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Alternative Monetary Policy Rules in an Imperfectly Competitive Dynamic Stochastic General Equilibrium Model

by

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Alternative Monetary Policy Rules in an Imperfectly Competitive Dynamic Stochastic General Equilibrium Model

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Abstract

In this paper we study optimal central bank interest rate policy, and compare it to interest rate rules, such as the Wicksell (1898), Fisher (1919) and Taylor (1993) rules, in an imperfectly competitive DSGE model of aggregate fluctuations. We demonstrate that in versions of the model with full price and wage adjustment, or staggered pricing, the optimal policy rule is the Fisher rule of absolute inflation stabilization. We also analyze a version of the model with exogenous inflation shocks, in which the "divine coincidence" does not apply. In this case, the optimal monetary policy rule takes the form of a Taylor rule, the parameters of which depend on the structural and policy parameters of the model.

Keywords: monetary policy, aggregate fluctuations, imperfect competition, Taylor rule.
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1 Introduction

The realization of the instability of the original Phillips curve it the late 1960s has led to a paradigm shift in the macroeconomics of aggregate fluctuations. The so called "neoclassical synthesis" was abandoned, in favor of dynamic stochastic general equilibrium models (DSGE) of aggregate fluctuations.

In one class of such models, all markets are assumed competitive, and wages and prices are assumed perfectly flexible to equilibrate both product and labor markets. In such models, only real shocks, such as shocks to productivity, can affect the fluctuations of output, employment and other real variables. Monetary shocks only affect nominal variables, such as the price level and inflation, and there are no effects from inflation to real output, even temporarily. The classical neutrality holds, and output and employment fluctuations are based on productivity shocks and inter temporal substitution in labor supply, as nominal shocks or monetary policy cannot affect fluctuations in real variables.

There is an alternative class of imperfectly competitive DSGE models in which firms cannot fully adjust prices and/or wages in every period. Thus, prices and/or wages are adjusted partially and in a staggered fashion. Monetary shocks and, therefore, monetary policy can affect the fluctuations of both nominal and real variables in such models, and there is a temporary positive relation between inflation and deviations of output from its "natural" rate.

In this paper we use an imperfectly competitive dynamic stochastic general equilibrium model of aggregate fluctuations, in order to study optimal central bank interest rate policy, and compare it to interest rate rules that have been proposed by monetary economists, such as the Wicksell (1898), Fisher (1919) and Taylor (1993). The model is a dynamic stochastic general equilibrium model based on monopolistic competition in product markets. We initially analyze it assuming full flexibility of prices and, subsequently, assuming staggered price adjustment. We abstract from distortions in the labor market.

The imperfectly competitive staggered pricing model has two important differences from the typical perfectly competitive model with perfectly flexible wages and prices.

First, because of the distortions of imperfect competition, the "natural" rate of employment, real output, consumption and real wages is determined at a lower level than in the corresponding competitive model, even when there is complete flexibility in prices and wages. However, by itself this difference does not result in major differences from the competitive model regarding the nature of macroeconomic fluctuations.

Second, if it is assumed that firms face price adjustment costs, the imperfectly competitive model can account for a short run "Phillips" type relation[1].

In the case of staggered pricing, it turns out that deviations of output from its "natural" level cause deviations of inflation from expected future inflation, as higher aggregate demand and output cause an increase in current nominal marginal costs and hence prices for firms able to adjust prices in the current period. This results in a positive short relation between output and inflation, i.e. a type of Phillips curve, usually referred to as the "new" Phillips curve.

In the model with full flexibility of wages and prices, fluctuations of all real variables are due to fluctuations of their "natural" rates, and monetary policy affects only nominal variables such as inflation and nominal interest rates. The optimal interest rate rule is the one which minimizes deviations of inflation from the central bank target, i.e. the Fisher rule of absolute inflation targeting.

In the model with staggered pricing there is a "Phillips curve" type relation between inflation and deviations of output from its "natural" level. Thus, nominal shocks and monetary policy have real effects on real output and employment, in addition to the effects of real shocks. However, even in this model, because there is a "divine coincidence" between inflation and output stabilization, the Fisher rule again turns out to be the optimal rule[2].

We finally analyze a version of the model in which the "divine coincidence" does not hold, in the sense that, in addition to the usual productivity and monetary policy shocks, there is an exogenous shock in the inflation process. In this more general version of the model, the Taylor rule turns out to be the optimal policy.

The parameters of the optimal Taylor rule depend on the slope of the Phillips curve, itself a function of the deeper structural parameters of the model, the responsiveness of aggregate demand to the real interest rate, equal to the elasticity of inter-temporal substitution in consumption, and the preferences of the central bank over their two objectives of stabilizing deviations of inflation from target and

[1] Two observationally equivalent versions of gradual price adjustment have dominated the literature. The one is the Rotemberg (1982 a,b) model of monopolistic price adjustment, and the second is the Calvo (1983) model of staggered pricing. In the Rotemberg model, firms balance the costs of adjusting prices against the costs of deviating from the profit maximizing optimal price. They end up gradually adjusting prices, so as to gradually approach the optimal price. In the Calvo model, it is assumed that only a fixed proportion of firms have the freedom to adjust prices in any given period. This results in the remaining firms not being able to adjust prices. Although optimal pricing takes this restriction into account in advance, the aggregate price level adjusts only gradually. These two alternative assumptions lead to models with price level stickiness, that result in observationally equivalent short run "Phillips" type relations.

[2] We use the terms "natural rate" or "natural level", in the sense of Wicksell (1898) and Friedman (1968). These are the values of real variables that would arise when prices, wages and interest rates fully adjust to equilibrate product, labor and asset markets. The "divine" coincidence arising in such models has been noted by, among others, Blanchard and Gali (2007).

deviations of output from its "natural" level.

The imperfectly competitive DSGE model with staggered pricing analyzed in this paper has the following structure:[3]

Deviations of inflation from the target of the central bank are determined by a dynamic "new" *Phillips curve*, and depend of expected future inflation deviations, and deviations of real output from its "natural" level, as the latter cause an increase in nominal marginal costs and hence prices.

The deviations of aggregate demand from the "natural" level of real output depend on the dynamic "new" *IS curve*, and, they depend on expected future deviations of aggregate demand and deviations of the current real interest rate from its "natural" level.

The nominal interest rate is determined by the central bank, which follows an interest rate rule. According to the interest rate rule, the nominal interest rate reacts positively to deviations of current inflation from the central bank target, and, possibly, on deviations of real output from its "natural" level.

After presenting the properties of this model, we analyze the effects of monetary and real shocks on the fluctuations in real output and the price level (inflation) and compare the properties of alternative interest rate rules to optimal monetary policy.

2 An Imperfectly Competitive Dynamic Stochastic General Equilibrium Model

In this section we examine in detail the structure of an imperfectly competitive dynamic stochastic general equilibrium (DSGE) model. The basic model that we analyze has two important differences from a dynamic stochastic competitive model.

First, instead of perfectly competitive markets for goods and services we assume that markets are characterized by conditions of imperfect (monopolistic) competition. Firms do not take prices as given, but have the power to determine prices that maximize profits. Because of imperfect competition, in an equilibrium with flexible prices, employment, real output, consumption and real wages are determined at a lower level than in the corresponding competitive model. However, by itself this difference does not result in material differences from the competitive model with respect to the nature of aggregate fluctuations.

[3]See Gali (2008, 2011) for an analysis of this class of models by numerical simulation methods. Smets and Wouters (2003) have estimated generalized versions of this class of models and also simulated the effects of monetary policy.The analysis of the present paper sticks to a version of the model that can be solved analytically, in order to concentrate on the analytics of the transmission mechanisms of monetary policy in this class of models.

Second, we assume that there is staggered price adjustment, i.e. that firms do not have the freedom to change their prices in every period. This assumption is what accounts for the non-neutrality of monetary shocks and monetary policy, as it leads to a model in which the price level adjusts gradually towards the equilibrium price level. As a result of gradual price adjustment, real variables deviate from their "natural" rates, and monetary shocks can have real effects.

2.1 The Representative Household

The problem of the representative household under monopolistic competition has one difference from the corresponding problem under perfect competition. The difference is that because of monopolistic competition, the household consumes differentiated products.

The representative household maximizes,

$$E_0 \sum_{t=0}^{\infty} (\frac{1}{1+\rho})^t u(C_t, N_t) \tag{1}$$

where C_t is consumption, N_t is labor supply, and ρ is the pure rate of time preference. Consumption consists of all produced goods, which are defined on the basis of a constant index j in the interval [0,1]. Aggregate consumption is thus given by,

$$C_t = (\int_{j=0}^{1} C_t(j)^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

where ε is also a parameter of the preferences of the representative household, and more precisely, the elasticity of substitution between goods. We assume that $\varepsilon>1$.

The sequence of budget constraints under which the household maximizes intertemporal utility is given by,

$$\begin{aligned} B_{t+1} &= (1+i_t)B_t + W_t N_t + T_t - \int_{j=0}^{1} P_t(j) C_t(j) dj && \text{(3a)} \\ \lim_{T\to\infty} E_t B_t &\geq 0 && \text{(3b)} \end{aligned}$$

where P(j) is the price of good j, W the nominal wage, i the nominal interest rate, B a nominal one period bond, and T an exogenous transfer of nominal income to the household (dividends, government transfers or taxes).

(3a) is the asset accumulation equation, under the assumption that the only household asset is one period bonds. Following Woodford (2003) we assume a cashless economy. (3b) is the appropriate transversality condition.

Apart from the decision about aggregate consumption and labor supply, the household must decide on the distribution of its consumption expenditure among the various goods. This requires the maximization of the consumption bundle (2) for any level of monetary expenditure. One can easily deduce that this implies,

$$C_t(j) = (\frac{P_t(j)}{P_t})^{-\varepsilon} C_t \tag{4}$$

for any good j in the interval [0,1], where P is the average price level, defined as,

$$P_t = (\int_{j=0}^{1} P_t(j)^{1-\varepsilon})^{\frac{1}{1-\varepsilon}} \tag{5}$$

In addition, when the household follows this optimal allocation policy, we also have that,

$$\int_{j=0}^{1} P_t(j) C_t(j) dj = P_t C_t \tag{6}$$

(6) suggests that total consumption expenditure can be written as the product of the aggregate consumption index (2) and the aggregate price index (5). Substituting (6) in the sequence of budget constraints (3a) and (3b), we get,

$$B_{t+1} = (1 + i_t) B_t + W_t N_t + T_t - P_t C_t \tag{7}$$

From the first order conditions for consumption and labor supply we get that,

$$-\frac{u_{N_t}}{u_{C_t}} = \frac{W_t}{P_t} \tag{8}$$

$$\frac{1}{1 + i_t} = \frac{1}{1 + \rho} E_t [\frac{u_{C_{t+1}}}{u_{C_t}} \frac{P_t}{P_{t+1}}] \tag{9}$$

The constraint (7) must also hold.

We assume that the utility function is defined by,

$$u(C_t, N_t) = \frac{C_t^{1-\theta}}{1-\theta} - \frac{N_t^{1+\lambda}}{1+\lambda} \tag{10}$$

where $1/\theta$ is the inter-temporal elasticity of substitution in consumption, and λ the Frisch elasticity of labor supply.

Assuming that preferences take the form of (10), the first order conditions (8) and (9) can be written in log-linear form as,

$$w_t - p_t = \theta c_t + \lambda n_t \tag{11}$$

$$c_t = E_t(c_{t+1}) - \frac{1}{\theta}(i_t - E_t\pi_{t+1} - \rho) \tag{12}$$

where lower case letters denote the logarithms of the corresponding variables. π is the rate of inflation, defined as π_t=p_t-p_{t-1}.

2.2 The Representative Firm and Optimal Pricing

We assume that output is produced by a set of firms denoted by a continuous index j defined in the interval [0,1]. Each firm produces a differentiated product under conditions of monopolistic competition. All firms have access to the same production technology, denoted by the production function,

$$Y_t(j) = A_t L_t(j)^{1-a} \tag{13}$$

where A>0 and 0<α<1 are exogenous technological parameters, common to all firms. L(j) is employment of labor by firm j. The parameter α is constant, while A is assumed to follow an exogenous stochastic process.

The optimal price of each firm, if it can choose its price in every period, is given by the maximization of its profits, under the constraint of the production function (13) and the demand function for its product (5). Each firm takes the average price P, the average wage W and the level of total demand C as given.

The per period profits of firm j are given by,

$$P_t(j)Y_t(j) - W_t L_t(j) \tag{14}$$

From the first order conditions for a maximum of (14), under the constraints (13) and (5), the optimal price is determined as,

$$P_t(j) = \frac{\varepsilon}{\varepsilon - 1}\left(\frac{W_t}{(1-a)A_t L_t(j)^{-a}}\right) \tag{15}$$

The optimal price is a fixed multiple of the firm's marginal cost, which equals the expression in brackets. The multiple depends on the elasticity of substitution between goods in the preferences of consumers, which determines the price elasticity of demand of their product, and therefore the profit margin of the firm. In the case of perfect competition, the elasticity of substitution tends to infinity, and the price tends to marginal cost. In the case of monopolistic competition with $\varepsilon > 1$, as we have assumed, the optimal price is higher than the marginal cost of labor.

As all firms have the same production function and face the same nominal wage and the same demand function for their product, they will all choose the same price. Consequently, the price level is defined as,

$$P_t = \frac{\varepsilon}{\varepsilon - 1}\left(\frac{W_t}{(1-a)A_t {L_t}^{-a}}\right) \tag{16}$$

Taking the logarithm of the production function (13) for the representative firm, and equation (16) for the price level under optimal pricing, we get,

$$y_t = a_t + (1-a)l_t \tag{17}$$

$$w_t - p_t = a_t - al_t - \mu \tag{18}$$

where,

$$a_t = \ln A_t, \mu = \ln\left(\frac{\varepsilon}{\varepsilon - 1}\right) - \ln(1-a)$$

α_t is the logarithm of the exogenous productivity shock, and the constant μ is the logarithm of the markup on marginal cost, minus the logarithm of the coefficient of decreasing returns to labor.

2.3 Equilibrium with Full Price Flexibility

Solving the model under the assumption of full flexibility of prices, one can show that fluctuations in employment, output, consumption and real wages are a function only of the exogenous shocks to productivity, while fluctuations in the real interest rate are a function of the expected change in productivity, just as in the classical model with the assumption of perfect competition.

To keep the analysis simpler, we shall assume that there are is no investment or public consumption. Thus, in equilibrium, labor supply would be equal to labor demand by firms, and consumption will be equal to output.

$$n_t = l_t \tag{19}$$

$$y_t = c_t \tag{20}$$

The model consists of the behavioral equations (11), (12), (17), (18), and the equilibrium conditions (19) and (20). The model determines employment, output, consumption, real wages and the real interest rate as functions of the exogenous shock to productivity α_t.

The real interest rate is defined by the Fisher equation as,

$$r_t = i_t - E_t \pi_{t+1} \tag{21}$$

Thus, to determine the rate of inflation, one needs to make an assumption about the determination of the nominal interest rate.

Solving the model for the five endogenous real variables, we get,

$$l_t^N = n_t^N = \phi a_t + \overline{n} \tag{22}$$

where, $\delta = \frac{1-\theta}{\theta(1-a)+a+\lambda}$ and $\overline{n} = -\frac{\mu}{\theta(1-a)+a+\lambda}$

$$y_t^N = c_t^N = \psi \alpha_t + \overline{y} \tag{23}$$

where, $\psi = 1 + (1-a)\delta = \frac{1+\lambda}{\theta(1-a)+a+\lambda}$ and $\overline{y} = (1-a)\overline{n}$

$$(w-p)_t^N = \chi a_t + \overline{\omega} \tag{24}$$

where, $\chi = 1 - \alpha\delta = \frac{\theta+\lambda}{\theta(1-a)+a+\lambda}$ and $\omega = (\theta(1-a)+\lambda)\overline{n}$

$$r_t^N = \rho + \theta\psi E_t \Delta a_{t+1} \tag{25}$$

(22), (23), (24) and (25), along with the equilibrium conditions (19) and (20), determine the five endogenous real variables as functions of the exogenous productivity shock. Superscript N (natural) denotes the equilibrium values of the relevant variables, which, according, to the Friedman (1968) definition, are their "natural" rates in this model.

Output, consumption and real wages are positive functions of the productivity shock α_t, while employment is a positive function of the productivity shock only if $\theta<1$, i.e. only if the elasticity of inter-temporal substitution is greater than one. If $\theta>1$ employment is a negative function of productivity, while if $\theta = 1$ employment is independent of productivity. This applies because if θ<1 the inter-temporal substitution effect dominates on the income effect, following a change in productivity and real wages. If θ>1 the income effect dominates on the inter-temporal substitution effect, which in the case where $\theta = 1$ the two effects cancel each other out, and employment is not affected.

For example, let us assume that the productivity shock follows a first order autoregressive process of the form,

$$a_t = \eta_a a_{t-1} + \varepsilon_t^a \tag{26}$$

where, 0< $\eta_a < 1$, and ε_t^a is a white noise process.

Then, all real variables follow first order autoregressive processes in this model.

No other factor affects fluctuations in real variables. We see that, as in a competitive real business cycle model, the classical dichotomy holds, and monetary factors such as nominal interest rates have no effect on the evolution of real variables.

2.4 The Inefficiency of the "Natural" Rate

However, in this model there is a significant difference from a competitive model. Because of monopolistic competition, which implies a positive margin of prices over marginal costs of firms, both employment and output, as well as consumption and real wages, are determined at a lower level than in the case of perfect competition. Monopolistic competition implies a distortion in the market of goods and services, which leads to lower equilibrium employment and output and to lower real wages than under perfect competition.[4]

If the productivity shock follows a stationary stochastic process with mean zero, then, from (22), the log of the steady state employment level will be equal to,

$$\overline{n} = -\frac{\mu}{\theta(1-a)+a+\lambda} = \frac{\ln(1-a)-\ln(\varepsilon/(\varepsilon-1))}{\theta(1-a)+a+\lambda} \tag{27}$$

If ε>1, the steady state employment level will be lower than in the case of perfect competition.

[4]See Akerlof and Yellen (1985), Mankiw (1985), Blanchard and Kiyotaki (1987) and Ball and Romer (1990) for the first generation of dynamic stochastic general equilibrium models that relied on monopolistic competition.

Under perfect competition, goods are perfect substitutes in the preferences of consumers. Thus, steady state employment would be equal to,

$$\lim_{\varepsilon \to \infty} \overline{n} = -\frac{\ln(1-a)}{\theta(1-a)+a+\lambda} \tag{28}$$

Thus, because of imperfect competition, this model implies under employment relative to a fully competitive model, even when there is full flexibility of prices and wages. Through (23) and (24), this under employment implies that steady state output and steady state real wages will also be lower compared to perfect competition.

In all other respects, this model resembles a competitive real business cycle model without capital.

3 The Role of Monetary Policy under Full Price Flexibility

We next turn to the role of monetary policy, in a model with full price flexibility. We assume a central bank that intervenes in the bond market to determine the path of nominal interest rates, so as to minimize an inter-temporal quadratic loss function that depends on deviations of the price level from a target price level p*, and deviations of output from its "natural" rate. This is assumed to take the form,

$$\begin{aligned} \Lambda_t &= \frac{1}{2} E_t \sum_{s=0}^{\infty} \beta^s [(p_{t+s} - p^*_{t+s})^2 + \zeta (y_{t+s} - y^N_{t+s})^2] \\ &= \frac{1}{2} E_t \sum_{s=0}^{\infty} \beta^s [(\pi_{t+s} - \pi^*_{t+s})^2 + \zeta (y_{t+s} - y^N_{t+s})^2] \end{aligned} \tag{29}$$

where β=1/(1+ρ) is the discount factor of the central bank, and where we have made use of the definition of inflation to substitute for the target price level. ζ measures the cost of deviations of output from its "natural" rate, relative to the cost of deviations of inflation from target. Under full price flexibility, output is always equal to its "natural" rate, so the second term of the loss function is always zero. Essentially, in a model with full price flexibility in all markets the loss function of the central bank depends only on expected deviations of actual inflation from target inflation.

This implies that in every period, the central bank would seek to ensure that,

$$E_t \pi_{t+s} = \pi^* = p^*_{t+s} - p_{t-1} \tag{30}$$

If inflation under the optimal policy follows a stationary stochastic process with mean π^*, then the losses of the monetary authority are proportional to the variance of inflation and are given by,

$$\Lambda_t = \frac{1}{2} E_t \sum_{s=0}^{\infty} \beta^s (\pi_{t+s} - \pi^*)^2 = \frac{1}{2(1-\beta)} E_t (\pi_t - \pi^*)^2 = \frac{1}{2(1-\beta)} Var(\pi_t) \tag{31}$$

Thus, the optimal monetary policy under full flexibility of prices is the one that minimizes the variance of inflation in the presence of stochastic shocks.

3.1 A Non Contingent Nominal Interest Rate Rule

If we assume that the central bank follows an exogenous path for the nominal interest rate, then, from the Fisher equation (21), it follows that,

$$E_t \pi_{t+1} = i_t - r_t \tag{32}$$

(32) does not determine inflation, but expected inflation, given the exogenous path of nominal interest rates. (32) is consistent with any price level path that satisfies,

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1} \tag{33}$$

where ξ is any shock that satisfies $\mathrm{E}_t \xi_{t+1} = 0$.

(33) suggests that there are multiple equilibria for the price level and inflation. Any zero mean stochastic ξ could affect inflation and the price level in this case. Consequently neither the price level, nor inflation can be determined uniquely when the central bank follows an exogenous path for the nominal interest rate.

However, not all interest rate rules result in price level indeterminacy. As suggested more than a century ago by Wicksell (1898), if the central bank conditions its nominal interest rate on the price level, or inflation, then price level and inflation indeterminacy does not necessarily follow.[5]

[5]Sargent and Wallace (1975) were the first to formally demonstrate that under rational expectations, a non-contingent interest rate target leads to price level indeterminacy and instability, citing similar conclusions by Wicksell (1898). However, it is now accepted that this problem does not

3.2 A Simple Wicksell Nominal Interest Rate Rule

Central banks predominantly follow policies according to which the path of nominal interest rates is not exogenous, but depends on past, current and expected future economic developments, mainly inflation. For example, if inflation rises, central banks usually raise nominal interest rates in order to reduce it, and vice versa.

Wicksell (1898) was probably the earliest advocate of a such a stabilizing interest rate rule. He suggested that to the extent that the price level exceeded the target price level of central banks, the nominal interest rate should rise above the "natural" rate of interest, in order to bring the price level back to target, and vice versa. In what follows, we shall examine the properties of a simple version of the Wicksell rule consistent with the presence of an inflation target equal to π^*, and the properties of our model.

Let us therefore assume the following simple rule for determining nominal interest rates,

$$i_t = \rho + \pi^* + \phi(\pi_t - \pi^*) + \nu_t \tag{34}$$

where $\rho + \pi^*$ is the steady state nominal interest rate target of the central bank, $\phi > 0$ is the reaction of the current central bank nominal interest rate to deviations of inflation from its target π^* and ν_t is a monetary policy shock that follows an exogenous AR(1) process of the form,

$$\nu_t = \eta_\nu \nu_{t-1} + \varepsilon_t^\nu \tag{35}$$

where $0 < \eta_\nu < 1$, and ε_t^ν is a white noise process.

From (34) and the Fisher equation (32), inflation is determined by,

$$\pi_t = \frac{\phi - 1}{\phi}\pi^* + \frac{1}{\phi}E_t\pi_{t+1} + \frac{1}{\phi}(r_t - \rho - \nu_t) \tag{36}$$

where the current equilibrium real interest r depends only on real factors, and is determined by (25).

Solving (36) under rational expectations,

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arise in the case of contingent interest rate rules that make the nominal interest rate depend on the price level (McCallum 1981), or a sufficiently sensitive positive function of inflation. See Clarida, Gali and Gertler (1999) and Woodford (2003) Ch.1 for a review of the relevant arguments. In addition, central banks, have been consistently using interest rates as their main monetary policy instrument.

$$\pi_t = \pi^* + \sum_{s=0}^{\infty}(\frac{1}{\phi})^{s+1} E_t(r_{t+s} - \rho - \nu_{t+s}), \quad if \quad \phi > 1 \tag{37a}$$

$$\pi_{t+1} = \phi\pi_t + (1-\phi)\pi^* - (r_t - \rho - \nu_t) + \xi_{t+1}, \quad if \quad \phi \leq 1 \tag{37b}$$

where ξ is any shock that satisfies $\mathrm{E}_t\xi_{t+1} = 0$.

Thus, if the reaction of the central bank nominal interest rates to inflation is sufficiently pronounced ($\phi > 1$), there is no indeterminacy problem for inflation. The fundamental solution is given by (37a). If the reaction of the nominal interest rates to inflation is not sufficiently pronounced ($\phi \leq 1$), then the problem of inflation indeterminacy remains.

Assuming that $\phi > 1$, and using (25) for the real interest rate, and (26) and (35) for the stochastic processes driving productivity shocks and interest rate shocks, inflation if determined by,

$$\pi_t = \pi^* - \frac{\theta\psi(1-\eta_\alpha)}{\phi - \eta_\alpha}\alpha_t - \frac{1}{\phi - \eta_\nu}\nu_t \tag{38}$$

Thus, under a simple feedback rule for the nominal interest rate, with a sufficiently strong response of the current interest rate to deviations of current inflation from the central bank target for inflation, again depends on both real (productivity) and nominal (monetary policy) shocks. Using (26) and (35), inflation follows a stochastic process of the form,

$$\pi_t = (1-\eta_\nu)(1-\eta_a)\pi^* + (\eta_\nu + \eta_a)\pi_{t-1} - \eta_\nu\eta_a\pi_{t-2} + z_t \tag{39}$$

where,

$$z_t = -\frac{\theta\psi(1-\eta_a)}{\phi - \eta_a}(\varepsilon_t^a - \eta_\nu\varepsilon_{t-1}^a) - \frac{1}{\phi - \eta_\nu}(\varepsilon_t^\nu - \eta_a\varepsilon_{t-1}^\nu) \tag{40}$$

From (39) and (40), steady state inflation is equal to π^*, the target of the central bank, and, in the absence of shocks, inflation gradually converges to this mean. However, although the rule is conditional on inflation, real shocks affect the inflation process, and there is significant persistence and high variance of inflation under this rule.

The variance of inflation is given by,

$$Var(\pi_t) = E(\pi_t - \pi^*)^2 = [\frac{\theta\psi(1-\eta_a)}{(1-\eta_a)(\phi-\eta_a)}]^2\sigma_a^2 + [\frac{1}{(1-\eta_\nu)(\phi-\eta_\nu)}]^2\sigma_\nu^2 \qquad (41)$$

where σ_a^2 and σ_v^2 are the variances of the innovation in productivity and the monetary policy shock, respectively.

From (31), the expected welfare losses of the monetary authority under the simple Wicksell nominal interest rate rule are given by:

$$\Lambda_t^W = \frac{1}{2(1-\beta)}\{[\frac{\theta\psi(1-\eta_a)}{(1-\eta_a)(\phi-\eta_a)}]^2\sigma_a^2 + [\frac{1}{(1-\eta_\nu)(\phi-\eta_\nu)}]^2\sigma_\nu^2\} \qquad (42)$$

where superscript W denotes the monetary policy under the simple Wicksell nominal interest rate rule.

Therefore, the expected losses of the monetary authority depend on the variance of both real and nominal shocks. Real shocks are not neutralized by monetary policy, as the central bank real interest rate does not react to fluctuations in the equilibrium real interest rate.

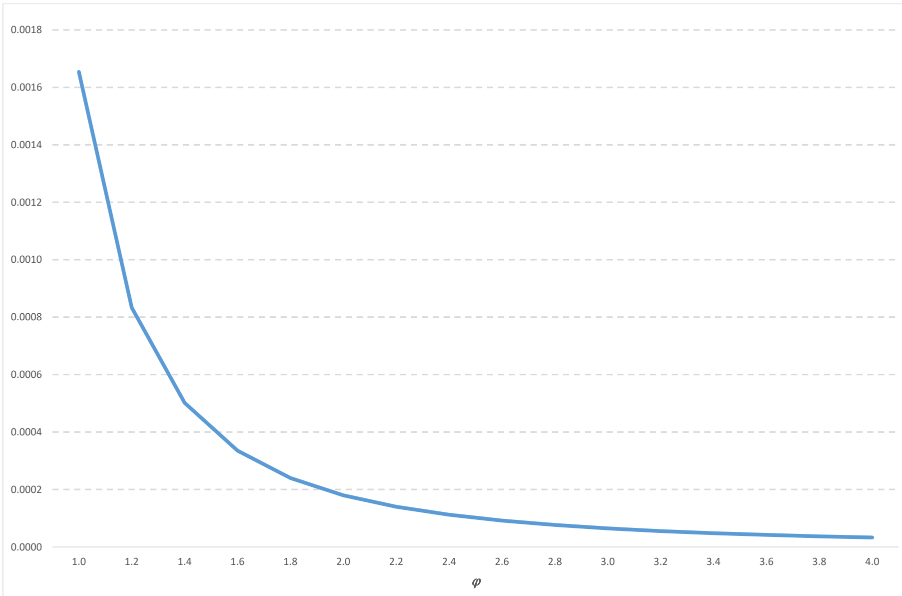
However, if the central bank's reaction to deviations of inflation from target was infinitely strong, then the expected welfare losses of the monetary authority could be driven to zero. It is straightforward to see from (42) that letting $\phi \to \infty$ would make the the expected losses of the central bank go to zero.

An interest rate policy of this form was proposed by Fisher (1919) and supported by Simons (1936). Fisher (1919) went further than Wicksell, by suggesting a policy of complete stabilization of the price level. In the context of our model this could be expressed as absolute inflation targeting, i.e. setting the nominal interest rate so that inflation is always equal to the central bank target π^*. This is the limit of the Wicksell rule (34), as the response of the central bank nominal interest rate to deviations of inflation from the central bank target tends to infinity. In this case, inflation would tend to be equal to π^* at all times.

The functional relation between Λ_t^W and ϕ is presented in Figure 1 assuming that ϕ, on horizontal axis, lies between 1 and 4. We have also assumed that the other structural parameters of the model take the values $\theta = 2/3$, $\alpha = 1/3$, $\lambda = 1.7$, $\eta_a = 0.75$, $\eta_\nu = 0.5$, $\beta = 0.98$, corresponding to a pure rate of time preference $\rho = 0.02$ and that $\sigma_a^2 = \sigma_\nu^2 = 0.01$.

Thus, in the context of a real business cycle model with full adjustment of wages and prices, the absolute inflation targeting rule of Fisher (1919) is the optimal interest rate rule, as it minimizes the variance of inflation and thus the expected welfare losses of the monetary authority.

Figure 1: Expected Losses of the Monetary Authority as a function of ϕ under a Simple Wicksell Rule



3.3 An Optimal Wicksell Interest Rate Rule

Could the same result be achieved if the Wicksell rule was conditioned on the equilibrium real interest rate instead of taking the simpler form in (34). From our analysis of the optimal central bank inflation policy, the central bank aims to keep inflation and expected inflation as close to π^* as possible. Using the Fisher equation (21), and the central bank optimal inflation target (30), the target nominal interest rate of the central bank should be equal to,

$$\bar{i}_t = r_t^N + \pi^* \tag{43}$$

Thus, the optimal nominal interest rate target should reflect the inflation target of the central bank, but also fluctuations in the equilibrium real interest rate. To the extent that current inflation deviates from target inflation, and to the extent that there are monetary policy shocks, the optimal nominal interest rate rule would thus take the form,

$$i_t = \bar{i}_t + \phi(\pi_t - \pi^*) + \nu_t = r_t + \pi^* + \phi(\pi_t - \pi^*) + \nu_t \tag{44}$$

Our objective again is to determine the optimal ϕ. We shall call this rule the optimal Wicksell rule.

Combining this rule with the Fisher equation, current inflation is then determined by,

$$\pi_t = \frac{\phi - 1}{\phi}\pi^* + \frac{1}{\phi}E_t\pi_{t+1} - \frac{1}{\phi}\nu_t \tag{45}$$

As can be seen from (45), under the optimal interest rate rule, current inflation does not depend on real shocks, as the impact of real shocks on current inflation has been neutralized by the reaction of the nominal interest rate to the current equilibrium real interest rate.

Assuming that $\phi > 1$, the rational expectations solution of (45) is given by,

$$\begin{aligned} \pi_t &= \pi^* - \frac{1}{\phi}\sum_{s=0}^{\infty}(\frac{1}{\phi})^s E_t\nu_{t+1} = \pi^* - \frac{1}{\phi - \eta_\nu}\nu_t = \\ &= (1-\eta_\nu)\pi^* + \eta_\nu\pi_{t-1} - \frac{1}{\phi - \eta_\nu}\varepsilon_t^\nu \end{aligned} \tag{46}$$

Inflation now does not depend on real shocks, but only nominal monetary policy shocks.

The variance of inflation is given by,

$$Var(\pi_t) = E(\pi_t - \pi^*)^2 = [\frac{1}{(\phi - \eta_\nu)(1-\eta_\nu)}]^2\sigma_\nu^2 \tag{47}$$

Therefore, the expected losses of the monetary authority under the optimal Wicksell nominal interest rate rule are given by:

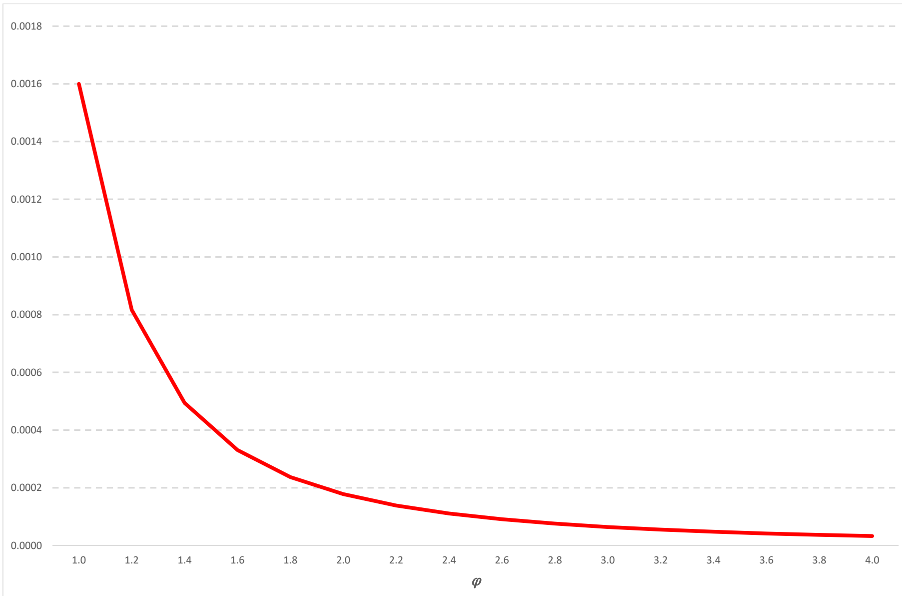
$$\Lambda_t^{W^*} = \frac{1}{2(1-\beta)}[\frac{1}{(\phi - \eta_\nu)(1-\eta_\nu)}]^2\sigma_\nu^2 \tag{48}$$

where superscript W* denotes the monetary policy under an optimal Wicksell interest rate rule.

If the central bank were to choose ϕ optimally, it should again allow it to be as large as possible. The policy that minimizes the variance of inflation and thus the expected losses of the central bank is the policy that requires that $\phi \to \infty$. Thus, we are back to the Fisher (1919) rule of absolute inflation targeting. In Figure 2, we depict the functional relation between $\Lambda_t^{W^*}$ and ϕ. All the parameters are as in Figure 1.

We have thus demonstrated that in the context of the model with full price flexibility, the Fisher rule of absolute inflation targeting is always the optimal policy.

Figure 2: Expected Losses of the Monetary Authority as a function of ϕ under an Optimal Wicksell Rule



4 An Imperfectly Competitive Model with Staggered Price Adjustment

We next introduce the assumption of gradual and not full adjustment of prices towards their equilibrium values. We shall utilize the Calvo (1983) model of staggered pricing.[6]

Following Calvo (1983), we shall assume that firms cannot freely adjust their prices in every period. For each firm, the probability of adjusting prices in any period is equal to 1-γ, which is constant and independent of the length of time that has elapsed since the last price adjustment by the firm. Thus, in each period, a proportion 1-γ of all firms adjust their prices, and the remaining proportion γ do not adjust their prices. This assumption has critical implications for the properties of the model, the nature of aggregate fluctuations and the effects of monetary shocks and monetary policy[7].

[6] An observationally equivalent model, the Rotemberg (1982 a,b) model of quadratic costs of adjusting prices, is analyzed in Appendix A to this paper and is shown to result in an observationally equivalent "new" Phillips curve with the Calvo (1983) model.

[7] See Yun (1996) for the first analysis of a dynamic stochastic general equilibrium model under the assumption that prices are set as postulated by Calvo (1983).

Under this assumption, in period t, the expected future duration of any price contract is given by,

$$(1-\gamma)\sum_{s=0}^{\infty} s\gamma^{s} = \frac{\gamma}{1-\gamma} \tag{49}$$

From the definition of the price level, and the fact that all firms that reset their prices in period t set the same price, it follows that,

$$P_t = (\gamma(\widehat{P}_{t-1})^{1-\varepsilon} + (1-\gamma)(\overline{P}_t)^{1-\varepsilon})^{\frac{1}{1-\varepsilon}} \tag{50}$$

where $\widehat{P}$ is the price level relative to the steady state price level, and $\overline{P}$ is the price set by the firms that freely reset their prices in the current period relative to the steady state price level.

From (50) one can show that the dynamic adjustment of the price level relative to the steady state price level is given by,

$$(\frac{\widehat{P}_t}{\widehat{P}_{t-1}})^{1-\varepsilon} = \gamma + (1-\gamma)(\frac{\overline{P}_t}{\widehat{P}_{t-1}})^{1-\varepsilon} \tag{51}$$

In the steady state with inflation equal to π^* we have that $\widehat{P}_t = \overline{P}_t = 1$ and the price level evolves as,

$$P_t = (1+\pi^*)P_{t-1} \tag{52}$$

A logarithmic approximation of (49) around a steady state inflation rate of π^* yields,

$$\widehat{p}_t - \widehat{p}_{t-1} \simeq (1-\gamma)(\overline{p}_t - \widehat{p}_{t-1}) \tag{53}$$

From (53) it follows that inflation exceeds it steady state level if firms that set prices in the current period set them at a higher level than the average price of the previous period adjusted for steady state inflation.

In order to analyze the adjustment of inflation, one thus has to examine how firms decide on their optimal price, taking into account the fact that for a period in the future they may not be able to readjust their prices, while some of their competitors have the option of readjusting their own prices.

4.1 Optimal Pricing with Staggered Price Adjustment

The problem of the firm that decides on the price it is about to set in period t is to set the price that maximizes the expected present value of its profits, given that the probability of readjusting its price in any future period is equal to 1-γ. Thus, all firms that readjust their prices in period t maximize,

$$\sum_{s=0}^{\infty} \gamma^s E_t(\prod_{z=0}^{s}(\frac{1}{1+i_{t+z}})(\overline{P}_t Y_{t+s}^t - \widehat{W}_{t+s} L_{t+s}^t)) \tag{54}$$

under the constraints of the production function,

$$Y_{t+s}^t = A_{t+s}(L_{t+s}^t)^{1-\alpha} \tag{55a}$$

$$Y_{t+s}^t = (\frac{\overline{P}_t}{\widehat{P}_{t+s}})^{-\varepsilon} Y_{t+s} \tag{55b}$$

where, Y_{t+s}^t and L_{t+s}^t is the volume of output and employment in period t+s, of the firm that has set its prices in period t. The higher the relative price of the firm in any period, the lower the demand for its product and thus the volume of its output and employment.

From the first order conditions for a maximum it follows that,

$$\sum_{s=0}^{\infty} \gamma^s E_t(\prod_{z=0}^{s}[(\varepsilon-1)(\frac{\overline{P}_t}{\widehat{P}_{t+s}})^{-\varepsilon} Y_{t+s} - \frac{\varepsilon}{1-a}(\frac{\overline{P}_t}{\widehat{P}_{t+s}})^{-\frac{1-a+\varepsilon}{1-a}} \frac{\widehat{W}_{t+s}}{\widehat{P}_{t+s}}(\frac{Y_{t+s}}{A_{t+s}})^{\frac{1}{1-a}}] = 0 \tag{56}$$

(56) implies that the expected present value of revenues from the optimal price is equal to the expected present value of the marginal cost of production, augmented by the profit margin $\varepsilon/(\varepsilon\text{-}1)$ of the firm.

It is worth noting that, as we have already shown (equation (16)), if the firm could determine its prices in every period, the price of the product in each period would be equal to the marginal cost of production plus the same profit margin. However, if the firm cannot adjust prices in every period, as is assumed in the Calvo (1983) model, pricing follows the rule (56).

Assuming that in the steady state inflation is equal to π^*, (56) can be transformed in logarithmic deviations from the steady state level, using a log linear Taylor approximation. Thus, in logarithms we shall have that,

$$\overline{p}_t \simeq (1-\beta\gamma)\sum_{s=0}^{\infty}(\beta\gamma)^s E_t[\widehat{p}_{t+s} + \omega(\mu + w_{t+s} - p_{t+s} + \frac{1}{1-a}(ay_{t+s} - a_{t+s}) \tag{57}$$

where: $\beta = \frac{1}{1+\rho+\pi^*}$ and $\omega = \frac{1-a}{1-a(1-\varepsilon)} < 1$.

Consequently, firms that reset their prices in period t will choose a price which corresponds to a weighted average of the current and expected future price levels, plus a margin μ on a weighted average of the current and expected future level of their real marginal costs. The discount factor of a future period t+s depends on the probability that the firm will not be able to reset its price in the future period t+s, which equals γ^s, times the pure discount factor β^s. Furthermore, the part of pricing which depends on the expected marginal cost of the firm depends negatively on the elasticity of demand for the product of the firm, through the parameter ω.

Using the future mathematical expectations operator F, (57) can be written as,

$$\overline{p}_t = \frac{1-\beta\gamma}{1-\beta\gamma F}\widehat{p}_t + \omega[\mu + \frac{1-\beta\gamma}{1-\beta\gamma F}(w_t - p_t + \frac{1}{1-\alpha}(ay_t - a_t)] \tag{58}$$

Substituting (58) in the equation for the adjustment of the average price level (53) we get that,

$$\widehat{p}_t = \gamma\widehat{p}_{t-1} + (1-\gamma)[\frac{1-\beta\gamma}{1-\beta\gamma F}\widehat{p}_t + \omega(\mu + \frac{1-\beta\gamma}{1-\beta\gamma F}(w_t - p_t + \frac{1}{1-\alpha}(ay_t - a_t))] \tag{59}$$

Multiplying both sides of (56) by 1-$\beta\gamma$F, after some rearrangements, we get that,

$$(1+\beta)\widehat{p}_t - \widehat{p}_{t-1} - \beta E_t\widehat{p}_{t+1} = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma}\omega[\mu + (w_t - p_t + \frac{1}{1-\alpha}(ay_t - a_t))] \tag{60}$$

(60) is the equation of adjustment of the price level towards the steady state price level, which depends on expected inflation and is a constant markup on the marginal cost of production.

In order to examine the short run behavior of the model, we must introduce the equilibrium conditions in the markets for goods and services and labor.

4.2 Equilibrium in the Market for Goods and Services and the "new" IS Curve

Equilibrium in the market for good j implies that,

$$Y_t(j) = C_t(j) \tag{61}$$

As a result, equilibrium in the market for all goods requires that,

$$Y_t = C_t \tag{62}$$

where Y is total output, defined in the same way as total consumption C in equation (2).

Substituting the Euler equation for consumption (13) in the equilibrium condition (62), the logarithm of real output is determined by,

$$y_t = E_t(y_{t+1}) - \frac{1}{\theta}(i_t - E_t\pi_{t+1} - \rho) \tag{63}$$

(63) is often referred to as the "new" IS curve, as it is derived from the equilibrium condition for the market for goods and services.[8]

4.3 Labor Market Equilibrium and the "New" Phillips Curve

We next turn to the equilibrium condition in the labor market. We assume that in contrast to product prices that adjust gradually, nominal wages adjust immediately in order to equate the demand and supply of labor in each period. Thus, the only stickiness which is analyzed in this version of the model is the gradual adjustment of prices rather than wages. This means that fluctuations in employment are the result of inter-temporal substitution by households and that no involuntary unemployment exists.[9]

Due to the gradual adjustment of prices, firms produce so as to satisfy aggregate demand at the given prices in each period. Aggregate output is determined at the level which is determined by aggregate demand, and differs from its "natural level", which is the level that would prevail if there was immediate price adjustment by all firms.

As a result, aggregate output, employment, consumption, real wages and the real interest rate, differ from their "natural levels" and display fluctuations which depend on nominal as well as real disturbances.

[8]Compared to the conventional IS curve, (63) contains the rational expectation about the future volume of output and depends on the real and not just the nominal interest rate. Its advantage over the conventional IS curve is that it has been derived from firm microeconomic foundations, and that its parameters depend on deep structural parameters, such as the pure rate of time preference of the representative household ρ, and the inter-temporal elasticity of substitution in consumption $1/\theta$.

[9]Gali (2008, 2011) and others have analyzed this model with the additional assumption of rigidity not only in prices but also in nominal wages. In this case there are fluctuations in the unemployment rate due to the fact that wages do not equate the demand with the supply of labor in each period, as is assumed here.

From the price adjustment equation (60), we can deduce an equation for fluctuations in inflation. Expressing (60) as an inflation equation we have that,

$$\pi_t = (1-\beta)\pi^* + \beta E_t \pi_{t+1} + \frac{(1-\gamma)(1-\beta\gamma)}{\gamma}\omega[\mu + (w_t - p_t + \frac{1}{1-\alpha}(ay_t - a_t))] \quad (64)$$

where $\pi_t = p_t - p_{t-1}$ is the rate of inflation.

(64) implies that deviations of current inflation from steady state inflation are greater than discounted expected future deviations of inflation from its steady state value, if the current marginal cost of labor, plus the margin μ is higher than the current price level p. The reason is that firms setting prices in the current period post larger price increases than (discounted) expected future inflation, in order to offset the higher current marginal cost of labor.

The assumption of equilibrium in the labor market means that we can substitute the real wage in (64) from the first order condition (11) for the representative household. Using (11), the condition for equilibrium in the market for goods and services c=y, and the production function (17), (64) can be rewritten as,

$$\pi_t = (1-\beta)\pi^* + \beta E_t \pi_{t+1} + \kappa(y_t - y_t^N) \quad (65)$$

where y^N is the "natural rate" of real output, i.e. the output that would be produced if there was full flexibility of prices, and is given by (23). The parameter κ is defined as,

$$\kappa = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma}\frac{\theta(1-a)+\lambda+a}{1-a(1-\varepsilon)} > 0$$

(65) is referred to as the new Phillips curve, and constitutes the second important behavioral equation of the imperfectly competitive dynamic stochastic general equilibrium model with staggered pricing.

The reason that deviations of output from its "natural rate" cause inflation to rise relative to expected future inflation is that higher output implies a higher current real marginal cost of labor, and thus induces firms that have the opportunity to change their current price, to post price increases which exceed discounted expected future inflation.

Like the "new" IS curve, the "new" Phillips curve has been derived from explicit macroeconomic foundations and labor market equilibrium, and its parameters are functions of the "deep" structural parameters describing the preferences of households, the technology of production and the price setting technology.

4.4 The Structure of the Model with Staggered Pricing

Equations (63) and (65), along with equations (23) and (25) for the "natural level" of real output and the real interest rate, constitute the basic structure of the imperfectly competitive model with staggered pricing.

Deviations of inflation from discounted expected future inflation are determined by the "new" Phillips curve (65), as a function of deviations of real aggregate demand and output from the "natural level" of output.

Deviations of real output from its "natural level" are determined by the "new" IS curve, which depends on deviations of the real interest rate from its "natural level". The "new" IS curve can be expressed as,

$$y_t - y_t^N = E_t(y_{t+1} - y_{t+1}^N) - \frac{1}{\theta}(i_t - E_t\pi_{t+1} - r_t^N) \tag{66}$$

where the natural levels of output and the real interest rate y^N, r^N are determined by (23) and (25).

In order to close the model we must consider the determination of the nominal interest rate. In contrast to the model with immediate price adjustment, in the model with staggered price adjustment, fluctuations in real variables cannot be determined without reference to monetary factors. The classical dichotomy breaks down during the adjustment towards the "natural" rate. Monetary factors and monetary policy determine not only the price level and inflation, but also fluctuations in real variables such as real output, consumption, employment, real wages and the real interest rate.

5 The Role of Monetary Policy under Staggered Pricing

Monetary policy will in this case determine not only the price level and inflation, but also deviations of real output and employment from their "natural" rates. In order to determine optimal monetary policy the central bank will thus need to consider both inflation and real variables. The second term in the loss function (29) will thus come into operation, as now output deviates from its "natural" level, and this deviation depends on nominal interest rates.

The central bank will thus seek to minimize (29) subject to the dynamic Phillips curve (65) and the dynamic IS curve (66).

From the first order conditions for a minimum, this implies that in every period it will seek to ensure that,

$$E_t(\pi_{t+1} - \pi^*) + \frac{\zeta}{\kappa} E_t(y_{t+1} - y_{t+1}^N) = 0 \tag{67}$$

The optimum policy, because of the trade-off between inflation and output, implies that the marginal cost of an expected deviation of inflation from target plus the marginal cost of an expected positive deviation of output from its "natural" rate must be equal to zero. At the optimum, the central bank will only choose to achieve its inflation target, if that is consistent with the equality of output to its "natural" rate. If output is lower than its "natural" rate the central bank will increase inflation above its target and vice versa.

Substituting this policy rule in the dynamic Phillips curve (65), it follows that the central bank should again seek to set inflation to π^* in all periods, as this would eliminate deviations of output from its "natural" rate. Substituting the π^* inflation target in the Fisher equation for the nominal interest rate, this implies a nominal interest rate target equal to,

$$\bar{i}_t = r_t^N + \pi^* \tag{68}$$

Thus, the optimal nominal interest rate target should reflect fluctuations in the "natural" real interest rate, plus the inflation target of the central bank.

5.1 A Non Contingent Nominal Interest Rate Rule

If we assume that the central bank follows an exogenous path for the nominal interest rate, then, the analysis is exactly the same as in section 3.1, for the model with perfectly flexible prices. From the Fisher equation (21), it follows that,

$$E_t(\pi_{t+1}) = i_t - r_t \tag{69}$$

As we have already mentioned, (69) does not determine inflation, but expected inflation, given the exogenous path of nominal interest rates. It is consistent with any price level path that satisfies,

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1} \tag{70}$$

where ξ is any shock that satisfies $\mathrm{E}_t\xi_{t+1} = 0$.

(70) suggests that there are multiple equilibria for the price level and inflation. Any zero mean stochastic ξ could affect inflation and the price level in this case. Consequently neither the price level, nor inflation can be determined uniquely when the central bank follows an exogenous path for the nominal interest rate.

5.2 The Taylor Interest Rate Rule with a Constant Intercept

We shall next analyze the model under the assumption that the central bank follows a standard constant intercept Taylor (1993) rule of the form,

$$i_t = \rho + \pi^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^N) + \nu_t \tag{71}$$

where ϕ_π and ϕ_y are positive coefficients, and ν is an exogenous stochastic disturbance in the nominal interest rate. It is worth noting that because the constant in this rule is equal to $\rho + \pi^*$, this rule is consistent with steady state inflation π^*.[10]

This rule implies a countercyclical monetary policy. When inflation is positive, the central bank increases nominal interest rates in order to reduce it. When employment is low, i.e. when output is lower than its "natural" level, the central bank reduces nominal interest rates in order to increase aggregate demand and employment and nudge output towards its "natural" level. As is well known, this feedback interest rate rule does not result in inflation and price level indeterminacy if the Taylor principle is satisfied, i.e. if the reaction of nominal interest rates to inflation is sufficiently strong.[11]

Having fully determined the model with staggered pricing, we can analyze how nominal and real disturbances produce aggregate fluctuations under the standard Taylor (1993) rule (71).

The full model consists of the "new" Phillips and IS curves (65) and (66), the Taylor rule (71), and the equation for the determination of the "natural" real interest rate (25). Thus, the model can be written as,

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \kappa \widehat{y}_t \tag{72}$$

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \frac{1}{\theta}(i_t - \pi^* - E_t \widehat{\pi}_{t+1} - r_t^N) \tag{73}$$

$$i_t = \rho + \pi^* + \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t + \nu_t \tag{74}$$

where, $\widehat{\pi}_t = \pi_t - \pi^*$, $\widehat{y}_t = y_t - y_t^N$ denote the deviations between current and target inflation, and between current (log) real output and its "natural" level.

[10]Note that the Taylor rule (71) is simpler than the optimal nominal interest rule (68). The nominal interest rate does to react to shocks that change the current real rate of interest, such as productivity on monetary policy shocks, and thus productivity shocks turn out to affect deviations of inflation from the central bank target and deviations of output from its "natural rate".

[11]See Woodford (2003), for an extensive analysis of the Taylor principle.

The percentage deviation between current real output and its "natural" level is often referred to as excess output, and its opposite is referred to as the *output gap*. When excess output is positive, the economy produces more than its "natural" level, while when it is negative (the output gap is positive) it produces less than its "natural" level.

The "natural" real rate of interest r^N is determined by (25) and depends only on the expected change in the productivity shock and the pure rate of time preference.

The model can be solved by substituting the Taylor rule for the nominal interest rate in the "new" IS curve (73) and solving for excess output. This substitution results in an aggregate demand function of the form,

$$\widehat{y}_t = \frac{\theta}{\theta+\phi_y} E_t \widehat{y}_{t+1} - \frac{\eta_\pi}{\theta+\phi_y}\widehat{\pi}_t + \frac{1}{\theta+\phi_y} E_t \widehat{\pi}_{t+1} - \frac{1}{\theta+\phi_y}(\theta\psi(1-\eta_\alpha)a_t + \nu_t) \quad (75)$$

Combined with the "new" Phillips curve (72), the model can determine excess output and inflation, as functions of the productivity shocks affecting the "natural" real rate of interest, and the monetary shocks, affecting the nominal interest rate.

One can then solve the two equation system (72) and (75) by substituting out for excess output from (75) into (72). This will, after some re-arrangement, result in an inflation equation of the form,

$$\begin{aligned} \widehat{\pi}_t \quad = \quad & \frac{\beta(\theta+\phi_y)+\theta+\kappa}{\theta+\phi_y+\kappa\phi_\pi} E_t\widehat{\pi}_{t+1} - \frac{\beta\theta}{\theta+\phi_y+\kappa\phi_\pi} E_t \widehat{\pi}_{t+2} \\ & - \frac{\kappa}{\theta+\phi_y+\kappa\phi_\pi}(\theta\psi(1-\eta_\alpha)a_t+\nu_t) \end{aligned} \quad (76)$$

As can be seen from (76), inflation depends on expectations about future inflation, productivity shocks, which affect the "natural" real rate of interest, and nominal interest rate shocks. Recalling the definition of κ, one can see that the parameters determining the inflation process depend on the preferences of the representative household, (θ, λ, ε and ρ), the technology of production (α), market structure (ε), the price adjustment mechanism (γ) and the parameters of the Taylor rule (ϕ_π and ϕ_y).

Since inflation is a non predetermined variable, both roots of (76) must be less than unity (inside the unit circle), which requires that the sum of the coefficients of the future expectations of inflation in (76) is less than one. Thus, for a stable inflation process a necessary and sufficient condition is that,

$$\frac{\beta(\theta+\phi_y)+\theta+\kappa-\beta\theta}{\theta+\eta_y+\kappa\eta_\pi} < 1 \quad (77)$$

Under the assumption that the coefficients of the Taylor rule ϕ_π and ϕ_y are positive, after some rearrangement of (77), one can see that a necessary and sufficient condition for a stable inflation process is,

$$\phi_\pi > 1 - \frac{(1-\beta)}{\kappa}\phi_y \tag{78}$$

Thus, for a stable inflation process, the *Taylor principle*, as implied by (78) must hold. The reaction of nominal interest rates to current inflation must be sufficiently high to satisfy (78). For example, if the reaction of the nominal interest rate to excess output is zero (ϕ_y =0), then the reaction of nominal interest rates to inflation must exceed unity.

Assuming that the central bank policy satisfies (78), (76) can be solved forward for inflation, which is a non pre-determined variable. Inflation will be determined by,

$$\widehat{\pi}_t = -\frac{\kappa}{\beta\theta}\frac{1}{\lambda_1\lambda_2}E_t\sum_{s=0}^{\infty}(\frac{1}{\lambda_1})^s\sum_{q=0}^{\infty}(\frac{1}{\lambda_2})^q(\theta\psi(1-\eta_\alpha)a_{t+q}+\nu_{t+q}) \tag{79}$$

where $1/\lambda_1$ and $1/\lambda_2$ are the two roots of the inflationary process (76), which are both lower than one, if the Taylor principle (78) is satisfied, and will be defined by,

$$\lambda_1 + \lambda_2 = \frac{\beta(\theta+\phi_y)+\theta+\kappa}{\beta\theta} > 2 \tag{80a}$$

$$\lambda_1\lambda_2 = \frac{\theta+\phi_y+\kappa\phi_\pi}{\beta\theta} > 1 \tag{80b}$$

Inflation will depend on future expectations of deviations of the "natural" real rate of interest from ρ and future expectations about nominal interest rate shocks ν. The solution for inflation, assuming that the two shocks follow AR(1) processes as in (26) and (35), takes the form,

$$\widehat{\pi}_t = -\theta\psi\kappa(1-\eta_a)\Lambda_a\alpha_t - \kappa\Lambda_\nu\nu_t \tag{81}$$

where:

$$\Lambda_a = \frac{1}{(1-\beta\eta_a)(\phi_y+\theta(1-\eta_a))+\kappa(\phi_\pi-\eta_\alpha)} \tag{82}$$

$$\Lambda_\nu = \frac{1}{(1-\beta\eta_\nu)(\phi_y+\theta(1-\eta_\nu))+\kappa(\phi_\pi-\eta_\nu)} \tag{83}$$

Note that the coefficients Λ_a and Λ_ν depend negatively on the Taylor rule parameters ϕ_π and ϕ_y which are in the denominator of both fractions. The higher the Taylor rule parameters, the lower the impact of real and nominal shocks on deviations of inflation from the central bank target.

By substituting the solution for inflation in the "new" Phillips curve (72), one can get the corresponding solution for the evolution of excess output. Excess output will be given by,

$$\widehat{y_t} = -\theta\psi(1-\eta_a)(1-\beta\eta_a)\Lambda_a\alpha_t - (1-\beta\eta_\nu)\Lambda_\nu\nu_t \tag{84}$$

From (84), because the coefficients Λ_a and Λ_ν depend negatively on the Taylor rule parameters ϕ_π and ϕ_y , the higher are the Taylor rule parameters, the lower the impact of real and nominal shocks on deviations of output from its "natural" level.

From (81) and (84), the variances of inflation and of deviations of output from its "natural" level will be given by,

$$Var(\widehat{\pi}_t) = E(\pi_t - \pi^*)^2 = (\theta\psi\kappa\Lambda_a)^2\sigma_a^2 + (\frac{\kappa}{1-\eta_\nu}\Lambda_\nu)^2\sigma_\nu^2 \tag{85}$$

$$Var(\widehat{y_t}) = E(y_t - y_t^N)^2 = (\theta\psi(1-\beta\eta_a)\Lambda_a)^2\sigma_a^2 + (\frac{1-\beta\eta_\nu}{1-\eta_\nu}\Lambda_\nu)^2\sigma_\nu^2 \tag{86}$$

Therefore, the expected losses of the monetary authority under a Taylor interest rate rule with constant intercept are given by:

$$\Lambda_t^T = \frac{1}{2}\frac{1+\rho}{\rho}\{(\theta\psi\Lambda_a)^2[\kappa^2 + \zeta(1-\beta\eta_a)^2]\sigma_a^2 + (\frac{\Lambda_\nu}{1-\eta_\nu})^2[\kappa^2 + \zeta(1-\beta\eta_\nu)^2]\sigma_\nu^2\} \tag{87}$$

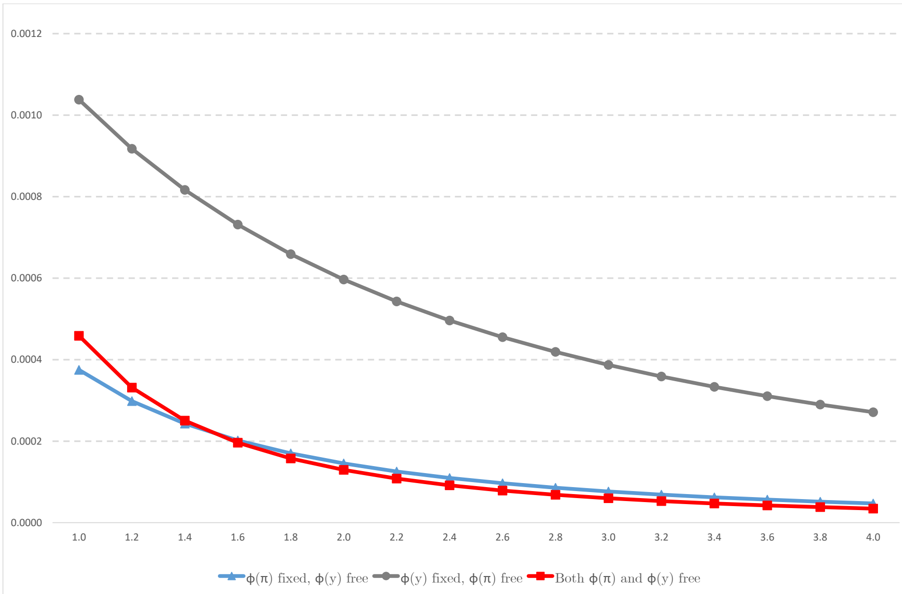
where superscript T denotes the monetary policy under a Taylor interest rate rule with a constant intercept.

Thus, the losses of the central bank, which depend on these two variances, will be affected by the variances of both real and nominal shocks. Real shocks are not neutralized by monetary policy, as the central bank real interest rate does not react to fluctuations in the equilibrium real interest rate.

However, if the central bank were to let $\phi_\pi \to \infty$ or $\phi_y \to \infty$, or both, then the variances of deviations of inflation from the central bank target, and of deviations of output from its "natural" level, would be driven to zero. The Fisher rule of absolute inflation targeting would thus be the optimal policy even in the presence of staggered pricing.

In Figure 3, we depict the functional relation between Λ_t^T and the two coefficients of the Taylor rule (71), ϕ_π and ϕ_y . We distinguish three cases. First, we keep ϕ_π

Figure 3: Expected Losses of the Monetary Authority as a function of ϕ_{pi} and ϕ_y under a Taylor Rule with constant intercept



fixed at value 1.5, as Taylor (1993) suggested, and we let ϕ_y lie between 1 and 4. Second, we keep ϕ_y fixed at value 0.5, as Taylor (1993) suggested, and we let ϕ_π lie between 1 and 4. Finally, we let both ϕ_π and ϕ_y lie between 1 and 4. We have assumed a portion of the firms that do not adjust their prices $\gamma = 2/3$, and a weight of output relative to inflation $\zeta = 0.25$. The other parameters are as in Figure 1. It is apparent that the optimal monetary policy is the one under the Fisher rule of absolute inflation targeting.

5.3 An Optimal Taylor Interest Rate Rule

From our analysis of the optimal central bank inflation policy, the central bank aims to keep inflation and expected inflation as close to π^* as possible. Using the Fisher equation (21), and the central bank optimal inflation target (30), the target nominal interest rate should be equal to,

$$\bar{i}_t = r_t^N + \pi^*$$

Thus, the optimal nominal interest rate target should reflect the inflation target of the central bank, but also fluctuations in the natural real interest rate. Would this make any difference in the model?

To the extent that current inflation deviates from target inflation, that current output deviates from its "natural" level, and to the extent that there are monetary policy shocks, the optimal nominal interest rate rule would thus take the form,

$$\begin{aligned} i_t &= \bar{i}_t + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^N) + \nu_t \\ &= r_t^N + \pi^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^N) + \nu_t \end{aligned} \tag{88}$$

Our objective is to determine the optimal ϕ's. We shall call this rule the *optimal Taylor rule.*

Combining this rule with the Fisher equation, and the "new" *Phillips* and *IS curves* (72) and (73), deviations of inflation from the central bank target are then determined by,

$$\widehat{\pi}_t = \frac{\beta(\theta + \phi_y) + \theta + \kappa}{\theta + \phi_y + \kappa\phi_\pi} E_t\widehat{\pi}_{t+1} - \frac{\beta\theta}{\theta + \phi_y + \kappa\phi_\pi} E_t\widehat{\pi}_{t+2} - \frac{\kappa}{\theta + \phi_y + \kappa\phi_\pi}\nu_t \tag{89}$$

As can be seen from (89), under the optimal Taylor rule, current inflation does not depend on real shocks, as the impact of real shocks on current inflation has been neutralized by the direct reaction of the nominal interest rate to the current equilibrium real interest rate.

Assuming that the Taylor principle, in the form of equation (78), holds, the rational expectations solution of (89) is given by,

$$\widehat{\pi}_t = -\frac{\kappa}{\beta\theta}\frac{1}{\lambda_1\lambda_2}E_t\sum_{s=0}^{\infty}(\frac{1}{\lambda_1})^s\sum_{q=0}^{\infty}(\frac{1}{\lambda_2})^q\nu_{t+q} \tag{90}$$

Inflation now does not depend on real shocks, but only nominal monetary policy shocks.

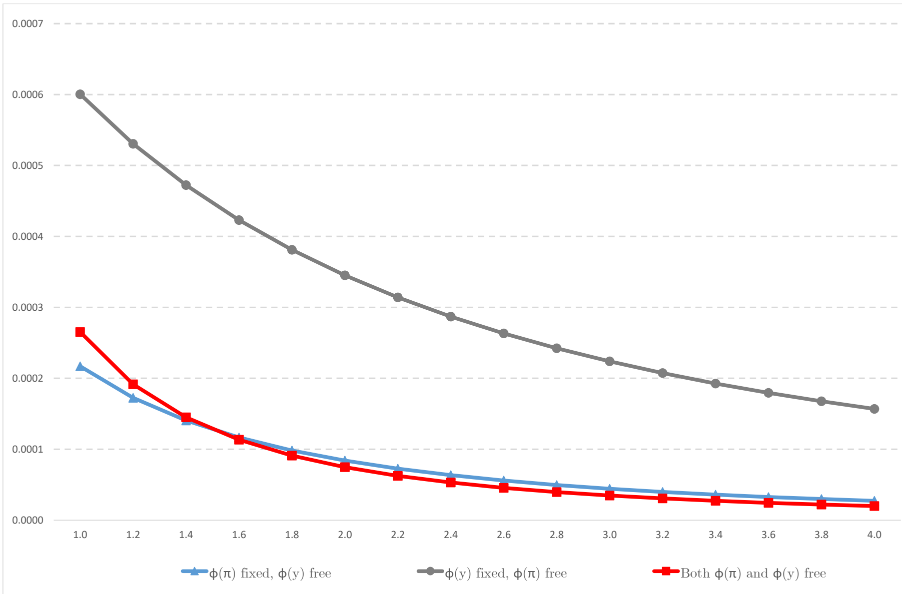
Solving for inflation and output, we get that,

$$\widehat{\pi}_t = -\kappa\Lambda_\nu\nu_t \tag{91}$$

$$\widehat{y}_t = -(1 - \beta\eta_\nu)\Lambda_\nu\nu_t \tag{92}$$

The variances of inflation and output are given by,

Figure 4: Expected Losses of the Monetary Authority as a function of ϕ_{pi} and ϕ_y under an Optimal Taylor Rule



$$Var(\widehat{\pi}_t) = E(\pi_t - \pi^*)^2 = (\kappa \Lambda_\nu)^2 \sigma_\nu^2 \tag{93a}$$
$$Var(\widehat{y}_t) = E(y_t - y_t^N)^2 = ((1 - \beta\eta_\nu)\Lambda_\nu)^2 \sigma_\nu^2 \tag{93b}$$

Now the variances of inflation and output depend only on monetary policy shocks, as the impact of real shocks is neutralized through the dependence of the optimal Taylor rule on the current real interest rate.

Therefore, the expected losses of the monetary authority under an optimal Taylor interest rate rule are given by:

$$\Lambda_t^{T^*} = \frac{1}{2}\frac{1+\rho}{\rho}\{\kappa^2 + \zeta(1 - \beta\eta_\nu)^2\}\Lambda_\nu^2\sigma_\nu^2 \tag{94}$$

where superscript T* denotes the monetary policy under an optimal Taylor interest rate rule.

However, even in this case, if the central bank were to choose the ϕ's optimally, it should allow them to be as large as possible. The policy that would minimize the variances of inflation and output and hence the expected losses of the central bank is the policy that requires that $\phi_\pi \to \infty$ or that $\phi_y \to \infty$, or both. The functional relation between $\Lambda_t^{T^*}$ and the coefficients of the Taylor rule (88) are depicted in Figure 4. All the parameters are as in Figure 3.

Thus, the Fisher rule of absolute inflation targeting would be optimal under the optimal Taylor rule specification too.

These results should not be too surprising. Absolute inflation targeting is optimal because of the "divine coincidence" that rules in this model. From the "new" Phillips curve specification (72), stabilizing inflation around the target of the central bank, immediately stabilizes output around its "natural" level. Thus, the policy of absolute inflation targeting also minimizes deviations of output from its "natural" level.

6 Optimal Monetary Policy in the Absence of the "Divine Coincidence"

Following Blanchard and Gali (2007), among others, we shall analyze a version of the model in which the divine coincidence does not hold.

Instead of the "new" Phillips curve (72), we shall now assume that,

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \kappa \widehat{y}_t + x_t \tag{95}$$

where x is an exogenous inflation shock, following an AR(1) process of the form,

$$x_t = \eta_x x_{t-1} + \varepsilon_t^x \tag{96}$$

It is assumed, as with the other exogenous shocks that $0<\eta_x<1$, and that ε_t^x is a white noise process.

We shall analyze the model under the "optimal" Taylor rule (88) and the "new" *IS curve* (73). Substituting the Taylor rule (89) in the "new" IS curve (73), and then using the "augmented new" Phillips curve (95), deviations of inflation from the central bank target are determined by,

$$\begin{aligned} \widehat{\pi}_t \quad = \quad & \frac{\beta(\theta+\phi_y)+\theta+\kappa}{\theta+\phi_y+\kappa\phi_\pi} E_t \widehat{\pi}_{t+1} - \frac{\beta\theta}{\theta+\phi_y+\kappa\phi_\pi} E_t \widehat{\pi}_{t+2} \\ & - \frac{\kappa}{\theta+\phi_y+\kappa\phi_\pi} \nu_t + \frac{\theta(1-\eta_x)+\phi_y}{\theta+\phi_y+\kappa\phi_\pi} x_t \end{aligned} \tag{97}$$

Now, both monetary policy shocks and exogenous inflation shocks affect the inflationary process.

Solving (97) under rational expectations, assuming that the Taylor principle holds, one gets,

$$\widehat{\pi}_t = -\kappa\Lambda_\nu\nu_t + (\theta(1-\eta_x) + \phi_y)\Lambda_x x_t \tag{98}$$

where:

$$\Lambda_x = \frac{1}{(1-\beta\eta_x)(\phi_y + \theta(1-\eta_x)) + \kappa(\phi_\pi - \eta_x)} \tag{99}$$

Λ_ν is defined as in (83).

From (98) the inflationary process now depends on both monetary policy and exogenous inflation shocks. Substituting the inflation process (98) in the "augmented new" Phillips curve (95), we can solve for deviations of output from its "natural" level as,

$$\widehat{y_t} = -(1-\beta\eta_\nu)\Lambda_\nu\nu_t - \frac{1-(1-\beta\eta_x)(\phi_y + \theta(1-\eta_x))\Lambda_x}{\kappa}x_t \tag{100}$$

Driving the parameters of the Taylor rule to infinity would no longer be optimal, as this would no longer eliminate deviations of output from its "natural" level. Even if Λ_ν and Λ_x are driven down to zero, the variance of deviations of output from its "natural level would be positive, and given by,

$$Var(\widehat{y_t}) = \frac{1}{\kappa^2(1-\beta\eta_x)^2}\sigma_x^2 \tag{101}$$

From (67), the optimal policy requires that,

$$\widehat{\pi}_t = -\frac{\zeta}{\kappa}\widehat{y_t} \tag{102}$$

Thus, from (98), (100) and (102), the "optimal" Taylor rule parameters should be such as to ensure that,

$$\Lambda_x = \frac{\zeta}{(\phi_y + \theta(1-\eta_x))(\kappa^2+\zeta)} \tag{103}$$

From (99) and (103) it follows that the parameters of the optimal Taylor rule should satisfy,

$$\phi_\pi = \eta_x + \frac{\kappa^2 + \zeta\beta\eta_x}{\kappa\zeta}(\phi_y + \theta(1-\eta_x)) > 1 \tag{104}$$

The two key parameters of the "optimal" Taylor rule are thus not uniquely defined, but must satisfy the unique linear relationship (104), which depends on all the structural parameters of the model. This is the same result as the non-uniqueness result derived in Alogoskoufis (2016, 2017), who analyzed the optimality of the Taylor rule in a different "new" Keynesian model, with competitive product markets and periodically determined nominal wage contracts.

To facilitate the interpretation of (104), let us assume that the persistence of the inflation shock x is equal to zero, i.e. that η_x=0. Then, (104) can be written as,

$$\phi_\pi = \frac{\kappa}{\zeta}(\theta + \phi_y) > 1 \tag{105}$$

From (105), the response of the optimal Taylor rule to deviations of inflation from target, relative to deviations of output from its "natural" level, should be higher the higher the slope of the "new" Phillips curve κ, the lower the weight of output relative to inflation in the preferences of the central bank ζ, and the lower the inter temporal elasticity of substitution in consumption $1/\theta$, which determines the semi-elasticity of aggregate demand with respect to the real interest rate in the "new" *IS curve.*[12]

In Figure 5 we present the optimal responses of nominal interest rates to inflation ϕ^*_π, measured on the vertical axis, as a function of the weight of output relative to inflation in the preferences of the Central Bank, ζ, measured on the horizontal axis. We have assumed a response to output $\phi_y = 0.5$, as suggested by Taylor (1993), and an intertemporal elasticity of substitution of consumption $1/\theta = 1.5$. A response of nominal interest rates to inflation equal to 1.5, as suggested by Taylor (1993), would be optimal if the Central Bank attached a weight of output relative to inflation equal to about 0.125.

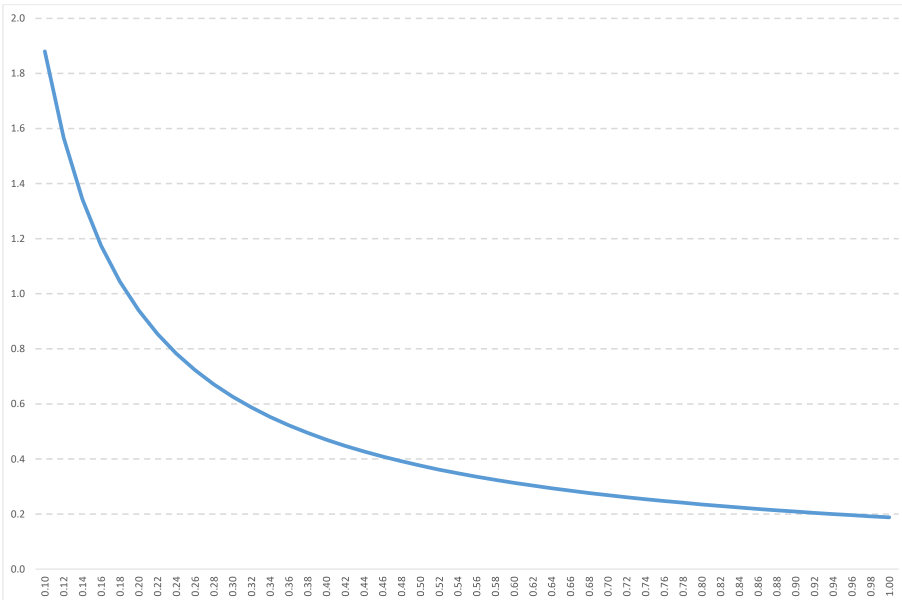
For the Taylor principle to be satisfied, ϕ_π must be greater than one, and the response of the nominal interest rate to excess output ϕ_y should satisfy,

$$\phi_y > \frac{\zeta}{\kappa} - \theta \tag{106}$$

For a sufficiently low weight of output relative to inflation ζ, the optimal policy could thus be parametrized as a Wicksell rule, with ϕ_y equal to zero.

[12] The parameters of the optimal Taylor rule in this model have the same properties with regard to the structural parameters of the model, as the parameters of the optimal Taylor rule in Alogoskoufis (2016, 2017), who relied on a different "new" keynesian model, with periodically determined nominal wage contracts, in which the "divine coincidence" does not arise. Thus, the analysis appears to be quite robust to model specification, as long as the "divine coincidence" is not present.

Figure 5: Optimal Response of Nominal Interest Rates to Inflation as a Function of the Weight of Output relative to Inflation in the preferences of the Central Bank



7 Conclusions

In this paper we use an imperfectly competitive dynamic stochastic general equilibrium model of aggregate fluctuations, in order to study optimal central bank interest rate policy, and compare it to interest rate rules that have been proposed by monetary economists, such as the Wicksell (1898), Fisher (1919) and Taylor (1993).

The model is a dynamic stochastic general equilibrium model based on monopolistic competition in product markets. We initially analyze it assuming full flexibility of prices and, subsequently, assuming staggered price adjustment.

We demonstrate that in versions of the model with full price and wage adjustment, or staggered pricing, the optimal policy rule is the Fisher rule of absolute inflation targeting, which is a special case of the other two. The reason is the "divine coincidence" characterizing the model with staggered pricing, according to which inflation stabilization automatically results in stabilization of fluctuations of output around its "natural" level.

We also analyze a more general version of the model, with exogenous inflation shocks, in which the"divine coincidence" does not apply. In this case, the optimal policy rule takes the form of a Taylor rule. The parameters of the optimal Taylor rule depend on the slope of the Phillips curve, itself a function of the deeper structural parameters of the model, the responsiveness of aggregate demand to the real interest rate, equal to the elasticity of inter-temporal substitution in consumption, and the preferences of the central bank over their two objectives of stabilizing deviations of inflation from target and deviations of output from its "natural" level.

Appendices

A Appendix: The Rotemberg Model of Convex Costs of Price Adjustment

An alternative model of sluggish price adjustment is the Rotemberg (1982 a,b) model of costly price changes.

For the representative monopolistically competitive firm, as the one examined in section 2.2, the optimal price is given by,

$$\overline{P}_t = \frac{\varepsilon}{\varepsilon - 1}\left(\frac{W_t}{(1-a)A_t {L_t}^{-a}}\right) \tag{A1}$$

The optimal price is a constant markup on marginal costs. Marginal costs are equal to wage costs over the marginal productivity of labor. Note that because of decreasing returns to employment, increasing employment and output implies declining marginal productivity of labor and increasing marginal costs of production. Using the production function to substitute out for labor, (A1) can also be expressed as,

$$\overline{P}_t = \frac{\varepsilon}{\varepsilon - 1}\left(\frac{W_t (Y_t)^{\frac{\alpha}{1-\alpha}}}{(1-a)(A_t)^{\frac{1}{1-a}}}\right) \tag{A1'}$$

An increase in output increases the marginal costs of production for given wages, because of the declining marginal productivity of labor. Hence, with higher output the optimal price must rise.

In logs, (A1) and (A1') imply,

$$\overline{p}_t = \mu + w_t - a_t + al_t = \mu + w_t + \frac{1}{1-a}(ay_t - a_t) \tag{A2}$$

where:

$$a_t = \ln A_t, \mu = \ln\left(\frac{\varepsilon}{\varepsilon - 1}\right) - \ln(1-a)$$

α is the logarithm of the exogenous productivity shock, and the constant μ is the logarithm of the markup on marginal cost, minus the logarithm of the coefficient implying decreasing returns to labor.

All firms, are assumed to be facing convex costs of adjusting prices. Rotemberg (1982 a,b) assumes that firms balance the costs of deviating from their optimal price against the costs of adjusting prices. In the model that follows, following Rotemberg, we assume that firms set current prices minimizing a quadratic cost function which penalizes both deviations of prices from the optimal price, and the adjustment of prices over steady state inflation from period to period. This takes the form,

$$\Lambda_t = E_t \sum_{s=0}^{\infty} \beta^s (\frac{1}{2}(p_{t+s} - \overline{p}_{t+s})^2 + \frac{\xi}{2}(p_{t+s} - p_{t+s-1} - \pi^*)^2 \tag{A3}$$

where p is the log of the actual price of the representative firm. ξ is a parameter measuring the cost of price adjustment relative to the cost of deviations from the optimal price and π^* is steady state inflation.

From the first order conditions for the minimization of (A3), it follows that,

$$\begin{aligned} p_t = & \frac{1}{1+\xi(1+\beta)}\overline{p}_t + \frac{\xi}{1+\xi(1+\beta)}p_{t-1} \\ & + \frac{\xi\beta}{1+\xi(1+\beta)}E_t p_{t+1} + \frac{\xi(1+\beta)}{1+\xi(1+\beta)}\pi^* \end{aligned} \tag{A4}$$

The current price, in logs, is a weighted average of the optimal price, the past price, the expected future price and steady state inflation. The firm is forward looking, and anticipates the future costs of adjusting prices, so its current price depends not only on its past price, but on its expected future price as well. Since this is the representative firm, we can take its price to be equal to the log of the price level.

Expressing (A4) as an inflation equation, one gets,

$$\pi_t - \pi^* = \beta(E_t \pi_{t+1} - \pi^*) + \frac{1}{\xi}(\overline{p}_t - p_t) \tag{A5}$$

where: $\pi_t = p_t - p_{t-1}$ is the rate of inflation.

Deviations of inflation from steady state inflation deviate from expected future deviations of inflation, to the extent that the optimal price exceeds the current price. Substituting for the optimal price from (A2), one gets,

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{1}{\xi}(\mu + w_t - a_t + al_t - p_t) \tag{A6}$$

where, $\widehat{\pi}_t = \pi_t - \pi^*$.

From (A6), deviations of inflation differ from discounted expected future deviations, to the extent that the marginal cost of production plus the optimal price markup exceeds the current price. Using the labor and product market equilibrium conditions to substitute out for the real wage and employment, as well as the definition of the "natural rate" of output, we can express (A6) as,

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \kappa \widehat{y}_t \tag{A7}$$

where $\kappa = \frac{\theta(1-a)+\lambda+a}{\xi(1-a)} > 0$ and $\widehat{y}_t = y_t - y_t^N$

(A7) has exactly the same form as the "new" Phillips curve (40) derived from the Calvo (1983) model of staggered pricing. The only difference is in the definition of κ which is now in terms of the parameter ξ of the Rotemberg model, instead of the parameter γ of the Calvo model. Thus, the two models of sluggish price adjustment, the Rotemberg model of costs of adjustment of prices and the Calvo model of staggered pricing are observationally equivalent at the aggregate level.

B Appendix: Expressing the "New Keynesian" Model in Matrix Form

A second way of solving the "new" Keynesian model (68), (69) and (70), is to write it in matrix form, and use a simultaneous equations solution method, such as the Blanchard Kahn (1980) method.

After substituting the Taylor rule (70) in (68) and (69), the model can be written in matrix form, as,

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{\pi}_t \end{pmatrix} = \frac{1}{\theta + \phi_y + \kappa\phi_\pi} \begin{pmatrix} \theta & 1 - \beta\phi_\pi \\ \theta\kappa & \kappa + \beta(\theta + \phi_y) \end{pmatrix} \begin{pmatrix} E_t\widehat{y}_{t+1} \\ E_t\widehat{\pi}_{t+1} \end{pmatrix} + \frac{r_t^N - \rho - \nu_t}{\theta + \phi_y + \kappa\phi_\pi} \begin{pmatrix} 1 \\ \kappa \end{pmatrix} \tag{B1}$$

We can confirm from (B1) that the fluctuations of excess output and inflation are driven by both types of shocks. Real shocks which affect $r_t^N - \rho$, and the nominal interest rate shock ν.

Recalling the definition of κ, one can also confirm that the parameters determining aggregate fluctuations depend on the preferences of the representative household, (θ, λ, ε and ρ), the technology of production (α), market structure (ε), the price adjustment mechanism (γ) and the parameters of the monetary policy rule (ϕ_π and ϕ_y).

Given that both excess output and inflation are non predetermined variables, the solution will be unique only if the matrix of coefficients of future expectations has both eigenvalues inside the unit circle.

Under the assumption that the coefficients of the Taylor rule ϕ_π and ϕ_y are positive, one can show that a necessary and sufficient condition for a unique solution is,[13]

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \tag{B2}$$

(B2) is the same as (73). It requires a sufficiently pronounced reaction of nominal interest rates to inflation, as, solving for ϕ_π , (B2) can be expressed as,

$$\phi_\pi > 1 - \frac{1 - \beta}{\kappa}\phi_y$$

which is non other than (74).

[13] See Bullard and Mitra (2002).

References

[1] Akerlof G. and Yellen J. (1985), "A Near-Rational Model of the Business Cycle with Wage and Price Inertia", *Quarterly Journal of Economics*, Vol. 100, Supplement, pp. 823-838.

[2] Alogoskoufis G. (2016), "Unemployment Persistence, Inflation and Monetary Policy in a Dynamic Stochastic Model of Natural Rate", Working Paper.

[3] Alogoskoufis G. (2017), "The Clash of Central Bankers with Labour Market Insiders, and the Persistence of Inflation and Unemployment", *Economica*, Vol. 85 (337), pp. 152-176.

[4] Ball L. and Romer D. (1990), "Real Rigidities and the Non-Neutrality of Money", *Review of Economic Studies*, Vol. 57, pp. 183-203.

[5] Bernanke B.S. (2006), "Monetary Aggregates and Monetary Policy at the Federal Reserve: A Historical Perspective", Speech at the 4th ECB Central Banking Conference, Frankfurt, Board of Governors of the Federal Reserve System.

[6] Blanchard O.J. and Gali J. (2007), "Real Wage Rigidities and the New Keynesian Model", *Journal of Money Credit and Banking*, Vol. 39, Supplement, pp. 35-65.

[7] Blanchard O.J. and Kahn C. (1980), "The Solution of Linear Difference Equations under Rational Expectations", *Econometrica*, Vol. 48, pp. 1305-1311.

[8] Blanchard O.J. and Kiyotaki N. (1987), "Monopolistic Competition and the Effects of Aggregate Demand", *American Economic Review*, Vol. 77, pp. 647-666.

[9] Bullard J. and Mitra K. (2002), "Learning about Monetary Policy Rules", *Journal of Monetary Economics*, Vol. 49, pp. 1005-1129.

[10] Calvo G. (1983), "Staggered Prices in a Utility Maximizing Framework", *Journal of Monetary Economics*, Vol. 12, pp. 383-398.

[11] Clarida R., Gali J. and Gertler M. (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature*, Vol. 37, pp. 1661-1707.

[12] Fisher I. (1896), Appreciation and Interest, Publications of the American Economic Association, Vol. 11, pp. 1-98.

[13] Fisher I. (1919), “Stabilizing the Dollar”, *American Economic Review*, Vol. 9, pp. 156-160.

[14] Fisher I. (1930), The Theory of Interest, New York, Macmillan.

[15] Friedman M. (1960), A Program for Monetary Stability, New York, Fordham University Press.

[16] Friedman M. (1968), “The Role of Monetary Policy”, *American Economic Review*, Vol. 58, pp. 1-17.

[17] Gali J. (2008), Monetary Policy, Inflation and the Business Cycle, Princeton N.J., Princeton University Press.

[18] Gali J. (2011), Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective, Cambridge Mass., The MIT Press.

[19] Hume D. (1752), “Of Interest; Of Money” in his Essays, Moral, Political and Literary, London, Longmann Green.

[20] Lucas R.E. Jr (1972), “Expectations and the Neutrality of Money”, *Journal of Economic Theory*, Vol. 4, pp. 103-124.

[21] Mankiw G. (1985), “Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly”, *Quarterly Journal of Economics*, Vol. 100, pp. 529-539.

[22] McCallum B.T. (1981), “Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations”, *Journal of Monetary Economics*, Vol. 8, pp. 319-329.

[23] Poole W. (1970), “Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model”, *The Quarterly Journal of Economics*, Vol. 84, pp. 197-216.

[24] Rotemberg J. (1982a), “Monopolistic Price Adjustment and Aggregate Output”, *Review of Economic Studies*, Vol. 44, pp. 517-531.

[25] Rotemberg J. (1982b), “Sticky Prices in the United States”, *Journal of Political Economy*, Vol. 90, pp. 1187-1211.

[26] Sargent T.J. and Wallace N. (1975), “Rational Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule”, *Journal of Political Economy*, Vol. 83, pp. 241-254.

[27] Simons H. (1936), “Rules versus Authorities in Monetary Policy”, *Journal of Political Economy*, Vol. 44, pp. 1-30.

[28] Smets F. and Wouters R. (2003), “An Estimated Dynamic Stochastic General Equilibrium Model for the Euro Area”, *Journal of the European Economic Association*, 1, pp. 1123-1175.

[29] Taylor J.B. (1993), “Discretion versus Policy Rules in Practice”, Carnegie-Rochester Conference Series on Public Policy, Vol. 39, pp. 195-214.

[30] Wicksell K. (1898), Interest and Prices, (English translation, Kahn R.F. 1936), London, Macmillan.

[31] Woodford M. (2001), “The Taylor Rule and Optimal Monetary Policy”, *American Economic Review*, Papers and Proceedings, Vol. 91, pp. 232-237.

[32] Woodford M. (2003), Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton N.J., Princeton University Press.

[33] Yun T. (1996), “Nominal Price Rigidity, Money Supply Endogeneity and Business Cycles”, *Journal of Monetary Economics*, Vol. 37, pp. 345-370.