Detecting Nonlinear Risk Exposures in Hedge Fund Strategies

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Abstract

This paper proposes a model that allows for nonlinear risk exposures of hedge funds to various risk factors. A flexible threshold regression model is introduced and a Bayesian approach is developed for model selection and estimation of the thresholds and their unknown number. Relevant risk factors and/or threshold values are identified through a computationally flexible Markov chain Monta Carlo stochastic search algorithm. Our analysis of several hedge fund returns reveals that different strategies exhibit nonlinear relations to different risk factors. We also explore potential economic impacts of our approach by analysing hedge fund strategy return series and by constructing style portfolios.

JEL Classification: G11, G12, C11.

K*eywords*: Hedge Funds, GARCH, MCMC methods, Model uncertainty, Risk factors, Style portfolio construction.

1 Introduction

High net worth individual investors, institutional investors and pension funds have shown a growing interest in investing into hedge funds, and have increased their allocation to hedge funds during the last decades. Hedge funds are alternative investment vehicles that involve highly dynamic, complex trading strategies, with great flexibility with respect to the types of securities they hold and the types of positions they take. They can take both long and short positions, use leverage, trade options and other derivatives, and invest in different asset classes in any market. They are not required to disclose their positions in specific securities, and they can change these positions frequently. These characteristics allow for investment strategies that offer low correlations to traditional portfolios of cash, bonds and equities, and differ significantly from traditional regulated investments, such as mutual funds; see, for example, Fung and Hsieh (1997) who studied the empirical characteristics of hedge funds.

There exist various hedge fund strategies which exhibit different statistical properties, different risk-return characteristics (Fung and Hsieh, 1999), and therefore different exposures to market risk factors. A large amount of research has been carried out to investigate the risk return characteristics of hedge fund strategies and to understand the exposures of hedge funds to risk factors. Several studies use linear regression techniques to examine the relationship of the hedge fund returns with a set of linear and nonlinear risk factors; see, for example, Fung and Hsieh (1997, 2001), Lhabitant (2001), Mitchell and Pulvino (2001), Agarwal and Naik (2004). In particular, Fung and Hsieh (2001) studied 'trend following' hedge fund strategies and found that the risk-return characteristics are nonlinear. Mitchell and Pulvino (2001) analysed 'risk arbitrage' hedge funds and their results reveal that there is a nonlinear - asymmetric relationship between risk arbitrage returns and market returns. Agarwal and Naik (2004) analysed equity-oriented hedge fund strategies and found evidence of nonlinear payoffs for a large number of hedge funds. The results of their analysis also indicate that many hedge fund strategies exhibit asymmetric betas or factor loadings in up-market vesrus down-market conditions. These studies highlight the asymmetric-nonlinear features of hedge funds and stress the importance of taking them into account when analysing hedge funds.

Although the estimation of standard risk factor models is straightforward, the identification of the most important set of risk factors in a highly nonlinear set up is not an easy task, due to the complex nature of hedge fund investments. Different model/variable selection strategies have been used in the literature to cope with this problem. The method of stepwise regression has been widely used in hedge fund pricing to identify significant pricing risk factors; see, for example, Liang (1999), Fung and Hsieh (2000), Agarwal and Naik (2004). The use of information criteria, such as Akaike's (1973) information criterion and Schwarz's (1978) Bayesian information criterion is one other approach to variable selection; see, for example, Bossaerts and Hillion (1999) who used different statistical criteria for the determination of valuable factors in equity pricing, and Vrontos *et* al. (2008) who used these information criteria in the pricing of hedge funds. However, these approaches do not take into account model uncertainty which is present in the context of hedge fund pricing. The solution to this problem, at least in principle, is to adopt a Bayesian approach that provides posterior model probabilities. Avramov (2002) and Cremers (2002) have considered Bayesian techniques using stock return series, while Meligkotsidou and Vrontos (2008), and Meligkotsidou *et* al. (2009) have developed Bayesian methods for the analysis of hedge fund returns. Vrontos *et* al. (2008) in a comparative study of different model selection techniques have found evidence that Bayesian methods outperform the other selection strategies in terms of predictive ability and performance of selected portfolios.

In this study, we propose a new class of flexible threshold risk factor models that can be used to better capture the features (nonlinear and dynamic nature) of hedge fund investments. Specifically, we consider threshold regression models which allow the dynamics of hedge fund returns to be asymmetric and different in different regimes/partitions of the data. We relax the assumption of the fixed number of regimes/partitions allowing for automatic model determination of the most important risk factors and estimation of the number and the values of the corresponding thresholds. We also consider GARCH-type conditional heteroskedasticity that allows for time-varying variances that are often present in financial series such as hedge funds. Note that the proposed model is new in the sense that, unlike the previous model specifications which allow for a simple threshold on the market risk factor only, our model specification allows for an unknown number of thresholds in every risk factor that affects hegde fund returns.

In our analysis we aim at simultaneously identifying significant pricing factors and determining the number of thresholds and the threshold values associated with the most relevant risk factors. We propose a Bayesian approach to model comparison which is based on calculating the posterior model probabilities for various competing models defined by different risk factors and different thresholds. To this end we develop a Markov chain Monte Carlo (MCMC) stochastic search algorithm which explores the model space by jumping between different threshold risk factor models. Within a specific model (i.e. specific risk factors and thresholds) the model parameters are integrated out using Laplace's approximation.

Several important results emerge from our analysis. First, we have found evidence that different hedge fund strategies exhibit asymmetric/nonlinear relations to different risk factors. Some of the risk factors at which thresholds are found to be present are the MSCI emerging market index, the MSCI world excluding USA index and the change in the S&P 500 implied volatility index. Second, we have found strong evidence for model uncertainty in all threshold risk factor models considered for hedge fund strategies. Finally, our results suggest that the threshold regression models improve our ability to construct style portfolios compared to the standard linear regression model.

The contributions of our work are several. First, we introduce a new class of flexible threshold regression models in the context of hedge fund pricing. In analysing hedge fund returns very little is known with regard to the risk factors that affect them, and the asymmetries of hedge funds to these factors. Second, we develop a Bayesian approach to inference with the aim to simultaneously identify the risk factors and the thresholds associated with these factors. This is very important in order to investigate whether risk exposures are different during up and down movements of relevant risk factors. We propose a stochastic search algorithm which is computationally less intensive than computing the collection of all models and provides posterior probabilities.

The remainder of the paper is organised as follows. In section 2 we introduce the proposed threshold risk factor model and in section 3 we describe the Bayesian approach to inference and present the stochastic search algorithm. Section 4 presents the data and the empirical application using hedge fund strategies, while section 5 concludes with a brief discussion.

2 The Threshold Regression Model

In this section we present the threshold regression model we use to analyse hedge funds. Suppose that we observe T consecutive realisations of hedge fund returns $\mathbf{r} = (r_1, ..., r_T)^{'}$ and that there are K available risk factors, $\mathbf{f} = (\mathbf{f}_1, ..., \mathbf{f}_K)^{'}$. Suppose also that there are n_i, $i = 1, ..., K$ possible threshold values associated with the ith risk factor. In this study we are interested in identifying the most relevant risk factors that should enter the regression model and simultaneously detecting the number and the values of the thresholds associated with these factors. Analytic evaluation of all models, with different risk factors and thresholds, is computationally very demanding, if not infeasible, because the number of all models, $\prod_{i=1}^{K} [2^{n_i} + 1]$, is huge. We propose a Bayesian approach to inference for this model selection problem.

To define our threshold regression models, we introduce a $K \times 1$ vector $\boldsymbol{\gamma} = (\gamma_1, ..., \gamma_K)'$, where $\gamma_i = 1$ if the ith factor is included in the model and $\gamma_i = 0$ if not. We also introduce a $K \times 1$ vector $\mathbf{p} = (p_1, ..., p_K)'$, where p_i denotes the number of partitions of the ith risk factor. For example, if the ith risk factor is not included in the model then $\gamma_i = 0$ and $p_i = 0$, if the ith risk factor is included in the model, but it has not any threshold then $\gamma_i = 1$ and $p_i = 1$, if the ith risk factor is included in the model and there exists one threshold associated with this factor then $\gamma_i = 1$ and $p_i = 2$, and so on. To define the threshold model we also consider a $K \times 1$ vector $\mathbf{t} = (\mathbf{t}_1, ..., \mathbf{t}_K)'$, which contains the threshold values which are associated with the K possible risk factors. Each $\mathbf{t}_i$, $i = 1, ..., K$, is a vector consisting of the $p_i - 1$ threshold values which subdivide the space of the ith risk factor in p_i partitions, i.e. $\mathbf{t}_i = (t_{i1}, ..., t_{i,p_i-1})$. Obviously, if $p_i = 0$ or $p_i = 1$, $\mathbf{t}_i$ is an empty vector.

The number of risk factors included in the model and the number of thresholds or equivalently the number of partitions defined by these thresholds determine the complexity of the threshold regression model. Inference is carried out assuming that the true threshold risk factor model is unknown. The parameter vector of the threshold structure is denoted by $\boldsymbol{\theta}_{th} = (\boldsymbol{\gamma}, \mathbf{p}, \mathbf{t})$ and determines uniquely the threshold model. Bayesian inference about $\boldsymbol{\theta}_{th}$ is based on the posterior distribution $p(\boldsymbol{\theta}_{th}|\mathbf{r})$ which can be written as

$$p(\boldsymbol{\theta}_{th}|\mathbf{r}) = p(\boldsymbol{\gamma}|\mathbf{r})\, p(\mathbf{p}|\mathbf{r}, \boldsymbol{\gamma})\, p(\mathbf{t}|\mathbf{r}, \boldsymbol{\gamma}, \mathbf{p}),$$

where $\mathbf{r}$ are the observed data. We generate a sample from the posterior distribution of $\boldsymbol{\theta}_{th}$ using an MCMC stochastic search algorithm.

The threshold regression model can be written as

$$\mathbf{r} = \mathbf{F}(\boldsymbol{\theta}_{th})\,\boldsymbol{\beta}(\boldsymbol{\theta}_{th}) + \boldsymbol{\varepsilon},$$

where $\mathbf{F}(\boldsymbol{\theta}_{th})$ is the design matrix and $\boldsymbol{\beta}(\boldsymbol{\theta}_{th})$ is the vector of the regression model parameters, which are both determined by the threshold structure $\boldsymbol{\theta}_{th}$. The hedge fund returns r_t at time t, can be modelled using the threshold regression by

$$r_t = \alpha + \sum_{i=1}^{K}\left[\sum_{j=1}^{p_i}\beta_{ij} f_{i,t} I\left(f_{i,t} \in P_{ij}\right)\right] + \varepsilon_t, \tag{1}$$

where r_t is the hedge fund excess return at time t, α is the intercept of the model, $f_{i,t}$, $i = 1, ..., K$ is the excess return of the ith factor at time t, p_i is the number of the nonoverlapping partitions P_{ij} of risk factor $\mathbf{f}_i$ which are defined by the $p_i - 1$ thresholds on $\mathbf{f}_i$, β_{ij}, $i = 1, ..., K$, $j = 1, ..., p_i$ are the factor loadings associated with factor i, and ε_t is the error term at time t. The proposed model allows the error term to be conditionally heteroscedastic, that is the conditional variance follows a GARCH(1,1) specification

$$\varepsilon_t | \Phi_{t-1} \sim N\left(0, \sigma_t^2\right)$$
$$\sigma_t^2 = \omega + b\varepsilon_{t-1}^2 + g\sigma_{t-1}^2$$

where Φ_{t-1} is the information set up to time $t-1$, the error process follows a conditional Normal distribution with conditional variance σ_t^2 at time t, and $\omega > 0$, $b \geq 0$, $g \geq 0$ in order to ensure that the conditional variance is allways positive. The GARCH specification allows for a more flexible structure than ARCH models (Engle, 1982) and captures some well known stylized facts of financial series such as volatility clustering and fat tails (Bollerslev, 1986). The parameter vector $\boldsymbol{\theta}$ of the proposed threshold regression model includes the vector of parameters $\boldsymbol{\theta}_{th}$ of the threshold structure as well as the vector $\boldsymbol{\theta}_{RG} = (\boldsymbol{\theta}_R, \boldsymbol{\theta}_G)$, which contains the parameters $\boldsymbol{\theta}_R = \left(\alpha, \beta_{11}, ...\beta_{1p_1}, ...\beta_{K1}, ...\beta_{Kp_K}\right)$ of the regression equation and the parameters $\boldsymbol{\theta}_G = (\omega, b, g)$ of the conditional variance specification. For homoscedastic return series with constant regression variance σ^2, the parameter vector $\boldsymbol{\theta}_G$ reduces to $\boldsymbol{\theta}_G = \left(\sigma^2\right)$. Therefore, the parameter vector of the threshold regression model is $\boldsymbol{\theta} = (\boldsymbol{\theta}_{th}, \boldsymbol{\theta}_{RG})$.

The proposed threshold regression model is very flexible and can be considered as a generalisation of the regression models proposed in the hedge fund literature. It is obvious that, if there is no threshold in the model it reduces to the regression model of Vrontos *et* al. (2008) with GARCH-type variances, while if there is not conditional heteroscedasticity it reduces to a threshold in mean model. Finally, if there is neither threshold structure nor heteroscedastic features, then the model reduces to a simple multifactor regression model. The proposed threshold structure allows for modelling the asymmetric behaviour of hedge fund returns, since it seems natural to allow for the existence of different states or partitions in different risk factors, and to model the underlying dynamics in a different way.

To clarify ideas we present a threshold regression model of the form

$$\begin{aligned} r_t &= \alpha + \beta_{11} f_{1,t} I\left(f_{1,t} \leq 0\right) + \beta_{12} f_{1,t} I\left(f_{1,t} > 0\right) + \beta_{31} f_{3,t} \\ &\quad + \beta_{41} f_{4,t} I\left(f_{4,t} \leq -0.3\right) + \beta_{42} f_{4,t} I\left(-0.3 < f_{4,t} \leq 0.3\right) + \beta_{43} f_{4,t} I\left(f_{4,t} > 0.3\right) + \varepsilon_t. \end{aligned}$$

In this example, the factors that are included in the model are the first one, $f_{1,t}$, the third, $f_{3,t}$, and the fourth, $f_{4,t}$. There is a single threshold in the first factor $f_{1,t}$ at the value 0, the third factor $f_{3,t}$ is included in the model but there is no threshold in this factor, while in the fourth factor $f_{4,t}$ there are two thresholds at the values -0.3 and 0.3, which divide the space of that factor in three partitions. Furthermore, if we consider that the total number of possible factors is $K = 5$, the above threshold regression model corresponds to the specifications $\boldsymbol{\gamma} = (1, 0, 1, 1, 0)'$, $\mathbf{p} = (2, 0, 1, 3, 0)'$ and $\mathbf{t} = (\mathbf{t}_1, ..., \mathbf{t}_5)'$ where $\mathbf{t}_1 = [0]$, $\mathbf{t}_2 = []$, $\mathbf{t}_3 = []$, $\mathbf{t}_4 = [-0.3\ 0.3]$, and $\mathbf{t}_5 = []$.

3 Bayesian Inference for threshold regression models

3.1 Model Comparison

This section considers the problem of model comparison which deals with the uncertainty about the set of risk factors that should enter the regression model, as well as the number and the values of the thresholds that allow for asymmetric effects in hedge fund return series. We propose a Bayesian approach to model comparison. This is a probabilistic approach to inference which is based on the calculation of the posterior distribution of $\boldsymbol{\theta}_{th}$, or equivalently on computing the posterior probabilities of different threshold regression models.

As described above, the risk factors included in the regression model identified by the vector $\boldsymbol{\gamma}$, the numbers of partitions, $\mathbf{p}$, and the threshold values, $\mathbf{t}$, determine uniquely the structure of the threshold model, which will be denoted by M_{th}. The posterior probability of model M_{th}, parametrised by $\boldsymbol{\theta}_{th} = (\boldsymbol{\gamma}, \mathbf{p}, \mathbf{t})$, is given by

$$p(M_{th}|\mathbf{r}) = \frac{p(\mathbf{r}|M_{th})p(M_{th})}{\sum_{m \in \mathbb{M}} p(\mathbf{r}|m)p(m)}, \tag{2}$$

for $M_{th} \in \mathbb{M}$, with $\mathbb{M}$ denoting the set of all possible threshold regression models, and where

$$p(\mathbf{r}|M_{th}) = \int p(\mathbf{r}|M_{th}, \boldsymbol{\theta}_{RG})p(\boldsymbol{\theta}_{RG}|M_{th})d\boldsymbol{\theta}_{RG}$$

is the marginal likelihood of model M_{th}, $p(\mathbf{r}|M_{th}, \boldsymbol{\theta}_{RG})$ is the likelihood of the model M_{th}, $p(\boldsymbol{\theta}_{RG}|M_{th})$ is the prior of the regression-GARCH parameters $\boldsymbol{\theta}_{RG}$ given the model M_{th}, and $p(M_{th})$ is the prior probability of model M_{th}.

In the threshold regression model with GARCH-type errors, analytic evaluation of the marginal likelihood is not possible. Kass and Raftery (1995) provide an extensive description of available numerical strategies that can be used to deal with this problem. Here, we use a variant of Laplace approximation based on the maximum likelihood

$$p\,\widehat{(\mathbf{r}|M_{th})}_{mle} = (2\pi)^{d_{\theta_{RG}}/2} \left|\widehat{\Sigma}\right|^{1/2} p\left(\mathbf{r}|\widehat{\boldsymbol{\theta}}_{RG}, M_{th}\right) p\left(\widehat{\boldsymbol{\theta}}_{RG}|M_{th}\right),$$

where $\widehat{\Sigma}$ is the inverse of the negative Hessian matrix of the log-likelihood evaluated at the maximum likelihood estimator $\widehat{\boldsymbol{\theta}}_{RG}$, $p\left(\mathbf{r}|\widehat{\boldsymbol{\theta}}_{RG}, M_{th}\right)$ and $p\left(\widehat{\boldsymbol{\theta}}_{RG}|M_{th}\right)$ are the likelihood and the prior, respectively, evaluated at $\widehat{\boldsymbol{\theta}}_{RG}$. Note that, for homoscedastic return series, the marginal likelihood of model M_{th}, $p(\mathbf{r}|M_{th})$, can be evaluated analytically since the model parameters are integrated out (see, for example, Zellner, 1971, O'Hagan and Forster, 2004). This method of approximation for the marginal likelihood has been used among others by Bauwens and Rombouts (2007) in the analysis of mixture GARCH models, Giannikis *et* al. (2008) in the analysis of threshold mixture GARCH models and Vrontos *et* al. (2003) and Dellaportas and Vrontos (2007) in the analysis of multivariate GARCH models.

3.2 MCMC stochastic search algorithm

When the number of available risk factors K in (1) and the number of possible threshold values associated with these factors is large, the set $\mathbb{M}$ of all threshold regression models is enormous and analytic evaluation of (2) is computationally infeasible. In this section, we describe a Markov chain Monte Carlo algorithm that moves through models of the same or different dimensionality, i.e. number of risk factors and different thresholds, to generate a sample from the posterior distribution of $(\boldsymbol{\gamma}, \mathbf{p}, \mathbf{t})$, or in other words to provide posterior probabilities for models M_{th}.

The proposed algorithm is based on the Metropolis-Hastings algorithm which can be used to explore the model space. We construct a Markov chain $\left\{M_{th}^{(j)},\ j = 1, 2, ...\right\}$ with state space $\mathbb{M}$, which under certain regularity conditions (see Tierney, 1994) converges to the equilibrium distribution $p\left(M_{th}|\mathbf{r}\right)$. The Metropolis-Hastings algorithm for simulating $M_{th}^{(1)}$, $M_{th}^{(2)}$,... is defined as follows. Starting with an initial model $M_{th}^{(1)}$ iteratively simulate the transitions from $M_{th}^{(i)}$ to $M_{th}^{(i+1)}$ by using two steps:

- generate a candidate model M'_{th} with probability distribution $q\left(M_{th}^{(i)}, M'_{th}\right)$
- accept $M_{th}^{(i+1)} = M'_{th}$ with probability

$$a\left(M_{th}^{(i)}, M'_{th}\right) = \min\left\{\frac{q\left(M'_{th}, M_{th}^{(i)}\right)}{q\left(M_{th}^{(i)}, M'_{th}\right)} \frac{p\left(\mathbf{r}|M'_{th}\right)}{p\left(\mathbf{r}|M_{th}^{(i)}\right)} \frac{p\left(M'_{th}\right)}{p\left(M_{th}^{(i)}\right)}, 1\right\}, \tag{3}$$

  otherwise set $M_{th}^{(i+1)} = M_{th}^{(i)}$. Note that the normalising constant for $p\left(M_{th}|\mathbf{r}\right)$ is not needed to compute (3).

Due to the complexity of the proposed class of models and the different dimensionality of different threshold models, we shall need to devise different types of moves between the subspaces. We consider transition kernels $q\left(M_{th}^{(i)}, M'_{th}\right)$ which generate candidate threshold models M'_{th} from $M_{th}^{(i)}$ by randomly choosing among the following steps.

- *Risk factor step*: with probability π choose to propose a different set of risk factors in the regression model, based on the following alternatives
  - *Birth of risk factor*: randomly add one of the remaining risk factors in the regression.
  - *Death of risk factor*: randomly delete a factor which is included in the regression and has no threshold.

  - *Change of risk factor*: randomly pick a factor which is in the regression and has no threshold and assign a new one from those available.

- *Threshold step*: with probability $1 - \pi$ choose to propose a different threshold structure in the regression model. First, choose randomly one of the risk factors that are included in the model. Then, proceed using one of the following alternatives

  - *Birth of threshold:* randomly add a new threshold (creates a new partition) from those possible for the selected factor.
  - *Death of threshold*: randomly pick a threshold on the selected factor and delete it (creates a model with one less threshold on the selected factor).
  - *Change of threshold*: randomly pick a threshold and assign a new one from those possible for the selected factor.

In the stochastic search algorithm, we propose a *Risk factor step* with probability $\pi = 1$ if there is no risk factor in the model, we use $\pi = 0$ if all factors are included in the model and have at least one threshold associated with them, while we use $\pi = 0.5$ in all other cases. In the *Risk factor step,* we assign probability 1 to perform a *Death of risk factor* step if all risk factors are included in the model, assign probability 1 to perform a *Birth of risk factor* step if there is no risk factor included in the model, while assign equal probabilities $\frac{1}{3}$ to perform a *Birth of risk factor*, *Death of risk factor* or *Change of risk factor* step in all other cases. In a similar way, when we perform a *Threshold step* at a selected risk factor i, we assign with probability 1 the *Birth of threshold* step if $p_i = 1$, we perform the *Death of threshold* step with probability 1 if $p_i = n_i + 1$, while we perform the *Birth of threshold, Death of threshold* and *Change of threshold* step with equal probability $\frac{1}{3}$ when $1 < p_i < n_i + 1$.

3.3 The Priors

To proceed with the Bayesian approach we have to assign a prior distribution to the parameter vector $\boldsymbol{\theta} = (\boldsymbol{\theta}_{th}, \boldsymbol{\theta}_{RG})$ which determines the threshold regression model. In this class of models we propose to use the following prior specification

$$p(\boldsymbol{\theta}) = p\left(\boldsymbol{\theta}_{th}, \boldsymbol{\theta}_{RG}\right) = p\left(\boldsymbol{\theta}_{th}\right) p\left(\boldsymbol{\theta}_{RG} | \boldsymbol{\theta}_{th}\right)$$

and specifying $p\left(\boldsymbol{\theta}_{th}\right)$ and $p\left(\boldsymbol{\theta}_{RG} | \boldsymbol{\theta}_{th}\right)$ seperately. This specification has the advantage that the prior of the threshold structure does not depend on the prior of the parameter vector $\boldsymbol{\theta}_{RG}$ and enhances posterior computation.

For the prior probability of the threshold structure model M_{th} which is determined by $\boldsymbol{\theta}_{th} = (\boldsymbol{\gamma}, \mathbf{p}, \mathbf{t})$ we use the following hierarchical structure

$$p(M_{th}) = p(\boldsymbol{\theta}_{th}) = p(\boldsymbol{\gamma}, \mathbf{p}, \mathbf{t}) = p\left(\boldsymbol{\gamma}\right) p\left(\mathbf{p} | \boldsymbol{\gamma}\right) p\left(\mathbf{t} | \boldsymbol{\gamma}, \mathbf{p}\right).$$

A prior distribution for $\boldsymbol{\gamma}$ provides information about the inclusion of a risk factor in the regression model. The prior distribution we adopt for $\boldsymbol{\gamma}$ is of the form

$$p(\boldsymbol{\gamma}) = \prod_{k=1}^{K} \pi_k^{\gamma_k} (1 - \pi_k)^{1-\gamma_k}$$

where $\pi_k \in [0, 1]$. Under this prior, each factor f_k, $k = 1, ..., K$ enters the regression model independently of the others with probability $p(\gamma_k = 1) = 1 - p(\gamma_k = 0) = \pi_k$. We assign $\pi_k = 0.5$ for all k, which yields the uniform prior, $p(\boldsymbol{\gamma}) = 1/2^K$, that is often used as a representation of ignorance, i.e. implies that the investor is neutral about the factors that will enter the model.

With regard to the prior distribution for the number of partitions $\mathbf{p}$ we adopt an independent prior of the form

$$p(\mathbf{p}|\boldsymbol{\gamma}) = p(p_1, ..., p_K|\boldsymbol{\gamma}) = p(p_1|\boldsymbol{\gamma}) ... p(p_K|\boldsymbol{\gamma})$$

where for each p_i, $i = 1, ..., K$, we assign a Poisson prior distribution truncated to one with p.d.f.

$$\begin{aligned} p(p_i|\gamma_i &= 1) = \frac{\lambda^{p_i}}{(e^{\lambda} - 1)p_i!}, \quad \lambda > 0 \\ p(p_i &= 0|\gamma_i = 0) = 1. \end{aligned}$$

Under this prior, if the ith risk factor is included in the model we require a minimum of one partition for this factor. In this case, we use $\lambda = 2$ so that we impose some penalty to models which tend to split the factors in many partitions. If the ith risk factor is not included in the model, i.e. $\gamma_i = 0$, it is obvious that $p_i = 0$.

For the threshold values $\mathbf{t}$, given $\boldsymbol{\gamma}$ and $\mathbf{p}$, we assign the following prior specification of the form

$$p(\mathbf{t}|\boldsymbol{\gamma}, \mathbf{p}) = p(\mathbf{t}_1, ..., \mathbf{t}_K|\boldsymbol{\gamma}, \mathbf{p}) = p(\mathbf{t}_1|\boldsymbol{\gamma}, \mathbf{p}) ... p(\mathbf{t}_K|\boldsymbol{\gamma}, \mathbf{p}) = p(\mathbf{t}_1|\boldsymbol{\gamma}, p_1) ... p(\mathbf{t}_K|\boldsymbol{\gamma}, p_K).$$

Then, if n_i denotes the number of potential thresholds of the ith risk factor, we use a discrete uniform prior distribution over the set of all possible thresholds, i.e. over the set of $\binom{n_i}{p_i - 1}$ possible ways of picking $p_i - 1$ thresholds out of n_i. Therefore,

$$p(\mathbf{t}_i|\boldsymbol{\gamma}, p_i) = p(t_{i1}, ..., t_{i,p_i-1}|\boldsymbol{\gamma}, p_i) = \frac{(p_i - 1)!(n_i - p_i + 1)!}{n_i!}.$$

For the parameters $\boldsymbol{\theta}_{RG}$ of the regression model we assume proper prior distributions to avoid resulting in an improper posterior. We consider the following two cases. In the case of homoscedastic return series i.e. constant regression variance σ^2, we assume independent conjugate prior distributions, that is a multivariate normal $N(\boldsymbol{\mu}, c\sigma^2\mathbf{V})$ for the vector $\boldsymbol{\theta}_R$, and an inverted Gamma $IG(d/2, \nu/2)$ prior for σ^2. We choose $\boldsymbol{\mu} = \mathbf{0}$, which reflects prior ignorance about the location of the means of the regression coefficients, $c = T$ and $\mathbf{V} = (\mathbf{F}'\mathbf{F})^{-1}$ where $\mathbf{F}$ is the design matrix, which replicates the covariance structure of the data and yield the g-prior of Zellner (1986). The hyperparameters d and ν are chosen such that the prior mean $E(\sigma^2) = \frac{v}{d-2}$, $d > 2$ equals to the maximum likelihood estimate of σ^2, i.e $\widehat{\sigma}^2$, and the prior variance $Var(\sigma^2) = \frac{2}{d-4}\left(\frac{\nu}{d-2}\right)^2$ equals to $100\widehat{\sigma}^2$. In the case of heteroscedastic return series, i.e. when the conditional variance is time-varying, we consider a unit information prior proposed by Kass and Wasserman (1995) and further applied by, for example, Dellaportas and Vrontos (2007), Vrontos *et* al. (2008) and Giannikis *et* al. (2008). After a possible transformation of the variance parameters $\boldsymbol{\theta}_{\mathbf{G}}$, to a vector taking values to $(-\infty, \infty)$, we use a multivariate Normal prior distribution, with covariance matrix equal to $T\widehat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\theta}}_{RG}}$, where T indicates the sample size and $\widehat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\theta}}_{RG}}$ is the covariance matrix of the maximum likelihood estimates of the model parameters.

4 Data and Empirical Analysis

4.1 The Data

In this section we illustrate the proposed threshold regression approach to analyse hedge fund single strategy indices. In particular, we analyse hedge fund data from Hedge Fund Research[1] (HFR), which are more relevant to style allocation decisions, as in Amenc and Martellini (2002), McFall Lamm (2003), Agarwal and Naik (2004), Morton *et* al. (2005), Giamouridis and Vrontos (2007), Meligkotsidou *et* al. (2009) among several others. The HFR indices are equally weighted average returns of hedge funds and are computed on a monthly basis. We consider different HFR single strategy indices in our empirical study of the detection of nonlinear risk exposures of hedge funds to different risk factors, but for reasons of space we present results for five hedge fund strategies[2]: Equity Hedge Macro, Relative Value Arbitrage, Event Driven, and Merger Arbitrage. Our study of these hedge funds uses net-of-fee monthly excess returns (in excess of the three month US Treasury Bill) from January 1994 to December 2005.

We model the hedge fund returns by using different risk factors as discussed in Agarwal and Naik (2004) and further used by Vrontos *et* al. (2008) and Meligkotsidou *et* al. (2009). These factors include returns on the Russel 3000 equity index (RUS), the Morgan Stanley Capital International (MSCI) world excluding the USA index (MXUS), the MSCI emerging markets index (MEM), the Salomon Brothers world government and corporate bond index (SBGC), the Salomon Brothers world government bond index (SBWG), the Lehman high yield index (LHY), the Goldman Sachs commodity index (GSCI), the Federal Reserve Bank competitiveness weighted dollar-index (FRBI); Fama and French's (1993) 'size' (SMB) and 'book-to-market' (HML) as well as Carhart's (1997) 'momentum' factors (MOM); the difference between the yield on the BAA-rated corporate bonds and the 10-year Treasury bonds (DEFSPR); and the change in equity implied volatility index VIX. This dataset also covers the period January 1994 to December 2005.

4.2 Empirical Analysis

In the following analysis we develop threshold regression risk factor models to simultaneously identify the relevant risk factors that affect hedge fund returns and determine the thresholds associated with

[1] Details can be found in Hedge Fund Research, www.hedgefundresearch.com

[2] The results for the other HFR single strategy indices are available upon request.

these factors. Next, we investigate whether the proposed threshold regression approach has economic significance by constructing style allocation portfolios.

4.2.1 Detecting Nonlinear risk exposures in hedge fund strategies

Here, we analyse monthly data on five hedge fund strategies i.e. Equity Hedge (EH), Macro (M), Relative Value Arbitrage (RVA), Event Driven (ED), and Merger Arbitrage (MA) from January 1994 to December 2005. We use the Bayesian approach described in Section 3 to simultaneously identify the most important risk factors and detect possible nonlinearities by using different threshold models. Due to the complexity of the threshold regression models we need a sufficient number of observations within each partition in order to estimate the model parameters, especially in the case of hedge funds due to the short history of data. For this reason we allow the elements t_{i1},...,t_{i,p_i-1}, $i = 1, ..., K$ of the threshold parameter vector $\mathbf{t}$, to take values from a prespecified finite grid. In the applications of this paper this grid consists of a set of empirical quartiles of the data at hand. This is a standard approach in this type of problems which involve complex models; see, for example, Audrino and Buhlmann (2001), Giannikis *et* al. (2008).

We also allow for accounting for several features that often are met in hedge fund returns such as autocorrelation of returns, time-varying variances and fat tails. In Table 1, we present the summary statistics for the returns of the analysed hedge funds, together with the Ljung-Box statistic computed for the return series, as well as for the absolute and the squared returns. The Ljung-Box statistic is computed based on 12 lags and shows that the null hypothesis of no autocorrelation in the return series is rejected at 5% level of significance for M, RVA and ED, while there is evidence for heteroscedastic effects in EH, M and RVA return series due to the presence of autocorrelation in the absolute and/or the squared returns. Note also that there is high kurtosis in hedge fund returns, varying from 3.59 for EH to 14.08 for RVA, indicating fat tails in most of the hedge fund return distribution. To capture these characteristics we add an additional pricing factor, r_{t-1}, the lag net-of-fee excess return if the return series is autocorrelated, while we model conditional heteroscedasticity using GARCH-type specification, if there is autocorrelation in the absolute or the squared returns.

Insert Table 1 about here

The features described above, i.e. taking the empirical quartiles of each risk factor as possible threshold values, and the inclusion of the lagged return in the set of risk factors, render the number of possible models to 9^{15}. Analytic evaluation of all models with different risk factors and different threshold values is infeasible in this set up. Traditional model selection techniques such as AIC or BIC, that evaluate all models, are computationally very intensive and can not be used due to the large model space. A solution to this problem is to apply the Bayesian stochastic search algorithm that explores the threshold regression model space and provides posterior model probabilities. A typical approach for this type of model selection problems with huge number of possible models is to implement the algorithm in two steps. First, we run the MCMC stochastic search algorithm for $10,000$ iterations to identify risk factors with high marginal posterior inclusion probabilities $p(\gamma_i|\mathbf{r})$, and we create a reduced model space consisting only of those factors whose marginal probabilities are above a specified value. Different values have used in the literature; Barbieri and Berger (2004) proposed 0.5 as a value for $p(\gamma_i = 1|\mathbf{r})$, while Fouskakis *et* al. (2006) used the value 0.3. In this study we adopt the lower value of 0.3, since our aim is to identify and eliminate factors that do not contribute to models with high posterior probabilities. Finally, we apply the stochastic search algorithm for $100,000$ iterations in the reduced model space to estimate posterior model probabilities. In the following analysis we develop threshold regression models for different hedge fund strategies to investigate if these strategies have different exposures to different risk factors and simultaneously if they have different nonlinear/asymmetric characteristics. The results for each index are presented in detail in the following paragraphs.

Equity Hedge. The HFR Equity Hedge strategy includes funds that take long and short positions in equities, based on the A.W. Jones model. From Table 1 we can see that the absolute and the squared EH returns are autocorrelated, providing evidence for conditional heteroscedasticity. Therefore, we use a GARCH specification for modelling the time-varying variances of the EH return series. The results of our empirical analysis, which are presented in Table 2, reveal that the most probable model, with posterior probability 0.03, includes (see, Table 2, column/vector $\boldsymbol{\gamma}$) the Russel 3000 equity index (RUS), the lagged once Russel return (RUS(-1)), the MSCI emerging markets index (MEM), the Fama and French (1993) 'size' (SMB) and 'book-to-market' (HML) factors, the Carhart (1997) 'momentum' factor (MOM), the Goldman Sachs commodity index (GSCI), and the change in equity implied volatility index (VIX). These findings are consistent with those of Agarwal and Naik (2004) and Meligkotsidou *et* al. (2009) who found that RUS, SMB, HML, GSCI and VIX are important pricing factors. In Table 2, we also present the number of partitions in each factor (vector $\mathbf{p}$), the corresponding threshold values (vector $\mathbf{t}$) and the estimated factor loadings together with their standard errors, in parenthesis. It can be seen that EH has positive (0.387) exposure to the Russel 3000 index, which is very close to 0.41 found by Agarwal and Naik (2004), positive exposure to Fama and French's size factor, suggesting that the managers tend to buy undervalued small stocks and go short on the large stocks. There is also positive exposure to the emerging market index, the 'momentum' factor and the commodity index. Of particular interest is the threshold value of -0.0167 found at the 'book-to-market' factor, which devides the space of this factor in two partitions. The beta coefficient takes a significant negative value of -0.215 if the HML factor is smaller than the threshold value -0.0167, while is positive (0.014) but not significant when the factor is bigger than the threshold. Agarwal and Naik (2004) have also found a negative exposure to HML factor but the estimating factor loading was smaller enough (-0.08), suggesting that the managers were long growth stocks. Our analysis reveals that there is nonlinear/asymmetric risk exposure of EH to 'book-to-market' factor, and that the above feature is met only for the first partition of the HML factor. Finally, the GARCH coefficients are significant, indicating time varying variances in the EH return series.

Insert Table 2 about here

Macro. The HFR Macro strategy funds make use of derivatives and leverage and invest on price movements of stock markets, interest rates, currencies and commodities. The Macro hedge fund strategy exhibits significant serial correlation on the return series as well as on the squared and the absolute returns (see Table 1). Hence we add the lagged Macro return, M(-1), as an additional pricing factor, and use the GARCH specification for modelling the conditional variance. The results of our analysis indicate that the most probable model has posterior probability 0.014, indicating strong evidence of model uncertainty. The most important risk factors include MXUS, MEM, SMB, MOM, SBGC, FRBI, VIX and M(-1). Most of these factors have been identified as significant risk factors by Meligkotsidou *et* al. (2009) who have used quantile regression and linear regression models to analyse hedge funds. From Table 3, we can see that there are nonlinear/asymmetric effects with respect to the MXUS factor and the change in volatility index. Both factors have one threshold which corresponds to two partitions. The factor loading regarding the MXUS is negative (-0.1408) when its value is less than its median 0.0034, while it takes a positive value (0.3068) when it exceeds its median. On the contrary, the factor loading with respect to VIX is positive (0.2091) when the change in implied volatility is negative and less than -0.0203, while when the change is more than -0.0203 the factor loading takes a negative value (-0.0825) but not significant at 5% level of significance.

Insert Table 3 about here

Relative Value Arbitrage. The HFR Relative Value Arbitrage strategy attempts to benefit from relative pricing discrepancies between instruments including equities, debt, options and futures. RVA returns exhibit autocorrelation in the level as well as in the absolute returns (Table 1), so the lagged RVA return series, RVA(-1), is included in the risk factors and the GARCH model is used to model conditional variances. The most probable model, which represents about 0.13 of the total posterior probability, includes MXUS, MEM, SBGC, LHY and RVA(-1) risk factors. We found strong evidence of nonlinear features of RVA with respect to MXUS and MEM factors. There are two thresholds in MXUS, i.e. -0.0302 and 0.0035 which correspond to three partitions, while there is one threshold at 0.0448 in MEM factor which creates two partitions. The estimated factor loadings and their standard errors for this model are illustrated in Table 4. The beta coefficient is negative (-0.0807) when the MXUS is smaller than the threshold -0.0302, is negative but higher in magnitude (-0.1924) when MXUS takes values between -0.0302 and 0.0035, and has positive exposure (0.0696) when MXUS is greater than 0.0035. With respect to the asymmetry in the MEM factor, the beta exposure is positive (0.0826) when MEM is less than 0.0448, while it is not significant when the MEM factor exceeds this threshold value.

Insert Table 4 about here

Event Driven. The HFR Event-Driven strategy involves investing in opportunities created by significant transactional events, such as mergers and acquisitions, bankruptcy reorganizations, recapitalizations and share buybacks. From Table 1 we see that its returns exhibit autocorrelation, but not GARCH effects. Hence we include the lagged returns, ED(-1), as potential variable in the variable selection exercise, but use constant variance. The factors which are included in the most probable model are, except for the lagged ED return series, the RUS, MEM, SMB, HML, MOM, LHY and VIX. These findings are intuitive and consistent with the results of Agarwal and Naik (2004) and Meligkotsidou *et* al. (2009). Moreover, we found a nonlinear risk-return relation with the change in the implied volatility index. The change in VIX influence the ED returns, and creates asymmetric effects with threshold value of -0.0203. From Table 5, which represents the parameters of the most probable model, we observe that all factors which are linearly modeled (i.e. RUS, MEM, SMB, HML, MOM, LHY) exhibit positive factor loadings. Concerning the implied volatility index we see that when the change in VIX takes a value less than -0.0203, the corresponding factor loading is positive (0.0840), while when the change in VIX has a value greater than -0.0203 the corresponding factor loading turns negative.

Insert Table 5 about here

Merger Arbitrage. The Merger Arbitrage strategy involves investment in event-driven situations such as leveraged buy-outs, mergers and hostile takeovers. Usually these funds invest in the securities of companies involved in a merger or acquisition by buying the stocks of the target and going short on the stocks of the acquirer. Mitchell and Pulvino (2001) analysed Merger Arbitrage funds using a piecewise linear regression with regard to the market factor and found that merger arbitrage returns are positively correlated with market returns when the markets experience a large decline (less than 4%), but are uncorrelated with market returns in flat and appreciating markets. From Table 1, we observe that Merger Arbitrage returns do not exhibit autocorrelation neither in the series nor in the absolute/squared returns. We present the most probable model produced by our stochastic search algorithm in Table 6. This model has posterior probability of 0.08 and identifies the RUS, RUS(-1) and MEM factors as the most important. In the proposed threshold regression model, we let the data to detect the risk factors and the corresponding thresholds; of particular interest is that the most probable model show asymmetric risk exposures with regard to the MEM factor. Furthermore, we see from Table 6, which also presents the estimated factor loadings of the most probable model, that the beta is positive (0.0856) when MEM is less than the threshold value of 0.0085, and negative (-0.0066) but not significant when MEM exceeds the threshold value of 0.0085. Moreover, MA has positive exposures to the Russel and lagged Russel returns. Note that, the model with a nonlinear/asymmetric effect in the market factor (RUS) ranks third with posterior probability of 0.03.

Insert Table 6 about here

These results reveal information about the presence of nonlinearities/asymmetries in hedge fund return series[3]. There are nonlinear/asymmetric effects of hedge funds to different risk factors and at different threshold values. The proposed stochastic search algorithm allows for automatic determination of the most important risk factors and detection of the corresponding threshold values. These thresholds are allowed to exist not only in the market factor, but in any factor that is defined by the data at hand.

[3]We have also analysed other twenty-one single strategy hedge fund indices, i.e. in total twenty-six series (see, next paragraph), and found evidence for nonlinearities/asymmetries in the 69% of the analysed series.

Actually, this is the case in the analysed hedge fund strategies, where thresholds were found to be present at different risk factors such as the MSCI emerging market index (MEM), the MSCI world excluding USA index (MXUS), the Fama and French's (1993) 'book-to-market' (HML), and the change in the implied volatility index (VIX). Another important result is that there is evidence for model uncertainty in all threshold risk factor models. This is reasonable since hedge fund managers deploy different and dynamic investment strategies.

4.2.2 Portfolio performance

In this section we extend our investigation to examine the usefulness of the most probable threshold regression models, obtained by our approach, by using them to construct hedge fund style portfolios in an out-of-sample fashion. The results are compared with those obtained by applying simple linear regression models that ignore the presence of thresholds in risk factors, i.e. ignore the presence of nonlinearities/asymmetries in hedge fund data generating process. For this exercise, we select hedge fund strategies on the basis of their manager skill, formulate style portfolios and investigate if incorporating nonlinear and asymmetric effects in the selection process has potential economic impact. To this end we split our sample into an estimation period, from January 1994 to December 2003, which is used to estimate our models, and an evaluation period, from January 2004 to December 2005, which is used for portfolio construction and performance evaluation.

Our implementation is concerned with the performance of hedge fund style portfolios for the last two years, i.e. for the last 24 months, based on (a) the most probable threshold regression models, (b) the most probable linear regression models, and (c) an equally weighted portfolio of all hedge funds, which is denoted as a benchmark portfolio. The estimation period from January 1994 to December 2003 is used to determine the most probable threshold regression models for the twenty six hedge fund single strategy indices considered[4], estimate 'alpha', rank the hedge fund strategies on the basis of their estimated alphas and allocate wealth in the top 8 and top 10 performing hedge funds. The estimation period is redefined iteratively, i.e. at iteration 2 to 24, the estimation sample is augmented to include one more monthly observation in order to utilize all the available information, estimate 'alpha' based on the most probable threshold regression models, rank the hedge fund strategies on the basis of their estimated alphas and allocate wealth in the top performing hedge funds. Whenever the out-of-sample time index is a multiple of 12, i.e. every year, the threshold structure is re-estimated, using the Bayesian stochastic search algorithm, to provide the most probable threshold models. In other words, we update the threshold structure every year, and we estimate the corresponding most probable threshold models

[4]We use the following hedge fund single strategy indices from HFR: Equity Hedge, Macro, Relative Value Arbitrage, Event-Driven, Merger Arbitrage, Convertible Arbitrage, Distressed Securities, Market Timing, Short Selling, Emerging Markets (Total), Emerging Markets: Asia, Equity Market Neutral, Equity Market Neutral: Statistical Arbitrage, Equity Non-Hedge, Fixed Income (Total), Fixed Income: Arbitrage, Fixed Income: High Yield, Fixed Income: Convertible Bonds, Fixed Income: Diversified, Fixed Income: Mortgage-Backed, FOF: Conservative, FOF: Diversified, FOF: Market Defensive, FOF: Strategic, Fund of Funds Composite, Weighted Composite.

every month in the out-of-sample period.

Insert Table 7 about here

We evaluate the style portfolios, constructed based on the different models, using unconditional measures. In particular, we consider the realised returns, and the risk adjusted realised returns. We calculate the mean excess return within the out-of-sample period and the cumulative excess return at the end of the period. In risk adjusted performance measurement the realised returns are related to a suitable risk measure. In the hedge funds performance literature[5] the most commonly used risk adjusted measure is the Sharpe ratio (Sharpe, 1966, 1994), which is calculated as the ratio of the average portfolio excess return and the portfolio's standard deviation of excess returns. However, due to the fact that the use of Sharpe ratio as an evaluation measure for hedge funds has been criticised (see, for example, Amin and Kat, 2003) we also use an alternative risk adjusted measure of performance, the Sortino ratio (Sortino and van der Meer, 1991, Sortino and Price, 1994) which is calculated as the ratio of the average excess return and the downside risk.

In Table 7 we present the results of our analysis. Specifically, we have calculated and present the average portfolios' excess returns, the cumulative excess returns and the risk adjusted performance measures, namely the Sharpe ratio and the Sortino ratio, for all the three methods of portfolio construction, i.e. using threshold regression models, using linear regression models, and using the benchmark. First, we observe that nonlinear modelling of the hedge funds outperform in all the cases the benchmark. Second, looking at the average portfolio returns and the cumulative portfolio returns we observe that the linear and the nonlinear regression models score equally well in the case of top 8 'alpha' style portfolios. However, the nonlinear models achieve higher values for the Sharpe and the Sortino ratio, i.e. 0.4596 and 0.8775, respectively, versus 0.4299 and 0.7829 for the corresponding values of the linear models, indicating the superiority of nonlinear threshold models. Finally, the top 10 'alpha' style portfolios constructed using the nonlinear modelling approach outperforms those based on the linear approach and the benchmark; the cumulative excess return is 14.26% for nonlinear models versus 13.37% and 10.65% for the linear models and the benchmark, respectively. From Table 7, it can be seen that the higher values of both the Sharpe ratio and the Sortino ratio are those obtained for the nonlinear regression models; the nonlinear regression approach outperform the linear regression methods in terms of the risk adjusted performance measures, with Sharpe and Sortino ratio 0.4819 and 0.9425 versus 0.3843 and 0.6819, respectively. These findings confirm the superior performance of hedge fund style portfolios constructed with the nonlinear modelling approach.

5 Conclusion

In this paper we have introduced a flexible threshold risk factor model which is appealing for modelling hedge fund returns since it captures the nonlinearitites/asymmetries in the risk exposures of hedge funds

[5] see, for example, Ackermann *et* al. (1999), Edwards and Liew (1999), Liang (1999), and Schneeweis *et* al. (2002)

to different risk factors. We have proposed a Bayesian approach to identify the relevant risk factors and simultaneously to detect possible thresholds in the model. This approach is based on the development of a stochastic search algorithm that explores the model space and computes posterior model probabilities. The proposed nonlinear risk factor models enable us to study the risk exposure of hedge fund returns and, most importantly, to examine if there are asymmetric risk exposures.

Several main results arose from our empirical investigation. First, in almost all of the analysed hedge fund return series we have found evidence for asymmetric risk exposures to different risk factors. Second, we found evidence that different hedge fund indices are affected by different risk factors. This seems reasonable since fund managers follow different investment strategies. Third, we found evidence that there are different nonlinear/asymmetric risk exposures of hedge funds to different risk factors, and that these asymmetries appeared to different risk factors than the market. Ignoring the presence of these thresholds to different than the market factor is likely to result in misleading conclusions about 'alpha' and about the risk exposures of hedge funds. Finally, from an out-of-sample exercise we have conducted, we have found that our threshold regression models perform better at constructing style hedge fund portfolios than standard linear regression models.

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Table 1: Summary statistics for the analysed hedge fund returns.

			return r_t		r_t^2	$\|r_t\|$
Hedge Funds	*Mean*	*Stdev*	*Kurtosis*	$LB(12)$	$LB(12)$	$LB(12)$
EH	0.0101	0.0250	4.48	13.85	39.83	31.23
M	0.0091	0.0239	3.59	24.85	30.12	28.71
RVA	0.0060	0.0103	14.08	23.51	7.23	41.28
ED	0.0083	0.0182	7.97	27.00	7.44	10.28
MA	0.0050	0.0108	13.38	15.32	1.76	8.97

This table presents summary statistics of monthly returns for five HFR indexes. The summary statistics include mean, standard deviation (Stdev), kurtosis, and the Ljung-Box statistic computed for the return series, the absolute and the squared returns.

Table 2: Analysis of the Equity Hedge Strategy.

Factors	γ	**p**	**t**	α, β_{ij}
Constant				0.0029 (0.001)
RUS	1	1	–	0.387 (0.033)
RUS (−1)	1	1	–	0.054 (0.022)
MXUS	0	0	–	–
MEM	1	1	–	0.066 (0.015)
SMB	1	1	–	0.212 (0.031)
HML	1	2	−0.0167	−0.215 (0.027) , 0.014 (0.034)
MOM	1	1	–	0.052 (0.015)
SBGC	0	0	–	–
SBWG	0	0	–	–
LHY	0	0	–	–
DEFSPR	0	0	–	–
FRBI	0	0	–	–
GSCI	1	1	–	0.040 (0.014)
VIX	1	1	–	0.084 (0.040)
ω				$1.99 \cdot 10^{-6}$ $\left(2.6 \cdot 10^{-6}\right)$
b				0.107 (0.038)
g				0.875 (0.044)

This table presents the most probable threshold regression model for the Equity Hedge Strategy. $\boldsymbol{\gamma}$ is the vector of inclusion of each factor, **p** is the vector with the number of partitions in each factor, **t** is the vector with the threshold values in the corresponding risk factor and α, β_{ij} denote the constant and the factor loadings of the risk factors, while the standard errors are presented in parenthesis. The risk factors are: the Russel 3000 equity index excess return (RUS) and the Russel 3000 equity index excess return lagged once [RUS(-1)], the Morgan Stanley Capital International world excluding USA index excess return (MXUS), the Morgan Stanley Capital International emerging markets index excess return (MEM), Fama and French's (1993) 'size' (SMB) and 'book-to-market' (HML) as well as Carhart's (1997) 'momentum' factors (MOM), the Salomon Brothers world government and corporate bond index excess return (SBGC), the Salomon Brothers world government bond index excess return (SBWG), the Lehman high yield index excess return (LHY), the difference between the yield on the BAA-rated corporate bonds and the 10-year Treasury bonds (DEFSPR), the Federal Reserve Bank competitiveness weighted dollar-index excess return (FRBI), the Goldman Sachs commodity index excess returns (GSCI), and the change in S&P 500 implied volatility index (VIX).

Table 3: Analysis of the Macro Strategy.

Factors	γ	$\mathbf{p}$	$\mathbf{t}$	α, β_{ij}
Constant				-0.0009 (0.0022)
RUS	0	0	–	–
RUS (-1)	0	0	–	–
MXUS	1	2	0.0034	-0.1408 (0.0570) , 0.3068 (0.0573)
MEM	1	1	–	0.1608 (0.0290)
SMB	1	1	–	0.1108 (0.0345)
HML	0	0	–	–
MOM	1	1	–	0.0541 (0.0183)
SBGC	1	1	–	0.6149 (0.1011)
SBWG	0	0	–	–
LHY	0	0	–	–
DEFSPR	0	0	–	–
FRBI	1	1	–	0.2715 (0.1040)
GSCI	0	0	–	–
VIX	1	2	-0.0203	0.2091 (0.0596) , -0.0825 (0.0595)
M (-1)	1	1	–	0.1183 (0.0588)
ω				$2.0 \cdot 10^{-7}$ $\left(4.9 \cdot 10^{-6}\right)$
b				0.0339 (0.0243)
g				0.9603 (0.0353)

This table presents the most probable threshold regression model for the Macro Strategy. $\boldsymbol{\gamma}$ is the vector of inclusion of each factor, $\mathbf{p}$ is the vector with the number of partitions in each factor, $\mathbf{t}$ is the vector with the threshold values in the corresponding risk factor and α, β_{ij} denote the constant and the factor loadings of the risk factors, while the standard errors are presented in parenthesis. The risk factors are: the Russel 3000 equity index excess return (RUS) and the Russel 3000 equity index excess return lagged once [RUS(-1)], the Morgan Stanley Capital International world excluding USA index excess return (MXUS), the Morgan Stanley Capital International emerging markets index excess return (MEM), Fama and French's (1993) 'size' (SMB) and 'book-to-market' (HML) as well as Carhart's (1997) 'momentum' factors (MOM), the Salomon Brothers world government and corporate bond index excess return (SBGC), the Salomon Brothers world government bond index excess return (SBWG), the Lehman high yield index excess return (LHY), the difference between the yield on the BAA-rated corporate bonds and the 10-year Treasury bonds (DEFSPR), the Federal Reserve Bank competitiveness weighted dollar-index excess return (FRBI), the Goldman Sachs commodity index excess returns (GSCI), the change in S&P 500 implied volatility index (VIX), and M(-1) is the lagged once Macro return series.

Table 4: Analysis of the Relative Value Arbitrage Strategy.

Factors	$\boldsymbol{\gamma}$	**p**	**t**	α, β_{ij}
Constant				0.0011 (0.0006)
RUS	0	0	–	–
RUS (−1)	0	0	–	–
MXUS	1	3	−0.0302, 0.0035	−0.0807 (0.0151) , −0.1924 (0.0444) , 0.0696 (0.0178)
MEM	1	2	0.0442	0.0826 (0.0076) , 0.0192 (0.0121)
SMB	0	0	–	–
HML	0	0	–	–
MOM	0	0	–	–
SBGC	1	1	–	0.0662 (0.0231)
SBWG	0	0	–	–
LHY	1	1	–	0.0539 (0.0141)
DEFSPR	0	0	–	–
FRBI	0	0	–	–
GSCI	0	0	–	–
VIX	0	0	–	–
RVA (−1)	1	1	–	0.2053 (0.0533)
ω				$2.6 \cdot 10^{-7}$ $\left(3.6 \cdot 10^{-7}\right)$
b				0.7525 (0.0637)
g				0.2474 (0.0637)

This table presents the most probable threshold regression model for the Relative Value Arbitrage Strategy. $\boldsymbol{\gamma}$ is the vector of inclusion of each factor, **p** is the vector with the number of partitions in each factor, **t** is the vector with the threshold values in the corresponding risk factor and α, β_{ij} denote the constant and the factor loadings of the risk factors, while the standard errors are presented in parenthesis. The risk factors are: the Russel 3000 equity index excess return (RUS) and the Russel 3000 equity index excess return lagged once [RUS(-1)], the Morgan Stanley Capital International world excluding USA index excess return (MXUS), the Morgan Stanley Capital International emerging markets index excess return (MEM), Fama and French's (1993) 'size' (SMB) and 'book-to-market' (HML) as well as Carhart's (1997) 'momentum' factors (MOM), the Salomon Brothers world government and corporate bond index excess return (SBGC), the Salomon Brothers world government bond index excess return (SBWG), the Lehman high yield index excess return (LHY), the difference between the yield on the BAA-rated corporate bonds and the 10-year Treasury bonds (DEFSPR), the Federal Reserve Bank competitiveness weighted dollar-index excess return (FRBI), the Goldman Sachs commodity index excess returns (GSCI), the change in S&P 500 implied volatility index (VIX), and RVA(-1) is the lagged once RVA return series.

Table 5: Analysis of the Event Driven Strategy.

Factors	γ	**p**	**t**	α, β_{ij}
Constant				0.0067 (0.0009)
RUS	1	1	–	0.2084 (0.0266)
RUS (−1)	0	0	–	–
MXUS	0	0	–	–
MEM	1	1	–	0.0493 (0.0139)
SMB	1	1	–	0.1921 (0.0238)
HML	1	1	–	0.0722 (0.0226)
MOM	1	1	–	0.0324 (0.0135)
SBGC	0	0	–	–
SBWG	0	0	–	–
LHY	1	1	–	0.0982 (0.0294)
DEFSPR	0	0	–	–
FRBI	0	0	–	–
GSCI	0	0	–	–
VIX	1	2	−0.0203	0.0840 (0.0333) , −0.0778 (0.0316)
ED (−1)	1	1	–	0.1898 (0.0360)
σ^2				$6.83 \cdot 10^{-5}$

This table presents the most probable threshold regression model for the Event Driven Strategy. $\boldsymbol{\gamma}$ is the vector of inclusion of each factor, **p** is the vector with the number of partitions in each factor, **t** is the vector with the threshold values in the corresponding risk factor and α, β_{ij} denote the constant and the factor loadings of the risk factors, while the standard errors are presented in parenthesis. The risk factors are: the Russel 3000 equity index excess return (RUS) and the Russel 3000 equity index excess return lagged once [RUS(-1)], the Morgan Stanley Capital International world excluding USA index excess return (MXUS), the Morgan Stanley Capital International emerging markets index excess return (MEM), Fama and French's (1993) 'size' (SMB) and 'book-to-market' (HML) as well as Carhart's (1997) 'momentum' factors (MOM), the Salomon Brothers world government and corporate bond index excess return (SBGC), the Salomon Brothers world government bond index excess return (SBWG), the Lehman high yield index excess return (LHY), the difference between the yield on the BAA-rated corporate bonds and the 10-year Treasury bonds (DEFSPR), the Federal Reserve Bank competitiveness weighted dollar-index excess return (FRBI), the Goldman Sachs commodity index excess returns (GSCI), the change in S&P 500 implied volatility index (VIX), and ED(-1) is the lagged once ED return series.

Table 6: Analysis of the Merger Arbitrage Strategy.

$Factors$	$\boldsymbol{\gamma}$	$\mathbf{p}$	$\mathbf{t}$	α, β_{ij}
$Constant$				0.0066 (0.0009)
RUS	1	1	–	0.0731 (0.0205)
$RUS\,(-1)$	1	1	–	0.0642 (0.0149)
$MXUS$	0	0	–	–
MEM	1	2	0.0085	0.0856 (0.0175) , −0.0066 (0.0204)
SMB	0	0	–	–
HML	0	0	–	–
MOM	0	0	–	–
$SBGC$	0	0	–	–
$SBWG$	0	0	–	–
LHY	0	0	–	–
$DEFSPR$	0	0	–	–
$FRBI$	0	0	–	–
$GSCI$	0	0	–	–
VIX	0	0	–	–
σ^2				$6.98 \cdot 10^{-5}$

This table presents the most probable threshold regression model for the Merger Arbitrage Strategy. $\boldsymbol{\gamma}$ is the vector of inclusion of each factor, $\mathbf{p}$ is the vector with the number of partitions in each factor, $\mathbf{t}$ is the vector with the threshold values in the corresponding risk factor and α, β_{ij} denote the constant and the factor loadings of the risk factors, while the standard errors are presented in parenthesis. The risk factors are: the Russel 3000 equity index excess return (RUS) and the Russel 3000 equity index excess return lagged once [RUS(-1)], the Morgan Stanley Capital International world excluding USA index excess return (MXUS), the Morgan Stanley Capital International emerging markets index excess return (MEM), Fama and French's (1993) 'size' (SMB) and 'book-to-market' (HML) as well as Carhart's (1997) 'momentum' factors (MOM), the Salomon Brothers world government and corporate bond index excess return (SBGC), the Salomon Brothers world government bond index excess return (SBWG), the Lehman high yield index excess return (LHY), the difference between the yield on the BAA-rated corporate bonds and the 10-year Treasury bonds (DEFSPR), the Federal Reserve Bank competitiveness weighted dollar-index excess return (FRBI), the Goldman Sachs commodity index excess returns (GSCI), and the change in S&P 500 implied volatility index (VIX).

Table 7: Realised returns and risk adjusted performance measures.

	Top 8 alphas portfolios		*Top 10 alphas portfolios*		*Benchmark portfolios*
Models	*Nonlinear*	*Linear*	*Nonlinear*	*Linear*	*Benchmark*
Mean Ret.	0.0062	0.0062	0.0059	0.0056	0.0044
Cum. Ret.	0.1495	0.1495	0.1426	0.1337	0.1065
ShR	0.4596	0.4299	0.4819	0.3843	0.4250
SoR	0.8775	0.7829	0.9425	0.6819	0.8208

This table presents performance measures for style allocation portfolios comprising of the top 8 'alpha' and top 10 'alpha' ranked hedge fund strategies. Portfolios are constructed for a period of two years using nonlinear and linear models obtained for the estimation period, estimating alpha, and allocating wealth in top-performing strategies. Portfolios are also constructed using an equally weighted portfolio of all hedge funds, which is denoted as a benchmark portfolio. The monthly mean excess return (Mean Ret.), the cumulative excess 2-year return (Cum. Ret.), the Sharpe ratio (ShR) and the Sortino ratio (SoR) are reported.