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Abstract

An empirical investigation of whether topological properties associated with point clouds formed by cryptocurrencies' prices could contain information on (locally) explosive dynamics of the processes involved. Those dynamics are associated with financial bubbles. The PSY method as well as notions associated with the TDA like persistent simplicial homology and landscapes are employed, on a data set consisting of the time series of daily closing prices of the Bitcoin, Ethereum, Ripple, and Litecoin. The note provides some empirical evidence that TDA could be useful in detecting and time-stamping financial bubbles. If robust, such an empirical conclusion opens some interesting paths of further research.

Key words: Financial Bubbles; Mild Explocivity, PSY, Bubble Detection and Time-stamping, Topological Data Analysis, Persistent Simplicial Homology, Persistent Landscapes, EGARCH, Cryptocurrencies.

1 Introduction

The present note provides with an empirical investigation of whether topological properties associated with point clouds formed by cryptocurrencies' prices could contain information on (locally) explosive dynamics of the processes involved. Those dynamics are associated

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with financial bubbles. The interest lies on both the issue of the statistical inference on the existence and timestamping of bubbles, as well as on the empirical predictability of their formation and/or termination dates. The analysis here, uses tools from the econometrics of locally explosive auto-regressive processes as well as from the topological data analysis-hereafter TDA.

Bubbles are known to form in the price processes of financial assets due to-among others-speculative behavior, see Diba and Grossman (1988) [8]. The determination of whether they have already occurred in some historical sample could of interest to theoretical and empirical finance and economics. Early empirical detection of the formation and more importantly of the collapse of a financial bubble could also be important to theoreticians and/or practitioners, since speculative bubbles may be associated to financial crashes, with sometimes detrimental effects for the functioning of the financial markets and of the real economy.

Bubbles are partially latent. One methodology for their empirical detection and time stamping-that is also used in this note-is proposed by Phillips, Shi and Yu (2015) [18], [19], hereafter PSY. The method is based on the Philips and Magdalinos (2007) [17] argument that underlying bubble behavior is signalled via locally explosive behavior of asset prices. The PSY method relies on right-tailed Dickey-Fuller unit root tests via a recursive estimation over rolling windows of increasing sizes. It can detect the existence of more than one bubble within a sample, as opposed to the method of Phillips, Wu and Yu [20], hereafter PWY. Also, PSY can consistently timestamp bubbles associated with mildly explosive autoregressive linear dynamics, i.e. in such probabilistic environments it can estimate consistently the origination and the termination date. It thus provides an empirical account for the existence, duration and time-stamping of in-sample speculative bubbles. One question regarding PSY is whether it is robust to local expocivity that deviates from linear dynamics.

TDA constitutes a recent and fast-growing branch of Computational and Applied Mathematics relying on the field of Algebraic Topology, see for example Hatcher (2001) [13], and Munkres (2018) [14]. Its applications spread out to several fields with highly significant contributions, such as the case of detecting a new subgroup of breast cancers, see Nicolau, Levine, and Carlsson (2011) [15], or the study concerning the spread of corona virus, see Chen and Volic (2021) [7]. It extracts robust topological information from complex and high dimensional datasets with noisy elements with computational convenience. It can thus provide with useful tools for data analysis, as it employs topological and geometric techniques, see Edelsbrunner, Letscher, and Zomorodian (2002) [9] in order to observe how data can be analyzed in specific spaces, how their analysis can be quantified and how statistics and other computational methods can be used for investigating a plethora of questions and topics in different fields and subfields, such as financial time series analysis see for example Gidea (2017) [11], Gidea and Katz (2018) [12], extracting, eventually, useful and robust conclusions.

The results of Gidea and Katz (2018) [12] essentially motivate the present empirical exercise. They find that topological information associated with persistent homology could provide with empirical early warning for financial crashes. The research question here, is whether there is empirical evidence on whether TDA could either provide with tools that could help in the detection and time-stamping of speculative bubbles, and/or provide with some early indicators for their initiation and/or burst. Given the non-parametric nature of the analysis, such tools could remain robust to deviations from linear locally explosive processes. Thus, here tools related to persistent homology are employed in order to investigate whether there is empirical topological information that signals the formation or the begining or collapse of financial bubbles, already empirically timestamped via the aforementioned PSY.

The PSY method and the TDA are employed on a data set consisting of the time series of daily closing prices for the four largest cryptocurrencies by market capitalization, i.e. the Bitcoin, Ethereum, Ripple, and Litecoin. The empirical cryptocurrencies' analysis occupies now a large strand of the empirical finance literature, given research questions about their potential diversification benefits, their relations to other asset classes, etc-see Anyfantaki et al. (2021) [1] along with the references therein. Cryptocurrencies empirically show asymmetric risk profiles since their returns exhibit high volatility along with significant (and often negative) empirical skewness and kurtosis (see Table 1 of summary statistics). Their dynamic behavior is consistent with the existence of bubbles and mild explosivity (see again Anyfantaki et al. (2021) [1]), making them an ideal dynamic empirical environment for the current research question.

The results do not seem to indicate that TDA could provide early warnings. They however provide some empirical evidence that TDA could be useful in detecting and time-stamping financial bubbles. If robust, such an empirical conclusion opens some interesting paths of further research.

The remaining note is organized as follow: in the following section the PSY and TDA methodologies used, are presented. In Sections 3 we describe our data and provide with the empirical analysis, and in the final section we conclude.

2 Methodology

Initially, the PSY algorithm is applied on the dataset consisting of time series of daily logarithmic prices of the aforementioned four cryptocurrencies. This intends to detect and time-stamp in-sample mildly explosive behavior-see Phillips and Magdalinos (2007) [17]. Any such period of explosive dynamics is interpreted as a speculative bubble. As mentioned before, the algorithm provides consistent estimates regarding the existence and the location of each bubble. Specifically, the methodology relies on the augmented Dickey-Fuller (ADF) test, which tests whether the prices follow a random walk, against the alternative of mild explocivity-see Phillips and Magdalinos (2007) [17]. The following formula describes the discrete time autoregressive dynamics in the presence of M multiple bubbles within the time interval $[1, T]$:

$$p_t = (p_{t-1} + \varepsilon_t) \cdot \mathbf{1}\{t \in T_0\} + (\delta_T \cdot p_{t-1} + \varepsilon_t) \cdot \mathbf{1}\left\{t \in B_j\right\}$$

$$+ \sum_{j=1}^{M} \left(\sum_{d=\tau_f+1}^{t} \varepsilon_d + p^*_{\tau_f} \right) \cdot \mathbf{1}\left\{t \in T_j\right\},$$

where $T_0 = [1, \tau_{1s}]$ and $T_j = \left(\tau_{jf}, \tau_{(j+1)s}\right)$, for $j = 1, 2, \ldots, M-1$ and $T_M = (\tau_{\mathrm{Mf}}, T)$, $B_j = [\tau_{js}, \tau_{jf}]$, for $j = 1, 2, \ldots, M$, $p^*_{\tau f} = p_{js} + p^*_j$ with $p^*_j = O_p\left(1\right)$ and $\delta_T = 1 + c \cdot T^{-\eta}$ with $c > 0$ and $\eta \in (0, 1)$. The algorithm conducts recursive ADF tests on a rolling window, which depends on the sample size. Then, a sequence of BSADF (Backward Supremum Augmented Dickey Fuller) test statistics is derived and the elementwise comparison of this sequence with the respective bootstrap critical values provides with consistent estimators of the bubbles' origination and termination dates.

Next, tools of TDA-related to the concept of persistent homology-are implemented on the logarithmic returns of the cryptocurrencies under consideration. We work either with pairs, or with the ensemble of cryptocurrencies, thus constructing via the relevant time series, point clouds inside $\mathbb{R}^2$ or $\mathbb{R}^4$, respectively-see Carlsson (2009) [4], and (2014) [5]. Specifically, selecting a sliding window $w \ll n$, where n is the size of the sample, we construct $n - w + 1$ point clouds, each of which has the form of the $w \times 2$ and $w \times 4$ matrix $X_i := \left(x_i, x_{i+1}, \ldots, x_{w+i-1}\right), i = 1, \cdots, n - w + 1$, where x_j is the column vector of the logarithmic returns in the analysis included cryptocurrencies observed at time j in the sample.

Then, for arbitrary $\varepsilon > 0$, each point cloud is transformed into an abstract simplicial complex. Specifically, the Vietoris-Rips complex-hereafter VRC- $\mathcal{R}\left(X_i, \varepsilon\right)$ of the point cloud is considered-see for example Ch. 2 of Ghrist (2014) [10]; there a k-simplex is actually the set of $k+1$ points in the cloud, if any, defined by the property that the Euclidean distance between each pair of points in the simplex is less than or equal to ε. Allowing the radius ε to vary, for each $i = 1, \cdots, n - w + 1$, a filtration of VRCs $\left(\mathcal{R}\left(X_i, \varepsilon\right)\right)_{\varepsilon > 0}$ is obtained, since $0 < \varepsilon_1 < \varepsilon_2$ implies that $\mathcal{R}\left(X_i, \varepsilon_1\right) \subseteq \mathcal{R}\left(X_i, \varepsilon_2\right)$.

For each VRC its' k-dimensional simplicial homology group $H_k(\mathcal{R}(X_i,\varepsilon))$ is considered-see Ch. 4 of Ghrist (2014) [10]; there $H_0(\mathcal{R}(X_i,\varepsilon))$ is the group generated by independent elements that correspond to connected components, $H_1(\mathcal{R}(X_i,\varepsilon))$ is generated by independent elements that correspond to loops, $H_2(R(X_i,\varepsilon))$ is the group generated by elements that correspond to voids, and generally, $H_k(R(X_i,\varepsilon))$ is the group generated by independent elements that correspond to k-dimensional holes. The simplicial homology (with integer coefficients) of $\mathcal{R}(X_i,\varepsilon)$ is then the sequence of homology groups$(H_k(R(X_i,\varepsilon)))_{k\in\mathbb{N}}$. The filtration property of the VCRs directly implies an analogous property for each level of homology; $0<\varepsilon_1<\varepsilon_2$ implies that $H_k(R(X_i,\varepsilon_1))\subseteq H_k(R(X_i,\varepsilon_2))$. This means that for any k there exist canonical inclusion homomorhisms $H_k(R(X_i,\varepsilon_1))\hookrightarrow H_k(R(X_i,\varepsilon_2))$, which along with obvious arguments of total boundedness for the point clouds at hand, then imply that for any $\varepsilon>0$, and any homology class $\mathfrak{c}\in H_k(R(X_i,\varepsilon))$, there exist $0<\varepsilon_1\leq\varepsilon<\varepsilon_2$ such that $\forall 0<\delta<\varepsilon_1$, $\mathfrak{c}\notin H_k(R(X_i,\varepsilon_1-\delta))$, $\mathfrak{c}\hookrightarrow\mathfrak{c}_{\varepsilon^\star}\neq 0$, $\forall\varepsilon_1\leq\varepsilon^\star<\varepsilon_2$, and $\mathfrak{c}\hookrightarrow 0$, $\forall\varepsilon^\star\geq\varepsilon_2$. In simple terms, $\mathfrak{c}$ is born at the time ε_1, and dies at the time ε_2. Hereafter, $b_c:=\varepsilon_1$ and $d_c:=\varepsilon_2$ denote the birth and the death of the topological features represented by the particular homology class, and the interval $[b_c,d_c]$ the lifespan that it persists. The accounting of the lifespan of the underlying homology classes is then termed persistent homology; this is thus an algebraic method that gives us information about the lifespans of the topological features that reside in the VRCs. If a topological feature 'lives' for a large time period, then it is considered as a significant feature. On the other hand, if its 'life' is small, then it is considered as a noisy one. In the present note analysis is restricted to $H_1(R(X_i,\varepsilon))$ for each VRC and ε as in Gidea and Katz (2018) [12].

A way to represent the persistence information of the generators of the order one homology group is via persistence diagrams of order one-see for example Carlsson et al. (2005) [6]. Those are two dimensional. Their horizontal axis shows birth values while their vertical one shows the death values. The diagrams contain the birth and death values of each homology class at the group, along with information about its multiplicity. They also contain the diagonal of $\mathbb{R}^2$, the points of which are interpreted as trivial homology generators with zero life span and infinite multiplicity. Analysis of the persistence diagrams can be facilitated by endowing the set of all possible suchlike diagrams with the Wasserstein metric of degree $p>1$-see Gidea and Katz (2018) [12] and the references therein; this has the advantage of pertubation robustness, yet it does not enjoy useful analytical properties like completeness that could facilitate statistical analysis. A way to overcome this, is by embedding the aforementioned space, to some complete function space-see Bubenik (2015) [3]. As in the present work the TDA methodology of Gidea and Katz (2018) [12] is followed, we work with such an embedding producing the notion of persistence landscapes of order one: if P is a persistence diagram (of order one) and $(b, d) \in P$ off the diagonal, define the piecewise linear function:

$$f(t) := \begin{cases} t-b & , t \in [b,(b+d)/2] \\ d-t & , t \in [(b+d)/2, d] \\ 0 & , t \notin [b, d]. \end{cases}$$

Then the first persistence landscape function is obtained as a pointwise maximum w.r.t. the off-diagonal elements of the persistence diagram:

$$\lambda_1(t) := \max\left\{f(t)|(b,d) \in P,\ (b,d) \text{ non diagonal}\right\}.$$

Whenever the maximum does not exist as a real number, $\lambda_1(t)$ is set equal to zero. This is not however relevant to our analysis which concerns by construction finite point clouds.

Each persistent diagram of order one is hence represented via a bounded integrable real valued function, and even though this representation is not full-see Par. 2 of Gidea and Katz (2018) [12], the representation's image becomes a complete metric space when endowed with the standard L^p-norm, $p \geq 1$, w.r.t. the Lebesgue measure, i.e. $\|\lambda_1\|_p^p := \int_{\mathbb{R}} |\lambda_1(t)|^p dt$. Thus, the topological information present in the persistent homology (of order one) of each point cloud in the analysis, is represented by a real number-the L^p norm of the associated persistent landscape of order one. When the above is performed at each point cloud defined in the rolling window, a time series of such norms is obtained $(\|\lambda_1\|_p)_{i=1,\cdots,n-w+1}$.

The analysis is then focused on properties of this series. Gidea and Katz (2018) [12] provide empirical evidence that the growth and the moving window variability of the L^p norms of particular financial time series seem to provide some early information of financial crashes. The present paper takes a somewhat different route. As in the previous work, the analysis is restricted to $p = 1, 2$. Instead of constructing explicit moving average variability filters for the times series of the associated norms, a volatility filter is provided via the maximization of the Gaussian (Quasi-) likelihood function of the EGARCH(1,1) model-see for example Straumann (2005) [21]. Specifically, the demeaned norms time series $(j_i)_{i=1,\cdots,n-w+1}$ is assumed to be approximated by a conditionally heteroskedastic process of the form: $j_i = z_i\sqrt{h_i}$, $h_i := \exp(\omega + \alpha z_{i-1} + \gamma|z_{i-1}| + \beta \ln(h_{i-1}))$, where z_i represents a martingale difference process, and h_i is a conditional volatility process obtained as a solution of the Stochastic Recurrence Equation above. Hence j is approximated by a martingale transform process, whereas both elements of the transform, along with the (pseudo-) true values of the associated parameter $\theta := (\omega, \alpha, \gamma, \beta)$. The particular conditionally heteroskedastic model is selected due to its versatility in embodying several stylized facts of conditional second moments of financial time series-see again Straumann (2005) [21]. The methodology however allows for the consideration of other suchlike models.

Given, the sample $(j_i)_{i=1,\cdots,n-w+1}$, and an initial condition $\hat{h}_1$ for the latent volatility process, the Gaussian Log-likelihood function $\ell(\omega, \alpha, \gamma, \beta; \hat{h}_0) := \sum_{i=1}^{n-w+1} \ln(\hat{h}_i(\theta)) + \frac{j_i^2}{\hat{h}_i(\theta)}$, $\ln(\hat{h}_i(\theta)) = \omega + \alpha \frac{j_{i-1}}{\sqrt{\hat{h}_i(\theta)}} + \gamma \frac{|j_{i-1}|}{\sqrt{\hat{h}_i(\theta)}} + \ln(\hat{h}_{i-1}(\theta))$, $i > 1$, is then maximized w.r.t. θ, to obtain the Gaussian QMLE $\hat{\theta}$ for the parameter, upon which and the initial condition the volatility filter $(\hat{h}_i(\hat{\theta}))_{i=1,n-w+1}$ is then constructed. Blasques et al. (2018) [2] provide with sufficient conditions that ensure the strong approximation of the volatility filter by a process that conveys probabilistic properties of the associated time series as $n \to \infty$, even if the EGARCH model is misspecified.

Finally, and in the spirit of Gidea and Katz (2018) [12], the time-path of the filter is contrasted to the PSY time stamping of the bubbles in order to descriptively extract empirical topological information about the bubbles formation and burst.

3 Empirical Analysis

3.1 Data

The financial time series used in the analysis consist of the four cryptocurrencies' daily closing prices (in US dollars) that span the period between August 07, 2015 and August 31, 2021. They are obtained from the Bitfinex exchange market through the CoinMarketCap. In total, the dataset involves 2217 daily observations on each cryptocurrency involved. TDA is performed on the relevant daily logarithmic returns; this sample thus contains 2216 observations for each cryptocurrency. Table 1 exhibits summary statistics for the latter.

	Mean	S.D.	Skewness	Kurtosis
Bitcoin	0	0.04	-0.81	11.77
Ethereum	0	0.07	-3.23	67.17
Ripple	0	0.07	2.08	33.87
Litecoin	0	0.06	0.35	11.93

Table 1: Summary statistics of the logarithmic returns of each cryptocurrency.

As mentioned in the introduction, those empirical moments suggest a risk profile that is characterized by high volatility (compared to the mean), and even higher levels of (absolute) skewness and kurtosis. It is noted though that the possibility of local non-stationarity for the returns could imply that those empirical moments may not approximate well the analogous population moments (even if asymptotically stationary versions of the later are well defined).

3.2 Numerical Environment

The numerical aspects of the analysis are mostly performed inside the programming environment of R. Specifically the package *psymonitor* is used for the PSY method, and the package *TDA* is used for the extraction of persistence homology and landscapes. The optimization of the Gaussian log-likelihood function, and the subsequent derivation of the filter was performed in Matlab, via the optimization routine *fmincon*.

3.3 Results

The PSY algorithm is applied to the daily logarithmic closing prices of each cryptocurrency. The minimum window size equals 106 implied by the algorithm's formula $T \cdot (0.01 + {}^{1.8}/\sqrt{T})$, where T is the length of logarithmic prices, i.e. 2217. For each cryptocurrency, the sequence of BSADF test statistics is a vector of dimension 2112. Its' elements are compared to the analogous critical values obtained via bootstrap, and every exceedance is counted as a bubble date. Table 2 presents the bootstrap critical values for each cryptocurrency for the 90%, 95% and 99% significance levels.

	Bitcoin	Ethereum	Ripple	Litecoin
90%	0.26	0.28	0.27	0.34
95%	0.58	0.72	0.74	0.60
99%	1.42	1.35	1.36	1.39

Table 2: The bootstrap critical values for Bitcoin, Ethereum, Ripple and Litecoin.

The first day the BSADF test statistic lies above the corresponding 95% level critical values is counted as the origination day of a bubble. Given an origination, the consequent first day at which the statistic lies below the critical value counts as the termination date for the particular epoch of mild explocivity. Figure 1 depicts the resulting inference for the in-sample bubbles superimposed to the time series path of the logarithmic closing prices for the Bitcoin, Ethereum, Ripple and Litecoin.

[Insert Figure 1 about here]

All cryptocurrencies seem to have several instances of explosive behavior for the time period at hand, and at least one period of significant duration initiated during 2017 in all cases. Given our larger and more recent dataset, the current results refresh the analogous and similar results of Par. 3.1 of Anyfantaki, Arvanitis and Topaloglou (2021) [1].

The TDA methodology is then implemented to the time series of the logarithmic returns of the four cryptocurrencies, each of 2216 observations. The data are transformed to sequences of point clouds via a. the choice of the set of cryptocurrencies included in the analysis, and b. the choice of the sliding window. As mentioned in the methodology section, the analysis is either performed on every possible pair of cryptocurrencies, hence the resulting clouds are in each such subsets of $\mathbb{R}^2$, or performed in the totality of the assets, hence the resulting clouds are subsets of $\mathbb{R}^4$. The results of the analysis of the pairs are presented below in some indicative cases-the remaining cases are similar and available to the interested reader upon request. For b., two values for w are investigated. The first is relevant to the window used in the PSY procedure, i.e. $w = 105$. In this case 2112 point clouds are obtained. The second choice of sliding window covers the duration of the largest bubble period as estimated in-sample by PSY; $w' = 200$. Then 2017 point clouds are obtained. For each choice of the sliding window, the time series of the L^p-norms and normalized L^p-norms of the persistent landscapes of order one, for $p = 1, 2$, are then derived.

Figures 2 and 3 show the paths of the aforementioned norms for each choice of the sliding window. Superimposed to the PSY estimates of the bubble periods, it is noted that the trajectories for both the normalized norms for the point clouds obtained from the totality of the cryptocurrencies, for both choices of the sliding window-see the fourth panel in Figure 3-seem to have a neighborhood of their maxima at which the paths assume quite large and volatile values, and those neighborhoods seem to lie in close vicinity to the large duration bubbles filtered by the PSY analysis.

[Insert Figures 2-3 about here]

Figures 4-7 depict the analogous analyses for the respective filtered EGARCH volatility paths of the associated norms. Those are superimposed in each case to the PSY timestamps of the relevant bubbles. Again, especially for the clouds that correspond to the entire set of cryptocurrencies, neighborhoods of "intensive activity" around the the maxima of the EGARCH volatilities seem to depict at least the large duration PSY bubbles in several cases.

[**Insert Figures 4-7 about here**]

4 Discussion

The abovementioned results seem somewhat dissimilar to the empirical results of Gidea and Katz (2018) [12]. Specifically, the intensive activity of the trajectories of the norms and of their volatilities seem to be related to the PSY timestamps of bubbles of considerable duration, even though they do not seem to provide some early warning for the formation and/or the burst of speculative bubbles.

The results-if robust w.r.t. the choice of the associated parameters-raise however the research question of whether-complementary to methodologies like the PSY-formal inferential procedures based on persistent homology and landscapes could be designed for the detection and timestamping of time series local explosive behavior. Such procedures if possible, could be related to more general forms of locally explosive behavior compared to the linear autoregressive ones used in PSY.

The robustness of the aforementioned descriptive results can be tested via further choices of the parameters; the sliding windows that specify the sequence of available point, the sets of assets that are included in the clouds, the orders of the homology groups utilized, the choice of norms, and of the volatility models. Moreover, further models of conditional heteroskewness and/or heterokurtosis can be employed in order to assess the behavior of the aforementioned conditional higher moments of the norms of the persistent landscapes during bubbles.

More generally the specification of the probabilistic properties of the persistent landscapes that may carry the topological information of local explocivity could be of central importance to the aforementioned research question and is thereby left to future research. Suchlike research could also be benefited from research on the derivation of the limiting properties of random elements associated with persistent homology in the spirit of Owada (2018) [16]. Especially cases where the point clouds employed are associated with non stationary time series and heavy tailed marginals seem quite an exciting line of further research.

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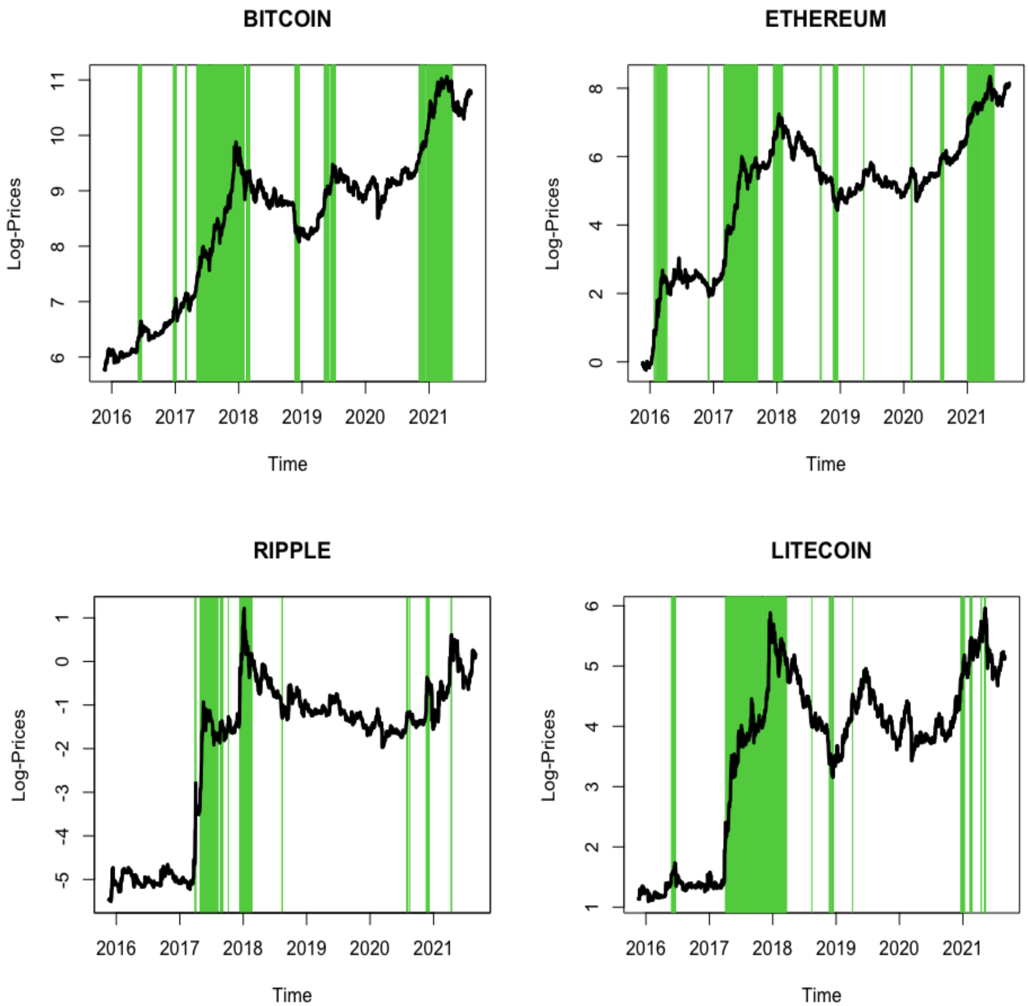

Figure 1: The green areas represent the periods of bubbles for Bitcoin (upper left), Ethereum (upper right), Ripple (lower left), and Litecoin (lower right).

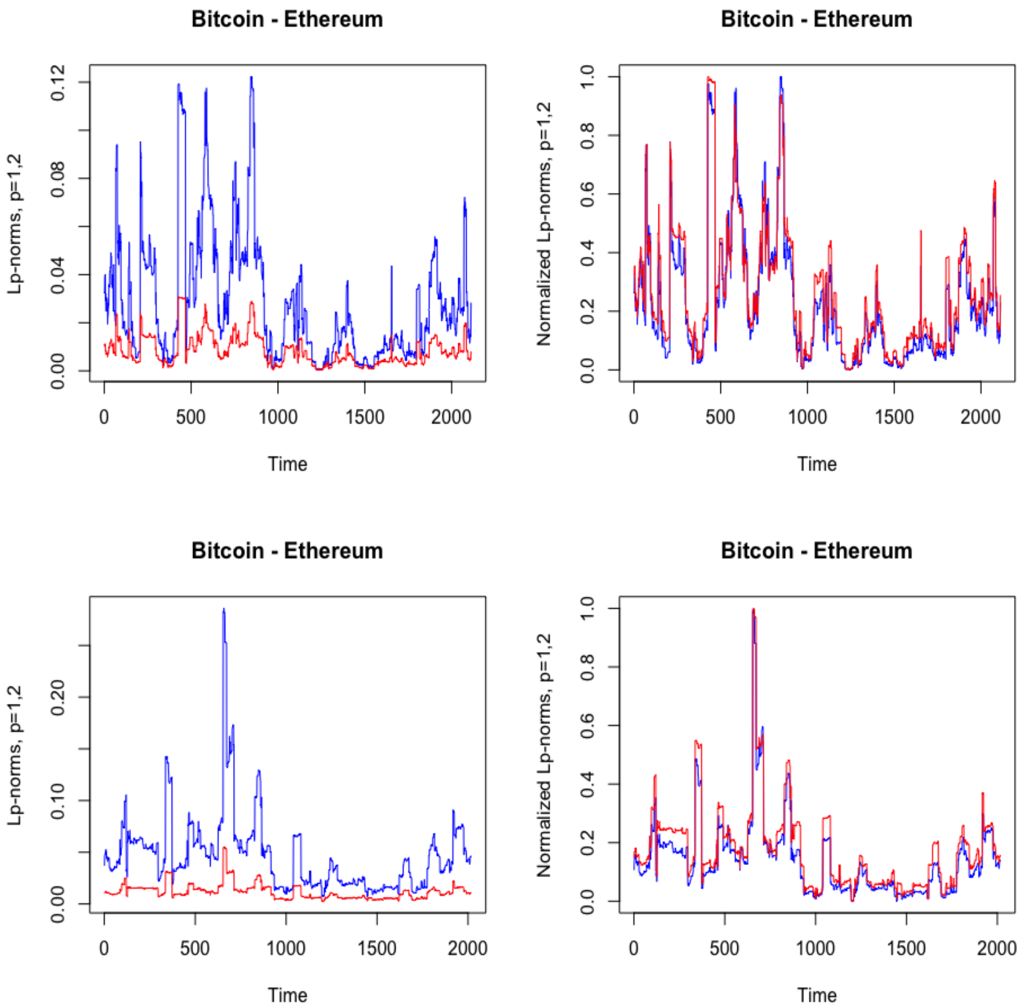

Figure 2: Time series of L^p-norms (left) and normalized L^p-norms (right), $p = 1$ (blue), $p = 2$ (red), for the pair Bitcoin - Ethereum, for $w = 105$ (up) and for $w' = 200$ (down).

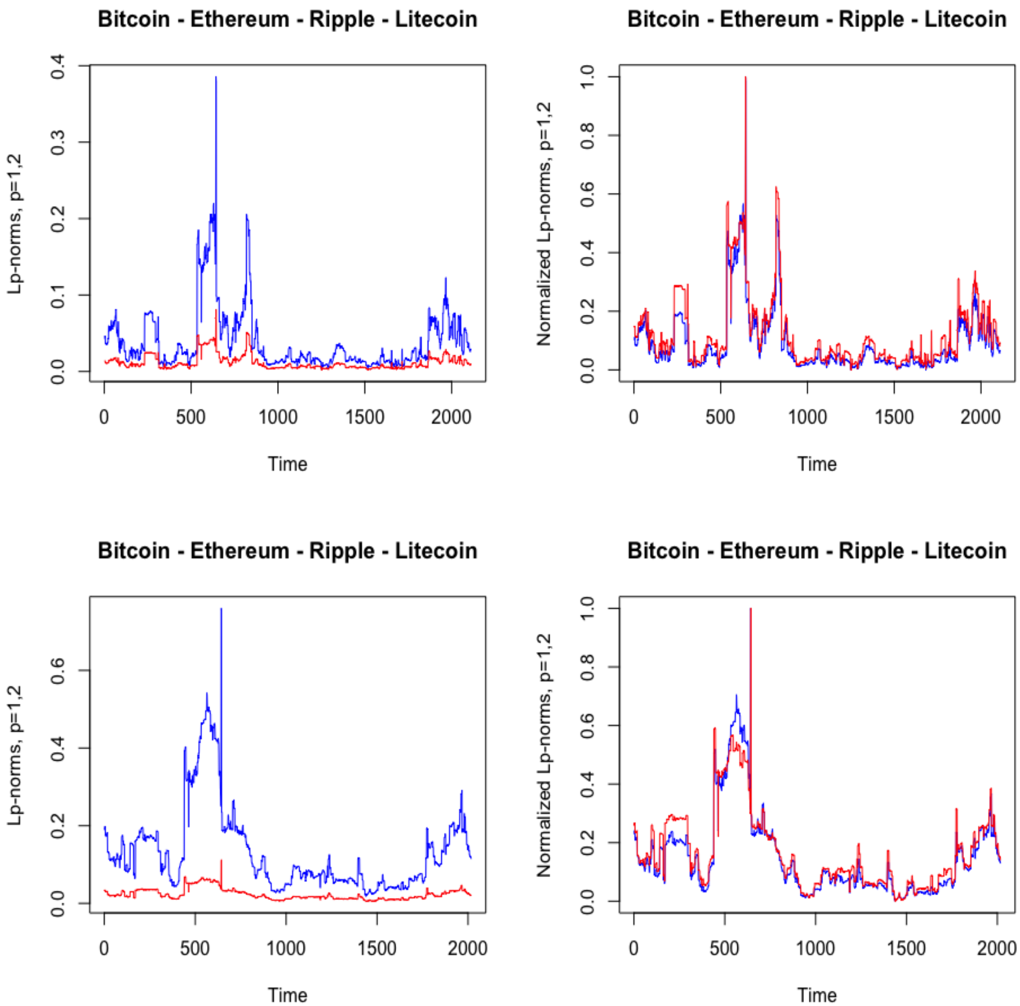

Figure 3: Time series of L^p-norms (left) and normalized L^p-norms (right), $p = 1$ (blue), $p = 2$ (red), for the ensemble of cryptocurrencies, for $w = 105$ (up) and for $w' = 200$ (down).

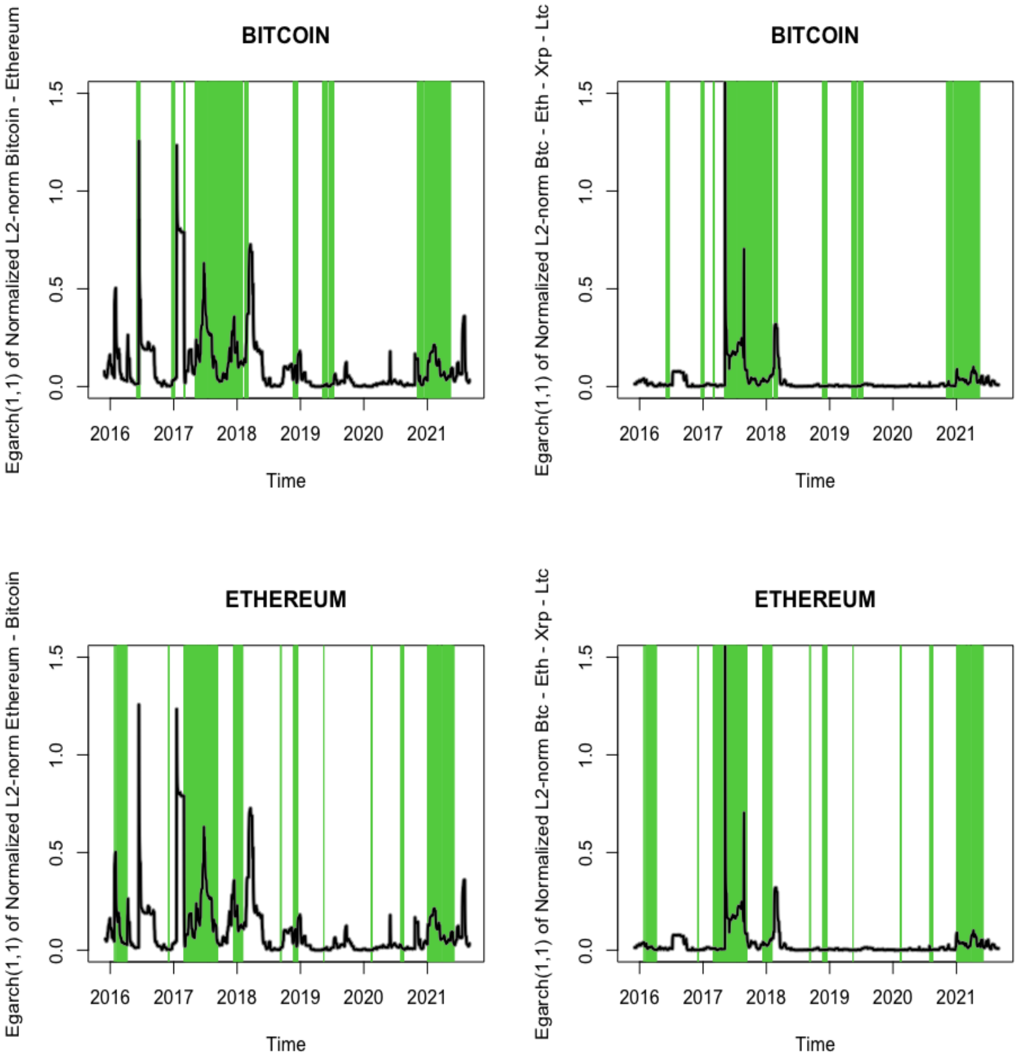

Figure 4: Time series of EGARCH(1, 1) filtered volatility of the normalized L^2-norms, for the pair Bitcoin - Ethereum and the ensemble of cryptocurrencies, and the periods of bubbles of Bitcoin and Ethereum ($w = 105$).

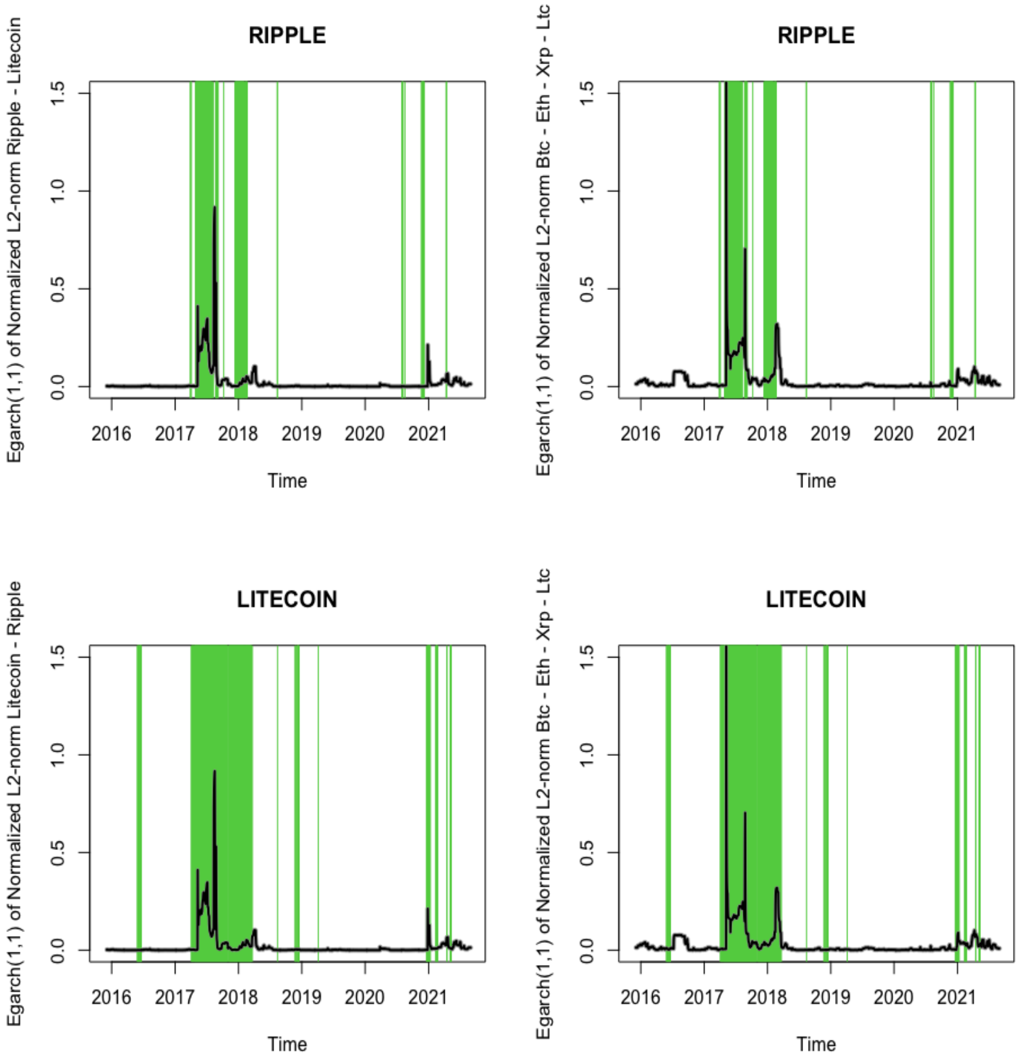

Figure 5: Time series of EGARCH(1, 1) filtered volatility of the normalized L^2-norms, for the pair Ripple - Litecoin and the ensemble of cryptocurrencies, and the periods of bubbles of Ripple and Litecoin ($w = 105$).

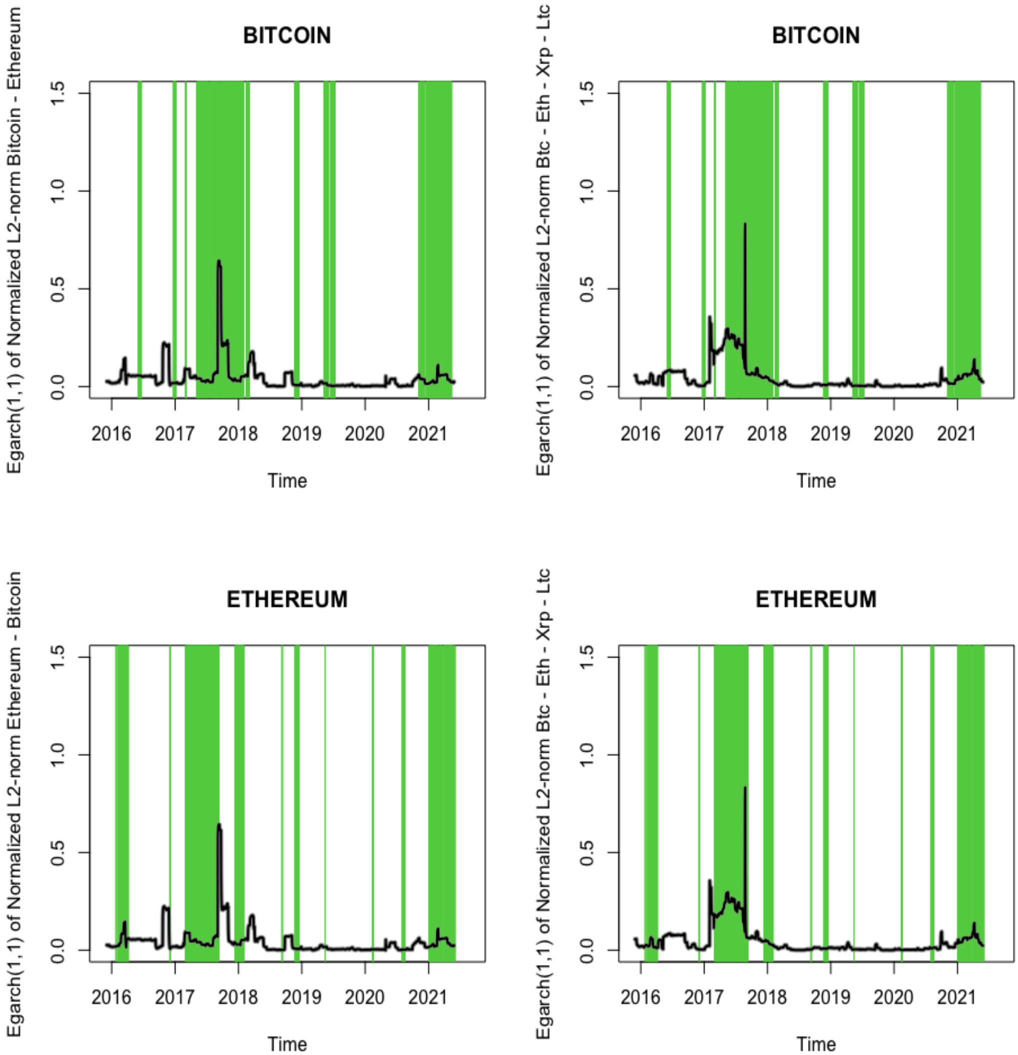

Figure 6: Time series of EGARCH(1, 1) filtered volatility of the normalized L^2-norms, for the pair Bitcoin - Ethereum and the ensemble of cryptocurrencies, and the periods of bubbles of Bitcoin and Ethereum ($w' = 200$).

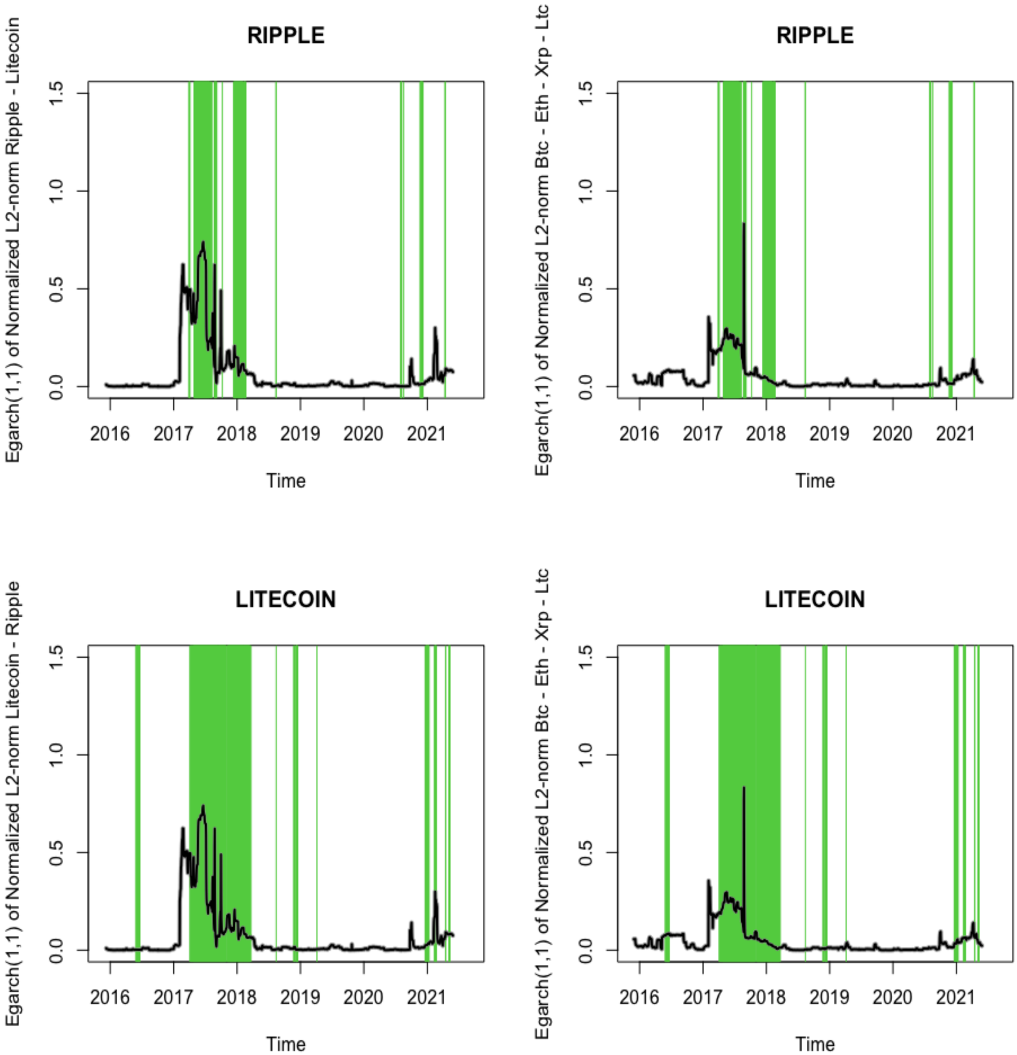

Figure 7: Time series of EGARCH(1, 1) filtered volatility of the normalized L^2-norms, for the pair Ripple - Litecoin and the ensemble of cryptocurrencies, and the periods of bubbles of Ripple and Litecoin ($w' = 200$).

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