

A DSGE MODEL OF INSIDERS – OUTSIDERS CAPITALISM: EMPIRICAL RESULTS FROM SEVEN EUROPEAN COUNTRIES.

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Abstract: We apply and estimate the "Insiders-Outsiders" model of Dimakopoulou et al. (2024) on seven European countries, using quarterly macroeconomic data for the period 1999-2018. The main purpose of the model is to explore and determine how non-competitive rigidities and frictions in the various industries and sectors of an economy manifest at the aggregate macroeconomic level, intertwined with government policy that is influenced by the diverging interests of the socio-economic groups of agents that operate in the economy. We find that countries differ as regards these structural aspects of their economies, but also, that the financial and public debt of 2009-2010 created a surge in these forces, which subsequently followed a downward trajectory, remaining non-negligible.

JEL classification codes: C32, E20, O41.

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I. Introduction.

The present work is the empirical companion of Dimakopoulou, Kollintzas, Papadopoulos and Vassilatos (2024) where the theoretical DSGE model of the Insiders-Outsiders Society (IOS) is developed and simulated in detail.¹ Here, we undertake the task of bringing the model to the data.

The IOS model pivots off the standard representative household growth-real business cycle macroeconomic model by including two sectors of intermediate goods production, one of which has a monopolistic structure in its product market but also in its labor market. The latter sector is the "Insiders" sector, while the perfectly competitive one is the "Outsiders" sector. The IOS model is first solved for the "Private Sector Equilibrium" (PSE) where the government does not optimize any objective function, and then for the "Political-Economic Equilibrium" (PE) where the government minimizes a weighted average of the deviations of attained utility/income from their point of bliss, of the two groups of agents in the economy, the Insiders and the Outsiders, respecting the conditions of the PSE. We will use an analogous approach in our empirical work.

From a mathematical point of view, a critical feature of the theoretical model is the presence of the "Insiders Index" $\chi_{t+1} \in [0,1]$, that formally measures the relative size of the Insiders sector as a proportion of the total intermediate goods sector of the economy. More generally, it can be thought as of an index for the relative intensity of rigidities and frictions in the economy that reduce the level and depth of economic competition. The Insiders Index is not fixed but time-variant. As detailed in Dimakopoulou et al. (2024), the essential difference between the PSE framework and the PE solution is that in the latter, the Insiders Index is set by the government through the optimization of its objective function, while in the PSE framework the Insiders Index is an exogenous state variable. The Insiders Index is present in all equilibrium relations characterizing the economy, and its steady-state level as well as its evolution through time is a prime estimation target. But the political power enjoyed by each of the two groups of agents, the Insiders and the Outsiders, as represented by the weights assigned to them in the government objective function, is also an important empirical goal.

¹ Previous incarnations of the model have been published in Kollintzas et al (2018) and Kollintzas et al. (2021)

A blessing (and maybe also curse) of the IOS model, is that despite the many elements of heterogeneity that are incorporated in its structure, it aggregates at the macro-level, and so it can be estimated by using aggregate macroeconomic data series, without the need for sectoral information.

In Section II we construct a state space model based on the expressions that characterize the private sector equilibrium of the theoretical model, and we obtain its reduced form. In Section III we present our data, comprised of quarterly samples of macroeconomic aggregates from seven European countries: France, Germany, Greece, Italy, Portugal, Spain, and the United Kingdom. In Section IV we present and discuss our empirical estimation strategy that includes the application of the Kalman Filter and of iterated Seemingly Unrelated Regression methods. In Section V we present the empirical results. In Section VI we confront the political-economic equilibrium of the model where the government optimizes an objective function, and, based on our empirical results, we obtain information per country as to the balance of political power between the Insiders and the Outsiders. The article is accompanied by a detailed Appendix that includes the related mathematical derivations.

II. From the theoretical solution to a state space model.

We construct a state-space model that has four observation equations and three unobservable state variables. From the theoretical model we take as-is the following three equilibrium expressions: the two first-order conditions from the intertemporal utility maximization of the representative household, namely the intratemporal condition that regulates the choice between labor-leisure and consumption, and the Euler equation. We also use the equilibrium aggregate final-good production function. For our fourth measurement equation, we deviate from the theoretical model as regards the government presence in the economy: the IOS model postulates a balanced government budget and the absence of transfers. Because in the real world the governments do not usually run balanced budgets, and transfers are an important part of government outlays, we formulate the related relation differently so as to allow for the existence of deficits/surpluses.

Specifically, from the intratemporal optimization condition of the Outsider household together with the equilibrium relation for the Outsider wage we get the relation

$$\frac{\mathcal{G}(1-\tau_{t+1}^L)}{(1-\mathcal{G})}(1-\alpha)\tilde{\omega}_{t+1}(1-h_{t+1}) = \frac{c_{t+1}h_{t+1}}{y_{t+1}} \quad (1)$$

where the various symbols are defined in Dimakopoulou et al. (2024). The Euler equation is

$$\frac{[c_t^\theta(1-h_t)^{1-\theta}]^{1-\gamma}}{c_t} = \frac{\beta}{(1+\eta)} E_t \left\{ \frac{[c_{t+1}^\theta(1-h_{t+1})^{1-\theta}]^{1-\gamma}}{c_{t+1}} \left[1 + (1-\tau_{t+1}^K) \left(\alpha \hat{\omega}_{t+1} \frac{y_{t+1}}{k_{t+1}} - \delta \right) \right] \right\} \quad (2)$$

The equilibrium aggregate production function is

$$y_{t+1} = \bar{\omega}_{t+1} \cdot k_{t+1}^\alpha h_{t+1}^{(1-\alpha)} \quad (3)$$

As for the government, we express the government deficit (seen as a positive number) as a linear function of the Insiders Index. Insiders can cause deficits by exploiting their economic and political power to extract rents or subsidies from the government or to direct subsidies to the Outsiders to maintain socio-political peace. But they may also cause surpluses if they have a sufficiently long time-horizon and they want to preserve the financial viability of the current economic-political structure, eyeing future gains from its exploitation. The related expression is

$$g_{t+1}^{data} = \tau_{t+1}^K \left[\alpha \hat{\omega}_{t+1} - \delta \left(\frac{k_{t+1}}{y_{t+1}} \right) \right] + (1-\alpha)\tau_{t+1}^L \tilde{\omega}_{t+1} + \hat{\psi} \chi_{t+1} \quad (4)$$

Here g_{t+1}^{data} is a GDP percentage and the first two components of the right-hand-side are the tax revenues as % GDP. These four equations are log-linearized around the steady-state (see Appendix A1-A3), and, re-arranging their order we obtain the following "measurement" structural equation, denoting the steady-state of a variable as \bar{z} and writing the log-deviations from the steady state as $\hat{z}_t \equiv \ln z_t - \ln \bar{z}$,

$$\Theta \mathbf{y}_{t+1} = \mathbf{d} + \Lambda \mathbf{y}_t + \mathbf{M} \mathbf{x}_{t+1} + \Xi \mathbf{s}_{t+1}$$

$$\begin{aligned}
& \begin{bmatrix} \theta_{11} & -1 & 0 & 1 \\ \theta_{21} & 1 & 0 & 0 \\ 0 & \theta_{32} & \theta_{33} & 0 \\ \theta_{41} & \theta_{42} & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{h}_{t+1} \\ \hat{y}_{t+1} \\ \hat{g}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} d_h \\ d_y \\ d_g \\ d_c \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \lambda_{41} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{h}_t \\ \hat{y}_t \\ \hat{g}_t \\ \hat{c}_t \end{bmatrix} \\
& + \begin{bmatrix} 0 & 0 & m_{13} \\ m_{21} & 0 & 0 \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & 0 \end{bmatrix} \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\tau}_{t+1}^K \\ \hat{\tau}_{t+1}^L \end{bmatrix} + \begin{bmatrix} 0 & \xi_{12} & \xi_{13} \\ 1 & \xi_{22} & \xi_{23} \\ 0 & \xi_{32} & \xi_{33} \\ 0 & \xi_{42} & \xi_{43} \end{bmatrix} \begin{bmatrix} \ln \tilde{A}_{t+1} \\ \ln B_{t+1} \\ \ln \chi_{t+1} \end{bmatrix}
\end{aligned} \tag{5}$$

The omega variables have been decomposed into their components during log-linearization and the system above includes the following unobservable states: the composite Total Factor Productivity process (TFP) $\tilde{A}_{t+1} \equiv A_{t+1} B_{t+1}^o$, i.e. the product of the TFP process in the Outsiders sector B_{t+1}^o times the TFP process in the final-goods production function A_{t+1} , the composite TFP ratio $B_{t+1} \equiv B_{t+1}^i / B_{t+1}^o$ where B_{t+1}^i is the TFP process in the Insiders sector, and the Insiders index χ_{t+1} . The structural coefficients of the matrices above are functions of the parameters of the theoretical model and of the steady states of the variables involved (see the Appendix A1-A3 for the detailed list and correspondence).

The matrix Θ is invertible, so we can obtain the reduced-form equation by inverting it (Appendix A4),

$$\begin{aligned}
& \mathbf{y}_{t+1} = \mathbf{c} + \Gamma \mathbf{y}_t + \Psi \mathbf{x}_{t+1} + \Pi \mathbf{s}_{t+1} + \boldsymbol{\varepsilon}_{t+1} \\
& \mathbf{c} = \Theta^{-1} \mathbf{d}, \quad \Gamma = \Theta^{-1} \Lambda, \quad \Psi = \Theta^{-1} \mathbf{M}, \quad \Pi = \Theta^{-1} \Xi \\
& \begin{bmatrix} \hat{h}_{t+1} \\ \hat{y}_{t+1} \\ \hat{g}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} c_h \\ c_y \\ c_g \\ c_c \end{bmatrix} + \begin{bmatrix} \gamma_{11} & 0 & 0 & \gamma_{14} \\ \gamma_{21} & 0 & 0 & \gamma_{24} \\ \gamma_{31} & 0 & 0 & \gamma_{34} \\ \gamma_{41} & 0 & 0 & \gamma_{44} \end{bmatrix} \begin{bmatrix} \hat{h}_t \\ \hat{y}_t \\ \hat{g}_t \\ \hat{c}_t \end{bmatrix} \\
& + \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ \Psi_{21} & \Psi_{22} & \Psi_{23} \\ \Psi_{31} & \Psi_{32} & \Psi_{33} \\ \Psi_{41} & \Psi_{42} & \Psi_{43} \end{bmatrix} \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\tau}_{t+1}^K \\ \hat{\tau}_{t+1}^L \end{bmatrix} + \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \\ \pi_{41} & \pi_{42} & \pi_{43} \end{bmatrix} \begin{bmatrix} \ln \tilde{A}_{t+1} \\ \ln B_{t+1} \\ \ln \chi_{t+1} \end{bmatrix} + \begin{bmatrix} \mathcal{E}_{t+1}^h \\ \mathcal{E}_{t+1}^y \\ \mathcal{E}_{t+1}^\tau \\ \mathcal{E}_{t+1}^c \end{bmatrix}
\end{aligned} \tag{6}$$

This is the reduced form of the measurement equation, where we have added stochastic errors in each equation to deal with the stochastic singularity issue.²

For the PSE estimation, all three state variables are assumed to follow stable AR(1) processes, since as already mentioned, we do not yet introduce an optimizing government. Then the "transition" equation of our state-space model is

$$\begin{bmatrix} \ln \tilde{A}_{t+2} \\ \ln B_{t+2} \\ \ln \chi_{t+2} \end{bmatrix} = \begin{bmatrix} d_A \\ d_B \\ d_\chi \end{bmatrix} + \begin{bmatrix} \rho^A & 0 & 0 \\ 0 & \rho^B & 0 \\ 0 & 0 & \rho^\chi \end{bmatrix} \begin{bmatrix} \ln \tilde{A}_{t+1} \\ \ln B_{t+1} \\ \ln \chi_{t+1} \end{bmatrix} + \begin{bmatrix} u_{t+1}^A \\ u_{t+1}^B \\ u_{t+1}^\chi \end{bmatrix} \quad (7)$$

Equations (6) and (7) form the state space empirical model.

III. Data

We have used the OECD data base to construct data samples for seven European countries: France, Germany, Greece, Italy, Portugal, Spain, and the United Kingdom. The data are quarterly and seasonally adjusted, and its real value is based on year 2010. All seven samples end at the 2nd quarter of 2018. The sample for Germany starts at the 1st quarter of 2002, while for the other six countries it starts three years earlier, at the 1st quarter of 1999. The variables we use in estimation are presented in Table 1.

² On the issue of stochastic singularity and on how it can be meaningfully solved beyond assuming the existence of measurement errors, see Sargent (1989), Ingram et al (1994), Ireland (2004).

Table 1. Variables comprising the data sets.

Variable	Definition (quarterly frequency)
h	Proportion of time working
y	Real GDP per efficiency-augmented hour available to allocate between work and leisure
g	Government expenses (consumption, investment, transfers) as % GDP
c	Real consumption (including net exports) per efficiency-augmented hour available to allocate between work and leisure
k	Real capital stock per efficiency-augmented hour available to allocate between work and leisure
τ^K	Effective capital tax rate
τ^L	Effective labor tax rate

We performed data imputation for the values of the capital and of the labor effective tax rate for Greece, for the two quarters of the year 2018, namely for four data points in total.

Since the model includes a leisure-labor choice for the individuals, and data is of quarterly frequency, the concept of "per capita" translates into "available hours to allocate between labor and leisure in a quarter". We set daily available such hours at 14, and the average days of a quarter at 91.

The proportion of time working was computed then as

$$h_{t+1} = \frac{\text{Aggregate hours worked per } Q_{t+1}}{\text{Employed heads per } Q_{t+1} \times 91 \times 14}.$$

The aggregate capital series per country was constructed as follows: we used the pre-sample period 1995-1998 to determine the initial value of capital at the beginning of 1999-Q1, and we applied the perpetual inventory method using the Investment series over the sample period. In this way we determined the average constant depreciation rate δ , per country.

We included in aggregate consumption also the aggregate net exports. Also, the rate of technical progress per country η was computed so as to remove trends from the output series.

We transformed aggregate output, consumption and capital by

$$x_{t+1} = \frac{X_{t+1}}{(1+\eta)^{t+1} \times \text{Employed heads per } Q_{t+1} \times 91 \times 14}, \quad x = y, c, k$$

and where the uppercase variable denotes the aggregate magnitude. In this way we obtain our variables "per efficiency-augmented hour available to allocate between work and leisure". In Table 2 we provide the empirical mean values for these series per country.

Table 2. Empirical means of the data series.

Variable	France	Germany	Greece	Italy	Portugal	Spain	UK
h	0.299	0.2735	0.4067	0.3499	0.3697	0.3378	0.3272
y	13.4	11.91	9.2940	12.7900	6.502	10.0500	11.2800
$g = G/Y$	0.5117	0.4511	0.4357	0.4219	0.3984	0.3806	0.3674
c	7.188	7.3760	5.6000	7.7500	3.822	5.6160	7.0490
k	120.8	148.2	97.78	156.0	80.27	126.3	94.8900
τ^K	0.3221	0.2584	0.1766	0.2462	0.2092	0.2106	0.2769
τ^L	0.4410	0.4232	0.3908	0.3922	0.2721	0.3507	0.2951
C/Y	0.5364	0.6193	0.6025	0.6059	0.5878	0.5588	0.6249
K/Y yearly	2.25	3.11	2.63	3.04	3.08	3.14	2.10

The two last lines show the mean value of consumption as a %GDP evaluated at the empirical means, and also the capital/output ratio (on a yearly basis). These variables are not used in estimation, but they are useful in comparing and contrasting the various countries.

Our empirical model is in log-deviations around the steady-state. It should then be the case, that the data can be seen as representing such a situation. In Table 3 we contrast the empirical time-series mean of our consumption data series against the steady-state value computed through

$$\bar{c} = \bar{y} - (\delta + \eta) \bar{k} - \bar{y} \cdot \bar{g}_{C+I}$$

Note that here \bar{g}_{c+I} is a percentage and includes only government consumption and investment, but not transfers, in order to avoid double-counting. The above relation does not include any estimated parameter, since both (δ, η) were calibrated in conjunction and during the construction of the data series as explained previously.

Table 3. Empirical mean of consumption vs. computed steady-state.

Country	Empirical mean of consumption	Computed SS value	% deviation of empirical mean from SS value
France	7.188	7.2747	−1.2%
Germany	7.376	7.1670	2.9%
Greece	5.600	5.5037	1.7%
Italy	7.750	7.8792	−1.6%
Portugal	3.822	3.7850	1.0%
Spain	5.616	6.06	−7.3%
UK	7.049	7.2505	−2.8%

We see that the two values differ little per country, with the exception perhaps of Spain. Still, being 7% away from the steady state is not a large deviation. So overall we can legitimately use the empirical time-series means as an estimate of the steady state,

$$\bar{z} = \frac{1}{n} \sum_{t=1}^n z_t, \quad \bar{z} = h, y, g, c, k, \tau^K, \tau^L.$$

Therefore, the actual data series used is the logarithms of the variables with the logarithm of their time-series mean subtracted.

IV. Estimation strategy

The model includes unobservable state variables. To obtain predicted time series for these unobservables, we will apply a linear Kalman Filter with time-invariant coefficient

matrices. Moreover, because our purpose is to reveal and interpret rather than forecast, we will use the series produced by the "smoothed" Kalman Filter, namely the series that are predicted by using all the sample information (i.e. by using "hindsight").³

IV.1 Likelihood methods

Our initial plan was to use likelihood methods for the estimation. In such a context the Kalman Filter, having been fed with starting/proposed values for the various parameters, completes the sample by providing predicted series for the unobservable states and computes the value of the likelihood, which is then used by the estimator for the next iteration. But here, the "curse" mentioned earlier most likely materialized: to begin with, all attempts at maximum likelihood estimation on all seven samples failed, even when we reduced the burden for the estimator by fixing for example the variance-covariance parameters. We were not surprised, because our sample is small (66 observations for Germany, 78 observations for the other six countries). The ML estimator failed consistently to converge, and when it thought it did, it produced either very large standard errors, or none at all. This is an indication of a "flat likelihood surface", and this raises the issue as to whether the parameters and the variables that we seek to estimate provide a strong enough "signal" inside the macroeconomic aggregate data series in order to be estimated reliably through likelihood methods.

Next we turned to Markov Chain Monte Carlo techniques, whose use has spread in macroeconometric studies during the last years.⁴ The method still uses the Kalman Filter for the purposes described above, but the next candidate set of parameter values is not decided through some maximum-seeking algorithm but using random draws and adding them to the previous accepted value. We used specifically the Metropolis-Hastings Random Walk (MHRW) algorithm. MCMC techniques are developed in a Bayesian framework, and what they provide is a posterior distribution for every parameter under estimation. They do not suffer from convergence failure in the same way as maximum likelihood estimation does. But the critical question is whether they have indeed converged to the *stationary* posterior distribution, given the prior distribution and the likelihood. The MCMC chain can run as long as we want. In our case, by the various diagnostics available in the literature, even for a run of 500,000 steps, the procedure had not converged to a stationary distribution. This again reflects

³ Harvey (1989), Durbin & Koopman (2012).

⁴ Fernández-Villaverde and Rubio-Ramírez (2007) Fernández-Villaverde (2010), Fernández-Villaverde et al (2010), Herbst and Schorfheide (2016).

the difficulty of extracting from macroeconomic data series the information and the estimates we are after. While we are still working on improving our MHRW procedure, we turned next to an iterative estimation procedure that uses non-linearity in order to increase identification power.

IV.2 Kalman Filter and Seemingly Unrelated Regression (SUR) system

In this estimation approach, the steps are as follows (per country):

1. By an initial explorative search, we determine a set of values for the various parameters, and we run the Kalman Filter on our state space model in order to obtain predicted series for the unobservable states.

2. We assess further the quality of the assumed values by predicting the *levels* of the four dependent variables, feeding the postulated parameter values, the data, and the predicted series for the states, into the original equilibrium expressions of the model (in their form before log-linearization), specifically into the aggregate production function and the intratemporal condition. In this way we obtain in-sample predicted series for y , c , h . But note that we may obtain good predictions by more than one set of assumed values for the parameters. This is an instance where the flexibility of the Kalman Filter to converge and provide predicted time series for different sets of coefficient values works against us, and requires more of us.

3. Once we have found a set of parameter values and predicted states that lead to good predictions for these variables, we form a Seemingly Unrelated Regression (SUR) system of equations, which is our reduced form measurement equation (6) with the predicted state series included as regressors, and we estimate applying iterated SUR estimation. This will give us estimates for the reduced-form coefficients.

We show in the Appendix (A5) that the elements of the matrix Θ that we have inverted in order to go from the structural form to the reduced form, can be obtained from the reduced form coefficients. Then we can obtain the full set of the structural coefficients of system (5) through pre-multiplication by Θ of the reduced form matrices of coefficients. In turn, the structural coefficients are non-linear functions of the unknown parameters that we want to estimate, and so by this reverse procedure we can obtain estimates for them (Appendix A6). But here, in many cases we will obtain nonsense results, e.g. estimates outside the feasible

range of a variable. In this way we reject proposed sets of parameter values and we move to the next candidate, until we get a set of parameter values that satisfy all the feasibility constraints and performs best in in-sample prediction terms.

One final note as regards the enhanced SUR system of equations: the theoretical reduced-form measurement equation (6) includes two columns of zeros in the coefficient matrix attached to the lags of the dependent variable (matrix Γ). This points towards estimating a restricted SUR system, by imposing these zero restrictions. But we decided to use the unrestricted model, in order to capture real-world effects that are not part of the theoretical model, like a lag in the collection of taxes (that would make the current effective tax rates to be partly dependent on output of the previous period), or anticipation effects that current government transfers may have on decisions regarding next period employment and consumption (making c_{t+1}, h_{t+1} dependent also on g_t). Moreover we run formal statistical tests for the restrictions and in all cases they were rejected.

V. Calibration and empirical results

V.1.1 Parameters calibrated in conjunction with the data set

We present in Table 4 the values of the two parameters that were calibrated in conjunction with the construction of the data sets, the labor-augmenting technical progress parameter and the capital depreciation rate.

Table 4. Technical progress and depreciation rates per country.

Country	Labor-augmenting technical progress η		Depreciation rate of Capital δ	
	quarterly	yearly	quarterly	yearly
France	0.00185	0.0074	0.0188	0.0731
Germany	0.00131	0.0053	0.01385	0.0543
Greece	0.000	0.000	0.0152	0.0594
Italy	0.000	0.000	0.0136	0.0553
Portugal	0.0018	0.0072	0.0135	0.0529
Spain	0.001524	0.0061	0.0126	0.0496
UK	0.0022	0.009	0.0146	0.057

We note the low level of technical progress, that ranges from zero to just shy of 1% yearly.

V.1.2 Common calibrated parameters for all countries

The IOS model uses the following functional forms for household utility and for the final-good production function:

$$u_t = \frac{[c_t^\theta (1-h_t)^{1-\theta}]^{1-\gamma}}{(1-\gamma)}, \quad (8)$$

$$y_t = A_t \left[\phi_1 \chi_t^{\phi_2} (Y_t^i)^\phi + (1-\phi_1)(1-\chi_t)^{\phi_2} (Y_t^o)^\phi \right]^{(1/\phi)}, \quad \phi < 1 \quad (9)$$

Here Y_t^i and Y_t^o are the aggregated outputs of the industries in the Insiders and the Outsiders sectors respectively.

Following the setup of the simulations in Dimakopoulou et al. (2024) we fix the three parameters shown in Table 5.

Table 5. Calibrated parameters common for all countries.

Parameter	Description	Value
γ	Reciprocal of the intertemporal elasticity of substitution of the representative household	2.00
ϕ_1	Final-good production function parameter	0.50
ϕ_2	Final-good production function parameter	0.00

We also fixed at a common value the autocorrelation coefficients of the \tilde{A}_{t+1} and χ_{t+1} processes. We used the value 0.975, which, since we work with quarterly data, corresponds to a yearly autocorrelation coefficient of ≈ 0.9 .

V.1.3 Variance-covariance matrices of the state-space model

The variance covariance matrices of the state model were computed and fixed as follows:

The variance-covariance matrix of the stochastic errors in the measurement equation was determined per country by running a SUR system on the measurement equation only with the observables, and obtaining its VCV matrix.

The variance covariance matrix of the stochastic errors in the transition equation, used calibrated values from Kollintzas et al (2021), but transformed them into quarterly values (see Appendix A7).

V.1.4 Empirical results for the Private Sector Equilibrium

Sanity checks. Before presenting our main empirical results, we perform the following two sanity checks: first we contrast the steady-state value of capital that is also based on estimates, with the empirical mean of the capital series which represented the steady-state in the model. Second we compute the pure discount factor of the representative household, based on the estimates.

Table 6. Capital values and discount factors per country.

Country	SS value of capital, based on estimates	Empirical mean of capital	% deviation of estimated SS from empirical mean	Implied value of household's pure discount factor (yearly)
France	116.93	120.8	-3.2%	0.9266
Germany	139.24	148.2	-6.0%	0.9797
Greece	94.607	97.78	-3.2%	0.9225
Italy	144.91	156.0	-7.1%	0.9786
Portugal	81.61	80.09	1.9%	0.9512
Spain	119.46	126.30	-5.4%	0.9520
UK	89.592	94.89	-5.6%	0.9511

The differences in the two values for the steady state of capital are not large. Also the (yearly) discount factors are realistic. So our estimation procedures pass these sanity checks.

We next present the estimates of the more important parameters of the theoretical model. These are presented in Table 7.

Table 7. Parameters under estimation.

Parameter	Description and comments
ϑ	Utility elasticity with respect to consumption. It regulates the choice between leisure-and labor.
α	Capital elasticity of output in the intermediate sectors.
θ	The term $1/(1-\theta)$ is the aggregation elasticity of substitution between the outputs of intermediate industries when used as aggregate input in the production of the final good . From an economic point of view, in equilibrium, $\theta = \text{APK}^o / \text{APK}^i$ i.e. it is the ratio of average capital products.
ξ	A composite parameter that is a function of α, θ and of the labor union parameters: of λ , the relative intensity of wage premium preferences in union objective function, and of μ , the relative bargaining power of Insiders' unions. From an economic point of view, in equilibrium $\xi = \text{APL}^o / \text{APL}^i$, namely, it is the ratio of average labor products of the Insiders over the Outsiders sector.
ϕ	The term $1/(1-\phi)$ is the elasticity of substitution across sectors in final good production.
$\hat{\psi}$	The rate of direct increase in government expenses as the Insiders Index changes. It multiplies χ_t , see equation (4).

In Table 8 we present the estimates of the parameters, per country.

Table 8. Estimated Parameters per country.

Parameter	France	Germany	Greece	Italy	Portugal	Spain	UK
ϑ	0.4683	0.45	0.466	0.3768	0.478	0.44	0.41
α	0.4712	0.3203	0.4297	0.269	0.443	0.4	0.32
θ	0.9399	0.8081	0.9553	0.8673	0.799	0.95	0.91
ξ	0.8427	0.9532	0.8882	0.7118	0.8402	0.8293	0.7965
ϕ	0.700	0.585	0.118	0.5535	0.500	0.4	0.3
$\hat{\psi}$	0.2900	0.220	0.34	0.18	0.3600	0.27	0.375

Some comments on the above estimates are in order. we note that the value of ϑ is in general estimated higher than what is the usual calibration value seen in the literature (which is around 1/3). A higher ϑ increases the relative utility valuation of consumption vis-a-vis

leisure and hence it promotes more work. This can easily be seen in equation (1) that clearly shows that the proportion of time working is an increasing function of ϑ .

We also note that the parameter ϕ was estimated in all cases as positive. If we look at the final-good production function (9), a positive value for ϕ implies that the "inputs" of the function are not "essential". In a standard microeconomic production function setting where the inputs would be capital and labor, a positive ϕ would imply that we could get positive output using capital only or labor only, something that does not appear realistic. But in our case, the "inputs" are the aggregated outputs of the two intermediate goods sectors, which have no relation to specific goods and services that are needed for the production of final goods. What they differ in, is institutional, structural aspects that do not constrain what it can be produced under them.

What therefore the estimated positive ϕ reveals here is that it is possible to have an economy where all industries are Insiders industries, having a non-competitive structure, or another economy where all industries are competitive. This accords with experience and logic.

Next, as mentioned in Table 7 in equilibrium we have the correspondence

$$\theta = \frac{APK^o}{APK^i} \quad \xi = \frac{APL^o}{APL^i} \quad \frac{\xi}{\theta} = \frac{(K/L)^o}{(K/L)^i} \quad (10)$$

Moreover, given the expression for ξ ,

$$\xi = \frac{(1-\alpha)\theta + (\mu/(1-\mu))\{(1-\alpha\theta) - \lambda[(1-\alpha\theta) + (1-\theta)]\}}{(1-\alpha)[1 + (1-\lambda)(\mu/(1-\mu))]} \quad (11)$$

one can show that the following relation holds:

$$\xi \geq \theta \Rightarrow \lambda \leq \frac{1}{2} \quad (12)$$

Namely, when $\xi > \theta$ we learn that the relative weight λ of the wage premium in the worker union preference function is smaller than 1/2, so the preference is relatively more in favor of maintaining employment.

Combined with the previous relation of the capital-labor ratio, this makes perfect sense: if the union preferences are more in favor of maintaining employment ($\xi > \theta$), we also have that the capital-labor ratio in the Insiders sector will be *smaller* than the capital labor ratio in the Outsiders sector: the level of the (preferred over the wage) employment in the Insiders sector will be higher relative to capital, compared to what happens in the Outsiders sector.

One could expect that if the Insiders Sector of the economy includes relatively more publicly controlled industries, the relative preference would be in favor of the wage premium, while if it includes mainly industries for the private sector (through many and strong labor unions) preferences could be more in favor of maintaining employment.

By using the relation for ξ and the fact that all five parameters involved are by design constrained to range in $(0, 1)$, we can obtain some information on the possible ranges for λ and μ given the estimates for ξ , θ , α . These are presented in Table 9.

Table 9. Relative capital-labor ratios and ranges for union preferences and bargaining power.

Country	$\frac{\xi}{\theta} = \frac{(K/L)^o}{(K/L)^i}$	Feasible range of λ (relative preference for wage premia over employment)	Feasible range of μ (relative bargaining power of unions)
France	0.8965	[0.66, 1]	[0.46, 0.97]
Germany	1.1796	[0, 0.32]	[0.51, 1.0]
Greece	0.9297	[0.66, 1]	[0.46, 0.97]
Italy	0.8207	[0.66, 1]	[0.46, 0.97]
Portugal	1.0516	[0, 0.47]	[0.11, 1.0]
Spain	0.8729	[0.72, 1]	[0.59, 0.97]
UK	0.8752	[0.66, 1]	[0.46, 0.97]

We turn now to the state variables. We have already mentioned that the autocorrelation coefficients for \tilde{A}_{t+1} and χ_{t+1} have been calibrated. More over, the steady-state of the process

\tilde{A}_{t+1} is a scaling factor that closes numerically the model, its magnitude is not especially meaningful. What matters mostly in Table 9 is the autocorrelation coefficient ρ^B of the other composite TFP process $B_{t+1} \equiv B_{t+1}^i / B_{t+1}^o$, and of course the steady-state level of the Insiders Index $\bar{\chi}$.

Table 9. Autocorrelation and steady-states of state variables.

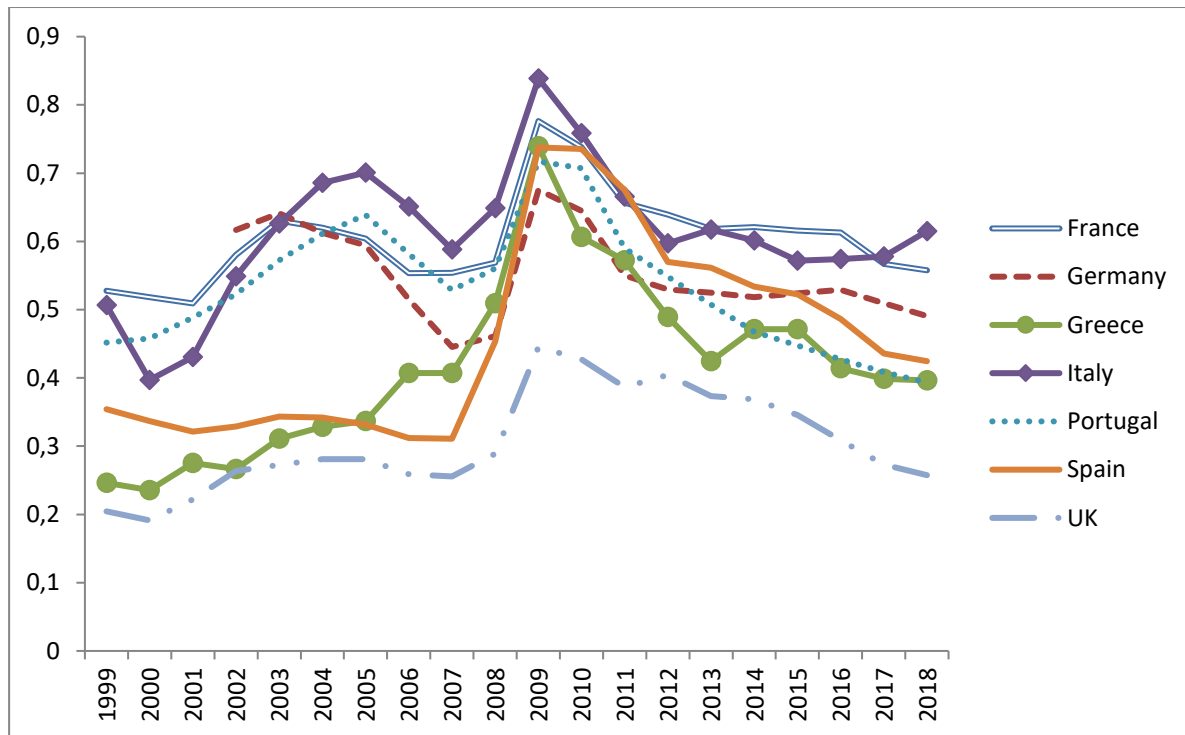
Variable	France	Germany	Greece	Italy	Portugal	Spain	UK
ρ^A	0.975	0.975	0.975	0.975	0.975	0.975	0.975
$\bar{A} = \overline{A \cdot B^o}$	6.16	14.60	9.42	14.65	4.10	6.45	12.21
ρ^B	0.2	0.5	0.15	0.5	0.1	0.05	0.3
$\overline{B^{i/o}} = \overline{B^i / B^o}$	0.8	1.0	1.0	1.0	0.9	0.8	1.0
ρ^χ	0.975	0.975	0.975	0.975	0.975	0.975	0.975
$\bar{\chi}$	0.6	0.55	0.4	0.6	0.525	0.44	0.3

For B_{t+1} we see that the autocorrelation is estimated as very low to almost non-existent, which was anticipated up to a point, because we have a ratio of two stochastic TFP processes, and ratios tend to magnify differences and break linear autocorrelations. But we did not expect to obtain such low values.

Finally, for the steady-state of the Insiders Index $\bar{\chi}$, we see that it varies per country and it is certainly not small in value. It reveals the degree to which socio-political forces and/or priorities move the economy away from its efficient best. Performing a cost-benefit analysis on such deviations is beyond the scope of this study.

We close the presentation of our main empirical results by providing a single Figure where we graph the Insiders Index processes of the seven European countries. We graph them together on purpose, in order to highlight their common trend despite their difference in level.

Figure 1. Estimated evolution of the Insiders Index process (yearly averages)



It is evident from the graph that in all cases countries reacted to the financial and public debt crisis of 2008-2010 with a move away from a competitive environment (a sudden increase in the Insiders Index). Whether this indicates a failure of the market economy (that needed to be corrected by government fiat or by the economic agents themselves through corporatist agreements), or a failure to trust the market economy, is a subject for ever-evolving research.

VI. The political-economic equilibrium: ranking countries according to the political power of Insiders.

The IOS model is completed with its political-economic equilibrium, where now, the government sets the level of the Insiders Index by optimizing over a weighted loss function, through which the government takes into account the distance of the welfare of Insiders and of Outsiders from their respective "bliss" points. This makes more clear the more general interpretation of the Insiders index as an intensity measure of the rigidities and interventions in the competitive functioning of the economy.

Important in this setting is how much weight is given to the Insiders welfare losses, denoted by $\rho \in (0,1)$, and how much to the Outsiders welfare losses. This succinctly characterizes the balance of political power in the macro-economy, beyond the rents that the Insiders manage to extract through the power that they enjoy at the sectoral level.

The asymptotic steady-state of the Political-Economic equilibrium is characterized by a series of equations that can be found in the Mathematical Appendix, Section V of the companion paper Dimakopoulou, et al (2024) (eq. A.22 through A.29). These relations incorporate and respect the conditions of PSE. Moreover, they include an "elasticity of substitution",⁵ denoted ε^ρ that measures the following: if the Insiders welfare relative to their bliss point increases, say, by 5%, how much (in percentage terms) must the corresponding measure for the Outsiders decrease in order to keep the overall value of the government objective function constant. Reversely, and rather more intuitively, if the welfare measure of the Insiders decreases how much should the welfare of Outsiders increase to keep the objective value constant. So ε^ρ is a measure of how much the government "cares" about welfare losses of the Insiders: if ε^ρ is "small", say smaller than unity, it cares relatively little since in the face of some welfare loss of the Insiders it must increase the welfare of the Outsiders by less in proportional terms. More over, ε^ρ is a monotonic increasing function of ρ , the actual weight given to Insiders welfare in the government's objective function.

We can compute ε^ρ using the obtained estimates from the previous estimation stage. They are given per country in Table 10, where we repeat also the steady-state of the Insiders Index, for convenience.

Table 10. Implicit weight given to Insiders welfare by the government.

Variable	France	Germany	Greece	Italy	Portugal	Spain	UK
ε^ρ	0.224	0.020	0.522	0.061	0.095	0.242	0.021
$\bar{\chi}$	0.6	0.55	0.4	0.6	0.525	0.44	0.3

⁵ The term "substitution" here is used in its general everyday meaning, not in the exact sense with which it is used in economic analysis.

There is not a strong co-variation between the two measures. If anything, Pearson's correlation coefficient is low and negative (-0.2), indicating that higher values of $\bar{\chi}$ tend to be associated with *lower* values of ε^p , but rather weakly.

In order to further understand the relation between the overall distance of the economy from its efficient best (as measured by $\bar{\chi}$) with the pure political balance of power in each economy (as indicated by ε^p), one should embark on a focused detailed and extensive country-per-country study to determine the historical, political and social idiosyncratic elements in each one that could affect the latter more strongly than the economic activity per se. This is beyond the scope of the present study, but certainly an interesting topic for future research.

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