The Dynamics of Informality and Fiscal Policy under Sovereign Risk*

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Abstract

This paper examines how the dynamics of informality affects optimal fiscal policy and default risk. We build a model of sovereign debt with limited commitment and informality to assess the consequences of dynamic distortions induced by fiscal policy. In the model, fiscal policy has a persistent impact on taxable activity, which affects future fiscal revenues and thus default risk. The interaction of tax distortions and limited commitment strongly constrains the dynamics of optimal fiscal policy and leads to (i) more frequent default episodes and (ii) costly fluctuations in consumption.

JEL: E26, E62, F34, F41, H20.

Keywords: fiscal policy, informal economy, sovereign default.

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1 Introduction

With limited commitment, sovereign default risk depends on future incentives for the government to raise fiscal revenues. At heart, a benevolent planner will face a trade-off between imposing the cost of a default to domestic agents or imposing the cost of repayment; the planner will choose to default on its obligations if the latter is high enough. This limited commitment to repay constrains consumption transfers across periods and affects the dynamics of fiscal policy, as shown in Eaton and Gersovitz (1981) or Arellano (2008). In these earlier contributions, however, transfers from and to households are not distortionary and do not affect domestic income. In practice, transfers involve taxation, taxation comes with well-known behavioral responses (e.g., labor supply, tax avoidance, or tax evasion; see Slemrod, 2001; Slemrod and Yitzhaki, 2002), and the cost of repayment comprises the static and dynamic distortionary costs of fiscal policy. How do such distortions affect the previous trade-off and the dynamics of optimal fiscal policy?

This paper fills a gap in the existing literature of sovereign default by studying the role of informality and its interplay with taxes and sovereign risk in environments where transfers from and to households are distortionary. Our contribution is to explicitly consider economic contexts with a non-negligible informal sector. In these contexts, higher taxes lead to an expansion of the informal sector, and the reallocation of the economy away from taxable activity affects fiscal revenues in the short run (a static distortion, as in Aruoba, 2021). These adjustments might outlive the initial contractionary fiscal policy and persistently lower returns to taxation (a dynamic distortion).[1]

We build a model of sovereign debt with limited commitment where local producers may operate in the formal or informal economy. We introduce the possibility for this choice to be sticky in order to understand how the *dynamics* of informality affects optimal fiscal policy and default risk. We then proceed in two steps. In a first step, we study the qualitative properties of a stylized, 2-period version of the model to highlight the respective role of static and dynamic distortions induced by fiscal policy. The latter are shown to further restrict the government in its ability to transfer consumption across periods. Intuitively, the prospect of a future default limits the set of feasible fiscal policies to contractionary policies; these policies however push producers towards the informal sector and persistently so, making it more costly for the government to repay and raising the prospect of a future default even further. In a second step, we quantify

[1]Tax evasion has been shown to be persistent due to the nature of filing costs (Erard and Ho, 2001), to the interaction of tax declarations with other frictional processes for businesses (e.g., consumer search or search for intermediaries, see Pomeranz, 2015), to dynamic incentives built within the tax monitoring system (Engel and Hines, 1999), or to the adjustment of the private sector to a general culture of informality (e.g., through bank loans, see Artavanis et al., 2016).

the ignored, yet non-negligible, welfare cost associated with the distortionary dynamics of informality. We exploit data on tax compliance across a sample of selected economies subject to some default risk in order to discipline the dynamics of informality in our stochastic model of sovereign default. We find that both the responsiveness and persistence of informality significantly constrain optimal fiscal policies which, ultimately, has an impact on welfare through more frequent default episodes, costly fluctuations in taxation, and the resulting volatility of household consumption.

Our main contribution is to uncover (i) the dynamics of fiscal policy and default risk and (ii) their welfare consequences, in environments where a non-negligible informal economy makes taxation distortionary in the short and medium run.[2] In particular, it is a well known empirical regularity that consumption is more volatile in emerging economies (Loayza et al., 2007). It is also known that informality constrains the set of feasible fiscal policies (see Elgin and Uras, 2013, for an empirical association between informality, financial instability, and default). Our theory connects these two empirical observations: informality limits the ability of the government to smooth consumption on behalf of households. In economies with higher incidence of informality, we should thus expect the volatility of consumption ("in excess of" the volatility of output) to be larger, in part because governments are expected to default more often and sustain higher debt-servicing costs—a theoretical insight that we bring to the data in Figure 1.

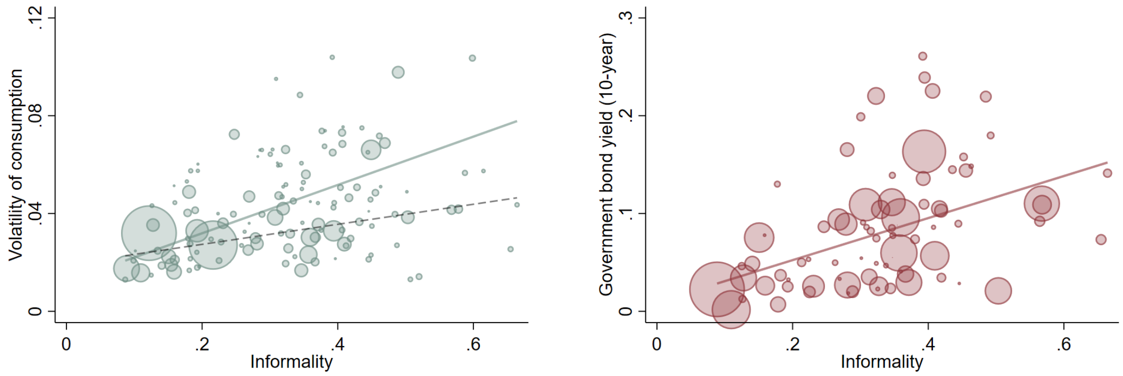
The key ingredients of our model are (i) limited commitment to repay sovereign debt and (ii) production distortions induced by fiscal policy. We build on Eaton and Gersovitz (1981) and consider the benevolent government of a small economy who stabilizes consumption on behalf of a representative household. The government has access to international markets but has limited commitment such that the debt price reflects future incentives to default.[3] The novelty of the model is to introduce dynamic distortions in production—as induced by fiscal policy—which interact with limited commitment. There is a mass of entrepreneurs, and they can adopt two technologies in order to produce the final good: an unobserved technology—which constitutes the informal sector of the economy—and a verifiable technology—the formal sector—in which there are production complementarities.[4] We assume that this choice is staggered, in a similar manner

[2]There is a literature studying the role of taxation and informality in transition economies (see, e.g., Ihrig and Moe, 2004; Saracoğlu, 2008). Our contribution differs from theirs in that we do not model growth or transitions towards a different steady state and rather focus on short to medium-run fluctuations in fiscal policy and their implications on the dynamics of consumption.

[3]Our modeling of sovereign debt follows the standards of the literature. First, there are two punishments following a default which give (limited) commitment to the government: market exclusion and a direct output cost possibly reflecting a failure of the domestic banking sector (Mendoza and Yue, 2012; Arellano et al., 2017). Second, we consider the long-term debt structure introduced in Chatterjee and Eyigungor (2012) to support higher debt levels while disciplining the fluctuations in debt prices.

[4]Our modeling of informality borrows from the literature on shadow economies with dual technology

Figure 1. Informality, the (excess) volatility of consumption, and long-term interest rates across countries.



(a) Volatility of consumption

(b) Long-term interest rate

Notes: Panel (a) displays the volatility of household consumption, calculated as the standard deviation of annual consumption growth per capita, against a measure of average informality between 1995–2018—using data from Elgin et al. (2021), who replicate the approach developed in Schneider et al. (2010). Dots are weighted by the country's population. The plain line shows the relationship between the standard deviation of annual *consumption* growth per capita, against a measure of average informality; and the dashed line shows the relationship between the standard deviation of annual *output* growth per capita, against the measure of average informality. Panel (b) displays the 10-year government bond yield over 1995–2018 (Source: OECD) against the same measure of average informality. The plain line shows the relationship between this average long-term interest rate and informality.

as Calvo (1983), and each entrepreneur may be able to reset technology with a certain probability in each period. These two technologies can be interpreted as two different production lines, where one involves intermediaries and thus needs to be transparent and the other one cannot be monitored. We assume that fiscal policy is the combination of exogenous, stochastic government expenditures and an endogenous, linear tax on production in the formal sector.

A stylized, deterministic, 2-period version of the model is sufficient to show how the (rigid) dynamics of informality qualitatively impacts optimal fiscal policy and the ability of the government to smooth consumption on behalf of households. We however quantify the possible importance of this novel interaction in an infinite-horizon model featuring both interest rate shocks—the average return expected by international financial markets once accounted for the endogenous default risk—and public expenditure shocks. The model is crucially disciplined by the volatility and persistence of informality, whose estimation requires rich variation in fiscal policy and tax compliance.[5] We

(Rauch, 1991; Enste and Schneider, 2000; Straub, 2005). We assume that tax monitoring is perfect for the formal technology and absent for the informal technology. This assumption can be relaxed, as long as the effort by the government in uncovering undeclared activity is exogenous. Another important feature of the model is that there exist strategic complementarities across producers of the formal sector (a feature also present in Dessy and Pallage, 2003). This could be generated by spillovers across production units through innovation, but also be related to increasing returns to tax compliance and built-in incentives to report Value-Added-Taxes. A recent contribution, Pomeranz (2015), presents evidence in favor of enforcement spillovers along the production chain.

[5]We use a measure of VAT (Value-Added-Taxes) compliance built in Pappadà and Zylberberg (2017)

calibrate our model using empirical, key moments from a selected sample of countries with non-negligible levels of informality and default risk. More specifically, our simulated model targets and matches three empirical moments characterizing tax compliance (in addition to an externally-derived fiscal multiplier)—an average tax compliance of about 72%, an elasticity with respect to tax rates of about -0.39 (an exercise in which we exploit tax reforms affecting the effective tax rate), and a persistence of about 0.85—to pin down four parameters characterizing domestic production: the location and shape of the distribution of private returns across entrepreneurs in the informal sector; the probability to reset technology; and the level of complementarities in the formal sector.[6] We calibrate the remaining parameters of our economy—pertaining to preferences, shocks, default punishments, and debt maturity—on a benchmark economy (Aguiar and Gopinath, 2006; Arellano, 2008; Chatterjee and Eyigungor, 2012).[7]

We study the dynamics of informality, fiscal policy, and default risk in the quantitative model and proceed in two steps. First, we analyze the dynamic policy response of fiscal policy and domestic production as a function of *exogenous* shocks, government expenditures in particular. We simulate many economies subject to the same initial expenditure shock (but allowed to differ in the path followed by subsequent shocks), and we consider the distributional response of the main endogenous variables (debt levels, tax rate, incidence of the informal sector, consumption, debt price) over a long horizon. As a response to the shock, the government accumulates more debt *and* increases taxation. While it takes on average about 5-6 years for debt to come back to pre-shock levels, the hike in informality (and the subsequent drop in consumption) is even more sluggish due to the technological frictions, and the government remains susceptible to default in the long run. We provide some empirical support for these model-based moments and identify empirical impulse responses to exogenous expenditure spikes within our panel

and consolidated in Pappadà and Rogoff (2023) as a proxy for general tax compliance. The rationale for analyzing VAT is manyfold: it constitutes a large share of tax revenues; it is an important adjustment tool at business cycle frequency; reconstructing counterfactual tax revenues under perfect tax compliance requires very few assumptions. The measure mostly contrasts two different data sources, i.e., taxes as received by the government and the reported consumption of goods at a highly disaggregated level (thus measuring frequent, minor changes in VAT), and captures any discrepancies between these two sources.

[6]The average incidence of the informal technology in the model is jointly characterized by the distribution of returns in the informal sector and the level of complementarities in the formal sector. While we do not explicitly model punishments for operating in the informal sector (monetary punishments from the government or the moral costs of tax evasion), one can interpret returns in the informal sector as being net of such expected punishments. In this interpretation, the idiosyncratic distribution of returns to the informal sector could partly reflect differences in such punishments across entrepreneurs.

[7]We also investigate how our calibrated model matches a few untargeted moments, most notably: the *overall* fluctuations in informality; the volatility of fiscal policy and default risk; the level and volatility of debt; and the volatility of household consumption. The model slightly overshoots for some of the untargeted volatilities; one explanation could be that we consider exogenous government expenditures and ignore monetary policy, thus limiting the set of stabilizers which could smooth fluctuations in fiscal quantities and in consumption.

of countries (following Born et al., 2020): we find evidence for an informality-driven "overhang" with high spreads, debt levels and tax rates. Second, we evaluate how the dynamic properties of informality affect optimal fiscal policy and welfare by comparing economies differing along the sensitivity of informality to fiscal policy shocks or the sluggishness of informality.[8] We find that both the sensitivity and the sluggishness of informality matter: the baseline economy with sensitive, sluggish adjustments in domestic production is more likely to default and is less able to smooth fluctuations. We use the model to quantify the welfare costs of imperfect consumption smoothing, as induced by the *combination* of sensitivity to taxes, technological rigidities, and imperfect commitment to repay.[9] We find that reducing dynamic distortions or providing full commitment to repay would lead to a 0.12-0.22% increase in certainty equivalent consumption.[10]

Related literature The theoretical environment builds on the literature on sovereign default. In models of sovereign debt with limited commitment (e.g., Eaton and Gersovitz, 1981; Arellano, 2008), default risk typically depends on an endogenous state variable, the debt level, and exogenous state variables, e.g., income shocks. In such a benchmark, a contractionary fiscal policy reduces debt levels and unambiguously lowers default risk. In contrast with these contributions, we explicitly model fiscal policy as a distortionary instrument that affects the future cost of raising tax revenues. A consequence is that another endogenous state variable—informality—affects default risk through the future cost of raising tax revenues.[11] In our setting, a contractionary fiscal policy diverts entrepreneurs away from the formal sector. This response decreases the contemporary

[8]The impact of dynamic distortions through tax evasion is not qualitatively different from that of more standard fiscal multipliers (Doda, 2007; Cuadra et al., 2010; Pouzo and Presno, 2022). Its quantitative implications are however very different. The standard behavioral response to tax policy is one order of magnitude lower than that implied by tax evasion in economies with imperfect tax enforcement, and the response is persistent. The magnitude and persistence of fluctuations in tax compliance imply sharply decreasing returns to taxes, and the economy may display a form of fiscal fatigue (Ghosh et al., 2013).

[9]While our primary research question is "What does (the dynamics of) informality bring to the understanding of fiscal policy and default risk?" and we consider counterfactual economies with different sensitivity to taxes or different technological rigidities to address it, our quantitative analysis can also shed light on how default risk shapes fiscal policy and the dynamics of informality. To do so, we consider an additional counterfactual economy with perfect commitment to repay and show that the government is then able to smooth costly fluctuations in taxation and informality, with a very significant stabilizing effect on household consumption.

[10]While this number might appear small, it reflects very significant consumption volatility in the baseline economy. Indeed, our measure is, in essence, akin to the welfare cost of business cycles derived in Lucas (1987): we consider the difference between our baseline economy and an economy in which all systematic risk in consumption would be eliminated, and we find that the welfare cost of consumption fluctuations in our economy is equivalent to about 0.246% of consumption. This number decreases to 0.027-0.126% in economies where we lower the sensitivity of the formal sector to taxes, where we lower rigidities in technological choice, or where we give perfect repayment commitment to the government.

[11]Reciprocally, default risk distorts the decision of entrepreneurs: (i) a future default would induce low taxes in the future; (ii) persistent default risk without default would induce high taxes in the future.

and future cost of raising tax revenues, because of staggered adjustments in technological choice. This indirect effect tilts the future trade-off between repayment and default, and mitigates the gains in debt service through the standard "fiscal surplus" effect.

One novelty of our approach, compared to numerous contributions (see for instance Aguiar et al., 2005; Cuadra et al., 2010; Bi, 2012; D'Erasmo and Mendoza, 2013; Arellano and Bai, 2016; Pouzo and Presno, 2022), is that distortions induced by fiscal policy do not only affect the contemporaneous choice of fiscal policies, but also future repayments. Through this inter-temporal distortion, the economy is at risk of falling into a tax evasion overhang and periods of contractionary fiscal policy *and* high default risk.[12]

Our calibration builds upon a measure of tax compliance constructed in Pappadà and Zylberberg (2017) (and extended in Pappadà and Rogoff, 2023), which highlights that tax compliance is responsive to tax rates and sluggish.[13] Our research thus relates to the literature discussing tax evasion and its dynamics during recent debt crises (see Pappa et al., 2015; Dellas et al., 2017). In Pappa et al. (2015), taxes increase incentives to produce in the (unproductive) informal sector. This mechanism explains the failure of the recent consolidation plans in Greece, Italy, Portugal and Spain. The contribution of the present analysis is to study the interaction of such distortions with sovereign default.

The paper relates to a wider literature on sovereign default and fiscal policy. A recent literature investigates the impact of sovereign default or fiscal policy on the private sector through the movement of factors: capital and the collapse of the banking sector (as in Mendoza and Yue, 2012; Bocola, 2016; Arellano et al., 2017), or workers (as in Bandeira et al., 2019; Alessandria et al., 2019). In our stylized model, we nest these default costs in an exogenous output cost that provides commitment for the government to repay. A large literature documents the use of pro-cyclical fiscal policies in developing economies (Kaminsky et al., 2004; Ilzetski and Vegh, 2008; Frankel et al., 2013; Vegh and Vuletin, 2015; Bianchi et al., 2023); and many theoretical mechanisms have been discussed to explain this observation.[14] In our model, limited commitment and the proximity to a "debt ceiling" induce the government to implement pro-cyclical fiscal policies (as in Aguiar et

[12] Dovis et al. (2015) develops a model in which a similar dynamic component affects the future cost of fiscal policies. Their framework relies on inequality across domestic citizens as the endogenous state variable affecting default risk, thereby describing an *inequality overhang*.

[13] These observations, also exploited in Pappadà and Zylberberg (2017) and Dellas et al. (2017), may help rationalize differences in estimates of fiscal multipliers across environments and fiscal policy tools (Alesina and Ardagna, 2009; Romer and Romer, 2010; Favero et al., 2011; Auerbach and Gorodnichenko, 2012; Ilzetzki et al., 2013; Alesina et al., 2015). In the same vein, two recent papers discuss the impact of consumption taxes in the presence of informality (see Bachas et al., 2020; Morrow et al., 2022, respectively constructing cross-country measures of tax compliance and informal consumption).

[14] See Kaminsky et al. (2004) for instance on international capital flows, or von Hagen and Harden (1995); Aaron Tornell (1999); Alesina et al. (2008) for explanations based on the redistribution effect of increasing taxes; the competition among taxpayers to receive the proceeds from the positive shock; or the desire to limit rents that politicians could capture.

al., 2005; Doda, 2007; Cuadra et al., 2010; Bianchi et al., 2023).

The remainder of the paper is organized as follows. In Section 2, we introduce a model of sovereign debt where the government has limited commitment and fiscal policy induces dynamic distortions. Section 3 derives qualitative predictions in a stylized version of the model. Section 4 discusses the dynamics of tax compliance and its welfare cost in a quantitative version of the dynamic, stochastic model. Section 5 concludes.

2 Environment

This section describes a model of a small open economy with a representative household and a government. The model embeds production with different technologies associated with different tax enforcement in an otherwise standard framework à la Eaton and Gersovitz (1981) where a benevolent government with limited commitment issues debt on behalf of the household.

2.1 Preferences and technology

The economy is populated by a continuum of infinitely lived households of measure one. Letting c_t denote consumption at time t, households in this economy maximize expected utility as given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s),$$

where $\beta < 1$ denotes the discount factor and $u(\cdot)$ represents the period utility function, which satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Since all households are identical, we refer to them throughout as the representative household.

There are two types of agents who populate households. Each household is composed of a unit mass of entrepreneurs who hold one unit of an intermediary good in each period, and a unit mass of producers. There is perfect redistribution within each household such that all agents consume the same in each period.

The consumption good, which is the numeraire, can be produced with two technologies. First, it can be assembled by producers using intermediary goods. We assume that there are many varieties of such intermediary goods, indexed by i, and there are complementarities between the intermediary goods when they are used as factors of production for the consumption good.[15] Producers assemble intermediary goods using the following

[15] The return to the formal technology increases with the mass of producers adopting this technology, possibly reflecting spillovers across transparent firms through innovation, complementarities along the production chain or (tax) enforcement spillovers along the production chain.

CES technology,

$$y_t = \left(\int_0^1 y_{it}^{\phi} di \right)^{\frac{1}{\phi}},$$

where $\phi < 1$ captures production complementarities. We assume that this technology is transparent and represents the formal sector of our economy.

Second, the consumption good can be directly produced by entrepreneurs. These entrepreneurs can transform their unit of the intermediary good into the consumption good using a private technology with period-specific net return R_{it}. We assume that R_{it} is known to the entrepreneur only and is independently drawn from a continuous probability distribution $H(.)$ in each period. This technology is not observable to the government; it represents the informal sector of our economy.[16] The heterogeneity in net return to the informal technology captures that entrepreneurs may differ in their current reliance on financial intermediation or in any other cost associated with running an informal business, including, for instance, punishments if detected. We assume that markets for intermediary goods and the final good are perfectly competitive.

Moreover, there are rigidities and technological choice is staggered over time, in the manner of Calvo (1983). In each period, there is an idiosyncratic draw determining whether an entrepreneur is allowed to change her technology.[17] With probability $1 - \theta$, the entrepreneur can choose and either (i) adopt the formal technology and sell her intermediary good to producers or (ii) adopt the private technology. With $\theta = 0$, this choice would be completely flexible across periods and the contemporary choice does not commit the entrepreneur in the future.

There is a benevolent government whose objective is twofold. First, in each period, the government needs to finance a public good whose cost g_t is exogenous and subject to shocks, following a Markov process. Second, the government maximizes the welfare of the representative household by issuing debt and purchasing assets on its behalf: the household's borrowing and saving are done exclusively through the government. In order to finance the public investment and transfer from and to the household, we assume

[16] We could allow for a less binary tax monitoring, i.e., there would be imperfect tax monitoring associated with both technologies and entrepreneurs would then pay a fine conditional on being detected. We could also allow for some degree of production complementarities in both technologies. The important feature is that the least monitored technology is the one with relatively low production complementarities such as to generate a gap between the social returns to the two technologies, a quantity directly related to the fiscal multiplier.

[17] The tractability of our framework partly relies on the assumptions that technology draws (both the productivity associated with the private technology and the possibility to reset technology) are independent across periods. This Calvo assumption might not be innocuous, e.g., compared to a Rotemberg framework, as illustrated in Lombardo and Vestin (2008). In particular, technology adoption could be motivated by changes in market conditions, technological improvements, or policy, and entrepreneurs might be more willing to incur the cost of considering such adoption in certain circumstances than others.

that the benevolent government can only levy indirect, linear taxes τ_t on the consumption good produced through the formal technology.

2.2 Financial markets

We assume that the economy is small relative to the international financial market, and that the government can issue and trade long-term bonds on these markets. In period t, the international financial market is willing and able to purchase any asset that yields an expected return at least as high as ρ_t, which is stochastic and follows a Markov process. At heart, one can think of public expenditures as a supply shifter for government bonds, while the interest rate shock is a demand shifter. Those will be the only sources of uncertainty in our framework.[18]

The government stabilizes the consumption of the representative household by issuing debt and purchasing assets on its behalf. The government has imperfect commitment and may default on its obligations. As in Chatterjee and Eyigungor (2012), debt is issued as a long-term debt contract with a probabilistic maturity: in each period, a fraction λ of outstanding debt matures, independently of its issuance date (any contract has the same forward-looking payoff structure); and we assume that there is a coupon rate, z, in each period. Let b_{t+1} and q_{t+1} respectively denote the amount and price of debt issued by the government at time t, and let $D_t \in \{0, 1\}$ denote the decision to default on previous obligations. If t_t denotes total indirect taxation levied by the government at time t, the resource constraint for the government is:

$$(1 - D_t) \left[(\lambda + (1 - \lambda) z) \, b_t - q_{t+1} \left(b_{t+1} - \lambda b_t \right) \right] = t_t - g_t \tag{1}$$

We suppose that there are two sources of punishment which give some (limited) commitment to the government. As in Arellano (2008), there is an exclusion from the international market following a default and reintegration is stochastic and occurs with probability ν in each period. We further assume that, during market exclusion, there is an exogenous default cost that is directly incurred by the household and is captured by a share of consumption, Δ. Since transfers from the government are made through a distortionary instrument, it is not innocuous to assume that the default cost is paid by the household: the punishment does not distort the choice of entrepreneurs through a direct effect. A government with distortionary tax instruments has incentives to default because a default is then a relatively efficient way to redistribute to the representative

[18] We consider risk-free rate shocks to better match the volatility in debt prices and debt levels, without modeling the dynamic interaction of those fluctuations with default risk (as in Stangebye, 2020, where volatility in debt prices partly arise from non-fundamental uncertainty). As we show in Appendix B.3, these shocks do not significantly alter our main quantitative insights.

household.[19]

In order to solve for the equilibrium of the economy, we need to understand how the government makes borrowing decisions. We turn to these next.

2.3 Timing of actions and equilibrium

We now specify the timing of actions within each period. At the beginning of each period, the aggregate shocks ρ_t and g_t are revealed and perfectly observed by all agents. If the government is excluded from international financial markets, a reintegration draw takes place. The government then decides to repay the debt arriving to maturity or default on all its past obligations, b_t, and commits to an indirect taxation rate τ_t. Entrepreneurs choose their technology, if this choice is available to them, with perfect knowledge of the aggregate shocks, their own idiosyncratic shock, and government policy. Production takes place and taxes are paid by agents. International financial markets open, and sovereign bonds are traded. Finally, households consume.

2.4 The programs

In a preliminary step, we derive the optimization problem of producers, entrepreneurs, the government and investors in order to characterize the equilibrium of this economy. We turn to the different programs in order.

Producers We first describe the (static) program of producers. In each period, producers take prices p_{it} of each intermediate good variety and taxes τ_t as given and maximize:

$$\max_{\{y_{it}\}} \left\{ (1-\tau_t) \left(\int_0^1 y_{it}^{\phi} di \right)^{\frac{1}{\phi}} - \int_0^1 p_{it} y_{it} di \right\}.$$

The resulting demand for variety i is characterized by the following equation:

$$(1-\tau_t) y_{it}^{\phi-1} \left(\int_0^1 y_{it}^{\phi} di \right)^{\frac{1}{\phi}-1} = p_{it}.$$

[19]This stylized modeling of default costs has another implication: the exogenous default cost equally affects consumption proceeds from the informal sector and from the formal sector. If these costs are thought to capture a shock to domestic intermediation through a capital flight (Mendoza and Yue, 2012; Bocola, 2016; Arellano et al., 2017), we might expect those to affect differentially entrepreneurs across different technologies: the formal sector should be more reliant on credit access. Embedding a proper credit channel in our quantitative model would be very challenging. In Appendix A.3, we do however explore how imposing a default cost on the production side, and differentially so across the two technologies, affects our theoretical insight within the lens of a stylized two-period model.

In equilibrium, all varieties will be produced at the same price, which will be denoted thereafter by r_t—the return to the formal sector. Entrepreneurs will decide whether to supply one unit of the variety or not, $y_{it} \in \{0, 1\}$, depending on this price. Letting γ_t denote the mass of entrepreneurs producing and selling their variety to producers, the price for each such unit is,

$$r_t = (1 - \tau_t)\gamma_t^{1/\phi - 1}.$$

Entrepreneurs We now focus on the program of entrepreneurs. In each period, there is a draw determining the return to the private technology, a random variable R, and an idiosyncratic draw determining whether an entrepreneur can set her technology. With probability $1-\theta$, the entrepreneur can choose to adopt any of the two technologies. With probability θ, the entrepreneur keeps the same technology as in the previous period.

The reader interested in the exact derivation of technological choices can refer to Appendix A.1; the description below simplifies the argument. An entrepreneur with the opportunity to re-optimize in period t and with unobserved individual net return in the informal sector, R, considers the path of future (and possibly stochastic) returns to the different technologies and chooses the technology with the highest expected revenues, which is equivalent to maximizing revenues in future states of nature where re-optimization is not possible. The discounted expected revenues in the formal sector *in those states of nature*, V^r, follow a recursive equation:

$$V_t^r = r_t + E_t\left[\theta\delta_{t,t+1}V_{t+1}^r\right],$$

where r_t is the price for one unit of the differentiated good in the formal sector in period t, θ is the probability of not having re-optimized after one period and $\delta_{t,t+1} = \beta\frac{u'(c_{t+1})}{u'(c_t)}$ is the discount factor between period t and period $t+1$. The equivalent discounted expected revenues in the informal sector are,

$$V_t^R = R + E_t\left[\theta\delta_{t,t+1}V_{t+1}^R\right].$$

Let $\tilde{R}_t$ denote the level of unobserved net return in the informal sector for which an individual is indifferent between the two technologies. The indifference threshold, $\tilde{R}_t$, verifies the following recursive equation,

$$\tilde{R}_t = r_t + E_t\left[\theta\delta_{t,t+1}(V_{t+1}^r - V_{t+1}^R)\right]. \quad (2)$$

Among entrepreneurs with the opportunity to modify their technology, the mass of them adopting the formal technology, γ_t^*, should be equal to the ones with sufficiently low net returns to the informal sector (those facing the highest cost of running an informal business), i.e.,

$$\gamma_t^* = H\left[\tilde{R}_t\right] \tag{3}$$

Finally, the aggregate share of entrepreneurs operating in the formal sector, γ_t, verifies the following dynamics:

$$\gamma_t = (1-\theta)\gamma_t^* + \theta\gamma_{t-1}. \tag{4}$$

We combine the previous equations to derive the (sluggish) dynamics of production across technologies (see Appendix A.1), which govern distortions on the production side of the economy, as a function of returns to the formal sector:

$$H^{-1}\left(\frac{\gamma_t - \theta\gamma_{t-1}}{1-\theta}\right) = r_t + \theta\beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\left(H^{-1}\left(\frac{\gamma_{t+1} - \theta\gamma_t}{1-\theta}\right) - \bar{R}\right)\right], \tag{5}$$

where the return to the formal sector, r_t, is equal to $(1-\tau_t)\gamma_t^{1/\phi-1}$ and $\bar{R} = \int RdH(R)$ is the expected return to the private technology in future periods, which is the same across entrepreneurs because these returns are drawn independently across periods.

Equation (5) describes the sluggish dynamics of technological choices: (i) some entrepreneurs have not been given the opportunity to respond to economic conditions (see Equation 4); and (ii) entrepreneurs with the opportunity to re-optimize internalize that they may not be able to do so in subsequent periods. The latter effect is best understood in Equation (2): the contemporary indifference threshold depends on the returns to the formal sector but requires a premium to compensate for future losses induced by not adjusting technology and the gap between the future indifference threshold and the expected returns to the informal technology. Two assumptions guarantee that the past allocation of entrepreneurs across technologies, γ_{t-1}, is sufficient to keep track of past choices of entrepreneurs: (i) the independence of draws determining whether an entrepreneur can re-optimize; and (ii) the independence of returns to the private technology across periods.

Equation (5) also shows how expectations about future technological choices influence their current choice. A large formal sector in future periods increase expected returns to the formal sector and induce more re-optimizing entrepreneurs to opt for the formal sector today. Through this channel, default risk directly affects the current choices of entrepreneurs, independently from the current level of taxes.

Households Households make no saving or borrowing decisions in our economy. Once they receive the output net of taxes, w_t, their consumption is given by,

$$c_t = w_t,$$

in each period t. In order to smooth consumption, the government needs to smooth output net of taxes through the level of these indirect taxes.

Bond prices Investors are ready to buy any bonds in period t as long as these bonds guarantee at least ρ_t in expectation. The bond price verifies,

$$q_{t+1} = E_t \left[\frac{1 - D_{t+1}}{\rho_t} \left(\lambda + (1 - \lambda)(z + q_{t+2}) \right) \right]. \tag{6}$$

Bond prices range between 0, when default is expected with certainty next period, and close to ρ_t^{-1}, when repayment is certain.

Government The government is assumed to be benevolent and to maximize the welfare of households in each period t by choosing the levels of taxes τ_t, public savings b_{t+1} and default decisions D_t, subject to the budget constraint (1), the endogenous response of entrepreneurs, as characterized by Equation (5), and the bond price equation (6).

We are now ready to define a recursive equilibrium of our economy and shed light on the main trade-off underlying government decisions.

Recursive equilibrium Our environment is one in which entrepreneurs and the government do not interact cooperatively over time: entrepreneurs form beliefs about future government decisions in order to set their current technology. Since we suppose limited commitment towards foreign investors, we also assume that the government does not have any commitment device towards domestic entrepreneurs about its future choices. The set of equilibria that will be considered are thus Markov perfect equilibria in which agents perfectly observe a common state vector.

Let us define the state of the economy. Given the assumptions that (i) interest rate and expenditure shocks follow a Markov process, (ii) technological choices are staggered in the manner of Calvo (1983), (iii) returns to the private sector are independently drawn across periods, (iv) market reintegration is idiosyncratic, the following quantities fully characterize the economy at the beginning of period t: the state of the economy with respect to international markets, $h_t \in \{A, X\}$, where A is access and X is exclusion; the inherited asset position b_t; the inherited state of technological choices γ_{t-1}; and current interest rate ρ_t and public expenditures g_t. For convenience, let $\mathbf{s_t} = (b_t, \gamma_{t-1}, \rho_t, g_t)$ denote the state of the economy, except from access to international bond markets.

A recursive Markov perfect equilibrium is given by a sequence of debt $\{b_{t+1}\}$, aggregate technological choices $\{\gamma_t\}$, default decisions $\{D_t\}$ and bond prices $\{q_{t+1}\}$ satisfying the following conditions in all periods and histories:

Definition 1. *In each period t, the government debt and default decisions maximize the representative household's welfare, given the state $(h_t, \mathbf{s_t})$ of the economy, and subject to the period budget constraint* (1) *and the bond price equation* (6)*. The entrepreneurs maximize their profits, and Equation* (5) *defines the resulting dynamics of production choices.*

3 A stylized two-period model

This section describes how limited commitment and technological choices shape optimal fiscal policy in a stylized two-period version of the previous model where the government aims to insulate households from variation in public expenditures across periods.

We consider a stylized, deterministic economy with the following features: all entrepreneurs can freely set their technology in the first period and the return to the private technology is drawn in the first period and constant from then on; initial debt is 0, the debt contract is a one-period contract (thus without coupon obligations), and $\rho_1^{-1} = \beta$; the proportional default cost, Δ, is incurred by the household directly and within the period of default; and the state of nature (g_1, g_2) is deterministic.

We further consider two polar cases: (i) technological choices are unconstrained and fiscal policy does not have distortionary effects beyond the contemporaneous period; and (ii) technological choices are rigid and fiscal policy distorts present and future technological choices.

Flexible technological choice The following proposition characterizes optimal fiscal policy when the technological choice is fully flexible (a polar case with $\theta = 0$).

Proposition 1 (Flexible technology)**.** *With perfect commitment to repay, the optimal fiscal policy would ensure that there is perfect consumption-smoothing, i.e., $c_1 = c_2$:*

$$b_2 = b_2^* = \frac{g_1 - g_2}{1 + \beta}, \tag{7}$$

and production is the same across periods, i.e., $\tau_1 = \tau_2$ and $\gamma_1 = \gamma_2$.

With limited commitment, perfect consumption smoothing is not always feasible,

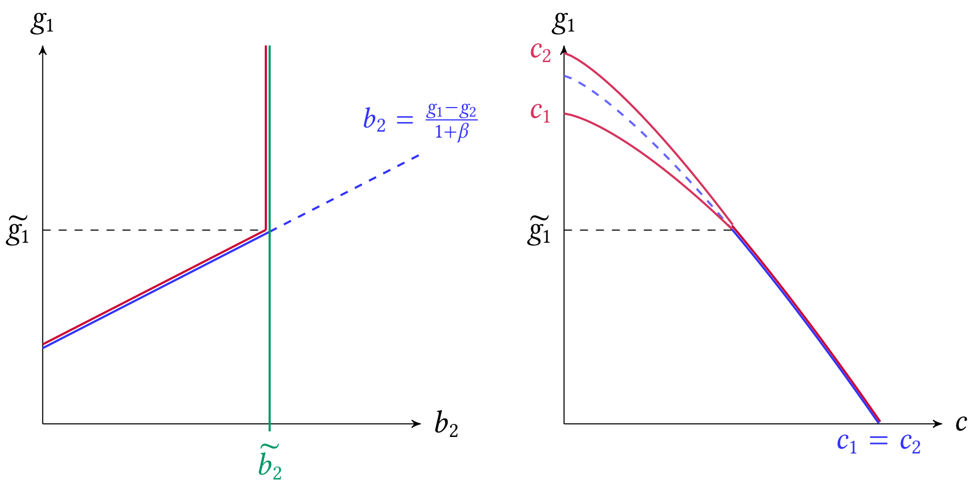
$$b_2 = \begin{cases} b_2^* & \text{if} \quad b_2^* \leq \widetilde{b}_2 \\ \widetilde{b}_2 & \text{otherwise} \end{cases}, \tag{8}$$

where $\widetilde{b}_2$ is a debt ceiling, which only depends on the state of nature in the last period (g_2) and the underlying parameters of the model (i.e., the production parameters, H and ϕ, and the default cost, Δ).

Proof. See Appendix A.2. □

To best understand how limited commitment and imperfect tax enforcement discipline fiscal policy, we display in Figure 2 how the state in the economy in the first period (g_1) affects debt (b_2, in panel a) and consumption (c_1 and c_2, in panel b).

Figure 2. Optimal fiscal policy and consumption in the flexible economy.



(a) Debt (b) Consumption

Notes: This figure represents debt (b_2, in panel a) and consumption (c_1 and c_2, in panel b) in the flexible economy ($\theta = 0$). In panel (a), the blue line represents the relationship between debt and the state of nature when the government has perfect commitment to repay its obligations; the green line represents the debt ceiling; and the red line represents the relationship between debt and the state of the economy under limited commitment. In panel (b), the blue line represents the relationship between consumption (in both periods) and the state of nature when the government has perfect commitment to repay its obligations; and the red lines represent the (possibly different) consumption schedules under limited commitment.

As apparent in Figure 2, the state of nature in the first period has different effects on fiscal policy, depending on the required level of debt to ensure perfect consumption smoothing. When the state of nature is not too bad, i.e., $g_1 \leq \widetilde{g}_1$, the government insures the households against higher expenditures by issuing more debt: the levels of consumption decrease, but similarly so across periods. As the state of nature deteriorates, it may reach a point where such fiscal policy is not feasible ($g_1 > \widetilde{g}_1$): the required debt level would be too high and the government would have incentives to default under limited commitment. After this point, the government incurs as much debt as it can in the first period, i.e., $b_2 = \widetilde{b}_2$, and there is a gap between the first-best debt level (blue line) and the one chosen under limited commitment (red line) that widens as g_1 increases. This gap affects welfare through two distinct effects. First, there is a welfare cost to imperfect consumption smoothing that would exist even with lump-sum payments (a *consumption-smoothing cost*). Second, raising revenue is distortive, and a lower consumption in period 1 comes with high taxation and a highly-distorted economy (a *distortion cost*).[20]

Persistent technological choice Consider now that technological choices from entrepreneurs in period 1 are fully persistent ($\theta = 1$) such that $\gamma_2 = \gamma_1$. The following proposition characterizes optimal fiscal policy in this other polar case.

Proposition 2 (Persistent technology). *With perfect commitment to repay, the optimal fiscal policy ensures that there is perfect consumption-smoothing, i.e., $c_1 = c_2$:*

$$b_2 = b_2^* = \frac{g_1 - g_2}{1 + \beta}, \tag{9}$$

and production is the same across periods, i.e., $\tau_1 = \tau_2$ and $\gamma_1 = \gamma_2$.

With limited commitment, perfect consumption smoothing is not always feasible,

$$b_2 = \begin{cases} b_2^* & \text{if} \quad b_2^* \leq \widehat{b}_2 \\ \widehat{b}_2 & \text{otherwise} \end{cases}, \tag{10}$$

where $\widehat{b}_2$ is a debt ceiling, which depends on the state of nature in both periods (g_1, g_2) and parameters of the model (i.e., the production parameters, H and ϕ, the default cost, Δ).

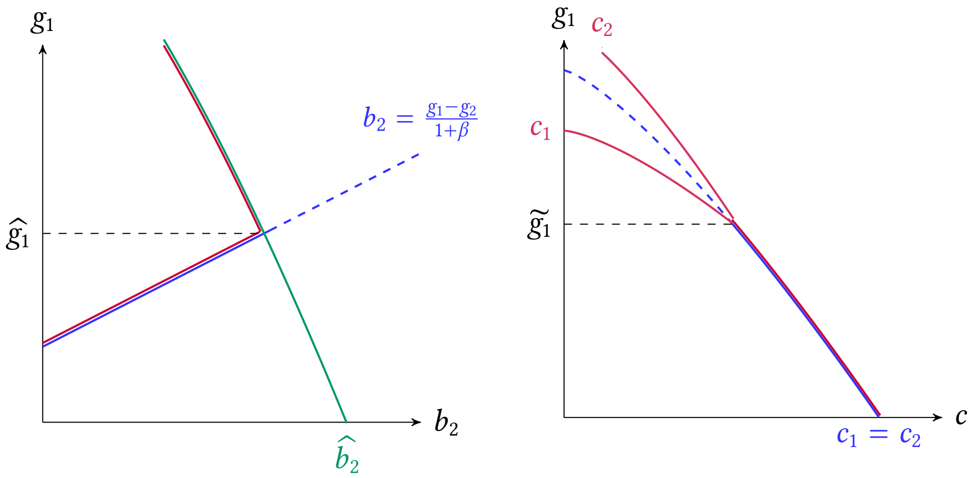
Proof. See Appendix A.2. □

Proposition 2 closely mirrors Proposition 1 in its structure, but the optimal fiscal policies in these polar cases markedly differ because of the nature of the present debt ceiling, $\widehat{b}_2$, versus the previous one, $\widetilde{b}_2$. We illustrate these differences in Figure 3.

Contrasting Figure 3 with Figure 2, we see that the persistent and flexible economies share two common aspects—the first-best policies are the same and fiscal policies are constrained by a debt ceiling when the state of nature is bad enough in the first period—and one main difference—the shape of the latter constraint. In the persistent economy,

[20] As will be shown later, the distortionary effect of taxation disciplines two aspects of the optimal fiscal policy under limited commitment: (i) it will affect the level of the debt ceiling—higher distortions leading to a lower debt ceiling; and (ii) it will affect the curvature of consumption as a function of government expenditures (through the mediation role of taxation).

Figure 3. Optimal fiscal policy and consumption in the persistent economy.



(a) Debt (b) Consumption

Notes: This figure represents debt (b_2, in panel a) and consumption (c_1 and c_2, in panel b) in the persistent economy ($\theta = 0$). In panel (a), the blue line represents the relationship between debt and the state of nature when the government has perfect commitment to repay its obligations; the green line represents the debt ceiling; and the red line represents the relationship between debt and the state of nature under limited commitment. In panel (b), the blue line represents the relationship between consumption (in both periods) and the state of nature when the government has perfect commitment to repay its obligations; and the red lines represent the (possibly different) consumption schedules under limited commitment.

the debt ceiling does not only depend on the state of nature in the second period; it crucially depends on the state of nature in the first period (panel a). When expenditures are high in the first period, the government will need to implement higher taxes. In turn, these taxes will affect the technological choices of entrepreneurs. Entrepreneurs will choose a production technology based on the relative returns of the two technologies in the first period and in the second period, as their current choice will fully persist in the subsequent period. Through these persistent technological choices, a poor state of nature in the first period makes reimbursement more costly in the second period and thereby reduces the government's commitment to repay. Beyond the point where the first-best fiscal policy is feasible ($g_1 > \widehat{g}_1$), the government incurs as much debt as it can in the first period, but this debt actually decreases with first-period expenditures—while the incentives to accumulate debt increase. The gap between the first- and second-best fiscal policies thus markedly widens with g_1. In turn, the welfare cost of limited commitment markedly increases through a more pronounced *consumption-smoothing cost* and a more pronounced *distortion cost*.

The distortionary effect of taxation shapes the debt constraint in two distinct ways: (i) higher distortions shift the debt ceiling schedule inwards; and (ii) higher distortions tilt the debt ceiling schedule which becomes more sensitive to the state of nature in the first period. We turn to these comparative statics next.

Comparative statics We now provide simple comparative statics, which further shed light on the implications of having distortionary redistribution between the government and households when there is limited commitment.

Proposition 3 (Comparative statics)**.** *The expenditure threshold from which first- and second-best fiscal policies differ is higher in the flexible economy than in the persistent economy, i.e.,* $\widetilde{g}_1 \geq \widehat{g}_1$*, and the debt ceiling is uniformly higher, i.e.,* $\widetilde{b}_2 \geq \widehat{b}_2\left(\widehat{g}_1\right)$*.*

Proof. See Appendix A.2. □

Proposition 3 illustrates the following mechanism. Higher expenditure implies higher taxation; and a higher taxation lowers the contemporary return r_1 to the formal sector through two channels. There is a direct effect deriving from higher taxes. There is an indirect channel deriving from entrepreneurs switching to the informal sector thereby lowering the returns to the formal sector. Through Equation (5), this affects the indifference threshold between the formal and the informal sector, and thus the number of entrepreneurs operating in the informal sector. With flexible technological choices, this response does not affect the incentives to repay in the second period. With persistent technological choices, this response increases the future costs of raising tax revenues.

While the previous mechanism is always at play, at least qualitatively, when there is some rigidity in technology, its quantitative relevance crucially depends on fundamentals of the economy: the sensitivity of technological choices to economic conditions, their persistence, and the nature of exogenous fluctuations. To shed light on such quantitative relevance, we now turn to analyzing the proper dynamic, stochastic model under a reasonable calibration.

4 Quantitative model

In this section, we analyze the full-fledged model developed in Section 2. We first calibrate the model to match key empirical moments, and we discuss how the resulting economy fares against a few untargeted moments; we then derive quantitative results about the joint dynamics of fiscal policy and default risk in the model; and we finally discuss the welfare implications of these dynamics.

4.1 Solution method and calibration

Value functions and solution method We solve for the government optimization program in a recursive fashion and as a fixed-point problem. We write the government problem as a Bellman equation and consider two value functions corresponding to different default histories. At any period t, the government may have access to international markets, A, or be excluded, X. Letting $\mathbf{s_t}$ denote the state of the economy, the government value function in period t is $v(\mathbf{s_t}) = \mathbb{1}_{h_t=A} \cdot v_A(\mathbf{s_t}) + \mathbb{1}_{h_t=X} \cdot v_X(\mathbf{s_t})$, where the conditional value functions v_A and v_X are defined as,[21]

$$v_A(\mathbf{s_t}) = \max_{D_t, b_{t+1}, \gamma_t, \tau_t} \left\{ u\left(c_t\right) + \beta(1 - D_t) E_t v_A(\mathbf{s_{t+1}}) + \beta D_t E_t v_X(\mathbf{s_{t+1}}) \right\},$$

and,

$$v_X(\mathbf{s_t}) = (1 - \nu) \left[u(c_t(1 - \Delta)) + \beta E_t v_X(\mathbf{s_{t+1}}) \right] + \nu \max_{b_{t+1}, \gamma_t, \tau_t} \left\{ u\left(c_t\right) + \beta E_t v_A(\mathbf{s_{t+1}}) \right\},$$

subject to Equations (1), (5), and (6). Note that, as the government is not allowed to return the proceeds of its external borrowing to the representative household as a lump-sum transfer, there is only one instrument to transfer from and to the household: in effect, the choice of future debt pins down the choice of the tax rate (through the budget constraint) and reciprocally. The government does face a debt price schedule which depends on: two endogenous state variables, $q(b_{t+1}, \gamma_t, \rho_t, g_t)$, but the two arguments are always tied through the budget constraint; and future debt prices.

We solve numerically for these value functions as a fixed-point problem. The government does not have commitment on future policy and takes it as given when deciding upon contemporary choices. The problem is solved by iteration, and we find the fixed point by looping over the future behavior of the government and debt pricing. More specifically, we initially set two policy functions $\mathbf{s} \mapsto \gamma(\mathbf{s})$ and $\mathbf{s} \mapsto q(\mathbf{s})$, which characterize future production and future expected default, both as a function of state variables ($\mathbf{s}$). Given the policy functions $\mathbf{s} \mapsto \gamma(\mathbf{s})$ and $\mathbf{s} \mapsto q(\mathbf{s})$, we solve the dynamic problem of the government through value function iteration. We then update the policy functions $\mathbf{s} \mapsto \gamma(\mathbf{s})$ and $\mathbf{s} \mapsto q(\mathbf{s})$, using the policy functions obtained in the previous step, and we iterate until convergence.

In practice, we need to keep track of four state variables: the inherited debt level, the inherited incidence of the formal sector and the contemporary values for the two exogenous stochastic processes. We create rough grids for each of the state variables: The AR(1) processes for interest rate and public expenditures are discretized using the Tauchen (1986) method. In each iteration, we use cubic spline interpolation to approximate the value function and the (given) policy functions, following Hatchondo et al. (2010), such that the government effectively solves a continuous problem.

[21]Without market access, the government makes no decisions because the period tax rate, and thus technological choices, are pinned down by the budget constraint $t_t = g_t$.

Calibrated parameters In order to calibrate the previous model, we proceed as follows. We first set the parameters characterizing preferences, default risk, debt maturity, and shocks *directly*, for a synthetic, average economy. We then estimate production parameters to *match* key empirical moments that we identify in a dataset of selected countries with non-negligible informality and default risk.

To discipline the inclination for consumption-smoothing in the quantitative model, we consider risk-averse and rather impatient households. The representative household enjoys consumption following a Constant Relative Risk Aversion (CRRA) utility function, $u = c^{1-\sigma}/(1-\sigma)$, and we set the curvature parameter towards the high-end in the spectrum of reasonable values ($\sigma = 2$, as in Aguiar and Gopinath, 2006; Arellano, 2008; Chatterjee and Eyigungor, 2012). The household is impatient, leading to debt accumulation—even absent any fluctuations: we set a discount factor (β) corresponding to an annual discount rate of 15%; external investors are more patient and we set the average annual risk-free interest rate ($E[1/\rho_t] - 1$) to be 5% (close to the quarterly rate used in Chatterjee and Eyigungor, 2012, for example).

Impatience is one factor leading to debt accumulation. Another, limiting factor is the (imperfect) commitment for the government to repay. In the model, commitment comes from two sources of punishment: one is temporary market exclusion preventing the government from bringing consumption forward for impatience-related or risk-related purposes; another one is a direct output cost possibly reflecting a capital flight from investors (Mendoza and Yue, 2012). We set the reintegration probability following a default, ν, to produce an average exclusion length of 8 years (slightly higher than the exclusion periods following Argentinian defaults in 1982 and 2001, see Chatterjee and Eyigungor, 2012). The proportional output cost, Δ, is calibrated at 15% in the high range of output losses for "hard defaults" (estimated to be between 6 and 15%, see Trebesch and Zabel, 2017). In our baseline model, each debt contract has a probabilistic maturity; and we set the parameter λ—i.e., the fraction of outstanding debt which matures in each period—equal to 0.40. In order to sustain higher debt levels with reasonable fluctuations in debt prices, we also add a small coupon rate of 0.05 such that the maturity—the average time for a debt unit to be repaid—is about 3 years.

The benevolent government issues debt to insulate households from exogenous fluctuations in economic conditions. We consider auto-correlated interest rate and spending shocks, whose logarithms follow AR(1) processes, and we set the standard deviations of innovation shocks to mimic the volatility of risk-free interest rates and public expenditures (as a share of output) in our baseline sample of economies. The bottom panels of Table 1 list these "non-estimated" parameter values.

Table 1. Parameters selected directly and by matching empirical moments.

Description	Parameter	Value
1. Preferences (directly)		
Discount factor	β	0.85
Average interest rate	$E[1/\rho] - 1$	0.05
Risk-aversion (CRRA)	σ	2
2. Default and debt (directly)		
Probability of reintegration	ν	0.125
Output cost	Δ	0.15
Maturity ("maturation" probability)	λ	0.40
Coupon rate	z	0.05
3. Shock (directly)		
Credit markets $\{\log \rho_t\}$, autocorrelation	s_ρ	0.656
Credit markets $\{\log \rho_t\}$, standard deviation	σ_ρ	0.010
Expenditures $\{\log g_t\}$, autocorrelation	s_g	0.656
Expenditures $\{\log g_t\}$, standard deviation	σ_g	0.020
4. Production (matched)		
Informal returns (Pareto, $H(x) = 1 - \left(\frac{a_1}{x}\right)^{a_2}$)	a_1	0.57
Informal returns (Pareto, $H(x) = 1 - \left(\frac{a_1}{x}\right)^{a_2}$)	a_2	5.48
Probability to set technology	θ	0.19
Complementarities	ϕ	0.91

Notes: See Appendix B.1 for a detailed description of the data sources and the derivation of empirical moments underlying the panel *4. Production (matched)*. Note that: we set the average level of public expenditures, $E[g_t]$, to equal 16% of output; and our time unit is a year such that the discount factor corresponds to a quarterly discount factor of around 0.96 and the autocorrelations of 0.656 correspond to quarterly autocorrelations of 0.90.

Targeted moments The model requires parameter values for fundamentals governing the production side of the domestic economy. Equation (5) characterizes the dynamics of domestic production as a function of fiscal policy—through the contemporary returns to the formal sector—, the dispersion of contemporary and future returns in the informal sector, and the future economic environment. The main fundamentals governing this relationship are: (i) the distribution of returns in the informal sector, $H(\cdot)$, mostly translating into an average level of informality and an elasticity of technological choices to economic conditions (typically, to the tax rate); and (ii) the Calvo parameter, θ, mostly translating into technology rigidities over time.

The variability of technological choices and their persistence are crucial to our analysis, and we put additional care into estimating them. To do so, we rely on data collection started in Pappadà and Zylberberg (2017) and consolidated in Pappadà and Rogoff (2023) that we harmonize across countries and over time to measure VAT compliance from the observation of consumption and tax rate across goods. The final dataset is an unbalanced panel of about 45 economies between 1995 and 2020. We complement the data with government expenditures, sectoral shares (services, agriculture, industry), output

(GDP per capita, constant USD, PPP), government debt stocks, and ten-year government bond rates, as collected from different sources (e.g., the Word Indicator Database, the OECD, and the IMF). Appendix B.1 further describes the different data sources and details how one can recover the incidence of informal transactions from observing actual consumption patterns against tax revenues.

A caveat with our data is that it relies on the availability of harmonized data on household consumption and tax receipts: our original sample thus disproportionately features rich economies. Our model is however designed to best capture economic contexts with relatively high informal sectors and default risk: we thus restrict the baseline sample to economies with high levels of informality and long-term sovereign debt spreads.[22] The resulting sample primarily features "middle-income" countries with high informality levels (e.g., Colombia, Chile, Russia, Serbia, South Africa), Eastern-European countries (e.g., Bulgaria, Croatia, Czech Republic, Hungary, Poland, Romania) and Southern-European countries (e.g., Greece, Italy, Portugal, Spain). For the sake of transparency, we however consider a sensitivity analysis in Appendix B.1 and report the selection of targeted moments for five alternative samples, e.g., including all economies or focusing on "middle-income" economies or peripheral European countries.

We exploit rich variation over time across our selected economies with non-negligible informality and default risk to identify the average elasticity of technological choices to tax rates and their average persistence. More specifically, we first identify the elasticity of tax compliance to tax rates by considering a country c in year t and estimating the following baseline specification in difference:[23]

$$\Delta \ln \gamma_{t,c} = \varepsilon_\gamma \Delta \ln \tau_{t,c} + \beta \mathbf{X}_{t,c} + \delta_t + \mu_c + e_{t,c}, \tag{11}$$

where: $\Delta \ln \gamma_{tc}$ is the annual change in (log) tax compliance; and $\Delta \ln \tau_{tc}$ is the annual change in (log) effective VAT rate. The vector $\mathbf{X}_{tc}$ includes the following set of time-varying controls: the annual change in (log) output per capita (in constant USD and PPP-adjusted); annual changes in the sectoral decomposition of economic activity across 1-digit sectors and in the ratio of exports and imports over GDP; and the annual changes in government expenditures as a ratio of output. These controls partly clean for confounding factors co-moving with tax compliance and fiscal policy. Second, we identify the persistence in tax compliance in a two-stage AR(1) specification where tax compli-

[22]More specifically, we drop countries that are either below median for the average level of informality or within the bottom quartile for average long-term sovereign debt spreads.

[23]We use a specification in difference to account for non-stationarity in informality (and, to a second order, in fiscal pressure). Note, however, that our estimates would not be markedly different in a level specification, which is the first-stage of our "persistence" specification (B1) (see Appendix B.1).

ance, $\ln \gamma_{t,c}$, is regressed on its lagged level, and the latter is instrumented in a first-stage by the lagged effective VAT rate. In this exercise, we consider the same controls as in Equation (11). We report these key targeted moments in Table 2, along with the average incidence of the formal sector (0.72) and an externally-derived targeted moment: a fiscal multiplier of 1.10 based on informal economies (Pappa et al., 2015).

Our approach minimizes the distance between moments of the simulated economy and these four targeted moments: (a) the average incidence of the formal sector, (b) the elasticity of such incidence to tax rates, (c) the persistence of formality, and (d) the fiscal multiplier. In effect, we allow for four degrees of freedom to characterize technological choices and match these moments: we use a Pareto distribution of returns H with location and shape parameters a_1 and a_2; and we then estimate these parameters (a_1, a_2), the degree of complementarity within the formal sector (ϕ), and the Calvo parameter (θ) to best match the four targeted moments. Intuitively, the average volatility of technological choices mostly pins down the dispersion of returns in the informal sector; the fiscal multiplier disciplines complementarities in the formal sector; and the persistence of technological choices pins down the period probability to reset technology. We report the distance-minimizing parameters under the heading "Production" in Table 1, and we contrast the targeted and simulated moments—based, for the elasticity and persistence, on regressions using simulated data—in Panel A of Table 2.

Non-targeted moments The previous calibration targets production parameters, but does not directly target a few important moments characterizing fiscal policy and default. In particular, we impose a structure of auto-correlated interest rate and spending shocks, and this structure crucially disciplines the uncertainty faced by the government.

In this section, we briefly probe whether our model is able to replicate the volatility of fiscal policy and default risk within our selected sample of countries (see Panel B of Table 2). We contrast the *overall* fluctuations in formality, the volatility of tax rates, the volatility of default risk, the average debt and its volatility, and the volatility of household consumption in 10,000 simulated economies over 50 years against those observed in our unbalanced panel of 20 economies between 1995–2020. We see that our structure of shocks, combined with our estimated production parameters, leads to an overshoot in terms of the volatility of fiscal policy and formality. A likely explanation is that government expenditure is exogenous in our model and we ignore other macroeconomic stabilizers, such that debt and a linear tax rate are the only "smoothing" instruments. Our structure of long-term debt contracts (borrowed from Chatterjee and Eyigungor, 2012) however creates acceptable levels of debt, default risk, and actual default incidence (see Panel C). In summary, our calibrated economy is reasonably close to the average econ-

Table 2. Simulated and empirical moments.

	Data	Model
Panel A: Targeted moments		
a. Incidence of the formal sector	0.720	0.712
b. Elasticity of the formal sector	-0.392	-0.409
c. Persistence of the formal sector	0.846	0.839
d. Fiscal multiplier	1.100	1.103
Panel B: Non-targeted moments		
e. Volatility of formality (SD)	0.044	0.077
f. Volatility of tax rates (SD)	0.022	0.053
g. Default risk (SD)	0.009	0.012
h. Debt to output (average)	0.484	0.522
i. Debt to output (SD)	0.088	0.100
j. Volatility of consumption to output (SD)	0.018	0.021
Panel C: Default		
k. Default incidence	-	0.0104
l. Market exclusion	-	0.0873

Notes: Column (*Data*) relies on our unbalanced panel of selected economies between 1995–2020. The empirical equivalent of the (log) incidence of the formal sector is the (log) tax compliance. The empirical elasticity of the formal sector is estimated following specification (11); and the persistence of the formal sector is estimated following the AR(1) specification (B1) (see Appendix B.1). Column (*Model*) provides the same moments from 10,000 simulated economies over 50 years. In practice, we run simulations across 100 years and only keep the last 50 years to limit the influence of initial conditions. In Appendix B.1 and Tables B1, B2, and B3, we report the selection of targeted and non-targeted moments across five alternative samples (S2: all economies, S3: high informality, S4: high default risk, S5: peripheral European economies, S6: middle-income economies). The model-based statistics in Panel A and Panel B are computed using the subsample of repayment periods.

omy of our baseline sample—notwithstanding the high(er) fluctuations in fiscal policy.

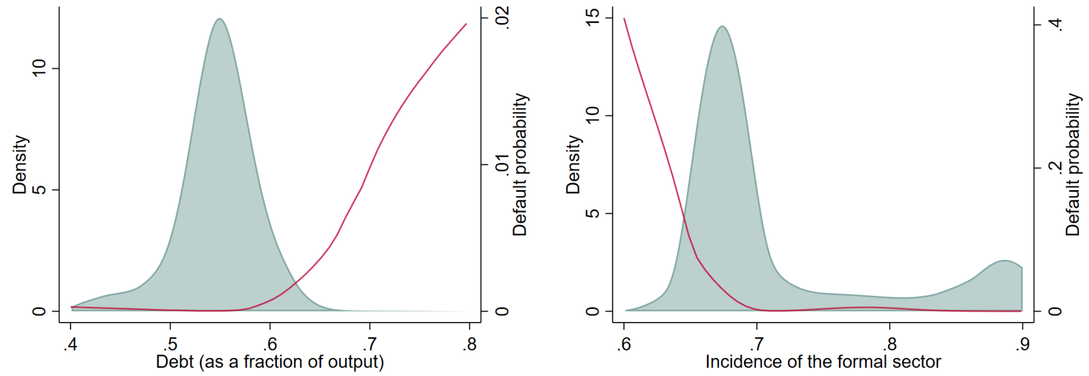
4.2 Properties of the calibrated economy

Default and state variables We first analyze the properties of the calibrated economy. To this purpose, we simulate 10,000 similar economies over 50 years, and we display the distribution of the two endogenous state variables, (b_t, γ_{t-1}), and their relationship with default in Figure 4.[24]

First, we find that the government has incentives to accumulate debt, but it comes at a cost. The government mostly accumulates debt over a range going from about 40 to 80% of output, but debt levels above 65% of output are rarely observed (the latter could be

[24]The interested reader can refer to Appendix B.1 and Appendix Figure B1 for a visualization of the joint distribution between debt, informality, and default.

Figure 4. Distribution of debt, technological choices, and incidence of default.



(a) Debt

(b) Formal sector

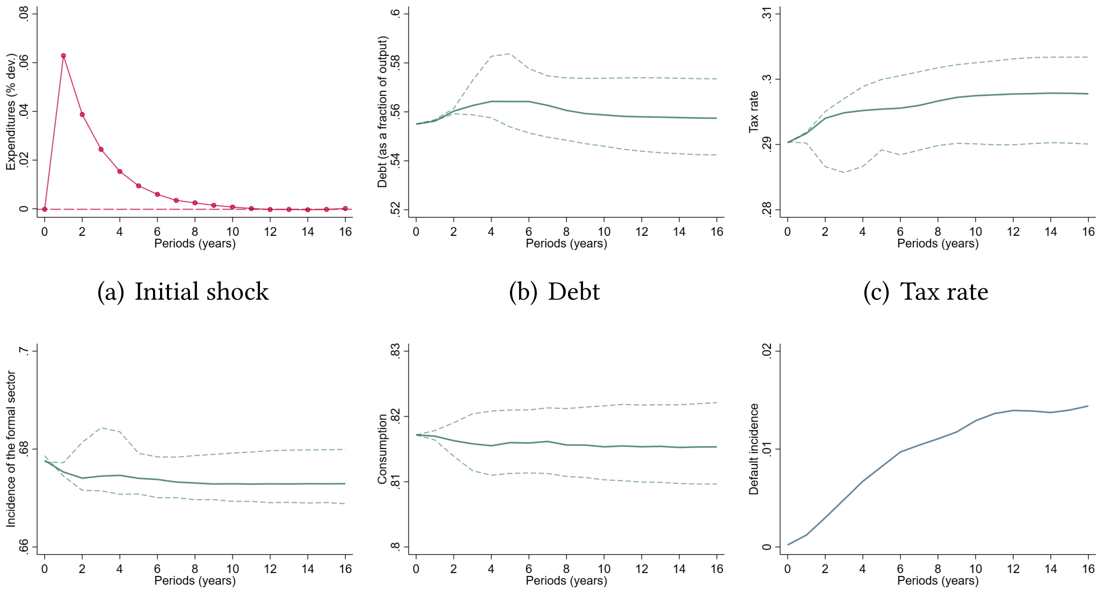
Notes: This Figure displays the distribution of the two endogenous state variables, (b_t, γ_{t-1}), and their relationship with default across 10,000 simulated economies over 50 years. In practice, we run simulations across 100 years and only keep the last 50 years to limit the influence of initial conditions. Panel (a) shows the distribution of debt (as a share of output) and the average default incidence for each debt level (in red)—as averaged over the other endogenous or exogenous state variables. Panel (b) shows the distribution of technological choice (the incidence of the formal sector). The "default probability" (reported in the second y-axis) is the average default incidence in period t (excluding periods of autarky); and the constructed distributions are based on simulated data *during periods of repayment.*

interpreted as a "sustainable" debt ceiling); the incidence of default is nil for low levels of inherited debt b_t but steadily increases for b_t between 60% and 80% of output. This steady increase of default incidence with inherited debt implies that: (i) there exists a significant region in the set of state variables where default is likely, and (ii) the government is still tempted to accumulate debt within this region, as the marginal cost of doing so is not immediately prohibitive.[25] Second, the dynamics of informality is key to understanding default decisions. The incidence of the formal sector ranges between 0.60 and 0.90 with an asymmetric, left mode, and default is predominantly observed towards the lower end of this distribution. The staggered technological adjustment of entrepreneurs implies that the dynamics of informality exhibits stickiness—a key property of the calibrated economy: high informality today induces low returns to taxation in the medium run, thereby giving strong incentives to default.

The response to shocks The complex dynamics followed by our economy as a function of exogenous fluctuations cannot be captured by the previous evidence. To shed some light on these dynamics, we produce impulse response functions following an ex-

[25] We find, however, that the tail of the distribution is quite thin above 65% of output. This pattern derives from two effects: (a) the threat of default restrains government choices; and (b) the government sometimes defaults and future debt is then reset to 0.

Figure 5. Impulse response functions—an illustration of dynamics in the simulated economy.



(a) Initial shock (b) Debt (c) Tax rate

(d) Formal sector (e) Consumption (f) Default incidence

Notes: This Figure displays the median response of 10,000 simulated economies to an expenditure shock over a period of 16 years. Panel (a) provides an illustration of the direct effect of the shock on average expenditures: in period 1, we set the level of expenditure to be about 1 percentage point of output above its long-term value (see period 0), which corresponds to a relative (log) deviation around 6%. We then simulate the stochastic, exogenous processes from period 1 onward across the 10,000 simulated economies. Panels (b), (c), (d), and (e) display the median response (plain line) and the 25-75% quantiles in green for the following variables: debt (as a share of output), the tax rate (τ), the incidence of the formal sector (γ), and consumption (c). In these panels, we compute these quantities while only including repayment periods; we provide a robustness check also including the (less frequent) default states in Appendix B.2. Finally, panel (f) displays the probability to observe a default across the 10,000 simulated economies. The initial conditions are: $\gamma_0 = 0.678$, $b_1 = 0.555$, and the other exogenous shock is initially centered around its average value, $dev(\rho_0) = 0$.

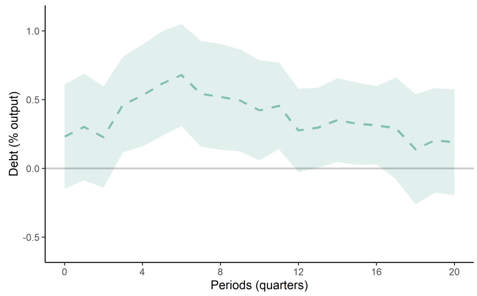
penditure shock within our dynamic system.[26] A limit of this exercise is that it depends on initial conditions and extracting an average response across those is not easy; for this reason, we use it as an illustration of the model dynamics rather than a proper validation against the data (as more explicitly done in Table 2).

The outcome of this exercise is reported in Figure 5. Panel (a) displays the evolution of the shocked, exogenous variable in (log) deviation from its mean. Panels (b) and (c) show the response of debt and taxation: the average simulated economy (plain line) fills the large gap in the government budget by incurring more debt *and* by increasing taxation. Along the median trajectory, the former covers about 50% of the gap, while the latter accounts for the remaining 50%. This median response masks wide heterogeneity across simulations (or states of nature): debt and tax rates significantly decrease

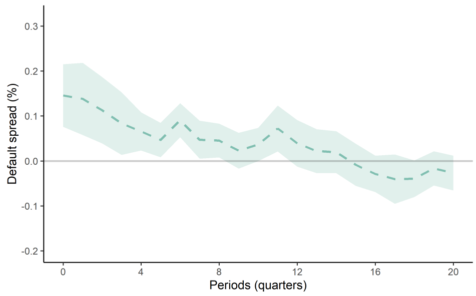
[26]We first simulate our economy over many periods and use the average of the state variables ($\bar{s}$) as initial conditions. We then deviate from this "steady-state" by setting exogenous expenditures above their long-term average by 1 percentage point of output. Finally, we simulate 10,000 economies affected by subsequent shocks and starting from this initial shock in period 1, and we report the distribution of their responses over a period of 16 years.

in those infrequent occurrences where the government ends up defaulting (see panel f, where default remains very likely 10 years after the initial shock); debt reverts to the mean in about 6-8 years; taxes however remain high in most cases and over the whole period—an informality-driven "overhang". This overhang is most visible when looking at technological choices (panel d, showing the incidence of the formal sector): formality remains persistently low—0.5 percent below its average value—in almost two-thirds of our simulation. Through this sluggish adjustment in (in)formality, the government remains susceptible to default in the medium run. Ultimately, what matters to welfare is consumption. Its evolution lies between the less sluggish debt response and the more sluggish technological response: there is a slight decrease in private consumption of about 0.2% of output on impact, which very slowly reverts to the mean.

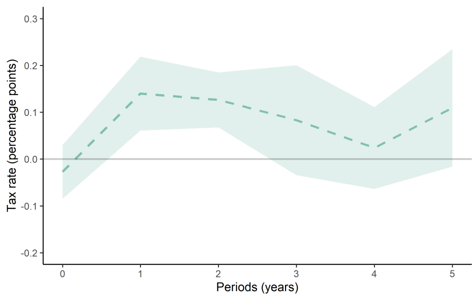
Figure 6. Impulse response functions—data.



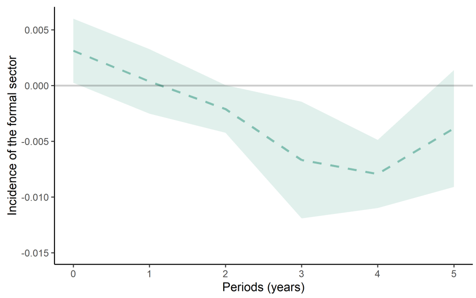
(a) Debt



(b) Default spread



(c) Tax rate



(d) Tax compliance

Notes: This Figure displays the estimated impulse response function to an increase of one percent of output in government expenditures (as in Born et al., 2020). Panels (a), (b), (c) and (d) display the estimated response (dashed line) and the 10-90% confidence interval for the following variables: debt as a percentage of output, the default spread on sovereign bonds, the average tax rate, and tax compliance. The estimation procedure is done at a quarterly level for debt and the default spread (panels a and b) and at a yearly level for the average tax rate and tax compliance (panels c and d).

We provide an empirical counterpart for selected variables in Figure 6 where we replicate the approach of Born et al. (2020) on our sample of countries across years.[27] The approach builds upon local projections (Jordà, 2005) and exogenous expenditure

[27] In effect, we cannot apply the selection implemented in Section 4.1 to our empirical computation of

shocks: forecast errors in government consumption. More specifically, we consider the following empirical specification,

$$y_{ct+h} = \Psi_h z^g_{ict} + \gamma_h \mathbf{x}_{ct} + \alpha_{ch} + \eta_{th} + \varepsilon_{ct+h}, \tag{12}$$

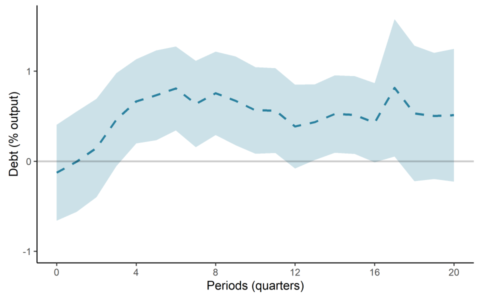
where c is a country, t is the time of the fiscal shock, z^g_{ict}, and h is the number of years/quarters after the initial shock. The fiscal shock, z^g_{ict}, is defined as the gap between actual government consumption and its forecast by Oxford Economics. In the specification, we also condition for control variables and their lags ($\mathbf{x}_{ct}$: corporate and income tax rates, year of entry into the UE), country fixed effects (α_{ch}) and year fixed effects (η_{th}). The coefficients of interest are the estimated Ψ_h at different horizons: $h \in \{1, 2, 3, 4, 5\}$ for yearly specifications and $h \in \{1, \ldots, 20\}$ for quarterly specifications.

Figure 6 shows the estimated effects, Ψ_h, of exogenous shocks in government expenditures over a 5-year horizon on: debt levels (b, quarterly), the default spread on sovereign debt (as a proxy for debt prices, q, quarterly), tax rate (VAT rate, as a proxy for τ, yearly), and tax compliance (VAT compliance, as a proxy for the incidence of the formal sector, γ, yearly). The initial shock is partly mitigated by higher debt levels (between 30% and 50% of the initial budget gap is covered by debt), but also by higher taxes. There is a significant tax overhang: fiscal policy remains contractionary in the medium run, leading to a persistent, delayed drop in VAT compliance. Five years after the initial expenditure shock (1% of GDP), tax compliance is still 0.6 percentage points below its average value—a persistence and magnitude close to the quantitative moments. These findings provide an empirical counterpart to the model-based response derived in Figure 5.

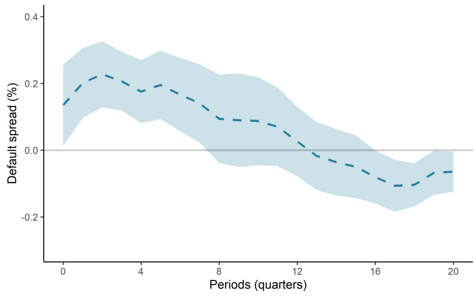
The previous empirical response is an average across different economies in different states of nature. In Figure 7, we provide a conditional impulse response based on two regimes: a low-γ_{t-1} regime, and a high-γ_{t-1} regime (calculated by the position of γ_{t-1} relative to its median value within each country). The responses to the expenditure shock markedly differ across the two regimes. In the low-γ_{t-1} regime (blue), both debt and the default premium increase, while debt increases without being associated to higher default risk in the high-γ_{t-1} regime (orange)—in line with the forces at play in our quantitative model. The dynamics of informality do not only respond to fiscal policy; it also alters its impact on the economy. These interactions between fiscal policy and the

impulse responses: once we match our full sample of economies (S2) to the fiscal shocks used in Born et al. (2020), we are left with only 18 economies, and half of them are outside of our baseline sample (S1). Implementing local projections requires sufficient variation, especially when estimated on annual data (Jordà, 2005). For this reason, we estimate Equation (12) on the largest possible sample (thus including countries with lower incidence of informality or low default risk). One needs to keep this limitation in mind when contrasting Figures 5 and 6.

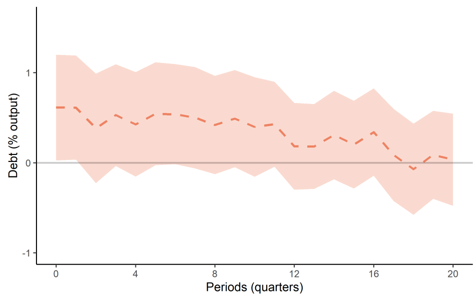
Figure 7. State-dependent impulse response functions—data.



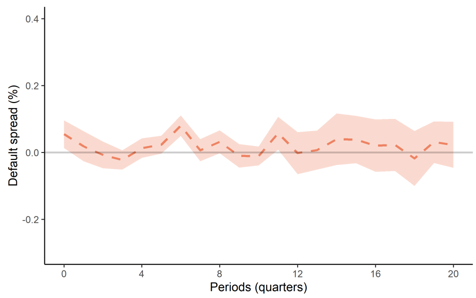
(a) Debt (low-γ_{t-1})



(b) Default spread (low-γ_{t-1})



(c) Debt (high-γ_{t-1})



(d) Default spread (high-γ_{t-1})

Notes: This Figure displays the estimated impulse response function to an increase of one percent of output in government expenditures (as in Born et al., 2020). Panels (a/c) and (b/d) display the estimated response (dashed line) and the 10-90% confidence interval for debt as a percentage of output and the default spread. The estimation procedure allows for regime switching between two regimes: low and high tax compliance in the previous period.

dynamics of formality may have non-negligible welfare consequences. We now quantify the impact of such interactions through the lens of our quantitative model.

4.3 The dynamic distortionary effect of fiscal policy and its welfare cost

To quantify the welfare consequences of dynamic distortions, we consider four different economies: [i. Baseline] the baseline economy calibrated in Section 4.1; [ii. Low-response] a lower-response economy with similar fundamentals but a lower elasticity of technological choices to taxation (-0.30), through a different distribution of returns in the informal sector, H; [iii. Flexible] a more flexible economy with similar fundamentals but a less persistent incidence of formality (0.64), through a different Calvo parameter, θ; and [iv. Commitment] a full-commitment economy similar to our baseline economy, but with infinite default costs—thus providing perfect commitment to repay to the government.[28] We normalize average output levels across counterfactuals, such that the average

[28]We consider additional experiments varying other parameters of the baseline model in Appendix B.3, which also indirectly shows the sensitivity of our calibration to preference parameters, the default and debt

level of consumption is not informative about welfare, and we focus on the welfare cost of consumption *fluctuations* in each economy (in the spirit of Lucas, 1987). For each of these economies, we simulate 10,000 stochastic processes and their associated responses over 100 periods (years) for which we focus on the last 50 years, using the same shock structure as in the baseline economy.

Table 3. The welfare cost of dynamic distortions and imperfect commitment.

	Baseline (i)	Low-response (ii)	Flexible (iii)	Commitment (iv)
Panel A: Fiscal policy				
Debt-to-output ratio	0.522	0.600	0.553	0.470
	(0.100)	*(0.046)*	*(0.047)*	*(0.030)*
Tax rates	0.284	0.279	0.298	0.295
	(0.053)	*(0.036)*	*(0.039)*	*(0.030)*
Formal sector	0.712	0.722	0.701	0.700
	(0.077)	*(0.036)*	*(0.046)*	*(0.035)*
Consumption	0.810	0.804	0.799	0.799
	(0.043)	*(0.018)*	*(0.015)*	*(0.013)*
Panel B: Default				
Default incidence	0.0104	0.0026	0.0002	0.0000
Market exclusion	0.0873	0.0209	0.0014	0.0000
Panel C: Welfare cost				
Cost of fluctuations (% cons.)	0.2460	0.1259	0.0274	0.0347

Notes: For each specification [i-iv], we simulate 10,000 stochastic processes and their associated responses over 100 periods (years), using the baseline shock structure and similar initial conditions. In practice, we run simulations across 100 years and only keep the last 50 years to limit the influence of initial conditions. Specification (i) is our baseline model (column 2 of Table 2), in which production parameters are estimated to produce an average incidence of formality of about 0.71, an elasticity of formality to tax rate of -0.40, a persistence of 0.84, and a fiscal multiplier of 1.10. Specification (ii) uses all parameters of specification (i), except that production parameters are set to match all previous moments but the elasticity of formality to tax rate (-0.30). Specification (iii) uses all parameters of specification (i), except that production parameters are set to match all previous moments but the persistence of formality (0.64). Specification (iv) is the same as specification (i), but with infinite default costs (thus providing perfect commitment to the government). For each specification, we report the average and standard deviation (in parentheses) of fiscal policy variables—debt to output, tax rate, incidence of the formal sector—and consumption in Panel A, and we report the default incidence and market exclusion incidence in Panel B. In Panel C, we produce the inferred cost of fluctuations in consumption. The latter is the percentage difference between an average certainty equivalent consumption from all simulations $s \in \{1, \dots, S\}$, $u^{-1}\left(1/\left(\sum_{\tau=t}^{100} \beta^{\tau-t}\right) \cdot \left(\sum_{\tau=t}^{100} \beta^{\tau-t} u(c_{\tau,s})\right)\right)$, and the average consumption $\bar{c} = 1/\left(\sum_{\tau=t}^{100} \beta^{\tau-t}\right) \cdot \left(\sum_{\tau=t}^{100} \beta^{\tau-t} c_{\tau,s}\right)$. The statistics about the debt-to-output ratio, the tax rate and the incidence of the formal sector are computed using the subsample of repayment periods, as in Table 2.

We report descriptive statistics about the distribution of fiscal policy variables and

structure, and the parametrization of shocks.

consumption across these different economies in Panel A of Table 3. Focusing on scenarios (ii) and (iii), we find that the less responsive and less rigid economies typically exhibit lower fluctuations in the incidence of the formal sector, as partly induced by a lower volatility of taxation (and a lower response to those fluctuations in the case of scenario ii). Fluctuations in fiscal policy might either stabilize consumption (when fiscal policy is counter-cyclical, as typically induced by interest rate shocks in our quantitative model) or amplify consumption volatility (when fiscal policy is pro-cyclical, as typically induced by government expenditure shocks in our constrained, baseline economy). We find that consumption appears to be less volatile in the less responsive and less rigid economies. In line with our qualitative insight of Section 3, default significantly decreases when domestic production is less responsive to taxes (see column ii, Panel B), but an even more acute decrease in default is observed in the flexible economy (see column iii, Panel B).[29]

The welfare cost of dynamic distortions arises from a combination of factors: dynamic distortions induce default, which has a mechanical effect on the *average* consumption throughout the period of market exclusion; dynamic distortions induce costly fluctuations in fiscal policy with repercussions on the (overall) incidence of the informal sector and output; and the same dynamic distortions induce costly fluctuations in consumption. We focus on (and summarize) the welfare cost of *consumption fluctuations* in Panel C of Table 3, where we compute the percentage difference between a certainty equivalent consumption and the average consumption across the different simulations and scenarios (an exercise resembling the computations in Lucas, 1987).[30] We find that, in our baseline economy, the welfare cost of consumption fluctuations would be equivalent to a percentage decrease in average consumption of 0.246%. These losses respectively fall to 0.125% and 0.027% in the low-response and flexible economies. Such differences are partly explained by the lower incidence of default, but the dynamics of informality also induces imperfect consumption smoothing outside of default episodes. Overall, the welfare cost of dynamic distortions arises from more frequent defaults, sustained fluctuations in taxation, both affecting the volatility of household consumption. Contrasting

[29]In a flexible economy, fiscal policy indeed has a smaller impact on *future* returns to taxes, and high tax rates are less likely to discourage future repayment. Rigidity in informality limits the commitment to repay, with important implications for welfare.

[30]To perform these calculations, we contrast the forward-looking welfare of the representative household using all realized values for consumption between period t and the final period 100 (allowing us to compute a certainty equivalent consumption) to the forward-looking average consumption values. We then compute the average of the percentage differences between certainty equivalent consumptions and average consumptions across simulations and across a range of initial periods $t = 29, \ldots, 59$. Given the low discount factor in all economies, this range for initial periods allows us to limit the influence of initial conditions and of the final horizon of our simulations. The second-order approximation used in Lucas (1987) would give slightly different welfare losses, because our economies experience very large fluctuations in consumption in periods of default or in periods of austerity.

the previous experiments (ii) and (iii) to the baseline experiment (i) sheds light on how the dynamics of informality—its elasticity combined with its sluggishness—affects fiscal policy and default risk. By contrast, the last experiment (iv) captures how default risk shapes fiscal policy and the dynamics of informality: we find that giving perfect commitment to the government strongly limits the observed fluctuations in taxation and in technological choices. Fiscal policy can then be properly used as a consumption stabilizer: the welfare cost of consumption volatility is an order of magnitude lower than in our baseline experiment.

Ultimately, these experiments illustrate that it is the *interaction* of dynamic distortions and imperfect commitment which strongly constrains the dynamics of optimal fiscal policy: toning down the impact of fiscal policy on future returns to taxation, either through less immediately responsive or less persistent technological choices, or tempering the threat of default would both lead to muted consumption fluctuations and non-negligible welfare gains.

5 Final remarks

In this paper, we study how the dynamic distortionary costs of fiscal policy interact with default risk. To do so, we introduce imperfect tax enforcement and some rigidities in the choice of entrepreneurs to operate in formal/informal sectors in an otherwise standard model of sovereign debt with limited commitment. We show that the interaction of limited commitment and imperfect tax enforcement strongly influences the dynamics of fiscal policy during default crises: fiscal policy impacts the technological choice of entrepreneurs, this choice affects the future trade-off between repayment and default through its persistence over time, and these prospective incentives to default impact contemporary debt prices and thus the inclination to bring consumption forward.

This channel is shown to strongly constrain the dynamics of optimal fiscal policies in a quantitative exercise where we calibrate the elasticity and persistence of technological choices by local entrepreneurs. We provide supporting empirical evidence for the sluggish response of informality and its further impact on default to an exogenous fiscal shock and show that the response crucially depends on inherited formality. The distortions associated with such sluggish dynamics of informality strongly constrain fiscal policy: in our quantitative exercise, the welfare costs associated with these distortions amount to a 0.12-0.22% loss in terms of certainty equivalent consumption.

A limitation of our study is to consider a stylized, theoretical representation of the economy. A more thorough quantitative exercise could consider: (i) a larger set of redistribution tools and/or endogenous government expenditures to better match the richness of actual fiscal policy; (ii) heterogeneous households with differential access to consumption-smoothing instruments; (iii) heterogeneous entrepreneurs and borrowing frictions; and (iv) a less stylized modeling of tax evasion (e.g., allowing for endogenous adjustments of tax enforcement), of the technological differences between the informal sector and the formal sector (e.g., with a more explicit modeling of strategic complementarities within the latter) and of the nature of sluggishness in entrepreneurial decisions. We leave these avenues for future research.

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APPENDIX

A Complements to the theory

In this section, we provide the following complements to Sections 2, 3, and 4: (i) we describe the detailed derivation of dynamic technological choices and their implication on production; (ii) we provide a characterization of the stylized model of Section 3 and the proofs of Propositions 1 and 2; and (iii) we shed light on default decisions in the quantitative model.

A.1 Technology choice, production and consumption

Technology choice Consider an entrepreneur with the opportunity to choose technology and with current net return to the private technology, R_t.

Let χ_τ denote her expected discounted revenues if she could re-optimize in any future period τ and let R_τ denote the random draw for the private technology in period τ. Her expected discounted revenues if she were to choose the private technology in period t would be:

$$R_t + E_t\left[\sum_{k=1}^{\infty}\theta^k\delta_{t,t+k}R_{t+k} + \sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)\delta_{t,t+k}\chi_{t+k}\right]$$

where θ is the probability of not having re-optimized after one period and $\delta_{t,t+k} = \beta^k\frac{u'(c_{t+k})}{u'(c_t)}$ is the discount factor between period t and period $t+k$.

Her expected discounted revenues if she were to choose the formal technology in period t would be:

$$r_t + E_t\left[\sum_{k=1}^{\infty}\theta^k\delta_{t,t+k}r_{t+k} + \sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)\delta_{t,t+k}\chi_{t+k}\right]$$

where $r_{t+k} = (1-\tau_{t+k})Y_{t+k}^{\frac{1}{\phi}-1}$ is the price for one unit of the differentiated good in the formal sector in period $t+k$.

Letting $\tilde{R}_t$ denote the indifference threshold, i.e., the level of private returns for which an entrepreneur is currently indifferent between the two technologies, and $\bar{R} = \int RdH(R)$ the average expected return to the private technology, we have:

$$\tilde{R}_t + \sum_{k=1}^{\infty}\theta^k\delta_{t,t+k}\bar{R} = E_t\left[r_t + \sum_{k=1}^{\infty}\theta^k\delta_{t,t+k}r_{t+k}\right],$$

which gives,

$$\tilde{R}_t - \bar{R} = E_t\left[\sum_{k=0}^{\infty}\theta^k\delta_{t,t+k}(r_{t+k}-\bar{R})\right].$$

The previous expression can be expressed in a recursive manner,

$$\tilde{R}_t - \bar{R} = r_t - \bar{R} + \theta E_t \left[\delta_{t,t+1} \left(\tilde{R}_{t+1} - \bar{R}\right)\right]$$

$$\tilde{R}_t - \bar{R} = r_t - \bar{R} + \theta \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\tilde{R}_{t+1} - \bar{R}\right)\right]$$

Among entrepreneurs with the opportunity to modify their technology, the share of them adopting the formal technology, γ_t^*, should be equal to the ones with sufficiently low returns to the informal sector, i.e., $\gamma_t^* = H\left[\tilde{R}_t\right]$. The aggregate share of entrepreneurs operating in the formal sector, γ_t, verifies the following dynamics:

$$\gamma_t = (1-\theta)\gamma_t^* + \theta \gamma_{t-1}.$$

We combine the previous equations to derive the (sluggish) dynamics of technological choices, which governs distortions on the production side of the economy, as a function of returns to the formal sector:

$$H^{-1}\left(\frac{\gamma_t - \theta\gamma_{t-1}}{1-\theta}\right) = r_t + \theta\beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(H^{-1}\left(\frac{\gamma_{t+1} - \theta\gamma_t}{1-\theta}\right) - \bar{R}\right)\right]$$

The previous equation describes the sluggish dynamics of technological choices due to the staggered nature of such choices. Two effects underlie these dynamics: (i) a backward-looking rigidity, because some entrepreneurs have not been given the opportunity to respond to the state of the economy in the current period; and (ii) a forward-looking rigidity, because entrepreneurs with the opportunity to re-optimize internalize that they may not be able to do so in subsequent periods.

The general formulation of current choices as a function of choices in $t-1$ and in $t+1$ (see Equation 5) relies on the independence of draws determining whether an entrepreneur can re-optimize as in the original model of staggered price-setting (Calvo, 1983). Importantly, the independence of returns to the private technology across periods ensures that production in the informal sector is independent of previous draws and its dependence on past choices is fully captured by the number of entrepreneurs operating with the private technology in the previous period.

Production, consumption and tax receipts Given the previous allocation of entrepreneurs across technologies, production in the formal sector equals $\gamma_t^{1/\phi}$ and production in the informal sector originates from two types of entrepreneurs:

- $\theta(1-\gamma_{t-1})\bar{R}$ is produced on average by an unbiased selection of entrepreneurs—the number of entrepreneurs operating with the private technology in the previous period and with expected return $\bar{R}$ in period t,

- $(1-\theta)\int_{\tilde{R}_t}^{\infty} RdH(R)$ is produced on average by a biased selection of entrepreneurs—the number of entrepreneurs having just re-optimized and with high returns to the private technology.

Consumption of the representative household thus satisfies,

$$c_t = (1-\tau_t)y_t^{1/\phi} + \theta(1-y_{t-1})\bar{R} + (1-\theta)\int_{H^{-1}\left(\frac{y_t-\theta y_{t-1}}{1-\theta}\right)}^{\infty} RdH(R) \tag{A1}$$

A convenient expression for consumption will be,

$$c_t = y_t^{1/\phi} + \theta(1-y_{t-1})\bar{R} + (1-\theta)\int_{H^{-1}\left(\frac{y_t-\theta y_{t-1}}{1-\theta}\right)}^{\infty} RdH(R) - t_t \tag{A2}$$

where $t_t = \tau_t y_t^{1/\phi}$ are tax receipts, which can also be written as a function of debt and government expenditures using the budget constraint.

Response to fiscal policy and fiscal multiplier We use the previous derivations to understand the impact of fiscal policy on fiscal receipts and consumption as a function of the production response $\partial y_t/\partial\tau_t$. Fiscal receipts depend on fiscal policy as follows,

$$\frac{\partial t_t}{\partial \tau_t} = y_t^{1/\phi} + \frac{\tau_t}{\phi} y_t^{1/\phi-1}\frac{\partial y_t}{\partial \tau_t}$$

Consumption depend on fiscal policy as follows,

$$\frac{\partial c_t}{\partial \tau_t} = -y_t^{1/\phi} + \frac{(1-\tau_t)}{\phi} y_t^{1/\phi-1}\frac{\partial y_t}{\partial \tau_t} - (1-\theta)\tilde{R}_t h(\tilde{R}_t)\frac{\partial \tilde{R}_t}{\partial \tau_t}$$

As $\widetilde{R}_t = H^{-1}\left(\frac{y_t-\theta y_{t-1}}{1-\theta}\right)$, we have $\frac{\partial \widetilde{R}_t}{\partial \tau_t} = \frac{1}{1-\theta}\frac{1}{h(\widetilde{R}_t)}\frac{\partial y_t}{\partial \tau_t}$, and the previous expression becomes:

$$\frac{\partial c_t}{\partial \tau_t} = -y_t^{1/\phi} + \frac{(1-\tau_t)}{\phi} y_t^{1/\phi-1}\frac{\partial y_t}{\partial \tau_t} - \tilde{R}_t\frac{\partial y_t}{\partial \tau_t}$$

The previous expressions allow us to quantify the production losses arising from tax distortions on technological choices:

$$\frac{\partial(t_t + c_t)}{\partial \tau_t} = \left(\frac{r_t}{\phi} - \tilde{R}_t\right)\frac{\partial y_t}{\partial \tau_t} < 0$$

These losses depend upon (i) the shift in the allocation of entrepreneurs between the formal and informal sector and (ii) the difference between the social returns in the formal and informal sectors for the indifferent entrepreneur, $r_t/\phi - \tilde{R}_t$. At the steady state, *private* returns in the two sectors should be equal, i.e., $r - \tilde{R} = 0$, but there would remain a gap between the social returns to the different technologies.

The previous approach allows us to derive a formula for the fiscal multiplier in the economy:

$$\frac{\partial c_t}{\partial t_t} = -1 + \left(\frac{r_t}{\phi} - \tilde{R}_t\right)\frac{\partial \gamma_t}{\partial \tau_t} < -1$$

With lump-sum transfers, a change in fiscal surplus would generate a one-to-one loss in consumption and total output would be left unchanged. Instead, when taxes are distortionary ($\partial \gamma_t/\partial t_t < 0$), consumption drops even further, and the size of leakages depends upon (i) the shift in the allocation of entrepreneurs between the formal and informal sector and (ii) the difference between the social returns in the formal and informal sectors $r_t/\phi - \tilde{R}_t$. The degree of complementarity in the formal sector impacts both quantities. When complementarity is high in the formal sector, there are large differences between the social returns in the formal and informal sectors at equilibrium and transparency sharply responds to changes in fiscal surplus. In our model with non-Ricardian households, the previous equation characterizes the size of the fiscal multiplier, which is always greater than 1.

A.2 Proofs

Proposition 1: the flexible economy We derive optimal policy in the flexible economy in two steps. In the first step, we consider the problem of a government with perfect commitment. In the second step, we analyze the restrictions on feasible policies imposed by limited commitment.

With perfect commitment, the government would maximize,

$$\max_{\{c_1, c_2, b_2, \gamma_1, \gamma_2\}} \{u(c_1) + \beta u(c_2)\},$$

subject to (i) a budget constraint in the first period,

$$c_1 = \gamma_1^{1/\phi} + \int_{H^{-1}(\gamma_1)} R dH(R) - g_1 + \beta b_2, \qquad [\lambda_1]$$

where the first term is total production in the formal sector, the second term is total production in the informal sector and the third term are net transfers to the government, (ii) a budget constraint in the second period,

$$c_2 = \gamma_2^{1/\phi} + \int_{H^{-1}(\gamma_2)} R dH(R) - g_2 - b_2, \qquad [\lambda_2]$$

the entrepreneurial choice in the first period (see the calculations in the previous section A.1 with $\theta = 0$, Equation (A2) in particular),

$$\gamma_1 H^{-1}(\gamma_1) = c_1 - \int_{H^{-1}(\gamma_1)} R dH(R), \qquad [\mu_1]$$

the entrepreneurial choice in the second period,

$$\gamma_2 H^{-1}(\gamma_2) = c_2 - \int_{H^{-1}(\gamma_2)} R dH(R). \qquad [\mu_2]$$

The first-order conditions with respect to $(c_1, c_2, b_2, \gamma_1, \gamma_2)$ give:

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial c_1} = u'(c_1) - \lambda_1 - \mu_1 = 0 & (C_1) \\ \frac{\partial \mathcal{L}}{\partial c_2} = \beta u'(c_2) - \lambda_2 - \mu_2 = 0 & (C_2) \\ \frac{\partial \mathcal{L}}{\partial b_2} = \beta\lambda_1 - \lambda_2 = 0 & (B_2) \\ \frac{\partial \mathcal{L}}{\partial \gamma_1} = \lambda_1 \left[\gamma_1^{1/\phi-1}/\phi - H^{-1}(\gamma_1)\right] - \mu_1 H^{-1}(\gamma_1) = 0 & (\Gamma_1) \\ \frac{\partial \mathcal{L}}{\partial \gamma_2} = \lambda_2 \left[\gamma_2^{1/\phi-1}/\phi - H^{-1}(\gamma_2)\right] - \mu_2 H^{-1}(\gamma_2) = 0 & (\Gamma_2) \end{cases}$$

We combine (C_t) and (Γ_t) to obtain $\lambda_t \frac{\gamma_t^{1/\phi-1}}{\phi} = H^{-1}(\gamma_t)u'(c_t)$ in both periods. From (B_2), we have $\beta\lambda_1 = \lambda_2$, which implies:

$$\beta \frac{H^{-1}(\gamma_1)}{\gamma_1^{1/\phi-1}} u'(c_1) = \frac{H^{-1}(\gamma_2)}{\gamma_2^{1/\phi-1}} u'(c_2). \tag{A3}$$

Equation (A3) is an Euler equation, weighted by the distortions induced by taxation in each period. Indeed, the terms in front of marginal utilities capture the relative gain in consumption of opting for the informal sector versus the formal sector.

In our stylized economy, the production sector is the same in both periods. The government separately smooths distortions across periods, i.e., $\gamma_1 = \gamma_2$, and consumption across periods, i.e., $c_1 = c_2$. The latter require:

$$b_2 = b_2^* = \frac{g_1 - g_2}{1 + \beta}.$$

The previous fiscal policy may not be compatible with limited commitment. As the economy is deterministic, borrowing in the first period is only possible if the default punishment is sufficient to deter the government from defaulting in the second period. The government will default if consumption is higher under default than under repayment. We now calculate consumption in those cases.

In the second period and in case of a default, consumption is given by,

$$\tilde{c}_2 = (1-\Delta)\left(\tilde{y}_2^{1/\phi} - g_2 + \int_{H^{-1}(\tilde{y}_2)} RdH(R)\right)$$

where $\tilde{y}_2$ solves $\tilde{y}_2 H^{-1}(\tilde{y}_2) = \tilde{y}_2^{1/\phi} - g_2$. In case of a repayment, consumption is given by,

$$c_2 = y_2^{1/\phi} - g_2 - b_2 + \int_{H^{-1}(y_2)} RdH(R) \tag{A4}$$

where y_2 solves,

$$y_2 H^{-1}(y_2) = y_2^{1/\phi} - g_2 - b_2 \tag{A5}$$

The decision to default or repay in the second period—depending on whether consumption under default $\tilde{c}_2$ is greater than consumption under repayment c_2—only depends on previous choices through the value of inherited debt, b_2, which needs to be lower than a threshold $\tilde{b}_2$ depending on g_2.[31]

The solution to the limited commitment problem is $b_2 = (g_1 - g_2)/(1-\beta)$ when feasible, i.e., when $(g_1 - g_2)/(1-\beta) \leq \tilde{b}_2$ and $b_2 = \tilde{b}_2$ otherwise. In the former case, both consumption and production are equalized across periods; in the latter case, consumption and production are too low in the first period. The production technology thus affects the levels of production and consumption, but does not have any impact on the optimal debt schedule chosen by the government. Note that this result may appear obvious, but actually depends on the relative strength of the production externality across the two periods. Indeed, entrepreneurs exert a positive externality on the return to the formal sector which they do not internalize. The government is tempted to lower taxes in order to further push entrepreneurs towards the formal sector. When $z_1 = z_2$, these incentives are the same across periods and do not give rise to a redistributive aspect of fiscal policy over time.

Proposition 2: the persistent economy We derive optimal policy in the persistent economy in two steps: we first consider the problem of a government with perfect commitment, and we then analyze the restrictions on feasible policies imposed by limited

[31] Letting $f(x) = x^{1/\phi} + \int_{H^{-1}(x)} RdH(R)$ and $x = g(y)$ the implicit function verifying the implicit equation $g(y)H^{-1}(g(y)) = g(y)^{1/\phi} - g_2 - y$, we can easily show that g is strictly decreasing in y and that f is increasing in regions where the previous implicit equation is verified. The latter directly relates to distortions: output in the economy increases with transparency at the equilibrium. These two monotonic relationships imply that we can define a threshold $\tilde{b}_2$ such that:

$$c_2 \geq \tilde{c}_2 \Leftrightarrow b_2 \leq \tilde{b}_2$$

commitment.

With perfect commitment, the government would maximize,

$$\max_{\{c_1,c_2,b_2,\gamma_1\}} \{u(c_1) + \beta u(c_2)\},$$

subject to (i) a budget constraint in the first period,

$$c_1 = \gamma_1^{1/\phi} + \int_{H^{-1}(\gamma_1)} RdH(R) - g_1 + \beta b_2, \qquad [\lambda_1]$$

(ii) a budget constraint in the second period,

$$c_2 = \gamma_1^{1/\phi} + \int_{H^{-1}(\gamma_1)} RdH(R) - g_2 - b_2, \qquad [\lambda_2]$$

the entrepreneurial choice in the first period accounting for its persistence (they choose technology to maximize overall returns in periods 1 and 2),

$$0 = u'(c_1)\left[c_1 - \gamma_1 H^{-1}(\gamma_1)\right] + \beta u'(c_2)\left[c_2 - \gamma_1 H^{-1}(\gamma_1)\right]. \qquad [\mu]$$

The first-order conditions with respect to $(c_1, c_2, b_2, \gamma_1)$ give:

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial c_1} = u'(c_1) - \lambda_1 - \mu\left[u''(c_1)\left(c_1 - \gamma_1 H^{-1}(\gamma_1)\right) + u'(c_1)\right] = 0 & (C_1) \\ \frac{\partial \mathcal{L}}{\partial c_2} = \beta u'(c_2) - \lambda_2 - \mu\left[u''(c_2)\left(c_2 - \gamma_1 H^{-1}(\gamma_1)\right) + u'(c_2)\right] = 0 & (C_2) \\ \frac{\partial \mathcal{L}}{\partial b_2} = \beta\lambda_1 - \lambda_2 = 0 & (B_2) \\ \frac{\partial \mathcal{L}}{\partial \gamma_1} = (\lambda_1 + \lambda_2)\left[\gamma_1^{1/\phi - 1}/\phi - H^{-1}(\gamma_1)\right] + \mu\left(u'(c_1) + \beta u'(c_2)\right)\left(H^{-1}(\gamma_1) + \frac{\gamma_1}{h(H^{-1}(\gamma_1))}\right) = 0 & (\Gamma_1) \end{cases}$$

We combine (C_1), (C_2) and (B_2) to get:

$$\mu = -\frac{f_1}{h_1 - \frac{r_1 u'(c_1) + r_2 \beta u'(c_2)}{u'(c_1) + \beta u'(c_2)}}$$

where $r_1 = 1 + \frac{u''(c_1)}{u'(c_1)}\left(c_1 - \gamma_1 H^{-1}(\gamma_1)\right)$, $r_2 = 1 + \frac{u''(c_2)}{u'(c_2)}\left(c_2 - \gamma_1 H^{-1}(\gamma_1)\right)$, $f_1 = \gamma_1^{1/\phi-1}/\phi - H^{-1}(\gamma_1)$ and $h_1 = H^{-1}(\gamma_1) + \frac{\gamma_1}{h(H^{-1}(\gamma_1))}$. This multiplier captures the impact of distortions on consumption smoothing, through the intertemporal technology choice from entrepreneurs. The Euler equation then verifies,

$$u'(c_1)\left[1 - \mu r_1\right] = u'(c_2)\left[1 - \mu r_2\right]$$

In our stylized economy, the production sector is the same in both periods. The government thus smoothes consumption across periods, i.e., $c_1 = c_2$, given that distortions will be symmetric across periods. The perfect-commitment policy is thus:

$$b_2 = b_2^* = \frac{g_1 - g_2}{1+\beta},$$

as in the perfect-commitment flexible economy.

With limited commitment, the previous fiscal policy may not be feasible, and we need to compare consumption levels in the second period when there is a default or when there is repayment. In the second period and in case of a default, consumption is $c_2 = (1-\Delta)\left(\gamma_1^{1/\phi} - g_2 + \int_{H^{-1}(\gamma_1)} RdH(R)\right)$. In the case of a repayment, we have instead:

$$c_2 = \gamma_1^{1/\phi} - g_2 - b_2 + \int_{H^{-1}(\gamma_1)} RdH(R)$$

Repayment thus requires that:

$$b_2 \leq \widehat{b_2} = \Delta\left[\widehat{\gamma_1}^{1/\phi} - g_2 + \int_{H^{-1}(\widehat{\gamma_1})} RdH(R)\right]$$

where $\widehat{\gamma_1}$ is pinned down by the previous equation together with the budget constraints in both periods and the equation characterizing the technological choice.

The solution to the limited commitment problem is thus $b_2 = (g_1 - g_2)/(1-\beta)$ when feasible, i.e., when $(g_1 - g_2)/(1-\beta) \leq \widehat{b_2}$ and $b_2 = \widehat{b_2}$ otherwise. With persistent technological choices, limited commitment imposes an "endogenous" constraint on fiscal policy in the first period: it should be that the long-run impact of fiscal policy—captured here by inherited debt, b_2, *and* inherited technology choices, γ_1—is such that repayment is desirable: consumption needs to be higher with repayment. The novel role of inherited technology choices implies that the debt ceiling now depends on the first-period state of nature: high expenditures would require high taxation, low transparency in the longer run, and thus higher incentives to default.[32]

A.3 The role of default costs

In the baseline model, we assume that the exogenous default cost is directly incurred by the household (i.e., it is a *consumption* discount), with two implications: (i) the exogenous default cost equally affects consumption proceeds from the informal sector and from the

[32] Note that, with persistent technological choices from entrepreneurs, the government has incentives to lower taxes in the first period in order to push entrepreneurs towards the formal sector (as the private returns to the formal technology are lower than the social return, and as fiscal policy is not distortive in the second period). However, the government is constrained in doing so because this would require a debt that may be infeasible.

formal sector; and (ii) the consumption discount applies to taxes, inducing no further direct, fiscal distortions. In this section, we explore how imposing a default cost on the *production* side, and differentially so across the two technologies, affects our theoretical insight within the lens of our stylized two-period model (see Section 3).

Consider an alternative structure of default costs, as captured by two parameters: (i) an iceberg loss for the output produced in the formal sector, Δ_f, i.e., the output produced in the formal sector is $(1-\Delta_f)\gamma^{1/\phi}$ (and the government is excluded from markets such that taxes applying to this output are set to finance current government expenditures, g); and (ii) an iceberg loss for the output produced in the informal sector, Δ_i. Any difference between the two values, e.g., $0 \leq \Delta_i < \Delta_f$, could capture—in a stylized manner—that: access to credit might be a benefit from operating in the formal sector, as indirectly shown by Artavanis et al. (2016) for instance; and a default could lead to a capital flight (Mendoza and Yue, 2012; Bocola, 2016; Arellano et al., 2017) which would disproportionately affect the returns to the formal technology.

The following proposition provides simple comparative statics in this model.

Proposition 4 (The role of default costs)**.** *Consider that the government has limited commitment to repay. With a rigid technology, fiscal policy satisfies,*

$$b_2 = \begin{cases} b_2^* & \text{if} \quad b_2^* \leq \widehat{b}_2 \\ \widehat{b}_2 & \text{otherwise} \end{cases},$$

where $\widehat{b}_2$, the debt ceiling, depends on the default costs as follows:

$$\frac{\partial \widehat{b}_2}{\partial \Delta_f} = \gamma_1^{1/\phi}, \quad \frac{\partial \widehat{b}_2}{\partial \Delta_i} = \int_{H^{-1}(\gamma_1)} R dH(R)$$

and the dependence of the debt ceiling in the first-period government expenditures (through the choice of technology, γ_1) varies with default costs as follows,

$$\frac{\partial^2 \widehat{b}_2}{\partial \Delta_f \partial \gamma_1} = \frac{1}{\phi}\gamma_1^{1/\phi - 1}, \quad \frac{\partial^2 \widehat{b}_2}{\partial \Delta_i \partial \gamma_1} = -H^{-1}(\gamma_1)$$

where $\left|\frac{\partial^2 \widehat{b}_2}{\partial \Delta_f \partial \gamma_1}\right| > \left|\frac{\partial^2 \widehat{b}_2}{\partial \Delta_i \partial \gamma_1}\right|$.

With a flexible technology, fiscal policy satisfies,

$$b_2 = \begin{cases} b_2^* & \text{if} \quad b_2^* \leq \widetilde{b}_2 \\ \widetilde{b}_2 & \text{otherwise} \end{cases},$$

where $\widetilde{b}_2$, the debt ceiling, depends on the state of nature in the last period (g_2) but not the state of nature in the first period (g_1), and,

$$\begin{cases} \dfrac{\partial \tilde{b}_2}{\partial \Delta_f} & = \dfrac{1-\beta}{1-\alpha} \tilde{\gamma}_2{}^{1/\phi} \\ \dfrac{\partial \tilde{b}_2}{\partial \Delta_i} & = \dfrac{1}{1-\alpha} \left[\int_{H^{-1}(\tilde{\gamma}_2)} R dH(R) + \beta \tilde{\gamma}_2 H^{-1}(\tilde{\gamma}_2) \right] \end{cases}$$

where $\tilde{\gamma}_2$ is the incidence of the formal technology in the occurrence of a default, and $0 < \alpha < 1$ and $0 < \beta$ depend on fundamentals and on the incidences of the formal technology across scenarios.

Proof. First, let us consider the fully rigid technology in which technological choices are made in period 1 and not re-adjusted in period 2 (as in Proposition 2). We compare the consumption level in the second period when there is a default,

$$\tilde{c_2} = (1-\Delta_f)\gamma_1^{1/\phi} - g_2 + (1-\Delta_i) \int_{H^{-1}(\gamma_1)} R dH(R),$$

to consumption when there is repayment:

$$c_2 = \gamma_1^{1/\phi} - g_2 - b_2 + \int_{H^{-1}(\gamma_1)} R dH(R).$$

Repayment requires that:

$$b_2 \leq \widehat{b_2} = \Delta_f \gamma_1^{1/\phi} + \Delta_i \int_{H^{-1}(\gamma_1)} R dH(R)$$

where γ_1 is pinned down by a complex system of equations (see Proposition 2). Importantly, γ_1 is not directly affected by the values of Δ_f and Δ_i, because there is no default on any of the feasible choices of fiscal policy (default can be considered an "off-equilibrium" occurrence). The independence of γ_1 with respect to default costs implies that the differentiation of the previous equation with respect to these costs and the first-period government expenditures (through the choice of technology, γ_1) gives,

$$\begin{cases} \dfrac{\partial \tilde{b}_2}{\partial \Delta_f} & = \gamma_1^{1/\phi} \\ \dfrac{\partial \tilde{b}_2}{\partial \Delta_i} & = \int_{H^{-1}(\gamma_1)} R dH(R) \end{cases}$$

and,

$$\begin{cases} \dfrac{\partial^2 \tilde{b}_2}{\partial \Delta_f \partial \gamma_1} & = \frac{1}{\phi} \gamma_1^{1/\phi - 1} \\ \dfrac{\partial^2 \tilde{b}_2}{\partial \Delta_i \partial \gamma_1} & = -H^{-1}(\gamma_1) \end{cases}$$

It is easy to prove that $\left| \dfrac{\partial^2 \tilde{b}_2}{\partial \Delta_f \partial \gamma_1} \right| > \left| \dfrac{\partial^2 \tilde{b}_2}{\partial \Delta_i \partial \gamma_1} \right|$, reflecting that the social return to the formal sector is higher than the social return to the informal sector. The latter implies that a uniform increase in default costs ($d\Delta_f = d\Delta_i > 0$) would both increase the level of debt sustainable by the government and the dependence of such debt on the first-period shock—the slope in Figure 3.

Second, let us consider a fully flexible technology such that entrepreneurs can reset their technological choices freely in periods 1 and 2 (as in Proposition 1). In the second period and in case of a default, consumption is given by,

$$\tilde{c}_2 = (1 - \Delta_f) \tilde{\gamma}_2{}^{1/\phi} - g_2 + (1 - \Delta_i) \int_{H^{-1}(\tilde{\gamma}_2)} R dH(R)$$

where $\tilde{\gamma}_2$ solves $(1 - \Delta_i) \tilde{\gamma}_2 H^{-1}(\tilde{\gamma}_2) = (1 - \Delta_f) \tilde{\gamma}_2{}^{1/\phi} - g_2$. In case of a repayment, consumption is given by,

$$c_2 = \gamma_2^{1/\phi} - g_2 - b_2 + \int_{H^{-1}(\gamma_2)} R dH(R)$$

where γ_2 solves $\gamma_2 H^{-1}(\gamma_2) = \gamma_2^{1/\phi} - g_2 - b_2$. The decision to default or repay in the second period—depending on whether consumption under default $\tilde{c}_2$ is greater than consumption under repayment c_2—only depends on previous choices through the value of inherited debt, b_2, which needs to be lower than a threshold $\tilde{b}_2$. A consequence of this lack of persistence is that we have:

$$\frac{\partial \tilde{b}_2}{\partial \gamma_1} = \frac{\partial^2 \tilde{b}_2}{\partial \Delta_f \partial \gamma_1} = \frac{\partial^2 \tilde{b}_2}{\partial \Delta_i \partial \gamma_1} = 0.$$

However, there remains a direct effect of default costs on the *level* of the debt ceiling. To derive it, one needs to differentiate the following equation,

$$(1 - \Delta_f) \tilde{\gamma}_2{}^{1/\phi} - g_2 + (1 - \Delta_i) \int_{H^{-1}(\tilde{\gamma}_2)} R dH(R) = \gamma_2^{1/\phi} - g_2 - \tilde{b}_2 + \int_{H^{-1}(\gamma_2)} R dH(R)$$

and to combine the outcome with the differentiation of the implicit equations defining technological choices under repayment (γ_2) and under default ($\tilde{\gamma}_2$). Following these steps, one can show that:

$$\begin{cases} \dfrac{\partial \tilde{b}_2}{\partial \Delta_f} & = \dfrac{1-\beta}{1-\alpha} \tilde{\gamma}_2{}^{1/\phi} \\ \dfrac{\partial \tilde{b}_2}{\partial \Delta_i} & = \dfrac{1}{1-\alpha} \left[\int_{H^{-1}(\tilde{\gamma}_2)} R dH(R) + \beta \tilde{\gamma}_2 H^{-1}(\tilde{\gamma}_2) \right] \end{cases}$$

where

$$0 < \alpha = \frac{\frac{1}{\phi} \gamma_2{}^{1/\phi - 1} - H^{-1}(\gamma_2)}{\frac{1}{\phi} \gamma_2{}^{1/\phi - 1} + H^{-1}(\gamma_2) + \gamma_2 / h\left(H^{-1}(\gamma_2)\right)} < 1$$

and

$$0 < \beta = \frac{\frac{(1-\Delta_f)}{\phi} \tilde{\gamma}_2{}^{1/\phi - 1} - (1-\Delta_i) H^{-1}(\tilde{\gamma}_2)}{\frac{(1-\Delta_f)}{\phi} \tilde{\gamma}_2{}^{1/\phi - 1} - (1-\Delta_i)\left(H^{-1}(\tilde{\gamma}_2) + \tilde{\gamma}_2 / h\left(H^{-1}(\tilde{\gamma}_2)\right)\right)} < 1.$$

□

Proposition 4 shows that allowing for differential default costs across technologies will induce different effects on fiscal policy, as disciplined by the differences in social returns across the two sectors. Within our quantitative model and given the previous formulas, a marginal increase in the output cost of default in the formal sector would typically have a stronger effect on the debt ceiling than a marginal increase in output cost for the informal sector. As default episodes are not frequent, it is however hard to calibrate the relative output costs Δ_f and Δ_i from the differential consumption responses of different economies. For this reason, we only consider the *consumption* discount in our quantitative model of Section 4.

A.4 Default sets

As in Eaton and Gersovitz (1981) or Arellano (2008), the decision to default can be fully described by a default set $D(b, \gamma)$, which is a set of states of nature (ρ, g) under which the government prefers to default, as a function of the endogenous state variables (b, γ). The recursive equilibrium of this economy is then defined as a set of price functions for bonds, policy functions for the government including $D(b, \gamma)$ such that (i) the government policy functions solve the government problem taking as given price functions for bonds and the dynamics of technological choices as defined by Equation (5), and (ii) bond prices reflect the default probabilities implied by the policy functions $D(b, \gamma)$.

These default sets defined above satisfy the following monotonicity property: default sets are monotonous in inherited debt. If $b_1 < b_2$, then $D(b_1, \gamma) \subseteq D(b_2, \gamma)$. The proof of this property is immediate by contradiction. Assume that there exists a state of nature (ρ, g) such that $(\rho, g) \in D(b_1, \gamma)$ but $(\rho, g) \notin D(b_2, \gamma)$. The maximum utility reached after a default is independent of current debt b_1 or b_2. By contrast, the maximum utility that can be reached with reimbursement depends on current debt. Let b' and γ' denote the chosen debt level, technological choices and indifference threshold conditional on reimbursing for the state (b_2, γ). We have that $u\left(w(b', b_2, \gamma, \rho, g)\right) + \beta E_t v_A(b', \gamma', \rho', g')$ is greater than the value of default because $(\rho, g) \notin D(b_2, \gamma)$ by assumption. However, the utility from reimbursement associated with inherited debt b_1 and the same targets (b', γ') would be $u\left(w(b', b_1, \gamma, \rho, g)\right) + \beta E_t v_A(b', \gamma', \rho', g')$ and would be higher than the utility from default because

$$w(b', b_1, \gamma, \rho, g) \geq w(b', b_2, \gamma, \rho, g)$$

As a consequence, reimbursement is preferred to default, and $(\rho, g) \notin D(b_1, \gamma)$, which contradicts the initial hypothesis.

B Complements to the quantitative analysis

In this section, we describe data sources, derive empirical moments which discipline the calibration of the model and provide a sensitivity analysis of the main quantitative analysis.

B.1 Calibration

Data sources To measure the incidence of informality (a crucial quantity in our quantitative model), Pappadà and Zylberberg (2017) construct a measure of tax compliance for a simple flat tax, the Value Added Tax, which is the preferred instrument to adjust fiscal policy to economic fluctuations. The measure of tax compliance mostly relies on OECD and Eurostat data and their harmonized Classification of Individual Consumption by Purpose (COICOP), which allows us to observe consumption across 48 sub-categories of goods. The data is most easily available for OECD countries and selected non-member economies reporting statistics through the same system. While we use the same sources to extract total VAT revenues, we extract data on tax *rates* between 1995 and 2020 from the European Commission documentation and national sources in about 45 of these countries—an exercise initiated in Pappadà and Zylberberg (2017) and consolidated in Pappadà and Rogoff (2023). More specifically, we identify good categories that are subject to each "VAT bracket" for each country/year (together with the level of these VAT brackets) and across these previous COICOP codes.

The measure of tax compliance compares tax receipts to expected receipts as predicted by tax rates, exemptions and actual expenditures. Intuitively, letting tr_{itc}, τ_{itc} and c_{itc} denote VAT revenues, VAT rate and consumption of good i in year t and country c, the measure of VAT compliance is defined as:

$$\gamma_{tc} = \frac{\sum_i tr_{itc}}{\sum_i (\tau_{itc} c_{itc})}.$$

When all exemptions are properly accounted for (see Pappadà and Rogoff, 2023, for a more extensive discussion), the gap between tax revenues and expected tax revenues, as captured by the distance between γ_{tc} and 1, reflects tax collection leakages from undeclared economic transactions.[33] In particular, the measure properly accounts for possible

[33] We implement the following corrections in order to clean our measure of tax compliance. First, we are interested in short-term fluctuations, and we cannot allow for "high-frequency" measurement error. Tax reforms are often implemented during the year, while national accounts are closed at the end of each period, i.e., year or quarter. When the tax structure is changed during the course of the year, we construct the annual effective tax rate by weighing each tax rate by the consumption observed during its spell. When consumption could not be observed at a higher frequency than annually, we construct the annual

changes in consumption patterns c_{itc} as a response to differential tax rates across goods such that fluctuations in tax compliance arise from changes in tax compliance *within* good categories. Finally, a common issue with measures of tax evasion is imperfect reporting. One advantage of the measure is that total tax revenues and reported consumption are reported by agents with limited incentives to under-report these quantities (see Bachas et al., 2020; Morrow et al., 2022, implementing similar approaches): tax receipts, $tr_{tc} = \sum_i tr_{itc}$, are reported by public authorities in national accounts; and actual consumption, c_{itc}, is extracted from annual household expenditure surveys, i.e., from the purchaser side, thereby alleviating under-reporting of undeclared transactions.[34]

The previous exercise provides tax compliance, standard/effective tax rates, and tax revenues across about 45 (mostly) high- and middle-income economies between 1995 and 2020. The coverage is however very heterogeneous, and the panel is highly unbalanced. We complement the data with: (i) other measures of informality (Schneider et al., 2010; Elgin et al., 2021); (ii) the 10-year government bond yield (OECD, IMF) and debt levels (OECD, IMF, World Development Indicators); (iii) government expenditure, household consumption, and measures of output (World Development Indicators); and (iv) sectoral shares (World Development Indicators).

Empirical moments We use the previous unbalanced panel data of countries to derive three empirical moments: (a) the average incidence of the formal sector, (b) the elasticity of such incidence to tax rates, and (c) the persistence of formality. In our baseline calibration, we restrict the sample to economies above median for the levels of informality and within the top-3 quartiles for long-term sovereign debt spreads. The

effective tax rate by weighting each tax rate with the time during which it was enforced. Second, some tax reforms may differentially affect sub-categories of goods within a 2-digit classification. Assume, for instance, that we do not observe consumption in art galleries, but we observe consumption for a larger category ("cultural goods"). For many countries entering in the European Union, art galleries would pass from category 1 (standard rate) to category 3 (super-reduced rate). We would reconstruct an expected tax revenue for cultural goods by considering the average share of sub-categories over the period. Along the same lines, VAT can be collected for all registered firms or there may exist a minimum threshold. In the case of a reform affecting this threshold, we recreate the new expected revenue by subtracting the average share of value added created by firms below the threshold. Third, some reforms modify the tax environment without modifying tax rates for any category of goods, e.g., adopting online forms. We collect this information and exclude such reform years from our empirical specifications. Lastly, one can construct a more precise measure if one can observe exports and imports within good categories (Pappadà and Rogoff, 2023): for this reason, we use the approach developed in Pappadà and Rogoff (2023) when available for a sufficient period of time (typically, for European economies), and we use the measure constructed in Pappadà and Zylberberg (2017) for the remaining economies.

[34]These household surveys are standardized across countries, and they follow similar methodology (i.e., sampling and questionnaire). The aggregate consumption constructed from these surveys strongly correlates with total output, but there exists (standard) measurement error (see Aguiar and Bils, 2015; Kolsrud et al., 2019, for a correction method and a comparison with registry data). This error will translate into a measurement error in the denominator of our expression for γ_{tc}.

sample includes: "middle-income" countries with high informality levels (e.g., Colombia, Chile, Russia, Serbia, South Africa); Eastern-European countries (e.g., Bulgaria, Croatia, Czech Republic, Hungary, Poland, Romania); Southern-European countries (e.g., Greece, Italy, Portugal, Spain); and a few richer economies (e.g., France, Netherlands). We label this sample (S1). In this Appendix, we derive the three empirical moments for five additional samples: (S2) all economies in our sample; (S3) economies with above-median level of informality; (S4) economies within the top 3 quartiles for long-term sovereign debt spreads; (S5) Southern and Eastern European economies; and (S6) middle-income countries.[35]

Table B1. Elasticity of tax compliance to tax rates across samples.

Samples	(S1)	(S2)	(S3)	(S4)	(S5)	(S6)
Tax rate	**-0.392**	-0.345	-0.366	-0.362	-0.382	-0.374
	(0.076)	(0.046)	(0.066)	(0.063)	(0.065)	(0.094)
Observations	298	664	331	477	318	80
Mean (tax compliance)	**0.720**	0.771	0.727	0.772	0.787	0.726

Notes: Standard errors are clustered at the country level and are reported between parentheses. The unit of observation is a country in a given year. The dependent variable is the yearly change in (log) tax compliance, while *Tax rate* is the yearly change in (log) VAT rate. All specifications condition on year- and country-fixed effects, the change in (log) GDP, annual changes in the sectoral decomposition of economic activity across 1-digit sectors, annual changes in trade (the ratio of exports and imports over GDP), and the annual change in the ratio of government expenditures to GDP. Samples are: (S1) economies with above median level of informality and within the top 3 quartiles for long-term sovereign debt spreads; (S2) all economies; (S3) economies with above median level of informality; (S4) economies within the top 3 quartiles for long-term sovereign debt spreads; (S5) Southern and Eastern European economies; and (S6) middle-income countries.

We identify the elasticity of tax compliance to tax rates by considering a country c in year t and estimating specification (11) in difference (see Section 4.1), where: (i) we control for confounding factors co-moving with tax compliance and fiscal policy; (ii) the inclusion of country fixed effects, μ_c, conditions the analysis on country-specific trends; and (iii) the inclusion of year fixed effects, δ_t, captures aggregate dynamics in tax compliance. In practice, there are frequent adjustments in the composition of exempted categories, and volatility in the effective VAT rate, $\sum_i(\tau_{itc}c_{itc})/\sum_i c_{itc}$, derives from large, infrequent changes in standard rates, and from smaller, frequent adjustments in the com-

[35] These countries are—in parentheses, the sample(s) to which they belong: Australia (S1, S2, S3, S4), Austria (S2), Belgium (S2), Bulgaria (S1, S2, S3, S4, S5, S6), Canada (S2), Chile (S1, S2, S3, S4, S6), Colombia (S1, S2, S3, S4, S6), Croatia (S1, S2, S3, S4, S5), Cyprus (S1, S2, S3, S4), Czech Republic (S2, S5), Denmark (S2), Estonia (S2, S5), Finland (S1, S2, S3, S4), France (S1, S2, S3, S4), Germany (S2), Greece (S1, S2, S3, S4, S5), Hungary (S1, S2, S3, S4, S5), Ireland (S1, S2, S3, S4), Italy (S1, S2, S3, S4, S5), Japan (S2), Korea (S2), Latvia (S1, S2, S3, S4, S5), Lithuania (S1, S2, S3, S4, S5), Luxembourg (S2), Macedonia (S1, S2, S3, S4, S5), Malta (S1, S2, S3, S4), the Netherlands (S1, S2, S3, S4), New Zealand (S2), Norway (S1, S2, S3, S4), Poland (S1, S2, S3, S4, S5), Portugal (S1, S2, S3, S4, S5), Romania (S1, S2, S3, S4, S5), Russia (S1, S2, S3, S4, S5, S6), Serbia (S1, S2, S3, S4, S5, S6), Slovakia (S1, S2, S3, S4, S5), Slovenia (S2, S5), South Africa (S1, S2, S3, S4, S6), Spain (S1, S2, S3, S4, S5), Sweden (S2), Switzerland (S2), and the United Kingdom (S2).

position of exempted or reduced-rates categories. For instance, categories like medical services, international public transport, basic food products or cultural services are often subject to reduced rates or exemptions, but might be moved across brackets. We find that the average elasticity of tax compliance to tax rates is around -0.39 in our preferred sample (column 1 of Table B1). We target this estimate by setting the production parameters accordingly in Section 4.1 and Table 1. The five subsequent columns of Table B1 report the estimates for: (S2) all economies; (S3) economies with above median level of informality; (S4) economies within the top 3 quartiles for long-term sovereign debt spreads; (S5) Southern and Eastern European economies; and (S6) middle-income countries.

In Table B1, we also report the average incidence of the formal sector across these different samples—the first empirical moment (a): our targeted moment is, again, in the first column of Table B1, $\bar{\gamma} = 0.720$, and reported in Section 4.1 and Table 2.

Table B2. Persistence of tax compliance across samples.

Samples	(S1)	(S2)	(S3)	(S4)	(S5)	(S6)
Tax compliance $(t-1)$	**0.846**	0.589	0.738	0.946	0.692	0.370
	(0.232)	(0.238)	(0.219)	(0.140)	(0.403)	(0.389)
Observations	250	583	279	419	282	68

Notes: Standard errors are clustered at the country level and are reported between parentheses. The unit of observation is a country in a given year. The dependent variable is (log) tax compliance. All specifications control for: year- and country-fixed effects; the change in (log) GDP; annual changes in the sectoral decomposition of economic activity across 1-digit sectors; and annual changes in trade (the ratio of exports and imports over GDP). Samples are: (S1) economies with above median level of informality and within the top 3 quartiles for long-term sovereign debt spreads; (S2) all economies; (S3) economies with above median level of informality; (S4) economies within the top 3 quartiles for long-term sovereign debt spreads; (S5) Southern and Eastern European economies; and (S6) middle-income countries.

Our third empirical moment is the persistence in tax compliance—as a proxy for the degree of rigidities in technological choices. We estimate such persistence in Table B2. To estimate this empirical persistence, we consider the following AR(1) specification:

$$\ln \gamma_{tc} = \alpha \ln \gamma_{t-1c} + \beta \mathbf{X}_{tc} + \delta_t + \mu_c + e_{tc}, \tag{B1}$$

where the vector $\mathbf{X}_{tc}$ includes the same time-varying controls as in specification (11), country and time fixed effects, and the lagged (log) tax compliance, $\ln \gamma_{t-1c}$, is instrumented by lagged (log) effective tax rates, $\ln \tau_{t-1c}$—a first stage which is quite similar in essence to Equation (11). Such a two-stage specification identifies the persistence in tax compliance from prior changes in fiscal policy. We find that tax compliance is persistent and follows an AR(1) process with a coefficient of 0.846—see column (1) of Table B2 for the estimate corresponding to the baseline sample (S1). We set production parameters to target the AR(1) coefficient of 0.846 in Section 4.1 and Table 1. Again, the five subsequent columns of Table B2 report the estimates for samples (S2) to (S6).

Finally, our fourth empirical moment is the average fiscal multiplier in the economy. We cannot reliably identify this quantity from our own data and rather rely on existing literature: we target a value of 1.10, a lower bound for informal economies (as estimated in Pappa et al., 2015).

Table B3. Untargeted moments across samples.

	(S1)	(S2)	(S3)	(S4)	(S5)	(S6)
e. Volatility of formality (SD)	**0.044**	0.027	0.045	0.032	0.034	0.037
f. Volatility of tax rates (SD)	**0.022**	0.023	0.025	0.022	0.029	0.020
g. Default risk (SD)	**0.009**	0.006	0.009	0.008	0.007	0.009
h. Debt to output (average)	**0.484**	0.597	0.558	0.552	0.542	0.324
i. Debt to output (SD)	**0.088**	0.064	0.081	0.069	0.092	0.074
j. Volatility of consumption (SD)	**0.018**	0.017	0.018	0.019	0.018	0.027

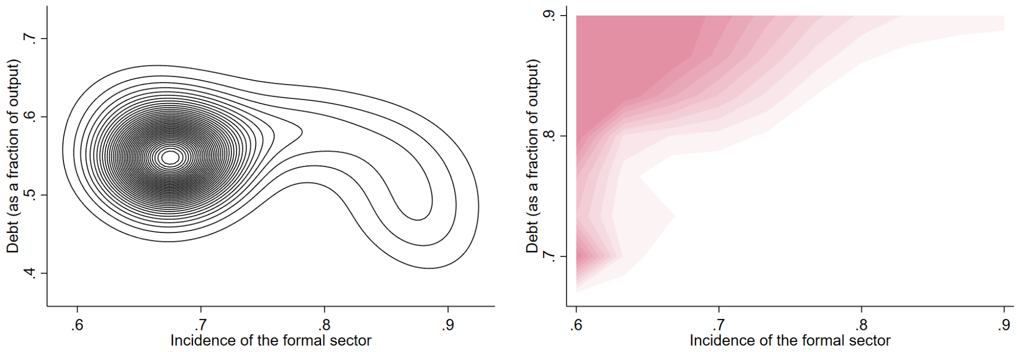
Notes: This table relies on different sub-samples from our full unbalanced panel of selected economies between 1995–2020. These sub-samples are: (S1) economies with above median level of informality and within the top 3 quartiles for long-term sovereign debt spreads; (S2) all economies; (S3) economies with above median level of informality; (S4) economies within the top 3 quartiles for long-term sovereign debt spreads; (S5) Southern and Eastern European economies; and (S6) middle-income countries.

Untargeted moments In Section 4.1, Table 2 contrasts the simulated moments of the calibrated model with their empirical counterparts, and Panel B explicitly focuses on those moments that are not directly targeted by the estimation of the production parameters. We provide a sensitivity analysis in Table B3 where we report the empirical moments observed across the different samples (S1)-(S6). One can see that adding richer economies, as in (S2), or restricting the sample to poorer economies, as in (S6), change default risk, the capacity of economies to accumulate debt, and the volatility in consumption.

Properties of the calibrated economy We finally provide additional evidence on the joint distribution of debt, informality, and default in our calibrated economy. The joint dynamics of debt and informality is important to understand default. Panel (a) of Figure B1 shows that, during periods of repayment, there is a "steady-state" or attractor for the joint distribution of debt and the incidence of the formal sector around ($b = 0.55, \gamma = 0.68$), and exogenous shocks generally induce converging fluctuations around this point. There are two instances in which governments stray away from this point: (i) there are (good) states of nature in which the government manages to lower debt while keeping tax evasion quite low, and (ii) the government might default.

The intuition behind the joint dynamics of debt and informality is the following. In

Figure B1. Distribution of debt and technological choices.



(a) Joint distribution

(b) Default incidence

Notes: Panel (a) displays 50 contour lines for the joint distribution of inherited debt and technological choices: the contour lines show the frequency of observing a given couple (b_t, γ_{t-1}) across 10,000 simulated economies over 50 years. In practice, we run simulations across 100 years and only keep the last 50 years to limit the influence of initial conditions. Panel (b) displays 10 contour areas (in ascending order) for the incidence of default as a function of inherited debt and technological choices.

general, a high degree of informality significantly lowers the returns to taxes: transfers from the household are then costly, fiscal surpluses are low, and debt levels end up being high with non-negligible default risk. In such states, debt remains at high levels because the government finds it too costly (in terms of tax distortions) to reduce it. As this situation is not desirable, the government is willing to avoid it: this explains the lower debt levels observed for low ranges of informality; the government then adopts a precautionary behavior. These responses give rise to costly, persistent fluctuations in consumption, as we see in Section 4.3.

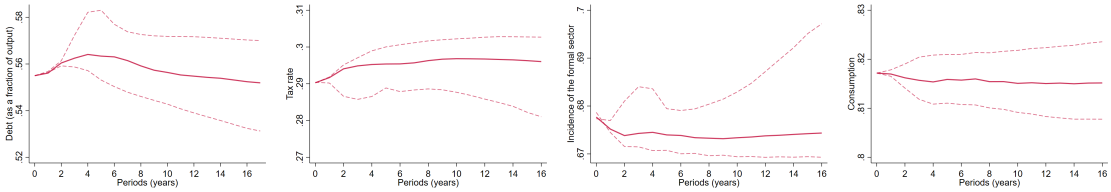
In Panel (b) of Figure B1, we further illustrate the constraints that informality put on debt accumulation and display the incidence of default as a function of inherited debt and informality: a higher degree of informality (or a low inherited incidence of the formal sector) makes default more likely, for a given level of debt.

B.2 The dynamic impact of fiscal shocks

In this section, we provide a sensitivity analysis for the derivation of our impulse functions.

In Section 4.2, we report impulse response functions following an expenditure shock within our dynamic system—an exercise in which the main outcomes are aggregated across periods and simulations. However, Figure 5 excludes all simulations involving default(s) and autarky periods, the latter being rarer but inducing very different dynamics than repayment periods. The resulting Figure 5 is thus showing the same economies

Figure B2. Impulse response functions—simulated economy (including autarky periods).



(a) Debt (b) Tax rate (c) Formal sector (d) Consumption

Notes: This Figure displays the average response of 10,000 simulated economies to an expenditure shock over a period of 16 years. In period 1, we set the level of expenditure to be about 1 percentage point of output above its long-term value (see period 0), which corresponds to a relative (log) deviation around 6%. We then simulate the stochastic, exogenous processes from period 1 onward across the 10,000 simulated economies. Panels (a), (b), (c), and (d) display the median response (plain line) and the 25-75% quantiles in red for the following variables: debt as a share of output, the tax rate (τ), the incidence of the formal sector (γ), and consumption (c). Note that the response in those panels mixes repayment states and (the less frequent) default states—which explains the rapid drop in average debt across states, compared to the lower quantiles. The initial conditions are: $\gamma_0 = 0.678$, $b_1 = 0.555$, and the other exogenous shock is initially centered around its average value, $dev(\rho_0) = 0$.

in each period, but the government is always repaying in these economies.

In Figure B2, we provide a sensitivity analysis in which we also include simulations in which (at least) one default occurs. Figure 5 differs from Figure B2 in two important ways. First, it selects favorable simulations in which the economy does not experience default. Second, and keeping the previous caveat in mind, the median non-defaulting economy sustains higher debt in the medium run; the debt absorption and sharper uptick in formality shown in Figure B2 are partly explained by default episodes. In general, however, the insights provided by the two Figures are qualitatively similar.

B.3 The welfare cost of dynamic distortions—sensitivity analysis

This section provides a sensitivity analysis of our quantitative exercise (Section 4.3). In particular, we derive the welfare cost of consumption fluctuations under alternative counterfactual experiments with: 1. different preferences (less impatient households; more patient investors; less risk-averse households); 2. different default punishments (ν, Δ) and maturity structure for the long-term debt (λ, z); and 3. a different structure of exogenous shocks (volatility and persistence).

This evidence also allows us to identify the role of other, important factors—e.g., preferences or nature of shocks—and their interaction behind the dynamic distortionary effect of fiscal policy. We report the main statistics and the cost of fluctuations for those alternative economies in Table B4. A summary of our findings is that the welfare loss widely ranges between 0.037 and 0.421% of consumption, depending on the incidence of default. One set of experiments is worth discussing: the sensitivity analysis changing the default cost and debt structure (see "R2. Default and debt structure" in Table B4).

Table B4. The welfare cost of dynamic distortions—sensitivity analysis.

Sensitivity	Debt	Tax	Formal	Consu.	Default	Fluct.
R1. Preferences						
Discount factor ($\beta = 0.90$)	0.544	0.278	0.722	0.809	0.008	0.185
	(0.091)	*(0.052)*	*(0.067)*	*(0.038)*		
Interest ($E[1/\rho] - 1 = 0.04$)	0.543	0.270	0.735	0.805	0.015	0.421
	(0.120)	*(0.071)*	*(0.079)*	*(0.045)*		
Risk-aversion (CRRA, $\sigma = 1.6$)	0.578	0.280	0.717	0.805	0.002	0.046
	(0.045)	*(0.037)*	*(0.038)*	*(0.017)*		
R2. Default and debt structure						
Reintegration ($\nu = 0.150$)	0.529	0.289	0.711	0.806	0.008	0.245
	(0.089)	*(0.053)*	*(0.069)*	*(0.041)*		
Cost ($\Delta = 0.18$)	0.495	0.307	0.676	0.800	0.001	0.037
	(0.030)	*(0.022)*	*(0.026)*	*(0.017)*		
Maturity ($\lambda = 0.30$)	0.544	0.269	0.752	0.804	0.003	0.103
	(0.067)	*(0.043)*	*(0.063)*	*(0.026)*		
Coupon ($z = 0$)	0.563	0.292	0.697	0.805	0.003	0.068
	(0.056)	*(0.030)*	*(0.040)*	*(0.025)*		
R3. Shocks						
Credit markets ($s_z = 0.85$)	0.502	0.293	0.699	0.807	0.009	0.154
	(0.081)	*(0.047)*	*(0.068)*	*(0.038)*		
Credit markets ($\sigma_z = 0.10$)	0.512	0.291	0.702	0.807	0.008	0.143
	(0.083)	*(0.047)*	*(0.068)*	*(0.037)*		
Expenditures ($s_g = 0.80$)	0.510	0.283	0.717	0.810	0.013	0.299
	(0.107)	*(0.057)*	*(0.084)*	*(0.047)*		
Expenditures ($\sigma_g = 0.05$)	0.545	0.293	0.701	0.805	0.006	0.118
	(0.079)	*(0.042)*	*(0.058)*	*(0.034)*		

Notes: For each alternative economy (in each row), we report the average and standard deviation of fiscal policy variables (debt to output, tax rate, incidence of the formal sector) and consumption (*Consu.*) in Panel A, and we report the default incidence (*Default*) in Panel B. In Panel C, we produce the inferred cost of fluctuations in consumption (*Fluct.*). The latter is the percentage difference between an average certainty equivalent consumption from all simulations $s \in \{1, \ldots, S\}$, $u^{-1}\left(1/\left(\sum_{\tau=t}^{100} \beta^{\tau-t}\right) \cdot \left(\sum_{\tau=t}^{100} \beta^{\tau-t} u(c_{\tau,s})\right)\right)$, and the average consumption $\bar{c} = 1/\left(\sum_{\tau=t}^{100} \beta^{\tau-t}\right) \cdot \left(\sum_{\tau=t}^{100} \beta^{\tau-t} c_{\tau,s}\right)$. The statistics about the debt-to-output ratio (*Debt*), the tax rate (*Tax*) and the incidence of the formal sector (*Formal*) are computed using the subsample of repayment periods, as in Table 2.

First, increasing the punishment for default alters consumption fluctuations. In theory, there exist two opposing forces: a higher punishment gives more commitment to the government (at the limit with infinite costs, it would give full commitment), but it also makes default costly and might induce costly precautionary behavior. We find the former to prevail. Second, longer-run debt tends to lower consumption fluctuations and the incidence of default.

In general, however, Table B4 provides support for our main analysis and shows that statistics about fiscal variables are not *too* dependent on specific aspects of our calibration—the cost of consumption fluctuations roughly being a quadratic function of consumption volatility, small differences in the latter would create larger differences in the former.