covers the period from January 1st 1990 to September 23rd 2014. Returns are calculated as logarithmic returns from stock indices. The stocks covered in this paper are Coca Cola, Proctor and Gamble and Walmart from Consumer Staples sector, Chevron, Conoco Phillips and Exxon from Energy sector, American Express, CitiBank and JP Morgan from Finance sector, Bristol-Squibb-Myers, Pfizer and Merck from Health sector, Boeing, Caterpillar and General Electrics from Industrials sector and finally IBM, Motorola and Oracle from Technology sector. We analyze each sector separately and as such the volatility of each stock is formed as a linear combination of the factor specific to the industry and its own idiosyncratic factor.

4 Empirical Results

We estimate a multivariate stochastic volatility model for a panel of stock returns for a number of S&P 500 firms from different industries. Considering the joint behavior of the volatility for different stocks in the same industry, we allow for a single common industry factor and J stock-specific factors, where J is equal to the number of stocks in each industry. Hence in total for each industry we have $J + 1$ factors for each industry. The results are presented in Tables 1 to

6, First of all, when trying to construct an optimal portfolio we need to take into account the fact that the results among sectors have important similarities and differences. For example for the consumer staples, industrials and technology sectors the results indicate that the industry specific factor δ displays high persistence that dominates any stock specific persistence parameters. On the other hand for the remaining sectors,the industry specific factor has low persistence and it is the stock specific $\delta's$ that exhibit high persistence rates. Furthermore, estimates of the latent factor variances $\nu's$ also follow a similar pattern, indicating similar magnitudes for the stock specific and industry specific cases. Hence again, the consumer staples, industrials and technology sectors behave as a similar group and the same is true for energy, finance and health. When we compare these results with the results of Ozturk and Richard (2015) who used a univariate framework of analysis, we can conclude that our results confirm their finding that volatilities of the stocks in the same industry behave similarly, yet we also show that when the industry factor is taken into consideration we can discern whether this similarity in behavior is industry specific or not. The consumer staples, industry and technology sectors are good examples where the similarity in behavior is mostly generated by the industry itself. The industrials sector seems to be the sector which is dominated by the industry specific factor the most. Only for one of the stocks, Boeing, the persistence parameter for the stock specific factor is significant. In the consumer staples sector for one of the stocks, PG, the stock-specific persistence is insignificant and for the technology sector it is significant for all stocks. Therefore our results suggest, that if we want to diversify risk constructing a portfolio from stocks in the same sector (consumer staples, industrials and technology sectors) will fail to achieve optimal risk diversification. However, for the remaining sectors, since the industry factor persistence is lower than that of individual stocks, it is possible to diversify risk within the same sector. Therefore this also indicates that a careful analysis of the specific sector is necessary during the portfolio allocation.

Our results reveal that there are important differences in the industry effects, something that suggests differential gains to portfolio allocations in the different industries that we examine. There are differences due to idiosyncratic factors and the common industry factors that suggest that each industry requires a separate treatment in arriving at portfolio allocations. Therefore investors should treat their portfolio allocation differently based on the industry they are investing in

5 Conclusion

We estimate a multivariate stochastic volatility model for a panel of stock returns for a number of S&P 500 firms from different industries. As in the case of a univariate model we use an efficient importance sampling (EIS) method to estimate the likelihood function of the given multivariate system that we analyze. As opposed to univariate methods where each return is estimated separately for each firm, our results are based on joint estimation that can account for potential common error term interactions based on industry characteristics that cannot be detected by univariate methods. In our analysis we follow a similar methodology of Jung et al. (2009). Considering the joint behavior of the volatility for different stocks in the same industry, we allow for a single common industry factor and J stock-specific factors, where J is equal to the number of stocks in each industry. Our results reveal that there are important differences in the industry effects, something that suggests differential gains to portfolio allocations in the different industries that we examine. There are differences due to idiosyncratic factors and the common industry factors that suggest that each industry requires a separate treatment in arriving at portfolio allocations. Therefore investors should treat their portfolio allocation differently based on the industry they are investing in.

6 Tables

Table 1. Results for Consumer Staples Sector

Consumer Staples			
	Coca Cola	**PG**	**Walmart**
μ	-0.529	-0.612	-0.514
	[0.078]	[0.077]	[0.083]
δ	0.407	0.0508	0.444
	[0.082]	[0.064]	[0.091]
ν	0.494	0.489	0.457
	[0.029]	[0.020]	[0.037]
γ	1	0.975	1.045
		[0.037]	[0.038]
Industry		**δ**	**ν**
Factor		0.989	0.118
		[0.002]	[0.011]

Table 2. Results for Energy Sector

Energy			
	Chevron	**Conoco**	**Exxon**
μ	-0.316	-0.762	-0.305
	[0.118]	[0.273]	[0.145]
δ	0.997	0.998	0.997
	[0.001]	[0.001]	[0.002]
ν	0.025	0.042	0.037
	[0.006]	[0.008]	[0.006]
γ	1	0.867	0.952
		[0.072]	[0.061]
Industry		**δ**	**ν**
Factor		0.386	0.613
		[0.046]	[0.029]

Table 3. Results for Finance Sector

Finance			
	Amex	**Citi**	**JP**
μ	-0.678	-0.722	-0.811
	[0.134]	[0.116]	[0.121]
δ	0.998	0.998	0.997
	[0.001]	[0.001]	[0.001]
ν	0.047	0.042	0.055
	[0.008]	[0.006]	[0.008]
γ	1	1.03	1.033
		[0.065]	[0.064]
Industry		**δ**	**ν**
Factor		0.345	0.639
		[0.049]	[0.033]

Table 4. Results for Health Sector

Health			
	BSM	**Pfizer**	**Merck**
μ	-0.625	-0.46	-0.567
	[0.079]	[0.147]	[0.128]
δ	0.998	0.997	0.997
	[0.001]	[0.001]	[0.001]
ν	0.044	0.045	0.042
	[0.006]	[0.005]	[0.006]
γ	1	0.947	1.053
		[0.044]	[0.047]
Industry		**δ**	**ν**
Factor		0.339	0.743
		[0.032]	[0.025]

Table 5. Results for Industrials Sector

Industrials			
	Boeing	Caterpillar	GE
μ	-0.477	-0.468	-0.639
	[0.076]	[0.074]	[0.098]
δ	0.459	0.079	0.08
	[0.107]	[0.099]	[0.214]
ν	0.485	0.578	0.369
	[0.043	[0.031]	[0.040]
γ	1	0.993	1.324
		[0.044]	[0.052]
Industry		δ	ν
Factor		0.978	0.145
		[0.004]	[0.015]

Table 6. Results for Technology Sector

Technology			
	IBM	Motorola	Oracle
μ	-0.734	-0.766	-0.947
	[0.068]	[0.068]	[0.082]
δ	0.764	0.883	0.34
	[0.065]	[0.021]	[0.098]
ν	0.445	0.36	0.602
	[0.058]	[0.036]	[0.032]
γ	1	0.911	1.308
		[0.068]	[0.066]
Industry		δ	ν
Factor		0.995	0.073
		[0.002]	[0.008]

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