Implementing Sustainable Development under Deep Uncertainty

Ambiguity Aversion, Modern Bayesianism and Small Worlds

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- Member of Greek Prime-Ministerial Committee on Recovery and Development Plan
- Member of the Greek Ministerial Climate Change Committee, Ministry of Environment and Energy

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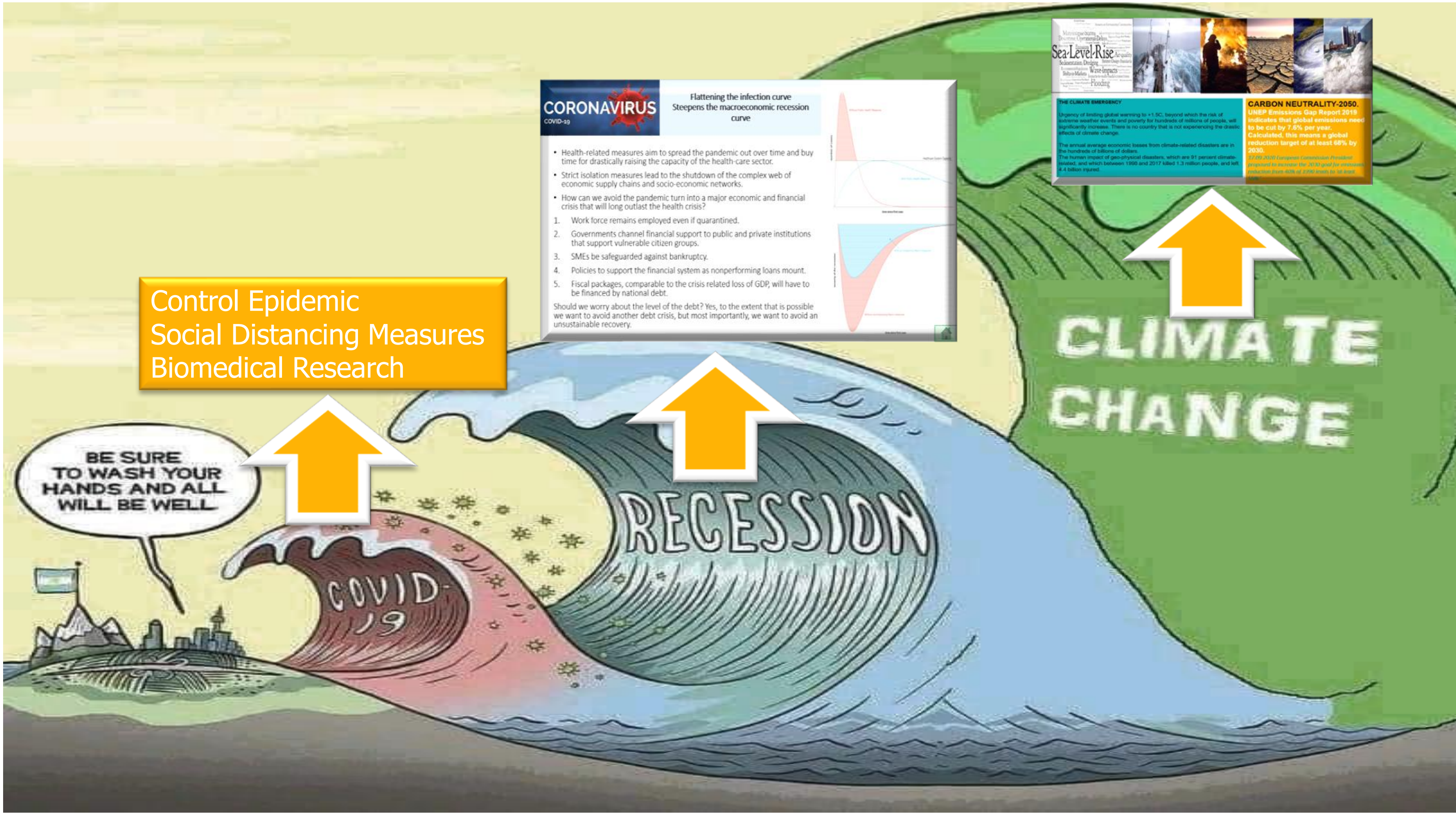
Climate-KIC is supported by the EIT, a body of the European Union



Control Epidemic
Social Distancing Measures
Biomedical Research
CORONAVIRUS
COVID-19
Flattening the infection curve
Steepens the macroeconomic recession curve
• Health-related measures aim to spread the pandemic out over time and buy time for drastically raising the capacity of the health-care sector.
• Strict isolation measures lead to the shutdown of the complex web of economic supply chains and socio-economic networks.
• How can we avoid the pandemic turn into a major economic and financial crisis that will long outlast the health crisis?
1. Work force remains employed even if quarantined.
2. Governments channel financial support to public and private institutions that support vulnerable citizen groups.
3. SMEs be safeguarded against bankruptcy.
4. Policies to support the financial system as nonperforming loans mount.
5. Fiscal packages, comparable to the crisis related loss of GDP, will have to be financed by national debt.
Should we worry about the level of the debt? Yes, to the extent that is possible we want to avoid another debt crisis, but most importantly, we want to avoid an unsustainable recovery.
Sea-Level-Rise
Wave-Impacts
Flooding
THE CLIMATE EMERGENCY
Urgency of limiting global warming to +1.5C, beyond which the risk of extreme weather events and poverty for hundreds of millions of people, will significantly increase. There is no country that is not experiencing the drastic effects of climate change.
The annual average economic losses from climate-related disasters are in the hundreds of billions of dollars.
The human impact of geo-physical disasters, which are 91 percent climate-related, and which between 1998 and 2017 killed 1.3 million people, and left 4.4 billion injured.
CARBON NEUTRALITY-2050.
UNEP Emissions Gap Report 2019 indicates that global emissions need to be cut by 7.6% per year.
Calculated, this means a global reduction target of at least 68% by 2030.
BE SURE TO WASH YOUR HANDS AND ALL WILL BE WELL
COVID-19
RECESSION
CLIMATE CHANGE

Sustainability Policy Framework

2015

UNITED NATIONS SUSTAINABLE DEVELOPMENT SUMMIT 2015
25-27 SEPTEMBER

193 Countries

17 SDGs

169 Targets

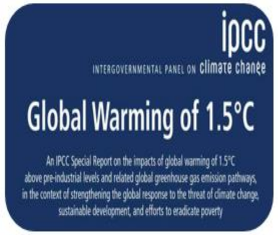
2015

PARIS2015
COP21·CMP11

197 Countries

Limiting global temperature to well below +2°C

2018



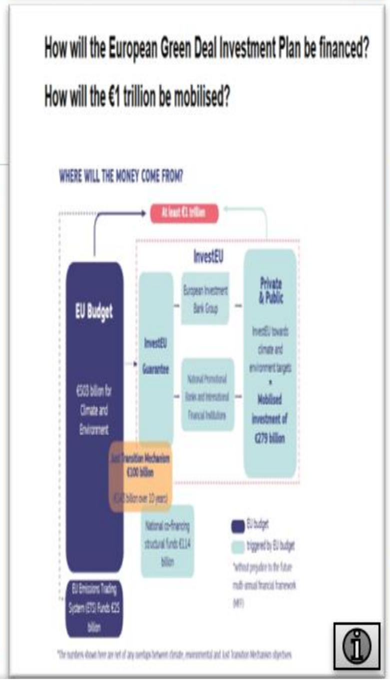
- Limiting global temp. to 1.5°c
- This implies zero net emissions globally by 2050

2019

6 Major Transformations to achieve SDGs

Dec 2019

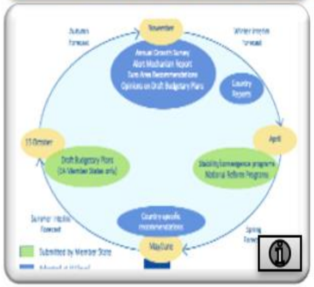
EGD Policies Overview



2020 …

Flattening the infection curve steepens the macroeconomic recession curve

Enhanced EU MFF & Recovery Plan Next Generation EU



Senior WG for the EU Green Deal

Top-Down Mobilization
Green New Deals around the World

Canada
The Pact for a Green New Deal
Proposed on May 2019

USA
Green New Deal
Proposed on March 2019

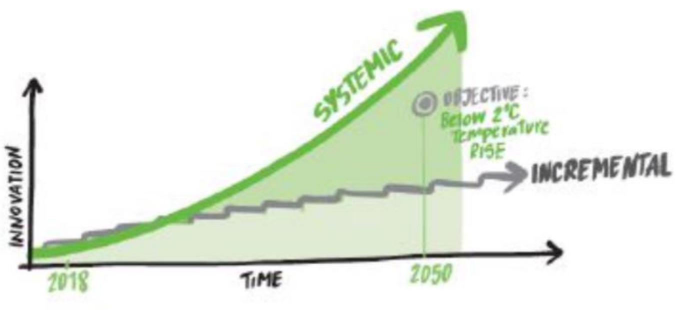
South Korea
Green New Deal
Agreed on 14 July, 2020
$94.5 billion

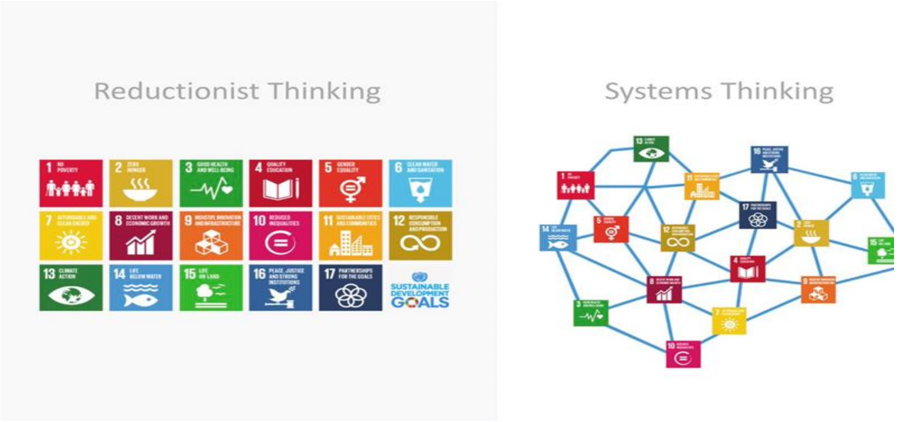
Israel
Green recovery plan
June 2020

China
Carbon neutral before 2060
Announced on 22 September, 2020

Systems Innovation co-designed with Problem Owners

Integrated & Coordinated Interventions in economic, financial, political and social systems and along whole value chains. In systems, by means of the relations, elements are arranged in such a fashion that gives rise to a **new structure** functioning.





Working through gradual, incremental changes is not enough!

What is needed now is a **fundamental transformation** of economic, social and financial systems that will trigger exponential change in decarbonisation rates and strengthen climate resilience – IPCC report: "**rapid, far- reaching and unprecedented changes in all aspects of society**".

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ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

SDSN GREECE

Cluster for Sustainability Transition

Transforming Research and Innovation into Climate Action

Director: Professor Phoebe Koundouri

The Cluster on Sustainability Transition (CST)

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ReSEES, AUEB
https://www.dept.aueb.gr/en/ReSEES

SDSN GREECE

UN SDSN GREECE
http://www.unsdsn.gr/

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EIT Climate-KIC HUB GR
https://www.athena-innovation.gr/en/eit-climate-kic-greece-hub

CLUSTER ON SUSTAINABILITY TRANSITION

Research - Innovation Acceleration
Deep Demonstration - Education & Training

Research and Innovation Projects Global Initiatives

BRIGAID

OpenAIRE

simra Social Innovation in Marginalised Rural Areas

AMAre

COASTAL Collaborative Land-Sea Integration Platform

eit Climate-KIC

SMIRES Science and Management of Intermittent Rivers and Ephemeral Streams

Interreg Balkan-Mediterranean RECONNECT

BlueBRIDGE

MERMAID

GENESIS groundwater and dependent ecosystems

TROPOS

GLOBAQUA

THESEUS

DAFNE

H2Ocean

Euro-limpacs

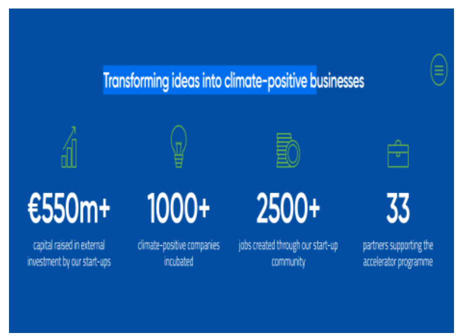
CIGESMED

AQUASTRESS

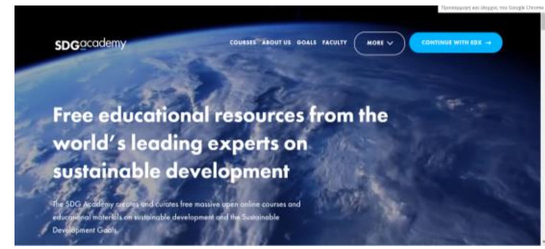
erc European Research Council Supporting top researchers from anywhere in the world

Smart Water Futures:

Innovation Acceleration Deep Demonstration



Education & Training



MSc in Law and Economics in Energy Markets

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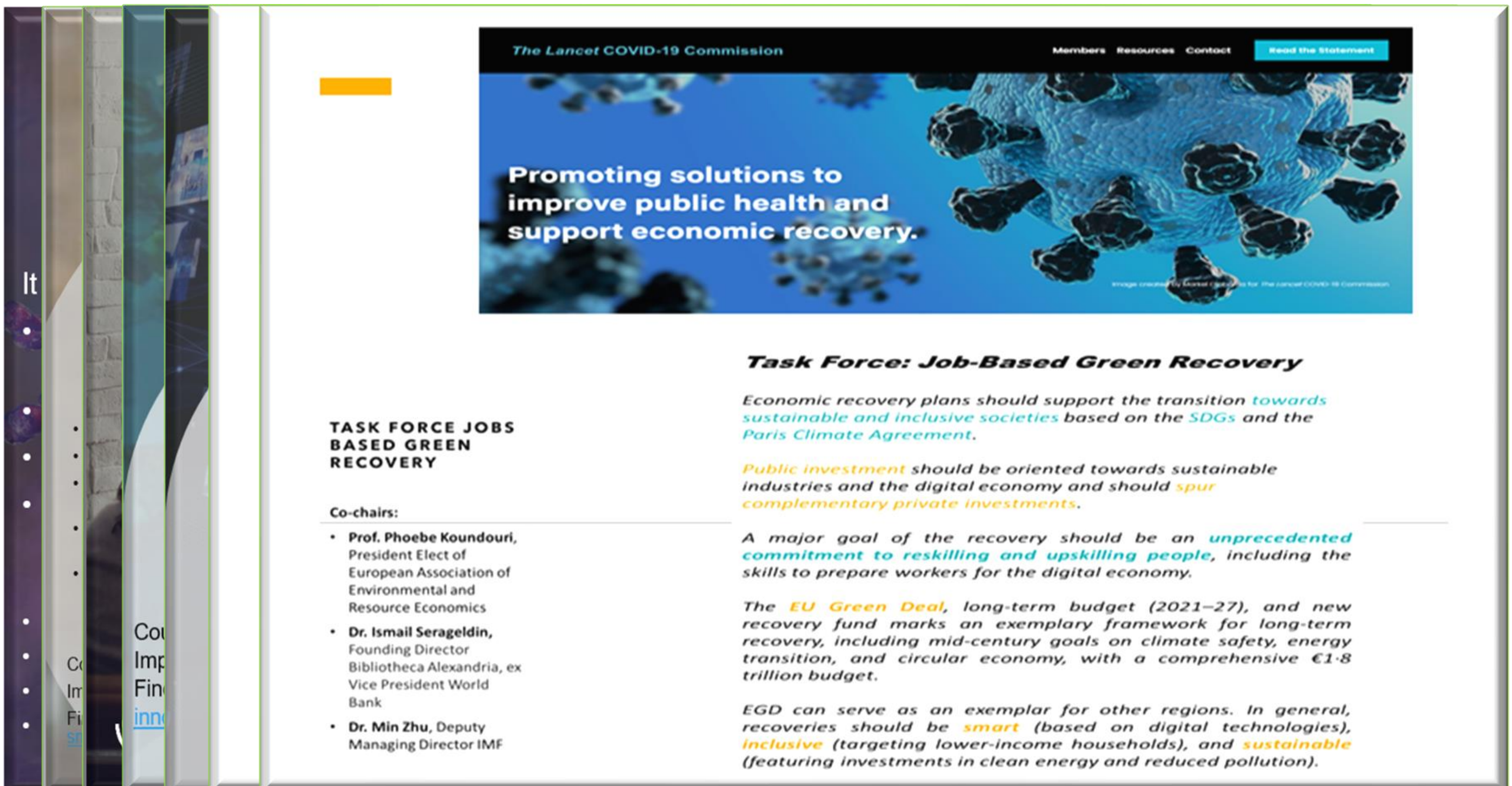
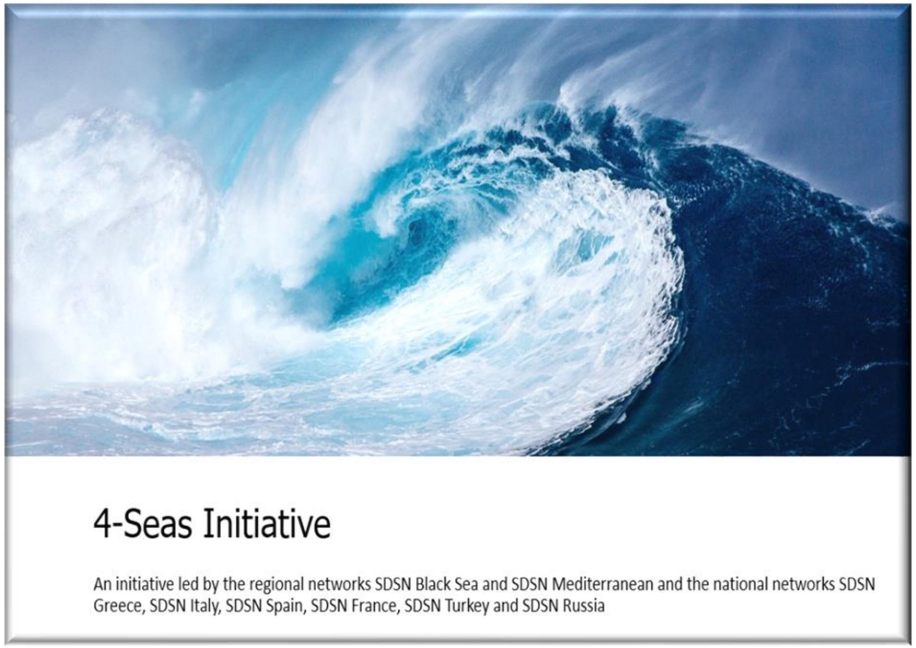
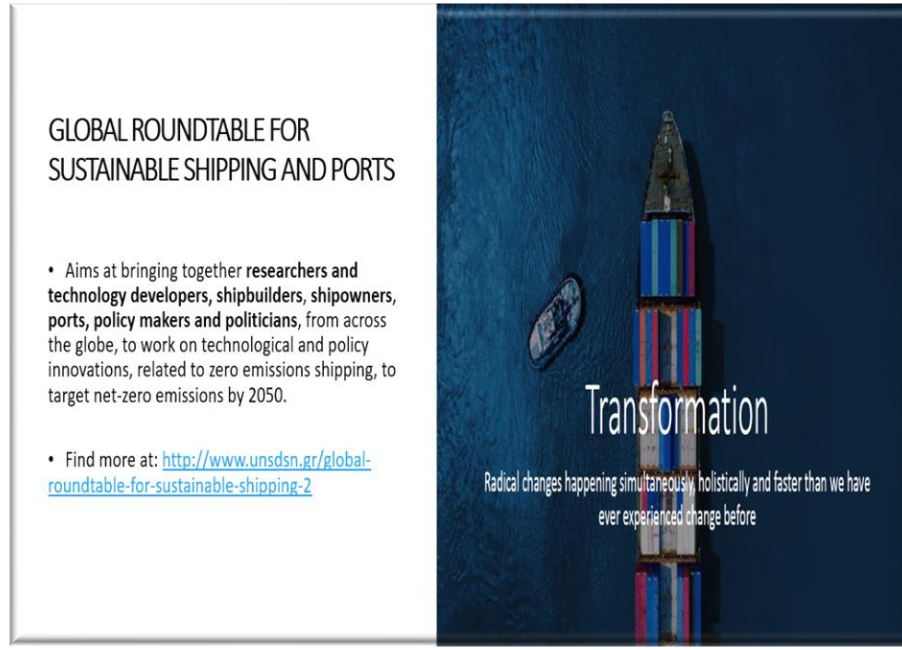
ΚΕΝΤΡΟ ΕΠΙΜΟΡΦΩΣΗΣ ΚΑΙ ΔΙΑ ΒΙΟΥ ΜΑΘΗΣΗΣ

Projects Green-Digital-Just Recovery / Circular Economy / Climate Change Mitigation and Adaptation







Projects

Blue Growth

Projects Water-Food-Energy Nexus
Smart Agriculture & Smart Urban Water Systems

Climate-KIC

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Successful Greek Start-ups

Cargo Share | citipost #greencity | CYRUS | ENALEiA redefining fishing | PARITY

UNISCRAP PBC. Social & Environmental Impact | Inteligg | Solmeyea | trustporter | swallows

Uncertainty affects Preferences and Decisions used to Estimate Total Economic Value Total Economic Value Shapes Policy Recommendations

Global climate change illustrates particularly well the importance of considering uncertainty when making a decision.

- **Do we face RISK (uncertainty but known probabilities)?**
- **Do we face DEEP UNCERTAINTY (unknown probabilities)?**
- **Decision making under deep uncertainty?**

- **IPCC (2007) wrote:**

"In most instances, objective probabilities are difficult to estimate. Furthermore, a number of climate change impacts involve health, biodiversity, and future generations, and the value of changes in these assets is difficult to capture fully in estimates of economic costs and benefits….. The literature on how to account for ambiguity in the total economic value is growing, even if there is no agreed standard."

How do agents make decisions under “deep uncertainty”?

- *Our Literature Review pointed to alternative decision-making rules away from Expected Utility Maximization Rule…*
- *…Instead of finding ideas to refine the theoretical underpinnings of our models and valuation methods, we ended up criticizing this literature!*

AMBIGUITY AVERSION, MODERN BAYESIANISM AND SMALL WORLDS

Phoebe Koundouri, Nikitas Pittis, Panagiotis Samartzis, Nikos Englezos

Outline

Ambiguity Aversion (AA): Aversion to Unknown Probabilities
Mathematical Economics – Decision Theory under Subjective Uncertainty

Modern Bayesianism (MB)
Bayesian Epistemology – Bayesian Confirmation Theory (BCT)

Small Worlds
Worlds where Small Number of Propositions
are required to cover all cases of Interest (Logic –BCT)

Main Result of the Paper:
AA under MB Collapses to
Dynamic Inconsistency of Beliefs

Also,
In Small Worlds, MB (apart from Normatively appealing)
is Descriptively plausible

Ambiguity Aversion: History

Origins of AA: Ellsberg Paradox (1961)

Interpretation of Ellsberg Paradox:
One (or more) of SEU axioms Fail

Evidence of Irrationality

SEU Conditions of Rationality too Demanding

New Axiomatization – Replacing SEU Axioms

Background Material for AA

Ambiguity Aversion

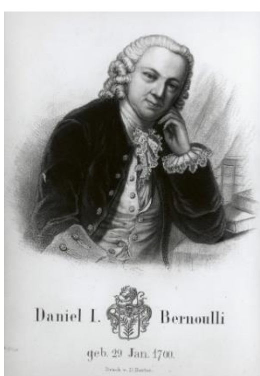
Ellsberg-type Behavior (1961)

Normative vs. Descriptive Status of SEU Axioms

Daniel Bernoulli St. Petersburg Paradox (1738)

Von-Neumann – Morgenstern EU Theory (1947)

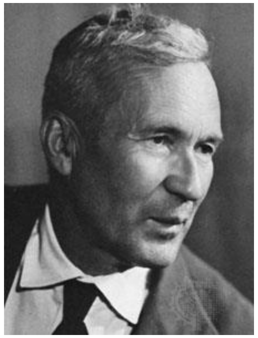
Savage's SEU Theory (1954)



Unknown Probabilities

Question: What is Probability?

Mathematician's Answer



Probability is a Measure Defined on a σ-field which satisfies additivity (finite or sigma)

Empiricist's Answer?

Empirical Interpretation(s) of Probability

OBJECTIVE

Relative Frequency (Actual or Hypothetical)

Propensity of a Chance Mechanism

Logical Relation between Propositions

and, finally…

Subjective Degree of Beliefs

Which of these Empirical Probabilities Obey the Mathematical Axioms?

Unknown Probabilities

Subjective Probabilities: Problems

Subjective Probability

⇕

Betting Behavior

Assuming that such a thing exists:

In order to be able to test whether it is consistent with Kolmogorov's axioms, it has to be measurable (in the sense of observable)

Frank Ramsey (1926)

Subjective probabilities can be inferred by observing actions that reflect individuals' personal beliefs.

The degree of probability that an individual attaches to a particular outcome can be measured by finding what odds the individual would accept when betting on that outcome

To be precise, given a subjective probability p for the proposition, you will accept odds of up to p : (1 – p) on its truth

Unknown Probabilities

Ramsey's Insights were ignored until the mid of 1950s

1954: Leonard Savage's
"The Foundations of Statistics"

More Appropriate Title:
"The Foundations of Economics"

Kreps (1988): "The Crowning Glory of Choice Theory"

Why is it so Important?

Simultaneous Axiomatization of Subjective Probability and Expected Utility Maximization

An Economic Agent does not Need Exogenous Objective Probabilities: All that is Needed is in his Own Mind

Axiomatization of Subjective Expected Utility: Savage 1954

Savage's Framework

Objects of Choice: Acts

Acts are functions from the Sample Space (source of uncertainty) to Outcome Space

Savage defines a preference relation on the set of (all) Acts

Axiomatization of Subjective Expected Utility: Savage 1954

Savage Representation Theorem
(Finite Outcomes Case)

A binary relation $\succeq$ defined on the set of Acts satisfies Savage's seven axioms,

iff there exist (i) A (subjective) probability function (defined on the sample space)

and (ii) a Utility function (defined on the outcome space)

such that for any two acts, f and g :

$$\boxed{f \succeq g \text{ iff } SEU(f) \geq SEU(g)}$$

All in One Package:

Proper Subjective Probability

Cardinal Utility

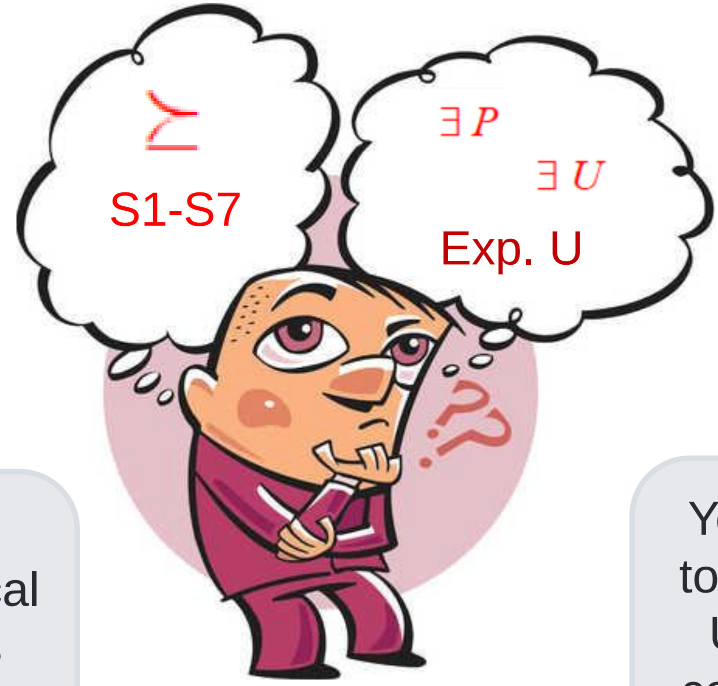
EU Form

Axiomatization of Subjective Expected Utility: Savage 1954

Does Savage’s Representation Theorem Address the Issue of Measurability of Subjective Probability?

Ask him questions of the type: Do you prefer this to that?

No Numerical Questions are Asked



You will be able to get Numerical Degrees of Belief which are Coherent

You will be able to get Numerical Utilities which contain Cardinal Information

Axiomatization of Subjective Expected Utility: Savage 1954

Profound Implications for:

Economic Theory	Bayesian Epistemology
⇓	⇓
Expected Utility Maximization Works even if Probabilities are not Known	The Founding Stone of BE, namely Prior Probability, Exists

Question: Where do the priors come from?

Answer: The same place Utility Function comes from!

Axiomatization of Subjective Expected Utility: Savage 1954

Savage's Representation Theorem provides a Separation of Tastes from Beliefs

Event A more probable to Event B

Act 1 more preferable to Act 2

$\succeq \;\to\; \succeq_p \;\Longrightarrow\; P_S : \mathcal{F} \to [0,1]$

$\succeq \;\to\;$ The rest $\;\Longrightarrow\; U_S : \mathcal{X} \to \mathbb{R}$

Preference Function – EU form

It also delivers three properties in one Package:

Subjective Tastes are represented by Utilities
Subjective Beliefs are represented by Probabilities
Subjective Probabilities and Utilities are used in conjunction with the maximization rule: You cannot have subjective probabilities and use them to maximize something else (other than expected utility)

Deviations from Savage's Axioms: Ellsberg Paradox

Do Economic Agents Behave according to Savage's Axioms?

Descriptive

Should Economic Agents Behave according to Savage's Axioms?

Normative

Evidence Against SEU Behavior

Ellsberg Paradox (1961)

People are Not Probabilistically Sophisticated

Deviations from Savage's Axioms: Ellsberg Paradox

An Urn with 90 Balls

$\square$	30 Balls	60 Balls	
$\square$	Red	Black	Yellow
$f_1(\cdot)$	\$100	\$0	\$0
$f_2(\cdot)$	\$0	\$100	\$0
$f_3(\cdot)$	\$100	\$0	\$100
$f_4(\cdot)$	\$0	\$100	\$100

No of Black: ???
No of Yellow: ???

In Savage's Framework:
Three States of Nature: R B Y
Four Acts
Monetary Outcomes

What is wrong with these choices?

Two Pairs of Choice:

(i) Do you prefer $f_1(\cdot)$ or $f_2(\cdot)$?

(ii) Do you prefer $f_3(\cdot)$ or $f_4(\cdot)$?

Most people prefer $f_1(\cdot)$ over $f_2(\cdot)$ and $f_4(\cdot)$ over $f_3(\cdot)$

Deviations from Savage's Axioms: Ellsberg Paradox

□	30 Balls	60 Balls	
□	Red	Black	Yellow
$f_1(\cdot)$	\$100	\$0	\$0
$f_2(\cdot)$	\$0	\$100	\$0
$f_3(\cdot)$	\$100	\$0	\$100
$f_4(\cdot)$	\$0	\$100	\$100

Probability of Winning
Known = 1/3

Probability of Winning
Unknown:[0,2/3]

Probability of Winning
Unknown = [1/3,1]

Probability of Winning
Known = 2/3

$f_1(\cdot)$ over $f_2(\cdot)$ and $f_4(\cdot)$ over $f_3(\cdot)$

Aversion to Unknown Probabilities

Ambiguity Aversion

Is there anything wrong in avoiding Ambiguity?

Where is the Paradox?

Deviations from Savage's Axioms: Ellsberg Paradox

□	30 Balls	60 Balls	
□	Red	Black	Yellow
$f_1(\cdot)$	\$100	\$0	\$0
$f_2(\cdot)$	\$0	\$100	\$0
$f_3(\cdot)$	\$100	\$0	\$100
$f_4(\cdot)$	\$0	\$100	\$100

The Paradox:

The Preferences $f_1(\cdot)$ over $f_2(\cdot)$ and $f_4(\cdot)$ over $f_3(\cdot)$ cannot be represented by an SEU Preference Function

Assume that such a representation exists:

$$f_1(\cdot) \succ f_2(\cdot) \Leftrightarrow SEU(f_1) \succ SEU(f_2)$$
$$SEU(f_1) = P(red) \times 100 + P(black) \times 0 + P(yellow) \times 0 = P(red) \times 100$$
$$SEU(f_2) = P(red) \times 0 + P(black) \times 100 + P(yellow) \times 0 = P(black) \times 100$$

Hence, $SEU(f_1) > SEU(f_2) \Leftrightarrow \mathbf{P(red) > P(black)}$

Similarly for the second pair:

$$f_4(\cdot) \succ f_3(\cdot) \Leftrightarrow SEU(f_4) \succ SEU(f_3)$$
$$SEU(f_3) = P(red) \times 100 + P(black) \times 0 + P(yellow) \times 100 = P(red) \times 100 + P(yellow) \times 100$$
$$SEU(f_4) = P(red) \times 0 + P(black) \times 100 + P(yellow) \times 100 = P(black) \times 100 + P(yellow) \times 100$$

Hence, $SEU(f_4) > SEU(f_3) \Leftrightarrow \mathbf{P(black) > P(red)}$

CONTRADICTION: No (Proper) Subjective Probability Exists.
Ellsberg-type Choices cannot be represented by a Coherent-Probability based SEU Rule

Response of the Literature to Ellsberg-type Behavior

Interpretations of Ellsberg Behavior

Irrationality

Lack of Probabilistic Sophistication is simply a Probabilistic Fallacy

If someone claims 1+1=3, we do not have to doubt Axioms of Arithmetic

Savage's Axioms too Demanding for Rationality

Which of S1-S7 is Violated by Ellsberg Behavior?

How Should we Represent Ellsberg Beliefs?

Prob. Sophistication:
(i) Unique and (ii) Additive

Non Uniqueness

Non Additivity

Response of the Literature to Ellsberg-type Behavior

Which of S1-S7 is Violated by Ellsberg Behavior?

S2: The Sure-thing Principle

S3: Strong Comparative Probability

A stronger version of Savage's weak Comparative Probability Axiom (Machina and Schmeidler 1992) allows qualitative probabilistic comparisons between events.

STP: If two subjective acts offer the same prize over some event,…

…then replacing it with any other prize will not change the ranking of the acts

□	30 Balls	60 Balls	
□	Red	Black	Yellow
$f_1(\cdot)$	\$100	\$0	\$0
$f_2(\cdot)$	\$0	\$100	\$0
$f_3(\cdot)$	\$100	\$0	\$100
$f_4(\cdot)$	\$0	\$100	\$100

$f_1(\cdot)$ over $f_2(\cdot)$ and $f_4(\cdot)$ over $f_3(\cdot)$

Violation of STP

Response of the Literature to Ellsberg-type Behavior

Schmeidler (1989): Non-additive probabilities (Capacities) - Choquet Integration

Gilboa and Schmeidler (1989): Multiple Probabilities - Maxmin Expected Utility

⇓

(i) For each act f compute all of its expected utilities for all $\mu_\theta(\cdot)$.
(ii) Find the minimum expected utility for act f
(iii) Compare all acts in terms of their minima. Choose the one with the maximum minimum

The Maxmin Expected Utility decision rule suggests that the decision maker can be characterized by a utility function and a set of prior probabilities, such that the chosen act maximizes the minimal expected utility, where the minimum is taken over the priors in the set

Criticism: Too Pessimistic

Source of Ambiguity – Ambiguity Aversion

□	30 Balls	60 Balls	
□	Red	Black	Yellow
$f_1(\cdot)$	\$100	\$0	\$0
$f_2(\cdot)$	\$0	\$100	\$0
$f_3(\cdot)$	\$100	\$0	\$100
$f_4(\cdot)$	\$0	\$100	\$100

Key Point of Ellsberg Paradox:

There are Two Kinds of Events for the Agent

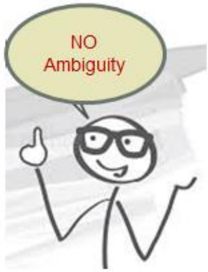
$$\mathcal{F} = \{\{R,B,Y\},\emptyset,\{R,B\},\{R,Y\},\{B,Y\},\{R\},\{B\},\{Y\}\}$$

Unambiguous

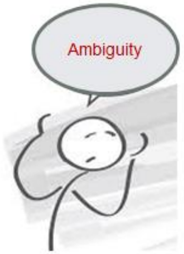
Ambiguous

$$\mathcal{F}^U = \{\{R,B,Y\},\emptyset,\{R\},\{B,Y\}\}$$

$$\mathcal{F}^A = \{\{R,B\},\{R,Y\},\{B\},\{Y\}\}$$



Preferring to bet on U rather than A events is inconsistent with Probabilistic Sophistication



Source of Ambiguity – Ambiguity Aversion

Source of Ambiguity:

At the moment of Formation of his Beliefs (Prior Beliefs), DM is faced with Asymmetric Information about Objective Probabilities (Chances)

$\mathcal{F} = \{\{R,B,Y\},\emptyset,\{R,B\},\{R,Y\},\{B,Y\},\{R\},\{B\},\{Y\}\}$

He has Specific Information (Chances) about Some events

I_B general characteristics of the chance set up: all H, E-relevant to H, entailment relations H-E

I_S actual outcomes or info on objective chances

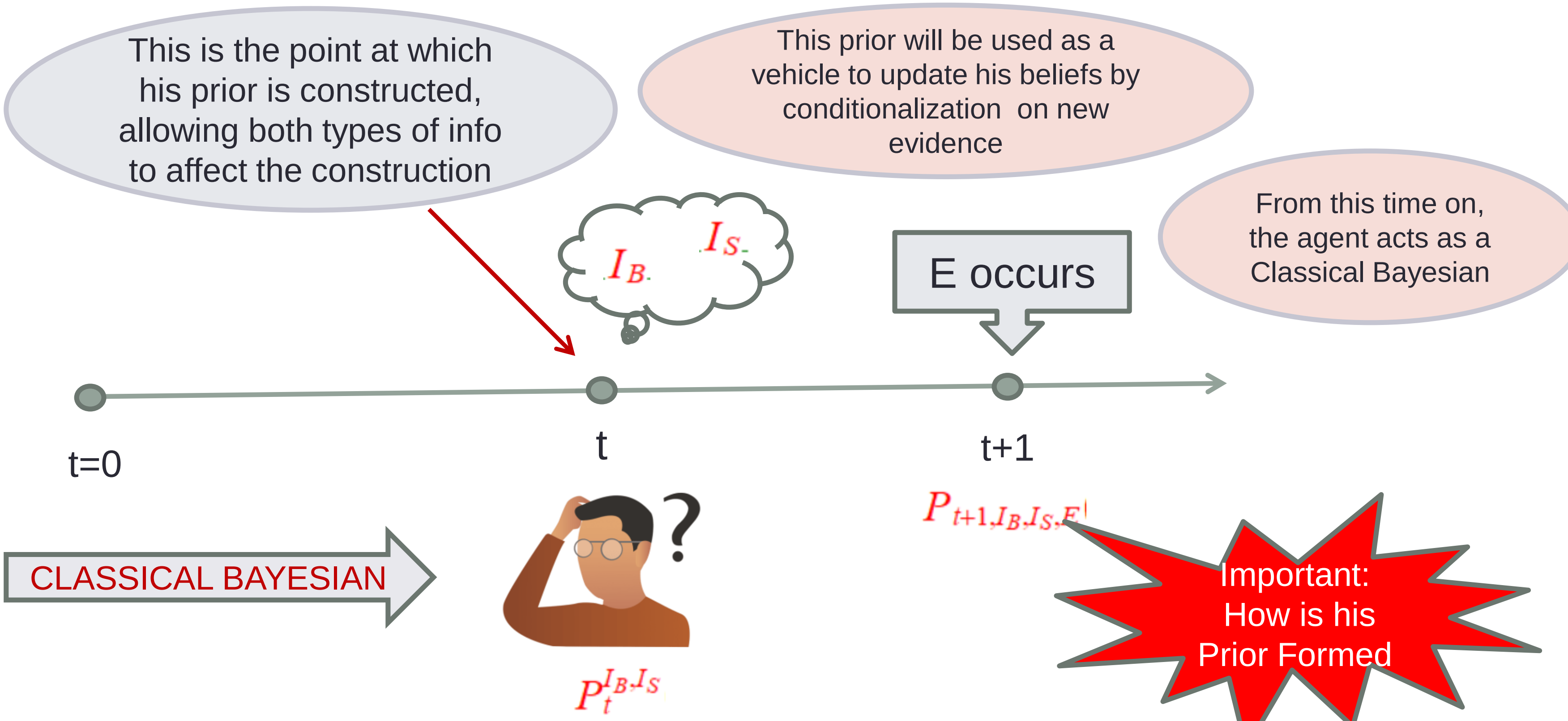
I_S will be utilized via conditionalization on the Prior

QUESTIONS

Should we allow Specific Information to Affect the Formation of Prior Beliefs?

Should we allow only Background Information to Affect the Formation of Prior Beliefs?

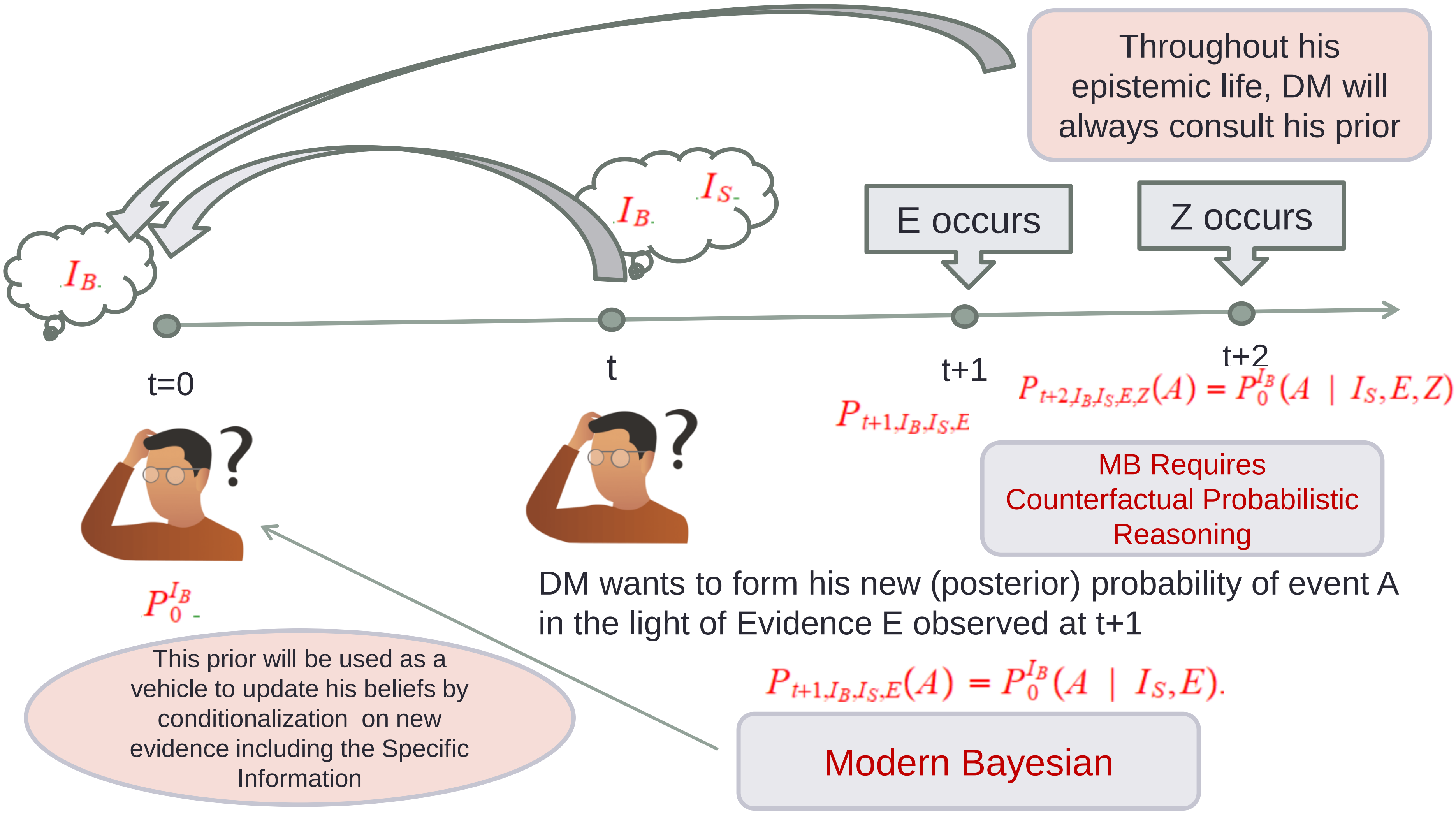
Background vs. Specific Information as Direct Determinants of Prior Beliefs



DM wants to form his new (posterior) probability of event A in the light of Evidence E observed at t+1

$$\underbrace{P_{t+1,I_B,I_S,E}}_{\text{Posterior}}(A) = \underbrace{P_t^{I_B,I_S}}_{\text{Prior}}(A \mid E)$$

Background vs. Specific Information as Direct Determinants of Prior Beliefs



Modern vs. Classical: Comparisons

Modern Bayesianism is adopted (almost without exception) from Philosophers of Science

It is a standard practice for Philosophy-of-Probability papers to start with a "reasonable initial Credence function" (before receiving any evidence)

Rudolph Carnap argues strongly in favor of MB
His monumental work on inductive logic (Carnap 1950) is based on the concept of hypothetical or counterfactual initial credence function that can be ascribed to the agent, before the collection of any evidence.

X's momentary inclination (to believe) at time t

vs.

X's permanent disposition to believe

Modern vs. Classical: Comparisons

There are Good Normative Reasons for Being Modern rather than Classical Bayesian

MB Dissolves the Problem of Ambiguity Aversion

m
hypo

...es the
problem of “old evidence”

contradictions arising
from Principal Principle

MB ensures faster convergence of opinions than CB

…and finally…

Main Result of the Paper:

In the context of MB, Ambiguity Aversion collapses to Dynamic Inconsistency of Beliefs

⇓

An MB Agent who exhibits AA is bluntly Irrational

I shall definitely sell my stock as soon as I earn 8%

t=0

I shall keep it for another 5% and then definitely sell

One month later… with 8% return being a fact

Schmeidler's Two-Coin Example

Schmeidler (1989) uses the following coin example, which aims at conveying the same message with Ellsberg's paradox

Assume that the agent X considers two coins A and B

He knows the objective probability features of A

Asymmetric Information about the Chances of the Events in the Domain of his Beliefs

He knows NOTHING about the physical probabilities of B

The two coins are about to be tossed and X has the option to bet either on A-related events or B-related events

Schmeidler's Two-Coin Example

X may choose bet A

$D_H^A = \{\text{coin A comes up heads}\} = \{H_A H_B, H_A T_B\}$

X wins 1$

$D_T^A = \{\text{coin A comes up tails}\} = \{T_A H_B, T_A T_B\}$.

X looses 1$

$D_H^A, D_T^A \in \mathcal{F}_X$

$D_H^B, D_T^B \in \mathcal{F}'_X$

X may choose bet B

$D_H^B = \{\text{coin B comes up heads}\} = \{H_A H_B, T_A H_B\}$

X wins 1$

$D_T^B = \{\text{coin B comes up tails}\} = \{H_A T_B, T_A T_B\}$

X looses 1$

Schmeidler's Two-Coin Example

X is forming his prior…

For example…

$D_H^A \rightarrow 0.5$

$D_T^A \rightarrow 0.5$

$D_H^B \rightarrow$? 0.4

$D_T^B \rightarrow$? 0.4

X is AA

Remember, Subjective Probabilities are measured via willingness to bet

Although X does not know the exact values of $P(D_H^B)$ and $P(D_T^B)$ he feels more willing to bet on D_H^A than D_H^B and on D_T^A than D_T^B

Schmeidler's Two-Coin Example

It is easy to show that such a P is non-additive. Indeed,

$$D_H^B \cap D_T^B = \emptyset$$

and

$$D_H^B \cup D_T^B = \Omega$$

where Ω is the relevant sample space, namely

$$\Omega = \{H_A H_B,\ H_A T_B,\ T_A H_B,\ T_A T_B\}$$

Assuming that P is additive,

$$1 = P(D_H^B \cup D_T^B) = P(D_H^B) + P(D_T^B) < 1$$

which is a contradiction.

What causes the violation of the additivity property in P?

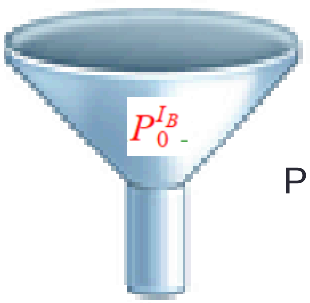
It is the fact that the agent allowed Specific Information
(Information about the Chances of A) to affect directly her probabilistic beliefs,
instead of utilizing (as she should) the specific information indirectly by conditionalization

Schmeidler's Two-Coin Example

…but in order to be able to conditionalize on the specific information : "the objective probability of Heads in A is 0.5"…

…he needs a (pre-existing) vehicle…

I_S



Prior Probability

I_S is not lost

$$P_{new, I_B, I_S}(A) = P_0^{I_B}(A \mid I_S)$$

It is treated as contingent, NOT actual information (being on par with any other conceivable piece of information)

I_S does not enjoy any special status

Schmeidler's Two-Coin Example

What was Agent's Background Information at t=0, that is at the beginning of his epistemic life, when no specific information was available?

For simplicity, we assume that concerning coin A, the agent knows with certainty that only one of the following three hypotheses is true:

$\mathcal{H}_1^A = \{Coin\ A\ is\ fair\}$

$\mathcal{H}_2^A = \{Coin\ A\ favors\ H\ (0.6-0.4)\}$

$\mathcal{H}_3^A = \{Coin\ A\ favors\ T\ (0.6-0.4)\}$

Similarly, for Coin B:

$\mathcal{H}_1^B = \{Coin\ B\ is\ fair\}$

$\mathcal{H}_2^B = \{Coin\ B\ favors\ H\ (0.6-0.4)\}$

$\mathcal{H}_3^B = \{Coin\ B\ favors\ T\ (0.6-0.4)\}$

Let $\mathbf{H}_A = \{\mathcal{H}_1^A, \mathcal{H}_2^A, \mathcal{H}_3^A\}$ and $\mathbf{H}_B = \{\mathcal{H}_1^B, \mathcal{H}_2^B, \mathcal{H}_3^B\}$

Partition of Event Space

Partition of Event Space

Schmeidler's Two-Coin Example

At t=0, the agent was not informed about the physical probabilities of Coin A. Hence, he treats both A and B symmetrically.

Principle of Insufficient Reason

Principle of Indifference

Maximum Entropy Principle

$$P_0(\mathcal{H}_1^A) = P_0(\mathcal{H}_2^A) = P_0(\mathcal{H}_3^A) = \frac{1}{3}$$

$$P_0(\mathcal{H}_1^B) = P_0(\mathcal{H}_2^B) = P_0(\mathcal{H}_3^B) = \frac{1}{3}$$

These are Prior Probabilities.
Any Specific Information will be Processed via Conditionalization using these priors

Important:

These assignments are part of Prior Probability Function (PPF)

PPF covers all Propositions:

1. Theoretical
2. Evidential
3. Partial Entailments between H and E

Once you have formed your PPF, you may sit back and enjoy the Bayesian ride!

Schmeidler's Two-Coin Example

Back to Schmeidler's events:

D_H^A = {coin A comes up heads} = $\{H_A H_B,\ H_A T_B\}$

D_T^A = {coin A comes up tails} = $\{T_A H_B, T_A T_B\}$

D_H^B = {coin B comes up heads} = $\{H_A H_B,\ T_A H_B\}$

D_T^B = {coin B comes up tails} = $\{H_A T_B,\ T_A T_B\}$

Using the Law of Total Probability, the Agent's Prior Probabilities are:

All this activity…

$$P_0(D_H^A) = \sum_{i=1}^{3} P_0(D_H^A \mid \mathcal{H}_i^A) P_0(\mathcal{H}_i^A) =$$

$$= 0.5 \times \frac{1}{3} + 0.6 \times \frac{1}{3} + 0.4 \times \frac{1}{3} = 0.5$$

In a similar fashion,

$$P_0(D_T^A) = P_0(D_H^B) = P_0(D_T^B) = 0.5$$

…takes place at t=0

Schmeidler's Two-Coin Example

Now we move in time

At t=1, the agent is informed (specific information) that the coin A is fair.
He still has no information about Coin B

$\mathcal{H}_1^A = \{Coin\ A\ is\ fair\}$

$\mathcal{H}_2^A = \{Coin\ A\ favors\ H\ (0.6-0.4)\}$

$\mathcal{H}_3^A = \{Coin\ A\ favors\ T\ (0.6-0.4)\}$

$\mathcal{H}_1^B = \{Coin\ B\ is\ fair\}$

$\mathcal{H}_2^B = \{Coin\ B\ favors\ H\ (0.6-0.4)\}$

$\mathcal{H}_3^B = \{Coin\ B\ favors\ T\ (0.6-0.4)\}$

Specific Information: $< \mathcal{H}_1^A$ is true $>$

How does he update?

The agent will Update his Old probabilities (prior→posterior) based on the Specific Information he has just received

By consulting his PPF. Whatever commitments he had made at t=0, he is obliged to respect them at t=1

Schmeidler's Two-Coin Example

Bayesian Conditionalization

$P_{new}(\mathcal{H}_1^A) = P_0(\mathcal{H}_1^A \ \lvert < \mathcal{H}_1^A$ is true $>)$	$= 1$
$P_{new}(\mathcal{H}_2^A) = P_0(\mathcal{H}_2^A \ \lvert < \mathcal{H}_1^A$ is true $>)$	$= 0$
$P_{new}(\mathcal{H}_3^A) = P_0(\mathcal{H}_3^A \ \lvert < \mathcal{H}_1^A$ is true $>)$	$= 0$

IMPORTANT:
All these are prior probabilities already formed at t=0

The New Probabilities for Coin-B Hypotheses:

$P_{new}(\mathcal{H}_1^B) = P_0(\mathcal{H}_1^B \ \lvert < \mathcal{H}_1^A$ is true $>)$	$= P_0(\mathcal{H}_1^B) = \frac{1}{3}$
$P_{new}(\mathcal{H}_2^B) = P_0(\mathcal{H}_2^B \ \lvert < \mathcal{H}_1^A$ is true $>)$	$= P_0(\mathcal{H}_2^B) = \frac{1}{3}$
$P_{new}(\mathcal{H}_3^B) = P_0(\mathcal{H}_3^B \ \lvert < \mathcal{H}_1^A$ is true $>)$	$= P_0(\mathcal{H}_3^B) = \frac{1}{3}$

Schmeidler's Two-Coin Example

The Posterior Probabilities of Schmeidler's Events are:
(once the specific information about Coin A has been allowed to play its role)

$$P_{new}(D_H^A) = P_0(D_H^A \;|< \mathcal{H}_1^A \text{ is true } >) \;=$$

$$\sum_{i=1}^{3} P_0(D_H^A \;|\; \mathcal{H}_i^A, < \mathcal{H}_1^A \text{ is true } >) \times P_0(\mathcal{H}_i^A \;|< \mathcal{H}_1^A \text{ is true } >)$$

$$\Downarrow$$

$$P_{new}(D_H^A) = P_0(D_H^A \;|< \mathcal{H}_1^A \text{ is true } >) \times 1 =$$

$$P_0(D_H^A \;|< \mathcal{H}_1^A \text{ is true } >) \;= 0.5$$

Schmeidler's Two-Coin Example

For the event D_H^B :

$$P_{new}(D_H^B) = P_0(D_H^B \;|< \mathcal{H}_1^A \text{ is true } >) =$$

$$\sum_{i=1}^{3} P_0(D_H^B \mid \mathcal{H}_i^B, < \mathcal{H}_1^A \text{ is true } >) \times P_0(\mathcal{H}_i^B \;|< \mathcal{H}_1^A \text{ is true } >)$$

Since $\mathcal{H}_i^B$ and $< \mathcal{H}_1^A$ is true $>$ are Independent Propositions

$$P_{new}(D_H^B) = P_0(D_H^B \;|< \mathcal{H}_1^A \text{ is true } >) =$$

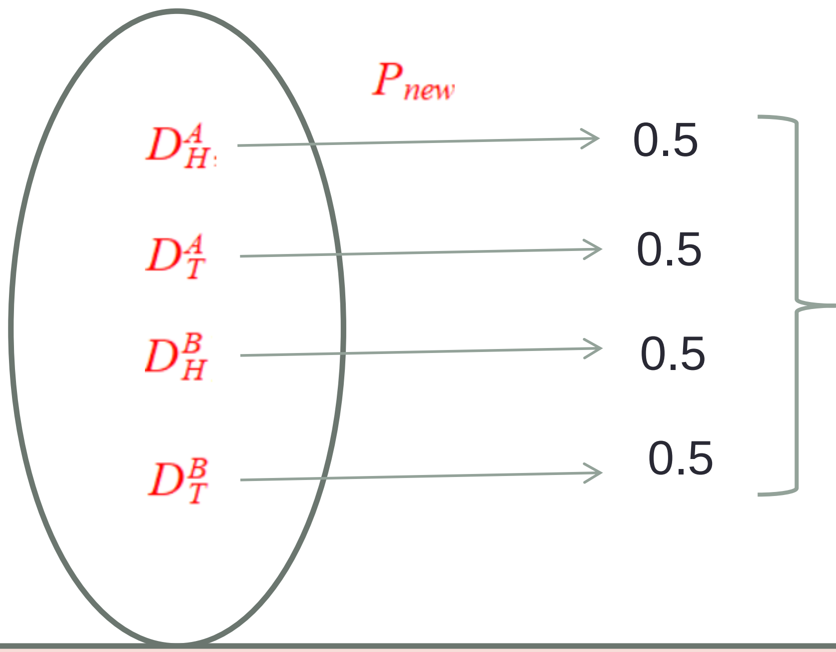
$$\sum_{i=1}^{3} P_0(D_H^B \mid \mathcal{H}_i^B) \times P_0(\mathcal{H}_i^B) = 0.5$$

Similarly, $P_{new}(D_T^B) = 0.5$

Schmeidler's Two-Coin Example

X has formed his posterior

Let us check whether it violates additivity (a-la Schmeidler)



If X is committed to consult his conditional priors each time that new information comes in, then in the case under study, he will see that his conditional priors for D^B_H and D^B_T coincide with the unconditional ones because the new information is independent to the events D^B_H and D^B_T

Posterior is Additive

Neither the prior nor the posterior violate additivity!

A MB does not exhibit AA, hence his probabilistic beliefs in the presence of "asymmetric information" about chances remain coherent

AA under MB is Equivalent to Dynamic Inconsistency of Beliefs

This is a commitment made at t=0 to be respected in future times

Specific Information:
Coin A is Fair

t=0

t=1

$$P_0(D_H^B \mid \mathcal{H}_1^A) = P_0(D_H^B) = 0.5$$

It reads as follows:

If, in the future, I get to know that the coin A is fair, I shall set (maintain) my subjective probability of H-event B equal to 0.5

Bayesian Conditionalization dictates:

$$P_{new}(D_H^B) = P_0(D_H^B \mid \mathcal{H}_1^A)$$

Hence,

$$P_{new}(D_H^B) = 0.5$$

If I set, instead

$$P_{new}(D_H^B) = 0.4$$

Not Dynamically Consistent

Modern vs. Classical Bayesianism

MB Benefits

MB Avoids Cognitive Dissonance

MB fits better within Bayesian Confirmation Theory

MB avoids certain contradictions arising from Principal Principle

MB handles the problem of “old evidence”

MB ensures faster convergence of opinions than CB

QUESTION: Is MB Plausible
(as a mode of probabilistic thinking)?

Modern Bayesianism and Small Worlds

ANSWER: In Small Worlds, YES

Small vs. Large Worlds

A Small World is a set of propositions whose number of elements is small

In a small world a Bayesian (modern or classical) can assign probabilities to all propositions of interest

Apart from Small, the World must be Non-Evolving

Modern Bayesianism and Small Worlds

Small and Non-Evolving Worlds

Moreover, apart from small, worlds must be Non-Evolving

Assume that at some point in time t>0,
you come up with a new hypothesis H-new

Motivated by H-new you acquire Evidence E-new

You attempt to Update your Beliefs based on E-new

As a good Bayesian (Modern or Classical) you consult your Prior

SURPRISE: Your Prior has not assigned probabilities to H-new, E-new Propositions (because back then they were unconceived)

Small vs. Large Worlds

- Binmore (2009): "Only in a small world, in which you can always look before you leap, it is possible to consider everything that might be relevant to the decisions you take."
- Indeed the "look before you leap" proverb is attributed to Savage who used it as antithetical to "cross that bridge when you come to it" that referred to the so called "large worlds".
- It is worth mentioning that Savage himself made quite clear that his own conception of subjective probability together with its axiomatization is relevant only for small worlds.
- This is because Savage's framework is essentially static in the sense that it does not allow for the so-called "concept formation", that is the formation of a new hypothesis or a new idea sometime in the future.
- Savage himself acknowledged the fact that the static nature of his theory makes it inapplicable in the case of large evolving worlds by referring to such an extension as "ridiculous" and "preposterous"

Conclusions

Modern Bayesianism Dissolves the Problem of Ambiguity Aversion

Under MB, AA collapses to Dynamic Inconsistency
(Non-negotiable Irrationality)

MB assumes that in forming his subjective priors, the agent makes use of Background Information only

Specific Information (e.g. information about chances) is allowed to affect beliefs only through conditionalization
(by means of the pre-existing prior)

Bayesianism in general (Modern or Classical) is Plausible only in Small Worlds

A third Option that relaxes BC: The Evolving Probability Model:
Change your probability (in the light of new evidence) anyway you want as long as you respect coherence