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WORKING PAPER no. 12-2025

Emergency Room General Equilibrium

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August 2025

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Abstract

The intention of this paper is to: (i) model a situation of randomly (in time and in space) picking and observing a number of patients inside some emergency room, of some hospital or some clinic, waiting to receive personalised (and individually optimal) treatments, (ii) define a Walrasian or a competitive general equilibrium for this situation and (ii) prove, under generic conditions, that such a general equilibrium exists for this emergency room and its patients, while it is Pareto optimal (i.e., efficient with respect to all the patients' treatments together) and individually rational.

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Key Words: Hospital or Clinic, Emergency Room, Patients, Allocations of Treatments, Waiting Times, Walrasian or Competitive General Equilibrium, Individually Rational, Pareto optimal, Berge Maximum Theorem, Gale-Nikaido-Debreu Lemma.

Classification: D5, D6, I10, I11.

Let the following emergency room of some clinic:

$$\underbrace{ER = \{\mathbb{R}^m_+; \Theta_i, u_i, h_i : i = 1, 2, ..., n\}}_{[0,1]}$$

Picture that here is a crowd (a finite number) of patients, $\{1, 2, ..., n\}$, waiting in this emergency room in order to receive some personalised urgent treatment. If 0 is the initiating time point that the medical affair under surveillance in this paper starts to unravel and, subsequently, begins to get modelled, then each patient sitting inside this emergency room waits and demands to receive her treatment within some sensible time interval $[0, 1]$, for example, within 1 hour. Beyond that time interval, the length of which is normalised and set here to 1, the average patient waiting in the ER suffers an irreversible damage or loss in his health (for instance, he dies). So there is a deadline that has to be met, and every client of the ER has to be necessarily served during that lapse of time. For this matter of interest, to make the script favourable, say that the hospital does not experience budget restrictions, and benevolently offers a copious (but finite) number (of kinds or types or variety) of treatments, $\{1, 2, ..., m\}$, of some typical or standard (and humanitarianly admissible) quality. There may not be, put it differently, unlimited supplies of medical services in the ER, but these services are certainly sufficient to cover, in theory, for all the patients in the ER, irrespectively of their circumstantially arisen number, within that window of time.

Crucially for the medical environment that is being gradually crafted, $h_i \in \mathbb{R}^m_{++}$ is (was) the initial endowment of the health status of the patient $i = 1, 2, ..., n$, before getting sick or injured, and had to seek for immediate medical attention and healing. More instructively, the coordinate $h_i^j > 0$ of the vector h_i is the original health status of the patient $i \in \{1, 2, ..., n\}$ with respect to the treatment $j \in \{1, 2, ..., m\}$ that the clinic is providing. This is a strictly positive number, because the patient did actually have an initial health status with respect to any nursing or curing service of the hospital.

Evidently now, dependently on his overall initial health status, $\sum_{j=1}^{m} h_i^j := \mathbf{h}_i > 0$, every patient $i \in \{1, 2, ..., n\}$ possesses a personalised (both quantified and qualified) overall treatment potential, or a set of potential treatment quantities (of some quality), $\Theta_i \subset \mathbb{R}^m_+$. Out of this set, alternative (both quantified and qualified) treatments $\theta_i \in \Theta_i$ can be delivered to $i = 1, 2, ..., n$. The coordinate $\theta_i^j \geq 0$ of the vector θ_i is the quantity (of some quality) of the treatment that is delivered to the patient $i \in \{1, 2, ..., n\}$ specifically with respect to the clinic's treatment service

$j \in \{1, 2, ..., m\}$, while $\sum_{j=1}^{m} \theta_i^j := \vartheta_i \geq 0$ is the aggregate treatment of the patient i over all the offered treatment services j. Note that the patient $i = 1, 2, ..., n$ may not actually be in need of some treatment service $j = 1, 2, ..., n$, i.e., $\theta_i^j = 0$. Simultaneously, $i = 1, 2, ..., n$ may not actually be in need of any treatment service, i.e., $\theta_i = \mathbf{0}$ and $\vartheta_i = 0$, even though i is sitting in the ER.[1] Concurrently, it is reasonable to assume that $h_i \in \Theta_i$, which then implies that $h_i \in Int(\Theta_i)$ because $h_i >> \mathbf{0}$, for every $i \in \{1, 2, ..., n\}$, meaning that the original health status of every patient does not fall out of the patient's set of treatment scenarios or prospects. As a consequence, $\Theta_i \neq \emptyset$, $i = 1, 2, ..., n$.

To gain more insight of the ER's working, each patient $i \in \{1, 2, ..., n\}$ waiting there is pre-assigned with a utility function

$$u_i : \Theta_i \rightarrow \mathbb{R}_+,\ u_i(\theta_i) \geq 0,$$

with $u_i(\theta_i) = 0$ meaning that the patient i does not get (within 1 hour) any benefit with the treatment θ_i. A patient may be receiving zero utility from an ineffective, inexpedient or inadequate (in quantity and/or quality) treatment. Or from a trivial, redundant or vacuous treatment; see back in footnote 1. Fallacious, faulty or erroneous treatments, which procure negative utility to the patients of the ER, are excluded on the basis of a minimum quality standard of the medical services that the emergency department of this hospital provides.

Consider, in the follow-up, the condition stated below:

$$\sum_{i=1}^{n} \theta_i \leq \sum_{i=1}^{n} h_i (>> \mathbf{0}) \iff \sum_{i=1}^{n} \theta_i - \sum_{i=1}^{n} h_i \leq \mathbf{0} \iff \sum_{i=1}^{n} (\theta_i - h_i) \leq \mathbf{0} \iff \sum_{i=1}^{n} z_i \leq \mathbf{0},$$

where in this occasion $\leq$ is the point-wise or coordinate-wise (partial) order on $\mathbb{R}^m$. This condition stands as a natural feasibility (allowability or admissibility) condition for this particular problem of Health Economics, that this paper addresses, as a whole. This condition is, in essence, a resources constraint for the ER of the clinic.

[1] Sometimes we are in pain or we have been injured and we show up, just to be sure, to the department of emergencies of some hospital mostly because we are afraid or we are stressed, but then eventually it turns out that it was nothing worrying or alarming, that required special attention and medical curing. In this case, we would have potentially been better off if we had not gone to the ER. This potential is true when $u_i(\mathbf{0}) = 0$, $i = 1, 2, ..., n$, because, then, the situation described in words just previously gets transformed in mathematical parlance as follows: $u_i(h_i) \geq u_i(\mathbf{0}) = 0$, $i = 1, 2, ..., n$, something that is true by default. See in the definition of the patient's utility right after.

As a matter of fact, the (negative) difference (spread or gap)

$$(\sum_{i=1}^{n}\theta_i - \sum_{i=1}^{n}h_i) := (\vartheta - \mathbf{h}) := z$$

shall be referred to as the aggregate (i.e., over all patients) hypo-treatment inside the ER, while the vector $z_i := (\theta_i - h_i) \in \mathbb{R}^m$ is denominated the *net treatment* of patient $i \in \{1, 2, ..., n\}$, that is delivered in the ER.

A brief explanation is in order regarding the former conceptualisations and definitions. It could be naturally assumed that, in principle, the average patient cannot receive an effectual or successful (positive utility yielding) rectifying treatment in the ER, i.e, a quick remedy or therapy, that will (at least in the short term) bring her into a better position than her original health status prior to coming to the ER, hence, it could logically hold that $\theta_i \leq h_i \iff z_i \in \mathbb{R}^m_-$, for the average patient i. Namely, the majority of patients could be expected to be, pragmatically, receiving negative net treatments in the ER. Remark however that, concurrently to the previous objective acknowledgement, patients in the ER usually have a blurry vision and thinking (judgement, assessment or evaluation) of their condition, because they are typically under severe fear, agony, ache and, generally, pressure. This means that, most likely, there exist patients that may be personally hyper-demanding an urgent treatment, above their initial health status, enforcing their subjective appraisal of their instantaneously unfavourable health position, and thereby reversing their (expected as being) negative net treatments into positive ones. This is a spurious treatment demand and an artificial treatment supply within the ER, in the sense that it is not actually needed at that time and place, but it may well be obtained by some patients, when the ER is balancing by distributing treatments to its patients, according to the treatments' average waiting-time schemes or schedules. In sum, both negative and positive net treatments, together of course with treatments that are not pair-wisely ordered and compared with the initial health statuses in the first place, may coexist for the patients of the ER, and they all-together imply the prequel feasibility condition for any allocation of treatments in the ER, but can never push the aggregate net treatment inside the ER to be positive, or lie elsewhere other than that the negative cone (or orthant) of the Euclidean space $\mathbb{R}^m$. In other words, given the (limited or restricted) human resources and the non-human resources of the ER of a clinic, where the latter include the available time and its organisation, the canonical (normal or regular), i.e., the feasible (realisable or attainable), medical operation of a hospital's ER is by aggregatively furnishing its patients with negative net treatments, which are nothing else than the most elementarily salutary ones.

Let us next assume that all the agents have private and/or public heath insurance that can cover for the pecuniary cost of their medical treatment by the ER of the hospital.[2] In the ER, thereupon, the (marginal) cost or the (unit) price of a quantitative treatment (of some quality) is the time a patient needs (and waits in pain, or at least in distress) so as to receive the required optimal-quantity (of some quality) of that treatment. This admission, basically, attaches a non-monetary cost (worth or value) to such medical services[3], making it clear that Health Economics may be fluently admitting a demonetised Theory of Value, due to the uniquely singular nature of the good (or product) 'health'. So let $t^j \in [0,1]$ be a potential average time needed for a patient to receive the treatment $j = 1, 2, ..., m$ in the ER of this clinic.[4] Recall that 1 (hour) is the totally available (quantity of) time in the ER for all partial treatments to be time-aggregatively executed, simultaneously for each one of the extant patients who is (partially) needing them. The hospital then has to make sure that all its resources will be employed (and managed) appropriately so that $\sum_{j=1}^{m} t_j = 1$, for the sake of that specific group of patients that wait in the ER within that specific hour, and who have to be attended to, and possibly get briefly nursed. This essentially means that for any time-vector (representing, here, a shadow price-vector) $t \in \mathbb{R}_+^m$ it holds that $t \in \Delta$, where Δ is the standard (i.e., the unit or the probability) $(m-1)$ - simplex of $\mathbb{R}^m$. That is,

$$(\emptyset \neq)\Delta = \{t \in \mathbb{R}_+^m : \sum_{\ell=0}^{m-1} t^\ell = \sum_{j=1}^{m} t^j = 1\}.$$

Moreover, given (or conditionally upon) his h_i and Θ_i, each patient $i \in \{1, 2, ..., n\}$ receives personally feasible alternative treatments, according to her following personalised time-constrained set of treatments:

$$T_i(t)|(h_i, \Theta_i) := T_i^{h_i,\Theta_i}(t) = \{\theta_i \in \Theta_i : t\theta_i \leq th_i\} \subseteq \Theta_i,\, t \in \Delta,$$

in which $\leq$ restores now to the usual total ordering of the real numbers.[5] This translates as: for a patient, the total cost of receiving a partial treatment in terms of (possibly painful) waiting-time cannot exceed the total cost, in terms of (possibly painful) waiting-time, of receiving the full-package treatment of bringing the patient

[2]Interchangeably, patients have purchased in advance, with their taxes (as public goods) and/or in the private markets (as private goods or commodities), the (sold to them) maximum quantity and quality of medical services that they could possibly need.

[3]Which cost may be also perceived as an opportunity cost.

[4]Two equivalences hold: (i) $|m| = 1 \iff t^1 = 1$ and (ii) $|m| = 0 \iff t^0 = 0$.

[5]In general, from now on, the context of the text will be determining the nature of $\leq$.

back to her original status. From now on, the notation of the time-constrained set of treatments for the patient $i \in \{1, 2, ..., n\}$ will be abbreviated to the notation $T_i(t)$, $t \in \Delta$. Observe at this point that, since $th_i = th_i$, for any patient $i \in \{1, 2, ..., n\}$, $[Int(\Theta_i) \ni] h_i \in T_i(t)$, $t \in \Delta$, for any i, thence, every patient's time-constrained set of treatments is non-empty. Of course, one may arrive to the same conclusion because (similarly) $\mathbf{0} \in T_i(t)$, $t \in \Delta$, for the patient $i = 1, 2, ..., n$. For the sake of its well-definition, it can be easily also checked that $T_i(t)$, $t \in \Delta$, is a convex subset of Θ_i, $i = 1, 2, ..., n$. Then, for each patient, since this convex set has at least two points (the elements $\mathbf{0}$ and h_i, $i = 1, 2, ..., n$), it is an uncountably infinite set.

To wind up hitherto, denote with

$$\theta \in \Theta := \prod_{i=1}^{n} \Theta_i$$

an allocation of treatments (remedies or therapies) that are distributed among the patients of the ER in reference, and within the time-slot in reference (herein, 1 unit of time, or 1 hour). Thereafter, the class of the aggregatively feasible treatment allocations of the ER contains all the primitively normative treatment allocations of the ER, while the family of both personally feasible and aggregatively feasible treatment allocations of the ER contains normative treatment allocations of the ER with further refinement or filtration. Remark that the second (thus, the first as well) class of allocations of treatments is not empty. Indeed, it contains the treatment allocation $h \in Int(\Theta)$[6]. Remark also that, for $t \in \Delta$, $\vartheta \geq \mathbf{0}$ and $\mathbf{h} >> \mathbf{0}$, both the condition '$\vartheta \leq \mathbf{h}$ over all i', which is of aggregate-and-vector scale, and the condition '$t\theta_i \leq th_i \iff tz_i \leq 0$ for every i', which is of individual-and-scalar level, where $i = 1, 2, ..., n$ in both of them, imply the same time-costly condition '$t\vartheta \leq t\mathbf{h} \iff tz \leq 0$', which is of aggregate-and-scalar level. Hence, one is consistent with the other.

Finally, with regard to the emergency room of this hospital, a negative (inverse or opposite) relationship between demanded (and consumed) treatments and their waiting-times is, effectively, recorded. Patients naturally exhibit a Mashallian (1890)

[6] This is the treatment allocation via which everybody walks out from the ER, within 1 hour, with her/his initial health status. This allocation, conveying the idea of indifference or neutrality, may be occurring for two reasons. The first explanation is due to the event that someone proved to have come to the ER for no reason, has wasted his (waiting) time in the ER, and did not eventually receive any novel or distinguishable treatment within the 1 hour time-frame. Or, according to the second rationale, someone could have kept her original health status after her ER visit and experience because she received a full(y effective and discerned) treatment, to wit, one that completely restored her post-treatment health position back to her pre-treatment health condition. For absolute conceptual-mathematical compatibility as far as these statements are concerned, see again in the prequel footnote 1, and in the prior text in general

demand with respect to the medical services (specifically, the urgent and brief treatments) of the ER of the clinic in reference. So, in congruence with the law of demand, which is one component of this demand function, the more the average waiting-times increase in this ER, the less often patients choose to visit this ER, thus, less quantities of treatment services are demanded in this ER; and, according to the substitution effect, more quantities of treatment services are demanded in other emergency rooms of other clinics. Conversely, it is impossible to justify the patients' income effect on demanded quantities of such hospitalised treatments, because they are stripped from any financial element. Clinical services are not only susceptible to substitutability and tradeoffs with respect to waiting-times. There exist, at the same time, complement services to any treatment service of any ER, which are also (potentially) captured in this demand function of the patients. As an exemplar, the medicines and their required own time consuming prescriptions, when medicines are (for the agenda of this paper) also prepaid (by taxes, or by market prices of merchandises), or are simply covered by health insurances. On the other hand, now, there is also a supply function for treatments in the clinic's ER. This supply is performed by the administration, the medical personnel and any other kind of production factor that is employed into the hospital's ER. This function naturally depends positively on the waiting-times of patients for treatment attainment, in compliance with the law of supply. Indeed, more health services will be produced and will be supplied (scilicet, will be made publicly available) in the clinic's ER if the patients sitting in it have (or are able) to wait more time for their collection, i.e., if the average treatment time is less pressing.

The ER framework as contextualised above can be simulated to a competition or contest à la Walras (1874), in which the patients' average waiting times, assigned to them contingently on the treatment that each patient separately needs, are generated endogenously by getting repeatedly (re)adjusted according to a tâtonnement process, which is in turn based onto the associated Walras law, as far the excess demands and excess supplies of treatment quantities are concerned.[7] A time-system clears the ER, with the law of Walras. More precisely, this procedure takes place when the ER functions as follows: the ER opens and we observe patients (or customers) sitting there in discontent and frustration, waiting all of them to be served, with some counter-fatality priority, and within exactly 1 hour, after which time instant the ER closes, namely, none treatment can be practically or efficaciously offered any more, because the patients' medical conditions cannot be repaired or fixed any longer. The patients (who are perhaps even fighting for their life in this regime) compete as a consequence over scarce treatment recourses. These resources are scarce (and

[7]In extreme cases of public health crises, in fact, the ER set up of the prequel analysis could be seen as a *mors tua vita mea* antagonism or struggle.

are suffering from excludability and rivalry) in this circumstance because they are limited (impaired or restrained) exclusively by their inter-contestable average time of provision within a critical time-period that they ought to be provided. Within that short time horizon, each one of the patients autonomously or independently pursues (or hopes) to attain an optimal personalised treatment, that is, does so without correlating or relating her own medical-case-based behaviour or choices with other patients' medical incidents, and hence actions or decisions. Under the neoclassical tenet, every rational patient is trying (or aspires) to have her utility from the receipt of treatment quantities (of some quality), subject to her time-constrained set of treatments, maximised. Every patient in the ER is a time-(price)-system taker, and takes the endogenously realised average waiting-times for granted. As the neoclassical tradition further has it, to get an equitable (fair or impartial) and ethical outcome for all the stakeholders of the medical problem under investigation, every moral patient oughts to be candid, and not attempt to gain priority (i.e., medical privilege or advantage) over the other patients and get served (or treated) before them, whenever his pain or discomfort is not transparent, and the medical staff has to be originally based onto what the patient (frankly or not) reports.

All the preceding discourse culminates with the following definition of a propitious, when conforming to the neoclassical creed, non-cooperative solution for the artisan ER competition:

Definition 1. The pair $(t, \theta) \in \Delta \times \Theta$ is called a Walrasian or competitive general equilibrium of/for the ER iff in the ER, the following two conditions hold:

1. For all $i \in \{1, 2, ..., n\}$ θ_i maximises u_i subject to $T_i(t)$, $t \in \Delta$.

2. $\sum\limits_{i=1}^{n} \theta_i \leq \sum\limits_{i=1}^{n} h_i$.

Definition 1 refers to a Walrasian or competitive general equilibrium that is customised to an ER medical backdrop. To articulate better the content of this definition, θ is a (i) rational and (ii) (both individually and aggregatively) feasible (Walrasian or competitive) *general equilibrium allocation of treatments* for the ER, while t is a (Walrasian or competitive) *general equilibrium time-system*, regarding (or supporting) the delivery of any (both individually and aggregatively) feasible allocation of treatments, of the ER. In general: (i) the time-system $t \in \Delta$ supports the treatment allocation $\theta \in \Theta$ iff for any patient $i \in \{1, 2, ..., n\}$, if there exists an allocation of treatments $\phi \in \Theta$ such that $u_i(\phi_i) \geq u_i(\theta_i)$, then $t\phi_i \geq t\theta_i$, while more specifically (ii) the time-system $t \in \Delta$ strictly supports the treatment allocation $\theta \in \Theta$ iff given the treatment allocation $\theta \in \Theta$, then for any patient $i \in \{1, 2, ..., n\}$, if there exists another allocation of treatments $(\theta \neq)\phi \in \Theta$ such that $u_i(\phi_i) > u_i(\theta_i)$, then $t\phi_i > t\theta_i$. Any Walrasian or competitive general equilibrium allocation shall be (strictly) supportable by some (general equilibrium) time-system. In the aftermath, after the completion of this paper, with specifically patients' strictly increasing utility functions: the aggregatively feasible (general equilibrium) allocation of treatments in the ER $\theta \in \Theta$ is announced Walrasian or competitive iff given the aggregatively feasible (general equilibrium) allocation of treatments in the ER $\theta \in \Theta$, then for any patient $i \in \{1, 2, ..., n\}$, if there exists another aggregatively feasible allocation of treatments $(\theta \neq)\phi \in \Theta$ such that $u_i(\phi_i) > u_i(\theta_i)$, then $(t\phi_i > th_i \geq t\theta_i) \Rightarrow (t\phi_i > t\theta_i)$, for some (general equilibrium) time-system $t \in \Delta$ iff given the aggregatively feasible (general equilibrium) allocation of treatments in the ER $\theta \in \Theta$, the fact that $\theta \in \Theta$ is strictly supportable by some (general equilibrium) time-system $t \in \Delta$ makes any other aggregatively feasible allocation of treatments not time-affordable, and in particular: if, for some patient, some other aggregatively feasible allocation of treatments $(\theta \neq)\phi \in \Theta$ is more healthily individually desirable (i.e., beneficial in terms of private utilitarian medical welfare) than θ, so that ϕ becomes strictly more valuable or worthy in terms of waiting time - and, automatically, strictly more expensive, costly or pricy in terms of pain and stress - than θ, then ϕ falls out of the time-constrained (i.e., personally feasible) set of treatments of this patient.

So far, normative allocations of treatments are characterised those who are (both individually and aggregatively) feasible, or accessible, in the ER. We now continue with another two paramount definitions as far as the extended normativeness of an endogenous medical outcome is concerned.

Definition 2. In the ER, an aggregatively feasible allocation of treatments $\theta \in \Theta$ is said to be Pareto optimal (that is, patient-widely efficient) iff given $\theta \in \Theta$, there does not exist one other aggregatively feasible allocation of treatments $(\theta \neq)\phi \in \Theta$ such that $u_i(\phi_i) \geq u_i(\theta_i)$, for all patients $i \in \{1, 2, ..., n\}$, with specifically at least one existing patient $i \in \{1, 2, ..., n\}$ among them such that $u_i(\phi_i) > u_i(\theta_i)$.

Definition 3. In the ER, an aggregatively feasible allocation of treatments $\theta \in \Theta$ is named individually rational (that is, patient-personally worth waiting for it) iff given $\theta \in \Theta$, $u_i(\theta_i) \geq u_i(h_i)$, for all patients $i \in \{1, 2, ..., n\}$.

Individually rational, i.e., individually acceptable, allocations of treatments are automatically aggregatively permissible as well, since for an individually rational θ it holds that

$$\sum_{i=1}^{n} u_i(\theta_i) \geq \sum_{i=1}^{n} u_i(h_i).$$

The (both individually feasible and aggregatively feasible) allocation h is a personally (hence, an aggregatively as well) acceptable allocation of treatments by definition, but is not necessarily a competitive allocation of treatments and, thus, is immediately not necessarily a Pareto optimal allocation of treatments (this is a corollary of the the concluding Theorem of the paper; see in the sequel).

Any Walrasian allocation of treatments in the ER is individually rational. This can be easily deduced by forcing a contradiction. Indeed, let $\theta \in \Theta$ be a competitive allocation of treatments in the ER. This means that, for every allocation $\phi \in \Theta$ with $\phi_i \in T_i(t), t \in \Delta$, $u_i(\theta_i) \geq u_i(\phi_i)$, for every $i \in \{1, 2, ..., n\}$. Say now that $u_i(\theta_i) < u_i(h_i)$ for some patient $i \in \{1, 2, ..., n\}$. This hypothesis then is a contradiction, because for the allocation $h \in Int(\Theta)$ we have that $h_i \in T_i(t), t \in \Delta$. Thence, every endogenous rational allocation of treatments as formulated in Definition 1, right after its individual and aggregative accessibility, fulfils individual (hence, aggregative) acceptability as well, for free.

So what remains to be proved is that there actually exists a Walrasian or competitive general equilibrium for the ER, which is the focal one, and then that if such a general equilibrium allocation of treatments is attained in the ER, it has to be Pareto optimal, namely, it can be centralised. This upshot cannot be obtained for free and in fact, to get down to this ultimate project, we need to infuse the patients' subjective *utility functions* and the patients' private *sets of alternative treatments* with normative mathematical properties. For the time being, towards this pursuit, consider first of all the standard topology on $\mathbb{R}^m$, with respect to which, if none other topology of $\mathbb{R}^m$ is specified, any mathematical property shall hold whenever this is appropriate or required. Then in principle, it will be championed that, for every patient $i \in \{1, 2, ..., n\}$, u_i is continuous, concave and increasing on its domain[8], while the non-empty Θ_i is a (convex) closed and above bounded (thus, norm bounded or

[8]Monotonicity of the patient's utility function is not needed at all for the existence of this general equilibrium notion. Strict monotonicity of the patient's utility function, nonetheless, is required for the Pareto optimality of the Walrasian or competitive general equilibria. For either of the two previous statements, see in the Theorem and its proof right ahead. For the time being, specifically with patients' strictly increasing utility functions, inside the patients' time-constrained sets of treatments, it necessarily holds, as a binding condition, that $t\theta_i = th_i$ ($\Rightarrow t\vartheta = t\mathbf{h}$), for every patient $i \in \{1, 2, ..., n\}$, for any time-system $t \in \Delta$, and for the treatment allocations $\theta \in \Theta$ and $h \in Int(\Theta)$. Heuristically, we would also sensibly expect for the weaker condition $\vartheta \leq \mathbf{h}$ to be pressed into the (more desirable) stricter condition $\vartheta = \mathbf{h}$. That is to say, we would expect for the patients' net-treatments to be balanced aggregatively, instead of being in deficit (of course, the patients' net-treatments can never aggregatively be in surplus). Notwithstanding, strong monotonicity of the patients' utility functions will not be altering (i.e., will not be specifying

order bounded) subset of $\mathbb{R}^m_+$.[9] The Heine-Borel Theorem implies afterwards that the convex $(\emptyset \neq)\Theta_i$, $i \in \{1, 2, ..., n\}$, is, specifically, compact on $\mathbb{R}^m_+$. Also, since the (convex and compact) set Θ_i, $i = 1, 2, ..., n$, contains an uncountably infinite subset [the set $T_i(t)$, $t \in \Delta$, $i = 1, 2, ..., n$], it is an uncountably infinite set itself. In general, these are the apposite conditions which would secure (quite straightforwardly) that (both individually and aggregatively) accessible and permissible allocations of treatments, apart from the allocation $h \in Int(\Theta)$, are viable in the ER when the patients have rational aspirations (i.e., are utility maximisers), but the feasible and acceptable allocations of treatments of that rational ilk which are, more plausibly, supported by time-systems, and which are put in this paper's analytical microscope, are not directly guaranteed to exist. Furthermore, if they exist, they are indeed individually-and-aggregatively accessible and permissible, but we do not know if they are optimal or efficient across all patients simultaneously (i.e., Pareto optimal). Note finally that a Walrasian or competitive general equilibrium allocation of/for the ER exists iff there exists (actually, if we can pick) a time-system $t \in \Delta$ for the ER such that the conditions 1 and 2 of Definition 1, which involve some $t \in \Delta$ and some $\theta \in \Theta$, hold. We know beforehand that $\Theta, \Delta \neq \emptyset$, so arbitrary treatment-allocations and time-systems exist in the the ER.

Thereupon, finally:

Theorem. If, for every $i \in \{1, 2, ..., n\}$, u_i is continuous and concave on its domain, and $(\emptyset \neq)\Theta_i$ is compact on $\mathbb{R}^m_+$, then a Walrasian or competitive general equilibrium exists for the ER. Further, if u_i is strictly increasing on its domain for every $i \in \{1, 2, ..., n\}$, then any Walrasian or competitive general equilibrium allocation of treatments of the ER is Pareto optimal.

Proof. Fix a patient $i \in \{1, 2, ..., n\}$. Consider the (non-empty and convex valued) correspondence $T_i : \Delta \rightarrow 2^{\Theta_i}$ that provides the time-constrained set of treatments, $T_i(t)$, $t \in \Delta$, of the patient i, given i's h_i and Θ_i. T_i will be exploited for the application of Berge's Maximum Theorem in the sequel of this proof, so, at first, befitting properties have to be built for this correspondence, which will then be compiled accordingly. To this end, to make a start, when (and since) $h_i >> \mathbf{0}$, it can be shown that T_i is lower hemi-continuous on its domain[10]. Continuing, since Θ_i is

always and only in pure balance) the aggregate feasibility condition in the ER, because (in reality) the specific health care unit of a hospital is very inelastic to this prospect, which can rarely occur.

[9]It is a convex set by default, because upon it is defined a concave utility function.

[10]To prove this conclusion, one can use the following definition: a correspondence $F : X \rightarrow 2^Y$ is lower hemi-continuous at $x \in X$ if and only if given any convergent sequence x_n of X to x, for any $y \in F(x)$ there exists a convergent sequence y_n of Y to y, such that $y_n \in F(x_n)$ for every $n \in \mathbb{N}$.

bounded on $\mathbb{R}_+^m$, its (non-empty and convex) subset $T_i(t)$, $t \in \Delta$, is also bounded on $\mathbb{R}_+^m$, so the previously defined correspondence is, additionally, bounded-valued. It needs to be proven now that $T_i(t)$, $t \in \Delta$, is a closed subset of $\mathbb{R}_+^m$ as well, so that the formerly defined correspondence is, in addition, closed-valued. Assume that it is not, thus, let θ_i° be a limit point of $T_i(t)$, $t \in \Delta$, such that $\theta_i^\circ \notin T_i(t)$, $t \in \Delta$, so that $t\theta_i^\circ > th_i$, given the h_i of the patient i. Since θ_i° is a limit point of $T_i(t)$, $t \in \Delta$, there exists a sequence $\{\theta_i\}$ of $T_i(t)$, $t \in \Delta$, such that $\{\theta_i\} \to \theta_i^\circ$, hence, $\{t\theta_i\} \to t\theta_i^\circ$, $t \in \Delta$. Because now $\{\theta_i\} \in T_i(t)$, $t \in \Delta$, we have that $t\theta_i \leq th_i$, $t \in \Delta$, for each $\theta_i \in \{\theta_i\}$, given the h_i of the patient i. Consequently, as $t \in \Delta$ and for some given h_i to patient i, the converging to $t\theta_i^\circ$ sequence $\{t\theta_i\}$ is order bounded, with upper bound (specifically supremum) the th_i, so that $t\theta_i^\circ \leq th_i$, which is a contradiction. By virtue of the Heine-Borel Theorem, eventually, $T_i(t)$, $t \in \Delta$, is a compact subset of $\mathbb{R}_+^m$, or, equivalently, T_i is compact valued. Since Δ is compact with respect to the product topology on $\mathbb{R}^m$, and Θ_i is compact, T_i has a closed graph. It then follows from the Closed Graph Theorem (of correspondences; see, for example, in Aliprantis and Border, 2006, Theorem 17.11, p. 574) that T_i is upper hemi-continuous on its domain. Thus, finally, T_i is continuous on its domain, and has a compact (Hausdorff) range $T_i(\Delta) = \bigcup_{t\in\Delta} T_i(t)$. Afterwards, for the fixed patient i, who is a time-taker and takes as given any endogenously generated time-system $t \in \Delta$, consider w.l.o.g. her utility function as the indirect (and constant on Δ, i.e., trivially dependent on time-systems) utility function $u_i : \Theta_i \times \Delta \to \mathbb{R}_+$, where $\Delta = \{t\}$, so that $u_i(\theta_i, t) = u_i(\theta_i) \geq 0$. Then, since u_i is continuous on Θ, u_i is immediately jointly continuous on both of its arguments (i.e., on its domain $\Theta \times \Delta$). Also, given some realised time-system $t \in \Delta$ for the fixed patient i, the naturally arising basal (i.e., defined on the time-systems of treatment allocations solely) Marshallian demand correspondence of i, $D_i : \Delta = \{t\} \to T_i(t)$, is constant, hence it is continuous (both upper and lower hemi-continuous) on its domain. It is also already proven that D_i is compact valued. Originally, however, the Marshallian demand of the fixed patient i is derived from the correspondence $D_i^\star : \Delta \to 2^{T_i(t)}$, which is defined by the set of time-constrained utility maximisers

$$D_i^\star(t) =$$
$$\arg \max_{(\theta_i,t)\in T_i(t)\times\Delta} u_i(\theta_i, t) = \{\theta_i \in T_i(t) : u_i(\theta_i, t) \geq u_i(\phi_i, t), \forall \phi_i \in T_i(t) = D_i(t) \subset \Theta_i\}$$
$$= \{\theta_i^\star \in T_i(t) : \theta_i^\star \text{ maximises } u_i \text{ subject to } T_i(t)\} \subseteq T_i(t).$$

For how to actually prove this result, in oder to save time and space here, the reader may refer, for example, to de la Fuente (2000, chapter 8), or to Mas-Colell et al (1995), or to other similar texts.

It then follows from Berge's (1963, p. 116) Maximum Theorem[11] that $D_i^\star$ is non-empty valued, compact valued and upper hemi-continuous on its domain. Moreover, since u_i is concave on its domain, it can be deduced that $D_i^\star(t)$ is a convex subset of $T_i(t)$, $t \in \Delta$,[12] so $D_i^\star$ is convex valued as well. Define, at this point, the ER's aggregate hypo-treatment correspondence $Z : \Delta \to 2^{\mathbb{R}^m}$, being specifically given by

$$Z(t) = \sum_{i=1}^{n} D_i^\star(t) - \{\sum_{i=1}^{n} e_i^m\} \subset \mathbb{R}^m.$$

Immediately then, Z_i is non-empty valued, compact valued, convex valued and upper hemi-continuous as a sum of such correspondences. By construction, at the same time, any $z \in Z(t)$, $t \in \Delta$, is defined as

$$z = [\sum_{i=1}^{n} \theta_i^\star - \sum_{i=1}^{n} h_i] \in \mathbb{R}^m.$$

Concurrently, in the main text of the paper it was explained that the individual time-constrained feasibility or accessibility of the fixed patient i implies the scalar condition $tz \leq 0$. The (correspondence version) Gale (1955) - Nikaido (1956) - Debreu (1959, chapter 5) Lemma[13] can be finally applied and guarantee that there exists a $t^\star \in \Delta$ such that $Z(t^\star) \cap \mathbb{R}_-^m \neq \emptyset$. Concluding, there exists a $(t^\star, \theta^\star) \in \Delta \times \Theta$, such that

1. For all $i \in \{1, 2, ..., n\}$ $\theta_i^\star$ maximises u_i subject to $T_i(t^\star)$, $t^\star \in \Delta$.

2. $\sum_{i=1}^{n} \theta_i^\star \leq \sum_{i=1}^{n} h_i$.

Let us move onto the second part of the proof now. With patients' strictly increasing utility functions, we have argued in the main text that (inside the patients' time-constrained sets of treatments) it is definitely the case that $t\varphi_i = th_i$, for every patient $i \in \{1, 2, ..., n\}$, for any time-system $t \in \Delta$, and for the treatment allocations $\varphi \in \Theta$ and $h \in Int(\Theta)$. Say that $\theta \in \Theta$ is an (aggregatively and individually) feasible Walrasian or competitive general equilibrium allocation, which is attained with patients' strictly increasing utility functions, so for θ it holds that $t\theta_i = th_i$, for

[11]For miscellaneous more contemporary variants of this theorem, look into Hu and Parageorgiou (1997), Aliprantis and Border (2006) and Efe Ok (2007), among a number of others.

[12]Again, *inter alia*, refer to Mas-Colell et al (1995).

[13]This list of names in that specific lemma, of course, can be continued and extended with the names of several other authors who (up until relatively recently) used fixed point theorems tailored to proving this lemma, or did not use fixed point theorems at all to proving it.

every patient $i \in \{1, 2, ..., n\}$, for some Walrasian or competitive general equilibrium time-system $t \in \Delta$, and for $h \in Int(\Theta)$. For θ, it is then ensued that

$$t \sum_{i=1}^{n} \theta_i = t \sum_{i=1}^{n} h_i.$$

Suppose then that θ is not Pareto optimal. This means that θ is Pareto dominated by an aggregatively feasible treatment allocation $(\theta \neq)\phi \in \Theta$, or equivalently, there exists a patient $i \in \{1, 2, ..., n\}$ for who the aggregatively feasible ϕ Pareto improves upon the Walrasian or competitive θ, meaning that $u_i(\phi_i) > u_i(\theta_i)$, for this i. This then implies that $t\phi_i > t\theta_i$, for a (Walrasian or competitive) general equilibrium time-system $t \in \Delta$ that supports θ (see in the main text). It then follows that

$$t \sum_{i=1}^{n} \phi_i > t \sum_{i=1}^{n} \theta_i = t \sum_{i=1}^{n} h_i.$$

Since ϕ is an aggregatively feasible allocation of treatments, we have that

$$\sum_{i=1}^{n} \phi_i \leq \sum_{i=1}^{n} h_i \Rightarrow t \sum_{i=1}^{n} \phi_i \leq t \sum_{i=1}^{n} h_i.$$

The last relationship contradicts the relationship just before it[14].□

As a general conclusion: The urgent health care of a medical organisation has the innate ability to self-regulate to an ideal *modus operandi*. It can by itself, i.e., by exclusively relying onto its interior qualities and intrinsic forces, deliver the best possible treatment outcomes, and the associated times that patients have to wait for them. Any external intervention or regulation in it, about who gets served first and how will he be attended to, is naturally expected to harm its operation. Emergency rooms (and their personnel) should be left alone to function freely, solely in accordance with the patients' posterior health needs, given their prior health conditions.

[14]Observe that exactly the same proof can be carried out if the aggregate feasibility condition of a treatment allocation were re-considered with strict equality, due to the strong monotonicity of the patients' utilities, although (as already mentioned in the main text) it is highly unlikely for the ER to respond aggregatively in this manner, even when the patients' utilities are so much pressing.

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