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Real interest rate and monetary policy in the post

Bretton Woods United States

George C. Bitros[1] and Mara Vidali[2]

Abstract

Using data for the United States over the period 1973-2019, we find that there is a long-run stable relationship showing that the real interest rate is determined by real investment, real money supply, the velocity of circulation, and the rates of wages, economic growth, technological change, and population. Moreover, upon an arbitrary shock in one of its aforementioned determinants, holding all the rest fixed, we find that the real interest rate returns to the long-run equilibrium within a few years. The estimated equation is derived by embedding monetary policy in a general equilibrium growth model of heterogeneous capital. From the latter's theoretical analysis, we expected that some instability might show up in the data because of the negative real interest rates, twice in the 1970s and then again in the early 2010s. However, no such evidence emerged, perhaps because the bias of monetary policy at those times was away from the zero bound and the real interest rate turned negative due to unexpected but transient shocks. The same of course may not be the case if the model is tested with data after 2019, when the bias of the monetary policy reversed, driving the nominal interest rates very close to the zero bound and thus, probably, giving rise to the current signs of economic instability.

Keywords: Real interest rate, growth model of heterogeneous capital, monetary policy, inflation, short- and long-run dynamics.

JEL Codes: E43, E52, C22, C51

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1. Introduction

For reasons that will become apparent shortly, we wish to start by drawing on a relevant well-researched country episode for reference and comparison. Suppose we lived in the proto-capitalist economy of classical Athens and observed the framework of state institutions and markets, within which money was produced and served as a medium of exchange and store of value, the *Attic* drachma. This currency was based on the silver standard. A state-owned mint cut and officially stamped coins of various denominations for the state, as well as for local and foreign economic agents who brought silver bullion for conversion into Athenian coins. Credit in the form of loans was furnished by bankers cum banks and much less so by individuals. There was no central bank and no lender of last resort. The state provided and enforced all necessary institutions for safeguarding the integrity of the currency and the process of competition as in a setup of free banking. Lastly yet importantly, the *nominal* and the *real* prices of goods and services, including the interest rate, coincided because, according to the evidence discussed in Bitros (2022, 18-20), there was no general inflation.

What was the nature of the interest rate and how was it determined? The answer is easy. The interest rate was the price of a scarce resource, that is, savings, and it was determined in a market where the supply of savings met their demand by households, enterprises, and the state, seeking to borrow for consumption and investment. The interest rate in this setting could never be zero, and much less negative. For, a zero interest rate would imply that savings had transformed into a free good, whereas a negative interest rate would signify that the political, economic, and social circumstances had become so pervert that the virtues of saving and lending as private activities promoting the best interests of society had turn into vices to be avoided. Alternatively, would savers become themselves investors? Some would even if they lacked the necessary know-how and risked losing their savings on top of wasting their precious time. But most would not and the only loser would be the economy and the society at large.

Now let us move fast forward by switching to the United States (US), the political institutions of which, as well as the currency, the US dollar, stand in the world today much like the Athenian drachma stood over the Eastern Mediterranean and beyond. What is the nature of the interest rate in the US and how is it determined? The answer is complicated, but not difficult to articulate by drawing on the narratives found in the macroeconomics textbooks. Authorized by the US Congress to handle all issues of money is a central bank called U. S. Federal Open Market Committee (FOMC) or the Fed. In 1972, the latter stopped converting the U. S, dollar into gold, and hence since then the currency has been based exclusively on the fiat standard. Aside from serving as lender of last resort to the banking system, the Fed is mandated by the U. S. Congress to pursue price stability, maximum employment, economic growth consistent with the economy's productive potential, and low long-term interest rate. Even though their achievement is scientifically doubtful, the Fed pursues these objectives using two main policy instruments, that is, the quantity of money supplied, up to the 1990s, and ever since the discount rate or the so-called Federal Funds Rate (FFR).

With these brief details in mind, let us return now to the question. The Fed has defined price stability to apply at an inflation rate in the neighborhood of 2%. Combining this target rate of inflation with an estimate of the rate of economic growth and the economy's productive potential, and hence maximum employment, the Fed arrives at an estimate of the "natural or neutral interest rate r^*.[3] In turn, the difference of this interest rate from some market proxy becomes the basis for injecting or withdrawing stimulus. While this setup aims at attaining the policy objectives over the business cycle, the Fed does not remain indifferent to its mandate for a low r^* in the long run. Figure 1 displays a possible sequence of actions in this regard. The Fed positions itself in the credit

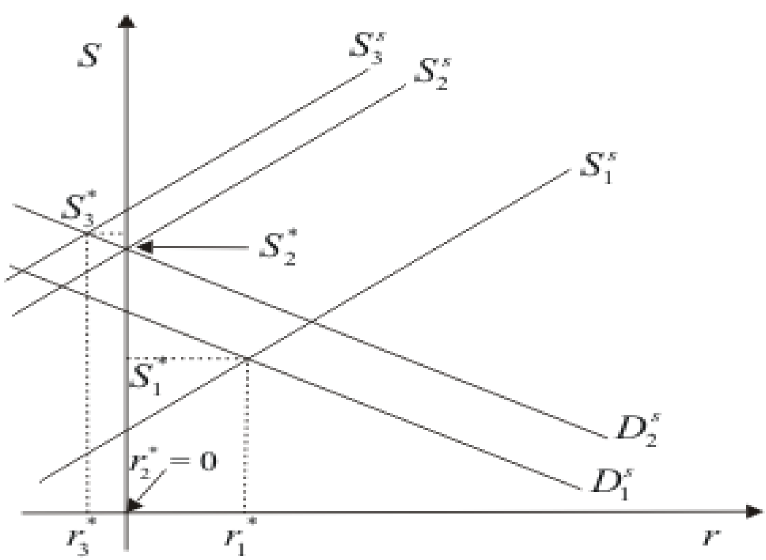

Figure 1: Supply and demand for "savings"

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[3] The great majority of researchers call r^* natural or neutral interest rate and define it as the equilibrium *real* interest rate that would prevail in the short- and in the long-run in an economy where money is managed by a central bank; inflation is kept at a constant rate targeting maximum employment commensurate with economic growth close to the economy's productive potential; there are no natural or man-made supply and demand shocks; prices and wages are flexible; and competition reigns in the markets for goods and services. This definition is purposely counterfactual to allow for the possibility of identification and estimation of the effects that transpire when its equilibrium is disturbed due to various shocks. As such, r^* is unobservable, and hence it must be inferred from the data.

market, along with household and enterprise savers, by offering "artificial" savings in the form of new money. Having done so persistently in recent decades, the supply curve of such "savings" has shifted from S_1^s, where the equilibrium interest rate was r_1^*, to the supply curve S_2^s, where the interest rate is now close to zero. Presumably this development has transpired because, as the supply of "savings" expanded, the demand for investment increased at a slower pace and, by shifting only from D_1^s to D_2^s, the interest rate declined from $r_1^*$to r_2^*. As for point S_3^s, this is reminiscent of Fed's policies in the 1970's, and less so in the late 2010s, which pushed $r*$ even to negative levels as depicted by r_3^*.

In sharp contrast to the above analysis, the bulk of the literature on this issue attributes the persistent decline of the interest rate since the mid-1980s to structural shifts that cannot be influenced by the Fed. For example, Brand, Bielecki, Penalver (2019) find that responsible are developments such as the slowdown in the growth rates of the labor force and the Total Factor Productivity (TFP), the decline in the labor participation rate, the ageing of the population, inequality, the flight to safe assets, and the global saving-investment imbalances. Certainly, some adverse structural shifts are not in the Fed's tasks to mind. Regarding them, we know that they lie in the domain of other institutions in free societies to control or even try to reverse. However, as suggested by Bitros (2018), the Fed cannot be absolved from all responsibility for the spectacular decline of the labor share in the US and, given that it manages the pre-eminent currency in the world, it cannot be considered alien to the global imbalances in the saving-investment nexus. Moreover, if Bernanke's (2005) hypothesis, while he was chair of the Fed, and the related findings by Rachel, Smith (2015) attest to a global glut of savings, then look at the data exhibited in Figure 2 and ask. How can we explain the negative interest rates that prevailed for several years five decades ago in the US and Europe? Was there at the time a savings glut in both sides of the Atlantic? In our view, there existed no glut of "true" savings then[4] and there is no glut of such savings now. As noted above, the negative interest rates then and now reflect a long-term deceleration of investment, the root causes of which have to do with at least two ominous trends. Namely, the worsening structure of Western type democracies, and the way in which elected governments have come to apply their monopoly power over money, especially after the abolishment in 1972 of the convertibility of the U. S. dollar into gold.

[4] To dispel any doubts that savings were in short supply on both sides of the Atlantic in those decades, see Blanchard, Summers (1984), and particularly the accompanying comments by the world-renowned economists who participated in the discussion of this paper.

Figure 2: Real ten-year sovereign bond yields

Source: Lane, P. R. (2019), "Determinants of the real interest rate," https://www.ecb.europa.eu/press/key/date/2019/html/ecb.sp191128_1~de8e7283e6.en.html.
Notes: Real sovereign bond yields are calculated as the difference between the nominal yield in year *t* and realized inflation in year *t*. The euro area series is based on GDP-weighted data for Germany, France, Italy, and Spain. Data for Italy before 1991, for Spain before 1980 and for Germany, France, and the United States before 1973 are based on Jordà et al. (2019), op. cit. Latest observation: 2019.

To corroborate this view, Figure 3 displays the average age of the capital stock in the U. S. private business sector over the last 60 years. The data are based on historical cost prices of structures, equipment, and intellectual property rights, but the graph remains unchanged if instead current cost prices are employed. Observe that before the mid-1980s the capital stock was getting younger, whereas since then it has been getting older. The reason for this development is profound. In the earlier period, even though according to Figure 2 the interest rate was trending upwards, investment accelerated both quantitatively and qualitatively, and incorporated into the capital stock vintages of producers' durables embodying the most recent achievements of science and technology.

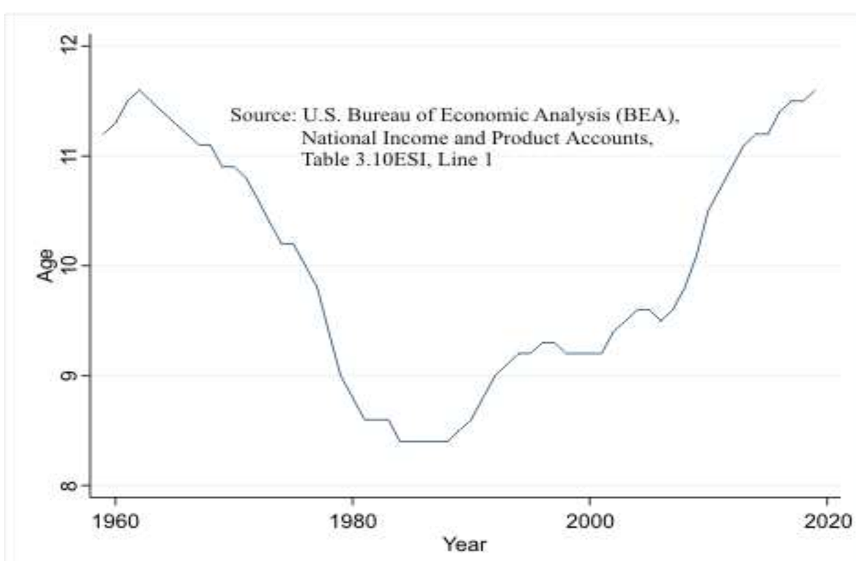

Figure 3: Average age of private business fixed assets
(Historical cost prices)

Thus the average age declined. On the contrary, after the mid-1980s, even though the interest rate trended downwards, gross investment decelerated, the process of speedy renewal of the productive capital reversed, and its average age has been rising ever since.

A convenient first step to explain these developments is to look through the lenses of the steady state solution of Solow's (1956) celebrated economic growth model. This takes the form of the following equation:

$$r = a(\frac{g+n+\delta}{s}), \quad (1')$$

where r corresponds to r^*, α denotes the marginal product of the accumulated capital stock, g and n stand respectively for the growth rates of output and population, δ is the depreciation rate, and s represents the rate of saving. Consider the developments since the mid-1980s. Since the average age of the capital stock was increasing, δ was falling. As the latter declined, given that $\partial r/\partial\delta > 0$, r declined as well. This first round effect though did not stop there. With the capital stock growing older and incorporating newer technologies at a slower pace, its marginal product, that is α, declined. In turn, this reduced the contribution of capital to the rate of economic growth and precipitated further declines in the interest rate, because $\partial r/\partial g > 0$. Moreover, since workers were working with implements that were getting older, the decline in the productivity of labor pushed g also downward. The interest rate depressing effects of the declining growth rate of the population have been confirmed by multiple sources, so we can turn immediately to the saving rate for which it holds that $\partial r/\partial s < 0$. In addition to loans from the banking system and the market, enterprises finance their investments through retained earnings and funds freed via the depreciation accounting process. In the US though the statutory depreciation service lives of capital goods are updated very slowly, so that depreciation allowances most likely exceeded significantly the necessary funds for replacement investment.[5] For, otherwise, the practice whereby corporations return capital to shareholders by buying back company shares would not have been adopted as widely as in recent decades. Apparently, abundant internal sources of funds, in conjunction with easy credit conditions that the Fed propagated throughout this period, pushed the rate of "artificial" and "true" savings systematically upwards and the interest rate downwards. On account then of all the above,

[5] Although this point derives from a long forgotten literature, it is still worth recalling that without any emphasis on the useful life of producers' goods and the crucial role of the interest rate, Eisner (1952) and Domar (1953) found that depreciation allowances exceed replacement requirements in a growing economy with stationary prices.

what would be surprising and in need of explanation is if the interest rate had not declined as spectacularly as exhibited in Figure 2.

Yet, irrespective of how informative they may be, inferences drawn from the steady state of an economic model limit their explanatory power to an environment where the effects from various shocks on the variables of interest have adjusted optimally and remain stationary over the long haul. Herein we are interested in the transition paths to the steady state and this renders it imperative to switch to a complete model which, in order to allow us to replicate the above analysis and at the same time highlight the short-run dynamics, it must be centered on the service life of producers' goods and the *real* interest rate.[6] Our research objective is to tweak a long forgotten general equilibrium model based on the useful life of an infinite series of heterogeneous capital goods; to derive an equilibrium relationship for the interest rate; and to estimate it in two consecutive phases. First, using data from the US, so that we may have the opportunity to study the effects of monetary policy on the interest rate in a world currency managing country, and secondly, to extend the estimations using panel data from a sizable sample of advanced, emerging, and underdeveloped countries. This paper focuses on the US and it is organized analogously.

2. The interest rate in the growth model with heterogeneous capital

In a series of papers presented several years ago, one of the co-authors drew on the dynamic general equilibrium model presented by Brems (1968, 473-503) to highlight certain important issues in the realm of capital theory. For example, in Bitros (2008) the model was employed to explain the reasons why the structure of capital and the useful lives of its constituent components may offer precious insights regarding the whims of the economy's business cycles. However, the focus in these papers was on the microeconomic dynamics that emanate from the presumed behavior of firms and their macroeconomic implications were held in abeyance. As a result, the strong potential of the model to shed light on such crucial contemporary issues as, for example, the slowdown of economic growth, the worrisome developments in the distribution of income, the destabilizing asymmetries of fiscal and monetary policies, etc. remained unexploited. Therefore, what we intend to do in this section is twofold. Initially, to shift attention from the microeconomic to the macroeconomic dynamics of this model, so as to shed light on the determinants of r^* in the absence of a central bank, and then, by relaxing this assumption, to bring the analysis closer to the U.S economy and the implications of the monetary policies conducted by the Fed.

[6] Unless indicated otherwise, henceforth the *real* interest rate will be referred to as interest rate and denoted by *r*.

2.1. The interest rate in the absence of a central bank

Section 2.1.1 summarizes the microeconomic foundations of the model. Its solution at the representative firm level enables us to derive an equilibrium relationship of the useful life of producers' goods, the interest rate, and the rate of technological change. In section 2.1.2, we present the macroeconomic analysis of the model. This leads to a supply and demand model centered on the equilibrium interest rate of loanable funds for investment purposes in the absence of a central bank in the economy. Lastly, in section 2.1.3, we comment on the nature of prices and money that pertain in the setting of this model.

2.1.1 Microeconomics

We consider a free market economy populated by any number of firms, one of which is representative of all others. Aside from its management, the representative firm comprises two distinct business units or divisions: That is, division C that produces consumer goods and division D that manufactures producers' goods for the C division. This organizational setup implies that the only markets that function in the economy are those of consumer goods and loans. Due to the nature of the issues of interest here, the analysis will focus on the equilibrium in the market where loans are exchanged for investment purposes.[7] However, this choice should have no bearing on the economy-wide equilibrium, since by Walras's law, if equilibrium holds in the one of the markets, it should hold in the other as well.

While for now we assume that in the economy there is neither central bank nor government, later on we shall relax the assumption with regard to the former of these institutions. During year υ, the firm's C division produces a basket of consumer goods in the quantity of $X(\upsilon)$ by combining $L(\upsilon)$ units of labor with $S(\upsilon)$ units of producer's goods newly built and supplied by division D.[8] The production of consumer and producer goods takes place under the following definitions, conventions, and functional specifications aimed at simplicity and generality of the analysis:

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[7] When investigating the determinants of the interest rate, contemporary macroeconomics literature follows two approaches. One, when the emphasis is on the *short run partial equilibrium analysis,* considering that prices may be sticky, and another when the emphais is on the *long-run general equilibrium analysis* and prices may be presumed to be flexible. The model adopted in the former case, called Liquidity Preference (LP), is based on the well-known Keynesian conceptualizations of the demand for money, whereas the model recommended in the latter case, called Loanable Funds (LF), is based on the well-known classical conceptualizations about the *neutrality* of money. In this paper, given that the Fed's target inflation rate is not differentiated between short and long run, we shall bridge the two models by assuming in the context of the LF model that the central bank maintains a given target rate of inflation throughout.

[8] Notice that the amount of newly constructed producer's goods is financed by an equivalent amount of loans extended to firms through saving by the workers who are employed in their D departments. Hence, the symbol for new investment $S(\upsilon)$ stands for the amount of loans.

- The unit of physical or real capital used in the production of consumer goods is defined as the quantity of producer's goods operated by one unit of labor. Hence, we set:

$$L(\upsilon) = S(\upsilon)\,. \tag{1}$$

- The capital-output coefficient is defined as:

$$b(\upsilon) = \frac{S(\upsilon)}{X(\upsilon)}\,. \tag{2}$$

- Owing to technological progress, which proceeds at the rate μ, the capital-output coefficient declines over time as follows:

$$b(t) = e^{\mu(t-\upsilon)} b(\upsilon), \text{ for } t > \upsilon \text{ and } \mu < 0\,. \tag{3}$$

- If k and u denote respectively the minimum labor required for building one unit of producer's goods and its useful or service life, the mechanism that governs their relationship is conceived to take the form:

$$k = \gamma_0 e^{-\gamma_1 u}, \quad \gamma_0, \gamma_1 > 0, \tag{4}$$

Parameter γ_0 reflects the balance of several factors. For example, the level of development of the country in which the representative firm operates is certainly one. Since the nature of productive techniques embodied in the capital stock, say, in the United Sates is more advanced than in Greece or Bangladesh, it takes more direct and indirect labor hours per unit to build. Another factor is the resource endowment of the country. In one rich in unskilled labor, building structures and equipment will rely more on simple labor hours relative to one rich in scientists and engineers. Still a third factor is the amount of resources a country devotes to Research and Development (R&D), given that it is hard scientific work that leads to discoveries of new products and productive techniques.

On the other hand, the rationale for introducing γ_1 is less easy to explain. We do not conceive of it as a means to capture differences in the wages paid to workers in the two departments because, say, of differences in their human capital. In both departments workers are paid the same wage rate $\tilde{w}$. But those in the D department are special because of their crucial contribution to the determination of the "durability" of capital goods they construct. Referring to the latter property, the case is that u is determined much less by the sturdiness of built than technological obsolescence, which, by reducing the revenue earning capacity of producers' goods, shortens their useful lives. On the consumption side, this effect is represented by μ and is captured in the model through equation (5) below. For citizens as consumers it represents a benefit. But for the management of the representative firm, which has to repay the loans through which the building of producers' goods is financed, losing revenues is a minus. For this reason, the management has a strong incentive to keep reducing the period over which funds are tied up in these productive assets. But shrinking *u* increases the minimum labor required per unit of durable goods built and raises the share of the wage bill that goes to the workers in the D department. Hence, the management faces a trade-off, which is reflected in the term $e^{-\gamma_1 u}$. Choosing a relatively high (low) γ_1, the minimum labor requirement does increase (decline), but at the same time u declines (increases) and the firm's productivity and competitiveness improves (worsens). The success or failure of management lies in selecting and adjusting from time to time the value of this parameter.

Lastly, to keep the analysis simple and tractable, we assume that durable goods are produced exclusively with labor.

- The price that the firm expects to emerge in the market for its output follows the rule:

$$P(t) = e^{\mu(t-\upsilon)} P(\upsilon), \quad \text{for } t>\upsilon \text{ and } \mu<0. \tag{5}$$

- The unit cost of producer's goods is described by:

$$p = k\tilde{w} \;, \tag{6}$$

in which $\hat{w}$ denotes the wage rate that serves also as a Walrasian *numeraire*.

- Assuming that capital goods are not subject to wear and tear due to utilization, the net worth $n(\upsilon)$ of each unit of producer's goods, put in place in year υ and lasting for u years, takes the following form:

$$n(\upsilon) = \int_{\upsilon}^{\upsilon+u} \left[\frac{P(\upsilon)}{b(\upsilon)} e^{\mu(t-\upsilon)} - \tilde{w} \right] e^{-r(t-\upsilon)} dt - p = \frac{P(\upsilon)}{b(\upsilon)} \frac{1-e^{(\mu-r)u}}{r-\mu} - \tilde{w}[\frac{1-e^{-ru}}{r} + \gamma_0 e^{-\gamma_1 u}] \,. \tag{7}$$

Looking closer at this expression, several noteworthy remarks are in order. First, observe that the stream of net revenues that the representative firm earns is discounted by the factor $(r-\mu)$ for $\mu < 0$. As ascertained by equation (5), this implies that the benefits from technological progress pass on to consumers in the form of reduced consumer prices. Second, in the absence of a central bank, the deflation that productivity improvements bring about drives a *positive* wedge between the current and the equilibrium values of r. On the contrary, in the presence of a central bank, monetary policy induced inflation drives a *negative* wedge between these values. The significance of this difference is that, whereas in the former case the wedge represents *real* benefits for consumers in the form of goods and services, in the latter case the wedge is purely *nominal* since it amounts solely to a change in the general level of prices. Third, on top of their wages, the workers in the D department may earn a second stream of income revenues. For, if they manage to improve on the management's choice of *u*, say $by - \Delta u$, they will be paid for extra time $\Delta k = e^{-\gamma_1 \Delta u}$ per unit of producers' goods built. The revenues from this stream, together with those that spring from the function of these workers as lenders of funds for the financing the firm's producers' good, may give rise to "income inequality". In the model, the latter is considered socially beneficial rather objectionable, because it mobilizes the "human capital" of these workers to the benefit of citizens at large.

- Dividing (3) and (5) side by side yields:

$$\frac{P(\upsilon)}{b(\upsilon)} = \frac{P(t)}{b(t)}. \tag{8}$$

Interpreted in conjunction with (8), this equation ascertains that the net worth of a unit of producers' goods is invariant with respect to the time it the firm put it in place. This is as it should be because the output price $P(\upsilon)$ and the capital-output coefficient $b(\upsilon)$ decline at the same rate, leaving the revenue stationary.

- Lastly, with an eye towards obtaining analytically manageable results, it is postulated that in the market for consumer goods the firm faces the following constant-elasticity-of-substitution demand curve:

$$X(\upsilon) = M(\upsilon)[P(\upsilon)]^{\eta}, \quad \text{for } \eta < -1, M(\upsilon) > 0, \ P(\upsilon) > 0, \ X(\upsilon) > 0. \tag{9}$$

Assuming that the firm behaves as if to maximize the net worth of an endless stream of investments $S(0), S(u), S(2u)\cdots$, it can be shown that the optimal solution is found by equating to zero the partial derivatives with respect to $P(0)$ and u of the following objective function:

$$A(0) = b(0)M(0)[P(0)]^{\eta}\left[\frac{P(0)}{b(0)}\frac{1}{1-\mu} - \frac{\tilde{w}}{r}\frac{1-e^{-ru}+r\gamma_0 e^{-\gamma_1 u}}{1-e^{(\mu-r)u}}\right]. \qquad (10)$$

Here, we shall focus first on the partial derivative of (10) with respect to u,[9] which is given by (11) below, and permits neither an explicit solution for u nor some other form of approximation:

$$r[e^{-(r-\gamma_1)u} - \gamma_0[(r-\mu)e^{-(r-\mu)u} + \gamma_1(1-e^{-(r-\mu)u})]]e^{-\gamma_1 u} - (r-\mu)e^{-(r-\mu)u} - \mu e^{-(2r-\mu)u} = 0 \qquad (11)$$

To characterize its properties in the setting of the US economy, we solved (11) numerically for u using the following empirically reasonable values for: $(r = 0.02,\ 0.03,\ 0.04,\ 0.05)$, $(\mu = -0.01, -0.015,\ -0.02,\ -0.025)$, $(\gamma_0 = 3,\ 3.5,\ 4,\ 4.5)$, while we adjusted the values of γ_1 to achieve convergence at $(\gamma_1 = 0.000,\ 0.0005,\ 0.001,\ 0.0015)$. As it can be ascertained by looking in Appendix A, from the solutions it emerged that u relates positively to r and negatively to μ for all values of (γ_0, γ_1). Is this finding consistent with the evolution of the interest rate as exhibited in Figure 2 for the post war period? The following analysis is in favor of a positive answer.

A good proxy for μ is the rate of change of total economy output per hour worked. According to Gordon (2015, 24), throughout the post war period in the US this index trended downwards, starting from around 3% in 1953 and declining down to around 1% in 2013. Placed in the context of our numerical solution of (11), this evidence suggests that the useful life of the capital goods would be expected to increase, the depreciation rate δ to decline, and hence by equation (1') the interest rate to trend downwards. Figures 2 and 3 show that, while this prediction is consistent with the data after the middle of the 1980s, the same does not hold in the earlier period during which r trended upwards even though $|\mu|$ was decreasing. Stated differently, it is odd that the useful life of the capital stock declined (see Figure 3) in the earlier years. The depreciation rate increased. And the interest rate climbed (see Figure 2), while technological progress was decelerating. We believe that this contradiction is more apparent than real, because the evolution of the growth rates of the other variables that are present in equation (1') have not been allowed for as yet. Thus, to facilitate the extension of our model in this direction, instead of (11) we adopted (12), which has similar properties.

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[9] To ascertain that the second-order conditions for a maximum are also satisfied, please see Brems (1968, 165-166).

$$u = \frac{1}{\mu - r + \lambda \frac{\gamma_0}{\gamma_1}}, \ \mu - r + \frac{\gamma_0}{\gamma_1} > 0, \ 0<\lambda<1.$$

$$\frac{\partial u}{\partial r} > 0, \ \frac{\partial u}{\partial \mu} < 0, \ \frac{\partial u}{\partial \gamma_0} > 0, \ \frac{\partial u}{\partial \gamma_1} < 0. \qquad (12)$$

As we turn to the derivation of the demand curve for loans in the money capital market,[10] this approximation, where λ stands as an appropriate scaling factor, will prove very useful.

2.1.2 Macroeconomics

Now, let the economy consist of firms all of which are exactly as the one described above. If so, within any year the economy will produce two goods, i.e. $X(v)$ *and* $S(v)$, of which the former remains the same from one year to the next whereas the latter changes according to (4). Additionally, we introduce the following conceptualizations. In the economy, there exists a money capital market where firms may borrow, and households may lend funds at the annual rate of interest r with continuous compounding. The labor force is a fix proportion α of the population $N(t)$, which grows at the annual rate n. Lastly, it is assumed that of the workers only those who are employed in the D divisions of the firms save money from their income and extend loans. The objective below is to trace the demand curve for loanable funds at the economy level.

2.1.2.1 The economy-wide demand for loanable funds

The mass of the physical or real capital stock $K(t)$ used in the economy may be obtained by means of the following integral:

$$K(t) = \int_{t-u}^{t} S(v)dv. \qquad (13')$$

In view of the controversies in the relevant literature regarding the units of measurement of this mass, it should be noted that while producers' goods of different vintages differ in quality, their quantity is well defined because of (1).[11] Moreover, it is assumed that producers' goods are physically and productively infinitely durable, and that the termination of their service life at u comes

[10] As we shall explain later on, money in this model is in the form of debits and credits in the accounting books of the firms that operate in the economy.

[11] It should be noted that, since by (1) the variable of gross investment is measured in labor units, no monetary values are involved in (13′), and hence, there arise no issues of aggregation.

about because the advancing technology renders them obsolete with zero resale value. [12] Considered in this way, let $S(\upsilon)$ grow at the rate n per annum, which coincides with the growth rate of the population $N(t)$:

$$S(t) = e^{n(t-\upsilon)} S(\upsilon) . \tag{13″}$$

Now, inserting (13″) into (13′) yields:

$$K(t) = \int_{t-u}^{t} e^{-n(t-\upsilon)} S(t) d\upsilon = \frac{1-e^{-nu}}{n} S(t) . \tag{13}$$

From this equation we see that since $S(\upsilon)$ is growing at the rate n per annum, the accumulated capital stock is growing at the same rate.

Next, using (13) in conjunction with (1) and (4), the structure of full employment is given by:

$$\bar{L}(t) + \bar{l}(t) = [\frac{1-e^{-nu}}{n} + \gamma_0 e^{-\gamma_1 u}] S(t) = \alpha N(t),\ 0<\alpha<1. \tag{14}$$

In this expression the symbols $\bar{L}(t)$ and $\bar{l}(t)$ stand for the workers employed in the industries of consumers and the producers goods, respectively, whereas α denotes the labor force participation rate. Observe from the middle expression that time enters only through $S(t)$, which by (13″) grows at the rate n per annum. This ascertains that full employment is guaranteed because by definition, with given α, the population $N(t)$ and the labor force are growing at the same rate.

Again though, equation (14) cannot be solved explicitly for u. Hence, linearizing it at $u = 0$, we obtained the following condition for the service life of the capital stock at the economy level:

$$u = \frac{\alpha N(t) - \gamma_0 S(t)}{(1+\gamma_1) S(t)},\ \text{for } \alpha N(t) - \gamma_0 S(t) > 0. \tag{15}$$

This equation, if satisfied, constitutes the equilibrium condition for the service life of capital at the full employment level of the economy, whereas equation (12) gives the equilibrium condition for the optimal service life of capital at the level of the representative firm. Therefore, for equilibrium to attain at the micro and macro levels of the economy, both equations should give the same value for u. Thus, substituting equation (12) into equation (15) and rearranging yields:

[12] Thus, absent from the determination of the useful life of capital are *operating policies* like the intensity of utilization and maintenance, and *capital policies* like abandonment and scrapping. For a detailed analysis of the latter policies, and how they affect the useful life of producer's goods, see Bitros, Flytzanis (2002, 2004).

$$r^d = \mu + \lambda\frac{\gamma_0}{\gamma_1} - \frac{(1+\gamma_1)S(t)}{\alpha N(t) - \gamma_0 S(t)}, \quad \frac{\partial r^d}{\partial S(t)} < 0. \tag{16}$$

Regarding this equation, two comments are in order. First, recall that both $S(t)$ and $N(t)$ grow at the rate n. Consequently, (16) remains stationary over time and only irregular changes in the rate of investment or the population may affect the demand price of loanable funds. Second, r^d relates positively to the population, $N(t)$, the labor participation rate, α, and the effective unit construction cost of producers' goods, $\lambda\gamma_0/\gamma_1$, and negatively to the loans demanded for business investment, $S(t)$. These findings are as they could be expected from economic theory. But why r^d relates positively to technical change μ is less obvious. An explanation is that, as technical change increases, the demand for replacement investment increases, the demand curve for loans in Figure 1 shifts to the right, and this in turn raises the interest demanded for all levels of loans.

Our next undertaking is to trace the equation for r^s, that is, the price of loans offered by savers for investment purposes.

2.1.2.2 The economy-wide supply of loanable funds

Recall that by assumption the workers in the C departments consume all their income, whereas those in the D departments save some part and lend the funds to firms to cover the expenses for new investment. Hence, the workers in the D departments of the economy receive each period three streams of revenues. The first consists of their wages:

$$Y_1 = \gamma_0 e^{-\gamma_1 u}\tilde{w}S(t). \tag{17'}$$

The second stream comprises the interest income that that they earn on the undepreciated value of the capital stock. Equation (13) expresses the entire physical capital stock in terms of the output of producers' goods of the latest vintage within it. Hence, multiplying it by $p = \gamma_0 e^{-\gamma_1 u}\tilde{w}$, the interest income earned by these workers is given by:

$$Y_2 = \left[\gamma_0 e^{-\gamma_1 u}\frac{1-e^{-nu}}{n}\tilde{w}S(t)\right]r \tag{17''}$$

Lastly, there is the issue of technological obsolescence. Due to the advancing technology, capital goods in place loose income-earning power. In actuality, firms accumulate funds in the so-called depreciation reserve account, and draw on them to replace capital goods at the end of their service lives. For simplicity, here we assume that depreciation funds due to technological obsolescence are used to repay debt, and hence, that the said workers receive each period:

$$Y_3 = \gamma_0 e^{-\gamma_1 u} \frac{1-e^{-nu}}{n}(-\mu)\tilde{w}S(t) \tag{17"}$$

In view of these three streams of revenues, the workers in the D departments of the economy are perceived to allocate them between consumption and saving to maximize their level over the useful life of the producers' goods, and at the same time to shift their composition in favor of incomes from capital. To achieve their plan, assuming that they behave rationally, it would be to the advantage of these workers to increase over time the share of $Y_2 + Y_3$ in their total income $Y_1 + Y_2 + Y_3$ by giving more weight to income from capital. They may do so by maximizing with respect to u the expression:

$$F(t) = \left[\gamma_0 e^{-\gamma_1 u}[(\frac{1-e^{-nu}}{n})(r-\mu)e^{(r-\mu)u} + 1] \right] \tilde{w}S(t)\,. \tag{17}$$

Observe that the weighting factor $e^{(r-\mu)u}$ increases over the service life of producers' goods, whereas the share of income from labor enters into the objective function with a weight of 1. This is the second source of benign or creative "income inequality" among the workers in the two departments of firms in the economy.

Linearizing the first order condition from (17) at $u = 0$ yields the expression (18') below. On

$$\begin{gathered} u = \frac{r-\mu+\gamma_1}{\gamma_1^2 + (r-\mu)[2(r-\mu+\gamma_1)-n]}, \\ \frac{\partial u}{\partial r} < 0, \ \ \frac{\partial u}{\partial \mu} > 0, \ \ \frac{\partial u}{\partial n} > 0, \ \ \frac{\partial u}{\partial \gamma_1} > 0 \end{gathered} \tag{18'}$$

closer observation, it turns out that the partial derivative with respect to the interest rate is opposite to that in equation (12). This suggests that the workers in the D departments take an opposite view to that of the managements regarding the interest rate and the useful life of producers' goods. In particular, at high interest rates they would bid for low service lives, whereas the management would pursue high service lives, and the same antithesis holds with reference to the rate of technological change μ. Moreover, observe that now present in (18') is the growth rate of the population, and hence of the labour force n, since the participation rate is held fixed at α. These considerations reflect the different perspectives that managements and workers in these departments take in the context of their cooperation within firms. Through the key decision on the service life of producers' goods each side pursues its interests. But their cooperation does not stop there, because by assumption the workers in the D departments of firms are the only ones that save part of their incomes and extend loans in the money market for the financing of the producers' good constructed each year. In turn, they are the only source of loans and this explains the linkage of savings to the useful life of producers' goods, which takes the form of the supply curve of loanable funds sought.

To this end we proceed as follows. For general equilibrium in the economy, it is imperative that the value of u from (18') is equal to that from (15). Yet, equating the two expressions gives an equation that cannot be solved explicitly for r. This difficulty led us to approximate (18') by the following equivalent expression in terms of the directions in the derivatives:

$$u = \frac{1}{r-\mu-n+\gamma_1}, \; r-\mu-n+\gamma_1 > 0 \qquad (18)$$
$$\frac{\partial u}{\partial r} < 0, \; \frac{\partial u}{\partial \mu} > 0, \; \frac{\partial u}{\partial n} > 0.$$

Thus, using (18) together with (15) we obtained the following supply curve for loanable funds:

$$r^s = \mu + n - \gamma_1 + \frac{(1+\gamma_1)S(t)}{\alpha N(t) - \gamma_0 S(t)}, \; \frac{\partial r^s}{\partial S(t)} > 0. \qquad (19)$$

2.1.2.3 The equilibrium interest rate in the economy

Equations (16) and (19) give respectively the demand and supply functions in the market for loans. For equilibrium in this market, it must hold that:

$$r^d = r^s \qquad (20)$$

Considering equations (16), (19), and (20) as a system, it should be possible in principle to solve for the equilibrium values $(r^*, S^*(t))$, as well as to examine the stability of this equilibrium. In symbolic terms the solution would take the form $r^* = r(S^*(t), N(t); \mu, n, \alpha; \lambda, \gamma_0, \gamma_1)$, whereas in graphic terms it would be given by the cross of the supply and demand curves for loans shown in Figure 4 by point A. Clearly, at this point, the interest rate is positive. So let us ask. In the absence of a central bank, can the interest rate decline to zero or become even negative? Observe that with the pair of demand and supply curves (r^d, r^s), for $S(t) = 0$, the interest rate is $\bar{r}^s = \mu + n - \gamma_1$. By the side condition in (18), this should be positive. Now for demonstration purposes suppose that $n = 0$. Then, ceteris paribus, the supply curve for loanable funds would shift to the position depict-

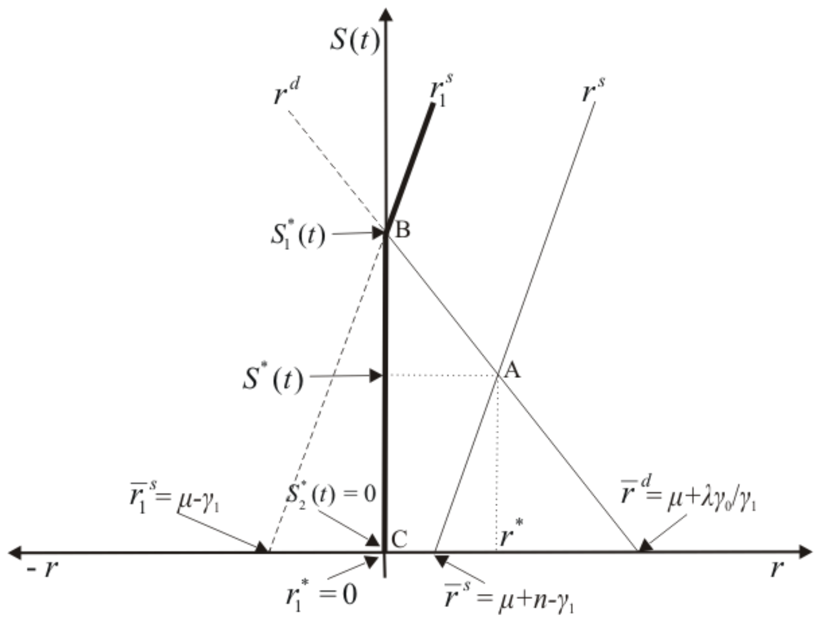

Figure 4: Supply and demand for loanable funds, no Central Bank

ed by r_1^s, thus establishing an interim equilibrium at B and eventually another at C. In the transition from points A to C, the equilibrium interest rate converges to $r_1^* = 0$ and remains there while the equilibrium gross investment declines, initially to $S_1^*(t)$ at point B, and ultimately to $S_2^*(t)$ at point C . This shows that, in the absence of a central bank, the interest rate in the economy may become zero but not negative. However, at points B and C there arise serious issues of economic and social instability that may act as deterrent from ever reaching a state where $r^* = 0$. We relegate their discussion in Subsection 2.3.

2.1.3 Nature of prices and money

In the economy analyzed above, producers' goods are built in the D departments of firms and transferred internally to the C departments at the price $p(t)$. In contemporary jargon the latter is a transfer price, i.e. an accounting price for intrafirm exchanges and record keeping purposes. The price of consumers' goods, as well as the general price level, is denoted by $P(t)$. From the first order conditions for the maximization of equation (10) with respect to consumer goods prices it follows that:

$$P(0) = \frac{\eta}{1+\eta} e^{-\mu u} b(0)\tilde{w} \quad . \tag{21'}$$

Among others, (21') reveals that the general price level depends on the numeraire wage rate $\tilde{w}$, the value of which is defined to be equal to the purchasing power in terms of consumer goods of one unit of labor. The question then that arises is how transactions are carried out in this economy. The answer is that money takes the form of entries in the accounting ledgers maintained by firms and all exchanges in the markets for consumer goods and loans are settled by debits and credits in those ledgers. By contrast, in today's market economies exchanges take place through a medium of exchange and store of value, the quantity or price of which is determined by a central bank. Therefore, the issue is how to tweak the model to render it capable to shed light on the influences that the central bank may exercise on the interest rate r^*.

Our objective in the sequel is to focus on this question by assuming that the loans market in the US is dominated by the Fed. But before turning to this task, two simplifications of (21') are in order. To allow for the first one, we assume that the market for consumer goods is perfectly competitive. This implies that $\eta \to -\infty$ and renders the term $\eta / 1+\eta$ equal to 1. As for the second, using equations (2) and (3) to write $P(0)/b(0) = P(t)X(t)/S(t)$, and substituting into (21'), yields:

$$P(t) = e^{-\mu u}\frac{S(t)}{X(t)}\tilde{w}. \tag{21}$$

This is of interest for the following reason.

So far, under the model, the rate of economic growth is $n-\mu$ and the available labor is fully employed because the capital stock of the economy increases at the rate n, which coincides with the growth rate of the population, and hence of the labor force. Consequently, with the two of the Fed's four mandates met, the ones that remain are price stability and low interest rate in the long-run. Equation (21) is central to tackling these two issues.

2.2. The interest rate in the presence of a central bank

Recall from equation (5) that the prices of consumer goods *P(t)* decline at the rate μ. In other words, under the model the economy experiences deflation because productivity improves at the rate of technological change μ. On the other hand, earlier we mentioned that the Fed defines price stability at $\theta = 0.02$. Therefore, in the light of this objective, let economic agents presume that the Fed follows Buiter's (2007) recommendation by introducing a mechanism relating the U. S. dollar to a unit of account such that at the new numeraire w, the prices of consumer goods increase at the rate $\theta - \mu$. Operationally this can be accomplished by setting $\tilde{w} = we^{(\mu-\theta)u}$. Moreover, let the firms presume that the Fed conducts monetary policies by following some version of the quantity theory of money $M(t)V(t) = P(t)X(t)$, where *M(t)* and *V(t)* stand respectively for the quantity of money and its velocity of circulation, and $P(t)X(t)$ denotes the Gross Domestic Product (GDP). In the model, the latter is given by:

$$P(t)X(t)=(\gamma_0 e^{-\gamma_1 u}+e^{-\mu u}\frac{1-e^{-(n-\mu)u}}{n-\mu})\tilde{w}S(t). \tag{22}$$

So, substituting first in (22) for $\tilde{w}$, and then into the quantity theory of money equation, leads to:

$$M(t)V(t)=(\gamma_0 e^{-(\gamma_1-\mu+\theta)u}+e^{-\theta u}\frac{1-e^{-(n-\mu)u}}{n-\mu})wS(t) \tag{23}$$

Lastly, upon linearization of (23) at $u=0$, we obtain:

$$u_F=\frac{M(t)V(t)-\gamma_0 wS(t)}{[\gamma_0(\theta-\mu+\gamma_1)-1]wS(t)},\ \gamma_0(\theta-\mu+\gamma_1)-1>0, \tag{24}$$

where the new symbol u_F stands for the average useful life of producers' durables as determined by allowing for effects of monetary policies that are channeled to the economy through the quantity of money, $M(t)$, the target rate of inflation, θ, the wage rate, w, and the velocity of circulation, $V(t)$. Equating (24) to (18) and rearranging yields the Fed-influenced supply of loanable funds:

$$r^s_{F(\theta,M)}=\mu+n-\gamma_1+\frac{[\gamma_0(\theta-\mu+\gamma_1)-1]wS(t)}{M(t)V(t)-\gamma_0 wS(t)},\ \frac{\partial r^s_F}{\partial S(t)}>0. \tag{25}$$

Figure 5 shows that for $S(t)=0$ the interest rates $\bar{r}^S_F$ and $\bar{r}^S$ coincide. Given that gross investment is bounded from below to zero, what this finding inplies is that in that event the interest rate $\mu+n-\gamma_1$ that would emerge in the presence of the Fed would be the same as that in its absence (see Figure 4). But, ceteris paribus, for values of $S(t)>0$, the two interest rates would diverge because the curve $r^S_{F(\theta,M)}$ stays always to the right of curve r^S. As a result, perhaps because of the uncertainty that central bank policies introduce in the economic system, and not just the targeted inflation rate, gross investment shrinks by the difference $S^*(t)-S^*_{F(\theta,M)}(t)>0$.[13] Additionally, the Fed strives to achieve a low long-term interest rate. To this effect, it expands money supply $M(t)$ by offering "ficticious" or "fake" savings to reduce the interest rate and stimulate investment. In Figure 5, this policy is represented by the shifts of the $r^S_{F(\theta,M)}$ curve to the two

[13] Evidence of a negative relationship between investment and inflation at various levels of aggregation has been reported by various studies. For example, Madsen (2003) has found such evidence at the level of OECD countries. However, more recent experiences from the case of Japan show that a low rate of inflation may stimulate investment. On this latter view, see Klein (2015).

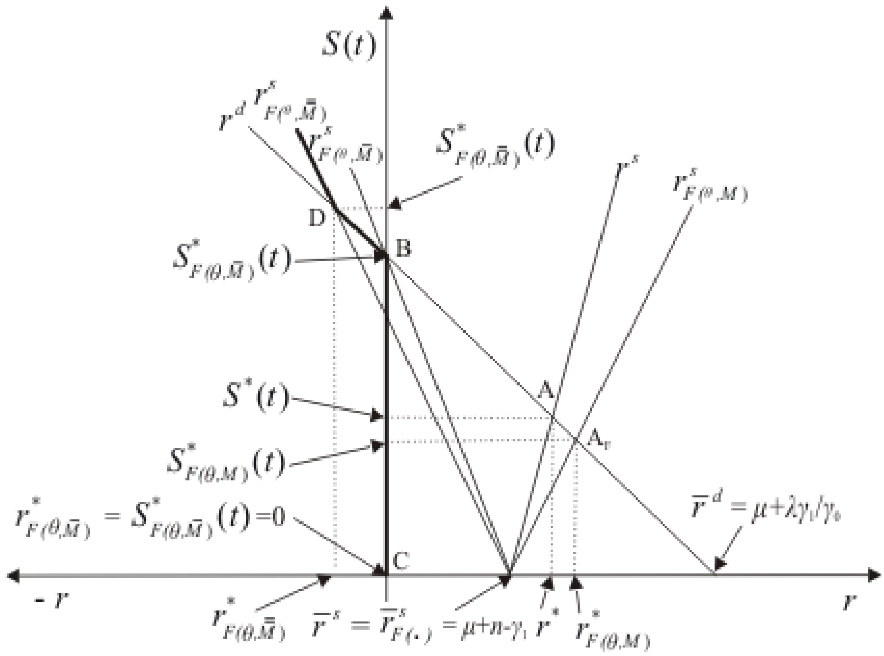

Figure 5: Supply and demand for loanable funds in the presence of a central bank

positions marked respectively $r^s_{F(\theta,\bar{M})}$ and $r^s_{F(\theta,\bar{\bar{M}})}$. These shifts, ceteris paribus, follow increases in the money supply to $\bar{M}(t)$ in the former case and to $\bar{\bar{M}}(t)$ in the latter. At point B the interest rate declines to zero, whereas at point D the interest rate becomes negative. Thus, in line with the data exhibited in Figure 2, the model shows that at times, in the presence of a central bamk, the interest rate may indeed become negative. Yet episodes of zero or negative interest rates can be pursued by central banks neither by designed nor for long because, as we shall argue in the next subsection, they may give rise to highly unsettling economic and social circumstances.

2.3. The instabilites of zero and negative interest rates

Let us return to Figure 5 and focus in particular on points B, C, and D. Observe that, while the curves r^d, $r^s_{F(\theta,\bar{M})}$ and $r^s_{F(\theta,\bar{\bar{M}})}$ are negatively sloped to the interest rate axis, the latter two curves cross the former from below. Drawing on Samuelson's (1941, 103) comparative static analysis, the equilibria at points B and D are unstable because the excess demand (supply) for loans before (after) these points declines (increases) while the interest rate declines (increases). Hence, the following question comes to mind: Given that the Fed faces no technical constraints regarding the quantity of the "artificial" savings that they may create, would it be a viable policy to reduce the interest rate to zero at point B, or even push it through the so-called zero bound to point D? Based on the following considerations, in both cases the answer is in the negative.

Relevant to the instability at point B are the questions: a) how might savers react to the zero interest rate? b) Would the demand for business loans remain unabated? And c) what course might we expect the instability to take under the economic and social circumstances that may emerge? Some savers may be expected to withdraw from the loans market and stash away their savings in safety boxes and bank deposits. These would absorb the safety and storage costs involved. Others, after weighing the benefits of forgoing storage and safety costs against the interest offered by higher risk borrowers, may continue lending their hard-earned savings just to avoid losing interest income. However, if anything is certain, this is that all savers, and particularly the retired people who depend through their pension funds on interest income, will suffer significant wealth losses. By implication, the latter may be expected to shrink aggregate consumption and discourage firms from investing. As a result, equilibrium at point B will slide gradually to point C, where $(r^{*}=0,\ S^{*}(t)=0)$, and somewhere along this path the worsening economic crisis will evolve into a severe political crisis.

In closed free market economies, equilibria like point B may occur at times due to unforeseen shocks either on the supply or the demand side of goods and services. They may transpire irrespective of whether there is or there is not a central bank. But as a rule they are more or less temporary because markets are resilient enough to absorb even non-systematic errors by a central bank, if there is one. This property functions particularly effectively, if the economy is open to international trade, like that in the U.S today. For then, before reaching point B, and certainly long before having to confront the social and political turmoil that would accompany the transition from B to C, loans would be redirected abroad and/or to consumer credit, curve $r^{s}_{F(\theta,\bar{\bar{M}})}$ would shift rightwards, and the interest rate would remain in or return to the positive territory. In the case of classical Athens, the interest rate was positive throughout because, there was no central bank to commit errors in the creation of "artificial" savings, the economy was open, consumer credit, including trade credit that had been officially outlawed, was regularly practiced, shocks from technological change were limited, if not nil, and "true" savings constituted a precious scarce resource.[14]

To conclude, the unsettling economic and social circumstances may be even worse in case the

[14] Referring to the interest rates in classical Athens, Bitros, Economou & Kyriazis (2020, 170) conclude as follows:

> The interest rates in the private sector of ancient Athens were two to four times those in the U.S. in the postwar period. Hence, drawing as we should on the established proposition that the interest rates are related inversely to the level of a country's economic development, it is rather surprising that based on this index, the Athenian economy stood relatively to the U.S. to where many emerging countries stand today.

central bank pursues for long systematic policies of negative interest rates. Then, most likely, equilibria like point D would stir the following sequence of developments; a) the interest income of workers in the producers' goods industries would turn negative (see equation (17")); b) since the stock of producers' goods has been financed by the savings of these workers, as we argued above, these workers would suffer major wealth losses; c) because of these losses, in addition to slowing down their consumption, these workers would lose all incentives to improve on the frontier of technological change; d) unemployment would increase, economic growth would decelerate, and deflation in the economy would require that the central bank continues reducing the policy rate further into the negative territory; and e) this spiral will evolve into a major economic and social crisis. The worst in this course of these events will be the indirect nationalization of the economy's capital stock, the true ownership of which will pass from citizen-savers to the central bank. This scenario corroborates that pushing deliberately the policy rate to the negative region cannot last for long. Otherwise, it will cut into the fundamental principles of the free market economy, which according to Bitros, Karayiannis (2013) is a necessary condition for the survival of democracy.

2.4. The theoretical model of the interest rate

All points of stable and unstable equilibria shown in Figure 5 correspond to solutions of a system of equations consisting of (16), (25) and the equilibrium condition:

$$r^d = r^s_{F(\theta,M)} \tag{26}$$

In principle, if it were possible to solve this system analytically, the solution would give us a relation for the Fed-influenced interest rate like the one below:

$$r^* = h(s^*(t), m(t), V(\mathrm{t}); \mu, n, w, \alpha; \lambda, \gamma_0, \gamma_1), \tag{27}$$

where $r^* = r^*_{F(\theta,M)} - \theta$ and the symbols $s^*(t)$ and $m(t)$ stand for the respective variables expressed in real money terms.

The predictions from (27) of the effects that the various determinants may exert on the interest rate are consistent with those from Solow's steady state relationship (1'). For example, a deceleration in the population growth rate *n*, ceteris paribus, would be expected to reduce $r^*_{F(\theta,M)}$. The reason is that with the savings rate kept fixed, the capital-labor ratio would increase, the marginal product of capital would decline, and this in turn would push $r^*_{F(\theta,M)}$ downwards. However, while from other economic growth models in the tradition of Diamond (1965) one derives similar predictions regarding this relationship, in the case of models following Ramsey (1928), predictions differ because in such models the savings rate adjusts and the capital-labor ratio remains fixed, so changes in the population growth rate leave r_F^* invariant. Ambiguities of this nature cannot be resolved by further refinements in the structure of the adopted models. For this reason, following Mankiw's remarks on the paper by Baker, Delong, Krugman (2005, 318), our research approach is to look at what the empirical evidence has to say. To this end, our objective in the sequel is to estimate (27) and experiment with it to highlight the short- and long-run dynamics of the adjustments. Moreover, at this stage, we plan to test for the significance of variables that do not appear in (27) but have been found to be empirically relevant in the extensive literature in this area.

3. Data, variables, and estimates

All raw data refer to the US economy and cover the period 1973-2019.[15] By implication, our sample includes 47 observations. We collected them from the databases of the World Bank (WB), the International Monetary Fund (IFM), the Organization for Economic Cooperation and Development (OECD), and the International Labor Organization (ILO). More details about the series that were selected from these sources are given in the notes section of Table B-1 in Appendix B. However, it should be noted that in the estimations all level variables have been expressed in constant 2015 prices.

Table 1 below reports the definitions and the measurement of the main variables used in the estimations, whereas Table 2 displays some summary statistics for these variables. Since the interest

Insert Table 1

Insert Table 2

rate $r_{F(\bullet)}$ is unobservable, as shown in the third row of Table 1, it is measured as $r = r_{F(\bullet)} - \theta$. At the same time, since by doing so the left hand side of (27) was expressed in real money terms, for consistency, $S(t)$ and $M(t)$ were normalized to allow for the impact of inflation.

[15] On August 15, 1971, the U.S. abandoned the gold standard and turned the U.S dollar into a purely fiat currency. But the burial of the Bretton Woods system came a few months later. In particular, in December 1971, monetary authorities from the world's leading developed countries met at the Smithsonian Institution in Washington, DC to rescue the international system of fixed exchange rates, which was rapidly disintegrating. They came up with the so-called Smithsonian Agreement. However, within fifteen months, the Bretton Woods system collapsed. This course of events explains why we decided to start collecting data from 1973.

3.1 Pre-estimation diagnostic tests

For (27) to be properly estimated, the first task required confronting the possibility of spurious regression issues. From a technical perspective this necessitated testing of the variables that enter into the estimations for stationarity. Consequently, unit root tests were performed at the levels and first differences of the variables using the criteria of Augmented Dickey–Fuller (ADF), Phillips–Perron (PP), and Kwiatkowski–Phillips–Schmidt–Shin (KPSS).[16] Under the ADF and PP criteria, the null hypothesis is that the time series considered are integrated of order $I(1)$, whereas the KPSS criterion assesses their trend stationarity. The results are presented in Table 3. They show that the dependent variable, r, is $I(1)$ while the set of right-hand regressors consists of stationary and non-stationary variables. In particular, the logarithms of gross investment[17] and money supply are $I(1)$; the velocity of circulation $V(t)$ and the index of the labour cost w are stationary in the first differences; the rates of economic growth and technological progress, g and μ, are stationary; and the rate of population growth n appears to be $I(1)$. Moreover, unit root tests were performed to investigate whether the variables under consideration have unit roots with structural breaks and the results are presented in Table 4. From them it turns out that the dependent variable has a break in 1980.

Insert Table 3

Insert Table 4

In view of the above findings, estimating (27) by a standard least-squares approach would be inappropriate because, as Jordan, Philips (2018) have argued, the probability of committing a Type I error, that is of accepting the existence of a statistically significant relationship, where there is none, would be high. Thus, to bypass this hurdle, it became imperative to adopt an estimating process based on the property of cointegration. In particular, according to Engle, Granger (1987), if, for example, two time series $Y(t)$ and $X(t)$ are $I(1)$, and the residuals from a regression between $Y(t)$ and $X(t)$ are $I(0)$ then, these two time series are said to be cointegrated and the difficulties posed by the finding that they lack stationarity can be avoided by switching to estima-

[16] These criteria have been developed the by authors after whom they are named in Dickey, Fuller (1979), Phillips, Perron (1988), and Kwiatkowski, Phillips, Schmidt, Shin (1992).

[17] In the model of the economy from which (27) derives and in the presence of a central bank, investment coincides with worker saving plus artificial saving created by the central bank.

tion based on the so-called Autoregressive Distributed Lag (ARDL) specification.

In the time since then, econometricians have proposed several alternative approaches to ARDL estimation. One was firstly introduced by Pesaran, Shin (1999) and further extended by Pesaran et al. (2001). This allows including a mix of $I(0)$ and $I(1)$ regressors. But, in order to test for cointegration, the dependent variable must be $I(1)$ and no regressors must be integrated of order greater than 1. In our case, this restriction is satisfied. Additionally, one of the advantages of this estimating approach is that it permits researchers to incorporate different number of lags in different variables, thus rendering it very flexible.

3.2 Estimation

The estimates to be presented derive from the ARDL specification that Kripfganz, Schneider (2018) introduced in Error Correction Form (ECF):

$$\Delta r_t = b_0 + b_1 t - \gamma(r_{t-1} - \delta'\mathbf{X}_{t-1}) + \sum_{i=1}^{p-1}\psi_{ri}\Delta r_{t-i} + \sum_{i=1}^{q_X-1}\psi'_{xi}\Delta\mathbf{X}_{t-i} + \omega'\Delta\mathbf{X}_t + e_t, \tag{28}$$

where Δ is the difference operator, r_t is the interest rate at time *t*, $\mathbf{X}_t$ is a vector of control variables, δ' is a vector of the long-run coefficients, γ is defined as the speed-of-adjustment coefficient, p and q_x are the optimal lag length operators selected by the Akaike information criterion, which can be different for each control variable, ψ_r, ψ'_x and ω' are vectors of coefficients showing the short-run effects, and e_t stands for the residuals at time *t*.

The preceding unit-root results indicated that r_t experienced a structural break in the year 1980. Most likely this related to the recession in the US economy that started in that year. But looking closer at the yellow line in Figure 2, the data show that potentially there might be two more structural breaks. One in the period 2008-2009, which coincided with the great recession that erupted in the period popularly known as the “global financial crisis” of 2008, and another relatively brief and shallow during the years 2015-2016. Thus, the issue of whether it would be appropriate to add in (28) one or more dummy variables in order to allow for these possible breaks was left to be resolved by reference to statistical criteria. On this ground, preliminary estimations suggested that it was adequate to insert just two dummy variables, one for the break in 1980 and another in 2009. Perhaps because the sample provided for too few observations after 2015-2016, including a dummy variable for a possible structural break in this period did not turn out to be statistically significant.

Table 5 presents the estimates of (28). Looking closer at them, several key features are worth

Insert Table 5

stressing. Observe that the estimated equation has remarkably high explanatory power. In particular, adjusted for degrees of freedom, the coefficient of determination displayed in the last row of the table explains 96.7%% of the variation in the interest rate. Of equal, if not higher, importance is that the coefficients of the long-term effects are all statistically significant at comfortable levels of confidence and their signs are consistent with those expected from theory. This assessment holds true even for the component of unexpected inflation, $\hat{\varepsilon}_{\theta,t-1}$, which was introduced on an ad hoc basis. Given that in the long run money would be expected to be neutral with respect to prices, the coefficient of this variable would be expected and it was found not to be different from zero. However, in the short run the coefficient of $\Delta\hat{\varepsilon}_{\theta,t-1}$ is statistically significant with a negative sign. What this finding implies is that deviations from the expected rate of inflation, while neutral in the long-run, in the short run they affect inversely the interest rate. By implication, economic agents do not appear to suffer from "interest rate illusion", and hence, monetary authorities should not rest assured that policy induced changes in the interest rate may escape unnoticed and without unintended consequences for the economy. With reference to monetary policy, it is also interesting to observe the strong negative long-run effect of money supply. In the light of this evidence, there is hardly any doubt that the issuing of voluminous "artificial savings" by the Fed has contributed to the downward trend of the interest rate. Last, but not least, notice that the coefficients of the dummy variables I_1 and I_2 are both statistically significant, thus ascertaining that in 1980 and 2008 the US economy did experience two structural breaks; and also that the coefficient of the Error Correction Term (ECT) is close to 1, which implies that after some disturbance the interest rate returns to equilibrium exceedingly fast.

Not shown in Table 5 are several ad hoc variables with which we experimented in the estimation stage. In particular, against our expectations, the variables of the exchange rate and inequality that are frequently found to influence the interest rate, did not turn out to be statistically significant. However, a set of tables including these variables is available on request from the authors.

3.3 Post-estimation diagnostic tests

The results from the unit root tests in conjunction with those discussed above regarding the estimation of (28) confirm with high levels of confidence that a long run relationship between the interest rate and its determinants does pervades in the data. This is an important finding. Indeed so important, that for further assurance we found it imperative to test for its presence by following the procedure that Pesaran, Shin, Smith (2001) have suggested, to be called hereafter PSS bounds test. In contrast to the earlier approach that was based on the significance of the lagged variables, these researchers focus on the asymptotic distribution of the popular *Wald* or *F-statistic* used to test the statistical significance of lagged variables. In conditional unrestricted correction models (ECM), these tests are "non-standard" under the null hypothesis of no-relationship in levels, irrespective of whether the regressors are purely *I(*0*)*, purely *I(*1*)*, or mutually cointegrated. Thus, they introduced the bounds test to investigate the existence of a relationship in the levels irrespective of the presence among the regressors of a mixture of *I(*0*)* and *I(*1*)* variables.

Additionally, they provide two sets of asymptotic critical values; one for the case that all regressors are purely *I(*1*)* and another in which all regressors are purely *I(*0*)*. These two sets of values allow for the application of the bounds test. More specifically, if the values of the *F-statistic* associated with the statistical significance of lagged independent variables, or the values of the *t-statistic* associated with the statistical significance of lagged dependent variable, fall outside the critical bounds of the test, the existence of a relationship in the levels is confirmed without knowing prior to testing the integration status of the underlying regressors. However, if the values of *F*- or *t*-statistic fall inside the bounds, the test is inconclusive and then it is necessary to know the order of integration.

Table 6 shows the results of the PSS bounds test and the respective critical values. The asymptotic approximation of critical values depends on the number of observations, the number of regressors that appear in the levels (in the present case is 8), and the restrictions (if any) on the constant

Insert Table 6

and the trend. The null hypothesis is that there is no relationship in the levels. If *F* (or *t*) is lower (or higher) than the critical values for *I(*0*)*, we do not reject the null hypothesis and hence there is no relationship in the levels. However, if *F* (or *t*) is higher (or lower) than the critical values for *I(*1*)* we do reject the null hypothesis of no long-run relationship. The results exhibited in Table 6 support the rejection of the null hypothesis at 10%, 5% and 1% level of significance, respectively. Therefore, the existence of a long-run relationship is confirmed.

Drawing on this finding, the next step in our research plan was to employ the estimated equation to conduct certain experiments to highlight the nature of possible effects of monetary policy on the interest rate. However, before turning to this task, following established practices, we considered it necessary to check on the robustness of the estimates presented in Table 5. To this end, we conducted a battery of popular post-estimation diagnostic tests,[18] the results from which are displayed in Table 7. In particular, lines 2 and 3 display the results for detecting heteroskedasticity. The tests

Insert Table 7

derive from Cameron, Trivedi's (1990) decomposition information matrix,[19] Breusch, Pagan (1979), and Cook, Weisberg (1983). On their account, it turns out that one fails to reject the null hypothesis that the residuals are homoscedastic. Lines 4, 5 and 6 of the table show the results of test for serial correlation or autocorrelation following Durbin's (1970) alternative test, Breusch's (1978) and Godfrey's (1978) test for high-order serial correlation, and Engle's (1982) Lagrange multiplier test for the presence of autoregressive conditional heteroskedasticity. Again, on the basis of the results we failed to reject the null hypothesis of no serial correlation or autocorrelation. Next, Ramsey's (1969) RESET test was computed to examine the possibility of omitted variables in the specification of (28). Under this test, the null hypothesis is that the estimated equation has no omitted variables. As the results in line 6 of this table attest, the null hypothesis cannot be rejected, thus implying that the equation is correctly specified.

Moreover, to be able to make statistical inferences, we need to validate the assumption regarding the distribution of the residuals. Relevant to this test are the values in the rows 7 and 8 of the table. The first test, suggested by D'Agostino, Balanger, D'Agostino (1990), examines the skewness and the kurtosis in the residuals, whereas the second is the popular Jarque-Bera (1987) asymptotic test for normality. On the basis of the results, the null hypothesis of normal distribution cannot be rejected. That this is the case can be further ascertained by examining the distribution of the residuals through plotting their quantiles, the quantiles of normal distribution estimates, and the standardized normal probability plot. Figures 6 and 7 below are consistent with this assessment.

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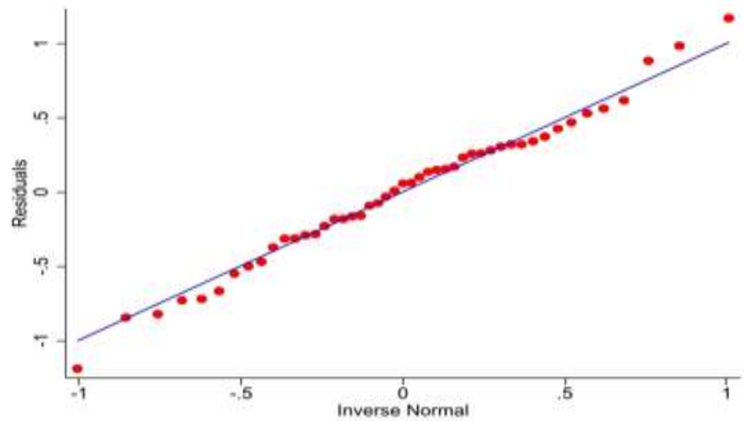
[18] The usefulness of these diagnostic tests is confirmed by many scholars. For example, see, Kripfganz, Schneider (2018), Sarkodie, Owusu (2020), and Pata, Isik (2021).

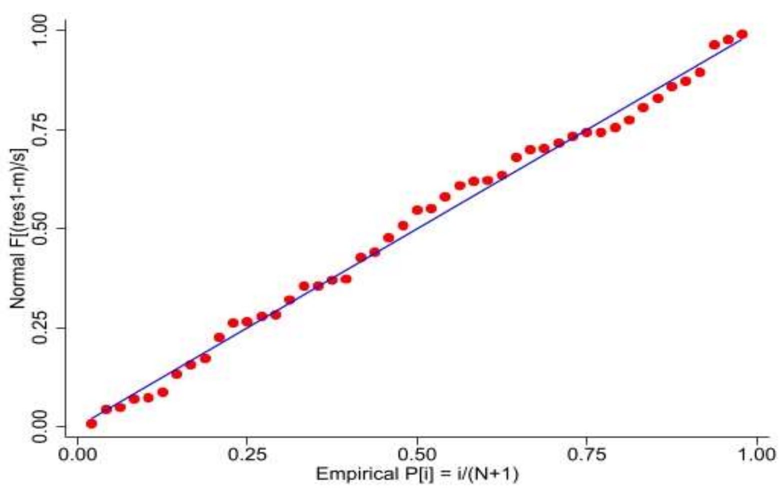
[19] This test is similar to the one suggested by White (1980).

Figure 6: Quantiles of residuals against quantiles of normal distribution



Notes: This figure plots the quantiles of residuals against the quantiles of normal distribution and the plot confirms that the residuals on the estimated model are normally distributed.

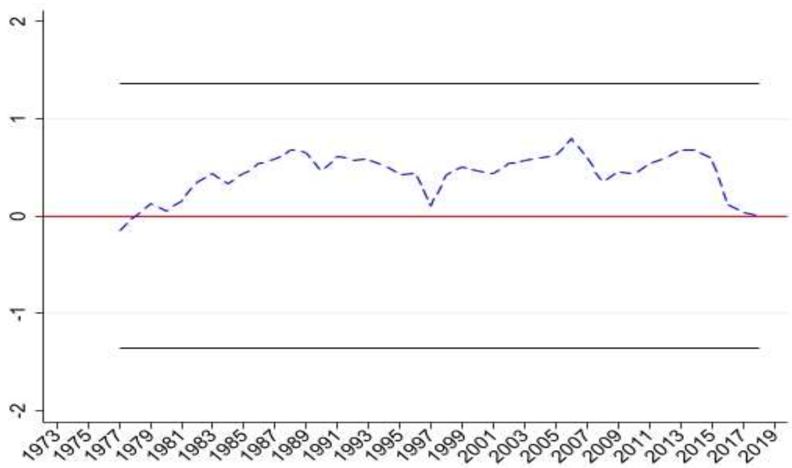
Figure 7: Standardized normal distribution



Notes: This figure plots the standardized normal distribution and shows that the residuals are normally distributed.

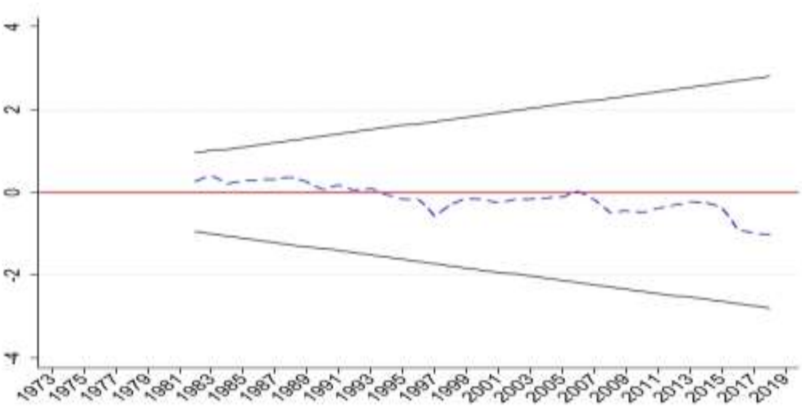
Finally, to test the stability of the parameters in the estimated equation, Figures 8 and 9 present the cumulative tests of recursive residuals, firstly introduced by Brown, Durbin, Evans (1975) and further enhanced by Ploberger, Kramer (1992). Since, the plots are within the 95% confidence interval, the stability of parameters is confirmed. Hence, all diagnostic tests indicate that the estimates of the ARDL model are reliable and robust.

Figure 8: Cumulative test using OLS CUSUM plot



Notes: CUSUM test for parameter stability. Black lines show the 95% confidence interval around the null hypothesis of non-existence of structural break. OLS residuals are used.

Figure 9: Cumulative test using recursive residuals



Notes: CUSUM test for parameter stability. Black lines show the 95% confidence interval around the null hypothesis of non-existence of structural break. Recursive residuals are used.

4. Short- and long-run dynamics

Having made sure through pre- and post-estimation tests that the estimated form of (28) represents a stable relationship, now it befits our research plan to use it to highlight the pattern and speed of adjustment of the interest rate to possible changes in its determinants in the short- and in the long-run. To this effect, we have chosen to employ the novel dynamic ARDL simulations procedure, which has been proposed by Jordan, Philips (2018).

Upon reparameterization of (28), we obtained the equivalent expression in the form of (29) below. Table 8 presents the dynamic ARDL simulation estimates of this equation. As explained in

$$\Delta r_t = b_0 + b_1 t + b_2 r_{t-1} + \sum_{k=1}^{K} \theta_k x_{k,t-1} + \sum_{i=1}^{p} \phi_i \Delta r_{t-i} + \sum_{j=0}^{q_1} \alpha_{1,j} \Delta x_{1,t-j} + \ldots + \sum_{j=0}^{q_k} \alpha_{k,j} \Delta x_{k,t-j} + e_t \qquad (29)$$

Note 1 at the bottom of the table, the optimal lag length in the estimation was chosen by Schwarz's information criterion; the optimal number of lags was selected by the dynamic ARDL estimating procedure itself; and the values of *F*- and *t*-tests from the PSS bounds test turned out to be equal to 20.74 and -10.85, thus ascertaining in comparison with the critical values presented in Table 6 the existence of (27) in the particular configuration that the coefficients of long term effects specify.

Insert Table 8

Now these estimates may be used to assess the possible effectiveness of monetary policy. To this end, a good indicator is the sensitivity of the interest rate to changes in those of its determinants that may be presumed to be controlled by the Fed. Table 9 reveals that such is the supply of money m_t. Expectedly, when the Fed decides to change this variable, the interest rate experiences a se-

Insert Table 9

quence of short-run changes, which after some time subside and give rise to a new long-run equilibrium. The elasticity in the cross of the extreme left column and the third row of the table shows how the interest rate would be expected to change in the long run when the Fed increases the m_t by 1%. Ceteris paribus, observe that it would decline 0.5%. What this elasticity implies is that monetary policies channelled to the economy through money supply may exercise a very large impact on the interest rate in the opposite direction. By itself, this is an important finding. However, given that the new long-run equilibrium may lie certain years into the future, thinking in ceteris paribus terms is a useful but utterly counterfactual thought experiment, because most likely the velocity of circulation *V* and other determinants of the interest rate will change, and indeed in ways unknown to and unpredictable by the Fed. Hence, in the light of this evidence, our first conclusion is that, if the Fed places the emphasis of its policies on achieving certain results in the long-run, their efficacy depends on how long the long-run is. For, if the transmission of the policy effects takes too long, with its manipulation of the money supply the Fed may risk doing more harm than good to the economy or even render monetary policy totally untenable. This explains the importance of the insights from the analysis of short-run dynamics.

Critical in this regard is the coefficient of the Error Correction Term (ECT). From Table 8 it turns out that its value is -0.93 and that it is statistically significant at a comfortable level of confidence. Its importance springs from two grounds. The first is that, since it has the expected negative sign, it provides extra reassurance that the estimated relationship does exist. As for the second ground, this has to do with its size. By being so close to 1, this finding suggests that, after a shock to any of the determinants of the interest rate, while all others remain fixed, convergence to the long-run equilibrium is very fast, with its major part being completed within the next 2-3 years. Hence, given that in this short horizon non-monetary policy related determinants of the interest rate may remain unchanged, our findings suggest that the Fed should be properly mindful of both the short- and the long-run effects of monetary policy.

To corroborate this assessment, the following indicative graphs display the adjustment paths of the interest rate, given an arbitrary increase once at a time to investment, money supply, and the rate of population growth. The cumulative plots of the adjustment paths are obtained by applying 5,000 simulations. With the exception of the control variable changed, all others are held fixed to their sample means and all differenced variables are set equal to zero.[20] Figure 10 shows the cumulative response of the interest rate to a 1% shock in the logarithm of investment at time t=5. At the end of the adjustment process, the value of the interest rate would have increased by 0.27%. This is analogous to the long-run effect of the ARDL model (see Tables 8 and 9). Black circles show the cumulative change in the predicted real interest rate every year and the spikes from darkest blue to the light blue show the 75%, 90%, and 95% confidence intervals, respectively. What is important to observe is that indeed, after the shock is applied, the interest rate increases by 0.25% in the first year and by 0.269% in the second year, which is very close to its long term change of 0.27.

[20] For more details, we encourage readers to study the work of Jordan and Philips (2018). To perform the analysis, we used the STATA package *dynardl.*

Figure 10: 1% shock in real investment

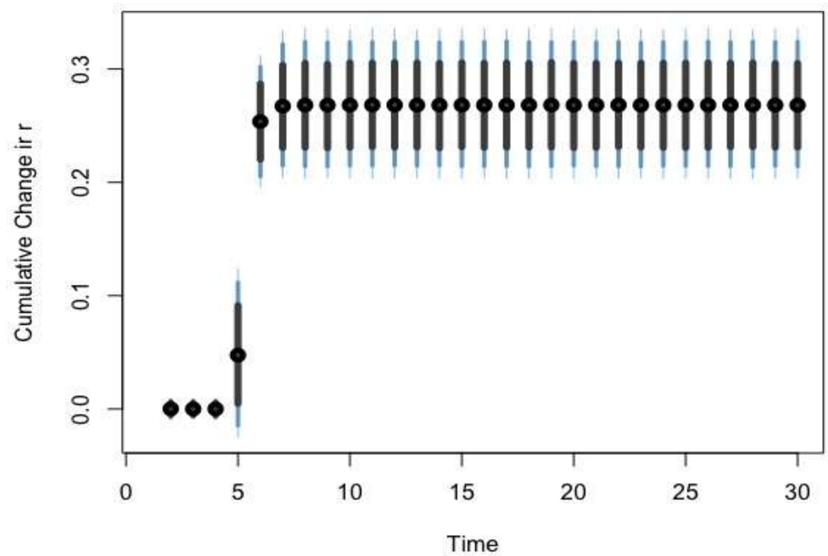


Figure 11: 1% shock in real money supply

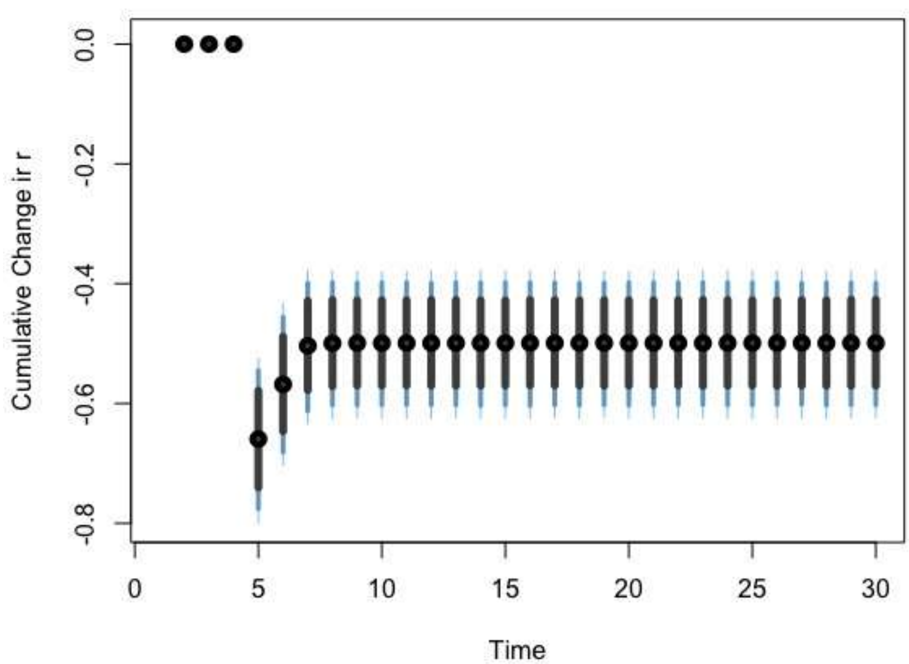
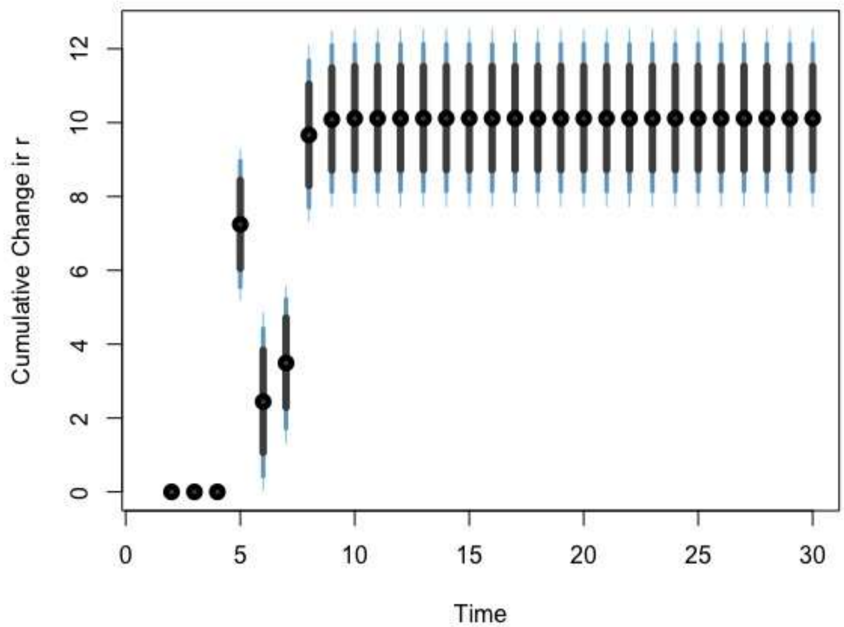


Figure 12: 1% shock in population growth rate.



Unlike its very fast adjustment given a change in investment, Figures 11 and 12 show that the interest rate will adjust slower after a change in the supply of money and much slower when the population rate changes. In particular, in the former case, the major part of the adjustment will take effect within three years whereas in the latter case within four years, but after taking a downward dip in the first year.

5. Conclusions

After the 2008 international financial crisis, central banks in Western type democracies adopted various forms of unconventional monetary policies. More recently, one of these policies assumed the form of zero and even negative central bank funds rate. In the U. S. the Fed came very close to adopting a zero Federal Funds Rate, but it did not breach this bound. On the contrary, central banks in the European Union, Switzerland and elsewhere apply negative rates to the present day. These policies have grave implications for saving and investing in free market economies because they distort the real interest rate which drives them. Our objective in this paper was to shed some light on the nature of possible distortions that may spring from monetary policies. For this purpose, we adopted a research strategy based on three tasks. The first one was to derive a relationship between the real interest rate and its determinants, including possible influences from the country's central bank. We obtained one by embedding monetary policy in a long-forgotten general equilibrium growth model of heterogeneous capital. The second task was to estimate this relationship using time series data for United States; and the third task was to employ the estimated relationship to highlight the short- and the long-term dynamics of the real interest rate, given an arbitrary shock to variables, related and unrelated to monetary policy.

Among other findings, the analysis on the theoretical plain indicated that, by injecting (withdrawing) "artificial savings", the central bank influences the real interest rate negatively (positively), thus creating conditions of economic instability by disturbing the long run plans of economic agents for saving and investing. On the empirical plain, using data over the period 1973-2019, we found that there is a long-run stable relationship showing that the real interest rate is determined by real investment, real money supply, the velocity of circulation, and the rates of wages, economic growth, technological change, and population. Also, upon an arbitrary shock in one of its aforementioned determinants, holding all the rest fixed, we found that the real interest rate returns to the long-run equilibrium within a few years. From the theoretical analysis we expected that some instability might show up in the data because of the negative real interest rates, twice in the 1970s and then again in the early 2010s. However, no such evidence emerged, perhaps because the bias of monetary policy at those times was away from the zero bound and the real interest rate became negative due to unexpected and transient shocks. The same of course may not be the case if the model is tested with data after 2019, when the bias of the monetary policy reversed, driving the nominal interest rates very close to the zero bound and thus, probably, giving rise to the current signs of economic instability.

6. References

1. Baker, D., Delong, B. J., Krugman, P. R., (2005), "Asset returns and economic growth," *Brookings Papers on Economic Activity*, 289-315.
2. Bernanke, B. S., (2005), "The global savings glut and the US current account deficit," https://econpapers.repec.org/paper/fipfedgsq/77.htm
3. Bitros, G. C., (2022), "Commodity money and the price level," *Quarterly Journal of Austrian Economics*, 25, 147-183, https://qjae.scholasticahq.com/article/35549-commodity-money-and-the-price-level
4. ---------------, (2018), **"**Monetary policy, market structure and the income shares in the US," *Open Economies Review*, 29, 383–413.
5. Bitros, G. C., (2008), "Why the structure of capital and the useful lives of its components matter: A test based on a model of Austrian descent," *Review of Austrian Economics*, 21, 301–328.
6. Bitros, G. C., Karayiannis, A. D., (2013), *Creative Crisis in Democracy and Economy*, Heidelberg: Springer Publishers.
7. Bitros, G. C. and Flytzanis, E., (2002), "Towards a General Theory of Real Capital", in George. C. Bitros and Yannis Katsoulakos (Eds.), *Essays in Economic Theory, Growth and Labor markets: A Festschrift in Honor of E. Drandakis*, Cheltenham, UK: Edward Elgar, 125-147.
8. Bitros, G. C. and Flytzanis, E (2004), "Operating policies in a model with terminal scrapping," *European Journal of Operational Research*, 194, 551-573.
9. Blanchard, O. J., Summers, L. H., (1984), "Perspectives on high world real interest rates," *Brookings Papers on Economic Activity*, 2, 273-324.
10. Brand, C., Bielecki, M., Penalver, A., (2019), "The natural rate of interest: Estimates, drivers, and challenges to monetary policy," National Bank of Poland, NBP Working Paper No. 308.
11. Brems, H., (1968), *Quantitative Economic Theory: A Synthetic Approach*, New York: John Wiley & Sons Inc.
12. Breusch, T. S., Pagan, A. R., (1979), "A simple test for heteroscedasticity and random coefficient variation," *Econometrica*, 47, 1287-1294.
13. Breusch, T. S., (1978)," Testing for autocorrelation in dynamic linear models," *Australian Economic Papers* 17, 334–355.

14. Brown, R. L., Durbin, J., Evans, J. M., (1975). “Techniques for testing the constancy of regression relationships over time,” Journal of the Royal Statistical Society, Series B 37, 149–192. https://doi.org/10.2307/2986299.
15. Buiter, W. H., (2007), “Is Numérairology the Future of Monetary Economics? Unbundling Numéraire and Medium of Exchange through a Virtual Currency and a Shadow Exchange Rate,” *Open Economies Review*, 18, 127–155.
16. Cameron, A. C., Trivedi, P. K., (1990), “The information matrix test and its applied alternative hypotheses,” Working Paper 372, University of California-Davis, Institute of Governmental Affairs.
17. Cook, R. D., Weisberg, S., (1983), “Diagnostics for heteroscedasticity in regression”, *Biometrika*, 70, 1-10.
18. D'Agostino, R. B., Belanger, A. J., D'Agostino and R. B., Jr., (1990), “A suggestion for using powerful and informative tests of normality,” *American Statistician*, 44, 316-321.
19. Diamond, P. A., (1965), “National debt in a neoclassical growth model,” *American Economic Review*, 55, 1126–50.
20. Dickey, D. A., Fuller, W. A., (1979), “Distribution of the estimators for autoregressive time series with a unit root,” *Journal of the American Statistical Association*, 74, 427–431.
21. Domar, E. D., (1953), “Depreciation, replacement, and growth,” *Economic Journal*, 63, 1-32.
22. Durbin, J., (1970), "Testing for serial correlation in least-squares regression when some of the regressors are lagged dependent variables," *Econometrica*, 38, 410-421.
23. Engle, R. F., (1982) “Autoregressive conditional heteroskedasticity with estimates of the variance of united kingdom inflation,” Econometrica. 50, 987–1007.
24. Engle, R. F., Granger, C. W. J., (1987), “Co-integration and error correction: representation, estimation, and testing,” *Econometrica* 55, 251–276.
25. Eisner, R., (1952), “Depreciation allowances, replacement requirements, and growth,” American Economic Review, 42, 820-831.
26. Godfrey, L. G., (1978), “Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables,” *Econometrica* 46, 1293–1301.
27. Gordon R. J., (2015), “Secular stagnation on the supply side: US productivity growth in the long Run,” *Digiworld Economic Journal*, 100, 20-45.

28. Grant, T., Lebo, M. J., (2016), "Error correction methods with political time series." *Political Analysis*, 24, 3–30.
29. Jarque, C.M., Bera, A. K., (1987), "A test for normality of observations and regression residuals," *International Statistical Review*, 55, 163-172; In Gujarati, D., (1995), *Basic Econometrics*, 143.
30. Jordan, S, Philips, A. Q., (2018), "Cointegration testing and dynamic simulations of autoregressive distributed lag models". *Stata Journal*, 18, 902-923.
31. Klein, M. C., (2015), "What is the relationship between inflation and investment?" Financial Times, January 2015, What is the relationship between inflation and investment? | Financial Times (ft.com).
32. Kripfganz, S., Schneider, D. C., (2020). Response surface regressions for critical value bounds and approximate p-values in equilibrium correction models. Oxford. Bulletin of Economics and. Statistics, 82, 1456–1481. https://doi.org/10.1111/obes.12377.
33. Kripfganz, S., Schneider, D. C., (2018). "Ardl: Estimating autoregressive distributed lag and equilibrium correction models," London Stata Conference. http://repec.org/usug2018/uk18_Kripfganz.pdf.
34. Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., Shin, Y., (1992). "Testing the null hypothesis of stationarity against the alternative of a unit root," Journal of. Econometrics. 54, 159–178, http://dx.doi.org/10.1016/0304-4076(92)90104-Y.
35. Lane, P. R., (2019), "Determinants of the real interest rate," https://www.ecb.europa.eu/press/key/date/2019/html/ecb.sp191128_1~de8e7283e6.en.html.
36. Madsen, J. B., (2003), "Inflation and investment," Scottish Journal of Political Economy, 50, 375-397.
37. Pata U. K., Isik, C., (2021), "Determinants of the load capacity factor in China: A novel dynamic ARDL approach for ecological footprint accounting," *Resources Policy*, 74, 102-313.
38. Perron, P., (1989),"The Great Crash, the oil price shock, and the unit root hypothesis," *Econometrica*, 57, 1361-1401.
39. Pesaran, M. H., Shin, Y., and Smith, J. R., (2001), "Bounds testing approaches to the analysis of level relationships". *Journal of Applied Econometrics*, 16, 289–326.

40. Pesaran, M. H., Shin, Y., (1999), "An autoregressive distributed lag modelling approach to cointegration analysis," in Strøm, S., (Ed.), *Econometrics and Economic Theory in the 20th Century*: *The Ragnar Frisch Centennial Symposium*, Cambridge: Cambridge University Press.

41. Phillips, P. C., Perron, P., (1988), "Testing for a unit root in time series regression," *Biometrika*, 75, 335–346, https://doi.org/10.1093/biomet/75.2.335.

42. Ploberger, W., Kramer, W., (1992), "The CUSUM test with OLS residuals," *Econometrica*, 60, 271–285. https://doi.org/10.2307/2951597.

43. Rachel, L., Smith, T. D., (2015), "Secular drivers of the global real interest rate, *Bank of England*, Staff Working Paper No 571.

44. Ramsey, F. P., (1928), "A mathematical theory of savings," *Economic Journal*, 38, 543-559.

45. Ramsey, J. B, (1969), "Tests for specification errors in classical linear least-squares regression analysis," *Journal of the Royal Statistical Society*, Series B, 31, 350-371.

46. Samuelson, P. A., (1941), The Stability of Equilibrium: Comparative Statics and Dynamics," *Econometrica*, 9, 97-120.

47. Sarkodie S. A., Owusu, P. A., (2020), "How to apply the novel dynamic ARDL simulations (dynardl) and Kernel-based regularized least squares (krls)," *Methods X*, 7, 101-160.

48. Solow, R. M., (1956), "A contribution to the theory of economic growth," *Quarterly Journal of Economics*, 70, 65–94.

49. Yule, G. U., (1926), "Why do we sometimes get nonsense-correlations between Time-Series?– a study in sampling and the nature of time-series." *Journal of the royal statistical society* 89, 1–63.

50. White, H., (1980), "A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity." *Econometrica*, 48, 817-838.

Appendix A

Results from the numerical solution of equation (11) in the text

$\gamma_0 = 3,\ \gamma_1 = -0.0000$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	25.4	26.4	27.5	28.7
-0.015	20.2	20.9	21.6	22.3
-0.020	17.2	17.7	18.2	18.7
-0.025	15.2	15.6	15.9	16.3

$\gamma_0 = 3,\ \gamma_1 = -0.0005$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	25.0	25.9	26.9	27.9
-0.015	20.0	20.6	21.2	21.8
-0.020	17.1	17.5	17.9	18.4
-0.025	15.1	15.4	15.8	16.1

$\gamma_0 = 3,\ \gamma_1 = -0.0010$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	24.6	25.4	26.3	27.1
-0.015	19.7	20.3	20.9	21.4
-0.020	16.9	17.3	17.7	18.1
-0.025	15.0	15.3	15.6	15.9

$\gamma_0 = 3,\ \gamma_1 = -0.0015$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	24.2	25.0	25.8	26.5
-0.015	19.5	20.0	20.5	21.1
-0.020	16.7	17.1	17.5	17.9
-0.025	14.8	15.1	15.4	15.7

$\gamma_0 = 3.5,\ \gamma_1 = -0.0000$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	27.5	28.7	30.0	31.4
-0.015	21.9	22.6	23.4	24.2
-0.020	18.6	19.1	19.7	20.3
-0.025	16.4	16.8	17.2	17.7

$\gamma_0 = 3.5,\ \gamma_1 = -0.0005$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	27.01	28.09	29.19	30.31
-0.015	21.57	22.26	22.97	23.69
-0.020	18.39	18.89	19.40	19.93
-0.025	16.25	16.64	17.03	17.44

$\gamma_0 = 3.5,\ \gamma_1 = -0.0010$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	24.6	25.4	26.3	27.1
-0.015	19.7	20.3	20.9	21.4
-0.020	16.9	17.3	17.7	18.1
-0.025	15.0	15.3	15.6	15.9

$\gamma_0 = 3.5,\ \gamma_1 = -0.0015$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	24.2	25.0	25.8	26.5
-0.015	19.5	20.0	20.5	21.1
-0.020	16.7	17.1	17.5	17.9
-0.025	14.8	15.1	15.4	15.7

$\gamma_0 = 4,\ \gamma_1 = -0.0000$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	29.5	30.8	32.3	33.9
-0.015	23.4	24.2	25.1	26.1
-0.020	19.9	20.5	21.1	21.8
-0.025	17.5	18.0	18.5	19.0

$\gamma_0 = 4,\ \gamma_1 = -0.0005$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	28.9	30.1	31.3	32.6
-0.015	23.0	23.8	24.6	25.4
-0.020	19.6	20.2	20.8	21.3
-0.025	17.3	17.8	18.2	18.7

$\gamma_0 = 4,\ \gamma_1 = -0.0010$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	28.4	29.4	30.5	31.4
-0.015	22.7	23.4	24.1	24.8
-0.020	19.4	19.9	20.4	21.0
-0.025	17.1	17.6	18.0	18.4

$\gamma_0 = 4,\ \gamma_1 = -0.0015$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	27.8	28.8	29.7	30.4
-0.015	22.4	23.0	23.7	24.3
-0.020	19.2	19.6	20.1	20.6
-0.025	17.0	17.4	17.7	18.1

$\gamma_0 = 4.5,\ \gamma_1 = -0.0000$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	31.3	32.9	34.5	36.3
-0.015	24.8	25.8	26.8	27.8
-0.020	21.1	21.7	22.4	23.2
-0.025	18.5	19.1	19.6	20.2

$\gamma_0 = 4.5,\ \gamma_1 = -0.0005$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	30.7	32.0	33.4	34.7
-0.015	24.4	25.3	26.2	27.0
-0.020	20.8	21.4	22.0	22.7
-0.025	18.3	18.8	19.3	19.8

$\gamma_0 = 4.5,\ \gamma_1 = -0.0010$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	30.0	31.2	32.3	33.3
-0.015	24.0	24.8	25.6	26.3
-0.020	20.5	21.1	21.7	22.2
-0.025	18.1	18.6	19.0	19.5

$\gamma_0 = 4.5,\ \gamma_1 = -0.0015$

$\mu \backslash r$	0.02	0.03	0.04	0.05
-0.010	29.5	30.5	31.4	32.2
-0.015	23.7	24.4	25.1	25.7
-0.020	20.3	20.8	21.3	21.8
-0.025	17.9	18.4	18.8	19.2

Appendix B

Data sources and conventions adopted in constructing the variables

Table B-1 below presents the list of the variables used in the estimations, along with certain other useful details for tracing their sources of origin and definitional content. Because of their sheer volume, the actual data of the variables will be posted separately.

Table B-1: Data sources of initial variables used in the estimations[a]

Names of variables	Symbols	Source[a]	Code of series[b]	Particular remarks
Nominal interest rate	r_F	IMF	FIGB_PA[1]	In current US dollars
Prices	$P(t)$	IMF	FP.CPI.TOTL[2]	Consumer Price Index (Base year 2010=100)
Nominal gross investment	$S(t)$	WB & OECD	NE.GDI.TOTL.CD[3]	In current US dollar prices
Real gross investment	$s(t)$	WB & OECD	NE.GDI.TOTL.KD[4]	In constant 2010 US dollars
Index of labor cost	w	AMECO	QLCD[5]	Base year 2015=100
Nominal Gross Domestic Product	$Y(t)$	WB & OECD	NY.GDP.MKTP.CD[6]	In current US dollars
Real Gross Domestic Product	$y(t)$	WB & OECD	NY.GDP.MKTP.KD[7]	In constant 2015 US dollars
Labor participation rate	a	FRED	CLF16OV[8]	Percentage of total population of ages 15+
Population	$N(t)$	WB	SP.POP.TOTL[9]	Midyear estimates
Nominal money supply	$M(t)$	WB	FM.LBL.BMNY.GD.ZS[10]	% of $Y(t)$

a. Notes: Abbreviations of sources

1. IMF - International Monetary Fund
2. WB – World Bank Open Data
3. OECD - Organization for Economic Cooperation and Development
4. ILOSTA - International Labor Organization
5. AMECO - Annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs
6. USCB - US Census Bureau
7. FRED – Federal Reserve Bank of St. Louis

b. Explanatory details:

1. Derived by International Financial Statistics database. It represents yields to maturity of government bonds or

other bonds that would indicate longer-term rates.

2. Consumer price index (2010=100) reflects the average cost to consumer for acquiring a basket of goods and services. The Laspeyres formula is generally used.
3. Gross capital formation (formerly gross domestic investment) consists of outlays on additions to the fixed assets of the economy plus net changes in the level of inventories. Fixed assets include land improvements (fences, ditches, drains, and so on); plant, machinery, and equipment purchases; and the construction of roads, railways, and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings. Inventories are stocks of goods held by firms to meet temporary or unexpected fluctuations in production or sales, and "work in progress." According to the 1993 SNA, net acquisitions of valuables are also considered capital formation.
4. Data as described under 3 above.
5. Real unit labour costs: total economy which is the ratio of compensation per employee to nominal GDP per person employed.
6. GDP at purchaser's prices is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources. Dollar figures for GDP are converted from domestic currencies using single year official exchange rates. For a few countries where the official exchange rate does not reflect the rate effectively applied to actual foreign exchange transactions, an alternative conversion factor is used.
7. Data are in constant 2015 prices, expressed in US dollars. Dollar figures for GDP are converted from domestic currencies using 2015 official exchange rates.
8. Retrieved from Federal Reserve Bank of St. Louis, only for United States, and is defined as the civilian labor force level, measured in thousands of persons, 16 years of age and older. The data are seasonally adjusted. We derived the annual data which is calculated on the average. The primary source is US Bureau of Labor Statistics.
9. Total population is based on the de facto definition of population, which counts all residents regardless of legal status or citizenship. The values shown are midyear estimates. World Bank data derived from (1) United Nations Population Division. World Population Prospects: 2019 Revision. (2) Census reports and other statistical publications from national statistical offices, (3) Eurostat: Demographic Statistics, (4) United Nations Statistical Division. Population and Vital Statistics Report (various years), (5) US Census Bureau: International Database, and (6) Secretariat of the Pacific Community: Statistics and Demography Program.
10. Broad money (IFS line 35L.ZK) is the sum of currency outside banks; demand deposits other than those of the central government; the time, savings, and foreign currency deposits of resident sectors other than the central government; bank and traveler's checks; and other securities such as certificates of deposit and commercial paper.

Table 1: Definition and measurement of variables

Symbol		Description	Derivation
$\dot{P}(t)$	θ	Observed rate of inflation	$\frac{P(t)-P(t-1)}{P(t-1)}$
	r_F	Nominal interest rate	Table B-1
	$s(t)$	Real loanable funds measured by real gross investment at time *t*	Table B-1
	$y(t)$	Real gross domestic product at time t	Table B-1
	$m(t)$	Real money supply at time *t*	Table B-1
$\dot{y}(t)$	g	Economic growth rate	$\frac{y(t)-y(t-1)}{y(t-1)}$
	w	Index of labor cost	Table B-1
	μ	Rate of technological progress	$\frac{y(t)}{total\ labor\ force(t)}$
$\dot{N}(t)$	n	Rate of population change	$\frac{N(t)-N(t-1)}{N(t-1)}$
	a	Labor force participation rate	Table B-1
	$V(t)$	Velocity of money at time *t*	$\frac{y(t)}{m(t)}$

Table 2: Summary Statistics

Variable	Mean	Std. Dev.	Min	Max
r	0.023	0.025	-0.035	0.081
$s(t)$	3.21	0.35	2.48	3.74
$m(t)$	2.07	0.4	1.42	2.75
$V(t)$	1.43	0.1	1.28	1.68
w	104.83	3.24	98.63	109.9
g	0.03	0.02	-0.03	0.07
μ	0.05	0.02	-0.02	0.1
n	0.01	0	0	0.01

Notes:The variables $s(t)$ and $m(t)$ are in logarithmic form.

Table 3: Unit root tests

	r	$s(t)$	$m(t)$	$V(t)$	w	g	μ	n	$\widehat{\varepsilon_\theta}$
Panel A: Levels									
ADF-test	-2.34	-0.86	0.07	-1.06	-0.99	-5.27***	-3.72***	0.11	-2.08
ADF-test with trend	-2.47	-0.252	-1.99	-1.15	-2.17	-5.28***	-5.46***	-0.82	-3.25*
PP-test	-2.35	-0.85	-0.09	-1.47	-1.04	-5.29***	-3.66***	-0.49	-1.99
PP-test with trend	-2.41	-2.73	-2.56	-1.52	-2.37	-5.27***	-5.42***	-1.21	-3.35*
KPSS	0.19**	0.13*	0.07	0.20**	0.19**	0.07	0.13**	0.25***	0.15**
Panel B: First Differences									
ADF-test	-6.92***	-5.85***	-4.44***	-5.36***	-6.36***	-9.05***	-9.51***	-4.43***	-6.7***
PP-test	-6.94***	-5.81***	-4.45***	-5.41***	-6.37***	-10.05***	-11.48***	-4.42***	-6.8***
KPSS	0.05	0.04	0.04	0.07	0.05	0.04	0.04	0.05	0.05

Notes:
The variables $s(t)$ and $m(t)$ are in logarithmic form. The null hypothesis for ADF and PP unit root tests is that the time series is non-stationary. $Z(t)$-*statistics* are presented for both tests. The null hypothesis for KPSS unit root test is that the time series is trend stationary. The *LM-statistics* are reported. The *maximum lag* for KPSS is 3 and has been chosen by the Schwarz criterion. The symbols ***, **, * denote the rejection of null hypothesis for α=1, 5, and 10%, respectively.

Table 4: Unit root results with structural break

	r	$s(t)$	$m(t)$	$V(t)$	w	g	μ	n	$\widehat{\varepsilon_\theta}$
ZA	-3.24	-2.43	-1.88	-2.84	-3.76	-5.82***	-6.19***	-2.65	-5.50***
Break	1980	1993	1994	2001	2001	2009	1984	2008	1980

Notes:
The null hypothesis is that the series have a unit root with a structural break against the alternative hypothesis that they are stationary with a break. The test is performed in Eviews using the command *buroot*. The optimal lag is chosen by Schwarz information criterion. This test is outlined based on the work of Perron (1989). The symbols ***, **, * denote the rejection of null hypothesis for α=1, 5, and 10%, respectively.

Table 5: ARDL estimates[1]

	Variables	Coefficients	T-ratios	95% CI
Short-run effects	Δs_t	0.05	1.27	[-0.03, 0.13]
	Δm_t	-0.66***	-9.28	[-0.81, -0.51]
	Δm_{t-1}	-0.06**	-2.40	[-0.11, -0.01]
	ΔV_t	-0.44***	-9.07	[-0.54, -0.34]
	Δw_t	-0.005***	-5.02	[-0.006, -0.003]
	Δw_{t-1}	-0.002**	-2.20	[-0.004, -0.0001]
	Δg_t	0.90***	5.32	[0.55, 1.25]
	$\Delta \mu_t$	-0.26*	-1.79	[-0.57, 0.04]
	$\Delta \mu_{t-1}$	0.313***	5.45	[0.19, 0.43]
	Δn_t	7.24***	6.90	[5.06, 9.54]
	Δn_{t-1}	-7.50***	-5.93	[-10.13, -4.87]
	Δn_{t-2}	-6.12***	-4.90	[-8.72, -3.52]
	$\Delta \hat{\varepsilon}_{\theta}$	-0.67***	-8.57	[-0.84, -0.51]
	$\Delta \hat{\varepsilon}_{\theta,t-1}$	-0.27***	-4.71	[-0.39, -0.15]
	$\Delta \hat{\varepsilon}_{\theta,t-2}$	-0.15***	-3.56	[-0.24, -0.06]
Long-run effects	s_{t-1}	0.27***	7.58	[0.20, 0.34]
	m_{t-1}	-0.50***	-7.34	[-0.64, -0.36]
	V_{t-1}	-0.46***	-8.68	[-0.56, -0.35]
	w_{t-1}	-0.003**	-2.90	[-0.005, -0.0007]
	g_{t-1}	0.96***	5.68	[0.61, 1.32]
	μ_{t-1}	-0.79***	-3.28	[-1.29, -0.29]
	n_{t-1}	10.12***	7.65	[7.37, 12.88]
	$\hat{\varepsilon}_{\theta,t-1}$	-0.06	-0.37	[-0.37, 0.26]
ECT	r_{t-1}	-0.93***	-11.22	[-1.09, -0.75]
Exogenous variables	$I_{1t} = 1[t \geq 1980]$	0.03***	7.33	[0.02, 0.04]
	$I_{2t} = 1[t \geq 2008]$	-0.01**	-2.33	[-0.03, -0.001]
	t	0.005***	5.40	[0.003, 0.008]
	b_0	0.76***	6.40	[0.51, 1.008]
ARDL(1,1,2,0,2,0,2,3,3)	Number of observations	47		
	R^2	0.985		
	$\overline{R}^2$	0.967		
	$\sqrt{MSE}$	0.003		

Notes:

1. The optimal lag length is chosen by Schwarz's information criterion; the symbols ***, **, * denote the rejection of null hypothesis that the coefficients are equal to zero for α=1, 5, and 10% confidence intervals, respectively; and lastly it is worth noting that on the basis of the critical values presented in Table 6, there is strong evidence of cointegration.

Table 6: PSS bounds testing

		10%		5%		1%	
	K	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
F	15.51	2.37	3.45	2.69	3.83	3.34	4.63
t	-4.83	-3.13	-4.53	-3.41	-4.85	-3.96	-5.49

Notes:
I(0) and *I(1)* denote the lower and upper bounds for asymptotical critical values at 10%, 5% and 1% significant level of Pesaran, Shin and Smith bound test.

Table 7: Post-estimation diagnostic tests

		Test statistics	p-value
Heteroskedasticity	Cameron-Trivedi IM, White, test	47.00	0.43
	Breusch-Pagan/Cook–Weisberg test	0	0.95
Serial Correlation/ Auto-correlation	Durbin's alternative test	2.15	0.16
	Breusch–Godfrey LM test	4.56	0.05
	LM test for ARCH	0.18	0.68
Misspecification	Ramsey RESET test	1.91	0.16
Normality	Skewness and kurtosis tests	0.15	0.93
	Jarque-Bera test	0.02	0.99

Table 8: Dynamic ARDL estimates

	Variables	Coefficients	T-ratios	95% CI
Short-run effects	Δs_t	0.05	1.27	[-0.03, 0.13]
	Δm_t	-0.66***	-9.28	[-0.81, -0.51]
	Δm_{t-1}	-0.06**	-2.40	[-0.11, -0.01]
	Δw_t	-0.005***	-5.02	[-0.006, -0.003]
	Δw_{t-1}	-0.002**	-2.20	[-0.004, -0.0001]
	$\Delta \mu_t$	-0.26*	-1.79	[-0.57, 0.04]
	$\Delta \mu_{t-1}$	0.313***	5.45	[0.19, 0.43]
	Δn_t	7.24***	6.90	[5.06, 9.43]
	Δn_{t-1}	-7.50***	-5.93	[-10.13, -4.87]
	Δn_{t-2}	-6.12***	-4.90	[-8.72, -3.52]
	$\Delta \hat{\varepsilon}_{\theta}$	-0.67***	-8.57	[-0.84, -0.51]
	$\Delta \hat{\varepsilon}_{\theta,t-1}$	-0.27***	-4.71	[-0.39, -0.15]
	$\Delta \hat{\varepsilon}_{\theta,t-2}$	-0.15***	-3.56	[-0.24, -0.06]
Long-run effects	s_{t-1}	0.25***	8.31	[0.19, 0.31]
	m_{t-1}	-0.47***	-7.88	[-0.59, -0.34]
	V_{t-1}	-0.43***	-9.01	[-0.53, -0.33]
	w_{t-1}	-0.003**	-2.99	[-0.004, -0.0007]
	g_{t-1}	0.90***	5.32	[0.55, 1.25]
	μ_{t-1}	-0.73***	-3.47	[-1.18, -0.30]
	n_{t-1}	9.46***	7.45	[6.82, 12.10]
	$\hat{\varepsilon}_{\theta,t-1}$	-0.05	-0.37	[-0.35, 0.24]
ECT	r_{t-1}	-0.93***	-11.32	[-1.11, -0.76]
Exogenous variables	$I_{1t} = 1[t \geq 1980]$	0.03***	7.33	[0.02, 0.04]
	$I_{2t} = 1[t \geq 2008]$	-0.01**	-2.33	[-0.03, -0.001]
	t	0.005***	5.40	[0.003, 0.008]
	b_0	0.76***	6.40	[0.51, 1.008]
ARDL(1,1,2,0,2,0,2,3,3)	Number of observations	47		
	R^2	0.986		
	$\bar{R}^2$	0.967		
	$\sqrt{MSE}$	0.003		

Notes:

1. The optimal lag length is chosen by Schwarz information criterion. ***, **, * declares the rejection of null hypothesis that the coefficients are equal to zero for α=1, 5, and 10%, respectively. The *F*- and *t-tests* are 20.74 and -10.85, respectively. Comparing with the critical values, presented in Table 6, there is strong evidence of cointegration.
2. We call "long-run" effects the estimates of lagged variables for representation purposes. However, these estimates do not directly correspond to the estimates of Table 5 and the vector, δ, of long-run parameters. One can compare the estimates of Table 5 with the estimates of Table 8 and verify that the estimates are identical by multiplying the opposite of the speed of adjustment, $\hat{\gamma}$, and the estimated long-run coefficients, $\hat{\delta}$, reported in Table 5. Or, alternatively, by dividing the long-run coefficients reported in this table by the opposite of the ECT coefficient.

Table 9: Long-run elasticities & semi-elasticities of the interest rate[1]

Variables	Coefficients	T-ratios	95% CI
$\hat{e}_{rs}$	0.27***	7.58	[0.20, 0.34]
$\hat{e}_{rm}$	-0.50***	-7.34	[-0.63,-0.37]
$\hat{e}_{rV}$	-0.45***	-8.68	[-0.55, -0.35]
$\hat{e}_{rW}$	-0.003***	-2.90	[-0.005, -0.01]
$\hat{e}_{rg}$	0.96***	5.68	[0.63, 1.29]
$\hat{e}_{r\mu}$	-0.79***	-3.28	[-1.26, -0.32]
$\hat{e}_{rn}$	10.12***	7.65	[7.53, 12.72]
$\hat{e}_{r\hat{e}_{\theta}}$	−0.06	0.37	[-0.35, 0.25]

Notes

1. The elasticities shown in this table have been calculated at the sample mean values of the variables using the estimates obtained from equation (29) and reported in Table 8. Given that the interest rate is measured in logarithms, when an independent variable x is expressed also in logarithms, the elasticities are calculated by applying the formula:

$$e_{rx} = \frac{d \ln r}{d \ln x} = \frac{\hat{\theta}}{\hat{b}_2}$$

Otherwise, the semi-elasticities are computed by the formula:

$$e_{rx} = \frac{d \ln r}{dx} = \frac{\hat{\theta}}{\hat{b}_2}$$

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