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**Binary public decisions with a status quo: undominated
mechanisms without coercion**

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The authors assume full responsibility for the accuracy of their paper as well as for the opinions expressed therein.

Binary public decisions with a status quo: undominated mechanisms without coercion.*

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Abstract

We discuss the problem of choosing between two public alternatives under the assumption that preferences are quasi-linear and that one of the two alternatives represents the status quo. We characterize the class of strategy-proof and feasible mechanisms satisfying Voluntary Participation, that are not dominated by another strategy-proof and feasible mechanism. These mechanisms form a n -parameter infinite family, the Unanimity mechanism is the only anonymous mechanism within this class.

JEL classification: D71; D82

Keywords: Pure public good, Unanimity, Strategy-proofness, Undominated mechanisms, Voluntary Participation.

1 Introduction

The prevalence of a status quo places a special burden on the problem of collective decision making. Whether due to entitlements, force of habit or inertia, a settled

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state of affairs is harder to upend. In the case of public decisions, in particular, it is not possible to exclude agents from the consequences of the decision. This exacerbates the problem. The principle of *Voluntary Participation* dismisses coercion as a means of bringing about change: in the presence of a status quo, change can only come about if all agents involved in the decision may partake in the gains it produces.

We study a simple model comprising two public alternatives. One constitutes the current state of affairs, the other represents a reform of the status quo. Consider a limited liability corporate partnership. Such decisions as the introduction of a new partner, the increase of share capital, changing the nature of the business or dissolving it, all conform to the paradigm we study. Agents have different preferences over the public alternatives. Moreover, preferences are quasi-linear and monetary transfers are possible. The transfers may serve to compensate agents who suffer from change favoring the search for solutions that do not rely on coercion.

The paper can be summarized in three findings. First, we characterize the set of mechanisms that satisfy Strategy-proofness, Feasibility and Voluntary Participation. The theorem demonstrates that Voluntary Participation is unrestrictive in our setting. A rich class of strategy-proof and feasible mechanisms satisfies it. However, the generic element of this class is *dominated* by some other strategy-proof and feasible mechanism.¹

Subsequently, we identify the class of Undominated mechanisms satisfying Strategy-proofness, Feasibility and Voluntary Participation. Each mechanism in this class upends the status quo if and only if a reference-dependent consensus is struck. Suppose that there are n agents, for each of whom the parameter $\theta_i \in \mathbb{R}$, with $i = 1, 2, \dots, n$, denotes their willingness to pay for the reform to ensue.² Any

¹A strategy-proof and feasible mechanism is dominated by another strategy-proof and feasible mechanism if each agent, at each preference profile, prefers the latter to the former and, at least for one agent, the preference is strict.

²A negative value of θ_i denotes a preference for the status quo, agent i is indifferent between the reform and the status quo if $\theta_i = 0$.

vector $(\tilde{\delta}_1, \dots, \tilde{\delta}_n) \in \mathbb{R}^n$, such that $\sum_{i=1}^n \tilde{\delta}_i = 0$, constitutes a reference point that may give rise to an Undominated mechanism satisfying Strategy-proofness, Feasibility and Voluntary Participation. Any such mechanism reforms the status quo if and only if, for each $i \in \{1, \dots, n\}$, $\theta_i \geq \tilde{\delta}_i$. Each agent pays the amount $\tilde{\delta}_i$ if the reform ensues and 0 otherwise. There are as many mechanisms in this class as there are reference vectors $(\tilde{\delta}_1, \dots, \tilde{\delta}_n) \in \mathbb{R}^n$, such that $\sum_{i=1}^n \tilde{\delta}_i = 0$.

Finally, we show that the Unanimity mechanism, associated with the reference vector $(0, \dots, 0) \in \mathbb{R}^n$, is the only mechanism in this class that satisfies Anonymity or a mild neutrality requirement.

A reference point may be construed as an acknowledgment of entitlements that may be objectively ascertained and verified. For instance, the partners in a limited liability corporate partnership are considering the motion of dissolving the partnership. Ahead of the decision, one of the partners wishes to register the fact that she has loaned money to the partnership. She asks that the outstanding debt is paid in full before the dissolution of the LLP takes effect. The paper does not tackle the issue of selecting the reference, other than remarking that any reference different from $(0, \dots, 0) \in \mathbb{R}^n$ inevitably introduces bias against some agent and some alternative. If any such bias is to be considered unacceptable, then the only available option is the Unanimity mechanism.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the notation. Section 4 provides some preliminary findings. Section 5 presents the results. Section 6 concludes. The proofs are relegated to the appendix.

Relation to the literature

In Athanasiou and Valletta [3] we show that, while it is natural to discard dominated mechanisms, the requirement is not particularly restrictive. There we provide a full characterization of the set of Undominated mechanisms in the class of mechanisms satisfying Feasibility, Strategy-proofness and Anonymity in two-agent economies. This set is very large, it even contains mechanisms that may decide

against the alternative that is unanimously preferred by all agents.

In Athanasiou and Valletta [2] we consider a binary public decision setting in economies comprising an arbitrary finite number of agents. We refrain from assuming that a status-quo exists but, beside Feasibility, Strategy-proofness and Anonymity, we consider two additional requirements: a weakening of Decision Efficiency and a weakening of No-Envy. We show that this family of mechanisms includes a large class of voting rules that is singled out by No-Envy. The Unanimity mechanism constitutes a limiting case in this class of voting rules. It is the only mechanism of that family that appears in the present paper.

The two papers cited above demonstrate that the Unanimity mechanism is one of infinitely many mechanisms that are Undominated. In this paper we show that adding Voluntary Participation to the class of Undominated mechanisms that are strategy-proof, feasible and anonymous, dramatically shrinks the available options: one is only left with the Unanimity mechanism.

Our results also relate to Osheto [23] and Bandhu et al. [5]. The former paper shows that the Unanimity mechanism dominates all the mechanisms satisfying Strategy-Proofness, Voluntary Participation and Citizen Sovereignty. However, the result is derived in a different setting. The critical difference is that in Osheto [23] all agents agree that the reform is preferable to the status quo. The problem at hand concerns the distribution of the cost of the reform among the agents. To that end, Osheto [23] confines his search to mechanisms that charge each agent with a share of the cost and that always balance the budget.³ Bandhu et al. [5] operate in a setting that is similar to ours however, in providing a characterization of Unanimity, they look at voting rules instead of mechanisms.

In yet a different setting, involving probabilistic mechanisms, Hashimoto and Shiozawa [15] show that the Unanimity mechanism is undominated along with

³It is well known that there is a tension between Strategy-proofness and Voluntary Participation. Saijo [24] shows that there does not exist a strategy-proof mechanism satisfying the stand alone test (a more demanding condition than Voluntary Participation) for the provision of non-excludable public goods in economies with production. In the setting we explore, the fact that we are considering two alternatives allows us to escape this impossibility.

the majority voting mechanism, the random serial dictatorship mechanism, and the Faltings mechanism (Faltings [8]). However, in their setting, welfare considerations can only be made in expected terms. We exclusively focus on deterministic allocation rules.

Apart from the few papers that use the same approach as we do, albeit in a different economic environment (Sprumont [25] and Athanasiou [1]), most of the attention of the literature on Strategy-proof mechanism design focuses on assignment-efficient mechanisms. Primarily, this involves Groves mechanisms (Groves [10]), rather than Undominated mechanisms, with an emphasis on possible solutions to the fact that they run a deficit.⁴ For instance, several papers propose the strategy to rank Groves mechanisms with respect to the deficit they entail, in order to arrive at the least wasteful among them. (Bailey [4], Cavallo [7], Guo and Conitzer [11],[12], Guo et al. [13], [14], [20], Guo et al. [14]). Some papers follow an approach that is similar to ours: they look for undominated Groves mechanisms (Moulin [18], Guo et al. [14]).

Our paper ultimately provides a new characterization of the Unanimity mechanism. The classic contributions by Wicksell [26] and Buchanan and Tullock [6] emphasize that unanimity is the only rule that ensures that collective action always constitutes a Pareto-improvement over the status quo. However, if utility is transferable, the wicksellian principle of universal consent, does not, on its own, suffice to isolate Unanimity as the unique option. As long as one commits to move away from the status quo only if a welfare gain is to be had, there are several ways to divide the gains in a manner that profits everyone.

⁴In a seminal contribution, Holmstrom [16] has shown that that the family of assignment-efficient and strategy-proof mechanisms coincides with the family of Groves mechanisms. Green and Laffont [9] have shown that, at some profile of preferences, they generate transfers whose sum is negative.

2 Notation

Let $N = \{1, 2, \dots, n\}$ denote a finite set of agents. The members of such population have to choose one of two non-excludable and non-rivalrous alternatives denoted respectively 0, the status-quo, and 1.⁵

Agents have *quasi-linear preferences* defined over the consumption space $\{0, 1\} \times \mathbb{R}$ comprising the two possible alternatives and a numeraire good, the instrument that renders transfers possible. For each $i \in N$, the parameter $\theta_i \in \mathbb{R}$, that is private information, fully describes i 's disposition over the public alternatives. For each bundle $(\lambda, z) \in \{0, 1\} \times \mathbb{R}$, the utility of an agent whose private valuation is $\theta \in \mathbb{R}$, is given by the expression $u((\lambda, z); \theta) = \theta\lambda + z$. The parameter θ_i represents the amount of the numeraire good that agent $i \in N$ is willing to pay in order to switch from the status quo to its alternative. Differently stated, agent i is indifferent between the bundle $(0, \theta) \in \{0, 1\} \times \mathbb{R}$ and $(1, 0) \in \{0, 1\} \times \mathbb{R}$. Hence, the sign of $\theta \in \mathbb{R}$ exposes the ordinal preference of the agent over the two alternatives. If $\theta > 0$, the agent prefers the amendment of the status quo over its preservation. If $\theta < 0$, the opposite is true. Finally, if $\theta = 0$, the agent is indifferent. An economy is characterized by the profile $\theta_N \equiv (\theta_i)_{i \in N} \in \mathbb{R}^n$.

The function $f : \mathbb{R}^n \rightarrow \{0, 1\}$ is a decision criterion that associates each economy $\theta_N \in \mathbb{R}^n$ with a *decision* between the two alternatives: if $f(\theta_N) = 1$, the status quo is upended, otherwise it is not. A function $t_i : \mathbb{R}^n \rightarrow \mathbb{R}$ associates each economy $\theta_N \in \mathbb{R}^n$ with a transfer for agent $i \in N$. A vector of transfers is denoted $t(\theta_N) \equiv (t_i(\theta_N))_{i \in N}$.⁶ An *allocation* comprises a decision over the two alternatives and a vector of transfers. A *mechanism* $\varphi = (f, t)$ is a function that assigns an allocation to each economy in the domain. Individual i 's bundle, at $\theta_N \in \mathbb{R}^n$, according to φ is denoted $\varphi_i(\theta_N) \equiv (f(\theta_N), t_i(\theta_N))$. As a consequence, the utility

⁵The possibility of exclusion somehow alleviates the incentive problem. Moulin and Shenker [21], Moulin [19] and Maniquet and Sprumont [17] are examples of papers that propose solutions in this vein.

⁶Wherever sets are concerned, \subset and \subseteq denote strict and weak inclusion respectively. For two sets A, B , with $A \subset B$, $B \setminus A \equiv \{x \notin A \text{ and } x \in B\}$. Along the same lines, for any set $A \subset N$, $\theta_{N \setminus A} \equiv (\theta_i)_{i \in N \setminus A}$. Whenever vector inequalities are employed, we make use of the

achieved by agent i , at the profile θ_N , under the mechanism $\varphi = (f, t)$ is equal to $t_i(\theta_N)$, if $f(\theta_N) = 0$, and $\theta_i + t_i(\theta_N)$, if $f(\theta_N) = 1$.⁷ We rely on the notation $(\theta'_i, \theta_{N \setminus \{i\}})$ to describe the profile obtained from the vector θ_N when the parameter θ_i is replaced by θ'_i . Accordingly, $u(\varphi_i(\theta'_i, \theta_{N \setminus \{i\}}); \theta_i)$ denotes the utility afforded to agent $i \in N$, whose preference parameter is θ_i , by the bundle $\varphi_i(\theta'_i, \theta_{N \setminus \{i\}})$.

3 Strategy-proof mechanisms

All the mechanisms we consider satisfy Strategy-proofness and Feasibility.

A mechanism $\varphi = (f, t)$ satisfies **Strategy-proofness** if and only if for each $\theta_N \in \mathbb{R}^n$, each $\theta'_i \in \mathbb{R}$ and each $i \in N$,

$$u(\varphi_i(\theta_i, \theta_{N \setminus \{i\}}); \theta_i) \geq u(\varphi_i(\theta'_i, \theta_{N \setminus \{i\}}); \theta_i).$$

A mechanism $\varphi = (f, t)$ satisfies **Feasibility** if and only if for each $\theta_N \in \mathbb{R}_+^n$, $\sum_{i \in N} t_i(\theta_N) \leq 0$.

All the results of the paper rely on a representation theorem due to Nisan [22]. The theorem outlines a simple algorithm that allows us to construct the set of strategy-proof mechanisms.

Let \mathcal{P} be the family of functions $P : \mathbb{R}^n \rightarrow \{0, 1\}$ that are weakly monotonic.⁸ A

following notation:

$$\begin{aligned} \theta_N \leq \theta'_N, & \quad \text{if for each } i \in N, \theta_i \leq \theta'_i, \\ \theta_N < \theta'_N, & \quad \text{if for each } i \in N, \theta_i \leq \theta'_i \text{ and } \theta_N \neq \theta'_N, \\ \theta_N \ll \theta'_N, & \quad \text{if for each } i \in N, \theta_i < \theta'_i. \end{aligned}$$

⁷In order to emphasize the prevalence of a status quo we normalize individual utilities so that, for each $\theta \in \mathbb{R}$, $u((0, 0); \theta_i) = 0$. However, since preferences are quasi-linear our results would carry over if we were to normalize utilities as in Athanasiou and Valletta [2] or in any other way for that matter.

⁸A function $P : \mathbb{R}^n \rightarrow \{0, 1\}$ is weakly monotonic if and only if for each $\theta_N, \theta'_N \in \mathbb{R}^n$, $\theta_N \leq \theta'_N$ implies $P(\theta_N) \leq P(\theta'_N)$.

mechanism $\varphi = (f, t)$ is strategy-proof only if there exists $P \in \mathcal{P}$ such that:

$$\begin{aligned} \{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} &= \{\theta_N \in \mathbb{R}^n : f(\theta_N) = 1\}, \\ \{\theta_N \in \mathbb{R}^n : P(\theta_N) = 0\} &= \{\theta_N \in \mathbb{R}^n : f(\theta_N) = 0\}. \end{aligned}$$

Each function $P \in \mathcal{P}$ is associated with a partition of the domain of preferences into two connected sets. For each $P \in \mathcal{P}$ and each $A \subseteq \mathbb{R}^n$, let $P(A)$ denote the image of A under P , namely, $P(A) = \{P(\theta_N) : \theta_N \in A\}$. For each $P \in \mathcal{P}$, each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, let

$$p_i(\theta_{N \setminus \{i\}}) = \begin{cases} \inf\{x \in \mathbb{R} : P(x, \theta_{N \setminus \{i\}}) = 1\} & \text{if } P(\mathbb{R}, \theta_{N \setminus \{i\}}) = \{0, 1\}, \\ 0 & \text{if } P(\mathbb{R}, \theta_{N \setminus \{i\}}) = \{k\}, \\ & \text{for some } k \in \{0, 1\}. \end{cases} \quad (3.1)$$

If for some $\hat{P} \in \mathcal{P}$, some $\theta'_N \in \mathbb{R}^n$ and some $i \in N$, $\hat{P}(x, \theta'_{N \setminus \{i\}})$ is constant in $x \in \mathbb{R}$, then agent $i \in N$ is *inconsequential* at profile $\theta'_N \in \mathbb{R}^n$. Namely, she cannot alter the public decision by acting unilaterally. If at the profile $\theta'_N \in \mathbb{R}^n$, agent i is inconsequential, then $\hat{p}_i(\theta'_{N \setminus \{i\}}) = 0$. Otherwise, if for some $\hat{P} \in \mathcal{P}$, some $\theta'_N \in \mathbb{R}^n$ and some $i \in N$, $\hat{P}(x, \theta'_{N \setminus \{i\}})$ is non-constant in $x \in \mathbb{R}$, then $\hat{p}_i(\theta'_{N \setminus \{i\}})$ represents the unique threshold such that:

$$\hat{P}(x, \theta'_{N \setminus \{i\}}) = \begin{cases} 1 & \text{if } x > \hat{p}_i(\theta'_{N \setminus \{i\}}), \\ 0 & \text{if } x < \hat{p}_i(\theta'_{N \setminus \{i\}}). \end{cases}$$

By Strategy-proofness, this threshold needs to be a component of the transfer that agent $i \in N$ receives if the status quo is upended. Finally, let \mathcal{G} be the set of functions $g : \mathbb{R}^{n-1} \rightarrow \mathbb{R}$.

Theorem 1 *A mechanism $\varphi = (f, t)$ satisfies Strategy-proofness if and only if there exist $P \in \mathcal{P}$ and $(g_i)_{i \in N} \in \mathcal{G}^n$ such that, for each $\theta_N \in \mathbb{R}^n$,*

$$\varphi(\theta_N) = \begin{cases} \left(1, (g_i(\theta_{N \setminus \{i\}}) - p_i(\theta_{N \setminus \{i\}}))_{i \in N}\right) & \text{if } P(\theta_N) = 1, \\ \left(0, (g_i(\theta_{N \setminus \{i\}}))_{i \in N}\right) & \text{if } P(\theta_N) = 0. \end{cases}$$

Henceforth, we refer to the pair $(P, (g_i)_{i \in N})$ as a payment scheme. The weak monotonicity of P is a consequence of Strategy-proofness. Both the statement and the proof of Theorem 1 follow from Nisan [22] (Theorem 9.36). We omit the proof.

4 Characterization results

Suppose that a group of countries are considering whether to form a customs union. In this case *yes* refers to signing a trade agreement and *no* entails leaving matters as they stand. Voluntary participation requires that each country should be at least as well off as it is at the status quo.

A mechanism $\varphi = (f, t)$ satisfies **Voluntary Participation** if and only if for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, $u(\varphi_i(\theta_N); \theta_i) \geq 0$.

Let $\Delta_N = \{\delta_N \in \mathbb{R}^n : \sum_{i \in N} \delta_i = 0\}$. Each $\delta_N \in \Delta_N$ uniquely determines the convex cone

$$C(\delta_N) = \{\theta_N \in \mathbb{R}^n : \forall i \in N, \theta_i \geq \delta_i\}.$$

The exercise of ascertaining whether a strategy-proof and feasible mechanism satisfies Voluntary Participation becomes one of answering the following question: Is it the case that for some $\delta_N \in \Delta_N$ the set of profiles at which the status quo is upheld constitutes a subset of $C(\delta_N)$?

Let $g^0 \in \mathcal{G}$ be the constant mapping $g^0 : \mathbb{R}^{n-1} \rightarrow \{0\}$.

Theorem 2 A mechanism $\varphi = (f, t)$, associated with the payment scheme

$$(P, (g_i)_{i \in N}) \in \mathcal{P} \times \mathcal{G}^n,$$

satisfies Strategy-proofness, Feasibility and Voluntary Participation if and only if $P \in \mathcal{P}$ is such that either $P(\mathbb{R}^n) = \{0\}$, or there exists $\delta_N \in \Delta_N$ such that,

$$\{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} \subseteq C(\delta_N),$$

and for each $i \in N$, $g_i = g^0$.

An illustration of the previous result is offered in Figure 1, where we depict two partitions of the domain of preferences, for economies comprising two agents, associated with the functions $P^1, P^2 \in \mathcal{P}$. In the Figure we draw the boundaries

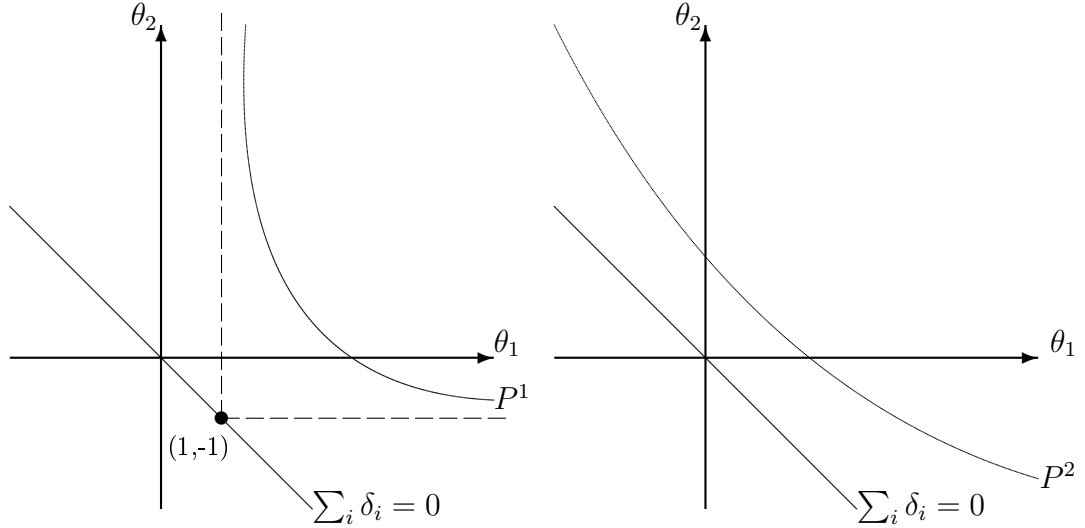


Figure 1: P^1 and P^2 can both be associated with a strategy-proof mechanism. Any feasible mechanism associated with P^2 will violate Voluntary Participation.

of the partitions of individual preference associated with each of the two functions. For each $\theta_N \in \mathbb{R}^n$ and each $j \in \{1, 2\}$, θ_N lying above the boundary associated with the partition function P^j entails $P^j(\theta_N) = 1$, while lying below the boundary associated with the partition function P^j entails $P^j(\theta_N) = 0$. We do not specify how the decision is resolved for profiles lying on the boundary. In light of Theorem 1, both P^1 and P^2 may be associated with a strategy-proof mechanism. However, in light of Theorem 2, any feasible mechanism associated with P^2 will violate Voluntary Participation. On the contrary, since

$$\{\theta_N \in \mathbb{R}^n : P^1(\theta_N) = 1\} \subset C((1, -1)),$$

the mechanism associated with the payment schemes (P^1, g^0) , satisfies Feasibility and Voluntary Participation.

Theorem 2 is instructive on how to construct a strategy-proof and feasible mechanism that satisfies Voluntary Participation. First, the designer needs to select some $\delta_N \in \Delta_N$. Subsequently, the designer needs to select some $P \in \mathcal{P}$ such that

$$\text{for each } \theta_N \in \mathbb{R}^n, P(\theta_N) = 1 \implies \theta_N \in C(\delta_N).$$

The matter of transfers is then fully resolved by Theorem 2. Although Voluntary

Participation is a notoriously strong requirement, it is not particularly restrictive in the context we are studying. Moreover, let us consider again the example depicted on the left-hand side of Figure 1. Since

$$\{\theta_N \in \mathbb{R}^n : P^1(\theta_N) = 1\} \subset C(-1, 1), \quad (4.1)$$

for any profile $\tilde{\theta}_N \in \mathbb{R}_N$, such that $P^1(\tilde{\theta}_N) = 1$, the transfers generated by the mechanism, in light of Theorem 2 (and 1), are such that, for each $i \in N$, $t_i(\tilde{\theta}_N) = -p_i^1(\tilde{\theta}_{N \setminus i}) < -\delta_i$ (this is due to the strict inclusion in equation 4.1). As a consequence, it would be possible to make all the agents better off by reducing the amount they have to pay (increasing the amount they receive) up to the value δ_i . This simple example highlights the fact that the typical mechanism belonging to the class described by Theorem 2, may be *dominated* by another strategy-proof and feasible mechanism.

The dominance criterion we employ compares pairs of strategy-proof and feasible mechanisms. Undominatedness is a weak efficiency requirement. All it requires is to make sure that there is no blatantly better way of doing things.

A mechanism φ' **dominates** φ , if and only if,

1. for each $\theta_N \in \mathbb{R}_+^n$ and each $i \in N$, $u(\varphi'_i(\theta_N); \theta_i) \geq u(\varphi_i(\theta_N); \theta_i)$,
2. for some $\tilde{\theta}_N \in \mathbb{R}_+^n$ and some $j \in N$, $u(\varphi'_j(\tilde{\theta}_N); \tilde{\theta}_j) > u(\varphi_j(\tilde{\theta}_N); \tilde{\theta}_j)$.

A mechanism φ is **Undominated** if and only if it is strategy-proof and feasible, and, moreover, there does not exist another strategy-proof and feasible mechanism φ' that dominates φ .

The dominance relation is a pre-order that compares pairs of mechanisms belonging to some set $\mathcal{M} \times \mathcal{M}$. In this paper \mathcal{M} consists of those mechanisms satisfying Strategy-proofness and Feasibility. In Athanasiou and Valletta [2] and [3], \mathcal{M} consists of those mechanisms satisfying Strategy-proofness, Feasibility and Anonymity.⁹ In Sprumont [25], \mathcal{M} consists of those mechanisms satisfy-

⁹For this specification of \mathcal{M} , Athanasiou and Valletta [3] describes the set of undominated mechanisms in economies comprising two agents.

ing Strategy-proofness, Feasibility, Anonymity and Envy-freeness. The standard that a mechanism needs to meet before it is deemed *undominated* is lower, the smaller \mathcal{M} becomes.

Adding the requirement of Undominatedness to the mechanisms singled out by Theorem 2 qualifies the design algorithm as follows. As before, one may select any $\delta_N \in \Delta_N$. Subsequently, and here lies the difference, the designer needs to select $P \in \mathcal{P}$ such that

$$\text{for each } \theta_N \in \mathbb{R}^n \setminus \{\delta_N\}, \quad P(\theta_N) = 1 \iff \theta_N \in C(\delta_N).$$

A mechanism $\varphi = (f, t)$, associated with the payment scheme $(P, (g_i)_{i \in N}) \in \mathcal{P} \times \mathcal{G}^n$, is a *reference-consensus* mechanism if and only if there exists $\delta_N \in \Delta_N$ such that

$$\{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} \setminus \{\delta_N\} = C(\delta_N) \setminus \{\delta_N\}$$

and for each $i \in N$, $g_i = g^0$.

Theorem 3 *A mechanism $\varphi = (f, t)$, associated with the payment scheme*

$$(P, (g_i)_{i \in N}) \in \mathcal{P} \times \mathcal{G}^n,$$

satisfies Strategy-proofness, Feasibility, Voluntary Participation and Undominatedness if and only if it is a reference-consensus mechanism.

Theorem 3 assigns to each mechanism satisfying Strategy-proofness, Feasibility, Voluntary Participation and Undominatedness a single element of Δ_N . Moreover, the mapping is *1-to-1* and *onto*. By Theorem 3, any mechanism in the class is fully described, decision and transfers, by the convex cone that underlies it. Figure 2 depicts two such mechanisms associated with $\delta_N^a, \delta_N^b \in \Delta_N$. Under the reference-consensus vector δ_N^a , agent 2 always receives a positive transfer ($t_2(\theta_1) = -\delta_2^a > 0$) if the status quo is upheld (so that she can be compensated in case $\theta_2 < 0$) while agent 1 is always charged the amount ($t_1(\theta_2) = -\delta_1^a < 0$) in case of reform. The opposite happens with the reference vector δ_N^b . In this

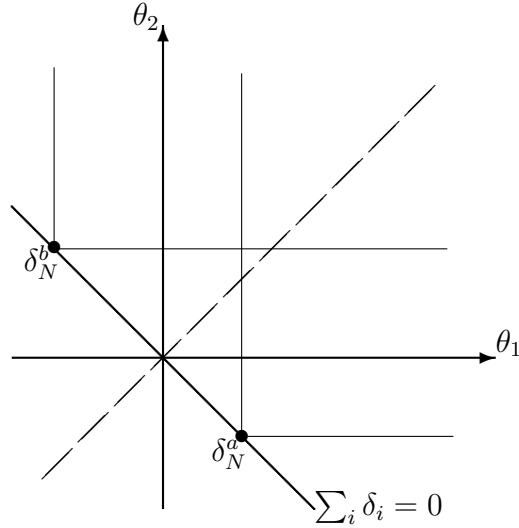


Figure 2: *Each $\delta_N \in \Delta_N$ can be associated with a unique strategy-proof, feasible and undominated mechanism that satisfies Voluntary Participation.*

respect, the former mechanism is somehow biased in favor of agent 2 while the latter is biased in favor of agent 1. Moreover, both mechanisms are biased in favor of the status-quo. Indeed, there are profiles at which the status-quo would not be upended even if both agents have a strictly positive willingness to pay for the reform.

The only reference vector avoiding both these occurrences is the reference-consensus vector

$$(0, \dots, 0) \in \Delta_N,$$

which generates the *Unanimity mechanism*. Indeed, at such reference-consensus vector the status-quo is upended only if all agents have a non-negative willingness to pay for the reform and no monetary transfers are performed. The following two well-known axioms put forward the idea of eliminating biases, both those that favor agents, as well as those that benefit some alternative.

Let Π be the family of bijective functions $\pi : N \rightarrow N$. For each $\pi \in \Pi$ and each $\theta_N \in \mathbb{R}_+^n$, $\theta_{\pi(N)}$ denotes the permutation of the elements of the vector θ_N according to π .

*A mechanism $\varphi = (f, t)$ satisfies **Anonymity** if and only if for each $\theta_N \in \mathbb{R}^n$,*

each $\pi \in \Pi$ and each $i \in N$,

$$\varphi_i(\theta_N) = \varphi_{\pi(i)}(\pi(\theta_N)).$$

A mechanism $\varphi = (f, t)$ satisfies **Weak Neutrality** if and only if for each $y \in \mathbb{R} \setminus \{0\}$,

$$f(y, \dots, y) \neq f(-y, \dots, -y).$$

Theorem 4 *A mechanism φ satisfies Strategy-proofness, Feasibility, Voluntary Participation, Undominatedness and Anonymity if and only if it is the Unanimity mechanism. Similarly, a mechanism φ satisfies Strategy-proofness, Feasibility, Voluntary Participation, Undominatedness and Weak neutrality if and only if it is the Unanimity mechanism.*

5 Concluding Remarks

Voluntary participation encompasses a fundamental prerogative of agents. In our setting the axiom becomes compelling due to the prevalence of some settled state of affairs. The status quo produces naturally a threshold to be met. If on the contrary, a group of agents must choose between two public alternatives, none of which corresponds to a status quo, the role of Voluntary Participation becomes opaque.

Suppose, for instance, that the two alternatives correspond to the two possible dates for some exam. It is still legitimate to require that the mechanism guarantees everyone a level of well-being above a certain threshold. However, the circumstances of the problem do not present us with an obvious way to set it. Suppose instead that a country is pondering whether to accept a set of commitments that aim to mitigate climate change. Suppose, moreover, that doing so would produce positive gains for society as a whole and the means exist to distribute these gains among the members of society. Under these assumptions, the objective of ensuring that everyone partakes in the gains becomes especially compelling.

Voluntary participation is known to be a restrictive axiom. Nonetheless, as Theorem 2 suggests, a plethora of strategy-proof and feasible mechanisms satisfy it. The addition of Undominatedness reduces the options available by discarding all the dominated mechanisms. Finally, adding either Anonymity or Weak neutrality dramatically shrinks the set of available options: one is only left with the Unanimity mechanism.

In this respect, an important insight this paper offers, we believe, is that Undominatedness constitutes a fundamental element of the normative argument for Unanimity. As much so, as Voluntary Participation.

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Appendix

In light of Theorem 1, for each strategy-proof mechanism $\varphi = (f, t)$, associated with the payment scheme $(P, (g_i)_{i \in N}) \in \mathcal{P} \times \mathcal{G}^n$, for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$$u(\varphi_i(\theta_N); \theta_i) = \begin{cases} \theta_i - p_i(\theta_{N \setminus \{i\}}) + g_i(\theta_{N \setminus \{i\}}) & \text{if } P(\theta_N) = 1, \\ g_i(\theta_{N \setminus \{i\}}) & \text{if } P(\theta_N) = 0. \end{cases} \quad (5.1)$$

Moreover, let

$$\beta(P) = \inf\{\theta \in \mathbb{R} : P(\theta, \dots, \theta) = 1\}. \quad (5.2)$$

For each $P \in \mathcal{P}$, $\beta(P)$ exists, unless

$$\text{either } P(\mathbb{R}^n) = \{0\} \text{ or } P(\mathbb{R}^n) = \{1\}.$$

If $\beta(P)$ exists, since $P \in \mathcal{P}$ is weakly monotonic, it is unique.

Proof of Theorem 2

Lemma 1 *If a mechanism $\varphi = (f, t)$, associated with the payment scheme*

$$(P, (g_i)_{i \in N}) \in \mathcal{P} \times \mathcal{G}^n,$$

satisfies Strategy-proofness, Feasibility and Voluntary Participation, then for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, $g_i(\theta_{N \setminus \{i\}}) = 0$.

Proof.

Step 1: *For each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, there exists $y_i \in \mathbb{R}$ such that $P(y_i, \theta_{N \setminus \{i\}}) = 0$.*

Suppose, for the sake of contradiction, that there exists $\theta'_N \in \mathbb{R}^n$ and $k \in N$ such that $P(\mathbb{R}, \theta'_{N \setminus \{k\}}) = 1$. Hence, by 3.1, $p_k(\theta'_{N \setminus \{k\}}) = 0$. Hence, by 5.1, for each $y \in \mathbb{R}$, $u(\varphi_k(y, \theta'_{N \setminus \{k\}}); y) = y + g_k(\theta'_{N \setminus \{k\}})$. By Feasibility, $g_k(\theta'_{N \setminus \{k\}})$ is finite. Hence, for $y \in \mathbb{R}$ sufficiently small we obtain $u(\varphi_k(y, \theta'_{N \setminus \{k\}}); y) < 0$, which contradicts Voluntary Participation.

Step 2: *For each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, $g_i(\theta_{N \setminus \{i\}}) = 0$.*

Consider some arbitrary $\theta'_N \in \mathbb{R}^n$ and $i \in N$. By Step 1, for some $y_i \in \mathbb{R}$, we obtain $P(y_i, \theta'_{N \setminus \{i}\}) = 0$. Hence, by equation 5.1,

$$u(\varphi_i(y_i, \theta'_{N \setminus \{i}\}); y_i) = g_i(\theta'_{N \setminus \{i}\}), \text{ and} \quad (5.3)$$

$$\forall j \in N \setminus \{i\}, u(\varphi_j(y_i, \theta'_{N \setminus \{i}\}); \theta'_j) = g_j(y_i, \theta'_{N \setminus \{i,j\}}). \quad (5.4)$$

By 5.3 and Voluntary Participation, $g_i(\theta'_{N \setminus \{i}\}) \geq 0$. Suppose that $g_i(\theta'_{N \setminus \{i}\}) > 0$. Hence, by 5.4 and Feasibility, there exists $k \in N \setminus \{i\}$ such that $g_k(y_i, \theta'_{N \setminus \{i,k\}}) < 0$. Hence, by 5.4, we obtain a contradiction with Voluntary Participation. Hence, $g_i(\theta'_{N \setminus \{i}\}) = 0$. ■

Let $g^0 \in \mathcal{G}$ be the constant mapping $g^0 : \mathbb{R}^{n-1} \rightarrow \{0\}$. Henceforth, in light of Lemma 1, all the mechanisms we consider are associated with a payment scheme $(P, (g_i)_{i \in N}) \in \mathcal{P} \times \{g^0\}$. Slightly abusing notation, we opt for the expression φ is associated with payment scheme (P, g^0) , instead of the more cumbersome expression φ is associated with the payment scheme $(P, (g_i)_{i \in N}) \in \mathcal{P} \times \{g^0\}$.

Lemma 2 *If a mechanism $\varphi = (f, t)$, associated with the payment scheme (P, g^0) , satisfies Strategy-proofness, Feasibility, Voluntary Participation and $\beta(P)$ exists, then $\beta(P) \geq 0$*

Proof. Suppose that $\beta(P) < 0$. Consider some $\epsilon \in (0, -\beta(P))$. By construction, $P(\beta(P) + \epsilon, \dots, \beta(P) + \epsilon) = 1$. By Theorem 1, Lemma 1 and equation 5.1, for each $i \in N$,

$$t_i(\beta(P) + \epsilon, \dots, \beta(P) + \epsilon) = -p_i(\beta(P) + \epsilon), \text{ and} \\ u(\varphi_i(\beta(P) + \epsilon, \dots, \beta(P) + \epsilon); \beta(P) + \epsilon) = \beta(P) + \epsilon - p_i(\beta(P) + \epsilon).$$

Hence, by Feasibility, for some $j \in N$, $p_j(\beta(P) + \epsilon) \geq 0$. Hence, as by construction $\beta(P) + \epsilon < 0$, we obtain a contradiction with Voluntary Participation.

Lemma 3 *If a mechanism $\varphi = (f, t)$, associated with the payment scheme (P, g^0) , satisfies Strategy-proofness, Feasibility and Voluntary Participation, then for each $\theta_N \in \mathbb{R}$,*

$$P(\theta_N) = 1 \implies \forall i \in N, P(\mathbb{R}, \theta_{N \setminus \{i}\}) = \{0, 1\}.$$

Proof.

Suppose that for some $j \in N$, $P(\mathbb{R}, \theta_{N \setminus \{j\}}) = \{k\}$, for some $k \in \{0, 1\}$. By assumption, $P(\theta_N) = 1$. Hence, $P(\mathbb{R}, \theta_{N \setminus \{j\}}) = \{1\}$. Hence, $p_j(\theta_{N \setminus \{j\}}) = 0$ and, by Lemma 1 and 5.1,

$$\text{for each } x \in \mathbb{R}, u(\varphi_i(x, \theta_{N \setminus \{j\}}); x) = x.$$

Hence, Voluntary Participation is violated. ■

Lemma 4 *If a mechanism $\varphi = (f, t)$, associated with the payment scheme (P, g^0) , satisfies Strategy-proofness, Feasibility, Voluntary Participation and $\beta(P)$ exists, then there exists $(\delta_1, \dots, \delta_n) \in \mathbb{R}^n$, with $\sum_{i \in N} \delta_i \geq 0$, such that for each $\theta_N \in \mathbb{R}^n$,*

$$P(\theta_N) = 1 \implies \langle \forall i \in N, p_i(\theta_{N \setminus \{i\}}) \geq \delta_i \rangle.$$

Proof.

Step 1: *For each $i \in N$ and each $\theta \in \mathbb{R}$ such that $\theta > \beta(P)$, $p_i(\theta, \dots, \theta)$ is non-increasing.*

Suppose, by way of contradiction, that there exists $j \in N$ and $\theta', \theta'' \in \mathbb{R}$ such that

$$\theta'' > \theta' > \beta(P) \tag{5.5}$$

and

$$p_j(\theta'', \dots, \theta'') > p_j(\theta', \dots, \theta'). \tag{5.6}$$

By construction, $P(\theta', \dots, \theta') = P(\theta'', \dots, \theta'') = 1$. Without loss of generality, let $j = 1$. By Lemma 3,

$$P(\mathbb{R}, \theta', \dots, \theta') = P(\mathbb{R}, \theta'', \dots, \theta'') = \{0, 1\}.$$

Hence, for each $y \in (p_1(\theta', \dots, \theta'), p_1(\theta'', \dots, \theta''))$,

$$P(y, \theta', \dots, \theta') = 1 \text{ and } P(y, \theta'', \dots, \theta'') = 0,$$

while by 5.5, $(y, \theta', \dots, \theta') < (y, \theta'', \dots, \theta'')$. Hence, $P \in \mathcal{P}$ is not weakly monotonic, a contradiction.

Step 2: For each $i \in N$, there exists $x_i \in \mathbb{R}$ such that, for each $\theta \in \mathbb{R}$ such that $\theta > \beta(P)$, $p_i(\theta, \dots, \theta) \geq x_i$.

Suppose, by way of contradiction, that for some $j \in N$,

$$\lim_{\theta \rightarrow +\infty} p_j(\theta, \dots, \theta) = -\infty. \quad (5.7)$$

For each $\theta > \beta(P)$, by definition, $P(\theta, \dots, \theta) = 1$ and, by Lemma 1,

$$\sum_{i \in N} t_i(\theta, \dots, \theta) = - \sum_{i \in N} p_i(\theta, \dots, \theta) \equiv \mu(\theta).$$

Moreover, by Step 1 and 5.7, for some $\tilde{\theta} > \beta(P)$ high enough, $\mu(\tilde{\theta}) > 0$, a violation of Feasibility.

By Step 2, for each $i \in N$,

$$\left\{ x \in \mathbb{R} : \forall \theta > \beta(P), p_i(\theta, \dots, \theta) \geq x \right\} \neq \emptyset.$$

and

$$\sup \left\{ x \in \mathbb{R} : \forall \theta > \beta(P), p_i(\theta, \dots, \theta) \geq x \right\} \text{ exists.}$$

Define, for each $i \in N$,

$$\delta_i \equiv \sup \left\{ x \in \mathbb{R} : \forall \theta > \beta(P), p_i(\theta, \dots, \theta) \geq x \right\}. \quad (5.8)$$

Step 3: $\sum_{i \in N} \delta_i \geq 0$.

Suppose, by way of contradiction, that $\sum_{i \in N} \delta_i < 0$. By definition and by Step 1, there exists $\bar{\theta} > \beta(P)$ such that, for each $i \in N$,

$$p_i(\bar{\theta}, \dots, \bar{\theta}) \text{ is infinitesimally close to } \delta_i.$$

Hence,

$$\sum_{i \in N} p_i(\bar{\theta}, \dots, \bar{\theta}) \text{ is infinitesimally close to } \sum_{i \in N} \delta_i < 0. \quad (5.9)$$

By construction, $P(\bar{\theta}, \dots, \bar{\theta}) = 1$ and $\sum_{i \in N} t_i(\bar{\theta}, \dots, \bar{\theta}) = - \sum_{i \in N} p_i(\bar{\theta}, \dots, \bar{\theta})$. By 5.9, we obtain a contradiction with Feasibility.

Step 4: For each $\theta_N \in \mathbb{R}^n$,

$$P(\theta_N) = 1 \implies \langle \forall i \in N, p_i(\theta_{N \setminus \{i\}}) \geq \delta_i \rangle.$$

Suppose not. Let there exist $\theta'_N \in \mathbb{R}^n$ and $j \in N$ such that

$$P(\theta'_N) = 1 \text{ and } p_j(\theta'_{N \setminus \{j\}}) < \delta_j.$$

Moreover, let $\theta'' \in \mathbb{R}$ be such that

$$\text{for each } i \in N, \theta'' > \max \{ \beta(P), \theta'_i \}.$$

Hence, by 5.8, for each $j \in N$, $p_j(\theta'', \dots, \theta'') \geq \delta_j$ and by construction,

$$p_j(\theta'_{N \setminus \{j\}}) < p_j(\theta'', \dots, \theta'').$$

Without loss of generality, let $j = 1$. Let $y \in (p_1(\theta'_{N \setminus \{1\}}), p_1(\theta'', \dots, \theta''))$. By Lemma 3, $P(\mathbb{R}, \theta'', \dots, \theta'') = \{0, 1\}$. Hence,

$$P(y, \theta'', \dots, \theta'') = 0. \tag{5.10}$$

By assumption, $P(\theta'_N) = 1$. Hence, either $P(\mathbb{R}, \theta'_{N \setminus \{1\}}) = \{0, 1\}$ or $P(\mathbb{R}, \theta'_{N \setminus \{1\}}) = \{1\}$. Hence,

$$P(y, \theta'_{N \setminus \{1\}}) = 1. \tag{5.11}$$

By construction,

$$(y, \theta'', \dots, \theta'') > (y, \theta'_{N \setminus \{1\}}). \tag{5.12}$$

By 5.10, 5.11 and 5.12, $P \in \mathcal{P}$ violates weak monotonicity, a contradiction. ■

Lemma 5 *If a mechanism $\varphi = (f, t)$, associated with the payment scheme (P, g^0) , satisfies Strategy-proofness, Feasibility, Voluntary Participation and $\beta(P)$ does not exist, then $P(\mathbb{R}^n) = \{0\}$.*

Proof.

Suppose that $P(\mathbb{R}^n) = \{1\}$. Hence, by Lemma 1 and 5.1, for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$$u(\varphi_i(\theta_N); x) = \theta_i.$$

Hence, we obtain a violation of Voluntary Participation. Since,

$$\beta(P) \text{ does not exist} \implies \langle \text{either } P(\mathbb{R}^n) = \{0\} \text{ or } P(\mathbb{R}^n) = \{1\} \rangle,$$

it must be $P(\mathbb{R}^n) = \{0\}$. ■

Proof of Theorem 2

Necessity

By Lemma 1, for each $i \in N$, $g_i = g^0$.

By Lemma 5,

$$\beta(P) \text{ does not exist} \implies P(\mathbb{R}^n) = \{0\}.$$

If, instead, $\beta(P)$ exists, then, by Lemmata 3 and 4, for some $(\delta'_1, \dots, \delta'_n) \in \mathbb{R}^n$, with $\sum_{i \in N} \delta'_i \geq 0$,

$$\{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} \subseteq \{\theta_N \in \mathbb{R}^n : \forall i \in N, \theta_i \geq \delta'_i\}. \quad (5.13)$$

Let $\Delta_N = \{\delta_N \in \mathbb{R}^n : \sum_{i \in N} \delta_i = 0\}$. Since $\sum_{i \in N} \delta'_i \geq 0$, there exists $\tilde{\delta}_N \in \Delta_N$ such that for each $i \in N$, $\delta'_i \geq \tilde{\delta}_i$. Hence, by 5.13,

$$\{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} \subseteq \{\theta_N \in \mathbb{R}^n : \forall i \in N, \theta_i \geq \tilde{\delta}_i\}.$$

Sufficiency

By Theorem 1, all the mechanisms characterized by Theorem 2 satisfy Strategy-proofness. It is easy to verify that they all satisfy Voluntary Participation and Feasibility as well. In particular, for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$$u(\varphi_i(\theta_N); \theta_i) = \begin{cases} \theta_i - p_i(\theta_{N \setminus \{i\}}) \geq \theta_i - \delta_i \geq 0 & \text{if } P(\theta_N) = 1, \\ 0 & \text{if } P(\theta_N) = 0. \end{cases}$$

Moreover, for each $\theta_N \in \mathbb{R}^n$,

$$\sum_{i \in N} t_i(\theta_N) = \begin{cases} -\sum_{i \in N} p_i(\theta_{N \setminus \{i\}}) \leq -\sum_{i \in N} \delta_i = 0 & \text{if } P(\theta_N) = 1, \\ 0 & \text{if } P(\theta_N) = 0. \end{cases}$$

■

Proof of Theorem 3

Necessity

Lemma 6 *If a mechanism $\varphi = (f, t)$, associated with the payment scheme (P, g^0) , satisfies Strategy-proofness, Feasibility, Voluntary Participation and $\beta(P)$ does not exist, then φ is dominated.*

Proof.

Suppose $\beta(P)$ does not exist. Hence, by Theorem 2, $P(\mathbb{R}^n) = \{0\}$. Hence, by 5.1 and Lemma 1, for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$$u(\varphi_i(\theta_N); \theta_i) = 0.$$

Let $P^u \in \mathcal{P}$ be such that $P^u(0, \dots, 0) \in \{0, 1\}$ and for each $\theta_N \in \mathbb{R}^n \setminus \{(0, \dots, 0)\}$

$$P^u(\theta_N) = \begin{cases} 1 & \text{if } \theta_N \in \mathbb{R}_+^n, \\ 0 & \text{otherwise.} \end{cases}$$

Let φ^u be a mechanism associated with the payment scheme (P^u, g^0) . By construction, for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$$p_i(\theta_{N \setminus \{i\}}) = 0 \text{ and } u(\varphi_i^u(\theta_N); \theta_i) = \begin{cases} \theta_i & \text{if } \theta_N \in \mathbb{R}_+^n \setminus \{(0, \dots, 0)\}, \\ 0 & \text{otherwise.} \end{cases}$$

By Theorem 1, φ^u satisfies Strategy-proofness and by construction it satisfies Feasibility. Moreover, by construction, for each $i \in N$,

$$u(\varphi_i^u(\theta_N); \theta_i) - u(\varphi_i(\theta_N); \theta_i) = \begin{cases} \theta_i & \text{if } \theta_N \in \mathbb{R}_+^n \setminus \{(0, \dots, 0)\}, \\ 0 & \text{otherwise.} \end{cases}$$

Hence, φ^u dominates φ . ■

Lemma 7 *If a mechanism $\varphi = (f, t)$, associated with the payment scheme (P, g^0) , satisfies Strategy-proofness, Feasibility, Voluntary Participation and for some $\delta_N \in \Delta_N$,*

$$\{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} \subset \{\theta_N \in \mathbb{R}^n : \forall i \in N, \theta_i \geq \delta_i\},$$

then φ is dominated.

Proof.

Suppose that for some $\delta_N \in \Delta_N$,

$$\{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} \subset \{\theta_N \in \mathbb{R}^n : \forall i \in N, \theta_i \geq \delta_i\}. \quad (5.14)$$

By Lemma 3, for each $\theta_N \in \mathbb{R}$,

$$P(\theta_N) = 1 \implies \forall i \in N, P(\mathbb{R}, \theta_{N \setminus \{i\}}) = \{0, 1\}. \quad (5.15)$$

Hence, by 5.14 and 5.15, for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$$u(\varphi_i(\theta_N); \theta_i) = \begin{cases} \theta_i - p(\theta_{N \setminus \{i\}}) \leq \theta_i - \delta_i & \text{if } P(\theta_N) = 1, \\ 0 & \text{otherwise.} \end{cases} \quad (5.16)$$

Let $P' \in \mathcal{P}$ be such that for each $\theta_N \in \mathbb{R}^n$,

$$P'(\theta_N) = \begin{cases} 1 & \text{if for each } i \in N, \theta_i \geq \delta_i, \\ 0 & \text{otherwise.} \end{cases}$$

Let φ' be a mechanism associated with the payment scheme (P', g^0) . By construction,

$$u(\varphi_i(\theta_N); \theta_i) = \begin{cases} \theta_i - \delta_i & \text{if } P(\theta_N) = 1, \\ 0 & \text{otherwise.} \end{cases} \quad (5.17)$$

By construction,

$$\{\theta_N \in \mathbb{R}^n : P'(\theta_N) = 1\} = \{\theta_N \in \mathbb{R}^n : \forall i \in N, \theta_i \geq \delta_i\}.$$

Hence,

$$\{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} \subset \{\theta_N \in \mathbb{R}^n : P'(\theta_N) = 1\}. \quad (5.18)$$

By 5.16, 5.17 and 5.18, φ' dominates φ . ■

Sufficiency

Let $\delta_N \in \Delta_N$. Let φ be some reference-consensus mechanism associated with the payment scheme (P, g^0) , where

$$P(\theta_N) = \begin{cases} 1 & \text{if for each } i \in N, \theta_i \geq \delta_i, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose, finally, that there exists a strategy-proof and feasible mechanism $\varphi' = (f', t')$, associated with the payment scheme $(P', (g'_i)_{i \in N}) \in \mathcal{P} \times \mathcal{G}^n$, that dominates φ . We will arrive at a contradiction after taking the following intermediate steps.

Step 1: For each $\theta_N \in \mathbb{R}^n$, if $\sum_{i \in N} \theta_i < 0$, then $P'(\theta_N) = 0$.

For the sake of contradiction, let there exist $\tilde{\theta}_N \in \mathbb{R}^n$ such that $\sum_{i \in N} \tilde{\theta}_i < 0$ and $P'(\tilde{\theta}_N) = 1$. Hence, for each $i \in N$, by 5.1,

$$u(\varphi'_i(\tilde{\theta}_{N \setminus \{i\}}); \tilde{\theta}_i) = \tilde{\theta}_i - p'_i(\tilde{\theta}_{N \setminus \{i\}}) + g'_i(\tilde{\theta}_{N \setminus \{i\}}).$$

By assumption, $\sum_{i \in N} \tilde{\theta}_i < 0$. Hence, $P(\tilde{\theta}_N) = 0$. By assumption, φ' dominates φ . Hence, for each $i \in N$,

$$\tilde{\theta}_i - p'_i(\tilde{\theta}_{N \setminus \{i\}}) + g'_i(\tilde{\theta}_{N \setminus \{i\}}) \geq 0.$$

Hence,

$$\sum_{i \in N} t'_i(\tilde{\theta}_N) = \sum_{i \in N} (-p'_i(\tilde{\theta}_{N \setminus \{i\}}) + g'_i(\tilde{\theta}_{N \setminus \{i\}})) \geq -\sum_{i \in N} \tilde{\theta}_i.$$

By assumption, $-\sum_{i \in N} \tilde{\theta}_i > 0$, thus Feasibility is contradicted.

Step 2: For each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, $g'_i(\theta_{N \setminus \{i\}}) = 0$.

Consider some arbitrary $\theta'_N \in \mathbb{R}^n$. For each $i \in N$, let $y_i \in \mathbb{R}$ be such that

$$y_i + \sum_{j \in N \setminus \{i\}} \theta'_j < 0. \tag{5.19}$$

By Step 1, for each $i \in N$,

$$P'(y_i, \theta'_{N \setminus \{i\}}) = 0.$$

By 5.19, for each $i \in N$,

$$P(y_i, \theta'_{N \setminus \{i\}}) = 0.$$

Hence, since φ' dominates φ ,

$$\text{for each } i \in N, g'_i(\theta'_{N \setminus \{i\}}) \geq 0.$$

Moreover, if for some $j \in N$, $g'_j(\theta'_{N \setminus \{j\}}) > 0$, then, by Feasibility, there must exist $k \in N$, such that $g'_k(y_j, \theta'_{N \setminus \{j, k\}}) < 0$. Hence,

$$u(\varphi'_k(y_j, \theta'_{N \setminus \{j\}}); \theta'_k) = g'_k(y_j, \theta'_{N \setminus \{j, k\}}) < 0 = u(\varphi_k(y_j, \theta'_{N \setminus \{j\}}); \theta'_k)$$

This violates the assumption that φ' dominates φ . Hence, for each $i \in N$, $g'_i(\theta'_{N \setminus \{i\}}) = 0$.

Step 3: For each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$\langle \text{for each } j \in N \setminus \{i\}, \theta_j \geq \delta_j \rangle \implies \langle P'(\mathbb{R}, \theta_{N \setminus \{i\}}) = \{0, 1\} \text{ and } p'_i(\theta_{N \setminus \{i\}}) = \delta_i. \rangle$

Suppose, first, that either $p'_i(\theta_{N \setminus \{i\}}) > \delta_i$ or $P'(\mathbb{R}, \theta_{N \setminus \{i\}}) = \{0\}$. Hence, for some $y \in (\delta_i, p'_i(\theta_{N \setminus \{i\}}))$, $P'(y, \theta_{N \setminus \{i\}}) = 0$. Hence, from Step 2 and by 5.1,

$$u(\varphi'_i(y, \theta_{N \setminus \{i\}}); y) = 0.$$

Moreover, by construction and by 5.1, $P(y, \theta_{N \setminus \{i\}}) = 1$ and

$$u(\varphi_i(y, \theta_{N \setminus \{i\}}); y) = y - \delta_i > 0.$$

This contradicts the assumption that φ' dominates φ .

Suppose, next, that $P'(\mathbb{R}, \theta_{N \setminus \{i\}}) = \{1\}$. Hence, for some $y' < 0$, $P'(y', \theta_{N \setminus \{i\}}) > P(y', \theta_{N \setminus \{i\}})$. Hence,

$$u(\varphi'_i(y', \theta_{N \setminus \{i\}}); y') = y' < 0 = u(\varphi_i(y', \theta_{N \setminus \{i\}}); y').$$

This contradicts the assumption that φ' dominates φ^u .

Suppose, finally, that $p'_i(\theta_{N \setminus \{i\}}) < \delta_i$. Hence, $P'(\delta_i, \theta_{N \setminus \{i\}}) = 1$. Hence, by Step 2, $t'_i(\delta_i, \theta_{N \setminus \{i\}}) = -p'_i(\theta_{N \setminus \{i\}}) > -\delta_i$. Hence, by Feasibility, since $\sum_{j \in N} \delta_j = 0$, there exists $k \in N \setminus \{i\}$ such that $t'_k(\delta_i, \theta_{N \setminus \{i\}}) < -\delta_k$. Hence,

$$u(\varphi'_k(\delta_i, \theta_{N \setminus \{i\}}); \theta_k) = \theta_k + t'_k(\delta_i, \theta_{N \setminus \{i\}}) < \theta_k - \delta_k. \quad (5.20)$$

Moreover, by construction, $P(\delta_i, \theta_{N \setminus \{i\}}) = 1$ and

$$u(\varphi_k(y, \theta_{N \setminus \{i\}}); \theta_k) = \theta_k - \delta_k. \quad (5.21)$$

By 5.20 and 5.21, the assumption that φ' dominates φ is again contradicted.

Step 4: For each $\theta_N \in \mathbb{R}^n$

$\langle \text{there exists } j \in N \text{ such that } \theta_j < \delta_j \rangle \implies \langle P'(\theta_N) = 0 \rangle$.

Let

$$\mathcal{Q} = \{\theta_N \in \mathbb{R}^n : \exists k \in N \text{ s.t. } \forall i \in N \setminus \{k\}, \theta_i \geq \delta_i \text{ and } \theta_k < \delta_k\}.$$

For each $\theta_N \in \mathcal{Q}$, by Step 3,

$$P'(\mathbb{R}, \theta_{N \setminus \{k\}}) = \{0, 1\} \text{ and } p'_k(\theta_{N \setminus \{i\}}) = \delta_k.$$

Hence,

$$\theta_k < \delta_k \implies P'(\theta_N) = 0.$$

Consider then some arbitrary $\hat{\theta}_N \in \mathbb{R}^n$ such that for some some $j \in N$, $\hat{\theta}_j < \delta_j$.

For some $\epsilon > 0$,

$$(\epsilon, \dots, \epsilon, \hat{\theta}_j, \epsilon, \dots, \epsilon) \in \mathcal{Q} \text{ and } (\epsilon, \dots, \epsilon, \hat{\theta}_j, \epsilon, \dots, \epsilon) \geq \hat{\theta}_N.$$

Hence, $P'(\epsilon, \dots, \epsilon, \hat{\theta}_j, \epsilon, \dots, \epsilon) = 0$ and, since $P' \in \mathcal{P}$, $P'(\hat{\theta}_N) = 0$.

Step 5: For each $\theta_N \in \mathbb{R}^n$,

$$\theta_N > \delta_N \implies P'(\theta_N) = 1.$$

Suppose that there exists $\tilde{\theta}_N > \delta_N$ such that $P(\tilde{\theta}_N) = 0$. By assumption, there exists $k \in N$ such that $\tilde{\theta}_k > \delta_k$. Hence, by Step 2,

$$u(\varphi'_k(\tilde{\theta}_N); \theta_k) = 0,$$

and by construction,

$$u(\varphi_k(\tilde{\theta}_N); \theta_k) = \tilde{\theta}_k - \delta_k > 0.$$

Hence, the assumption that φ' dominates φ is contradicted.

By Steps 4 and 5, for each $\theta_N \in \mathbb{R}^n \setminus \{\delta_N\}$, $P(\theta_N) = P'(\theta_N)$. Hence, by Step 2, φ' and φ are welfare equivalent, a contradiction. ■

Proof of Theorem 4

Proof.

We only prove the necessity part of both statements. Without loss of generality, let $N = \{1, 2\}$. Suppose that $\varphi = (f, t)$, associated with the payment scheme

$$(P, (g_i)_{i \in N}) \in \mathcal{P} \times \mathcal{G}^n,$$

is a reference-consensus mechanism with $\delta_N \in \Delta_N \setminus \{(0, 0)\}$.

Hence, for some $x > 0$, $P(x, 0) \neq P(0, x)$, which violates Anonymity and $P(x, x) = P(-x, -x) = 0$, which violates Weak Neutrality. ■



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