Department of Economics

Athens University of Economics and Business

WORKING PAPER no. 09-2022

A panel bounds testing procedure

Georgios Bertsatos, Plutarchos Sakellaris, Mike G. Tsionas

March 2022

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 8203303-5. E-mail: econ@aueb.gr / www.aueb.gr
76, Patission Street, Athens 104 34 Greece. Tel.: (+30) 210 8203303-5

A panel bounds testing procedure

Georgios Bertsatos[1], Plutarchos Sakellaris[2,*], Mike G. Tsionas[3]

March 2022

Abstract

We propose a bounds testing procedure (BTP) with a battery of tests for the existence of a non-degenerate co-integrating relationship in levels, for long panels. It is a natural extension to panel data of the respective approach in time series as described by Pesaran, Shin and Smith (2001) and extended by Bertsatos, Sakellaris and Tsionas (2022). Simulations suggest that standard inference is not valid for at least one of the tests in our proposed panel BTP. Codes that generate sample-specific critical values are also provided.

JEL classification: C12; C21; C22; C23;

Keywords: Bounds Testing Procedure; Conditional Quantile Regressions; Fixed Effects; Panel Co-integration

---

[1] Post-doctoral researcher at *Athens University of Economics and Business, School of Economic Sciences, Department of Economics*, Patission 76, Athens, 10434, Greece.
E-mail: bertsatosg@aueb.gr ORCID: https://orcid.org/0000-0003-4009-6564

[2] Professor at *Athens University of Economics and Business*, *School of Economic Sciences*, *Department of Economics*, Patission 76, Athens, 10434, Greece.
E-mail: plutarch@aueb.gr ORCID: https://orcid.org/0000-0002-1313-6586
* Corresponding author. Tel.: +30 210 8203353; fax: +30 210 8203499.

[3] Professor at *Lancaster University Management School*, *Department of Economics*, LA1 4YX, Lancaster, United Kingdom. E-mail: m.tsionas@lancaster.ac.uk ORCID: https://orcid.org/0000-0003-3761-8106

1. Introduction

We introduce a bounds testing procedure (BTP), in the spirit of Pesaran, Shin and Smith (2001, hereafter PSS) and Bertsatos, Sakellaris and Tsionas (2022, hereafter BST), for the existence of a non-degenerate co-integrating relationship in levels of the dependent variable and its forcing variables for "large *N* large *T*" panel datasets. We allow for homogeneous responses of the key variable of interest using a mean-based framework, as well as for heterogeneous responses of the dependent variable operating in conditional quantile environment.

The dynamic fixed effects (DFE) estimator is easy to run and has been utilized in many papers (see e.g. Pesaran et al., 1999, Clements et al., 2019) as a benchmark or main model to analyze panel datasets and derive long-run multipliers, where a panel autoregressive distributed lag (PARDL) model is implied. When the time dimension, *T*, is large then, it is legit to use this estimator since the familiar downward lagged dependent bias, known as "Nickel bias" (see Nickel, 1981), is eliminated given a large number of cross sections, *N*.[4] For small *T* on the other hand, dynamic panel generalized method of moments (GMM) estimators have been proposed (see e.g. Arellano and Bond, 1991). Homogeneity across cross sections is assumed in both cases. However, the long-run effects based on the DFE estimator may be spurious if there is no co-integration, i.e. this is a degenerate case and the long-run coefficients could be misleading. Therefore, based on the (one-way or two-way) DFE estimator we propose a bounds testing procedure similar to that of PSS and BST in the time-series environment, and provide coefficients interpretation depending on the treatment of fixed effects.

However, homogeneity across individuals may be restrictive for panels and if heterogeneity holds then, heterogeneity bias (see Pesaran and Smith, 1995) emerges and the results may be misleading. To this reason, several appealing dynamic estimators for heterogeneous panels have been proposed with the mean group (MG) estimator of Pesaran and Shin (1995) being the cornerstone.[5] Though, DFE and the family of MG estimators are mean-oriented estimators that

---

[4] Judson and Owen (1999) show that the "Nickel bias" of the least squares dummy variable (LSDV) estimator (or DFE) is non-negligible even when *T* is 20 or 30. Hence, as we will present later we focus on simulations with *T* greater than or equal to 50. Also, in cases with unbalanced panels when *T* is greater than 30, they show that the LSDV estimator is the preferred estimator amongst others they examine.

[5] Pesaran and Smith (1995) propose the mean group (MG) estimator that allows for full heterogeneity, i.e. for heterogeneous long-run and short-run responses. Pesaran et al. (1999) propose the pooled mean group (PMG) estimator, which restricts the long-run responses to be homogeneous, while it allows for heterogeneous short-run

rely on OLS (PMG is based on ML for the long-run part), and the covariates are allowed to shift only the location or the scale of the dependent variable. Also, the responses of the examined key variable to its covariates may not be representative by the average effect and thus, the outcome is distorted. Moreover, OLS estimates are sensitive to outliers and OLS does not perform well when the variable of interest exhibits skewness or kurtosis. On the other hand, conditional quantile regressions (CQREGs) deal with these "problems" since they permit estimation of the relationship at different quantiles and spotting any differences. Furthermore, with CQREGs one can explore the shape of the distribution and allow for heterogeneous effects of the independent variables to the dependent variable.[6] Every part of the conditional distribution of the dependent variable and not just the mean can be explored with CQREG, while a fruitful discussion about the spectrum of the responses to different covariates may arise.

We diverge from this strand of literature of (P)MG estimators whose coefficients interpretation is like in a time-series model. In particular, we introduce a dual source of heterogeneity based on a Canay (2011) type estimator for dynamic quantile modelling, and propose a BTP for the existence of co-integration at a selected percentile with CQREGs. As far as we are aware the suggested in this paper BTP is the first venture for establishing quantile co-integration for panel data. Our method brings two advances. First, quantile-dependent coefficients allow for heterogeneous responses across the conditional distribution of the dependent variable. Second, duality in interpretation of the state-dependent coefficients that can be seen as time-varying or individual-specific coefficients as we will show below.

We define the state as the relative performance (overperformance or underperformance) of the dependent variable and the state depends on the fixed effects. According to our method in conditional quantile framework, we examine how a change of a covariate over time or in the

---

responses, i.e. there is partial heterogeneity. Chudik and Pesaran (2015) expand the estimator of common correlated effects (CCE) of Pesaran (2006) to dynamic modelling (DCCE). Chudik et al. (2016) refer to DCCE as cross-sectionally augmented ARDL or CS-ARDL, and propose an alternative estimation strategy by the name of cross-sectionally augmented distributed lag or CS-DL, which complements CS-ARDL. Briefly, CS-ARDL and CS-DL are expansions of the MG estimator that allow for heterogeneous cross correlations. Finally, the assumption that a long-run relationship exists is the *Achilles heel* of (P)MG estimators (see Pesaran et al., 1999).

[6] With OLS the average prediction error is zero by construction, whilst with conditional quantiles underprediction is observed at low percentiles (the average prediction error is positive) and overprediction at high percentiles (the average prediction error is negative). This property of CQREGs is used in business; for example, delivery firms employ high quantiles to predict delivery time and risk managers low quantiles for value-at-risk of stock returns.

cross section (dual interpretation of coefficients) affects the response of the key variable of interest (dependent variable). This is important as it offers flexibility to researchers and practitioners in answering their research questions because the coefficients interpretation varies with the presence of cross-sectional and time fixed effects in the model specification.

Finally, our paper is also related to the literature of panel co-integrating tests and especially to Westerlund (2007). He proposes four different tests, which are based on unrestricted error-correction models (UECMs) and focus on the error-correction coefficient. If the null hypothesis of no error correction is rejected then, that of no co-integration is also rejected. However, even if these tests are robust to several error specifications, they are limited to $I(1)$ variables and are subject to problems related to pretesting for unit roots.

To summarize, for the existence of co-integration we propose first, a BTP based on the DFE estimator in unrestricted error-correction form to detect situations, where the long-run effects could be questionable (degenerate case of co-integration). Second, we propose a BTP for a selected percentile using a Canay (2011) type estimator for unrestricted error-correction models (UECMs) at conditional quantile framework. For its completeness we also propose an interpercentile BTP to detect similarities of long-run components across quartiles (25^{th}, 50^{th} and 75^{th} percentiles) and deciles (10^{th}, 20^{th}, …, 80^{th}, 90^{th} percentiles) in the spirit of BST. Third, we provide coefficients interpretations – depending on the treatment of cross-sectional and/or time fixed effects – in both conditional mean and quantile setups for our methodologies.

The rest of the paper is as follows: *Section 2* presents the framework of unrestricted error-correction models that is required for the bounds testing procedure and, *Section 3* describes the simulations used and presents the results. *Section 4* is dedicated to coefficients interpretation and treatment of fixed effects, and *Section 5* concludes and offers future suggestions. Finally, there is an *Appendix* incorporating tables with the results of the employed simulations and a demonstration for the treatment of fixed effects.

2. The unrestricted error-correction framework

In this section we present the unrestricted error-correction models to be estimated before executing the BTP. We state the assumptions and pose the associated tests. First we deal with the conditional mean environment and next, we move to the conditional quantile setup. We work with panel datasets, where the number of cross sections, N, and time periods, T, are large.

We employ panel ARDL models in unrestricted error-correction form as PSS and BST in time-series level. UECMs are attractive since they permit examination of both short-run and long-run multipliers in one step efficiently, as well as of transitional dynamics towards the steady-state equilibrium (if any). Moreover, they allow regressors to be a mixture of stationary variables, i.e. $I(0)$ processes, and variables with unit root, i.e. $I(1)$ processes (see e.g. Pesaran et al., 1999, PSS and Chudik et al., 2016), and allow for one co-integrating relationship that runs from the forcing variables to the dependent variable.[7]

2.1 Basic setup

We begin with the conditional mean setup. Given a PARDL (p, q, … , q) model in levels:

$$Y_{i,t} = c + a \cdot t + b \cdot t^2 + \sum_{j=1}^{p} \lambda_j \cdot Y_{i,t-j} + \sum_{j=1}^{q} \boldsymbol{\delta}'_j \cdot \mathbf{X}_{i,t-j} + d_i + d_t + e_{i,t} \quad (1)$$

Our methodology for the panel BTP is built on the ARDL (p-1, q-1, … , q-1) model in unrestricted error-correction form that is equivalent of the ARDL (p, q, … , q) model in levels:[8]

$$\Delta Y_{i,t} = c + a \cdot t + b \cdot t^2 + \left(-1 + \sum_{j=1}^{p} \lambda_j \right) \cdot Y_{i,t-1} + \left(\sum_{j=0}^{q} \boldsymbol{\delta}'_j \right) \cdot \mathbf{X}_{i,t-1}$$

$$+ \left[\sum_{j=1}^{p-1} \left(-\sum_{m=j+1}^{p} \lambda_m \right) \right] \cdot \Delta Y_{i,t-j} + \boldsymbol{\delta}'_0 \cdot \Delta \mathbf{X}_{i,t} + \left[\sum_{j=1}^{q-1} \left(-\sum_{m=j+1}^{q} \boldsymbol{\delta}'_j \right) \right] \cdot \Delta \mathbf{X}_{i,t-j} + d_i + d_t + e_{i,t} \quad (2)$$

$$\Leftrightarrow$$

$$\Delta Y_{i,t} = c + a \cdot t + b \cdot t^2 + \varphi_y \cdot Y_{i,t-1} + \boldsymbol{\gamma}'_{\mathbf{x}} \cdot \mathbf{X}_{i,t-1} + \sum_{j=1}^{p-1} \lambda_j^* \cdot \Delta Y_{i,t-j} + \sum_{j=0}^{q-1} \boldsymbol{\delta}_j^{*'} \cdot \Delta \mathbf{X}_{i,t-j} + d_i + d_t + e_{i,t}$$

where, Y is the main variable of interest, φ_y is the error-correction parameter, $\mathbf{X}$ is a column vector of k stochastic regressors (forcing variables) and $\boldsymbol{\gamma}'_{\mathbf{x}}$ is the raw vector with the associated k coefficients, c is the constant term, a is the coefficient of the linear trend, b is the coefficient of quadratic trend, and d_i and d_t are the cross-sectional and fixed time effects, respectively. The error structure is $e_{i,t}$. For ease of exposition, we assume that the lags for all forcing variables equal q however in practice, the lags of the stochastic regressors may vary and not be the same.

---

[7] However, there are some caveats as with the (P)MG estimators. $I(2)$ variables or of higher order of integration are not supported and if the dependent variable affects some of the forcing variables in the long run then, ARDL models may yield spurious results and a new estimator with multiple co-integrating vectors should be employed.

[8] We should notice that an ARDL in unrestricted error-correction form is not the same with a first-difference model, since the error term and the deterministic factors remain unchanged. BST discuss this issue thoroughly.

Next, we move to conditional quantile framework and present the panel quantile ARDL, PQARDL, (p-1, q-1, … , q-1) in unrestricted error-correction form employing first, a Canay (2011) type approach for dynamic modelling and second, the standard minimization routine and "check function" of Koenker and Bassett (1978):

$$\Delta\widehat{Y}_{i,t} = c(\tau) + a(\tau)\cdot t + b(\tau)\cdot t^2 + \left(-1+\sum_{j=1}^{p}\lambda_j(\tau)\right)\cdot Y_{i,t-1} + \left(\sum_{j=0}^{q}\boldsymbol{\delta}_j^{'}(\tau)\right)\cdot \mathbf{X}_{i,t-1}$$
$$+\left[\sum_{j=1}^{p-1}\left(-\sum_{m=j+1}^{p}\lambda_m(\tau)\right)\right]\cdot\Delta\widehat{Y}_{i,t-j} + \boldsymbol{\delta}_0^{'}(\tau)\cdot\Delta\mathbf{X}_{i,t} + \left[\sum_{j=1}^{q-1}\left(-\sum_{m=j+1}^{q}\boldsymbol{\delta}_j^{'}(\tau)\right)\right]\cdot\Delta\mathbf{X}_{i,t-j} + e_{i,t}(\tau)$$
$$\Leftrightarrow \tag{3}$$
$$\Delta\widehat{Y}_{i,t} = c(\tau) + a(\tau)\cdot t + b(\tau)\cdot t^2 + \varphi_y(\tau)\cdot\widehat{Y}_{i,t-1} + \boldsymbol{\gamma}_{\mathbf{x}}^{\prime}(\tau)\cdot\mathbf{X}_{i,t-1} + \sum_{j=1}^{p-1}\lambda_j^{*}(\tau)\cdot\Delta\widehat{Y}_{i,t-j}$$
$$+\sum_{j=0}^{q-1}\boldsymbol{\delta}_j^{*'}(\tau)\cdot\Delta\mathbf{X}_{i,t-j} + e_{i,t}(\tau)$$

where, $\widehat{Y}_{i,t} = Y_{i,t} - d_i^{est} - d_t^{est}$ is the Canay-type new dependent variable for the conditional quantile estimation, d_i^{est} and d_t^{est} are the consistently estimated cross-sectional and time fixed effects from Equation (2), $\tau\in(0,1)$ is the selected quantile and $e_{i,t}(\tau)=\widehat{Y}_{i,t}-Q_{Y_t}(\tau\,|\,\Xi_{t-1})$ is the error structure, whose second term, i.e. $Q_{Y_t}(\tau\,|\,\Xi_{t-1})$, is the τ^{th} quantile of $\widehat{Y}_{i,t}$ conditional on the sigma field $\Xi_{t-1}=\sigma\left\{\widehat{Y}_{i,t-1},\Delta\widehat{Y}_{i,t-1},...,\Delta\widehat{Y}_{i,t-p+1},\mathbf{X}_{i,t-1},\Delta\mathbf{X}_{i,t},...,\Delta\mathbf{X}_{i,t-q+1}\ \forall\ i\right\}$. Note that, even if Canay's (2011) estimator assumes that the fixed effects are location shifters, i.e. non-quantile dependent, the constant term is distribution shifter, i.e. it varies across quantiles.

If the roots of the characteristic polynomial $\sum_{j=1}^{p}\lambda_j\cdot w^j=1$, $\sum_{j=1}^{p}\lambda_j(\tau)\cdot w^j=1$ lie outside the unit circle[9] then, the ARDL models are stable. So, if we find evidence in favor of the existence of a non-degenerate co-integrating relationship, running from **X** to Y with the panel BTP as we discuss later, this long-run equilibrium relation will also be stable. Namely, the actual values of Y converge eventually to their steady-state values, where the long-run multipliers of forcing **X** are consistently calculated via delta method as $\boldsymbol{\theta}^{'}=-\frac{\boldsymbol{\gamma}_{\mathbf{x}}^{\prime}}{\varphi_y}$, $\boldsymbol{\theta}^{'}(\tau)=-\frac{\boldsymbol{\gamma}_{\mathbf{x}}^{\prime}(\tau)}{\varphi_y(\tau)}\ \forall\ \tau\in(0,1)$.

[9] This condition also ensures that the error-correction parameter is negative (see Pesaran et al., 1999).

The errors have to be free of serial correlation and cross-sectional dependence so that consistency of estimates is achieved. The first condition can be met with a sufficiently large number of lags.[10] The second condition under homogeneity of cross sections can be met either with inclusion of fixed time effects or by expressing variables as deviations from their cross-sectional means (see Pesaran et al., 1999). Also, it could be met with inclusion of common variables, such as global macro variables (e.g. gold, or kerosene, prices) that enter the equilibrium relationship, or with inclusion of a time trend (see Pesaran and Smith, 1995). One other way to capture time-variant unobserved characteristics is to express the variables as deviations from their cross-sectional means.[11]

We focus on Equation (2) to execute the panel BTP as we will present next for the conditional mean environment. In practice, there may be fixed cross-sectional and/or time effects and thus, practitioners and researchers have to purge them. In this study, we suggest prior to the execution of our BTP the use of DFE estimator for conditional mean models and an expansion of Canay's (2011) estimator for conditional quantile models to get rid of fixed effects.[12] We are interested in dynamic models and to tackle some estimation problems we

---

[10] The use of lags in ARDL models also alleviates endogeneity concerns (see e.g. PSS, and Clements et al., 2019). Particularly, short-run reverse causality is resolved with the error-projection technique given that regressors are represented by finite-order autoregressive processes (see Pesaran and Shin, 1999, Pesaran et al., 1999, PSS, Shin et al., 2014, Cho et al., 2015).

[11] Pesaran et al. (1999) also suggest to augment the model with cross-sectional means of the variables used as additional regressors however in this case, the simulated critical values have to be modified. For example, these variables have to be excluded from the *F*-tests since they do not have any meaningful interpretation. So, even if the degrees of freedom of the numerator remain the same, this is not the case for the denominator, where the degrees of freedom become smaller; this effect may be trivial because the number of observations after the estimation process range from 2,500 (when both N and T equal 50) to 1,000,000 (when both N and T equal 1,000) in our setup. This can be also the case when we treat macro variables as deterministic factors and we do not include them in the co-integrating relationship. Therefore, we suggest two options along with the use of cross-sectional fixed effects. First, the use of fixed time effects and second, the model augmentation with macro variables that affect the dependent variable along the path towards the steady-state equilibrium relationship.

[12] In *Section 4* we discuss about the inclusion of fixed effects in the specification and how it alters the interpretation of the coefficients. In the conditional mean setup, the interpretation is like that in the time-series case when cross-sectional fixed effects are added, whilst with fixed time effects the interpretation is like that in the cross-sectional case. Regarding the two-way fixed effects, the interpretation is more complicated (see Kropko and Kubinec, 2018) however, we derive a much easier interpretation. Regarding the conditional quantile setup, the coefficients interpretation with respect to fixed effects is dual.

focus on "large N large T" datasets. Therefore, equations (1) and (2) can be estimated consistently with the DFE estimator, and Equation (3) with conditional quantile estimator of Koenker and Bassett (1978) within a Canay (2011) type approach for dynamic modelling, as the familiar lagged dependent bias vanishes in long panels (see Nickel, 1981, for conditional mean environment, and Galvao, 2011, and Covas et al., 2014 for conditional quantile setup).

Finally, the specification of Equations (2) and (3) can be easily expanded in the spirit of Shin et al. (2014) so that non-linear responses of Y to its covariates are allowed.[13] Therefore, that would be the panel non-linear ARDL, or PNARDL. In the quantile setup that would be the panel quantile non-linear ARDL, or PQNARDL.

2.2 Presentation of the tests

In this subsection we focus on the unrestricted models of Equations (2) and (3), and highlight eleven cases following BST. In particular, these cases are based on the specification of the deterministic components, i.e. the constant term, the linear trend and the quadratic trend.[14] Furthermore, we present for each case the involved tests of the BTP and their hypotheses. As previously, we begin with the conditional mean framework and next, we move to the conditional quantile setup.

Case I involves the following specification:

$$\Delta Y_{i,t} = \varphi_y \cdot Y_{i,t-1} + \boldsymbol{\gamma}'_{\mathbf{x}} \cdot \mathbf{X}_{i,t-1} + \sum_{j=1}^{p-1} \lambda_j^* \cdot \Delta Y_{i,t-j} + \sum_{j=0}^{q-1} \boldsymbol{\delta}_j^{*'} \cdot \Delta \mathbf{X}_{i,t-j} + d_i + d_t + e_{i,t} \tag{4}$$

For Case I, the panel BTP includes testing:

(i) $\mathrm{H}_0: H_0^{F_{yx}}: \varphi_y = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.I} = 0$ with a F_{yx}-test

(ii) $\mathrm{H}_0: \varphi_y = 0$ with a t_{y}-test,

(iii) $\mathrm{H}_0: \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.I} = 0$ with a F_{x}-test,

(iv) $\mathrm{H}_0: \gamma_j = 0$ with a t_{x}-test for $j = 1$ to k.

Cases II and III deal with the following specification:

---

[13] When the threshold is known *a priori*, either with zero or non-zero value, and there are no large differences in the regime probabilities then, no estimation or inference issues arise (see e.g. Greenwood-Nimmo and Shin, 2013).

[14] BST discuss in detail the null and alternative hypotheses for all cases.

$$\Delta Y_{i,t} = c + \varphi_y \cdot Y_{i,t-1} + \boldsymbol{\gamma}'_{\mathbf{x}} \cdot \mathbf{X}_{i,t-1} + \sum_{j=1}^{p-1} \lambda_j^* \cdot \Delta Y_{i,t-j} + \sum_{j=0}^{q-1} \boldsymbol{\delta}_j^{*\prime} \cdot \Delta \mathbf{X}_{i,t-j} + d_i + d_t + e_{i,t} \tag{5}$$

For Case II, the panel BTP includes testing:

(i) $H_0: \varphi_y = c = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.II} = 0$ against with a F_{yx}-test

(ii) $H_0: \varphi_y = 0$ with a t_y-test,

(iii) $H_0: c = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.II} = 0$ with a F_x-test,

(iv) $H_0: \gamma_j = 0$ with a t_x-test for $j = 1$ to k.

For Case III, the panel BTP includes testing:

(i) $H_0: \varphi_y = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.III} = 0$ with a F_{yx}-test

(ii) $H_0: \varphi_y = 0$ with a t_y-test,

(iii) $H_0: \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.III} = 0$ with a F_x-test,

(iv) $H_0: \gamma_j = 0$ with a t_x-test for $j = 1$ to k.

Case II is subsumed by Case III for the *t*-tests, which are the same in these cases. On the other hand, the *F*-tests are different in these cases.

Cases IV to VII are based on the following specification:

$$\Delta Y_{i,t} = c + a \cdot t + \varphi_y \cdot Y_{i,t-1} + \boldsymbol{\gamma}'_{\mathbf{x}} \cdot \mathbf{X}_{i,t-1} + \sum_{j=1}^{p-1} \lambda_j^* \cdot \Delta Y_{i,t-j} + \sum_{j=0}^{q-1} \boldsymbol{\delta}_j^{*\prime} \cdot \Delta \mathbf{X}_{i,t-j} + d_i + d_t + e_{i,t} \tag{6}$$

For Case IV, the panel BTP includes testing:

(i) $H_0: \varphi_y = a = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.IV} = 0$ with a F_{yx}-test

(ii) $H_0: \varphi_y = 0$ with a t_y-test,

(iii) $H_0: a = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.IV} = 0$ with a F_x-test,

(iv) $H_0: \gamma_j = 0$ with a t_x-test for $j = 1$ to k.

For Case V, the panel BTP includes testing:

(i) $H_0: \varphi_y = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.V} = 0$ with a F_{yx}-test

(ii) $H_0: \varphi_y = 0$ with a t_y-test,

(iii) $\mathrm{H_0}: \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.V} = 0$ with a F_x-test,

(iv) $\mathrm{H_0}: \gamma_j = 0$ with a t_x-test for j = 1 to k.

For Case VI, the panel BTP includes testing:

(i) $\mathrm{H_0}: \varphi_y = c = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.VI} = 0$ with a F_{yx}-test

(ii) $\mathrm{H_0}: \varphi_y = 0$ with a t_y-test,

(iii) $\mathrm{H_0}: c = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.VI} = 0$ with a F_x-test,

(iv) $\mathrm{H_0}: \gamma_j = 0$ with a t_x-test for j = 1 to k.

For Case VII, the panel BTP includes testing:

(i) $\mathrm{H_0}: \varphi_y = c = a = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.VII} = 0$ with a F_{yx}-test

(ii) $\mathrm{H_0}: \varphi_y = 0$ with a t_y-test,

(iii) $\mathrm{H_0}: c = a = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.VII} = 0$ with a F_x-test,

(iv) $\mathrm{H_0}: \gamma_j = 0$ with a t_x-test for j = 1 to k.

Note that in Cases IV to VII, the t-tests are the same, while the F-tests differentiate.

Cases VII to XI cope with the following specification:

$$\Delta Y_{i,t} = c + a \cdot t + b \cdot t^2 + \varphi_y \cdot Y_{i,t-1} + \boldsymbol{\gamma}'_{\mathbf{x}} \cdot \mathbf{X}_{i,t-1} + \sum_{j=1}^{p-1} \lambda_j^* \cdot \Delta Y_{i,t-j} + \sum_{j=0}^{q-1} \boldsymbol{\delta}_j^{*\,'} \cdot \Delta \mathbf{X}_{i,t-j} + d_i + d_t + e_{i,t} \quad (7)$$

For Case VIII, the panel BTP includes testing:

(i) $\mathrm{H_0}: \varphi_y = c = a = b = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.VIII} = 0$ with a F_{yx}-test

(ii) $\mathrm{H_0}: \varphi_y = 0$ with a t_y-test,

(iii) $\mathrm{H_0}: c = a = b = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.VIII} = 0$ with a F_x-test,

(iv) $\mathrm{H_0}: \gamma_j = 0$ with a t_x-test for j = 1 to k.

For Case IX, the panel BTP includes testing:

(i) $\mathrm{H_0}: \varphi_y = a = b = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.IX} = 0$ with a F_{yx}-test

(ii) $\mathrm{H_0}: \varphi_y = 0$ with a t_y-test,

(iii) $\mathrm{H_0}: a = b = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.IX} = 0$ with a F_x-test,

(iv) $\mathrm{H}_0: \gamma_j = 0$ with a t_x-test for j = 1 to k.

For Case X, the panel BTP includes testing:

(i) $\mathrm{H}_0: \varphi_y = c = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.X} = 0$ with a F_{yx}-test

(ii) $\mathrm{H}_0: \varphi_y = 0$ with a t_y-test,

(iii) $\mathrm{H}_0: c = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.X} = 0$ with a F_x-test,

(iv) $\mathrm{H}_0: \gamma_j = 0$ with a t_x-test for j = 1 to k.

For Case XI, the panel BTP includes testing:

(i) $\mathrm{H}_0: \varphi_y = \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \varphi_y = \boldsymbol{\varphi}'_{\mathbf{x},C.XI} = 0$ with a F_{yx}-test

(ii) $\mathrm{H}_0: \varphi_y = 0$ with a t_y-test,

(iii) $\mathrm{H}_0: \boldsymbol{\gamma}'_{\mathbf{x}} = 0 \Leftrightarrow \boldsymbol{\varphi}'_{\mathbf{x},C.XI} = 0$ with a F_x-test,

(iv) $\mathrm{H}_0: \gamma_j = 0$ with a t_x-test for j = 1 to k.

Note that in Cases VII to XI, the *t*-tests are the same, while the *F*-tests differentiate. The alternative hypothesis of each *F*-test denotes that at least one coefficient tested in the null is different from zero. Furthermore, the alternative of t_y-test denotes that the coefficient of the error-correction term (ECT) is negative ($\mathrm{H}_A: \varphi_y < 0$) and that of a t_x-test denotes that the associated coefficient is different from zero ($\mathrm{H}_A: \gamma_j \neq 0$ for j = 1 to k).

When fixed time effects are taken into account, 3 cases remain for the BTP, i.e. Cases I, II and III. Also, when time trends are included to handle time-variant unobserved heterogeneity then, the first 3 cases are non-eligible, i.e. Cases I, II and III. Finally, when the specification is augmented with common variables, which enter the co-integrating relationship, to deal with unobserved heterogeneity in the time dimension then, Cases I, II and III are eligible.

As discussed previously, when we move to the conditional quantile framework we use a Canay (2011) type approach for dynamic modelling. So we have to transform the dependent variable *Y* into $\widehat{Y}_{i,t} = Y_{i,t} - d_i^{est} - d_t^{est}$ given that the fixed effects are consistently estimated.

The corresponding Equation (4) in conditional quantile framework for Case I is:

$$\Delta\widehat{Y}_{i,t}=\varphi_y(\tau)\cdot\widehat{Y}_{i,t-1}+\boldsymbol{\gamma}'_{\mathbf{x}}(\tau)\cdot\mathbf{X}_{i,t-1}+\sum_{j=1}^{p-1}\lambda_j^*(\tau)\cdot\Delta\widehat{Y}_{i,t-j}$$
$$+\sum_{j=0}^{q-1}\boldsymbol{\delta}_j^{*'}(\tau)\cdot\Delta\mathbf{X}_{i,t-j}+e_{i,t}(\tau) \tag{8}$$

where, the consistently estimated fixed effects from Equation (4) are obtained to transform the dependent variable in levels.

Similarly, using the consistently estimated fixed effects from Equation (5), Cases II and III in conditional quantile framework involve the following specification:

$$\Delta\widehat{Y}_{i,t}=c(\tau)+\varphi_y(\tau)\cdot\widehat{Y}_{i,t-1}+\boldsymbol{\gamma}'_{\mathbf{x}}(\tau)\cdot\mathbf{X}_{i,t-1}+\sum_{j=1}^{p-1}\lambda_j^*(\tau)\cdot\Delta\widehat{Y}_{i,t-j}$$
$$+\sum_{j=0}^{q-1}\boldsymbol{\delta}_j^{*'}(\tau)\cdot\Delta\mathbf{X}_{i,t-j}+e_{i,t}(\tau) \tag{9}$$

Cases IV to VII in the conditional quantile setup employ the consistently estimated fixed effects from Equation (6) for the transformation of *Y* ending up with:

$$\Delta\widehat{Y}_{i,t}=c(\tau)+a(\tau)\cdot t+\varphi_y(\tau)\cdot\widehat{Y}_{i,t-1}+\boldsymbol{\gamma}'_{\mathbf{x}}(\tau)\cdot\mathbf{X}_{i,t-1}+\sum_{j=1}^{p-1}\lambda_j^*(\tau)\cdot\Delta\widehat{Y}_{i,t-j}$$
$$+\sum_{j=0}^{q-1}\boldsymbol{\delta}_j^{*'}(\tau)\cdot\Delta\mathbf{X}_{i,t-j}+e_{i,t}(\tau) \tag{10}$$

Finally, using the consistently estimated fixed effects from Equation (7) we get the following specification for Cases VIII to XI in the conditional quantile environment:

$$\Delta\widehat{Y}_{i,t}=c(\tau)+a(\tau)\cdot t+b(\tau)\cdot t^2+\varphi_y(\tau)\cdot\widehat{Y}_{i,t-1}+\boldsymbol{\gamma}'_{\mathbf{x}}(\tau)\cdot\mathbf{X}_{i,t-1}+\sum_{j=1}^{p-1}\lambda_j^*(\tau)\cdot\Delta\widehat{Y}_{i,t-j}$$
$$+\sum_{j=0}^{q-1}\boldsymbol{\delta}_j^{*'}(\tau)\cdot\Delta\mathbf{X}_{i,t-j}+e_{i,t}(\tau) \tag{11}$$

The tests involved in the BTP for a selected quantile $\tau\in(0,1)$ share the same null hypotheses with these described previously in the conditional mean environment. For example in Case III, the panel BTP for PQARDL models includes testing:

(i) $H_0: \varphi_y(\tau)=\boldsymbol{\gamma}'_{\mathbf{x}}(\tau)=0\Leftrightarrow\varphi_y(\tau)=\boldsymbol{\varphi}'_{\mathbf{x},C.III}(\tau)=0$ with a F_{yx}-test

(ii) $H_0: \varphi_y(\tau)=0$ with a t_y-test,

(iii) $H_0: \boldsymbol{\gamma}'_{\mathbf{x}}(\tau)=0\Leftrightarrow\boldsymbol{\varphi}'_{\mathbf{x},C.III}(\tau)=0$ with a F_x-test, and

(iv) $H_0: \gamma_j = 0$ with a t_x-test for j = 1 to k.

Table 1 summarizes the aforementioned cases for the panel BTP.

Table 1. Cases for the panel bounds testing procedure

$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}'\right)'$ and $w_{i,t} = 0$	Case I
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}', 1\right)'$ and $w_{i,t} = 0$	Case II
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}'\right)'$ and $w_{i,t} = 1$	Case III
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}', t\right)'$ and $w_{i,t} = 1$	Case IV
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}'\right)'$ and $w_{i,t} = (1, t)'$	Case V
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}', 1\right)'$ and $w_{i,t} = t$	Case VI
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}', 1, t\right)'$ and $w_{i,t} = 0$	Case VII
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}', 1, t, t^2\right)'$ and $w_{i,t} = 0$	Case VIII
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}', t, t^2\right)'$ and $w_{i,t} = 1$	Case IX
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}', 1\right)'$ and $w_{i,t} = \left(t, t^2\right)$	Case X
$\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}'\right)'$ and $w_{i,t} = \left(1, t, t^2\right)$	Case XI

Notes: This Table shows all the cases of the panel bounds testing procedure we examine. Regarding the generation of the simulated critical values (SCVs), we run the model in Equation (12) with OLS and in Equation (13) with conditional quantile regressions of Koenker and Bassett (1978). The SCVs are the empirical quantiles of the following computed test-statistics. The tests involved in each case are: *i.* $\mathbf{\Phi}' = \mathbf{0}$ (F_{yx}-test), where $\mathbf{\Phi}'$ equals to $\mathbf{\Phi}' = \left(\varphi_y, \boldsymbol{\varphi}_{\mathbf{x}}'\right)$, includes the coefficients of $\mathbf{z}_{i,t-1}$ according to this Table and φ_y is the coefficient of $y_{i,t-1}$, *ii.* $\varphi_y = 0$ (t_y-test), *iii.* $\boldsymbol{\varphi}_{\mathbf{x}}' = \mathbf{0}$ (F_x-test), which includes the coefficients of $\mathbf{z}_{i,t-1}$ except for that of $y_{i,t-1}$, and *iv.* $\varphi_j = 0$ (t_x-test), where j = 1 to k. Note that the t_y-test is the same for Cases II and III (we keep Case III), Cases IV to VII (we keep Case V), and Cases VII to XI (we keep Case XI). The same holds for the t_x-test. The t_y-test is a left-oriented test and the t_x-test is a two-tailed test.

To sum up, there are four common steps involved in the proposed panel BTP for the two different frameworks we examine.

Step 1: If the F-statistic of F_{yx}-test is greater than the simulated critical bounds then, move to the next step. If on the other hand, it is lower the simulated critical bounds, there is no co-integrating relationship with this specification under the selected Case. The F_{yx}-test tests if the lagged dependent variable in levels, the lagged stochastic regressors in levels and any deterministic factor (intercept, linear trend, quadratic trend) are jointly zero.

Step 2: If the t-statistic of t_y-test is smaller than the simulated critical values then, we move to the next step. If there is a positive or a small negative t-statistic, which is greater than the simulated critical bounds then, there is no long-run relationship with the examined specification under the selected Case. The t_y-test tests if the lagged independent variable in levels equals zero.

Step 3: If the F-statistic of F_x-test is greater than the simulated critical bounds then, the BTP is successfully passed in this specification under the selected Case and the long-run, or co-integrating, multipliers, $\boldsymbol{\theta}' = -\frac{\boldsymbol{\gamma}'_{\mathbf{x}}}{\varphi_y}$ and $\boldsymbol{\theta}'(\tau) = -\frac{\boldsymbol{\gamma}'_{\mathbf{x}}(\tau)}{\varphi_y(\tau)}$ $\forall\ \tau \in (0,1)$ can be successfully assessed with the delta method since the sample size is large (both N and T are large). The F_x-test tests if the lagged stochastic regressors in levels and any deterministic factor (intercept, linear trend, quadratic trend) are jointly zero.

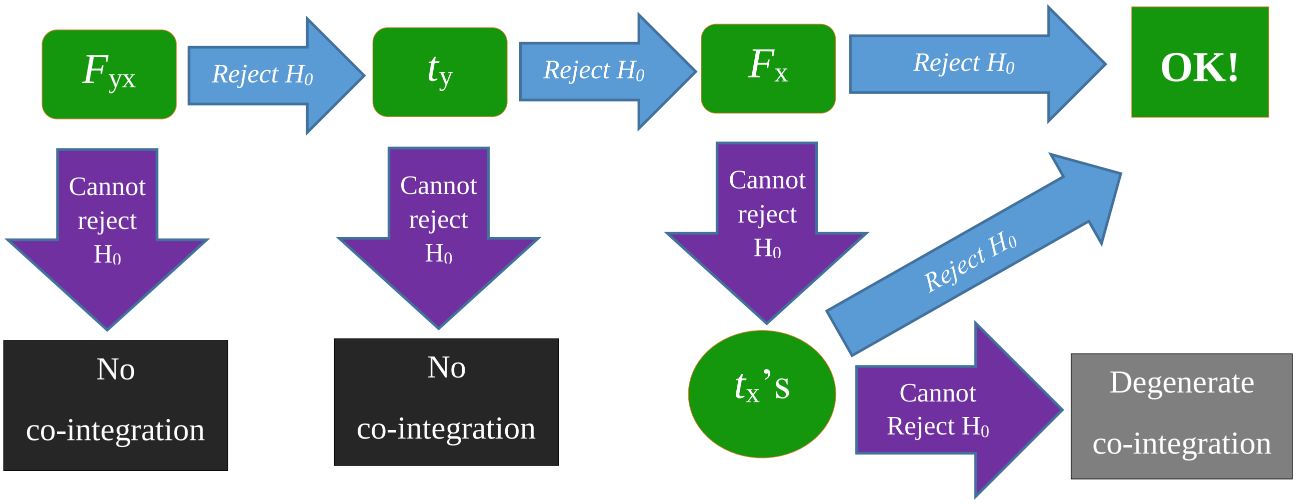
Step 4: If the F_{yx}-test and t_y-test reject their null hypotheses, and the F_x-test does not reject its null, and the γ's are individually significant (t_x-tests), $H_0 : \gamma_j = 0$ against $H_A : \gamma_j \neq 0$ for $j = 1, \ldots, k$, then, there is a long-run relationship running from X's to Y and the long-run coefficients can be evaluated with the delta method. The t_x-tests test if the lagged stochastic regressors in levels are equal to zero individually. Furthermore, if both the F_x-test and the t_x-tests in Step 3 do not reject their null hypothesis, and the delta method generates statistically significant long-run multipliers then, we argue this is a degenerate case of co-integration.

The first two steps (F_{yx} and t_y tests) are the same as in PSS and BST. The F_x and t_x tests help us towards the path to co-integration as in BST and filter out degenerate cases. After the rejection of the null hypotheses of F_{yx}-test and t_y-test, there are two paths towards the existence of a non-degenerate co-integration. The first one is through joint testing with the F_x-test and if this leads to a dead end, there is one alternative through individual testing with the t_x-tests.

If we reject the joint null hypothesis of interest given by $H_0 = H_0^{F_{yx}} \cap H_0^{t_y} \cap H_0^{F_x}$ against the alternative that at least one of the three null hypotheses cannot be rejected then, we find evidence that a non-degenerate co-integrating relationship in levels between Y and forcing $\mathbf{X}$ exists. On the other hand if we fail to reject only the null of F_x-test, we could focus on the joint null of $H_0 = H_0^{F_{yx}} \cap H_0^{t_y} \cap H_0^{t_x}$ whose rejection also implies the existence of a non-degenerate co-integration between Y and forcing $\mathbf{X}$. Moreover, if we fail to reject the null hypothesis of the F_{yx}-test or that of the t_y-test then, there is no evidence of co-integration for the examined sample and model specification.

Graphically, our suggested BTP is depicted in Figure 1.

Figure 1: Graphical illustration of the tests involved in the bounds testing procedure



Finally, one extra set of tests arises for the BTP in the conditional quantile environment as a natural consequence of quantiles themselves. The main advantage of quantile regressions over mean-based regressions is that the former explores the whole distribution of the dependent variable, allowing at the same time for heterogeneous, or quantile-driven covariates' effects, and does not just focus on the mean of the distribution as the latter. Therefore, it makes sense to test whether there are heterogeneous responses of the dependent variable.

As discussed in the previous subsection for Equation (3), the use of a Canay (2011) type estimator for dynamic modelling involves a new dependent variable $\widehat{Y}$, which is the deviation of Y from the consistently estimated fixed effects of Equation (2). However, the covariates $\mathbf{X}$ remain unaltered and are not transformed as in the case of the one-way or two-way DFE. This is a very attractive feature since a change of $\mathbf{X}$, stemming from either the time-series dimension or the cross-sectional dimension, can affect the new dependent variable $\widehat{Y}$.

If we consider the time-series dimension of individual *i* then, the response of $\widehat{Y}$ at period *t* to X_j, for *j* = 1 to *k*, may be time-varying as it depends on the conditional distribution of $\widehat{Y}$ with respect to the history of individual *i* given characteristics $\mathbf{x}_{-j,i,t}$, i.e. $\widehat{Y}_i \left| \mathbf{X}_{-j} = \mathbf{x}_{-j,i,t} \right.$. Namely, periods with conditional high values of $\widehat{Y}$ (that correspond to high quantiles of the conditional distribution of $\widehat{Y}$) could be more responsive than periods with conditional low values (that correspond to low quantiles of the conditional distribution of $\widehat{Y}$). This implies a "panel-invariant" heterogeneity across time, i.e. evidence for time-varying coefficients (see also Xiao, 2009) that are the same for all individuals.[15]

On the other hand, if we consider the cross-sectional dimension of individual *i* then, the response of $\widehat{Y}$ to X_j may differ across individuals as it depends on the conditional distribution of $\widehat{Y}$ with respect to the rest individuals ($-i$) given characteristics $\mathbf{x}_{-j,i,t}$, i.e. $\widehat{Y}_{-i} \left| \mathbf{X}_{-j} = \mathbf{x}_{-j,i,t} \right.$. Namely, a panel with conditional high values of $\widehat{Y}$ (that correspond to high quantiles of the conditional distribution of $\widehat{Y}$) could be more responsive than a panel with conditional low values (that correspond to low quantiles of the conditional distribution of $\widehat{Y}$). This denotes "time-invariant" heterogeneity across cross-sections, i.e. evidence for individual-specific coefficients that are the same for all periods.[16]

Hence in our setup, testing for coefficients equality across quantiles is equivalent to testing for time-varying or individual-specific coefficients (duality in interpretation). In other words, quantile-dependent coefficients imply heterogeneous coefficients across time or cross sections.

We focus on testing if there are quantile-dependent co-integrating relationships employing a series of Wald tests like BST. This is the interpecentile BTP. Specifically, we are interested

---

[15] For example, two individuals at percentile τ of their conditional distributions and at a given period share the same coefficients ("panel-invariant" part). But an individual at different periods may have different coefficients ("heterogeneity across time" part)

[16] For example one panel at percentile τ of its conditional distribution and at a given period, and one other panel at percentile τ of its conditional distribution at a different period share the same coefficients ("time-invariant" part). But at the same period two individuals may have different responses if one is at the top quantiles of its conditional distribution and the other is at the lower part ("heterogeneity across cross-sections" part).

in Cases III, V and XI (the "fully unrestricted" cases) since the intercept and the time trends are unlikely to be equal across quantiles and so, the tests could be skewed against the null hypothesis. Hence, we focus on the three quartiles (interquartile BTP), i.e 25th, 50th and 75th percentiles, and the nine deciles (interdecile BTP), i.e. 10th, 20th, … 80th, 90th percentiles.

We demonstrate the interpercentile BTP for the three quartiles and similarly, the corresponding tests of the interdecile BTP for the nice deciles are derived.

1. The Wald test, W_{yx}, for

$$\begin{aligned} H_0:\ & \varphi_y(0.25) = \varphi_y(0.50) = \varphi_y(0.75) \\ & \gamma_1(0.25) = \gamma_1(0.50) = \gamma_1(0.75) \\ & \gamma_2(0.25) = \gamma_2(0.50) = \gamma_2(0.75) \\ & \vdots \\ & \gamma_k(0.25) = \gamma_k(0.50) = \gamma_k(0.75) \end{aligned}$$

   against the alternative hypothesis that at least one equality does not hold. If we reject the null hypothesis then, we move on to the next test of the interquartile BTP. On the other hand, if we fail to reject the null then, there is no evidence for quantile-dependent co-integrating relationships. The interquartile W_{yx} test tests if the lagged dependent variables in levels at the 25th, 50th and 75th percentiles are equal, and if the lagged stochastic regressors in levels at the 25th, 50th and 75th percentiles are equal.
2. The Wald test, W_y, for

$$H_0 : \varphi_y(0.25) = \varphi_y(0.50) = \varphi_y(0.75)$$

   against the alternative that at least one equality does not hold. The null hypothesis tests whether the error-correction terms are equal across the three quartiles, i.e. whether the speed of adjustment to equilibrium is the same at the 25th, 50th and 75th percentiles. If we reject the null, then we move to the last test of the interpercentile BTP. If we fail to reject the null then, we argue there are no heterogeneous long-run equilibrium relations.
3. The Wald test, W_x, for

$$\begin{aligned} H_0:\ & \gamma_1(0.25) = \gamma_1(0.50) = \gamma_1(0.75) \\ & \gamma_2(0.25) = \gamma_2(0.50) = \gamma_2(0.75) \\ & \vdots \\ & \gamma_k(0.25) = \gamma_k(0.50) = \gamma_k(0.75) \end{aligned}$$

against the alternative that at least one equality does not hold. The interquartile W_x test tests if the lagged stochastic regressors in levels at the 25th, 50th and 75th percentiles are equal. If we cannot reject the null then, there is no evidence in favor of quantile-dependent co-integration. If we reject the null then, we conclude there is evidence of quantile-dependent co-integration.

In each test of the BTP, should we have a test statistic that lies between the simulated critical values for $I(0)$ and $I(1)$ regressors at a given size then, this is the so-called "grey area". Inference is inconclusive for that test at that size and we have to switch to a different level of statistical significance.

3. Monte Carlo simulations and results

We run stochastic simulations to obtain our test statistics with 50,000 replications using OLS regressions for the mean-oriented environment and conditional quantile regressions (CQREGs; see Koenker and Bassett, 1978, and Koenker, 2005) for the quantile setup in the spirit of BST. The regression required for the simulations for the conditional mean setup is shown in Equation (12) and for the conditional quantile environment in Equation (13):

$$\begin{aligned} &OLS: \\ &\Delta y_{i,t} = \mathbf{\Phi}' \cdot \mathbf{z}_{i,t-1} + \mathbf{a}' \cdot \mathbf{w}_{i,t} + \xi_{i,t}, \quad i = 1, 2, \ldots, N \;\; \& \;\; t = 1, 2, \ldots, T \end{aligned} \tag{12}$$

$$\begin{aligned} &CQREG: \\ &\Delta y_{i,t} = \mathbf{\Phi}'(\tau) \cdot \mathbf{z}_{i,t-1} + \mathbf{a}'(\tau) \cdot \mathbf{w}_{i,t} + \xi_{i,t}(\tau), \quad i = 1, 2, \ldots, N \;\; \& \;\; t = 1, 2, \ldots, T \end{aligned} \tag{13}$$

where, $\mathbf{z}_{i,t-1} = \left(y_{i,t-1}, \mathbf{x}_{i,t-1}{}'\right)'$, $\mathbf{x}_{i,t} = \left(x_{1,i,t}, \ldots, x_{k,i,t}\right)'$, $\mathbf{w}_{i,t} = \left[1,\, t,\, t^2\right]'$, the variables $y_{i,t}$ and $\mathbf{x}_{i,t}$ are generated from $y_{i,t} = y_{i,t-1} + \varepsilon_{1,i,t}$ and $\mathbf{x}_{i,t} = \mathbf{P}\mathbf{x}_{i,t-1} + \boldsymbol{\varepsilon}_{2,i,t}$ with $y_{i,0} = 0$ and $\mathbf{x}_{i,0} = \mathbf{0}$, $\boldsymbol{\varepsilon}_{i,t} = \left(\varepsilon_{1,i,t}, \boldsymbol{\varepsilon}_{2,i,t}{}'\right)'$ is drawn as $(k+1)$ independent standard normal variables, and τ is the user-specified percentile, where $\tau \in (0,1)$.[17] The same data generation process for y_t and $\mathbf{x}_t$ holds

[17] By definition from the minimization problem of CQREG, estimating the model at the τ^{th} quantile implies that underprediction is weighted by (100τ)% and overprediction is weighted by [100(1-τ)]%. Furthermore, the positive residuals are almost [100(1-τ)]% of the total residuals, whilst the negative residuals consist nearly (100τ)% of the

in the quantile environment too. Also, $\xi_{i,t}(\tau) = \Delta Y_{i,t} - Q_{\Delta Y_{i,t}}(\tau \mid \Theta_{t-1})$ in Equation (2), where $\Theta_{t-1} = \sigma\{\mathbf{z}_{i,t-1}, \mathbf{w}_{i,t}\}$, i.e. the sigma field generated by $\{\mathbf{z}_{i,t-1}, \mathbf{w}_{i,t}\}$.[18] *T* is the number of observations (periods) for each cross section, *N*. If $\mathbf{x}_{i,t}$ is purely *I*(1) then, $\mathbf{P} = \mathbf{I}_k$, while if $\mathbf{x}_{i,t}$ is purely *I*(0) then, $\mathbf{P} = \mathbf{0}$. The raw vectors, in Equations (1) and (2), $\mathbf{\Phi}'$ consists of $\left(\varphi_y, \boldsymbol{\varphi}'_{\mathbf{x}}\right) = \left(\varphi_y,\ \varphi_1, \ldots, \varphi_k\right)$ and $\mathbf{a}' = (c,\ a,\ b)$, where *c* is the constant term, *α* is the coefficient of the linear trend and *b* is the coefficient of the quadratic trend. Moreover, we study "large *N* large *T*" datasets. Specifically, we study combinations of *N* and *T*, where *N* takes the values of 50, 100, 200, 300, 400, 500 and 1,000, *T* equals to 50, 100 and 1,000, while *k* (row size of **x**) ranges between 0 and 13.

According to Table 1, we obtain the simulated critical values (SCVs) for the following test statistics. First, for the *F*-statistics (F_{yx} and F_x) for testing *i.* $\mathbf{\Phi}' = 0$ (F_{yx}-test), where $\mathbf{\Phi}'$ equals to $\mathbf{\Phi}' = \left(\varphi_y, \boldsymbol{\varphi}'_{\mathbf{x}}\right)$, includes the coefficients of $\mathbf{z}_{i,t-1}$ and φ_y is the coefficient of $y_{i,t-1}$, and *ii.* $\boldsymbol{\varphi}'_{\mathbf{x}} = \mathbf{0}$ (F_x-test), where $\boldsymbol{\varphi}'_{\mathbf{x}}$ includes the coefficients of $\mathbf{z}_{i,t-1}$ except for that of $y_{i,t-1}$. Second, for the *t*-statistics (t_y and t_x) for testing $\varphi_y = 0$ (t_y-test) and $\varphi_j = 0$ (t_x-test), where *j* = 1 to *k*. A set of critical values is obtained when the stochastic regressors **x** are stationary and another one when they contain a unit root. These are the bounds of the critical values from the simulations we run, and include the case when the stochastic regressors are mutually co-integrated (see PSS). The SCVs are the empirical quantiles of the computed test-statistics.[19]

---

total residuals. Also, the average prediction error is positive at low percentiles and turns to negative as we move on to high percentiles.

[18] We follow Cho et al. (2015) for the definition of the error term in the quantile framework. However, we could also follow Xiao (2009) and rewrite Equation (2) for panels as:

$Q_{\Delta y_{i,t}}(\tau) = \mathrm{A}^{-1}_{i,\xi}(\tau) + C + A(\tau)\cdot t + B(\tau)\cdot t^2 + \mathbf{\Phi}'(\tau)\cdot \mathbf{z}_{i,t-1}$, where $\mathrm{A}(.)$ is the cumulative distribution function of $\xi_{i,t}$.

[19] For the calculation of the simulated critical values we have created 3 codes in EViews (9th edition) based on 50,000 replications: one for the BTP in the OLS case, one for the BTP in the CQREG case and one for the interpercentile BTP.

To see whether SCVs are required for the tests in the BTP, we compare them with the typical or conventional critical values (CCVs). Specifically, we calculate the absolute percentage deviations (APDs) of CCV from the SCV relative to CCV for the 4 mostly-used sizes (1%, 2.5%, 5% and 10%) for all the cases (11 for the *F*-tests and 4 for the *t*-tests).

$$APD = \left| \frac{SCV - CCV}{CCV} \right| \cdot 100\% \tag{14}$$

We observe that most of the times for a given significance level or for a given case, the APDs are very close to each other. So, for simplicity and to save space, we calculate the average absolute percentage deviations (AAPDs) for all the aforementioned cases and significance levels.[20] We consider a threshold of 5% such that any AAPD, which is greater than that, suggests that we employ the SCVs for the proposed panel BTP and that standard inference could lead to misleading results. We do not use a lower or a higher threshold because a higher value (e.g. the liberal 10%) could bias our results towards the use of the CCVs, whilst a lower value (e.g. the conservative 1%) may favor our SCVs against the use of the CCVs.

We examine combinations of cross-sectional dimension, *N*, and time-series dimensions, *T*, where *N* equals to 50, 100, 200, 300, 400, 500 and 1000, while *T* equals to 50, 100 and 1000 after the estimation process. We find that when *N* is equal to or greater than 100, the CCVs should be used for the *F*-tests and the t_x-tests. However, when *N* is 50 the SCVs can be useful for the inference of these tests, as well as when $N \in [50, 100)$. Yet, regarding the t_y-test, we show that the CCVs are not appropriate and that SCVs should be employed for every pair *N* and *T* we examine. These results hold for the BTP both in the OLS framework and the 50th decile (median) in the conditional quantile environment.[21]

When we look beyond the 50th percentile, we find not so different results for the Cases III, IV, V, IX, XI for the *F*-tests (i.e. a constant term is included in the specification however, it is not included in the long-run equation), and for the Cases III, V, XI for the *t*-tests.[22] We name these cases as the “unrestricted intercept” cases. Specifically, the CCVs can be used for the t_x-

[20] Should we employ the median instead of the average, results do not change.

[21] In *Section I* and *Section II* of the Appendix, Tables I.1 to I.7 and Tables II.1 to II.8 respectively, contain the AAPDs for every (*N*, *T*) we examine for the conditional mean and for the conditional quantile environment.

[22] These AAPDs are calculated for Cases III, IV, V, IX and XI for the *F*-tests, and for Cases III, V and XI for the *t*-tests.

tests, while the SCVs are required for the t_y-tests for every combination we examine. Moreover, when T equals 50, or 100, and $N \in [50, 100)$, the SCVs should be used for the F-tests. However, when N is at least 100, or when T is 1000, CCVs can be used for the F-tests. In the rest cases, i.e. when there is no intercept in the specification (Case I for F-tests and t-tests), or it is included in the steady-state equilibrium equation (Cases II, VI, VII, VIII and X for the F-tests) we argue that there cannot be evidence of co-integration because the AAPDs skyrocket. So, when a researcher/practitioner intends to estimate CQREGs beyond the median we suggest he/she should rather focus on the "unrestricted intercept" cases, i.e. when there is an intercept in the specification and the steady-state equilibrium relationship does not contain that quantile-specific constant term.

Yet, when it comes for the interpercentile BTP we find that the SCVs are highly approximated by the CCVs. The CCVs should be employed in every combination of cross sections and time periods we examine.

When we compare the AAPDs and the SCVs we see that the AAPDs of the OLS model are larger than these of the conditional quantile models. This implies that the critical values depend on the estimation strategy. Furthermore, we see that the SCVs at the τ^{th} percentile, where $\tau \in (0,1)$, are very close to these at the 1-τ^{th} percentile.

3.1 Lag augmentation

To save space and for presentation reasons we

i. used equations (12) and (13) for the calculation of the critical values, where no lagged values of Δy, and contemporaneous or lagged values of $\Delta \mathbf{x}$, are taken into account, and
ii. calculated the average deviations of the simulated critical values from the conventional critical values [see Equation (14)].

In this way we provide notable evidence that there are several circumstances, where the conventional critical values exhibit great divergences from the simulated critical values.

The observations after the estimation of Equations (12) and (13) range from 2,500 (when both N and T equal 50) to 1,000,000 (when both N and T equal 1,000) leading to large degrees of freedom. Therefore, lag augmentation could have a weak effect, if any at all, for the generation of the simulated critical values. To alleviate any concern, we developed "lag and stochastic regressor" specific codes for greater accuracy of the bounds testing procedure, as well as for completeness of our work.[23] These codes generate strictly speaking, sample-specific critical values unless there are extra variables, which affect only the short-run or long-run path[24] of the dependent variable in the examined empirical model of researchers and practitioners.

4. Coefficients interpretation and treatment of fixed effects

In this Section we discuss coefficients interpretation that depends on the treatment of fixed (cross-sectional and/or time) effects. The interpretation of DFE coefficients is as if we estimated a time-series model since the elimination of cross-sectional fixed effects removes the cross-sectional variation. So, the temporal variation is left. That is, with this estimator we study the response of the dependent variable over time to a change of a covariate over time. If we use the two-way fixed effect estimator, or if time dummies are added too then, we show that a different interpretation holds. In particular, how the time-series variation of X affects the cross-sectional variation of Y, or how the cross-sectional variation of X (regressor) affects the time-series variation of Y (regressand). Moreover, with the one-way or two-way DFE estimator, results hold on average for every cross section and every time period. There are equal responses across individuals and time periods, and this is also the same case for the widely used dynamic panel GMM estimators.[25] Hence, the treatment of fixed effects must be taken into consideration every time when a researcher/practitioner designs his/her project goals, and this is something that Kropko and Kubinec (2018) have pointed out.

The interpretation of the (P)MG estimators is like that in time-series environment. If the goal of the researcher/practitioner is to analyze results in the cross-sectional level (e.g. like a fixed time effect estimator, or as if you run cross-sectional regressions for every period) then, these estimators are not appropriate. Also, results with MG hold on average for every time

---

[23] Specifically, three codes are available accounting for sample size, number of regressors and their lagged terms. One code for the BTP in the OLS case, one code for the BTP in the CQREG case and one code for the interpercentile BTP. In these codes we also allow for one cross section and this is an extension of the BST time-series codes. Furthermore, besides a standard normal distribution we provide the option to the user to select a student-*t* distribution with 5 degrees of freedom to account for fat tails.

[24] We discussed this issue at footnote #11.

[25] Including only fixed time effects in a model (or using a demeaning process with cross-sectional averages) cancels out the temporal variation and the interpretation of the DFE coefficients is as if we run cross-sectional regressions.

period (for both long-run and short-run coefficients), while with PMG estimator long-run results hold on average for every time period and every cross section, and short-run results hold on average for every time period. Finally, as with DFE estimators, there is one co-integrating vector from forcing variables to dependent variable and no time-varying coefficients.

The two-way fixed effect estimator (TW) for balanced panels is:[26]

$$\beta_{TW}^{*} = \frac{\sum_i \sum_t \left(X_{i,t} - \overline{X_i} - \overline{X_t} + \overline{X_{i,t}} \right)\left(Y_{i,t} - \overline{Y_i} - \overline{Y}_t + \overline{Y_{i,t}} \right)}{\sum_i \sum_t \left(X_{i,t} - \overline{X_i} \right)\left(X_{i,t} - \overline{X_t} \right)} \quad (15)$$

where, Y is the dependent variable and X the independent variable. Kropko and Kubinec (2018) show that formula (15) can be written as (see Equation 12 in their paper):

$$\beta_{TW}^{*} = \frac{\sum_i \sum_t \left(X_{i,t} - \overline{X_i} \right)\left(Y_{i,t} - \overline{Y}_t \right)}{\sum_i \sum_t \left(X_{i,t} - \overline{X_i} \right)\left(X_{i,t} - \overline{X_t} \right)} \quad (16), \quad \text{or} \quad \beta_{TW}^{*} = \frac{\sum_i \sum_t \left(X_{i,t} - \overline{X_t} \right)\left(Y_{i,t} - \overline{Y_i} \right)}{\sum_i \sum_t \left(X_{i,t} - \overline{X_i} \right)\left(X_{i,t} - \overline{X_t} \right)} \quad (17)$$

Generally, they argue that this OLS estimator is difficult to interpret since it is a complex amalgamation of variation in time and cross-sectional dimensions. We expand the two previous formulas and after a series of simple calculus we end up with a more concrete and dual interpretation. In particular, Equations (16) and (17) are, respectively, equivalent to:

$$\beta_{TW}^{*} = \frac{\sum_i \sum_t \left(X_{i,t} - \overline{X_i} \right)^2 \frac{\sum_t \left(X_{i,t} - \overline{X_i} \right)\left(Y_{i,t} - \overline{Y_t} \right)}{\sum_t \left(X_{i,t} - \overline{X_i} \right)^2}}{\sum_i \sum_t \left(X_{i,t} - \overline{X_i} \right)\left(X_{i,t} - \overline{X_t} \right)} = \frac{\sum_i \sum_t \left(X_{i,t} - \overline{X_i} \right)^2 \beta_i^{*}}{\sum_i w_i} = \frac{\sum_i V_i(X) \cdot (T-1) \cdot \beta_i^{*}}{\sum_i w_i} \quad (18),$$

$$\beta_{TW}^{*} = \frac{\sum_t \sum_i \left(X_{i,t} - \overline{X_t} \right)^2 \frac{\sum_i \left(X_{i,t} - \overline{X_t} \right)\left(Y_{i,t} - \overline{Y_i} \right)}{\sum_i \left(X_{i,t} - \overline{X_t} \right)^2}}{\sum_t \sum_i \left(X_{i,t} - \overline{X_t} \right)\left(X_{i,t} - \overline{X_i} \right)} = \frac{\sum_t \sum_i \left(X_{i,t} - \overline{X_t} \right)^2 \beta_t^{*}}{\sum_t z_i} = \frac{\sum_t V_t(X) \cdot (N-1) \cdot \beta_t^{*}}{\sum_t z_t} \quad (19)$$

Intuitively, we argue that <u>the two-way fixed effect estimator has an attractive dual interpretation</u>. First, according to formula (18), the two-way estimator is the weighted average

[26] For unbalanced panels there are matrix-algebra techniques that eliminate the fixed effects (see Davis, 2002). However, the intuition for the interpretation of the fixed effect estimator remains the same.

(part of the weights consists of the period, or within or temporal, variance) of the coefficients from time-series regressions for each cross section of the excess cross-sectional *Y* on excess historical *X*, or the "between *Y*" on "within *X*". Alternatively, the two-way estimator captures the effect of the historically adjusted *X* on the cross-sectionally adjusted *Y*. We see that there is a blend of cross-sectional and temporal variation and specifically, <u>how the between dimension responds to the within dimension</u>. Second, according to formula (19), the two-way estimator is the weighted average (part of the weights consists of the contemporaneous, or between or cross-sectional, variance) of the coefficients from cross-sectional regressions for each period of the excess historical *Y* on excess cross-sectional *X*, or the "within *Y*" on "between *X*". In other words, the two-way estimator measures how the historically adjusted *Y* responds on the cross-sectionally adjusted *X*. Again there is a mixture of cross-sectional and temporal variation and particularly, <u>how the between dimension affects the within dimension</u>.

Moving to quantile environment and using Canay's (2011) methodology to two-way fixed effects, we need to transform the dependent variable as deviations from the consistently estimated fixed effects $\widehat{Y_{i,t}} = Y_{i,t} - d_i^{est} - f_t^{est}$ from the two-way fixed effect estimator given a "large *N* and large *T*" dataset.[27]

The new dependent variable is free of temporal and contemporaneous trends. <u>It can be treated as the relative performance of *Y* with respect both to cross-sectional performance and its average, or historical, performance</u>. We name it as the model-specific "total performance" of *Y* because this transformation provides "full information" about the track of *Y*. Namely, we get the deviation of *Y* from model implied time-series and cross-sectional means. Increased values of the new dependent variable denote a great performance and decreased values a poor performance. Also, since the *X*'s are not transformed then, with this specification we study the effect of a change in *X* either in the time, or cross-sectional, dimension (dual interpretation) on the total performance of *Y*. Hence, the researcher/practitioner explores the effects of changes in *X* and he/she is not restricted to one dimension (see e.g. MG and PMG estimators, one-way fixed effect estimators). Particularly, he/she can detect the responses of *Y*, relative to the cross section and its average performance, on variations of *X*. This is a very promising tool for researchers and practitioners, as well as for policy makers since a change in *X* can truly improve

[27] In the Appendix we elaborate on two more cases of fixed effects. One case with cross-sectional fixed effects and one case with time fixed effects under the Canay (2011) approach.

Y. Furthermore, low percentiles denote overperformance of Y relative to historical and cross-sectional performance, and high percentiles, where underperformance of Y relative to historical and cross-sectional trends. Also, the responses at these quantiles can be different implying time-varying or individual-specific coefficients. Alternatively, high percentiles correspond to poor total performance of Y (total underperformance) relative to the predicted total performance of Y (overprediction), and low percentiles to high total performance of Y (or total overperformance) relative to the predicted performance of Y (underprediction).

Canay's (2011) estimator accounting for two-way fixed effects is very attractive from the point of view of policy implications since the coefficients offer flexibility in the effects from shocks in X and these effects are essentially imprinted on the model-specific performance of Y relative to historical and cross-sectional trends. So, there is full information about an upward, or downward, movement of Y.

5. Concluding remarks

In this paper we propose for large panel datasets a bounds testing procedure for the existence of a non-degenerate co-integrating relationship running from the forcing variables to the main variable of interest. We start with the conditional mean environment employing the dynamic fixed effect estimator, which can be used as a benchmark for the transition to conditional quantile setup. Previous findings that used the dynamic fixed effect estimator for studying long-run multipliers may contain degenerate cases of co-integration, which can be detected with our proposed method. Elaborating on the interpretation of fixed effects, we also show that the answers to research questions heavily depend on the treatment of fixed effects.

Next, we proceed with the conditional quantile setup employing a Canay (2011) type estimator for dynamic modelling. To the best of our knowledge, this is the first attempt for the establishment of a panel co-integrating relation in quantile framework. Fixed effects are treated as location shifters, i.e. they do not vary across quantiles, and provide attractive interpretations for the coefficients of forcing variables allowing for a dual source of heterogeneity across the time dimension or the cross-sectional dimension.

Finally, we notice that there is plenty of room for advances and future research could investigate the following directions. First, focusing on the time-series interpretation of the dependent variable, each individual could have its own time-varying coefficients, i.e. an individual at a given percentile τ faces different coefficients relative to other individuals. This feature could be incorporated by expanding the family of the (pooled) mean group estimators in the conditional quantile framework. Second, fixed effects could be modelled to differ across quantiles rather than affect all quantiles the same way, resulting to an extra source of heterogeneity in panel co-integration. Yet, an even more challenging and daunting task would be the establishment of panel co-integration at the unconditional quantile setup, but such a proposal would be a major breakthrough in the econometrics field.

Acknowledgments

Bertsatos and Sakellaris thank the Laboratory of Econometrics and the Laboratory of Economic Policy Studies (EMOP) for their hospitality when this paper was written.

References

Arellano, M., Bond, S. (1991). Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations, *The Review of Economic Studies*, 58 (2), April, pp. 277-297.

Bertsatos, G., Sakellaris, P., Tsionas, M. G., (2022). Extensions of the Pesaran, Shin and Smith (2001) bounds testing procedure, *Empirical Economics*, 62 (2), February, pp. 605-634.

Canay, I. A., 2011. A simple approach to quantile regression for panel data, *The Econometrics Journal*, 14 (3), October, pp. 368-386.

Cho, J. S., Kim, T., Shin, Y., 2015. Quantile Cointegration in the Autoregressive Distributed-Lag Modeling Framework, *Journal of Econometrics*, 188 (1), September, pp. 281-300.

Chudik, A., Mohaddes, K., Pesaran, M. H., Raissi, M., 2016. Long-Run Effects in Large Heterogeneous Panel Data Models with Cross-Sectionally Correlated Errors, *Essays in Honor of man Ullah* (Advances in Econometrics, Vol. 36), Emerald Group Publishing Limited, pp. 85-135.

Chudik, A., Pesaran, M. H., 2015. Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors, *Journal of Econometrics*, 188 (2), October, pp. 393-420.

Clements, B., Gupta, S., Khamidova, S., 2019. Is Military Spending Converging Across Countries? An Examination of Trends and Key Determinants, *International Monetary Fund*, WP 19/196, September, p. 21.

Covas, F. B., Rump, B., Zakrajsek, E., 2014. Stress-testing US bank holding companies: A dynamic panel quantile regression approach, *International Journal of Forecasting*, 30 (3), July-September, pp. 679-713.

Davis, P., 2002. Estimating multi-way error components models with unbalanced data structures, Journal of Econometrics, 106 (1), January, pp. 67-95.

Galvao, A. F. Jr., 2011. Quantile regression for dynamic panel data with fixed effects, *Journal of Econometrics*, 164 (1), September, pp. 142-157.

Greenwood-Nimmo, M., Shin, Y., 2013. Taxation and the Asymmetric Adjustment of Selected Retail Energy Prices in the UK, *Economics Letters*, 121 (3), December, pp. 411-416.

Kropko, J., Kubinec, R., 2018. Why The Two-way Fixed Effects Model is Difficult to Interpret, and What to Do About It, *SSRN Working Paper*, May, p. 27.

Koenker, R., Bassett, G. Jr., 1978. Regression Quantiles, *Econometrica*, 46 (1), January, pp. 33-50.

Koenker, R., 2005. Quantile Regression, New York: *Cambridge University Press,* 9 May, p. 366.

Judson, R. A, Owen, A. L., 1999. Estimating Dynamic Panel Data Models: A Guide for Macroeconomists. *Economic Letters* 65 (1), October, pp. 9-15.

Nickel, S., 1981. Biases in Dynamic Models with Fixed Effects. *Econometrica* 49 (6), November, pp. 1417-1426.

Pesaran, M. H., 2006. Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure. *Econometrica* 74 (4), July, pp. 967-1012.

Pesaran, M. H., Smith, R., 1995. Estimating long-run relationships from dynamic heterogeneous panels, *Journal of Econometrics*, 68 (1), pp. 79-113.

Pesaran, M. H., Shin, Y., 1999. An Autoregressive Distributed Lag Modelling Approach to Cointegration Analysis. In: S. Strom (ed.), chapter 11. *Econometrics and Economic Theory in the 20th Century: The Ragnar Frisch Centennial Symposium*, Cambridge University Press, Cambridge, pp. 371-413.

Pesaran, M. H., Shin, Y., Smith, R. P., 1999. Pooled Mean Group Estimation of Dynamic Heterogeneous Panels, *Journal of the American Statistical Association*, 94 (446), June, pp. 621-634.

Pesaran, M. H., Shin, Y., Smith R. J., 2001. Bounds Testing Approaches to the Analysis of Level Relationships, *Journal of Applied Econometrics*, 16 (1), May, pp. 289-326.

Shin, Y., Yu, B., Greenwood-Nimmo, M. J., 2014. Modelling Asymmetric Cointegration and Dynamic Multipliers in a Nonlinear ARDL Framework, in: *Horrace, William, C., Sickles, Robin, C. (Eds.), Festschrift in Honor of Peter Schmidt*. Springer Science & Business Media, New York.

Westerlund, J., 2007. Testing for Error Correction in Panel Data, Oxford Bulleting of Economics and Statistics, 69 (6), July, pp. 709-748.

Xiao, Z., 2009. Quantile Cointegrating Regression, *Journal of Econometrics*, 150 (2), June, pp. 248-260.

Appendix

I. *OLS results in detail*

Based on our simulations and the combinations (*N*, *T*) we examine, we argue that for the t_y-test of the coefficient of the error-correction term (ECT) simulated critical values (SCVs) are required, especially as the number of stochastic regressors, *k*, increases. The maximum AAPD is 67.10%. Also, when $N = 50$ for the *I*(1) case, the AAPDs for the t_x-test of the regressors and both the *F*-tests (F_{yx} and F_x) exceed the 5% threshold as *k* increases. On the other hand, when *N* is greater than or equal to 100, the t_x-tests and the *F*-tests can be evaluated with the CCVs. The AAPDs range from 0.13% to 4.56%. Moreover, the AAPDs for the *I*(1) case are increasing in *k* for given *N* and *T*.

We see that the cross-sectional dimension plays a crucial role for the determination of the SCVs. As *N* increases for fixed *k* and *T*, the divergence of CCVs from SCVs diminishes. Specifically, the AAPDs, which exceed the threshold of 5% for the *I*(1) case, become smaller and there is a tendency for convergence towards the CCVs. On the other hand, the time dimension does not contribute a lot to the determination of the SCVs. There is no clear evidence that the SCVs for the *I*(1) case are increasing or decreasing in *T* for given *k* and *N*. They are rather very close to each other.

Table I.1	***k*** **= 1**		***k*** **= 5**		***k*** **= 9**		***k*** **= 13**	
	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**
				N **= 50 & *T*** **= 50**				
F_{yx}	1.01%	2.70%	0.38%	4.67%	0.18%	**6.31%**	0.16%	**8.34%**
F_x	0.84%	2.80%	0.71%	4.78%	0.35%	**6.57%**	0.26%	**8.71%**
t_y	**8.60%**	**12.90%**	**8.55%**	**30.17%**	**8.57%**	**47.41%**	**8.58%**	**65.36%**
t_x left	0.40%	1.78%	0.30%	3.44%	0.25%	**5.56%**	0.23%	**7.65%**
t_x right	1.27%	1.86%	1.29%	3.71%	1.25%	**5.21%**	1.28%	**7.62%**
				N **= 50 & *T*** **= 100**				
F_{yx}	1.38%	2.50%	0.72%	4.32%	0.36%	**6.18%**	0.43%	**8.42%**
F_x	0.85%	2.46%	0.44%	4.72%	0.19%	**6.59%**	0.26%	**8.82%**
t_y	**8.22%**	**12.68%**	**8.16%**	**30.26%**	**8.19%**	**48.10%**	**8.15%**	**66.15%**
t_x left	0.65%	1.34%	0.85%	3.75%	0.90%	**5.37%**	0.89%	**7.58%**
t_x right	0.56%	2.40%	0.42%	4.17%	0.54%	**6.47%**	0.53%	**8.77%**
				N **= 50 & *T*** **= 1000**				
F_{yx}	1.74%	2.90%	0.87%	4.81%	0.38%	**6.91%**	0.42%	**8.90%**
F_x	0.83%	3.21%	0.40%	**5.17%**	0.24%	**7.33%**	0.23%	**9.34%**
t_y	**9.10%**	**13.56%**	**9.14%**	**30.98%**	**9.14%**	**48.86%**	**9.13%**	**67.10%**
t_x left	0.60%	1.72%	0.70%	3.79%	0.70%	**5.48%**	0.79%	**7.67%**
t_x right	0.53%	2.27%	0.57%	4.15%	0.55%	**6.18%**	0.56%	**8.49%**

Table I.2	k = 1		k = 5		k = 9		k = 13	
	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)
N = 100 & T = 50								
F_{yx}	0.43%	0.76%	0.26%	1.27%	0.30%	2.37%	0.13%	3.81%
F_x	0.39%	1.30%	0.33%	1.94%	0.33%	2.99%	0.16%	4.31%
t_y	**5.47%**	**8.14%**	**5.48%**	**19.81%**	**5.50%**	**31.34%**	**5.45%**	**43.08%**
t_x left	0.24%	0.60%	0.31%	1.39%	0.36%	2.18%	0.39%	3.29%
t_x right	0.24%	1.50%	0.23%	2.26%	0.29%	2.99%	0.21%	3.81%
N = 100 & T = 100								
F_{yx}	0.35%	1.10%	0.45%	1.75%	0.60%	2.89%	0.78%	3.86%
F_x	0.41%	1.45%	0.57%	1.89%	0.62%	3.00%	0.76%	4.03%
t_y	**5.33%**	**8.18%**	**5.29%**	**19.94%**	**5.28%**	**31.85%**	**5.32%**	**44.00%**
t_x left	0.28%	0.41%	0.18%	1.09%	0.18%	2.22%	0.16%	3.11%
t_x right	0.30%	1.13%	0.29%	1.87%	0.32%	3.21%	0.35%	4.39%
N = 100 & T = 1000								
F_{yx}	1.27%	1.94%	0.69%	2.40%	0.30%	3.22%	0.23%	4.25%
F_x	0.78%	1.95%	0.48%	2.33%	0.24%	3.41%	0.24%	4.56%
t_y	**6.67%**	**9.71%**	**6.71%**	**21.76%**	**6.68%**	**34.07%**	**6.70%**	**46.15%**
t_x left	0.41%	1.06%	0.40%	1.91%	0.44%	2.77%	0.47%	3.85%
t_x right	0.70%	1.16%	0.70%	2.00%	0.70%	3.01%	0.74%	4.19%

Table I.3	k = 1		k = 5		k = 9		k = 13	
	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)
N = 200 & T = 50								
F_{yx}	0.39%	0.81%	0.40%	1.06%	0.45%	1.63%	0.61%	2.05%
F_x	0.47%	0.51%	0.25%	1.10%	0.32%	1.63%	0.51%	2.03%
t_y	4.01%	**6.06%**	4.02%	**13.91%**	4.03%	**21.82%**	4.01%	**29.67%**
t_x left	0.21%	0.57%	0.34%	0.76%	0.29%	0.94%	0.22%	1.17%
t_x right	0.45%	0.86%	0.43%	1.07%	0.41%	1.57%	0.48%	2.11%
N = 200 & T = 100								
F_{yx}	0.51%	0.92%	0.25%	1.43%	0.28%	1.83%	0.13%	2.15%
F_x	0.85%	1.01%	0.35%	1.73%	0.25%	2.14%	0.17%	2.38%
t_y	4.59%	**6.65%**	4.58%	**14.75%**	4.59%	**22.84%**	4.61%	**31.25%**
t_x left	1.23%	0.34%	1.25%	0.83%	1.22%	1.23%	1.29%	1.56%
t_x right	0.22%	0.52%	0.22%	0.87%	0.26%	1.29%	0.27%	1.71%
N = 200 & T = 1000								
F_{yx}	0.43%	0.55%	0.31%	0.78%	0.32%	1.39%	0.25%	2.12%
F_x	0.64%	0.70%	0.36%	0.94%	0.47%	1.47%	0.29%	2.17%
t_y	4.06%	**6.21%**	4.06%	**14.29%**	4.06%	**22.68%**	4.05%	**30.96%**
t_x left	0.40%	0.47%	0.43%	0.76%	0.36%	1.30%	0.34%	1.74%
t_x right	0.70%	0.45%	0.70%	0.52%	0.68%	0.58%	0.69%	0.85%

Table I.4	k = 1		k = 5		k = 9		k = 13	
	I(0)	*I*(1)	*I*(0)	*I*(1)	*I*(0)	*I*(1)	*I*(0)	*I*(1)
***N* = 300 & *T* = 50**								
F_{yx}	0.46%	0.34%	0.67%	0.65%	0.75%	0.90%	0.36%	1.40%
F_x	0.91%	0.58%	0.93%	0.89%	0.77%	0.90%	0.48%	1.37%
t_y	3.20%	4.79%	3.23%	**11.38%**	3.19%	**17.85%**	3.28%	**24.45%**
t_x left	0.19%	0.55%	0.22%	0.51%	0.20%	0.56%	0.25%	0.97%
t_x right	0.61%	0.17%	0.62%	0.28%	0.66%	0.65%	0.70%	1.05%
***N* = 300 & *T* = 100**								
F_{yx}	0.51%	1.13%	0.27%	0.99%	0.20%	0.70%	0.21%	1.40%
F_x	0.69%	0.64%	0.27%	0.92%	0.16%	0.86%	0.29%	1.51%
t_y	3.53%	**5.31%**	3.58%	**11.64%**	3.61%	**18.01%**	3.56%	**24.63%**
t_x left	0.73%	0.24%	0.74%	0.40%	0.81%	0.79%	0.79%	1.19%
t_x right	0.36%	0.66%	0.34%	0.81%	0.32%	1.09%	0.42%	1.33%
***N* = 300 & *T* = 1000**								
F_{yx}	0.63%	0.77%	0.70%	0.40%	0.31%	0.99%	0.21%	1.56%
F_x	0.58%	0.99%	0.82%	0.45%	0.36%	1.16%	0.32%	1.58%
t_y	3.49%	**5.18%**	3.51%	**11.56%**	3.51%	**18.11%**	3.51%	**24.82%**
t_x left	0.62%	0.77%	0.65%	0.74%	0.69%	1.16%	0.68%	1.39%
t_x right	0.64%	0.13%	0.63%	0.44%	0.60%	0.81%	0.57%	1.16%

Table I.5	$k = 1$		$k = 5$		$k = 9$		$k = 13$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$N = 400$ & $T = 50$								
F_{yx}	0.38%	0.42%	0.37%	0.77%	0.23%	0.75%	0.43%	1.04%
F_x	0.51%	0.56%	0.45%	0.74%	0.18%	0.55%	0.27%	0.99%
t_y	2.38%	3.66%	2.39%	**9.24%**	2.34%	**14.75%**	2.35%	**20.39%**
t_x left	0.54%	0.48%	0.48%	0.36%	0.48%	0.40%	0.48%	0.27%
t_x right	0.25%	0.56%	0.21%	0.40%	0.21%	0.42%	0.22%	0.66%
$N = 400$ & $T = 100$								
F_{yx}	0.55%	0.60%	0.35%	0.65%	0.26%	0.96%	0.54%	1.45%
F_x	0.95%	0.75%	0.28%	0.80%	0.37%	0.89%	0.46%	1.42%
t_y	2.87%	4.25%	2.85%	**9.86%**	2.87%	**15.59%**	2.86%	**21.40%**
t_x left	0.87%	0.14%	0.89%	0.36%	0.85%	0.41%	0.87%	0.75%
t_x right	0.65%	0.51%	0.62%	0.60%	0.62%	0.69%	0.62%	0.99%
$N = 400$ & $T = 1000$								
F_{yx}	0.57%	0.75%	0.25%	1.10%	0.21%	1.17%	0.21%	1.28%
F_x	0.35%	0.54%	0.27%	0.86%	0.13%	1.25%	0.23%	1.28%
t_y	2.66%	4.28%	2.65%	**10.02%**	2.66%	**15.59%**	2.66%	**21.17%**
t_x left	0.35%	0.22%	0.36%	0.35%	0.37%	0.35%	0.39%	0.53%
t_x right	0.42%	0.40%	0.44%	0.84%	0.42%	1.08%	0.42%	1.27%

Table I.6	*k* = 1		*k* = 5		*k* = 9		*k* = 13	
	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**
***N* = 500 & *T* = 50**								
F_{yx}	0.55%	1.16%	0.25%	0.29%	0.22%	0.17%	0.19%	0.29%
F_x	0.57%	0.96%	0.32%	0.26%	0.20%	0.20%	0.21%	0.43%
t_y	2.07%	3.23%	2.03%	**8.02%**	2.02%	**12.73%**	2.04%	**17.75%**
t_x left	0.29%	0.64%	0.35%	0.48%	0.29%	0.30%	0.31%	0.31%
t_x right	0.84%	0.89%	0.88%	0.80%	0.85%	0.69%	0.83%	0.77%
***N* = 500 & *T* = 100**								
F_{yx}	0.34%	0.35%	0.32%	0.65%	0.34%	0.26%	0.14%	0.62%
F_x	0.48%	0.37%	0.29%	0.79%	0.19%	0.35%	0.16%	0.66%
t_y	2.63%	3.83%	2.65%	**8.80%**	2.71%	**13.92%**	2.66%	**18.93%**
t_x left	0.31%	0.22%	0.38%	0.29%	0.34%	0.37%	0.30%	0.75%
t_x right	0.61%	0.24%	0.62%	0.40%	0.54%	0.36%	0.51%	0.34%
***N* = 500 & *T* = 1000**								
F_{yx}	0.75%	1.04%	0.69%	0.48%	0.19%	0.75%	0.13%	0.47%
F_x	0.37%	0.37%	0.47%	0.31%	0.16%	0.66%	0.24%	0.44%
t_y	2.73%	4.06%	2.73%	**9.03%**	2.73%	**14.03%**	2.75%	**19.18%**
t_x left	0.36%	0.22%	0.36%	0.43%	0.39%	0.69%	0.38%	0.81%
t_x right	0.38%	0.13%	0.35%	0.20%	0.38%	0.32%	0.37%	0.63%

Table I.7	***k* = 1**		***k* = 5**		***k* = 9**		***k* = 13**	
	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**
***N* = 1000 & *T* = 50**								
F_{yx}	0.39%	0.49%	0.30%	0.81%	0.24%	0.71%	0.19%	0.43%
F_{x}	0.58%	0.34%	0.30%	0.85%	0.29%	0.69%	0.19%	0.43%
t_{y}	2.40%	3.29%	2.36%	**6.88%**	2.36%	**10.17%**	2.41%	**13.70%**
t_{x} **left**	0.90%	0.46%	0.87%	0.48%	0.93%	0.44%	1.00%	0.39%
t_{x} **right**	0.38%	0.72%	0.43%	0.84%	0.41%	0.83%	0.37%	1.01%
***N* = 1000 & *T* = 100**								
F_{yx}	0.33%	0.93%	0.28%	0.40%	0.25%	0.40%	0.28%	0.43%
F_{x}	0.46%	0.48%	0.25%	0.26%	0.33%	0.23%	0.33%	0.28%
t_{y}	2.81%	3.78%	2.81%	**7.07%**	2.84%	**10.71%**	2.84%	**14.19%**
t_{x} **left**	0.18%	0.31%	0.16%	0.39%	0.16%	0.27%	0.17%	0.17%
t_{x} **right**	0.18%	0.49%	0.18%	0.66%	0.19%	0.73%	0.19%	0.68%
***N* = 1000 & *T* = 1000**								
F_{yx}	0.43%	0.86%	0.41%	0.64%	0.18%	0.21%	0.15%	0.43%
F_{x}	0.58%	1.16%	0.40%	0.63%	0.23%	0.22%	0.19%	0.41%
t_{y}	1.93%	2.86%	1.94%	**6.37%**	1.95%	**9.87%**	1.96%	**13.48%**
t_{x} **left**	0.52%	0.27%	0.56%	0.49%	0.54%	0.50%	0.53%	0.59%
t_{x} **right**	0.22%	1.17%	0.21%	1.35%	0.19%	1.40%	0.22%	1.52%

Tables I.1 to I.7 correspond to the AAPDs when researchers and practitioners employ the DFE estimator in unrestricted error-correction form. We use bold and purple font when the AAPDs are greater than 5%.

II. *CQREG results in detail*

Analyzing the results of our simulations for all the examined combinations of *N* and *T* for the 50th percentile, i.e. median conditional regression, we find that the CCVs should be used for the *F*-tests and the t_x-tests when *N* is greater than or equal to 100. The AAPDs range from 0.14% to 2.87%. Only when *N* is 50 and for big values of *k* we spot AAPDs greater than 5%. On the other hand, when we look at the t_y-test of the coefficient of the ECT we find that for every pair (*N*, *T*) we examine the AAPDs are greater than 5% suggesting that the SCVs should be used rather than the CCVs. The maximum AAPD is 53.85%. Specifically, we see that for given *N* and *T*, the AAPDs for the *I*(1) case are increasing in *k*. Also for the *I*(1) case, for given *T* and *k*, as *N* increases the AAPDs fall. For fixed *N* and *k* though, there is no clear pattern for the AAPDs as *T* changes for the *I*(1) case again.

When we examine what happens beyond the 50th percentile, we also find that the SCVs should be used for the *F*-tests when *N* equals 50 and *T* is 50, or 100. However, for *T* 50, or 100, and when *N* is equal to, or greater than 100 then, CCVs should be used for the *F*-tests. Yet, when *T* is 1000 and for every *N* we examine, the CCVs can be used for the *F*-tests. Moreover, the inference for the t_y-test should be relied on the SCVs and that of the t_x-tests on the CCVs for every pair *N* and *T* we consider. However, these findings hold for Cases III, IV, V, IX, XI for the *F*-tests and Cases III, V and XI for the *t*-tests. Namely, these are the cases, where there is a quantile-dependent constant term included in the specification and it is not included in the long-run relationship. These are the "unrestricted intercept" cases as stated earlier. The rest cases, where the constant term is not included in the specification, i.e. Case I for the *t*-tests and *F*-tests, or it is included in the *F*-tests, i.e. Cases II, VI, VII, VIII and X then, we observe a different behavior of the SCVs. We name these as the "restricted cases". In these cases, we argue that there cannot be evidence in favor of co-integrating relationships since the AAPDs, corresponding to the SCVs for these cases, are so big and thus, the null hypotheses cannot be rejected in practice. Therefore, we suggest when a researcher/practitioner estimates CQREGs beyond the median, he/she should rather focus on the "unrestricted intercept" cases, i.e. when the constant term, which is not a location shifter, is not part of the steady-state equilibrium relationship.

Turning next to the interpercentile BTP, we have to note that since the simulations (50,000 repetitions for each pair *N* and *T*) for the calculation of the SCVs are computationally intensive and time consuming, we have focused on the scenario, where the time dimension is 100.

However, the main findings seem also to hold for $T = 50$ and $T = 1000$, i.e. we should use the CCVs for the interpercentile tests. When T equals to 1000, we have results for $N = 50$, 100 and 200, where the maximum AAPD is 1.10% and the minimum 0.08%, and for $T = 50$ we have results when N equals to 50, 100, 200, 300, 400 and 500, where the maximum AAPD is 2.44% and the minimum 0.10%.

We focus on the interdecile BTP, i.e. the 10^{th}, 20^{th}, … , 80^{th}, 90^{th} percentiles, and on the interquartile BTP, i.e. the 25^{th}, 50^{th} and 75^{th} percentiles, for Cases III, V and XI. We allow for a constant term in the model, and these cases involve no deterministic restrictions under the null of no co-integration when we test for the long-run relationship in levels between the dependent and independent variables, i.e. the equilibrium relationship does not include an intercept and/or trends. Furthermore, in Cases III, V and XI, the model includes an intercept, an intercept and linear trend, and an intercept, a linear and quadratic trend, respectively. Specifically we test with Wald tests first, whether the lagged dependent variable in levels is jointly equal across the quantiles and the lagged regressors in levels are jointly equal across the quantiles (W_{yx} test), second whether the lagged dependent variable in levels implies homogeneous convergence rates across quantiles (3 quartiles or 9 deciles, W_{y} test) and third, whether the lagged stochastic regressors in levels are location shifters (W_{x} test).

Regarding the interquartile Wald tests, we find that the AAPDs for the W_{yx} test are less than 1% when is N equal to or greater than 200. Yet, when N is 50, or 100, the maximum AAPD is 1.57% and the minimum 0.48%. We conclude we can use the CCVs for the W_{yx} test. Examining the W_{y} test we see that the AAPDs are less than 1% when N equals or is greater than 300. Also, when N is 50, 100 and 200, the maximum AAPD is found to be 1.96% and the minimum 0.92%. The CCVs can be employed for the W_{y} test. Finally, we argue that we can also rely on the CCVs for the W_{x} test since the AAPDs are above the threshold of 1% only when N is 50 or 100 or 400. Specifically the maximum AAPD for these scenarios are 1.34% and the minimum is 0.23%.

Regarding the interdecile Wald tests, we find that all the AAPDs are less than 1%. In particular, the maximum AAPD is 0.90% and the minimum 0.08%. So, we see that the smaller the percentile distance is the smaller the AAPDs become. For the interquartile tests the distance is 0.25, while for the interdecile tests the distance is 0.1. Therefore, our main conclusion from the interpercentile tests of the BTP is that the CCVs should be used since we find that the CCVs are approximated very well by the SCVs.

Table II.1	k = 1		k = 5		k = 9		k = 13	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
N = 50 & T = 50								
F_{yx}	1.02%	0.84%	1.59%	0.94%	1.79%	2.19%	1.89%	3.45%
F_x	1.40%	1.07%	1.65%	1.08%	1.98%	2.37%	2.06%	3.67%
t_y	**5.23%**	**8.43%**	**5.11%**	**21.62%**	**5.25%**	**35.53%**	**5.18%**	**49.46%**
t_x left	0.34%	0.42%	0.53%	1.09%	0.31%	2.14%	0.44%	3.17%
t_x right	1.28%	0.55%	1.20%	0.68%	1.46%	1.86%	1.66%	3.45%
N = 50 & T = 100								
F_{yx}	0.47%	0.59%	0.28%	1.56%	0.42%	3.26%	0.40%	**5.24%**
F_x	0.89%	0.68%	0.59%	1.81%	0.51%	3.30%	0.46%	**5.38%**
t_y	**5.97%**	**9.13%**	**6.01%**	**22.53%**	**5.82%**	**36.39%**	**5.69%**	**51.05%**
t_x left	0.54%	0.74%	0.74%	0.91%	0.75%	2.33%	0.93%	3.64%
t_x right	0.56%	0.18%	0.55%	1.21%	0.52%	2.61%	0.46%	3.58%
N = 50 & T = 1000								
F_{yx}	1.05%	1.50%	0.41%	1.98%	0.29%	3.82%	0.16%	**5.24%**
F_x	1.11%	1.17%	0.42%	2.08%	0.29%	4.14%	0.20%	**5.61%**
t_y	**6.93%**	**10.47%**	**6.84%**	**24.32%**	**6.91%**	**38.85%**	**6.76%**	**53.85%**
t_x left	0.57%	0.86%	0.46%	2.35%	0.61%	4.37%	0.50%	**5.60%**
t_x right	1.19%	0.88%	1.30%	2.52%	1.24%	3.75%	1.35%	4.59%

Table II.2	$k = 1$		$k = 5$		$k = 9$		$k = 13$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$N = 100$ & $T = 50$								
F_{yx}	0.99%	0.57%	0.94%	0.36%	0.75%	0.43%	0.75%	1.67%
F_x	1.42%	0.62%	1.15%	0.29%	0.88%	0.26%	0.78%	1.62%
t_y	4.01%	**6.17%**	4.01%	**15.20%**	4.01%	**24.41%**	3.95%	**34.02%**
t_x left	1.32%	0.48%	1.34%	0.29%	1.56%	0.57%	1.72%	1.14%
t_x right	0.24%	0.40%	0.38%	0.88%	0.24%	1.18%	0.37%	1.72%
$N = 100$ & $T = 100$								
F_{yx}	0.56%	0.55%	0.40%	0.91%	0.61%	1.30%	0.42%	1.76%
F_x	1.00%	0.64%	0.61%	1.13%	0.77%	1.32%	0.49%	1.89%
t_y	**5.03%**	**7.60%**	**5.36%**	**16.56%**	**5.13%**	**25.73%**	**5.12%**	**34.96%**
t_x left	0.29%	1.10%	0.36%	1.65%	0.49%	2.12%	0.63%	2.60%
t_x right	0.89%	0.51%	0.75%	0.42%	0.68%	0.88%	0.68%	1.54%
$N = 100$ & $T = 1000$								
F_{yx}	0.65%	0.55%	0.31%	1.58%	0.22%	2.34%	0.28%	2.53%
F_x	0.78%	0.63%	0.53%	1.85%	0.34%	2.62%	0.35%	2.65%
t_y	4.04%	**6.46%**	3.95%	**15.99%**	3.90%	**25.36%**	4.06%	**35.14%**
t_x left	0.29%	0.48%	0.29%	1.23%	0.26%	2.17%	0.35%	2.87%
t_x right	1.09%	0.38%	1.15%	0.66%	1.17%	1.25%	1.25%	1.76%

Table II.3	k = 1		k = 5		k = 9		k = 13	
	I(0)	*I*(1)	*I*(0)	*I*(1)	*I*(0)	*I*(1)	*I*(0)	*I*(1)
***N* = 200 & *T* = 50**								
F_{yx}	0.85%	1.01%	0.46%	0.24%	0.47%	0.97%	0.22%	1.13%
F_{x}	1.11%	0.67%	0.76%	0.25%	0.70%	1.13%	0.26%	1.28%
t_{y}	2.57%	4.14%	2.44%	**10.32%**	2.52%	**16.56%**	2.81%	**23.16%**
t_{x} left	1.27%	0.77%	1.31%	1.23%	1.50%	1.52%	1.36%	1.47%
t_{x} right	0.25%	0.83%	0.21%	0.50%	0.42%	0.36%	0.45%	0.20%
***N* = 200 & *T* = 100**								
F_{yx}	0.38%	0.43%	0.37%	0.56%	0.28%	0.89%	0.27%	1.42%
F_{x}	0.95%	0.65%	0.32%	0.46%	0.31%	0.70%	0.32%	1.36%
t_{y}	3.42%	**5.03%**	3.23%	**11.36%**	3.42%	**17.91%**	3.55%	**24.44%**
t_{x} left	1.07%	0.36%	0.94%	0.26%	0.85%	0.17%	0.64%	0.33%
t_{x} right	0.55%	0.47%	0.40%	0.26%	0.46%	0.63%	0.40%	0.78%
***N* = 200 & *T* = 1000**								
F_{yx}	1.09%	0.97%	0.77%	0.76%	0.49%	0.86%	0.38%	0.73%
F_{x}	0.62%	0.66%	0.57%	0.56%	0.32%	0.81%	0.28%	0.69%
t_{y}	3.61%	**5.42%**	3.61%	**11.77%**	3.62%	**18.52%**	3.58%	**25.01%**
t_{x} left	0.77%	0.55%	0.63%	0.70%	0.68%	0.72%	0.66%	0.96%
t_{x} right	0.38%	0.49%	0.31%	0.55%	0.31%	0.87%	0.40%	1.25%

Table II.4	***k* = 1**		***k* = 5**		***k* = 9**		***k* = 13**	
	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**
***N* = 300 & *T* = 50**								
F_{yx}	0.48%	0.39%	0.26%	0.36%	0.19%	0.46%	0.19%	0.55%
F_x	0.61%	0.58%	0.45%	0.42%	0.24%	0.60%	0.16%	0.67%
t_y	2.87%	4.22%	2.89%	**9.14%**	3.13%	**14.14%**	3.07%	**18.90%**
t_x left	0.39%	0.98%	0.43%	0.69%	0.51%	0.38%	0.50%	0.64%
t_x right	0.65%	0.37%	0.57%	0.64%	0.54%	0.69%	0.44%	0.74%
***N* = 300 & *T* = 100**								
F_{yx}	0.41%	0.39%	0.55%	0.32%	0.54%	0.41%	0.56%	0.72%
F_x	0.61%	1.10%	0.70%	0.42%	0.61%	0.28%	0.67%	0.55%
t_y	2.46%	3.73%	2.38%	**8.63%**	2.30%	**13.93%**	2.36%	**19.35%**
t_x left	0.68%	0.62%	0.76%	0.32%	0.84%	0.23%	0.83%	0.33%
t_x right	0.36%	0.59%	0.53%	0.62%	0.34%	0.57%	0.45%	0.49%
***N* = 300 & *T* = 1000**								
F_{yx}	0.49%	0.65%	0.38%	0.23%	0.31%	0.36%	0.29%	0.69%
F_x	0.57%	0.68%	0.54%	0.19%	0.41%	0.32%	0.31%	0.77%
t_y	3.93%	**5.43%**	3.92%	**10.62%**	3.85%	**15.63%**	3.93%	**20.68%**
t_x left	0.44%	0.34%	0.56%	0.52%	0.52%	0.68%	0.49%	0.82%
t_x right	0.40%	0.21%	0.37%	0.44%	0.42%	0.40%	0.41%	0.46%

Table II.5	***k* = 1**		***k* = 5**		***k* = 9**		***k* = 13**	
	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**
***N* = 400 & *T* = 50**								
F_{yx}	1.10%	0.66%	0.95%	0.28%	0.58%	0.21%	0.36%	0.56%
F_x	0.81%	0.44%	0.77%	0.18%	0.54%	0.35%	0.39%	0.55%
t_y	1.00%	1.88%	0.97%	**6.13%**	1.00%	**10.79%**	1.01%	**15.17%**
t_x left	0.27%	0.40%	0.23%	0.38%	0.36%	0.44%	0.31%	0.36%
t_x right	0.96%	0.62%	1.02%	0.58%	1.05%	0.73%	1.08%	0.68%
***N* = 400 & *T* = 100**								
F_{yx}	0.35%	0.47%	0.33%	0.36%	0.38%	0.53%	0.28%	0.46%
F_x	1.06%	0.49%	0.43%	0.24%	0.52%	0.50%	0.32%	0.66%
t_y	2.69%	3.86%	2.81%	**8.33%**	2.83%	**12.81%**	2.68%	**17.22%**
t_x left	1.00%	0.34%	0.89%	0.33%	0.86%	0.55%	0.72%	0.40%
t_x right	0.60%	0.38%	0.52%	0.45%	0.49%	0.49%	0.58%	0.35%
***N* = 400 & *T* = 1000**								
F_{yx}	0.74%	0.30%	0.26%	0.35%	0.35%	0.28%	0.41%	0.55%
F_x	1.34%	0.46%	0.48%	0.42%	0.52%	0.33%	0.62%	0.59%
t_y	2.70%	3.85%	2.60%	**8.27%**	2.76%	**12.79%**	2.68%	**17.48%**
t_x left	1.36%	0.25%	1.24%	0.28%	1.32%	0.36%	1.06%	0.48%
t_x right	0.54%	0.45%	0.47%	0.27%	0.42%	0.38%	0.44%	0.35%

Table II.6	*k* = 1		*k* = 5		*k* = 9		*k* = 13	
	I(0)	*I*(1)	*I*(0)	*I*(1)	*I*(0)	*I*(1)	*I*(0)	*I*(1)
N = 500 & T = 50								
F_{yx}	0.83%	0.74%	0.69%	0.45%	0.87%	0.22%	0.57%	0.16%
F_x	0.97%	1.38%	0.69%	0.67%	0.89%	0.27%	0.63%	0.24%
t_y	2.19%	3.36%	2.32%	**7.09%**	2.19%	**11.11%**	1.95%	**14.94%**
t_x left	0.52%	1.05%	0.51%	0.83%	0.56%	0.61%	0.46%	0.55%
t_x right	1.45%	1.14%	1.37%	0.72%	1.22%	0.72%	1.17%	0.75%
N = 500 & T = 100								
F_{yx}	1.31%	0.57%	0.37%	0.56%	0.50%	0.29%	0.59%	0.17%
F_x	1.61%	0.70%	0.32%	0.68%	0.41%	0.36%	0.49%	0.16%
t_y	1.30%	2.31%	1.29%	**6.29%**	1.35%	**10.31%**	1.40%	**14.25%**
t_x left	1.18%	0.81%	1.08%	0.50%	1.24%	0.49%	1.21%	0.27%
t_x right	0.75%	0.40%	0.89%	0.55%	0.72%	0.69%	0.65%	0.67%
N = 500 & T = 1000								
F_{yx}	0.41%	0.57%	0.28%	0.56%	0.25%	0.40%	0.26%	0.47%
F_x	0.47%	0.46%	0.34%	0.85%	0.14%	0.44%	0.30%	0.62%
t_y	2.05%	2.94%	2.01%	**6.90%**	1.97%	**10.84%**	1.97%	**14.94%**
t_x left	0.29%	0.32%	0.29%	0.25%	0.29%	0.25%	0.39%	0.16%
t_x right	0.46%	0.53%	0.51%	0.68%	0.43%	0.41%	0.50%	0.51%

Table II.7	***k* = 1**		***k* = 5**		***k* = 9**		***k* = 13**	
	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**
				***N* = 1000 & *T* = 50**				
F_{yx}	0.51%	0.55%	0.49%	0.39%	0.67%	0.40%	0.54%	0.22%
F_x	0.56%	0.65%	0.38%	0.32%	0.57%	0.37%	0.63%	0.20%
t_y	0.92%	1.46%	0.88%	4.33%	0.90%	**7.03%**	1.08%	**9.79%**
t_x **left**	0.28%	0.40%	0.35%	0.40%	0.28%	0.48%	0.41%	0.46%
t_x **right**	0.61%	0.25%	0.71%	0.27%	0.73%	0.34%	0.78%	0.29%
				***N* = 1000 & *T* = 100**				
F_{yx}	1.17%	0.34%	0.40%	0.44%	0.36%	0.36%	0.20%	0.29%
F_x	1.13%	0.69%	0.40%	0.75%	0.31%	0.47%	0.25%	0.43%
t_y	0.74%	1.32%	0.70%	4.02%	0.72%	**6.73%**	0.70%	**9.36%**
t_x **left**	0.99%	0.44%	1.02%	0.36%	0.96%	0.34%	0.90%	0.19%
t_x **right**	0.70%	0.55%	0.66%	0.68%	0.64%	0.61%	0.63%	0.85%
				***N* = 1000 & *T* = 1000**				
F_{yx}	0.38%	0.35%	0.29%	0.31%	0.32%	0.16%	0.41%	0.16%
F_x	1.36%	0.59%	0.34%	0.35%	0.31%	0.18%	0.51%	0.19%
t_y	1.05%	1.79%	1.14%	4.56%	1.08%	**7.41%**	1.02%	**10.21%**
t_x **left**	0.62%	0.46%	0.59%	0.56%	0.57%	0.41%	0.49%	0.28%
t_x **right**	1.47%	0.86%	1.40%	0.83%	1.42%	0.86%	1.43%	1.06%

Tables II.1 to II.7 correspond to the AAPDs when researchers and practitioners use a PQARDL at the 50th percentile. We use bold and purple font when the AAPDs are greater than 5%.

Below, Table II.8 shows the AAPDs for the interpercentile BTP. We use bold and purple font when the AAPDs are greater than 1%.

Table II.8	***k* = 1**		***k* = 13**		***k* = 1**		***k* = 13**	
	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**	***I*(0)**	***I*(1)**
				***N* = 50 & *T* = 100**				
	interquartile				***interdecile***			
W_{yx}	0.48%	**1.03%**	**1.25%**	0.64%	0.57%	0.60%	0.90%	0.85%
W_y	0.97%	**1.20%**	**1.17%**	**1.17%**	0.62%	0.47%	0.76%	0.86%
W_x	0.59%	**1.58%**	**1.31%**	0.82%	0.34%	0.70%	0.87%	0.89%
				***N* = 100 & *T* = 100**				
	interquartile				***interdecile***			
W_{yx}	**1.57%**	**1.04%**	0.83%	0.51%	0.37%	0.56%	0.50%	0.58%
W_y	**1.64%**	**1.96%**	**1.01%**	**1.63%**	0.49%	0.40%	0.53%	0.74%
W_x	**1.34%**	**1.06%**	0.98%	0.46%	0.41%	0.36%	0.50%	0.67%
				***N* = 200 & *T* = 100**				
	interquartile				***interdecile***			
W_{yx}	0.82%	0.49%	0.44%	0.22%	0.37%	0.28%	0.50%	0.28%
W_y	**1.07%**	0.92%	**1.01%**	**1.14%**	0.37%	0.29%	0.43%	0.36%
W_x	0.48%	0.32%	0.47%	0.22%	0.16%	0.61%	0.54%	0.32%
				***N* = 300 & *T* = 100**				
	interquartile				***interdecile***			
W_{yx}	0.45%	0.17%	0.28%	0.54%	0.15%	0.27%	0.17%	0.21%
W_y	0.38%	0.69%	0.69%	0.23%	0.24%	0.36%	0.23%	0.26%
W_x	0.26%	0.27%	0.33%	0.85%	0.24%	0.16%	0.20%	0.26%
				***N* = 400 & *T* = 100**				
	interquartile				***interdecile***			
W_{yx}	0.65%	0.31%	0.17%	0.25%	0.35%	0.25%	0.11%	0.09%
W_y	0.46%	0.45%	0.37%	0.60%	0.38%	0.32%	0.37%	0.24%
W_x	**1.07%**	**1.02%**	0.23%	0.38%	0.39%	0.40%	0.14%	0.08%
				***N* = 500 & *T* = 100**				
	interquartile				***interdecile***			
W_{yx}	0.69%	0.26%	0.09%	0.34%	0.19%	0.14%	0.09%	0.09%
W_y	0.57%	0.39%	0.58%	0.86%	0.32%	0.22%	0.20%	0.22%
W_x	0.31%	0.32%	0.07%	0.39%	0.33%	0.30%	0.08%	0.13%
				***N* = 1000 & *T* = 100**				
	interquartile				***interdecile***			
W_{yx}	0.58%	0.40%	0.29%	0.18%	0.31%	0.26%	0.16%	0.11%
W_y	0.43%	0.46%	0.80%	0.26%	0.73%	0.54%	0.44%	0.44%
W_x	0.37%	0.23%	0.18%	0.24%	0.41%	0.53%	0.11%	0.13%

III. *Coefficients interpretation and fixed effects*

In this Appendix section we offer two examples both in the conditional mean and conditional quantile environments demonstrating the interpretation of coefficients, which as we argue in main text, depends on the treatment of fixed (cross-sectional and/or time) effects.

Moving to quantile environment with using Canay's (2011) estimator, we first express the dependent variable as deviation from the consistently estimated fixed effects, $\widehat{Y_{i,t}} = Y_{i,t} - d_i^{est}$ (both the cross-sectional and time dimensions should be large), as Canay suggests.

The new dependent variable *Y* is purged from cross-sectional fixed effects and cross-sectional (or contemporaneous) variation. It can be seen as the excess historical performance of *Y*, or the "within *Y*", since *Y* is net off contemporaneous trends. Namely, we get the deviation of *Y* from model-implied time-series means, or the model-specific relative performance of *Y* with respect to its history. On the other hand, the *X*'s are not subject to any transformation and thus, an increase in *X* is considered, either in the cross section or in the time dimension (dual interpretation), to affect the "net off contemporaneous trends" *Y*. In other words, it can be seen how *Y* responds over time on changes in *X*. However, since we are in the quantile environment, we do not study the results on average as in the OLS case, and the covariates' effect may be quantile-dependent. Low (high) percentiles denote overperformance or underprediction (underperformance or overprediction) of "within *Y*" and indicate how the historically adjusted *Y* responds to changes in *X*.

If we adjust Canay's (2011) methodology to fixed period effects, we should first transform the dependent variable as deviations from the consistently estimated fixed time effects $\widehat{Y_{i,t}} = Y_{i,t} - d_t^{est}$ from the fixed time effect estimator given large *N* and large *T*.

Under this scenario, we assume that there are only common time-variant unobservable factors and no time-invariant factors exist. The new dependent variable is cleansed from fixed time effects and temporal (or time-series) variation. It could be seen as the excess cross-sectional performance of *Y*, or the "between *Y*", since *Y* is net off time-series trends. Namely, we get the deviation of *Y* from model-implied cross-sectional means, or the model-specific relative performance of *Y* with respect to the cross sections in sample. Moreover, the *X*'s are not subject to any transformation. Therefore, we study the effect of variation in *X* (either in the cross section or in the time dimension; duality in interpretation) on *Y* in the cross section. Low (high) percentiles correspond to overperformance or underprediction (underperformance or overprediction) of "between Y" and indicate how a change in X affects the cross-sectionally adjusted Y.

Example #1: GDP's effect on tourist arrivals

Suppose that we examine a panel with tourist arrivals, Y, at a given destination country and GDP levels, X, of origin country, employ the two-way fixed effect estimator (TW) and estimate the expected positive coefficient of GDP on tourist arrivals.

According to the first interpretation of TW [see equation (18) in main text], if a country exhibits a high GDP relative to its historical performance then, the tourist arrivals from that country to destination country, relative to the cross-sectional performance of tourist arrivals, will increase. According to the second explanation of TW [see equation (19) in main text], the richer the origin country relative to the cross section of countries in sample the higher the historically adjusted tourist arrivals from that country to destination country. So, it will be in destination's country interest that the origin country exhibits either a historical high record at GDP levels, or GDP levels that are higher than the contemporaneous GDP levels of the rest origin countries in sample.

Next, we use the Canay (2011) type approach for dynamic modelling and allow for cross-sectional fixed effects. In such a case, we can examine how GDP of origin country over time, or in the cross section, affects tourist arrivals at the destination country over time. Low percentiles correspond to overperformance (or underprediction) of "within tourist arrivals", and we can explore the response of excess historical performance of tourist arrivals on GDP variations either over time, or in the cross section. On the other hand, with low (high) percentiles we produce pessimistic (optimistic) predictions of the "within tourist arrivals" and study how they respond to GDP variations.

Now we use the Canay (2011) type approach for dynamic modelling and allow for time fixed effects. We examine how a change in GDP of an origin country affects the cross-sectional performance of tourist arrivals from that country to the destination country. Specifically, high percentiles correspond to underperformance (or overprediction) of "between tourist arrivals" from an origin country, and indicates how the GDP of that origin country affects the excess cross-sectional performance of tourist arrivals. In other words, with low (high) percentiles we produce pessimistic (optimistic) predictions of the "between tourist arrivals" and study how they respond to GDP variations.

Finally, we use the Canay (2011) type approach for dynamic modelling and allow for both cross-sectional and time fixed effects. We argue that high percentiles denote tourist arrivals' underperformance (or overprediction) relative to historical and cross-sectional trends, and indicate how a change in GDP either in time-series level, or in the cross section, affects the total performance of tourist arrivals. Also, it could be the case that overprediction, or optimistic predictions, of tourist arrivals is differently affected by GDP shocks of the origin country than underprediction, or pessimistic predictions, of tourist arrivals. For example, the coefficients at the high quantiles could be less positive than the respective ones at the low deciles.

Example #2: Sales' growth effect on firm size

Suppose that we have a panel analyzing sales' growth rates of firms and their response to shocks in firm size captured by book value of assets.

First we begin with the cross-sectional fixed effect estimator. It shows us how a firm's growth rate, relative to its average performance, responds to firm size. Alternatively, we study how a firm's excess historical sales' growth responds to size over time. If size increases comparatively to its average performance then, the coefficient tells us how a firm's sales' growth rate will move relative to its average performance. So we do not know if a relative improvement of sales' growth (with respect to its average sales' growth rates) from relative size (with respect to its average size) will bring this firm in a better position relative to its competitors and rivals. This is a very important question for short-term investors and the researcher/practitioner cannot answer it using the cross-sectional fixed effect estimator.

Second, we allow only for fixed time effects. Specifically, with the fixed time effect estimator we examine how relative size (with respect to the cross-sectional firm sizes) affect relative improvement of sales' growth (with respect to its cross-sectional performance). However, there is no information about the relative performance with respect to the firm's average/historical sales' growth rate. The fixed time effect estimator does not reveal what happens to the excess historical performance of sales' growth rate and this is a very crucial question for long-term debt and equity holders.

Finally, employing the TW estimator, we can see whether a relative improvement of sales' growth (with respect to its cross-sectional performance) stems from a relative change of size (with respect to its historical performance), or whether a relative improvement of sales' growth (with respect to its average performance) stems from a relative change of size (with respect to its cross-sectional performance). With these explanations we are able to discover whether an improvement of sales' growth rate is actually essential, or not, since we account for both model-specific time-series and cross-sectional trends of sales' growth.

The improvement of growth rate may differ when we focus on periods of overprediction and total underperformance (or periods of underprediction and total overperformance), of sales' growth rates since the coefficients are allowed to vary across quantiles of the conditional distribution of sales' growth. Namely, high quantiles imply an optimistic prediction of the sales' growth rate and low quantiles a pessimistic one. Assuming a positive coefficient of size then, the firm's balance-sheet expansion indicates that the firm's sales' growth will essentially increase. This increase is totally informational (e.g. to policy makers, consultants and investors) because both average/historical sales' growth rates and contemporaneous/cross-sectional sales' growth rates are taken into consideration in the estimation process. Also, during total underperformance, firm size may be more influential than during periods of total overperformance, i.e. the coefficient gets smaller as we move down to the quantiles implying a dual source of heterogeneity across time and cross sections.

Department of Economics
Athens University of Economics and Business

List of Recent Working Papers

2020

01-20 Incubated Early Stage Startuppers in Athens and their Initiatives during the Crisis (2010-2016), Ioannis Besis and Ioanna Sapfo Pepelasis
02-20 Alternative Monetary Policy Rules in an Imperfectly Competitive Dynamic Stochastic General Equilibrium Model, George Alogoskoufis and Stelios Giannoulakis
03-20 Quantifying Qualitative Survey Data: New Insights on the (Ir)Rationality of Firms' Forecasts, Alexandros Botsis, Christoph Görtz and Plutarchos Sakellaris
04-20 Can Country-Specific Interest Rate Factors Explain the Forward Premium Anomaly?, Efthymios Argyropoulos, Nikolaos Elias, Dimitris Smyrnakis and Elias Tzavalis
05-20 Signaling product quality through prices in an oligopoly model with costly consumer search, Minghua Cheny, Konstantinos Serfes and Eleftherios Zacharias
06-20 Thinking ahead of the next big crash: Clues from Athens in classical times, George C. Bitros
07-20 Financial crises, firm-level shocks, and large downturns: Evidence from Greece, Stelios Giannoulakis and Plutarchos Sakellaris
08-20 Notes on the Demand Side Secular Stagnation, George D. Demopoulos and Nicholas A. Yannacopoulos
09-20 Institutions and Macroeconomic Performance: Core vs Periphery Countries in the Eurozone, Tryfonas Christou, Apostolis Philippopoulos and Vanghelis Vassilatos
10-20 Macroeconomic Policy Lessons for Greece, George Economides, Dimitris Papageorgiou and Apostolis Philippopoulos
11-20 Energy Transitions and the role of the EU ETS: The case of Greece, Andriana Vlachou and Georgios Pantelias
12-20 Measuring the Default Risk of Small Business Loans: Improved Credit Risk Prediction using Deep Learning, Yiannis Dendramis, Elias Tzavalis and Aikaterini Cheimarioti
14-20 Saving Democracy by Switching to Corporate-like Governance, George C. Bitros
15-20 The impact of the lockdown on the Greek economy and the role of the Recovery Fund, George Economides, Apostolis Philippopoulos and Vanghelis Vassilatos

2021

01-21 Historical Cycles of the Economy of Modern Greece From 1821 to the Present, George Alogoskoufis
02-21 Greece Before and After the Euro: Macroeconomics, Politics and the Quest for Reforms, George Alogoskoufis
03-21 Commodity money and the price level, George C. Bitros
04-21 Destabilizing asymmetries in central banking: With some enlightenment from money in classical Athens, George C. Bitros
05-21 Exploring the Long-Term Impact of Maximum Markup Deregulation, Athanasios Dimas and Christos Genakos

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 8203303-5. E-mail: econ@aueb.gr / www.aueb.gr
76, Patission Street, Athens 104 34 Greece. Tel.: (+30) 210 8203303-5

06-21 **A regularization approach for estimation and variable selection in high dimensional regression models, Y. Dendramis, L. Giraitis, G. Kapetanios**
07-21 **Tax Competition in the Presence of Environmental Spillovers, Fabio Antoniou, Panos Hatzipanayotou, Michael S. Michael, Nikos Tsakiris**
08-21 **Firm Dynamics by Age and Size Classes and the Choice of Size Measure, Stelios Giannoulakis and Plutarchos Sakellaris**
09-21 **Measuring the Systemic Importance of Banks, Georgios Moratis, Plutarchos Sakellaris**
10-21 **Firms' Financing Dynamics Around Lumpy Capacity Adjustments, Christoph Görtz, Plutarchos Sakellaris, John D. Tsoukalas**
11-21 **On the provision of excludable public goods General taxes or user prices? George Economides and Apostolis Philippopoulos**
12-21 **Asymmetries of Financial Openness in an Optimal Growth Model, George Alogoskoufis**
13-21 **Evaluating the impact of labour market reforms in Greece during 2010-2018 , Georgios Gatopoulos, Alexandros Louka, Ioannis Polycarpou, Nikolaos Vettas**
14-21 **From the Athenian silver to the bitcoin standard: Private money in a state-enforced free banking model, George C. Bitros**
15-21 **Ordering Arbitrage Portfolios and Finding Arbitrage Opportunities. Stelios Arvanitis and Thierry Post**
16-21 **Inconsistency for the Gaussian QMLE in GARCH-type models with infinite variance, Stelios Arvanitis and Alexandros Louka**
17-21 **Competition and Pass-Through: Evidence from Isolated Markets, Christos Genakos and Mario Pagliero**
18-21 **Exploring Okun's Law Asymmetry: An Endogenous Threshold LSTR Approach, Dimitris Christopoulos, Peter McAdam and Elias Tzavalis**
19-21 **Limit Theory for Martingale Transforms with Heavy-Tailed Multiplicative Noise, Stelios Arvanitis and Alexandros Louka**
20-21 **Optimal taxation with positional considerations, Ourania Karakosta and Eleftherios Zacharias**
21-21 **The ECB's policy, the Recovery Fund and the importance of trust: The case of Greece, Vasiliki Dimakopoulou, George Economides and Apostolis Philippopoulos**

2022

01-22 **Is Ireland the most intangible intensive economy in Europe? A growth accounting perspective, Ilias Kostarakos, KieranMcQuinn and Petros Varthalitis**
02-22 **Common bank supervision and profitability convergence in the EU, Ioanna Avgeri, Yiannis Dendramis and Helen Louri**
03-22 **Missing Values in Panel Data Unit Root Tests, Yiannis Karavias, Elias Tzavalis and Haotian Zhang**
04-22 **Ordering Arbitrage Portfolios and Finding Arbitrage Opportunities, Stelios Arvanitis and Thierry Post**
05-22 **Concentration Inequalities for Kernel Density Estimators under Uniform Mixing, Stelios Arvanitis**
06-22 **Public Sector Corruption and the Valuation of Systemically Important Banks, Georgios Bertsatos, Spyros Pagratis, Plutarchos Sakellaris**
07-22 **Finance or Demand: What drives the Responses of Young and Small Firms to Financial Crises? Stelios Giannoulakis and Plutarchos Sakellaris**
08-22 **Production function estimation controlling for endogenous productivity disruptions, Plutarchos Sakellaris and Dimitris Zaverdas**

Department of Economics
Athens University of Economics and Business

The Department is the oldest Department of Economics in Greece with a pioneering role in organising postgraduate studies in Economics since 1978. Its priority has always been to bring together highly qualified academics and top quality students. Faculty members specialize in a wide range of topics in economics, with teaching and research experience in world-class universities and publications in top academic journals.

The Department constantly strives to maintain its high level of research and teaching standards. It covers a wide range of economic studies in micro-and macroeconomic analysis, banking and finance, public and monetary economics, international and rural economics, labour economics, industrial organization and strategy, economics of the environment and natural resources, economic history and relevant quantitative tools of mathematics, statistics and econometrics.

Its undergraduate program attracts high quality students who, after successful completion of their studies, have excellent prospects for employment in the private and public sector, including areas such as business, banking, finance and advisory services. Also, graduates of the program have solid foundations in economics and related tools and are regularly admitted to top graduate programs internationally. Three specializations are offered:1. Economic Theory and Policy, 2. Business Economics and Finance and 3. International and European Economics. The postgraduate programs of the Department (M.Sc and Ph.D) are highly regarded and attract a large number of quality candidates every year.

For more information:

https://www.dept.aueb.gr/en/econ/