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Public investment multipliers revisited: The role of production complementarities[1]

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Abstract

This paper revisits the issue of the public investment multiplier through the lens of complementarity or substitutability between private inputs and public infrastructure capital. Our main result is that when private inputs and public capital are good complements relative to the canonical Cobb-Douglas case in which the degree of complementarity is unity, an increase in public investment, not only increases public capital which is the "weak link" in production in Jones' (2011) terminology, but also enhances substantially the usage of private inputs; all this generates public investment multipliers much larger than in the literature and, actually, the stronger the degree of complementarity, the larger the size of the multiplier. And vice versa: when private inputs and public capital are good substitutes relative to the canonical Cobb-Douglas case, public investment spending multipliers are much smaller than in the literature.

JEL classification: E62, E23, D21

Keywords: Fiscal policy, Production, Firm behavior

1 Introduction

Most policymakers and politicians seem to have a great faith in spending on public infrastructure as a means to support durable economic growth and hence avoid unpleasant public finance decisions. Examples include the policy agenda of the new Labour government in the UK, the recent Draghi Report on the European Economy, and the big infrastructure stimulus of the previous US administration. In the academic literature, on the other hand, this is less clear. The macroeconomic impacts of higher public investment spending, and hence the size of the public investment multiplier, are sensitive to a wide range of factors such as the public financing scheme, implementation lags, the stance of monetary policy, the maturity of public debt, the state of the economy, political (in)stability, etc (see e.g. Leeper (2010) for a review). Nevertheless, although public investment multipliers can differ, a rather general result is that the neoclassical growth model, as well as its popular extensions, cannot produce large multipliers.[1]

This paper adds another factor that appears to be crucial to the size of the public investment multiplier. Building upon Casseli's (2005) and Jones' (2011) insight about the role of complementarity in production, the paper shows that the degree of complementarity or substitutability between private inputs on one hand and public capital on the other hand, where the latter is augmented by public investment spending, is a crucial determinant of the public investment multiplier.

We work in steps. We start with the canonical neoclassical growth model with public investment (see e.g. Leeper et al (2010)). Then, in this well-known model, we simply replace the Cobb-Douglas production function, which is typically used by most of the literature, with a more flexible CES production function that allows for various degrees of substitutability or complementarity between private inputs and public capital.

Our main result is that when private inputs and public infrastructure capital are good complements relative to the canonical Cobb-Douglas case in which the degree of complementarity is unity, an increase in public investment, not only increases public capital which is the "weak link" in production in Jones' (2011) terminology, but also enhances substantially the usage of private inputs. These combined effects result in public investment spending multipliers much larger than the canonical model and, actually, the stronger the degree of complementarity, the larger the size of the multiplier. Or, reversing the argument, the public investment multiplier turns negative when public capital and private inputs are good substitutes and,

[1]For instance, Leeper et al (2010), Coenen et al (2013), Buakez et al (2017), Sims and Wolff (2018), Ramey (2020), Malley and Philippopoulos (2023), among many others, report cumulative public investment multipliers around 1, especially when it is a distorting public financing instrument, like a tax rate, that reacts to rising public debt to restore stability and debt boundedness.

actually, the stronger the degree of substitutability, the more negative is the multiplier. These results are consistent with both Casseli (2005) and Jones (2011) who demonstrate that, with strong complementarity, allocative efficiency calls for boosting the usage of "the low-efficiency factor" (Caselli) or of "the weak link" (Jones); the latter is public capital in the model as well as in the data in the sense that its stock is less than the stock of private capital. We show that these results hold both in steady state and along the transition.

We wish to clarify two things from the outset. First, as said in the opening paragraph, the size of fiscal policy multipliers is uncertain depending on a plethora of factors. Thus, here we do not wish to take a stance on the exact size of the public investment multiplier. Instead, what we want to show is that ignoring the complementarity (resp. substitutability) between public capital and private inputs can severely distort downwards (resp. upwards) its value. Second, we deliberately build on the stylized neoclassical growth model. Using a simple model makes our main points clearer. On the other hand, when we need to use data for our solutions, we use data from the UK economy.

Our paper belongs to the vast and still growing literature on fiscal policy multipliers and their determinants. By allowing for various degrees of complementarity or subsitutability between public infrastructure and private inputs, we shed light on another important determinant and thus enrich the literature on public investment multipliers (see e.g. the papers in footnote 1). Our work also differs from e.g. Bouakez and Rebei (2007), Feve et al (2013), Leeper et al (2017), Sims and Wolff (2018), Jalles and Karras (2022), Bouakez et al (2023) and Malley and Philippopoulos (2023) who allow government consumption spending to complement or substitute private consumption and thus focus on public consumption multipliers. Further, we differ from Casseli (2005) and Jones (2011) who analyse the relationship between intermediate privately-produced inputs. Our paper also builds upon the econometric findings of e.g. An et al (2019) and Jalles and Karras (2022). An et al investigate the relationship between private capital and public capital in the data and provide some evidence of their complementarity at least in advanced economies and when they exclude cyclical effects. Jalles and Karras, on the other hand, investigate the relationship between private and public consumption and conclude that, while private and public consumption can be thought of as complements in the aggregate, the estimated degree of complementarity or substitutability differs considerably between different sub-categories of public consumption spending (see also the earlier paper by Fiorito and Kollintzas (2004)).

The rest of the paper is as follows. Section 2 presents the canonical model and section 3 adds a more flexible CES production function. Results are in section 4. Section 5 concludes.

2 The canonical model

This section presents a simplified version of the model used by Leeper et al (2010), which is a well cited paper in the literature on public investment multipliers. We adopt the model as it is except that, to make our main points as clear as possible, we abstract from various frictions that are not important to our main subject (results, with frictions included, are available upon request).

2.1 Households

There is a representative household whose aim is to maximise:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right) \tag{1}$$

where c_t and l_t are consumption and work hours, and $0 < \beta < 1$ is the time discount factor.

The utility function is:

$$u\left(c_t, l_t\right) = \frac{(c_t)^{1-v}}{1-v} - u\frac{(l_t)^{1+\kappa}}{1+\kappa} \tag{2}$$

where v is a risk-aversion parameter, $u > 0$ is a preference parameter and $\kappa > 0$ is the inverse of the Frisch elasticity of labour supply.

The budget constraint is:

$$(1+\tau_t^c)c_t + i_t + b_t = (1-\tau_t^y)(w_t l_t + r_{t-1}^k k_{t-1} + \pi_t) + (1+r_{t-1}^b)b_{t-1} + g_t^t \tag{3}$$

where i_t is investment in physical capital, b_t is one-period government bonds purchased at t, w_t is the wage rate, r_{t-1}^k denotes the return to accumulated physical private capital, r_{t-1}^b is the return to outstanding bonds, π_t is dividends distributed by firms, g_t^t is a lump-sum transfer from the government, and $0 \leq \tau_t^c$, $\tau_t^y < 1$ are tax rates on consumption and personal income. We assume that interest income from bonds is untaxed.

Private capital evolves as:

$$k_t = (1-\delta_p)k_{t-1} + i_t \tag{4}$$

where $0 < \delta_p < 1$ is a parameter.

Using (4), the standard first-order conditions for c_t, l_t, b_t and k_t are respectively:

$$\lambda_t = \frac{1}{(1+\tau_t^c)c_t^v} \tag{5a}$$

$$u(l_t)^\kappa = \lambda_t(1-\tau_t^y)w_t \tag{5b}$$

$$\lambda_t = \beta\lambda_{t+1}(1+r_t^b) \tag{5c}$$

$$\lambda_t = \beta\lambda_{t+1}\left[1-\delta_p + (1-\tau_{t+1}^y)r_t^k\right] \tag{5d}$$

where λ_t is the multiplier associated with (3).

2.2 Firms

There is a representative firm. Its production function is (see also e.g. Baxter and King (1993), Leeper et al (2010), Sims and Wolff (2018) and Ramey (2020)):

$$y_t = A_t k_{t-1}^{a} l_t^{1-a} k_{g,t-1}^{\sigma} \tag{6}$$

where $k_{g,t-1}$ is the outstanding stock of public capital acting as a positive production externality, $0 < \alpha,\ \sigma < 1$ are efficiency parameters, and $A_t > 0$ is TFP.

The firm's profit is:

$$\pi_t = y_t - w_t l_t - r_{t-1}^{k} k_{t-1} \tag{7}$$

and the standard first-order conditions for the private inputs are:

$$w_t = \frac{(1-a)y_t}{l_t} \tag{8a}$$

$$r_{t-1}^{k} = \frac{\alpha y_t}{k_{t-1}} \tag{8b}$$

2.3 Government budget constraint

The budget identity of the government is:

$$g_t^i + g_t^c + g_t^t + (1 + r_{t-1}^b) b_{t-1} \equiv b_t + \tau_t^c c_t + \tau_t^y (w_t l_t + r_{t-1}^k k_{t-1} + \pi_t) \tag{9}$$

where g_t^i is public investment and g_t^c is public consumption.[2] Along the transition to the steady state, it will be b_t that closes this constraint.

Public investment augments the stock of public capital whose motion is:

$$k_{g,t} = (1 - \delta_g) k_{g,t-1} + g_t^i \tag{10}$$

where $0 < \delta_g < 1$ is a parameter.

2.4 Macroeconomic system

Collecting equations, the equilibrium system consists of 12 equations, (3), (4), (5a-d), (6), (7), (8a-b), (9) and (10), in 12 variables, $\{c_t,\ l_t,\ k_t,\ i_t,\ r_t^b,\ \lambda_t,\ y_t,\ w_t,\ r_t^k,\ \pi_t,\ b_t,\ k_{g,t}\}_{t=0}^{\infty}$. This is given initial conditions for the state variables and the values of the independently set policy instruments, $\{g_t^t,\ g_t^i,\ g_t^c,\ \tau_t^y,\ \tau_t^c\}_{t=0}^{\infty}$. In what follows, to bring the model closer to the fiscal data, we re-express the public spending items as shares of GDP. In particular, we define $g_t^t = s_t^t y_t$, $g_t^i = s_t^i y_t$ and $g_t^c = s_t^i y_t$ so that the policy instruments are $\{s_t^t,\ s_t^i,\ s_t^c,\ \tau_t^y,\ \tau_t^c\}_{t=0}^{\infty}$ and in the equations above we replace g_t^t, g_t^i and g_t^c with these definitions.

[2] Government consumption is included for numerical/calibration reasons. We could allow it to be an argument in households' utility but here we are not interested in welfare comparisons.

3 The same model with a more flexible CES production function

Say now that the production function has the more flexible CES form:

$$y_t = A_t[x(k_{t-1}^a l_t^{1-a})^\eta + (1-x)(k_{g,t-1})^\eta]^{1/\eta} \tag{11}$$

where the parameter $\eta < 1$ governs the degree of substitution or complementarity between the bundle of private inputs on one hand and public infrastructure capital on the other hand so that their elasticity of substitution is $1/(1-\eta) > 0$, and $0 < x < 1$ is the relative importance of the bundle of private inputs in production. This form is also known as a power or generalized mean of the inputs used.

As explained by e.g. Casseli (2005), Acemoglu (2009, chapter 15) and Jones (2011), such a CES function delivers several popular cases as special cases. For instance, when $\eta \to 0$ so that the elasticity of substitution approaches 1, the production function reduces to Cobb-Douglas like in the canonical model above. Also, following the terminology of e.g. Acemoglu (2009, chapter 15) and Aghion et al (2017), when $0 < \eta < 1$, so that $1/(1-\eta) > 1$, private inputs and public capital are gross substitutes relative to the Cobb-Douglas case, while, when $\eta < 0$, so that $1/(1-\eta) < 1$, private inputs and public capital are gross complements relative to the Cobb-Douglas case. As η becomes more and more negative, substitution becomes more and more difficult so that more and more weight is put on the smallest level of the inputs used or what Jones (2011) calls the weak links. In other words, as η becomes more and more negative, output converges to the minimum of inputs (we report that indeed this will be the case in equilibrium where all inputs in (11) are endogenously and jointly determined).

The firm's first-order conditions for the two private inputs are now:

$$w_t = y_t^{1-\eta} x(1-\alpha) k_{t-1}^{a\eta} l_t^{(1-a)\eta-1} \tag{12a}$$

$$r_{t-1}^k = y_t^{1-\eta} x \alpha k_{t-1}^{a\eta-1} l_t^{(1-a)\eta} \tag{12b}$$

Nothing else changes. Thus, in the new equilibrium system, (11) and (12a-b) replace (6) and (8a-b) from the previous model.

4 Solutions

4.1 Parameters and policy variables

Regarding parameter values, in the canonical model of section 2, following e.g. Leeper et al (2010) and Malley and Philippopoulos (2023), we set $v = 1.5$, $\kappa = 1.4$, $a = 0.36$ and $\sigma = 0.05$. We normalize the steady state value of the TFP parameter, A, at unity and keep it at this value over time except otherwise said. Regarding the private and public capital depreciation rates, δ_p and δ_g respectively, we calibrate them so as to target the gross private capital to output ratio, k/y, and the gross public capital to output ratio, k_g/y, in the UK data (3.3 and 0.69 respectively); this implies $\delta_p = 0.055$ and $\delta_g = 0.05$. In other words, in the initial steady state, $k_g/k = 0.2091$ as in the data, which means that k_g is smaller than k and we report that k_g is also smaller than the bundle of private inputs, $k_{t-1}^a l_t^{1-a}$. Hence, public capital is the weak link. Moving on to other parameters, in the more flexible model of section 3, we calibrate x in (11) so as the steady state public investment multiplier in the flexible model when η is close to zero to become equal to that in the canonical model in which case, as said above, the production functions in the two models become identical; this gives $x = 0.9507$. The discount rate, β, follows from the steady-state version of equation (5c) and UK data for the 10-year government real bond yield; this implies $\beta = 0.9788$. Regarding the agnostic value of parameter $\eta < 1$, following Casseli (2005), we will experiment with different values defined below.

Regarding policy variables, in the initial steady state, we set $s^c = 0.212$, $s^t = 0.158$, $s^i = 0.033$, $\tau^c = 0.152$ and $b/y = 1.006$, as in the current UK data, and allow the income tax rate, τ^y, to adjust to close the government budget constraint in (9). We work similarly when there is a permanent change, like a permanent rise in s_t^i, so that the economy ends up at a new terminal steady state; in other words, in the latter, we will again set $b/y = 1.006$ and allow τ^y to adjust to close (9). Thus, as in Leeper et al (2017), we have "distorting steady-state taxes".[3] By contrast, along the transition from the initial to the terminal steady state, as is typically the case, τ_t^y and b_t change places so that it is b_t that becomes endogenous to close (9); further details are provided below.

4.2 Policy experiment and how we will work

Using the above parameter and policy values, we will first solve for the steady states of the two models presented above and for various values of $\eta < 1$ in the second model. In particular, we will experiment with η at 0.9, 0.5, ≈ 0, -0.5, -1, -1.5 so that the elasticity of substitution, $1/(1-\eta)$, is respectively 10, 2, ≈ 1, 0.67, 0.5, 0.4 (results for a wider range of η are available upon request). Then, departing from the initial steady state, we will assume a permanent rise in public investment spending as share of GDP by, say, 1pp and examine what happens depending on the different values of $\eta < 1$. We will study both steady states and transition paths.

[3] We report that our main results are not affected when it is lump-sum income transfers that adjust to close the government budget constraint at steady state.

4.3 Steady state results

In this subsection, we solve for the initial steady state and compare it to the new steady state where s_t^i is higher by 1pp. We compute the output multiplier defined as $(y - y_0)/(g^i - g_0^i)$, where variables subscripted with 0 refer to initial steady-state values before the increase in public investment spending and variables without subscripts refer to new steady-state values with the increase in public investment spending. We do so for the two models presented above and for various values of $\eta < 1$ in the second model. Results for the multiplier are reported in Table 1.

Table 1: Steady state public investment multipliers

Canonical model	More flexible production function					
	$\eta = 0.9$	$\eta = 0.5$	$\eta \approx 0$	$\eta = -0.5$	$\eta = -1$	$\eta = -1.5$
1.01	-0.24	0.26	1.01	1.93	3.02	4.56

As can be seen, the public investment multiplier gets bigger and bigger as $\eta < 1$ becomes smaller and smaller.

Consider first the benchmark case in which $\eta \approx 0$ (from above or below). In this case, the multiplier with the CES function in the more flexible model approaches that with the Cobb-Dounglas function in the canonical model; this is as it should be. Notice also that in this case, the size of the multiplier is within the usual range in the literature (1.01), where recall that, in all our steady state solutions, the financing of the increase in public investment is financed by an increase in the income tax rate which distorts private incentives to work and save.

When $\eta = 0.9$, the multiplier is negative meaning that an increase in public investment causes a decrease in long-run output; thus, it is harmful for the economy. The intuition of this seemingly paradoxical result is as follows. When private inputs and public capital are strong substitutes vis-a-vis the Cobb-Douglas case ($1/(1-\eta) = 10$, when $\eta = 0.9$), an increase in public investment, that augments public capital, causes a decrease in the usage of private inputs, namely, capital, k, and labor, l (see below on this). This adverse effect on private inputs more than offsets the growth-enhancing impact of public investment stimulus. This result is consistent with Casselli (2005, p. 732) who says that when "the factors are good substitutes, it makes sense to try to increase the usage of the most efficient factor", and the latter, both in the model and in the data, are the private ones (recall the value of $x = 0.9507$ and the initial public to private capital ratio $k_g/k = 0.2091$).

As Table 1 shows, this situation is gradually reversed as η gets smaller and, after a model-specific critical value, the multiplier becomes positive (see for instance the case when $\eta = 0.5$ or $\eta \approx 0$). However, as long as $\eta > 0$, the multiplier is small.

The growth footprint of an increase in public investment becomes substantial, that is the multiplier becomes clearly higher than unity, when $\eta \lessapprox 0$, in which case private and public inputs are good complements vis-a-vis the Cobb-Douglas case The reason for this is symmetrically opposite to that when $\eta > 0$. Namely, when they are good complements, an increase in public investment, that augments public capital (k_g), not only increases the weak link (namely, k_g) but it also enhances the usage of private inputs and this reinforces the positive effect of the fiscal stimulus. This is again consistent with Casseli (2005, p. 732) who says that with $\eta < 0$, "allocative efficiency calls for boosting the overall efficiency units provided by the low-efficiency factor", and the latter is public capital. This result is also consistent with Jones (2011, p. 7) who says that "a stronger degree of complementarity puts more weight on the weakest links". Notice the high values of the multiplier when η equals -0.5, -1 and -1.5.[4] Therefore, good production complementarity can restore the social desirability of public investment spending and, actually, this happens even if it is the distorting income tax rate that finances the increase in public investment spending in our steady state solutions.

To support the above narrative, Table 2 reports the effects on private inputs, namely, capital and labor. As can be seen, the gaps between the-end-of-horizon private capital and labor (i.e. after the stimulus) and the initial private capital and labor (i.e. before the stimulus) are negative in the range $0 \lessapprox \eta < 1$, while, they turn to positive when $\eta < 0$ and, in particular, they become more and more positive (especially, capital) as η becomes more and more negative. This explains the output multipliers in Table 1.

Table 2: Changes of inputs
(final steady-state value relative to initial one, %)

Variable	Canonical	More flexible production function					
	model	$\eta = 0.9$	$\eta = 0.5$	$\eta \approx 0$	$\eta = -0.5$	$\eta = -1$	$\eta = -1.5$
Private capital	-0.63	-2.79	-1.97	-0.63	1.17	3.59	6.84
Labour	-0.15	-0.42	-0.32	-0.15	0.09	0.42	0.86
Public capital	31.68	30.00	30.64	31.68	33.04	34.82	37.15

4.4 Transition results

In this subsection, we again compare the two models and for various values of η in the second model as we did above, except that now we study transition dynamics. We depart from the initial steady state at $t = 0$ (i.e. $s_0^i = 0.033$ at $t = 0$) and then at all $t \geq 1$ we assume a permanent rise in public investment

[4]In their econometric study for 151 countries, An et al (2019) find an estimate of around -0.5 for the substitution parameter between private and public capital when they focus on advanced economies and use 5-year average growth rates so as to exclude cyclical effects. On the other hand, in most of their regressions, these estimates are not significant at least at conventional levels.

by 1pp (i.e. $s_t^i = 0.043$ at $t \geq 1$). As a measure, following the literature (see e.g. Uhlig (2010), Leeper et al (2010), Coenen et al (2013) and Malley and Philippopoulos (2023)), we compute cumulative present value output multipliers defined as:

$$M_t \equiv \sum_{j=1}^{t} \left(R^{-j}\right)(y_j - y_0) / \sum_{j=0}^{t} \left(R^{-j}\right)\left(g_j^i - g_0^i\right) \tag{13}$$

where $R = (1 + r^b)$ is the steady state value of the sovereign interest rate and variables subscripted with 0 refer to initial steady-state values at $t = 0$ before the exogenous increase in s_t^i. Also recall that $g_t^i = s_t^i y_t$.

As is well known (see e.g. the same papers cited above), usually, a stable and determinate transition solution with bounded public debt necessitates that at least one of the exogenous policy instruments follows a debt contingent feedback rule. This is also the case here. Thus, if we assume that this role is played by the income tax rate, τ_t^y, then, at $t \geq 1$, we have the policy rule:

$$\tau_t^y = \tau^y + \rho^y \left(\frac{b_{t-1}}{y_{t-1}} - b_y \right) \tag{14}$$

where b_y is a target for the public debt to output ratio (we set b_y at 1.006 as in the initial steady state), τ^y is the income tax rate in the new steady state (as said, in all steady state solutions, with the public debt to GDP ratio set at 1.006, τ^y adjusts to close the government budget constraint), and $\rho^y \geq 0$ is a feedback policy coefficient (we set it at 0.3 which is the minimum value required for stability across various experiments).[5]

We present results for M_t for the two models above and for various values of η in the second model. The number of periods over which we compute the cumulative present value output multipliers is, for instance, 50.[6] Results are reported in Table 3.

Table 3: Cumulative public investment multipliers (50 periods)

Canonical model	More flexible production function					
	$\eta = 0.9$	$\eta = 0.5$	$\eta \approx 0$	$\eta = -0.5$	$\eta = -1$	$\eta = -1.5$
0.26	-0.34	-0.10	0.28	0.75	1.34	2.06

[5] Notice that, in this rule, τ^y is the value of the tax rate in the new steady state so this rule holds at $t \geq 1$; at $t = 0$, the tax rate is as in the initial steady state (using an $AR(1)$ process would not affect our main results). We also report that our key results do not change when it is, for example, lump-sum income transfers that play this role; the only difference is that, with non-distorting transfers, all multipliers get bigger (see e.g. Malley and Philippopoulos (2023) for a detailed analysis).

[6] We report that, as the time horizon gets large enough, the cumulative multipliers converge to the steady state multipliers in Table 1 above.

As can be seen, and as it was the case in the previous subsection, the cumulative multiplier gets bigger and bigger as η becomes more and more negative.[7]

4.5 A policy experiment: Public investment during downturn

Here we examine the case in which there is an adverse TFP shock that triggers an economic downturn and investigate what an increase in public investment spending can do to counter this and how this depends on the degree of substitutability or complementarity. In particular, we assume a temporary fall in A_t in (6) and (11) by say 1pp and, as we did above, we compare the two models and for various values of η in the second model when, at the same time, there is a permanent increase in public investment spending as share of output by 1pp. For benchmark, we repeat the same when public investment spending as share of output remains unchanged, which means that only A_t falls and there is no fiscal counter reaction to this shock. As a measure, we compute, in each case, the present value of cumulative output gaps (COG_t) as a percent from the initial steady state defined as:

$$COG_t \equiv \left[\sum_{j=1}^{t} \left(R^{-j} \right) \left(\frac{y_j - y_0}{y_0} \right) \right] x100 \tag{15}$$

Results for, say, the first 50 periods are reported in Table 4. As can be seen, and as is probably expected given the results presented above, when η is high (e.g. 0.9 or 0.5), the use of public investment as a countercyclical fiscal policy instrument does not benefit the economy. Instead, its use makes the recession even deeper relative to the no policy scenario and this is due both to the substitution of private inputs for public capital as explained above and the increase in the income tax rate needed to finance the rising public debt over time. In other words, in case private inputs are good substitutes to public inputs, it could be better to leave the economy absorb the recessionary shock on its own. By contrast, as η starts falling, and in particular when it becomes negative, this is reversed and expansionary policy becomes very beneficial.

[7] In Table 3, the Cobb-Douglas function and the CES function do not generate the same result when $\eta \approx 0$. This happens simply because we have calibrated x so as to make these two functional specifications identical in the steady state. By contrast, here we compute transition results for the first 50 periods so the two solutions do not coincide when $\eta \approx 0$.

Table 4: Cumulative output gaps after an adverse TFP shock (50 periods)
(change from initial steady state, %)

	$\eta = 0.9$	$\eta = 0.5$	$\eta \approx 0$	$\eta = -0.5$	$\eta = -1$	$\eta = -1.5$
No policy	-2.91	-3.35	-3.88	-4.42	-4.95	-5.48
Policy	-13.09	-6.41	4.56	19.14	38.22	62.90

4.6 An alternative public policy and a sensitivity check

So far we have investigated what happens when public investment spending increases by 1pp of GDP given that, as is the case in the data, public capital is less that private capital so that the former is the weak link in Jones' (2011) terminology. In this subsection, we check the role of these assumptions and initial conditions. We first assume an increase in subsidies to private capital instead of an increase in public investment spending, other things equal relative to the main scenario assumed so far. Second, to understand the working of the model better, we make the counter-factual assumption that, initially, public capital is more abundant than private capital so that now it is the latter that plays the role of the weak link, again other things equal. We focus on steady state solutions to save on space.

Regarding the first experiment, Table 5 is like Table 1 above except that now, instead of increasing public investment as a fraction of output by 1pp, we assume a quantitatively equivalent subsidy to firms' private capital (modeling details are available upon request). As can be seen, although the main qualitative result in Table 5 is similar to that in Table 1, in the sense that as η decreases, the public spending multiplier increases, there are also differences. In particular, when $\eta \leq 0$, multipliers are much smaller in Table 5 than in Table 1 where we augmented public capital, while the opposite happens when $\eta > 0$. Thus, to the extent that public capital is the weak link, it is more efficient to augment public capital (resp. private capital) when public capital complements (resp. substitutes) private inputs.

Table 5: Steady state private capital subsidy multipliers

$\eta = 0.9$	$\eta = 0.5$	$\eta \approx 0$	$\eta = -0.5$	$\eta = -1$	$\eta = -1.5$
0.72	0.73	0.74	0.76	0.78	0.81

Regarding the second experiment, recall that, in the solutions so far, we started with a gross public capital to private capital ratio equal to $k_g/k = 0.2091 < 1$ as in the UK data (actually, this is also the case in most countries; see e.g. Malley and Philippopoulos (2023) for the US). In other words, public capital was the weak link in Jones' (2011) terminology. Now, we imagine the opposite, counter-factual case: we depart from a k_g/k higher than unity (for instance, we set $k_g/k = 1.1$ in the initial steady state, although the exact number is not important to our results). Results are reported in Table 6, which is like Table 1 except that now we start with $k_g/k > 1$. As can be seen, the results of Table 1 are reversed. Specifically, now, the public investment multiplier is higher in the range $1 < \eta < 0$ than in the range $\eta < 0$ and becomes smaller and smaller as η becomes smaller. This happens because now public capital is not the factor in shortage or the weak link. As said above, when the inputs are poor substitutes, i.e. when $\eta < 0$, allocative efficiency calls for increasing the units of the low-efficiency factor or, in our model, the factor that is provided inadequately. That role was played by public infrastructure in the main results above. By contrast, when we make the counter-factual assumption that the factors that are provided inadequately are the private ones, the results are reversed.

Table 6: Steady state public investment multipliers
when private capital is the weak link

$\eta = 0.9$	$\eta = 0.5$	$\eta \approx 0$	$\eta = -0.5$	$\eta = -1$	$\eta = -1.5$
1.71	1.56	1.011	0.46	0.04	−25.91

5 Conclusions and possible extensions

This short paper revisited the issue of the public investment multiplier through the lens of production complementarity/substitutability between private inputs and public infrastructure capital. We showed that, other things equal, when there is a good degree of complementarity (resp. substitutability), the multiplier becomes substantially larger (resp. smaller or even negative).

We did so in a stylized model. This was deliberate to make our arguments clearer. An extension could be to embed this story into a DSGE model carefully calibrated and/or estimated for an actual economy; this will also allow us to get a data-consistent value of the key parameter η (see e.g. Leeper et al (2017) for the US economy although in the presence of consumption complementarities). Actually, although the empirical evidence seems to suggest that public capital is a gross complement to private inputs, $\eta < 0$, this is at aggregate level only, since the estimated degree of complementarity or substitutability can differ considerably across different types of public capital and hence different categories of public investment spending (see the paper by Jalles and Karras (2022) discussed in the Introduction). This implies that the DSGE model should be rich enough to permit different categories of public infrastructure (e.g. hard or physical infrastructure, education, health, government institutions, etc) to play distinct roles. We leave these extensions for future work.

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