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From debt arithmetic to fiscal sustainability and fiscal rules: Taking stock*

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Abstract

We start by clarifying the role of the interest rate-growth rate differential in debt arithmetic with a numerical example for the Greek economy. In turn, building upon this standard and popular approach to fiscal sustainability, which is based on the intertemporal government budget constraint only, we make a number of methodological points that question its quantitative usefulness. A structural approach is thus needed and this reveals the necessity of fiscal rules according to which fiscal instruments react to public debt imbalances. This naturally enables us to evaluate the EU’s fiscal rules and to suggest simple and implementable alternatives. Throughout, we confront our arguments with data from the Euro Area.

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1 Introduction

Fiscal sustainability is a necessary condition for macroeconomic stability which, in turn, is a prerequisite for economic growth and the funding of social policies. But how can we judge fiscal sustainability? The popular approach, at least in policy reports and public debates, is based on the inter-temporal government budget constraint (IGBC); see, for example, the European Commission’s long-term fiscal sustainability indicators S1 and S2, as well as its recommendations for the public finances of EU countries (European Commission (2023a)). This means that one calculates the primary fiscal balance required to hit a target value for the public debt to GDP ratio after a certain number of time periods. As is known, in this kind of analysis which is also known as public debt arithmetic, the required primary fiscal balance and hence fiscal sustainability depend critically on the comparison between the real interest rate on sovereign bonds and the economy’s real growth rate, both of which are assumed to be exogenous variables.

We will therefore start with this standard approach to fiscal sustainability. Using data, for example, from Greece which is the country with the highest public debt to GDP ratio in the EU, we will provide a numerical solution that illustrates the importance of the interest rate-growth rate differential. In particular, we will show that when the growth rate is assumed to exceed the interest rate by one percentage point, the public debt ratio can be brought down from its current level of 171% to 100% in say 35 years from now without any extra fiscal effort, simply by keeping the primary fiscal balance almost balanced on average over time. By contrast, when the interest rate is assumed to exceed the growth rate by one percentage point, other things equal, a primary fiscal surplus of around 3.4% of GDP is required on average in each year for the same goal. A 3.4% primary surplus is rather demanding! This kind of arithmetic also illustrates the importance of public policies that enhance growth and trust for fiscal sustainability (a loss of trust is immediately reflected in an increase in sovereign interest rates).

In turn, building upon the above, we will make four points.

First, we argue that, for countries with official obligations to EU institutions, things are more demanding. In Greece, for example, standard calculations, like the above, do not take into account the country’s obligations to EU institutions – specifically, that the outstanding debt to ESM, EFSF, etc, from the three official fiscal bailouts of the previous decade amounting to around 250 billion euros, has been agreed to be repaid by around 2070. Once this is taken into account, the required fiscal surplus is higher than that implied by the standard calculations.

Second, one should be careful how to read the ranking of countries according to the European Commission’s long-term fiscal sustainability indicators S1 and S2, because they are sensitive to the assumption that the fiscal situation in the departure year will not change over time. This can contribute to explaining why, for example, Greece is ranked as a low fiscal risk country in the long run, while, Germany is ranked as a medium fiscal risk country in the European
Commission’s (2023, chapter 3) report.

Third, and more importantly, we stress that debt arithmetic analyses like the above, although conceptually and educationally useful, are less reliable quantitatively because they suffer from the Lucas critique (see also D’Erasmo et al. (2016)). This is because the real interest rate on sovereign bonds, the economy’s real growth rate, as well as most items incorporated in the primary fiscal balance (tax revenues for instance), are all endogenous variables depending on a number of factors and policies including the level of public debt and fiscal policy reactions to it. Hence, a structural approach is needed and this rationalizes the use of dynamic general (dis)equilibrium macroeconomic models. Then, a common implication of such models is that dynamic stability and hence fiscal sustainability require debt-contingent fiscal rules according to which fiscal instruments (like public spending items and tax rates) react to the gap between the outstanding public debt and a policy target value; simply, this is a necessary condition to get a solution.

Fourth, the necessity of debt-contingent fiscal reaction functions allows us to contribute to the current debate on the EU’s fiscal rules. After evaluating the EU’s recently proposed expenditure rule (see European Commission (2023b)), we suggest that, in addition to the public debt gap, fiscal instruments should be contingent on the interest rate-growth rate differential and the gap between the primary fiscal balance from its medium-term objective (these are simply the variables that jointly shape the public debt dynamics).

Throughout the note, we confront our arguments with data from the Euro Area (EA).

Section 2 presents the government budget constraint and solves it depending on the interest rate-growth rate differential. Section 3 provides numerical solutions, or debt arithmetic, using Greek data. Section 4 presents data on real interest rates and growth rates in the EA countries. Section 5 evaluates some of the EC’s fiscal sustainability criteria that rely on the government budget constraint. Section 6 adds debt-based rules to debt arithmetic. Section 7 argues for the use of dynamic general (dis)equilibrium models. Section 8 presents evidence of fiscal reaction to public debt in EA countries. Section 9 evaluates the EU’s fiscal rules and suggests alternatives. Section 10 concludes.

2 Fiscal sustainability through the lens of the government budget constraint

Most policy reports (see e.g. European Commission (2023a) for a recent example) analyse the issue of fiscal sustainability through the lens of the government budget constraint. We therefore start by presenting the government budget constraint, writing it in terms of GDP and then solving the resulting difference equation for public debt as proportion of GDP in two different ways depending on the interest rate-growth rate differential. This is pretty standard macroeconomics but it will be necessary for the debt arithmetic that follows in the next
section. It will also allow us to evaluate some popular indicators of fiscal sustainability as well as to understand what type of policies can reduce the debt ratio.

2.1 Government budget constraint

The within-period government budget constraint is (see e.g. Walsh (2017, chapter 4) and Buiter (2021, chapters 1 and 2) for details):

\[ G_t + i_t B_{t-1} = (B_t - B_{t-1}) + T_t + N_t \]  

where \( G_t \) is total government spending except interest payments, \( T_t \) is total tax revenues, \( B_t \) is the end-of-period total public debt, \( N_t \) is transfers from the CB to its government (if any)\(^1\) and \( i_t \) is the nominal interest rate on outstanding one-period bonds, \( B_{t-1} \). All variables are expressed in nominal terms. Notice that for simplicity we assume that bonds have one period maturity.

If we express nominal quantities as shares of nominal GDP, we have:

\[ \frac{B_t}{Y_t} = R_t \frac{B_{t-1}}{Y_{t-1}} + \left( \frac{G_t}{Y_t} - \frac{T_t}{Y_t} - \frac{N_t}{Y_t} \right) \]

where \( R_t \equiv \frac{1 + i_t}{(1 + \pi_t)(1 + \gamma_t)} \), \( \pi_t \equiv \frac{p_t - p_{t-1}}{p_{t-1}} \) is the inflation rate and \( \gamma_t \equiv \frac{y_t - y_{t-1}}{y_{t-1}} \) is the growth rate of real GDP. Notice that, approximately, \( R_t \approx \frac{1 + i_t}{(1 + \pi_t)(1 + \gamma_t)} \equiv 1 + i_t - \pi_t - \gamma_t \), where \( i_t - \pi_t \) is the usual definition for the real interest rate.

In a shorter notation, we have:

\[ b_t \equiv R_t b_{t-1} + d_t \equiv R_t b_{t-1} - s_t \]  

where \( b_t \equiv \frac{B_t}{Y_t} \) is the end-of-period public debt to GDP ratio, \( b_{t-1} \equiv \frac{B_{t-1}}{Y_{t-1}} \) is the beginning-of-period public debt to GDP ratio and \( d_t \equiv \left( \frac{G_t}{Y_t} - \frac{T_t}{Y_t} - \frac{N_t}{Y_t} \right) \) is the primary fiscal deficit as share of GDP or, symmetrically, \( s_t \equiv -d_t \equiv \left( \frac{G_t}{Y_t} + \frac{N_t}{Y_t} - \frac{T_t}{Y_t} \right) \) is the primary fiscal surplus.

Therefore, as equation (2) shows, the drivers of the debt ratio over time are the interest rate-growth rate differential, as captured here by \( R_t \), and the primary balance, \( d_t \) (or \( s_t \)). Technically speaking to the extent that \( R_t \) and \( d_t \) (or \( s_t \)) are assumed to be exogenous and in particular independent of public debt (see below for this assumption), this is a first-order linear difference equation in the path of \( b_t \), whose stability, and hence the conditions for fiscal sustainability, depend crucially on the value of \( R_t \). Typically, we distinguish two cases, \( R_t < 1 \) and \( R_t > 1 \) (see e.g. Blanchard et al (1990) and Wickens (2008, chapter 5)).

\(^1\)This transfer consists of seigniorage revenue and interest income from the central bank’s net assets. See Buiter (2021) for details and numbers; seigniorage revenue can be large at the effective lower bound, i.e. in liquidity traps, but very small in more normal periods.
2.2 Favorable interest rate-growth rate differential

If $R_t < 1$, namely if the interest rate-growth rate differential is favorable, equation (2) is stable. A model that satisfies this condition is said to be stationary. In this case, since the cost of inherited debt steadily declines over time, the government does not need to generate primary surpluses to achieve sustainability. To the extent that the sequence $\{d_t\}$ is bounded, the government can simply roll over its debt, issuing new debt to pay for the interest, without the need to cut spending or raise taxes in the future for the debt to GDP ratio to remain finite (see e.g. Blanchard et al (1990), Wickens (2008, chapter 5.4) and Blanchard (2019)). Note however that, in practice, even if the government can run permanent primary deficits and these deficits can lead to a finite public debt to GDP ratio, there might be fears of default if this finite ratio is considered to be “too” high. But, even without default fears, a high debt ratio limits the room for fiscal maneuver and support of the economy in case of downturns in the future. These can provide extra arguments for upper limits on the debt-to-GDP ratio like those of the Stability and Growth Pact in the EU even when the differential is favorable (see Wickens (2008, chapter 5.4.2) and Blanchard (2019) for the economic intuition behind the stable case).

Since $R_t < 1$, equation (2) can be solved backward. By repeated substitutions, we get (here, for simplicity, we assume that the exogenous $R_t$ and $d_t$ remain constant over time):

\[ b_t \equiv d \sum_{i=0}^{t-1} R^i + R^t b_0 \]

or in a simpler way:

\[ b_t \equiv d \left( \frac{1 - R^t}{1 - R} \right) + R^t b_0 \]

where $b_0$ is the initial debt to GDP ratio.

Summing up, when the interest rate-growth rate differential is favorable and the path of primary fiscal deficits is simply bounded, the debt to GDP ratio will remain finite and hence the fiscal stance is sustainable.

Notice that the above relates to the so-called "revised S1 indicator" used for long-term fiscal sustainability analysis by the EC (see European Commission (2023, chapter 3 and Annex A5.3)). In particular, our equations (3)-(4) can be compared to equations (4)-(5) in the EC’s Annex A5.3. Details on S1 are provided in Section 5 below.

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As Jones (2008, chapter 13) points out, there is no magic level of the debt to GDP ratio that triggers such a calamity. The level depends on a number of economic and political fundamentals.

See e.g. the early papers by Buiter and Kletzer (1992) and Friedman (1992) for the consequences of fiscal deficits and public debt.

We work as in e.g. Sargent (1987, chapter IX) and Azariadis (1993, chapter 2). Appendix A at the end of this note provides algebraic details for the more general case where $d_t$ changes over time.
2.3 Unfavorable interest rate-growth rate differential

If $R_t > 1$, the public debt ratio is not stationary meaning that, given $d_t$, its path is explosive over time. In this case, there are two ways to restore stability and hence sustainability (see e.g. D’Erasmo et al (2016)).

First, we can introduce a feedback fiscal policy rule according to which a fiscal instrument reacts to outstanding public debt so that the “effective” coefficient on outstanding debt becomes less than one; this is analysed in detail below.

Second, since $R_t > 1$, equation (2) can be solved forward. By repeated substitutions, we get (here, as above, for simplicity, we assume that $R_t$ and $s_t = -d_t$ remain constant over time):

$$b_{t-1} = \frac{s}{R} \sum_{i=0}^{T} \left( \frac{1}{R} \right)^i + \left( \frac{1}{R} \right)^{T+1} b_{t+T}$$  \hspace{1cm} (5)

or in a simpler way:

$$b_{t-1} = \frac{s}{R} \left[ 1 - \left( \frac{1}{R} \right)^{T+1} \right] + \left( \frac{1}{R} \right)^{T+1} b_{t+T}$$  \hspace{1cm} (6)

so that current liabilities are equal to the present discounted value (PDV) of expected future fiscal surpluses plus the discounted value of the end-of-horizon debt ratio.

If no other side condition is imposed, we usually assume an infinite time horizon and impose the so-called transversality condition $\lim_{T \to \infty} \left( \frac{1}{R} \right)^{T+1} b_{t+T} = 0$.

From an economics point of view, such a condition excludes Ponzi-type games (see e.g. Wickens (2008, p. 100) and European Commission (2023, Annex A5)). From an algebraic point of view, this condition looks innocent at first sight since $\frac{1}{R} < 1$ is raised to a large number, but this presupposes that the future debt ratio, $b_{t+T}$, is finite; if, however, this forward-looking variable is thought of as an asset price, then self-fulfilling rational bubbles cannot be excluded so that the expected value of the debt ratio can become explosive over time (see Blanchard and Fischer (1989, chapter 5) for a proof). In what follows, as in most policy papers, we will assume that there is a side condition that ties down the future value of $b_{t+T}$ in the intertemporal government budget constraint (5)-(6).

Summing up, when the interest rate-growth rate differential is unfavorable, a finite debt to GDP ratio (and hence fiscal sustainability) requires not only that the path of primary fiscal balances is bounded as in the favorable case studied above, but also that the PDV of expected future primary fiscal surpluses plus the discounted value of the end-of-horizon debt ratio (where, in the research appendices (5) and (6), we again work as in e.g. Sargent (1987, chapter IX) and Azariadis (1993, chapter 2). Appendix B at the end of this note provides algebraic details for the more general case where $d_t$ changes over time.

6 See also Obstfeld and Rogoff (1996, chapter 2) for the intuition of such terminal conditions although in a different setup.
literature, the latter is typically set to zero) are large enough to meet the current liabilities of the government.

Notice that the above relates to the so-called "S2 indicator" used for long-term fiscal sustainability analysis by the EC (see European Commission (2023), chapter 3 and Annex A5.4)). In particular, our equations (5)-(6) can be compared to equations (7)-(9) in the EC’s Annex A5.4. Details on S2 are provided in Section 5 below.

3 Numerical solutions and public debt arithmetic

Using the above analytical framework, we will now illustrate the quantitative importance of the interest rate-growth rate differential for the fiscal balance required for fiscal sustainability. That is, using the simple tool of the government budget constraint, we will quantify the required fiscal adjustment under different assumed scenarios regarding the interest rate-growth rate differential as well as the target for the public debt to GDP ratio at some future time. This is known as debt arithmetic. As an example, as said above, we will refer to the case of Greece being the country with the highest public debt to GDP in the EU.

3.1 The interest rate-growth rate differential and its role in the standard debt arithmetic

We start with a scenario of an unfavorable interest rate-growth rate differential. Thus, we work with equation (6). In particular, let us say that the outstanding public debt to GDP ratio is 171% as it was the case in Greece at the end of 2022. Also say that the nominal interest rate is 4%, the inflation rate is 2%, so that the real interest rate is 2%, and that the growth rate of real GDP is 1% (these are the numbers also used by Buiter (2021, chapter 1) for the EA). In other words, we assume \( R = 1.01 > 1 \). We also assume a time horizon of say 35 years, i.e. \( T = 35 \), at the end of which the public debt ratio is simply set at its starting value, i.e. \( b_{t+T} = b_{t-1} = 1.71 \). That is, we ask what primary balance is needed to keep the debt ratio after 35 years equal to the debt ratio today (see also Blanchard and Das (2017) for similar exercises). This is a case of debt stabilization. Then, solving equation (6) for \( s \) gives \( s \approx 0.017 \).\(^7\) In other words, public debt stability requires a primary annual surplus of 1.7% of GDP on average over the coming 35 years. If, on the other hand, we assume that, at the end of the 35 years, the public debt ratio is lower than its starting value as recommended by the EC, say 100% of GDP, then \( s \approx 0.034 \).\(^8\) This is a case of debt consolidation. In other words, according to this more ambitious scenario where the end-of-horizon debt is lower than the current one, the average surplus should be 3.4% of GDP. Note that these numbers are close to those reported\(^7\) That is, \( s \) solves \( 1.71 = \frac{1}{1.01} \left( \frac{1}{1.01} \right)^{36} + \left( \frac{1}{1.01} \right)^{36} \cdot 1.71 \).

\(^8\) That is, \( s \) solves \( 1.71 = \frac{1}{1.01} \left( \frac{1}{1.01} \right)^{36} + \left( \frac{1}{1.01} \right)^{36} \cdot 1 \).
by the EC in its Post Programme Surveillance Report on Greece published in Autumn 2022 (see European Commission (2022b, p. 18)); the latter reports numbers between 1.4% (under a relatively optimistic scenario about the gap between the real interest rate and the growth rate) and 3.1% (under a relatively pessimistic scenario about the same gap).

The above can be compared to a favorable interest rate-growth rate differential. We use the same parameter values as above except that now we set, for example, \( R = 0.99 < 1 \). Thus, now we work with equation (4). Focusing on the relatively ambitious case in which the end of period debt is 100% of GDP, solving equation (4) for \( d \) gives \( d \equiv -0.007 \) or \( s = -d \equiv 0.7\% \) of GDP.\(^9\) This primary surplus of 0.7% is much smaller than 3.4% which was the solution under the adverse differential above, other things equal. A favorable interest rate-growth rate differential erodes the burden of outstanding public debt so that public debt can be brought down without any extra fiscal effort (here by just keeping the primary fiscal balance almost balanced).

In sum, as is well recognized, the interest rate-growth rate differential makes a lot of difference in terms of the fiscal effort needed to achieve fiscal sustainability. This is in particular so in high public debt countries like Greece. At this point, it is also useful to compare the fiscal effort required for fiscal sustainability to the actual data.\(^10\) If big primary fiscal deficits - like those experienced during the global financial crisis, the pandemic crisis and the energy-food crisis - become a normality, then the numbers for fiscal effort that come out from debt arithmetic exercises like the above are quite demanding, even if we assume a favorable interest rate-growth rate differential. The climate crisis and the ageing problem are additional fiscal risks (see e.g. Schuknecht (2022) for a discussion of risks and public finances).

### 3.2 How inflation erodes the real debt burden - alas, temporarily

Say that \( b_{t-1} \) is 193% of GDP, the nominal interest rate on outstanding debt, \( i_t \), is 2%, the inflation rate, \( \pi_t \), is 10% and the real GDP growth rate, \( \gamma_t \), is 4%. Let us also assume a primary deficit of around 2% of GDP. These numbers are very close to the actual Greek data in the year 2022. Then, equation (6) above implies that at the end of 2022 the public debt ratio would be around 175%, which is substantially below the starting value of 193% and, actually, is close to the Greek data (see European Commission (2022c)). That is, even with a fiscal deficit, the debt to GDP ratio has decreased over time. In this example, this happens thanks to growth but mainly thanks to inflation which erodes the real burden of outstanding public debt and hence reduces the end-of-period public debt to GDP ratio.

\(^9\)That is, \( d \) solves \( 1 = d \left( \frac{1-0.99^{35}}{1-0.99} \right) + 0.99^{35} \times 0.171 \).

\(^{10}\)For primary fiscal balances in the EA and the EU, see e.g. the Economic Forecasts of the EC over the years. Wyplosz (2014) and Alesina and Passalacqua (2016) summarize historical data for fiscal deficits in OECD countries.
As already mentioned above, this has been one of the classic ways to reduce public debt burdens and hence public debt ratios in the world history of debt (see e.g. Dornbusch and Draghi (1990)). But, as is widely recognized, this is a short-term resolution only to debt stabilization. In addition to the standard redistributive and aggregate costs associated with high inflation, high inflation also means that the government will sooner or later have to make concessions. The latter typically include a mix of rising interest rate premia on long-term bonds, a shift to shorter maturities and the issuance of indexed bonds (of course, all this applies to newly issued bonds). If such things occur, sooner or later, the burden of adjustment will shift to higher taxes and/or spending cuts.

In sum, unexpected inflation can help the public finances but, alas, temporarily only. High inflation and low nominal interest rates just buy time. See also International Monetary Fund (2023, chapter 3) for the undesirable effects of high inflation as a means of reducing debt ratios.

3.3 A more careful debt arithmetic for countries with obligations to EU institutions

The above may be helpful to understand the standard methodology used in policy circles to evaluate debt sustainability but it ignores the extra obligations of some highly indebted countries to EU public institutions. Using again Greece as an example, a large part of the Greek public debt is in the hands of non-market EU institutions (ESM, EFSF, etc) as a result of the three official fiscal bailouts in the 2010s amounting to around 290 billion euros, and the country’s obligation is that all this has to be paid back between 2060 and 2070 (by the year 2060 for ESM loans and by 2070 for EFSF loans). In this section, we will add this to the previous analysis. Note that we keep working with the government budget constraint only.

Decomposing the total public debt into that held by private agents/banks and that held by non-market EU institutions, we rewrite (2) as:

\[ b^p_t + b^{eu}_t = R^p_t b^p_{t-1} + R^{eu}_t b^{eu}_{t-1} - s_t \]  

where the superscripts \( p \) and \( eu \) refer to public debt owed to private agents/banks and non-market EU institutions respectively and \( s_t \) denotes the total primary fiscal surplus (we again assume one period debt maturity for simplicity).

We will work in two steps. In the first step, we will calculate the average over time fiscal surplus needed to pay back the debt to the EU in, say, 35 years. Then, in the next step, we will check what this implies for the other part of the debt (namely, the private one) and hence for the total primary fiscal surplus required for sustainability.

Regarding the fraction of Greek public debt in the hands of non-market EU institutions this has been estimated to be at least 70% of the total Greek public debt (see Dimakopoulou et al (2022)). If we assume that the non-market nominal interest on this part of the debt is 1%, and, as assumed above, inflation is 2% and the growth rate is 1%, and that all of them remain constant over time,
this implies $R^u_t = 0.98 < 1$, which in turn means that the associated difference equation for this part of the debt, $b^u_t \equiv R^u_t b^u_{t-1} - s^u_t$, is dynamically stable. Thus, we can use equations (3)-(4) above. Setting $b^u_t = 0.7x1.71$ and assuming that after 35 years this part of debt is fully repaid, equation (4) implies that, other things equal, this requires an average primary fiscal surplus of $s^u_t \approx 0.023$ or 2.3% over the next 35 years.\footnote{Thus, $s^u$ solves $0 = -s^u \left(1 - x 0.98^{35} \right) + (0.98^{35} x 0.7x1.71).$} Before we move on, it is worth examining what $s^u_t$ would be in the counter-factual case in which the nominal interest rate on the EU debt were the market one. In particular, we examine what happens if $b^u_t \equiv R^u b^u_{t-1} - s^u_t$, where, by using the same numbers as in the previous section, $R^u_t = 1.01 > 1$. In this case, we have to use equation (6) which gives $s^u_t \approx 0.04$ or 4%.\footnote{Thus, now $s^u$ solves $0.7x1.71 = s^u \left(1 - x 0.98^{35} \right) + \left(1 - (1 + 0.01)^{36} \right)$.}

\footnote{Thus, now $s^p$ solves $0.3x1.71 + 0.023 \left(1 - x 0.98^{35} \right) = s^p \left(1 - x 0.98^{35} \right) + \left(1 - (1 + 0.01)^{36} \right) + 1.71 \left(1 - (1 + 0.01)^{36} \right).$} This is much higher than 2.3%.

In turn, we plug this value of 0.023 as a constant term (denoted as $a$) on the expenditure side of equation (B.3) in Appendix B which is a generalization of equation (6) above. Thus, now, the associated difference equation is $b^p_t \equiv R^p b^p_{t-1} - s^p_t + 0.023$, where $R^p_t = 1.01 > 1$. As in the previous section, let us first study the less ambitious scenario where the end-of-period private debt (which will also be the total public debt, since the EU public debt will have been repaid in 35 years) remains as it is today, namely, $b^p_t = b_{t+T} = 1.71$. Then, equation (B.3) implies $s^p_t \approx 0.13$\footnote{Thus, $s^p$ solves $s^p \left(1 - x 0.98^{35} \right) + \left(1 - (1 + 0.01)^{36} \right) = s^p \left(1 - x 0.98^{35} \right) + \left(1 - (1 + 0.01)^{36} \right) + 1.71 \left(1 - (1 + 0.01)^{36} \right).$} In other words, the total primary fiscal surplus required for sustainability if the public debt after 35 years simply remains as it is today (namely, 171% of GDP) is 2.3% $(2.3 + 0 = 2.3)$, which should be compared to 1.7% in the experiment above which did not take into account the obligations of Greece to the EU institutions. If, on other hand, again as we did above, the public debt after 35 years is assumed to be 100% of GDP, the same calculations imply $s^p_t \approx 0.017$\footnote{Thus, now $s^p$ solves $0.3x1.71 + 0.023 \left(1 - x 0.98^{35} \right) = s^p \left(1 - x 0.98^{35} \right) + \left(1 - (1 + 0.01)^{36} \right) + 1.71 \left(1 - (1 + 0.01)^{36} \right)$.} In other words, the total primary fiscal surplus required for sustainability in the more ambitious case in which the public debt after 35 years will be 100% of GDP, is 0.023 + 0.017 = 0.04 or 4%, which should be compared to 3.4% in the experiment above which did not take into account the obligations of Greece to the EU institutions.

In sum, the primary fiscal surplus should be systematically higher (by around 0.6 percentage points) than that implied by standard-type calculations which do not take into account the obligations of the country to the EU, specifically, that the outstanding debt to ESM, EFSF, etc, has been agreed to be paid back by around 2070. This happens because, in the standard-type calculations found in most policy reports, the surplus needs to adjust merely to the differential between the real interest rate and the economy’s growth rate. By contrast, when there are debt repayment deadlines like in the case of Greece, the surplus should adjust, not only to this differential but also to the extra expenditure due to repayment of the EU debt. This is a methodological issue which means that –
irrespectively of the primary fiscal surplus implied by the standard calculations that unavoidably depend on the assumed values of the real interest rate and the economy’s growth rate – the inclusion of these repayments increases the required surplus by around 0.5 percentage point on average over time. Note that all this happens even if the EU debt is paid back at policy interest rates which are lower than the market ones. Actually, our calculations have shown that had the interest rate on the EU debt been the same as the market one, the required primary fiscal surplus would have been considerably higher (around 1.7 percentage points higher on average over time, other things equal).

4 The interest rate-growth rate differential: a look at the data

As shown above, assumptions about the interest rate-growth rate differential are crucial for stability and hence the fiscal primary balance needed for sustainability. But what happens in practice? Table 1 reports data for the real interest rate on 10-year sovereign bonds, the real growth rate and their resulting difference (the so-called $r - g$ differential) in 18 EA countries. These are averages of annual data over 2001-2022 for each country. The interest rates are those in the secondary market. The numbers in parentheses for Cyprus, Greece, Ireland and Portugal exclude the sovereign debt crisis years during which these countries were shut down from sovereign bond markets and had to resort to official financial aid from the EC, the ECB and the IMF.

As can be seen in the third column, which covers the full euro period, growth rates have exceeded interest rates in most countries except in Greece, Italy and Portugal, where the differential has been unfavourable. However, once we exclude the sovereign debt crisis years as defined above, the differential ceases to be positive in Greece and Portugal and becomes even more negative in Cyprus and Ireland (see the numbers in parentheses in the third column). Thus, at first sight, things are not bad, with the exception of Italy where the differential has been clearly unpleasant. However, the last column repeats the same exercise except that now we cover the period 2001-2014 only, namely we leave aside the period of the ECB’s large-scale purchases of sovereign bonds (the so-called quantitative easing, QE) that started officially in the beginning of 2015 as well as the recent year of 2022 during which high inflation has led to negative real interest rates in most countries. Comparison of the numbers in the last two columns reveals that, in most cases, the interest rate-growth rate differential turns from negative to positive, or to less negative, in the last column, which illustrates the beneficial effect of the ECB’s massive bonds purchases on bonds prices and their yields. Since such large-scale QE policies cannot continue for ever, now things look worse.

\[\text{15}\] That is, for these countries, we have recalculated the averages excluding their debt crisis years during which their nominal interest rate on sovereign bonds in the secondary market exceeded 6 percent. These years are 2012-14 for Cyprus, 2010-17 for Greece, 2011-12 for Ireland and 2011-13 for Portugal.
Therefore, the evidence is mixed with both positive and negative differentials over time and across countries. Also, if we think of the period since 2015 as being temporary, in the sense that sooner or later the ECB will embark on a gradual quantitative tightening, and that high inflation as a result of Russia’s invasion of Ukraine will not continue to erode the real interest rates, then unfavourable differentials can be expected in several countries especially if economic growth slows down.

<table>
<thead>
<tr>
<th>Country</th>
<th>Real Interest Rate</th>
<th>Real Growth Rate</th>
<th>Interest Rate – Growth Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.3</td>
<td>1.5</td>
<td>−1.2</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.4</td>
<td>1.6</td>
<td>−1.2</td>
</tr>
<tr>
<td>Cyprus</td>
<td>2.5(1.9)</td>
<td>2.6(3.6)</td>
<td>−0.1(−1.7)</td>
</tr>
<tr>
<td></td>
<td>1.7</td>
<td>1.4</td>
<td>−0.6</td>
</tr>
<tr>
<td>France</td>
<td>0.9</td>
<td>1.2</td>
<td>−0.3</td>
</tr>
<tr>
<td>Germany</td>
<td>0.4</td>
<td>1.2</td>
<td>−0.7</td>
</tr>
<tr>
<td>Greece</td>
<td>4.4(1.3)</td>
<td>0.4(2.3)</td>
<td>4.0(−0.4)</td>
</tr>
<tr>
<td>Ireland</td>
<td>1.4(1.0)</td>
<td>5.5(6.0)</td>
<td>−4.1(−5.0)</td>
</tr>
<tr>
<td>Italy</td>
<td>1.6</td>
<td>0.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Latvia</td>
<td>−0.2</td>
<td>3.4</td>
<td>−3.6</td>
</tr>
<tr>
<td>Lithuania</td>
<td>0.4</td>
<td>4.0</td>
<td>−3.6</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>0.1</td>
<td>2.6</td>
<td>−2.5</td>
</tr>
<tr>
<td>Malta</td>
<td>1.2</td>
<td>4.0</td>
<td>−2.7</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.1</td>
<td>1.5</td>
<td>−1.4</td>
</tr>
<tr>
<td>Portugal</td>
<td>2.1(1.3)</td>
<td>0.8(1.3)</td>
<td>1.2(0)</td>
</tr>
<tr>
<td>Slovakia</td>
<td>−0.3</td>
<td>3.5</td>
<td>−3.7</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.8</td>
<td>2.5</td>
<td>−1.6</td>
</tr>
<tr>
<td>Spain</td>
<td>1.0</td>
<td>1.4</td>
<td>−0.5</td>
</tr>
</tbody>
</table>

Source: Eurostat, OECD and World Government Indicators.

5 The EC’s fiscal sustainability indicators

Before we move on, it is useful to clarify how standard debt arithmetic like the above relates to the EC’s S1 and S2 indicators usually used for the evaluation of long-term fiscal sustainability in EU countries (see e.g. European Commission (2023, chapter 3)).

We start with S2 since, according to the EC, "the S2 indicator is the central element of the long-term sustainability analysis" (see European Commission (2022a, p. 97)). Recall first that working with this indicator presupposes $R > 1$. Also recall that our equations (5)-(6) above are like equations (7)-(9) in European Commission (2023, Annex A5.4). More specifically, if we rewrite our
equation (6) as \( s = \left( \frac{R(1-R)}{(1+\tilde{\pi})^{T+1}} \right) \left( b_{t-1} - \left( \frac{1}{R} \right)^{T+1} b_{t+T} \right), \) this is like equation (11) in that Annex (except that here we include the debt ratio at the end of the projection period and we also leave aside ageing costs, etc, which enter separately the EC’s formula). Notice that our \( s \) in equation (6) corresponds to the sum of the structural primary balance at the departure point (denoted as \( PB_{t_0} \) in the EC’s Annex) and the additional fiscal adjustment that would be required for debt sustainability (which is the EC’s \( S_2 \) indicator). In other words, one should be careful how to read country rankings based on \( S_2 \). The value of the latter depends crucially on the initial budgetary position, \( PB_{t_0} \), which, in the EC’s calculations, is assumed to remain constant and unchanged during all years of the experiment. But this means that if \( PB_{t_0} \) is say 1.8\%, which is the forecasted value for the 2023 structural primary balance in Greece as provided by the EC’s (2023) forecasts, then \( S_2 < 0 \) since \( s \cong 0.017 = 0.018 + S_2 \). While, if \( PB_{t_0} \) is say −1.7\%, which is the forecasted value for Germany or the Euro Area as a whole for 2023, then \( S_2 > 0 \), since \( s \cong 0.017 = −0.017 + S_2 \). That is, the initial budgetary position is critical to the extent that it is assumed to remain unchanged throughout the calculations. In other words, if a country happens to enjoy a structural primary surplus in the departure year, this naturally means that the extra fiscal effort can be small or even negative in the years to come; and vice versa. This methodological issue (jointly with forecasted developments in ageing costs, etc) can perhaps explain why Greece is ranked as a low fiscal risk country in the long run, while, Germany is ranked as a medium fiscal risk country and Luxembourg as a high fiscal risk country by the EC.

The same concerns apply to the \( S_1 \) indicator. Recall first that working with this indicator presupposes \( R < 1 \) which means that we work with equation (4). Also recall that our equation (3)-(4) above are like equations (4)-(5) in European Commission (2023, Annex A5.3). In particular, if we rewrite our equation (4) as \( -d = s = \left( \frac{R(1-R)}{1-R} \right) b_0 - \left( \frac{1-R}{1-R} \right) b_t, \) this is like equation (6) in that Annex (except that again here we leave aside ageing costs, etc, which enter separately the EC’s formula). Then, the same remark as in the case of \( S_2 \) applies. Namely, one should be careful how to read \( S_1 \) since its value depends crucially on the initial budgetary position, \( PB_{t_0} \), which, in the EC’s calculations, is assumed to remain constant and unchanged during all years of the experiment. Also, for the same reasons as in the case of \( S_2 \), this is why, if we use \( S_1 \) as a criterion for fiscal sustainability, Greece is again ranked as a low fiscal risk country in the long run, while, Germany is ranked as a medium fiscal risk country and Luxembourg as a high fiscal risk country by the EC.

\footnote{Recall that \( s \cong 0.017 \) was our baseline solution above.}
\footnote{The European Fiscal Board has also pointed out that "the EC’s estimates are subject to notorious uncertainty surrounding output gap estimates" (see European Fiscal Board (2022, chapter 2, p. 36)).}
6 The connection between debt-based fiscal rules and debt arithmetic

As said in subsection 2.3 above, in the case in which the path of public debt is unstable, there are two ways to restore stability. First, to solve the debt equation (2) forward and work with the IGBC as we did above and, second, to allow for fiscal reaction to inherited public debt. Here, we study the latter. In particular, we clarify the connection between debt arithmetic and debt-based fiscal rules, and how this connection affects the debt dynamics.

Following D’Erasmo et al (2016), let us say that the primary fiscal deficit, $d_t$, is contingent on outstanding public debt, that is, $d_t = d_0 - \mu b_{t-1}$, where the term $d_0$ includes determinants of the primary balance such as exogenous factors and counter-cyclical fiscal policies, while $\mu b_{t-1}$ is the debt-contingent part of the policy instrument with $\mu \geq 0$ being a feedback policy coefficient (thus, this is a Taylor-type fiscal rule). Say that the interest rate exceeds the growth rate so that the coefficient on outstanding debt is higher than 1 in equation (2) above, $R_t > 1$; then, if $\mu$ is set high enough so as $(R_t - \mu) < 1$, equation (2) can become stable from unstable meaning that the debt arithmetic changes from unpleasant to pleasant. Note however that, even in the favorable case in which the growth rate exceeds the interest rate in the first place, $R_t < 1$, a feedback reaction to outstanding debt can help the economy to converge to a lower public debt ratio and at a faster pace other things equal.\footnote{Thus, from equation (2), in the long run, we have $(1 - R + \mu) b = d_0$ or $b = \frac{d_0}{1 - R + \mu}$, so the higher is $\mu$, the lower $b$ can be other things equal.}

But, of course, feedback reactions to debt imbalances are not a free lunch. A fiscal reaction like $-\mu b_{t-1}$ represents a fiscal cost, namely, the deficit has to become smaller or the surplus has to become larger other things equal. And this cost needs to be compared to the benefit from the switch to a more favorable debt arithmetic.

Formally, if $\tilde{R}_t = (R_t - \mu) < 1$, while $R_t > 1$, we can use equation (4) instead of (6) and solve for $d_0$ but, on the other hand, now the actual primary fiscal deficit needed to support this policy is $d_t = d_0 - \mu b_{t-1}$ instead of $d_0$ only. In other words, now $d_0$ solves (we again assume that $\tilde{R}_t$ is constant over time):

$$b_t = d_0 \left(\frac{1 - \tilde{R}_t}{1 - R}\right) + \tilde{R}^t b_0$$  

and in turn the path of $d_t$ follows from:

$$d_t = d_0 - \mu b_{t-1}$$  

which, using the government budget constraint, $b_t = \tilde{R} b_{t-1} + d_0$, implies by repeated backward substitutions that, at some time $T$, the primary fiscal deficit will be:
\[ d_T = \left[ 1 - \mu - \mu \sum_{i=1}^{T-2} \bar{R}^i \right] d_0 - \mu \bar{R}^{T-1} b_0 = \]
\[ = \left[ 1 - \mu - \frac{\mu \bar{R} \left( 1 - \bar{R}^{T-2} \right)}{1 - \bar{R}} \right] d_0 - \mu \bar{R}^{T-1} b_0 \]  
(9b)

where \( b_0 \) is the initial debt to GDP ratio.

What does all this imply numerically? Say that initially we had \( R = 1.01 > 0 \) as in the unpleasant case studied in subsection 4.1 above, but now, thanks to the feedback reaction to debt with \( \mu = 0.02 \), we switch to the pleasant case, \( \bar{R}_t \equiv (R_t - \mu) = 0.99 < 1 \). Let us also repeat the same policy experiment as above where the public debt is initially 171% and we want to reduce it to 100% in 35 years from now. Then, as we have already seen in subsection 4.1, and as also follows from (8), we have \( s_0 \equiv -d_0 \equiv 0.007 \). Then, using these values, equation (9b) can give the time path of primary deficits. For example, in ten periods from now, we have \( s_{10} \equiv -d_{10} = 0.037 \), in twenty periods \( s_{20} \equiv -d_{20} = 0.033 \), in 30 periods \( s_{30} \equiv -d_{30} = 0.029 \), etc. These numbers should be compared to \( s = -d \equiv 0.034 \), which is the primary fiscal surplus as share of GDP required when debt sustainability is achieved by a constant or flat over time, non-debt contingent policy when \( R \) is 1.01 (see subsection 4.1 above). In other words, there is an intertemporal tradeoff as typically happens in cases of reforms. If the government follows the same, flat fiscal policy in each period, it spreads its fiscal cost out equally over time or across generations.\(^{19}\) If, on the other hand, it follows a debt-contingent fiscal policy, it front-loads the cost of the fiscal adjustment, with higher short-term costs in terms of surpluses and smaller sacrifices in the later periods (this happens because of the more favorable interest rate-growth differential).

Note that similar results follow when \( R = 0.99 < 1 \) so that we start with the pleasant case. That is, now, again with \( \mu = 0.02 \), the differential becomes even more favorable, \( \bar{R}_t \equiv (R_t - \mu) = 0.97 \). Then, working similarly, we get \( d_0 \equiv 0.019 \) from (4) and in turn, using (9b), we have \( s_{10} \equiv -d_{10} = 0.01 \) in ten years from now, \( s_{20} \equiv -d_{20} = 0.005 \) in twenty years, \( s_{30} \equiv -d_{30} = 0.0024 \) in thirty years, etc. These numbers should be compared to \( s = -d \equiv 0.007 \), which is the primary fiscal surplus as share of GDP required when debt sustainability is achieved by a constant over time, non-debt contingent policy (see subsection 4.1 above).

In sum, a debt-contingent fiscal policy - according to which fiscal instruments react to public debt imbalances at a constant rate - front-loads the costs of fiscal adjustment but, after a point in time, the required surpluses get smaller and smaller as the benefits of a more favorable interest rate-growth differential build up. More loosely speaking, the main benefit from front-loading is credibility of fiscal policy and hence lower interest rates. The main risk of front-loading is that fiscal austerity may lead to a recession and vicious cycles in the short

\(^{19}\)Perhaps there is an analogy between this and Barro’s (1979) tax smoothing result.
7 Is public debt arithmetic reliable?

Calculations like the above, based on the government budget constraint only, are popular in policy papers but are sensitive to assumptions about sovereign interest rates and growth rates over time. More importantly, sovereign interest rates and growth rates are endogenous variables, and the same applies to several items included in the primary fiscal balance (think of tax revenues, social expenditure programs, etc). In reality, all these variables are endogenous and hence - in the absence of Ricardian Equivalence - depend, directly or indirectly, on the inherited public debt itself. Such endogeneity implies that the debt dynamics, and hence what is needed for debt stability and fiscal sustainability, are more complicated than those implied by the above popular policy analysis. Note that the same applies when we introduce fiscal reaction functions to restore stability as we did above; the behaviour of economic agents is affected by fiscal (re)actions and this can again shape the growth rate, the interest rate, tax bases, etc. All this, as pointed out by D’Erasmo et al (2016), is a reflection of the Lucas critique.\(^{20}\)

The above imply that a reliable quantitative fiscal sustainability analysis requires a structural approach (see also e.g. Alesina and Passalacqua (2016) and D’Erasmo et al (2016)). This means the use of macroeconomic models where these three key drivers of public debt dynamics (the sovereign real interest rate, the growth rate of real GDP and most items included in the primary fiscal balance) are all endogenous variables whose paths over time are affected - among other things - by the accumulated public debt as well as by policy reactions (if any) to it. In other words, in such models, private decisions, and in turn macroeconomic outcomes like growth rates and market interest rates, are not invariant with respect to state variables and policy actions. There are many quantitative dynamic general (dis)equilibrium models of this type in the academic literature but also by researchers in the EC, the ECB, the IMF, etc. The QUEST model used by the EC over the years is a well-known example.

Then, to the best of our understanding, there are some common messages from the macroeconomic literature: First, given the current situation, in almost all cases, if a shock hits the economy, macroeconomic stability and determinacy can be guaranteed only if some fiscal policy instruments react systematically to public debt imbalances and, specifically, if these instruments react to deviations of the outstanding public debt to GDP ratio from a policy target value.\(^{21}\) In

\(^{20}\) See e.g. Sargent (2023) for a recent technical paper on the Lucas critique.

\(^{21}\) For the US economy, see e.g. Leeper et al (2010), Davig et al (2010), Davig and Leeper (2011) and Malley and Philippopoulos (2023). For the Euro Area as a whole, see e.g. Dimakopoulou et al (2023) and the references therein, while see e.g. Malley et al (2009) for a study of the big EU countries before the global financial crisis. For the Greek economy, see e.g. Papageorgiou (2014), Dellas et al (2017), Economides et al (2021, 2022), Dimakopoulou et al (2022) and, for an econometric model, Dendramis et al (2022).
other words, as Sims (2017) points out, to ensure stability and hence get a solution, we cannot assume that government spending and/or tax rates are independent of the public debt path.\textsuperscript{22} Second, it is hard to find self-financing fiscal expansions even when the latter are in the form of an increase in public investment. In other words, even when an increase in public debt is used to finance an increase in public investment, which augments public infrastructure and enhances economic growth and tax bases in the medium term, a cut in another public spending item, and/or a rise in a tax rate, are also unavoidable at least in the early period during which the public debt is rising.\textsuperscript{23} Third, the effect of a cut in public spending, or a rise in tax rates, on the debt to GDP ratio is far from obvious. Depending on the instrument used for fiscal consolidation, the latter may reduce the level of debt but it can also reduce the level of GDP so that the debt ratio can increase (for recent empirical evidence and how to tackle soaring public debt, see International Monetary Fund (2023, chapter 3)). In other words, one should carefully check the size of the multiplier of each policy instrument used to bring the debt ratio down.\textsuperscript{24}

But, regarding the first message, what happens in practice? Do we observe fiscal reactions to rising public debt? This is addressed next.

8 Fiscal reaction functions: a look at the data

Although it is widely recognized that debt-contingent fiscal policy rules are needed for stability in structural macroeconomic models, the empirical evidence is mixed to say the least. The European Commission itself reports the lack of fiscal reaction to public debt imbalances (see European Commission (2015, part IV; and 2021, part IV)). Actually, as it admits, "the debt ratio in particular does not seem to have played any role in determining the fiscal effort required, which is interesting considering not only that debt is the centre of focus of the existing literature on the fiscal reaction function but also considering the legislation itself, which states that the medium-term debt position (its dynamics and sustainability) constitutes the key factor in determining the recommendation" (see European Commission (2021, p. 132). D’Erasmo et al (2016) also provide estimates of fiscal reaction functions for a number of countries and find that debt stabilization reactions become much weaker when post-2008 are added to the sample. In particular, they report that there is a structural break after 2008 in the response of the primary balance to high debt both in the European economies and the USA. On the other hand, Attinasi et al (2019) provide evi-

\textsuperscript{22}Sims (2017) uses an educational simple general equilibrium model to show how feedback fiscal rules (in the sense that tax rates rise, and/or public spending falls, in response to rising public debt) can avoid an explosive path of debt. As he elaborates, this response should be high enough so that the debt ratio does not explode upward but, at the same time, not too high that the debt ratio explodes downward (see also our third message below).

\textsuperscript{23}See e.g. Malley and Philippopoulos (2022) and the references there for the US economy. See e.g. Dimakopoulou et al (2022) for the Greek economy.

\textsuperscript{24}See e.g. Philippopoulos et al (2017) for various fiscal policy scenario used to bring the public debt ratio down. In the final section, we summarize some related results.
dence that, in high-debt EA countries, cyclically-adjusted primary balances do react to inherited public debt, although this is significant only during bad times when the output gap is negative (see their Table 7 in particular).

We also provide own evidence. In Table 2, we calculate the correlation between current public debt as share of GDP and next year’s primary fiscal surplus as share of GDP in 18 EA countries.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>75.5</td>
<td>−0.26</td>
</tr>
<tr>
<td>Belgium</td>
<td>101.7</td>
<td>−0.13</td>
</tr>
<tr>
<td>Cyprus</td>
<td>78.7</td>
<td>0.22 (*)</td>
</tr>
<tr>
<td>Finland</td>
<td>55.0</td>
<td>−0.66</td>
</tr>
<tr>
<td>France</td>
<td>85.2</td>
<td>−0.44</td>
</tr>
<tr>
<td>Germany</td>
<td>68.7</td>
<td>0.39</td>
</tr>
<tr>
<td>Greece</td>
<td>149.4</td>
<td>0.19 (*)</td>
</tr>
<tr>
<td>Ireland</td>
<td>60.8</td>
<td>−0.07</td>
</tr>
<tr>
<td>Italy</td>
<td>123.7</td>
<td>−0.55</td>
</tr>
<tr>
<td>Latvia</td>
<td>30.7</td>
<td>−0.01</td>
</tr>
<tr>
<td>Lithuania</td>
<td>31.3</td>
<td>0.30</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>16.7</td>
<td>−0.11</td>
</tr>
<tr>
<td>Malta</td>
<td>60.3</td>
<td>0.20</td>
</tr>
<tr>
<td>Netherlands</td>
<td>55.2</td>
<td>−0.32</td>
</tr>
<tr>
<td>Portugal</td>
<td>101.8</td>
<td>0.39 (*)</td>
</tr>
<tr>
<td>Slovakia</td>
<td>46.2</td>
<td>0.31</td>
</tr>
<tr>
<td>Slovenia</td>
<td>51.0</td>
<td>0.08</td>
</tr>
<tr>
<td>Spain</td>
<td>77.0</td>
<td>−0.21</td>
</tr>
</tbody>
</table>

Source: Eurostat and own calculations.

As can be seen in the last column of Table 2, and for most of the EA countries, the correlation is negative meaning that an increase in public debt to GDP ratio in the current period is associated with a lower primary fiscal surplus, or a higher primary fiscal deficit, in the next period. Exceptions include Germany, which has a relatively high positive coefficient, as well as Cyprus, Greece and Portugal (these three countries are marked with an asterisk). However, recall that in the 2010s Cyprus, Greece and Portugal had been in enforced fiscal austerity programs as a condition for their official bailout from EU public institutions. We additionally report that, for the EA as a whole, the correlation coefficient is also negative, around −0.2.

In sum, we think it is fair to say that there is little evidence of systematic stabilizing fiscal reaction to debt imbalances. Given this, if, in practice, we do not observe any systematic fiscal reaction to public debt imbalances, then, quoting Leeper et al (2010) in their study for the US, a natural question to ask
ourselves is "Why do forward-looking agents continue to purchase bonds with relatively low interest rates?". The answer given by Leeper and his co-authors is that - to the extent that we want to maintain the assumption of rationality - agents believe that current inaction is temporary and it will be replaced by necessary policy corrections in the future. This is why trust, expectations about the future, and what is signaled by policymakers in the present, are crucial.

9 OK, fiscal rules, but which ones?

As said above, a common property of most structural macroeconomic models is that Taylor-type feedback rules, according to which tax-spending instruments respond to outstanding public debt among other state variables, are needed to avoid an explosive path of public debt. As also said above, this is the type of fiscal rules employed in most research papers. Other types of fiscal rules usually met in policy circles, which are believed to limit the accumulation of excessive deficits and debts, include numerical targets (like a balanced budget rule, a debt ceiling, a limit on public spending, etc) or the so-called golden rule according to which budget deficits are allowed to finance public investment only (see e.g. Alesina and Passalacqua (2016) for a review of fiscal rules).

The debate for fiscal rules has been particularly hot in the EU. Since the Maastricht Treaty in 1992, the agreement has been that, in a second-best world, fiscal rules at national level are needed for the viability of the single currency. Various rules have been introduced and debated over the years without much success or agreement. It is fair to say that the EU’s fiscal rules, past and present, have been of four kinds: deficit-based, debt-based, expenditure-based and structural balance-based (see e.g. European Commission (2021, part IV)).

Most of the time so far, the policy emphasis and compliance have been on deficit-based rules like the 3% ceiling. By contrast, in the new economic governance framework recently presented by the European Commission (2022b and 2023b), although references to the 3% ceiling for fiscal deficits and the 60% target for the debt ratio remain, the emphasis has shifted from limits on budget deficits to limits on public expenditure.

To the best of our understanding, the recently proposed expenditure rule by the EC works as follows. We first calculate the country’s annual growth rate of primary government expenditure, net of discretionary revenue measures and one-off. Second, we calculate the ten-year average growth rate of potential nominal GDP minus the convergence margin necessary to ensure an adjustment of the country’s structural budget deficit of the general government in line with the country’s structural balance rule (this adjustment implies that we take into account whether the structural budget balance of the general government is above, or below, the country’s Medium-Term Objective, MTO). Finally, if the country’s annual growth rate of primary government expenditure from the first

For the history of EU fiscal rules as well as for the current state of affairs and controversies, see e.g. Wyplosz (2014, 2021, 2023), Beetsma and Larch (2019), Bilbiie et al (2021), Beetsma (2022) and the references cited there.
step exceeds the adjusted ten-year average growth rate from the second step, a fiscal correction or effort is needed. Before we assess this rule, we should recall, as Wyplosz (2023) points out, that the expenditure rule is not new. It was introduced in 2011 as part of the preventive arm of the SGP; the difference is that now the EC proposes to make it the only, or the main, measure to be used.

The general idea behind the emphasis on an expenditure rule is that primary public expenditures are directly under the control of national governments so such a rule has to do with a policy instrument rather than an intermediate target like the budget balance or the cyclically adjusted budget balance which have been the main measures in the past. As such, the rule has been received positively (see e.g. the European Central Bank (2023) and Wyplosz (2023)). On the other hand, it has also received criticism. For example, the suggested expenditure rule is too complex; it has to rely on unobservable variables that are not readily available (like cyclical unemployment, potential growth, discretionary revenue changes, etc); its methodology needs clarification; etc (again see e.g. the European Central Bank (2023) and Wyplosz (2023)). In other words, the usual broader concerns about simplicity, transparency and effectiveness of policy rules continue to apply.

The discussion about the optimality, or simply the effectiveness, of the EU’s fiscal rules (previous and new) has been big and we cannot review it here (see the references above for good reviews). Nevertheless, to this literature, we wish to add three points that might be useful.

Our first point is obvious and general but we feel that sometimes is forgotten in the heat of political debates. All policy rules used in practice are, by definition, suboptimal (and this includes the famous Taylor rule for monetary policy). Ramsey, and especially time-consistent, optimal policy rules would be too complicated, even computationally, to be useful for policy-making since they follow from a solution of a large general equilibrium model where the government acts as a Stackelberg leader which means that a complete description of optimal policy results in complicated feedback rules where policy instruments react to a very large number of state variables including auxilliary multipliers (see e.g. Benigno and Woodford (2006) and Schmitt-Grohe and Uribe (2006) among many others). This is why policymakers need to resort to "simple and implementable" rules meaning that policy instruments react to a small number of observable variables only (see e.g. Schmitt-Grohe and Uribe (2006, 2007) and Kliem and Kriwoluzky (2014)). On the other hand, although the EU’s fiscal rules cannot, by definition, be immune to the standard criticism about optimality, the calculation of the recommended maximum growth rate of primary public expenditure relies on too many unobservable variables so it is hard to be characterized as "simple and implementable".

Our second point is that, as already reported above, the lesson from most structural macroeconomic models is that macroeconomic stability requires policy reaction to the public debt ratio itself. We report that, by making use of rather conventional DSGE models like those listed in section 7 above, our simulation experiments systematically imply that by simply keeping the deficit below the numerical value of 3%, or by restricting the growth rate of primary
expenditure, or by reacting to flow variables (like the budget balance, or the cyclically adjusted balance, or the MTO) do not seem able to restore stability in an otherwise unstable economy. And this is hardly surprising: public debt is a state stock variable so, if it happens to be explosive, reaction to this very variable is necessary to restore dynamic stability. This means that a policy rule should be contingent (perhaps among other things as discussed next) on the outstanding public debt ratio or the deviation of the latter from a target value, at least for some time. This is simply necessary for stability.

Third, a measure of fiscal stance - and it is the fiscal stance that dictates the macroeconomic variables that policy instruments should be contingent on - should include those variables that shape the dynamics of the debt ratio. And, as we saw in detail in section 2 above and as is well known, these variables are the primary fiscal balance, the outstanding public debt to GDP ratio and the interest rate-growth rate differential. Among them, reaction to the outstanding debt ratio is necessary, for the reasons said just above. But reaction to the other two (which could make the fiscal effort smaller, or bigger, depending on their evolution) can be desirable in terms of the required fiscal adjustment. For example, if the interest-rate growth rate differential is favorable so that the economy grows out its debt ratio, fiscal reaction to public debt (which would mean unpopular, and perhaps recessionary, fiscal consolidation) can be milder. In other words, the fiscal stance could be a weighted average of the gap between the debt ratio and its target value, the gap between the primary fiscal balance and its target value, and the difference between the sovereign interest rate and the economy’s growth rate. In turn, the expenditure instrument(s) should be contingent on this stance.

Formally, the fiscal stance can be defined as:  

$$ \left( \frac{b_{t-1}}{b_t^{\arg \text{ct}}} \right)^{a_1} \left( \frac{d_t}{d_t^{\arg \text{ct}}} \right)^{a_2} \left( \frac{r_t}{\gamma_t} \right)^{1-a_1-a_2} \leq 1 $$  

(10)

where $0 < a_1, a_2 < 1$ are policy weights that the EU can decide on, while the policy targets, again decided by the EU, could be, for example, $b_t^{\arg \text{ct}} = 0.95 b_{t-1}$ in case we want the debt ratio to fall over time and $d_t^{\arg \text{ct}} = MTO$. In turn, public spending (being a component of $d_t$) can follow residually to satisfy the above, written as a binding equality, to give a relatively simple and implementable feedback fiscal rule for primary expenditure.

10 Policy conclusions

Sovereign interest rates, growth rates and most items included in primary fiscal balances are all endogenous variables which are jointly determined. It is also obvious that all of them are affected by economic policies. This is another application of the Lucas critique, which naturally implies that approaching the

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26 Recall that $r_t = i_t - \pi_t$ is the real interest rate on sovereign bonds and $\gamma_t$ is the growth rate of real GDP. It is important to report that this is very similar to Korea’s recent fiscal rules.
issue of fiscal sustainability through the lens of the government budget constraint only is not reliable for quantitative conclusions.

Therefore, a more reliable analysis of fiscal sustainability necessitates the use of structural dynamic general (dis)equilibrium macroeconomic models. In these models, all the above key variables that shape the public debt dynamics are endogenous variables and, as such, are affected by policy actions and the public debt itself. Then, a common finding from this literature is that if we assume that fiscal policies remain unchanged as in the current data or more generally are exogenous, the path of public debt is explosive over time and this applies to most countries. Hence, debt-contingent fiscal rules are necessary according to which fiscal instruments react to the gap between the outstanding public debt and a policy target value. We also contributed to the debate on fiscal rules in the EU by suggesting that, in addition to the public debt gap which is necessary for stability and fiscal sustainability, fiscal rules should also be contingent on the interest rate-growth rate differential and the gap between the primary fiscal balance from its medium-term objective. Such a rule can give more flexibility and explicitly reward those countries that, although have high public debt, they manage to grow it out and/or enjoy the trust of the markets as reflected in relatively low sovereign interest rates.

Finally, we should recall that which particular fiscal policy instrument is being used to bring public debt down is essentially a fiscal policy multiplier problem. The macroeconomic literature (see e.g. Philippopoulos et al (2017) and the references therein) suggests that a damage-minimizing policy mix is the one in which we use fiscal instruments with small output multipliers to bring public debt down and - once public debt has been brought down - we allow fiscal instruments with large output multipliers to take advantage of the fiscal space created; the anticipation of the latter, if credible, shapes private incentives and may mitigate the recessionary effects even in the short term. This is consistent with the “expenditures” rules suggested recently by the EC, although one has to be clear regarding the kind of public expenditures that should be used to stabilize public debt.

References


[41] International Monetary Fund (2023): World Economic Outlook: A Rocky Recovery.


Appendices

Appendix A

Consider the first-order linear difference equation in $Y_t$:

$$Y_t = RY_{t-1} + a + bX_t \quad (A.1)$$

where $R < 1$.

By repeated backward substitutions (see e.g. Sargent (1987, chapter IX) and Azariadis (1993, chapter 2)), we get:

$$Y_t = a \sum_{i=0}^{t-1} R^i + b \sum_{i=0}^{t-1} R^i X_{t-i} + R^t Y_0$$

where $Y_0$ is a given initial value.

Since $R < 1$, this can be written as:

$$Y_t = a \frac{1 - R^t}{1 - R} + b \sum_{i=0}^{t-1} R^i X_{t-i} + R^t Y_0 \quad (A.2)$$

which, if we assume for simplicity that $X$ is constant, simplifies to:

$$Y_t = a \frac{1 - R^t}{1 - R} + b X \frac{1 - R^t}{1 - R} + R^t Y_0 \quad (A.3)$$

or:

$$Y_t = a + bX \frac{1 - R^t}{1 - R} + R^t \left( Y_0 - a + bX \frac{1 - R^t}{1 - R} \right) \quad (A.4)$$

This generalizes (3)-(4) in the text.

Note that the process of the driving force, $X_t$, can also be important for convergence. For example, say that now $X_{t+1} = \theta X_t$, where $\theta$ is a parameter. Then, equation A.2 becomes:

$$Y_t = a \frac{1 - R^t}{1 - R} + bX_t \sum_{i=0}^{T} \left( \frac{R}{\theta} \right)^i + R^t Y_0 \quad (A.2a)$$

so that, in this more general case, stability also requires $|\frac{R}{\theta}| < 1$ which puts additional restrictions on the exogenous variable. Recall that the exogenous variable is the primary fiscal deficit in our analysis.

Appendix B

Consider the same equation in $Y_t$:

$$Y_t = RY_{t-1} + a + bX_t \quad (B.1)$$
except that now $R > 1$.

By repeated forward substitutions (see e.g. Sargent (1987, chapter IX) and Azariadis (1993, chapter 2)), we get:

$$Y_{t-1} = -\frac{a}{R} \left( 1 + \frac{1}{R} + \cdots + \left( \frac{1}{R} \right)^T \right) - b \sum_{i=0}^{T} \left( \frac{1}{R} \right)^i X_{t+i} + \left( \frac{1}{R} \right)^{T+1} Y_{t+T}$$

or

$$Y_{t-1} = -\frac{a}{R} \left( \frac{1 - \left( \frac{1}{R} \right)^{T+1}}{1 - \frac{1}{R}} \right) - b \sum_{i=0}^{T} \left( \frac{1}{R} \right)^i X_{t+i} + \left( \frac{1}{R} \right)^{T+1} Y_{t+T} \quad (B.2)$$

which, if we assume for simplicity that $X$ is constant, simplifies to:

$$Y_{t-1} = -\frac{a}{R} \left( \frac{1 - \left( \frac{1}{R} \right)^{T+1}}{1 - \frac{1}{R}} \right) - b X \left( \frac{1 - \left( \frac{1}{R} \right)^{T+1}}{1 - \frac{1}{R}} \right) + \left( \frac{1}{R} \right)^{T+1} Y_{t+T} \quad (B.3)$$

or

$$Y_t = \frac{a + bX}{(1 - R)} + \left( \frac{1}{R} \right)^{T+1} \left( Y_{t+T} - \frac{a + bX}{(1 - R)} \right) \quad (B.4)$$

This generalizes (5)-(6) in the text.

Note that the process of the driving force, $X_t$, can also be important for convergence. For example, say that now $X_{t+1} = \theta X_t$, where $\theta$ is a parameter. Then, equation B.2 becomes:

$$Y_{t-1} = -\frac{a}{R} \left( \frac{1 - \left( \frac{1}{R} \right)^{T+1}}{1 - \frac{1}{R}} \right) - b X \theta \sum_{i=0}^{T} \left( \frac{\theta}{R} \right)^i + \left( \frac{1}{R} \right)^{T+1} Y_{t+T} \quad (B.2a)$$

so that, in this more general case, stability also requires $\left| \frac{\theta}{R} \right| < 1$ which puts additional restrictions on the exogenous variable (see also e.g. Obstfeld and Rogoff (1996, p. 729)). Recall that the exogenous variable is the primary fiscal deficit in our analysis.
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