

## Math for Data Science

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### Overview

The course is a brief overview of the basic tools from Linear Algebra and Multivariable Calculus that will be needed in subsequent courses of the program.

### Key Outcomes

By completing the course the students will have been reminded of the basic tools of Linear Algebra and Multivariable Calculus needed in subsequent courses in the program, notably:

- Fundamental properties of matrices, their norms, and their applications
- Differentiating/integrating multiple variable functions, and the role of the gradient and the hessian matrix.
- Basic properties of optimization problems involving matrices and functions of multiple variables.

### Requirements and Prerequisites

This course is meant to be a refresher. Students are assumed to have developed some mathematical maturity through a few mathematically oriented classes that involve the standard undergraduate treatment of linear algebra and single-variable calculus.

### Bibliography

1. Gilbert Strang, *Linear Algebra and Its Applications*, Thomson/Brooks Cole (Available in a Greek Translation)
2. Thomas M. Apostol, *Calculus*, Wiley, 2nd Edition, 1991, ISBN 960-07-0067-2.
3. Michael Spivak, *Calculus*, Publish or Perish, 2008, ISBN 978-0914098911.
4. Ross L. Finney, Maurice D. Weir, and Frank R. Giordano, *Thomas's Calculus*, Pearson, 12th Edition, 2009.
5. David C. Lay, *Linear Algebra and Its Applications*, 4<sup>th</sup> Edition.
6. Yousef Saad, 'Iterative Methods for Sparse Linear Systems'

### Grading

There will be a written examination at the end of the course.

## Course Syllabus

The course comprises twelve one-hour lectures which will be spread out in the first two weeks of the first semester.

### Lecture 1: Matrices and Basic Operations, Special structures

Matrices and Basic Operations, interpretation of matrices as linear mappings, and some examples

### Lecture 2: Square Matrices, Determinants

Properties of determinants, singular and non-singular matrices, examples, finding an inverse matrix

### Lecture 3: Eigenvalues and Eigenvectors

Characteristic Polynomial, Definition of Left/right Eigenvalues and Eigenvectors, Caley-Hamilton theorem, Singular Value Decomposition, interpretation of eigenvalues/vectors

### Lecture 4: Normed Spaces, Vector Spaces, Matrix Norms

Definition of complete normed and vector spaces and some examples. Matrix norms and properties, applications to series of matrices and their convergence

### Lecture 5: The Range and the Null space of a Matrix

Definition and basic properties, orthogonality, Gram-Schmidt algorithm

### Lecture 6: Positive-Definite Matrices and the Taylor Expansion of a two-variable function

Definition of positive-definiteness and the role of the eigenvalues. Physical meaning and importance in real-life problems

### Lecture 7: Linear Systems

Definition, applications, solving linear systems, linear inequalities, linear programming

### Lecture 8: Real-valued functions of two or more variables.

Definition, examples, simple demos, applications

### Lecture 9: Analysis elements

Distance, Limits, continuity, differentiability, the gradient and the Hessian

### Lecture 10: Optimization problems

Simple examples, motivation, the role of the Hessian, maxima and minima and related extrema conditions

### Lecture 11: Integration

Double integrals, Fubini's theorem, properties, applications

### Lecture 12: Elements of Convex Optimization

Functions of  $n$  variables. Convex sets, convex functions, convex problems, and their basic properties. Examples of convex problems, convexity versus non-convexity