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Cryptocurrency Market**

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# Behavioral Personae, Stochastic Dominance, and the Cryptocurrency Market

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## Abstract

We investigate whether Cryptoassets enhance optimal portfolio performance for the most prominent investor personae in the Behavioral Finance literature, namely, the Cumulative Prospect Theory, the Markowitz and the Loss Averse types of investors. We frame our analysis on the grounds of risk aversion w.r.t. perceived returns, and thus remain consistent with Second order Stochastic Dominance. Using the Stochastic Spanning criterion, we construct optimal portfolios with and without cryptoassets, allowing however for local non-stationarities and bubbles in the dynamics of the returns process. Additionally, we take into consideration the most exploited safe-haven asset, namely Gold, as well as the Covid-19 outbreak's effect on the markets. Our out of sample comparative performance analysis indicates that investors impression of gains and losses affects significantly the aggregate performance of optimal portfolios and that cryptoassets are an attractive option for the examined investor types.

**Keywords and phrases:** Parametric and Non-parametric tests, Second Order Stochastic Dominance, Stochastic Spanning, Cumulative Prospect Theory, Loss Aversion, Markowitz Theory, Probability Weighting, Linear Programming, Portfolio Performance, Cryptoassets, Gold, Covid-19, Bubbles, Mildly Explosive dynamics.

**JEL Classification:** C12, C13, C15, C44, D81, G11, G14.

## 1 Introduction

The issue of whether cognitive biases in the perception of gains and losses, actually affect the decision-making process when forming an investment strategy, is a point of disagreement between Classical and Behavioral Finance (henceforth BF). Under Classical Finance, the formation of portfolios is based on investors' preferences towards risk; the usual standard being global risk aversion. Under BF, attitude towards risk is the impact of investors' sentiment (behavioral trigger), as well as their perception of gains and losses. It is now experimentally recognized that when individuals

face a prospect, they assess subjectively the potential outcomes (Kahneman and Tversky, 1979). Subjectivity usually implies differing assessments and choices.

The normative classical approach is to regard each investor as rational (von Neumann and Morgenstern, 1953), and the Market as efficient (Bachelier, 1900). Thereby, cognitive biases, i.e. systematic deviations from rationality, are *de jure* neglected and investors are treated as perfectly tuned automata. They are never being carried away by their emotions, and their reasoning is always objective and grounded on the available information. Their goal is to maximize their portfolios' expected utility under their budget constraints and their dispositions towards risk (Markowitz, 1952b).

A large body of positive empirical evidence (among others Kahneman and Tversky, 1979; Kahneman, Knetsch and Thaler, 1991) has however demonstrated that cognitive biases are main drivers of financial decision-making. They are expressed via subjective *heuristics-decision shortcuts* (Camerer et al., 1998), that have the advantage of neutralizing emotions (doubts, etc.), in decision making.

In this paper we are interested in assessing whether and how the introduction of heuristics affects the construction and performance of optimal portfolios, when the associated portfolio spaces are allowed to contain assets from the fin-tech expanded asset class universe. We employ heuristics on the cross-section of the assets' returns, in the form of three different investor personae. Those arise from prominent theoretical frameworks in the BF literature; namely, Cumulative Prospect Theory (CPT, Tversky and Kahneman 1992; henceforth TK, 1992), Markowitz (Markowitz, 1952 a & b) and Loss Aversion (henceforth LA, Benartzi and Thaler, 1995; Barberis Huang and Santos, 2002). CPT agents employ subjective probability transformations along with loss aversion, while LA agents focus solely on averting losses; those are fundamental elements of both theories. A key behavioral aspect about LA is that it minimizes future regret, i.e. the frustration related to the belief that a different action would lead to a more desirable outcome. In the case of the Markowitz persona, probability distortions are not part of the relevant theoretical context, nor is loss aversion. A Markowitz investor exhibits risk aversion for losses and is a risk seeker for gains.

For both CPT and LA, loss aversion and probability distortion stem from experimentally and empirically observed phenomena of choice (resulting from cognitive biases) such as narrow framing, overconfidence and mental accounting together with myopic investment decisions. For LA we follow the work of Benartzi and Thaler (1995), who argue that investors are primarily interested in avoiding losses and when they do, they search for ways to improve their investing performance. Availability of time also plays a significant role in decision making, especially when it comes to the discretion of a strategy to mature. When there is "time shortage", effort reasoning is disabled and heuristics take action. Heuristics are decision making shortcuts supported by sentimental factors such as anxiety, herd behavior, etc. When there is abundance of time, temperance and prudence can be the main drivers of decision making, resulting into more sober choices and/or practises.

In general, short term gambles are supported by risk seeking behaviors (convex part in the value function), while long term gambles by risk averting (concave part in the value function). One way or another, there is strong evidence that loss aversion is always present, making the value function

steeper near the origin over the losses domain. Loss aversion is a documented phenomenon of choice where potential or realized losses loom larger than gains. Thereby, while classical theory in Finance sets the risk averse type of investor as the norm, CPT, LA and Markowitz personae partly emerge descriptively.

For the CPT persona we deploy the particular value function along with the relevant probability weighting functions (henceforth PWF). The PWF are subjective transformations of objective probability distributions and are used to form the so-called "capacities" (i.e. decision weights). These distortions result in over-weighting small probabilities of large gains and under-weighting moderate and large probabilities of losses (TK, 1992). Both the value function and the PWF are defined simultaneously over the losses and the gains domain, and their combination gives rise to specific attitudes towards risk (Baucells and Heukamp, 2006), as well as to behavioral/cognitive biases. Some of the most common are: the disposition effect, mental accounting, narrow framing and overconfidence (Barberis and Thaler, 2003). For example, the disposition effect is the observed phenomenon where investors tend to keep assets that have lost their value and sell the ones whose values have increased and that its magnitude depends on the size of prior gains and/or losses. This situation may create future lower returns, initiating stronger aversion to losses and enhancing risk seeking behaviors in order to mitigate portfolio's potential out-of-target performance. Another example is overconfidence; the phenomenon where some investors tend to believe that either their information are more accurate, or their skills are superior, or both, compared to the rest of the market. This results in over-trading with poor future returns, on average. Even though many scholars argue that loss aversion is more than enough in "capturing" real time financial behavior (Barberis and Thaler, 2003), in this work we employ it along with PWF, in order to examine how likely they are to play a combined significant role in the investors' decisions and subsequently on their portfolios' performance under the stochastic dominance paradigm.

All experiments focus on the formation of buy-and-hold portfolios, where weights are chosen on a business day basis (myopic framing). Re-balancing occurs at every point in time (every day), apart from some initial "training period". This does not necessarily reflect actual practises; in several cases re-balancing occurs once and annually, when investors take the report sheet of the annual performance of their investments (Barberis and Thaler, 2003). How investors *frame* gains and losses is plausibly influenced by the way relevant information is presented to them. One could argue that our approach introduces *myopic-loss aversion* (Benartzi and Thaler, 1995), because of the high frequency of valuation combined with loss aversion, in the CPT case. However, we point out that we are not interested in re-balancing itself, but rather on the behavior (actions) of the three investor types, as these are depicted on the weights of their optimal portfolios.

The first portfolio universe (henceforth Traditional), is structured by convex combinations of traditional assets (stocks and bonds), and Gold. The expanded second portfolio universe (henceforth Augmented) additionally includes cryptocurrencies in its base assets.<sup>1</sup> We employ the four largest

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<sup>1</sup>We also conduct empirical analyses re-combining traditional assets, Gold and the cryptoassets to different base asset sets in structuring the portfolio universes of interest. For instance, the base set for the first universe may contain only the traditional asset classes, while the enhanced is structured via Gold and cryptoassets. In every such case we

cryptoassets by market capitalization (BTC: Bitcoin, ETH: Ethereum, LTC: Litecoin and XRP: Ripple). Cryptoassets tend to have an asymmetric risk profile because they exhibit an extremely high return volatility together with positive skewness and kurtosis (see Table 4 of descriptive statistics). Furthermore, their dynamic behavior is consistent with the existence of bubbles and mild explosivity (see Anyfantaki et al. (2021)-AAT21). Following the reasoning of Odean (1999), we analogously believe that cryptoassets have raised an attention effect that has resulted to excessive trading (i.e. overconfidence bias). Due to their extreme past performance they have caught the attention of investors. Despite this, it is also possible that their markets would be non-attractive for "rational" highly risk averting investors, due to the characteristics of the dynamics of their returns, their complicated technology and their connection to the not (yet?) mainstream decentralized Finance.

Thus, the comparative consideration of those portfolio universes is suitable for analysis that includes several cognitive biases like the above. For example "narrow framing" is triggered because both portfolio universes are treated in isolation from the rest of the market; we remain indifferent about the markets' aggregate performance. It is a fact that the market's aggregate performance affects an investor's sentiment, and subsequently her actions, through the herd behavior mechanism. The taxonomy to Traditional and Augmented, could also introduce "mental accounting" because we classify their performance separately, irrespective of the fact that they both contain some assets in common.

The optimal portfolios in each universe are constructed in the framework of Second order Stochastic Dominance (SSD) (Levy, 2006) and Stochastic Spanning (henceforth StSp) (Arvanitis et al. 2019); those are actually applied on the persona distorted values of the base assets returns. We thus work under the assumption that each persona decides portfolio weights optimally, exhibiting non-satiation and risk aversion. Those however are not evaluated on the distribution of the original return process, but on its transformation according to the persona's cognitive biases. We work in the non-parametric framework of SSD since we want to remain agnostic on the non-satiation and risk aversion characteristics of investors' preferences. Optimal weights are constructed for both portfolio types via the nested optimizations related to the construction of the functional that decides whether the traditional portfolio universe (empirically) stochastically spans the augmented one. Whether spanning is the case or not, the resulting optimal weights for both types can be perceived as optimal choices of an optimal risk averse utility.

In order to avoid conceptual and statistical problems with (locally) non-stationary returns, we use the AAT21 extension of the SSD and StSp, to the mildly explosive framework for logarithmic prices. This allows for the existence of multiple and possibly interconnected non-stationary bubbles, under appropriate limiting sparsity conditions that render them asymptotically negligible. We extend the AAT21 stochastic dominance relation under local non-stationarity to the present framework that additionally involves the associated value transformations. Exploiting continuity

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repeat our analysis regarding the formation of optimal portfolios. For the sake of brevity we omitted the presentation of those cases in the main text. The results are presented in the online appendix; the associated analysis can be provided upon request.

properties of the transformations involved we construct a related statistical machinery for inference about stochastic spanning under the extended dominance relation, and derive its first order limit theory; we show that the subsequent test is asymptotically conservative and consistent.

We additionally take into consideration the Covid-19 outbreak's effect on the markets. We do so by focusing our analysis separately on the period prior of the COVID-19 outbreak (January 2020) and on the one afterwards. This partitioning enables us to focus on the pandemic's and the associated quarantines' effect on the financial behavior of the three personae.

Summarizing, for all personae we construct empirically optimal Augmented and Traditional portfolios using the StSp machinery, that are business-daily re-balanced, in a mildly explosive framework of asset returns, and taking account of the pandemic. We assess their out-of-sample comparative performance, using a battery of non-parametric and parametric tests. To the best of our knowledge this combination of BF personae value function transformations with stochastic spanning is novel. Our empirical results suggest that the Augmented portfolios outperform the optimal Traditional ones, in most cases.

The rest of the paper is organized as follows: in the following section we describe in detail our methodology, derive the limit theory that supports it, and present our empirical findings. In the final section we conclude while also discussing some behavioral implications. In the appendices we provide the proofs of our asymptotic analysis, as well as further information related to our empirical work. Finally, a separate Online Appendix contains the results of extra experiments regarding different variations of the traditional as well as the augmented choice sets.

## 2 Methodology and Empirical Results

Our empirical application begins with the manipulation of the dataset, which covers the period from mid-August 2015 to end-of August 2021. Our dataset contains business days closing prices for the following "traditional" assets (stock and bond indices): The S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. It additionally contains the aforementioned four cryptoassets, as well as 1M Gold Futures closing prices. We first "align" the dataset by keeping the closing prices of cryptoassets for the business days only, and afterwards we estimate the returns of all assets. This is because the most common practise for investors is to consider the aggregate performance of specific assets, or groups of assets.

We apply on the dataset the distorted value transformation associated with each BF persona. We conduct both in-sample as well as out-of sample tests. In the out-of-sample analysis, we solve the analogous stochastic spanning optimization problem in a rolling window pattern - thus obtain the empirically optimal portfolios - and then assess their performance. At each stage we exemplify related theoretical concepts and subsequently present the relevant empirical results.

## 2.1 Distorted Value Function Transformation

We are "distorting" the data according to cognitive biases that fall within any of the aforementioned personae. The data transformations are employed on the cross-section of returns. A new dataset is thus obtained where now the returns have been transformed into the relevant "behavioral" values, at which optimal investment strategies are then decided and executed.

We consider, for analytical convenience, a general form of distorted value function transformation (DVT);  $v : \mathbb{R}^d \rightarrow \mathbb{R}^d$ ,  $v_j(x) := c_j x_j^{\alpha_j} \mathbb{I}(x_j \geq 0) + c_j^*(L(x_j))^{\beta_j} \mathbb{I}(x_j < 0)$ , where  $c_j, c_j^*, \alpha_j, \beta_j$ ,  $j = 1, \dots, d$ , are appropriate constants and  $L : \mathbb{R} \rightarrow \mathbb{R}$  is a linear transformation. It encompasses all three transforms that we analytically present below, and thus facilitates derivations.

### 2.1.1 Cumulative Prospect Theory type of persona

CPT bypasses the drawbacks of Prospect Theory (henceforth PT) (Kahneman and Tversky, 1979). PT employs a monotonic transformation of outcome probabilities and this monotonic transformation can not be applied to prospects (i.e. portfolios) with any number of outcomes (is restricted to two). Also, PT does not always satisfy Stochastic Dominance (SD) and hence investors may choose dominated prospects (a mild form of irrationality).

Each of the  $d$  base assets is considered as a risky prospect. The set of risky projects is considered endowed with the discrete uniform distribution. Decision weights (or "capacities"), which are non-additive set functions that generalize the standard notion of probabilities, are applied to the prospects. Specifically, in the definitions below, the capacity  $\pi_i^+$  mathematically exemplifies the assertion "the outcome is at least as good as  $x_i$ " minus "the outcome is strictly better than  $x_i$ ". Analogously,  $\pi_i^-$  denotes that "the outcome is at least as bad as  $x_i$ " minus "the outcome is strictly worse than  $x_i$ ", with  $x_i$  being the return of the  $i^{\text{th}}$  prospect. In order to specify the above, we introduce transformations  $w^+, w^-$  which are both non-decreasing, while being inverse S-shaped respectively with  $w^+(0) = 0 = w^-(0)$  and  $w^+(1) = 1 = w^-(1)$  such that  $w^+, w^- : [0, 1] \rightarrow [0, 1]$ .

Given the cross-section of returns, we initially rank the latter in increasing order for every  $t$ . Suppose that  $m$  of these returns are negative, while the remaining  $n = d - m$  are positive. We label the sorted returns using  $r_{-m}$  for the minimum negative return and through to  $r_n$  for the maximum positive. The cross-sectional return distribution is then described by the array of pairs,

$$(r_{-m}, p_{-m}; r_{-m+1}, p_{-m+1}; \dots; r_{-1}, p_{-1}; r_0, p_0; \dots; r_{n-1}, p_{n-1}; r_n, p_n),$$

where  $p_j$  is the associated probability from the discrete uniform. Given  $w^+, w^-$ , the capacities are then defined by

$$\pi_n^+ = w^+(p_n) \quad \text{and} \quad \pi_{-m}^- = w^-(p_{-m})$$

$$\pi_i^+ = w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n) \quad \text{where} \quad 0 \leq i \leq n$$



and

$$\pi_j^- = w^-(p_{-m} + \dots + p_j) - w^-(p_{-m} + \dots + p_{j-1}) \quad \text{where} \quad -m \leq j < 0$$

In our analysis the PWF used is,

$$w^+(p) = \frac{p^{0.61}}{[p^{0.61} + (1-p)^{0.61}]^{\frac{1}{0.61}}} \quad \text{and} \quad w^-(p) = \frac{p^{0.69}}{[p^{0.69} + (1-p)^{0.69}]^{\frac{1}{0.69}}}.$$

This is justified by Ingersoll (2008), who provides a proof that for a range of values, the exponents in the PWF can induce non-monotonicity. This can lead to negative decision weights and preference for first order stochastically dominated (FSD) prospects. As Ingersoll (2008) recommends, the problem is tackled if both exponents of  $w^+$  and  $w^-$  are strictly above 0.279. Moreover, TK (1992) propose the following *S-shaped* functional form for the value function  $u : \mathbb{R} \rightarrow \mathbb{R}$ ,

$$v_P(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\beta & \text{if } x < 0 \end{cases},$$

where  $\lambda$  is the loss aversion coefficient ( $\lambda = 2.25$ , because losses 'hurt' twice as more than gains on average, TK 1992) and  $\alpha, \beta < 1$ ; where we use the same values as TK (1992), and set  $\alpha = \beta = 0.88$ .

We employ the estimates of TK (1992) for the parameter and the PWF exponent. Subsequent to TK (1992), several papers have used more sophisticated techniques, in conjunction with new experimental data, to estimate these parameters (Gonzalez and Wu 1999; Abdellaoui et al. 2007). Their estimates are similar to those obtained by TK (1992). Since in our experiments we do not need to estimate the overall value of a mixed prospect but rather the individual "behavioral" value of every asset, the transformation  $v_P : \mathbb{R}^d \rightarrow \mathbb{R}^d$  is

$$v_P(x) := (\pi_i^+ v_P(x_i), -m \leq i < 0, \pi_j^- v_P(x_j), 0 \leq j < n).$$

For  $c_j := \pi_j^+$ ,  $c_j^* := -\lambda \pi_j^-$ ,  $\alpha_j = a$ ,  $\beta_j = b$  and  $L(x) = -x$  our general DVT reduces to CPT.

### 2.1.2 Markowitz type of persona

We follow the work of Markowitz (1952a) who argues that the utility function of Friedman and Savage (1948) must have convex, as well as concave segments, near the point of origin. In this regard an inverse S-shaped value function emerges where the curvature changes at the point of origin which separates gains from losses. This type of investor is averting losses in the losses domain and seeking risk in the gains domain. Investors employ a reverse S-shaped value function, when it comes to bets whose outcomes are moderate and not too extreme. Under this context, no PWF is applied nor any loss aversion coefficient. We thus employ the following transformation  $v_M : \mathbb{R}^d \rightarrow \mathbb{R}^d$ :

$$v_{M,j}(x) = \begin{cases} x_j^\alpha & \text{if } x_j \geq 0 \\ -(-x_j)^\beta & \text{if } x_j < 0 \end{cases}, \quad j = 1, \dots, d,$$

where  $\alpha, \beta > 1$ ; thus, in order to satisfy the curvature as described in theory, we are allowed to set  $\alpha = \beta = 2$ , without harming generality. For  $c_j := 1, c_j^* := -1, \alpha_j = a, \beta_j = b$  and  $L(x) = -x$  our general DVT reduces to the Markowitz transform.

### 2.1.3 Loss averse type of persona

Following the work of Barberis, Huang and Santos (2002) we employ a value function that refers to investors who specialize in averting losses. The aforementioned authors argue that loss aversion is by itself sufficient in capturing and explaining major phenomena of choice, such as the disposition effect. Moreover, Benartzi and Thaler (1995) use the loss aversion behavioral bias to explain the equity premium puzzle. They find that investors evaluate the aggregate stock market by computing the PT value of its historical distribution. Loss aversion is regarded as the most important bias in the BF literature and the novelty of our approach stems from its use on the cross-section of returns. Avoiding losses is a major concern in investing because if they are realized, they affect expectations (developed endogenously) about future performance. These rational expectations serve as "anchors" (Kőszegi and Rabin, 2006), provide *wishful* targets to be met, and thus operate as bounds between gains and losses. We work with the following LA type of transformation  $v_{LA} : \mathbb{R}^d \rightarrow \mathbb{R}^d$ :

$$v_{LA,j}(x) = \begin{cases} x_j & \text{if } x_j \geq 0 \\ 2.25x_j & \text{if } x_j < 0 \end{cases}, j = 1, \dots, d.$$

Under this setting, the value function is piecewise with its lower set being convex. It is increasing, implying explicitly that investors are non-satiated, while its segmental linearity can implicitly imply risk aversion. It is kinked at the origin, where gains are zero which is the most often used boundary between gains and losses. For  $c_j := 1, c_j^* := \lambda, \alpha_j = 1, \beta_j = 1$  and  $L(x) = x$  our general DVT reduces to LA.

## 2.2 Mildly explosive framework with multiple bubbles

In the next stage we employ the concept of spanning on the personae transformed dataset. The incorporation of cryptoassets' returns suggests that we may need to account for potentially parodically explosive dynamics in the associated DGP of base asset returns as far as the limiting properties of our statistical procedures are concerned. In this paragraph we describe our assumption framework that deals with this.

We work with a portfolio space defined as the set of positive convex combinations of the DVT transform on the returns of  $d$  base assets and represented by the  $d-1$  simplex  $\{\boldsymbol{\lambda} \in \mathbb{R}_+^d : \boldsymbol{\lambda}^T \mathbf{1}_d = 1\}$ . The framework is broad enough to allow for base assets that are themselves constructed via complicated portfolio constraints on deeper underlying individual securities, like short sales, position limits, and restrictions on factor loadings. Some of the base assets are cryptoassets. Their empirical locally explosive behavior necessitates the considerations of (locally) non stationary return processes. We employ the framework of AAT21 of mildly explosive VAR(1) processes for the logarithmic prices, along with almost exact versions of their ME and AN Assumptions. In what follows

and depending on the context,  $\|\cdot\|$  denotes either the Euclidean norm on  $\mathbb{R}^d$  or the Frobenius norm on the space of  $d \times d$  real matrices. Moreover,  $c$  denotes a generic positive constant that may assume different values in different occurrences. We also denote with  $\delta$  some constant greater than or equal to  $\max(\max_j \max(\alpha_j, \beta_j), 1)$ .

**Assumption ME.**

1.  $(\varepsilon_t)_{t \in \mathbb{N}}$  which is an  $\mathbb{R}^d$ -valued stationary and strong mixing process with mixing coefficient sequence  $(\alpha_m)_{m \in \mathbb{N}}$  that satisfy  $\alpha_m = O(m^r)$  for some  $r > 1$ . Furthermore,  $\exists L, q > 0$  such that for large enough  $t > 0$ ,  $\mathbb{E}[\exp(t\|\varepsilon_0\|)] \leq \exp(Lt^q)$ .
2. For the sample size  $T \in \mathbb{N}^*$ ,  $\{0, \dots, T\}$  is partitioned in  $K$  mild-explosivity periods  $B_k$ ,  $k = 1, \dots, K$  and the remaining stationary periods  $\cap_{k=1}^K B_k^c$ .
3. The logarithmic prices  $\mathbb{R}^d$ -valued process sequence satisfies the recursion  $\mathbf{X}_t = \left( Id + \sum_{k=1}^K \frac{C_k}{M(T,k)} \mathbb{I}\{t \in B_k\} \right) \mathbf{X}_{t-1} + \varepsilon_t$ ,  $t > 0$ , where  $C_k$  is a positive  $d \times d$  explosivity coefficient matrix at the  $k^{\text{th}}$  explosive period, and  $M(T, k) > 0$  and diverging to infinity as  $T \rightarrow \infty$ , which represents the rate at which the  $k^{\text{th}}$  explosive behaviour vanishes as a function of  $T$ . The process is initiated by  $\mathbf{X}_0$  and  $\exists L^*, q^* > 0$  such that for large enough  $t > \delta$ ,  $\mathbb{E}[\exp(t\|\mathbf{X}_0\|)] \leq \exp(L^*t^{q^*})$ .

*Remark 1.* ME.1 allows for a large variety of linear and/or conditionally heteroskedastic models typically used for the stationary parts of logarithmic returns in empirical finance (see, for example, Drost and Nijman, 1993; Basrak et al., 2002). This allows for models that exhibit stationary, ergodic and geometrically mixing temporal dynamics, along with innovation distributions that have densities (see AAT21 for examples). ME.2 allows for the existence of  $K$  sub-periods of non-stationary bubbles in parts of the base assets process. It is also allowed that  $K \rightarrow \infty$  as  $T \rightarrow \infty$ . This implies that the number of bubbles need not asymptotically stabilize, though the following assumption will specify bounds on the intensity of the bubbles as  $T$  grows. In the third part, at each bubble period  $k$  the structure of the explosiveness coefficient matrix  $C_k$  is general enough to allow for intra-bubble dependence of currently explosive base assets on the dynamics of other currently and/or previously explosive assets, as well as on assets that are never explosive. Obviously for the latter the relevant blocks of  $C_k$  are zero for all  $k$ . The moment generating function conditions that appear in ME.(1),(3) are compatible with sub-Gaussian and sub-exponential distributions for the random variables involved (see, for example, Chapter 2 of Vershynin, 2018)-see AAT21 for a Gaussian example. They do not allow for multivariate distributions that do not possess moment generating functions; e.g. non-Gaussian stable distributions. Due to the presence of  $\delta$  in their formation, those conditions may be slightly stronger than AAT21.

The base assets DVT of the locally non-stationary net return process is thus  $v(\mathbf{R}_t) := v\left(\exp^*\left(\sum_{k=1}^K \frac{C_k}{M(T,k)} \mathbb{I}\{t \in B_k\} \mathbf{X}_{t-1} + \varepsilon_t\right) - \mathbf{1}\right)$ ,  $t > 0$  with  $\exp^* : \mathbb{R}^d \rightarrow \mathbb{R}^d$  defined by  $\exp^*(\mathbf{y}) :=$

$(\exp(\mathbf{y}_1), \dots, \exp(\mathbf{y}_d))^T$  and  $\mathbb{R}^d \ni \mathbf{1} := (1, \dots, 1)^T$ .  $\mathcal{X}$  denotes the convex hull of the union of the supports of the elements of  $v(\mathbf{R}_t)$ . It is bounded from below by  $\min_j c_j^* L(-1)^{\beta_j}$ .

Given that  $K$  is allowed to diverge, the following assumption prescribes restrictions between the singular values of the explosivity coefficient matrices  $C_k$ , the degree of the return to the random walk dynamics  $M(T, k)$  and the maximal bubble time instance  $\max_{t,k} B_k$ . It is identical to the homonymous assumption of AAT21.

**Assumption AN.**  $\exists c > 0, \epsilon > 0$  such that  $\max_k \frac{\|C_k\|^{(d+\epsilon)^{\max_{t,k} B_k}}}{M(T,k)} \leq c$  for all  $T \geq 0$ . As  $T \rightarrow \infty$ ,  $\frac{\max_{t,k} B_k}{\sqrt{T}} = o(1)$ .

*Remark 2.* AN is satisfied when  $\|C_k\|$  is bounded in  $k$ ,  $\max_{t,k} B_k \sim c \ln T$  and  $M(T, k) = \delta_k T^{a_k}$ , with  $\min_k \delta_k, \min_k a_k > 0$  (such choices are compatible with the bubble duration conditions of Phillips et al., 2015). The condition is not empirically identifiable and it concerns the future behaviour of bubbles. As remarked in AAT21, it allows for slowly diverging bubble durations compatible with future improvements in the technology and the regulation framework associated with the creation and circulation of cryptoassets. AAT21 point out that this is compatible with future improvements in the technology and the regulation framework associated with the creation and circulation of cryptoassets; e.g. stricter regulation as a result of series of severe bubbles, or the development of derivative markets on cryptoassets, as long as those deter investors from forming expectations via bubble producing sunspot processes, or force investors to correct parameters associated with fundamentals. AN also allows for more complicated behaviours; for example it is compatible with unbounded  $\|C_k\|$  as  $k$  grows, thus intense intra-bubble feedbacks between explosive assets, mitigated by stronger degrees of the return to the random walk dynamics due to learning mechanisms like the above. In any case, AN prescribes the exact conditions on the explosive dynamics parameters of the logarithmic prices process, that imply asymptotic dominance of the stationary dynamics in the formation of the lower partial moments differentials employed in the SD relations.

### 2.2.1 In-sample empirical results: evidence of bubbles and mild explosivity

Phillips and Magdalinos (2007) argue that when asset prices exhibit explosive behavior this signals an underlying bubble behavior. Within this framework, we test the existence of multiple speculative bubbles in the cryptoassets market by using the PSY methodology (Phillips et al., 2015).

The PSY test, relies on right-tailed Dickey-Fuller tests via a recursive estimation over rolling windows of increasing sizes. It is solely applied to cryptoassets since their market has experienced several and severe turmoils; the relevant literature concurs that this may be a quite strong indication for the presence of bubbles. The smallest window width fraction is manually chosen, in order to initiate computations, while the largest is the total sample size. We follow Phillips and Shi (2020) and use 10% as a starting fraction of the cryptoassets dataset. The test's null hypothesis assumes that logarithmic prices have a unit root random walk; the alternative assumes that there exists at least one subperiod exhibiting mild explosivity. When the null is rejected, the procedure is also used for date-stamping multiple bubbles.

The shaded green areas in the figures in Appendix 2 are the identified multiple bubble periods, obtained by using the the 95% bootstrap critical values, for BTC, ETH, XRP and LTC respectively. Moreover, Table 1 reports the total number of explosivity days that each cryptoasset exhibits, which stems from the number of times the test statistic exceeds the critical value. BTC exhibits the highest number (365) of explosivity days; it has attracted most invested capital in the crypto-market and has been also the starring cryptoasset for many scams. The second is ETH (335), with LTC (230) and XRP (192) following. The bubble periods for all cryptoassets do not in general coincide, however they do intersect on numerous dates. On average, it seems that the period starting from early 2017 till end-of 2018 has been for all four cryptoassets the period with the highest frequency of multiple bubbles.

Table 1: Number of explosive days

BTC	ETH	XRP	LTC
365	335	192	230

Entries report the number of days characterised by explosive price behaviour for the four analysed cryptoassets over the period from mid-of August 2015 to end-of August 2021, for a total of 1527 business days returns.

The bubble tests along with their time-stamping provide some evidence in favour of Assumption AN: given the large sample period under study it is seen that for all cryptoassets' bubbles are sparse enough and of durations consistent with AN.

### 2.3 Stochastic dominance and stochastic spanning on distorted values

An SSD approach (Levy, 2006) on the behaviorally modified returns is used due to its non-parametric nature and its relation with risk aversion. To relate risk aversion (i.e. the traditional approach in Modern Portfolio Theory) with the aforementioned personae, we follow various works (Barberis, Mukherjee and Wang, 2016; Koszegi and Rabin 2006, among others) who argue that a better description of reality is the one where behavioral investors pay at least some attention to traditional factors. Additionally, it seems that a significant part of investors exhibit risk aversion. For example, the fact that the riskless interest rate is generally lower than the cost of capital of most firms, is an indication that investors are risk averse and require a risk premium (Levy, 2006).

SD ranks investments based on general regularity conditions (Hadar and Russel, 1969; Hanoch and Levy 1969; Rothchild and Stiglitz, 1970) for decision making under risk and it can be seen as a model-free alternative to Mean-Variance (M-V) dominance (Levy, 2006). SD is quite appealing because it accounts all moments of the returns' distribution without assuming any particular family of distributions nor specific preferences on behalf of agents. Because of its non-parametric nature, it is quite useful for assets with asymmetric risk profiles, like cryptoassets.

StSp (w.r.t. SSD) occurs if introducing new securities or relaxing investment constraints does not improve the investment opportunity set, uniformly over the class of increasing and concave utilities. StSp can be seen as a model-free alternative to M-V spanning (Huberman and Kandel, 1987) that accounts for higher-order moment risk in addition to variance. StSp involves the comparison of two

choice sets, not necessarily disjoint (e.g. nested). It evaluates all feasible portfolios, even the ones that include a relatively small number of assets and thus are more susceptible to higher moment risk. In other words, StSp is basically an SSD order-preserving reduction of the portfolio opportunity set.

### 2.3.1 Stochastic dominance in the mildly explosive framework

We employ SSD in our mild explosivity framework. We follow closely the constructions of AAT21. Our main difference lies in the fact that the associated lower partial moments that define the order, are w.r.t. the DVT of the net returns process, instead of the returns process per se. We thus modify the analogous definitions and notations of AAT21 as follows; for  $z \in \mathcal{X}$  and  $\boldsymbol{\kappa}, \boldsymbol{\lambda}$  elements of the unit  $d - 1$  simplex define, and for  $v$  the general form of DVT defined above:

$$\begin{aligned} D(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R}_t)) &:= (z - \boldsymbol{\kappa}^\top v(\mathbf{R}_t))_+ - (z - \boldsymbol{\lambda}^\top v(\mathbf{R}_t))_+, \\ D_T(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})) &:= \frac{1}{T} \sum_{t=1}^T D(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R}_t)), \\ D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})) &:= \lim_{T \rightarrow \infty} \mathbb{E}[D_T(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R}))]. \end{aligned}$$

Due to stationarity,

$$D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\boldsymbol{\varepsilon})) = \mathbb{E} \left[ (z - \boldsymbol{\kappa}^\top v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1}))_+ \right] - \mathbb{E} \left[ (z - \boldsymbol{\lambda}^\top v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1}))_+ \right],$$

which is the standard LPM differential employed in SSD, yet here w.r.t. the DVT of the stationary part of the returns.

As in AAT21, the assumptions above, imply an asymptotic negligibility property for the totality of bubble periods, with respect to some of the functionals above, when properly scaled. This is essentially represented by the following general result of central importance, the proof of which is quite similar to the proof of Proposition 3 of AAT21:

**Proposition 1.** *Suppose that Assumptions ME and AN hold. Then uniformly in  $z, \boldsymbol{\lambda}$  as  $T \rightarrow \infty$ ,*

$$\mathbb{E} \left[ \left| \frac{1}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} (z - \boldsymbol{\lambda}^\top v(\mathbf{R}_t))_+ - \frac{1}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} (z - \boldsymbol{\lambda}^\top v(\exp^*(\boldsymbol{\varepsilon}_t) - \mathbf{1}))_+ \right| \right] = o(1).$$

Then, Assumption ME, Proposition 1 and dominated convergence imply that the functional  $D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R}))$  is well defined, bounded and continuous in  $(z, \boldsymbol{\kappa}, \boldsymbol{\lambda})$ . We thus apply the AAT21 definition of SSD, which is compatible with the framework of multiple non-stationary bubbles (MESSD), for the DVT of the returns as follows:

**Definition 1.**  $\boldsymbol{\kappa} \succeq_{v\text{-MESSD}} \boldsymbol{\lambda}$  iff  $\forall z \in \mathcal{X}, D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})) \leq 0$ .

The Cezaro-limit based definition of  $D^*$  is similar to Definition 5.1 of Jin *et al.* (2017) that handles distributional heterogeneity in the context of forecast comparison. It corresponds to a limiting Lebesgue–Stieltjes (discrete) integration (across time) of the LPM differentials that collapses to the standard definition of SSD under stationarity. Then, the auxiliary Proposition 2 (see Appendix) directly implies that:

**Corollary 1.** *Under Assumptions ME and AN,  $\boldsymbol{\kappa} \succeq_{v\text{-MESSD}} \boldsymbol{\lambda}$  iff  $\forall z \in \mathcal{X}, D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\boldsymbol{\varepsilon})) \leq 0$ .*

Thus, Assumption AN ensures that  $v$ -MESSD is essentially (DVT-) SSD between the DVTs of the stationary part of the returns. As in AAT21, the assumption forces the non-stationary contributions to  $D_T(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R}))$  to asymptotically vanish.

### 2.3.2 Stochastic spanning in the mildly explosive framework

Consider two non-empty subsets of the general portfolio space,  $K \subset \Lambda$ , which are also assumed to be closed and simplicial, to facilitate among others the invocation of properties of convex optimisation. We employ the concept of MESSD spanning of AAT21, to the DVTs of the associated returns, obtaining:

**Definition 2.**  $K \succeq \Lambda$  iff  $\forall \boldsymbol{\lambda} \in \Lambda, \exists \boldsymbol{\kappa} \in K : \boldsymbol{\kappa} \succeq_{v\text{-MESSD}} \boldsymbol{\lambda}$ .

Arguments involving continuity and compactness imply that  $K \succeq_{v\text{-MESSD}} \Lambda$  iff

$$\eta^* := \sup_{\Lambda} \inf_K \sup_{\mathcal{X}} D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})) = 0.$$

Then auxiliary Proposition 2 (see Appendix) implies that  $v$ -MESSD spanning equivalently holds iff  $\eta := \sup_{\Lambda} \inf_K \sup_{\mathcal{X}} D(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1})) = 0$ . Consider then statistically testing the hypothesis structure  $\mathbf{H}_0 : K \succeq_{v\text{-MESSD}} \Lambda$  vs.  $\mathbf{H}_1 : K \not\succeq_{v\text{-MESSD}} \Lambda$ . Since under Assumption AN, the null hypothesis is equivalent to that  $\eta^* = 0$ , auxiliary Proposition 3 (see Appendix) and the latency of the stationary part of the logarithmic returns process,  $(\varepsilon_t)_t$ , imply that, under Assumptions ME and AN, the spanning test statistic of AAT21 evaluated at the DVTs of the non-stationary returns sample is usable. We thus employ a scaled empirical analogue of  $\eta^*$ , namely,

$$\eta_T^* := \sup_{\Lambda} \inf_K \sup_{\mathcal{X}} \sqrt{T} D_T(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})).$$

The asymptotic decision rule is to reject  $\mathbf{H}_0$  in favor of  $\mathbf{H}_1$  iff  $\eta_T^* > q(\eta_{\infty}^*, 1 - \alpha)$ , which is the  $(1 - \alpha)$  quantile of the distribution of the null limiting distribution of the statistic at a significance level  $\alpha \in (0, 1)$ . The quantile is expected to-among others-depend on latent parameters, like the dependence structure of  $(\varepsilon_t)_t$  (see also Theorem 1 below and the auxiliary Proposition 4 in the Appendix). As in AAT21, we approximate the quantile via the use of a subsampling procedure.

A choice of the subsampling rate,  $1 \leq b_T < T$ , generates maximally overlapping subsamples  $(R_s)_{s=t}^{t+b_T-1}$ ,  $t = 1, \dots, T - b_T + 1$ . We evaluate the test statistic on each subsample, thereby obtaining  $\eta_{b_T; T, t}^*$  for  $t = 1, \dots, T - b_T + 1$ , obtaining  $q_{T, b_T}(1 - \alpha)$ , the  $(1 - \alpha)$  quantile of the empirical distribution of  $\eta_{b_T; T, t}^*$  across the subsamples. Using the above, the modified decision rule is to reject  $\mathbf{H}_0$  in favor of  $\mathbf{H}_1$  iff  $\eta_T^* > q_{T, b_T}(1 - \alpha)$ .

The following result exemplifies the first order limit theory of the procedure and asymptotically rationalizes the empirical results of the following section. Under Assumptions ME-AN and a standard subsampling rate restriction we obtain:

**Theorem 1.** As  $T \rightarrow \infty$ , under Assumptions ME and AN, and if  $(b_T)$ , possibly depending on  $(R_t)_{t=1,\dots,T}$ , satisfies  $\mathbb{P}(l_T \leq b_T \leq u_T) \rightarrow 1$ , where  $(l_T)$  and  $(u_T)$  are real sequences such that  $1 \leq l_T \leq u_T$  for all  $T$ ,  $l_T \rightarrow \infty$  and  $\frac{u_T}{l_T} \rightarrow 0$  as  $T \rightarrow \infty$ :

1. Under  $\mathbf{H}_0 : K \succeq_{v\text{-MESSD}} \Lambda$ ,

$$\eta_T^* \rightsquigarrow \sup_{\Lambda} \inf_K \sup_{\mathcal{X}} \mathcal{L}_v(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}), (z, \boldsymbol{\kappa}, \boldsymbol{\lambda}) \in CS,$$

where the and the limiting Gaussian process  $\mathcal{L}_v(z, \boldsymbol{\kappa}, \boldsymbol{\lambda})$  is defined in Proposition 4, and the contact set CS is defined by

$$CS := \{(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}) : \boldsymbol{\lambda} \in \Lambda, \boldsymbol{\kappa} \in \mathbf{K}, \boldsymbol{\kappa} \succeq_{v\text{-MESSD}} \boldsymbol{\lambda}, z \in \mathcal{X}, D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\varepsilon)) = 0\}.$$

2. Under  $\mathbf{H}_0 : K \succeq_{v\text{-MESSD}} \Lambda$ , and if  $\exists (z^*, \boldsymbol{\kappa}^*, \boldsymbol{\lambda}^*) \in CS : \text{Var}(\mathcal{L}_v(z^*, \boldsymbol{\kappa}^*, \boldsymbol{\lambda}^*)) > 0$  then the testing procedure is asymptotically exact if  $\alpha < 0.5$ .
3. Under  $\mathbf{H}_1 : K \not\succeq_{v\text{-MESSD}} \Lambda$ , the testing procedure is consistent.

Theorem 1 is the direct analogue of Theorem 7 of AAT21, given that the testing procedure is applied on the DVTs of the locally non-stationary base return process, instead of the returns per se. The existence of a non trivial-in terms of asymptotic variance-condition in part 2 ensures exactness under every empirically plausible choice of significance level. The existence is ensured when there exists a pair  $\boldsymbol{\kappa}^*, \boldsymbol{\lambda}^*$  such that the  $\boldsymbol{\kappa}^*$  DVT based portfolio may be chosen over  $\boldsymbol{\lambda}^*$  by any Russell-Seo (Russell and Seo, 1989) elementary utility, except for at least one utility that chooses both, and this corresponds to a threshold level at the interior of intersection of the supports of the two portfolios. When the existence of a non trivial contact condition does not hold, arguments similar to Assumption 4.1.4 of Arvanitis *et al.* (2020) can be employed to ensure asymptotic conservatism.

### 2.3.3 Empirical results: in-sample spanning tests

We test whether the traditional portfolio universe, consisting of Gold, stock and bond indices,  $v$ -MESSD spans the augmented, with added cryptoassets, for each investor persona, under the framework of mild explosivity developed previously.  $\Lambda$  denotes the augmented portfolio universe having as vertices the crypto-assets in addition to the traditional assets.  $\mathbf{K}$  is the one has only the traditional assets as vertices. We partition our initial dataset into two parts, namely the one before the COVID-19 outbreak (January 2020) and the one afterwards. We do so in order to focus on the pandemic's effect in the financial behavior of the three personae. We employ the Linear Programming (LP) formulations as in Arvanitis *et. al* (2019), on the distorted dataset depending on the various investor types. The subsampling distribution of the test statistic is derived for subsample size  $b_T \in [T^{0.6}, T^{0.7}, T^{0.8}, T^{0.9}]$ .

Table 2 reports the test statistics  $n_T^*$ , the associated critical values  $q_T^{BC}$  together with the relevant decisions, for the three investor personas. To mitigate specification error of the subsample length



Table 2: Covid-19 outbreak in-sample performance with  $v$ -MESSD spanning test.

	Before			After		
	$n_T^*$	$q_T^{BC}$	Decision	$n_T^*$	$q_T^{BC}$	Decision
CPT	0.01236	0.01846	Spanning	0.00717	0.01172	Spanning
Markowitz	-0.05359	-0.14836	Reject	-0.09755	-0.45392	Reject
Loss Aversion	-0.02368	-0.05804	Reject	-0.04487	-0.08191	Reject

Entries report test statistics and critical values for stochastic spanning test of the augmented portfolios with respect to the traditional portfolio. The dataset spans the period from mid-of August 2015 to end-of August 2021, separated on January 2020, for a total of 1527 business days returns.

and correct for bias in finite samples, we employ the bias-correction method of Arvanitis et. al. (2019), where the regression estimates  $q_T^{BC}$  are given for significance level  $\alpha = 0.05$ .

The null hypothesis is rejected for the Markowitz and Loss Averse investor types, not for the CPT type though. Thus, we can argue that for the chosen significance level, the performance of traditional portfolios can be improved with the inclusion of the four cryptoassets for some Markowitz and Loss Averse investors; this does not seem to hold for CPT investor types.

### 2.3.4 Out-of-sample analysis

For each investor type, we are exploring the out-of-sample performance of her augmented optimal portfolio in comparison to the traditional one. We apply DVT on the raw returns on each business day and hence we obtain a "behavioral" new dataset, either for the before- or after- era of the COVID-19 outbreak (January 2020), in the Western world. We construct optimal portfolios based on the behavioral information up to time  $t$  by evaluating the spanning statistic, and we then reap their actual returns, given the actual data at time  $t+1$ . We record optimal portfolios separately for the traditional and the augmented opportunity sets, where the relevant optimisation problem is solved for the stochastic spanning test. The clock is set forward and we collect the realized returns of the optimal portfolios. This procedure is repeated for all subsequent business day returns till the end-of August 2021. In all cases we follow the reasoning that investors separate gains and losses, as well as they re-balance daily, after the first training year. We thus focus on the decisions investors would take about the optimal choice of portfolio weights, decisions that stem from the various behavioral elements that are being employed. We do not apply the DVTs to the returns of the optimal portfolios; we evaluate their performance using their realized actual returns.

This out-of-sample exercise involves a rolling window of 1 year (the most usual time period for assessing performance and rebalancing, see for example Barberis and Thaler 2003) and covers the time span from August 2016 till end of August 2021. We are primarily interested in observing how, during these two aforementioned characteristic periods, the three investor personae allocate weights in their optimal portfolios, on average. Hence, one can observe not only how aggregate gains and losses, but also overall sentiment, affect their decision making process.

We begin our analysis with the pro-outbreak period where the realized returns time series for all cases are presented in Figure 1. Despite the differences in aggregate returns, the traditional underperforms the augmented for the CPT and LA cases, while for the Markowitz case their performance

is more or less the same, on average, except from a breakout in the performance of the augmented in the last few months. For the CPT and LA, the augmented portfolio performs on average 5% more than the traditional, while for Markowitz the spread between the augmented and the traditional is about 1,5%. Another interesting feature is that for all investor types, the augmented portfolios include relatively small weights for the cryptoassets (see Table 6 in the Appendix).

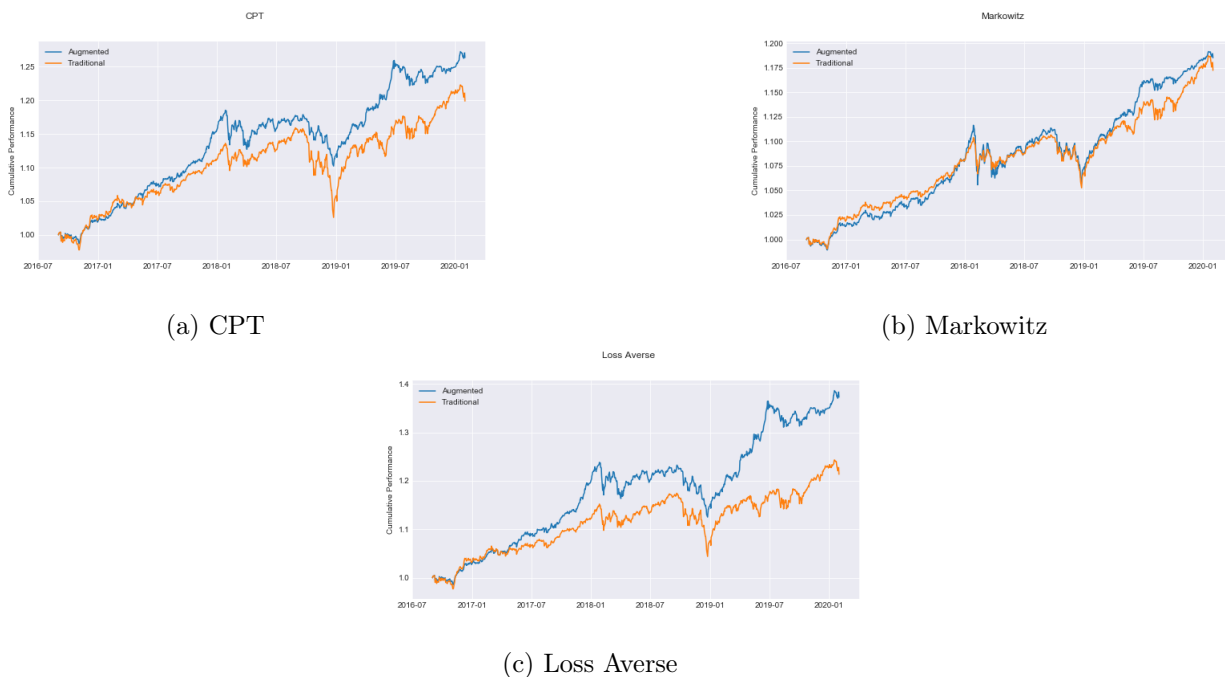


Figure 1: Before January 2020 (pandemic outbreak)

We then move on to the after outbreak period where the realized returns time series for all cases are presented in Figure 2. Now all investor types perform better when they include cryptoassets in their portfolios. The average outperforming spread ends up being about 60% for CPT, 40% for LA, and finally 10% for Markowitz. Again, the augmented portfolios include relatively small weights for the cryptoassets.

Overall, we observe that in the pre-outbreak era the average total weight on cryptoassets was 2% for CPT, 0.5% for Markowitz and 2.5% for LA. While, in the post-outbreak period we obtain 0.6%, 0.1% and 0.9%, respectively. This "depreciation" could be partially explained on the grounds that given the "exotic" nature of the cryptoassets in combination with the uncharted waters of a global economy entering into a deceleration, because of the pandemic, investors withdrew capital and turned to more typical investment vehicles. Another interesting outcome is that no matter the relatively safe nature of Gold as an investment (basically because of its properties as a material), it did not manage to attract capital but rather repel it. During these two periods under examination we observe that after January 2020 the average optimal weight on Gold was 0.018% while before January 2020 was 0.061%. The asset that attracted the most capital in order to bounce this new form of uncertainty was the 1M-TBill. It climbed from an average of 44% to 52%, among all three

investor types, with CPT being the one investing the most, LA followed while Markowitz on the contrary withdrew funds. Overall, we could say that the Markowitz type exhibited a temperate behavior, regarding rebalancing between her Traditional and Augmented portfolios, after January 2020. The usual practice of *not-acting-till-we-see*, when entering into unprecedented situations, led to average lower aggregate returns for her.

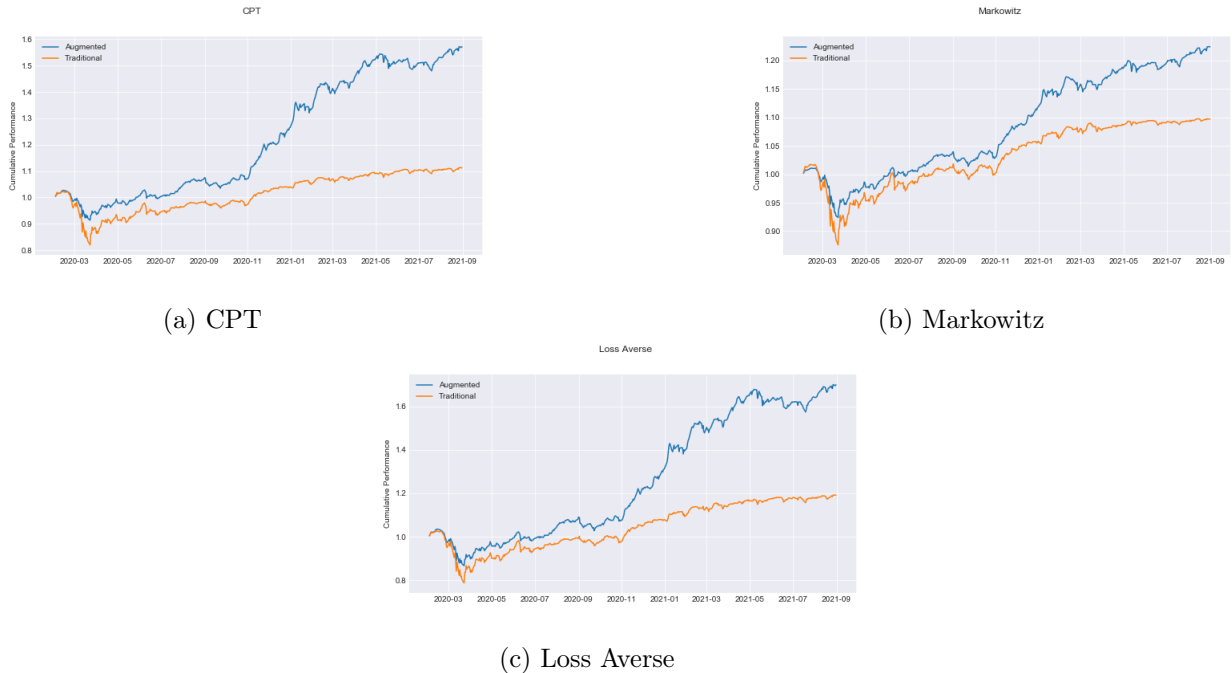


Figure 2: After January 2020 (pandemic outbreak)

For all three personae, anticipated gains and losses play a tremendous role in the formation of portfolios, out-of-sample. Interestingly, we observe in Table 8 that all investor types prefer to invest lightly on cryptoassets, even though their average returns are higher compared to all other asset classes (Table 4). Thus, we could say that the main drivers of portfolio choice are not the high returns per-se but rather low dispersion (S.D.), which is the highest in the case of cryptoassets. Hence, large standard deviations indicate abrupt gains and losses. Although all three types exhibit different behaviors in gains and losses, they all depart from averting risk. That is why we observe high fund concentration on safer asset classes. These asset classes demonstrate low S.D. and relative high mean. In this case, these are: the S&P 500 Index, the Barclays Bond Index and the 1M T-Bill. They attract an average capital of 71% in the traditional portfolios for all three personae, for both periods. Subsequently, when cryptoassets come on the foreground this percentage climbs to an average of 76.5%, again by considering both periods. This 5.5% difference is a strong indication of skepticism towards the crypto-market. Investors do want to benefit from the inclusion of cryptoassets but at the same time they are aware of the risk inherent in these exotic products, let alone the novel global status-quo of the pandemic with the associated all-time high financial uncertainty.

### 2.3.5 A conservative test for pairwise (non-) dominance

We perform a non-parametric out-of-sample performance comparative assessment of the optimal portfolios, using the Davidson and Duclos (2013) pairwise (non-) dominance test, as modified by AAT21 to allow for the mildly explosive framework. The modified test retains as null the less logically strict hypothesis of non-dominance, yet in a composite form concerning the comparison between every possible cluster point of the empirically optimal portfolio weights sequences.

The procedure allows for processes that appear in the context of Assumptions ME-AN, and for stochastic portfolio weights that may not be consistent or convergent at all. This is generally expected to be the case for the empirically optimal portfolio emerging from the stochastic spanning criterion; it need not have unique optimisers.

The test is applied exactly in its AAT21 form, on the returns of the empirically optimal portfolios, and not on their DVTs. Thereby, for brevity, we do not present the details on the hypothesis structure, the form of the test statistic and the rejection region, and its limiting properties. Those are discussed in AAT21 and the interested reader is referred there. We however point out the following:

The null hypothesis is that for any pair of cluster points between the associated portfolio, the cluster point of the first, does not MESSD the second; MESSD is now the dominance relation defined before for  $v$  equal to the identity. The alternative posits the existence of a pair for which the first dominates the second.

The test statistic is essentially a supremum of easily computable  $t$  statistics over  $\mathcal{X}$ , and the rejection region is based on the standard normal distribution. Thereby, the procedure is independent of the choice of numerical approximation parameters (like the subsampling length) as resampling based approximations of the limiting rejection region are avoided.

Under assumptions involving properties of the associated cluster points and the LPMs around them, and if a consistent estimator for long run covariances is used, the test is shown to be asymptotically conservative and consistent.

### 2.3.6 Out of sample empirical results: pairwise (non-) dominance

We apply our modification of the Davidson and Duclos (2013) pairwise (non-) dominance test on the two optimal portfolios derived in the previous rolling analysis for experiment 1.

Table 3 reports the quartile  $p$ -values from the distribution of daily portfolio returns, for the null hypothesis that the augmented portfolio does not stochastically dominate the traditional one by second order (see Davidson and Duclos, 2013). The results entail  $T - 1$  (1157 in total) overlapping periods for the in-sample fitting of the two portfolios with corresponding out-of-sample comparisons. The  $T - 1$   $p$ -values are considered from September 1, 2016 to January 31, 2020 and February 2021 to August 31, 2021, using overlapping periods of 100 daily returns in all three cases, in both cases. The quartile  $p$ -values from the distribution of the  $T - 1$  modified t-test statistics are computed.

We observe that, for the 25% and 50% (in 5%) quartile  $p$ -values, the null hypothesis that the augmented optimal portfolio does not stochastically dominate the traditional one by second order

is rejected in all cases. Hence, the out-of-sample performance of optimal portfolios constructed by every investor persona, that include cryptoassets seems dominant to the performance of the corresponding benchmark.

Table 3: Out-of-sample performance: Non-parametric stochastic dominance test

Traditional vs Augmented	Before	After
<b>CPT</b>		
Quartile		
25% Rejection rate	33.76%	28.45%
50% Rejection rate	41.57%	39.42%
75% Rejection rate	47.55%	48.55%
<b>Markowitz</b>		
Quartile		
25% Rejection rate	9.89%	23.42%
50% Rejection rate	13.35%	30.22%
75% Rejection rate	23.92%	41.23%
<b>Loss aversion</b>		
Quartile		
25% Rejection rate	41.96%	30.11%
50% Rejection rate	47.78%	38.22%
75% Rejection rate	52.18%	45.92%

Entries report quartile rejection rates from the distribution rejection rates across out-of-sample periods under the null hypothesis that the augmented cryptocurrencies optimal portfolio does not second order stochastically dominate the optimal traditional portfolio using a modification of the Davidson and Duclos (2013) test statistic, over the period from are considered from September 1, 2016 to January 31, 2020 and February 2021 to August 31,2021.

### 2.3.7 Further out-of-sample empirical results: parametric tests

We apply a set of commonly used parametric performance measures: the Sharpe ratio, the downside Sharpe ratio (DS) (Ziemba, 2005), the upside potential (UP) and downside risk ratio (Sortino and van den Meer, 1991), the opportunity cost (Simaan, 1993), the portfolio turnover (P.T.) and a measure of the portfolio risk-adjusted returns net of transaction costs (RL). Since assets' returns exhibit asymmetric return distributions, the downside Sharpe and UP ratios are more appropriate measures than the typical Sharpe ratio. For the compatibility of those with Assumptions ME-AN see AAT21, as well as the online appendix for their formulas.

Table 5 reports the parametric performance measures, where the higher the value of each one of these measures, the greater the investment opportunity for including cryptoassets. The results show that the inclusion of cryptoassets into the opportunity set, increases all performance measures for all three investor types in the period prior to January 2020, as well as after January 2020. We observe that since in all cases the UP ratio increases, all investors exhibit a benefit from the inclusion of cryptoassets. Furthermore, we observe that portfolios with only traditional assets induce less portfolio turnover than the ones with cryptoassets, which analogously creates more portfolio turnover for all types. Additionally, we can see that the return-loss measure is positive for all types. Thus, the traditional portfolio has to increase its return in order to perform equally "well" with the augmented. Finally, in all cases we find positive opportunity costs  $\theta$  for CPT, LA and

Markowitz investors. Hence, one needs to give a positive return equal to  $\theta$  to a CPT, LA and Markowitz investor who optimizes in the augmented opportunity set, so that she becomes willing not to include cryptoassets.

## 2.4 Discussion: the COVID-19 pandemic

When the pandemic broke out on January 2020, no one really knew what that meant at the time. Officials throughout the world took instant measures in order to try to restrain it. Lock-downs, in most countries, were the first line of defence against the new threat and this propagated a severe contraction of the global economy. Where it was possible, working-from-the-office switched to working-from-home. This new remote way of doing many of our tasks resulted in taking almost all our daily routines and habits inevitably online and thus a new group of investors revealed itself. It was the group of people that could afford placing a relatively small amount of capital on investments, given that consumption was restricted to the basics like food and utilities. This fact created financial space and now that most individuals had more free time, or more free-riding at work, they had the time to think about their financial, or simply gamble. Subsequently, the usual suspect that attracted most attention and even at a greater scale at the times of mandatory staying-at-home was, the cryptoassets market. This happened naturally because most individuals had heard about cryptoassets as alternative investment vehicles, their up-side potential to make a profit fast and the ease to enter the market as investors, but in most cases they had limited time to take a closer look or even simple enjoyed trading cryptoassets as a pandemic pastime. Then, the opportunity presented itself.

At the same time, the traditional markets exhibited sharp decline, as for example the S&P500 which dropped 1,150 points in just one month, from January 2020 to February 2020. Then it was the time when economic sentiment was at its lowest while economic uncertainty at its highest. Governments were still parsimonious in their announcements towards any economic stimulus packages. In parallel, the market witnessed the fastest-ever bear market sell-off while the job market its immense layoffs. A global pandemic was a highly stressful experience and this most likely affected investors' ability to make prudent and tempered decisions. It is a fact that investing is, for the majority, an emotional experience, especially during periods of turmoil. Another fact is that investors pay closer attention (narrow framing) to markets when volatility is high, their available information may be out of sync or false (even for the financial news), this results into high levels of anxiety, thus more frequent alterations (overreaction) in their investing strategies and eventually excessive trading affects prices in the short-term (losses are very common in the short-term due to excessive volatility) and forms expectations (usually misleading) for the long-term (moderate to low volatility). BF argues that when it comes to investing, in order not to initiate regret and all its by-products, investment strategies must provide emotional comfort and confidence. Thus, the reduction of the number of financial decisions when things are not going well seems imperative, because the temptation to make urgent changes may lead to destructive outcomes.



Figure 3: Price (green shaded area) and Trade Volume (gray bars at the bottom) for BTC, ETH, XRP and LTC covering the period from January 2020 till December 2021. The graphs were obtained from <http://www.coinmarketcap.com>

Figure 3 presents prices (green areas), as well as 24h trade volumes (gray bars located on the x-axis of the graphs), of the four cryptoassets we employ. Our analysis focused primarily for the period when most lock-downs (or strict restrictions) took place, mostly during the winters of 2020 and 2021. As the graphs show, indeed excessive trading took place during those periods. Especially during the winter of 2021 this excessive trading pushed prices up and the crypto-market became even more attractive due to its bullish trend. A potential explanation is that part of this excessive trading was conducted by the aforementioned new pool of investors who now had the capability to enter in the crypto-market.

In our analysis, the three different investor personae express more interest, in terms of optimal weights in their augmented portfolios compared to the afore COVID 19 era, in the crypto-market during the period of February 2020 till June of 2021. Thus, we find support that the associated price increments, possibly due to-among others-the reasons stated above, attracted investors' interest and capital. In our out-of-sample analysis we found that during the pandemic period, the portfolios' aggregate performance after a brief and sharp decline, just after the official announcement of the pandemic, started their soar, for all three investor types.

### 3 Further discussion and future research

We examined how different investor personae "behave" when forming optimal portfolios of different asset classes. We were primarily interested in the performance of the three main decision-making personae in Behavioral Finance namely, CPT, Markowitz and LA type.

We worked with two classes of portfolio spaces, where in every case the one (traditional) was a proper subset of the other (augmented). In our experiment, we used a stochastic spanning methodology to test whether cryptoassets offer diversification benefits to some risk averse investors after

returns are adjusted for the cognitive biases for each of the BF persona involved. We allowed for an empirically plausible framework of multiple - possibly interdependent - bubbles in cryptoassets. We conducted our analysis both in- and out-of-sample, by also taking into account the occurrence of the Covid-19 pandemic, via constructing and comparing optimal portfolios derived from the two respective asset universes.

Even though, the in-sample results suggest that for the Markowitz and LA personae no investment opportunities exist in augmented portfolio space, compared to the traditional, the out-of-sample analysis suggest that optimal augmented portfolios stochastically dominate the respective traditional one for CPT and LA, and generally outperform them w.r.t. every parametric criterion that we employed, for both periods. For the Markowitz type, the out-of-sample analysis suggests that while in the afore-outbreak period the augmented portfolio does not provide different yields, on average, in the post-outbreak period it does and similarly, the other parametric criteria we employed seem to consent.

The asymptotic stationarity framework implied by our assumption on bubble sparsity is not generally empirically identifiable, and its validity is essentially based on backward looking historical arguments. If this does not hold and more generally the (essentially forward looking) condition is not valid, then our statistical inference becomes ambiguous. It may be thus of interest to extend the testing methodologies presented above in asymptotically persistent non-stationary frameworks for the returns, like the weakly non-stationary processes of Duffy and Kasparis (2021). This non trivial task is left for future research.

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## Appendix 1: Proofs and auxiliary results

The appendix contains the proofs of the main results, as well as the derivation of several auxiliary results used in the proofs.

**Proof of Proposition 1.** In what follows H. in. abbreviates the Holder inequality, CS in. the Cauchy-Schwarz inequality, tr. in. the triangle inequality and tr. in.\* its dual form, D-R Mink. in. the dual form of the reverse Minkowski inequality, J. in. Jensen's inequality, and norms in. the bounding from above of the max-norm by a constant multiple of the Euclidean norm in  $\mathbb{R}^d$ .  $\odot$  denotes the Hadamard product.  $c$  denotes a generic positive constant that may change its value at different occurrences. Notice first that

$$\begin{aligned} & \| (v(\exp^*(\mathbf{X}_t) - \mathbf{1}) - v(\exp^*(\varepsilon_t) - \mathbf{1})) \| \\ \stackrel{\text{norms in.}}{\leq} & c \max_i |c_i| (p) \sum_{i=1}^d \left| \begin{array}{l} |\exp(\mathbf{X}_{t(i)}) - \mathbf{1}|^{\alpha_i} \mathbb{I}((\exp(\mathbf{X}_{t(i)}) - \mathbf{1}) \geq 0) \\ - |\exp(\varepsilon_{t(i)}) - \mathbf{1}|^{\alpha_i} \mathbb{I}((\exp(\varepsilon_{t(i)}) - \mathbf{1}) \geq 0) \end{array} \right| \\ & + c \max_i |c_i^*| \sum_{i=1}^d \left| \begin{array}{l} |\exp(\mathbf{X}_{t(i)}) - \mathbf{1}|^{\beta_i} \mathbb{I}((\exp(\mathbf{X}_{t(i)}) - \mathbf{1}) < 0) \\ - |\exp(\varepsilon_{t(i)}) - \mathbf{1}|^{\beta_i} \mathbb{I}((\exp(\varepsilon_{t(i)}) - \mathbf{1}) < 0) \end{array} \right|. \end{aligned}$$

Then, for  $\delta_i = \max \alpha_i, \beta_i$ , the rhs of the previous display is then less than or equal to

$$\begin{aligned} & c \sum_{i=1}^d \left| |\exp(\mathbf{X}_{t(i)}) - \mathbf{1}|^{\alpha_i} - |\exp(\varepsilon_{t(i)}) - \mathbf{1}|^{\alpha_i} \right| \\ & + c \sum_{i=1}^d \left| |\exp(\mathbf{X}_{t(i)}) - \mathbf{1}|^{\beta_i} - |\exp(\varepsilon_{t(i)}) - \mathbf{1}|^{\beta_i} \right| \\ & + c \sum_{i=1}^d \left| \exp(\varepsilon_{t(i)}) - \mathbf{1} \right|^{\alpha_i} \left| \mathbb{I}((\exp(\mathbf{X}_{t(i)}) - \mathbf{1}) \geq 0) - \mathbb{I}((\exp(\varepsilon_{t(i)}) - \mathbf{1}) \geq 0) \right| \\ & + c \sum_{i=1}^d \left| \exp(\varepsilon_{t(i)}) - \mathbf{1} \right|^{\beta_i} \left| \mathbb{I}((\exp(\mathbf{X}_{t(i)}) - \mathbf{1}) < 0) - \mathbb{I}((\exp(\varepsilon_{t(i)}) - \mathbf{1}) < 0) \right| \\ \stackrel{\text{D-R Mink. in. or tr.* in.}}{\leq} & c \sum_{i=1}^d \left| \exp(\mathbf{X}_{t(i)}) - \exp(\varepsilon_{t(i)}) \right|^{\delta_i} + c \sum_{i=1}^d \left| \exp(\varepsilon_{t(i)}) - \mathbf{1} \right|^{\delta_i}, \end{aligned}$$

where the final inequality in the last display follows from the application of dual form if the Minkowski reverse inequality, or the dual form of the triangle inequality, depending on whether the relevant exponent is less or greater than or equal to one, respectively. Then, due to the Lipschitz continuity property of  $x \rightarrow (x)_+$ , and the above display, for  $\delta := \max_i \delta_i$ ,

$$\begin{aligned} & \mathbb{E} \left[ \left| \frac{1}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \left( z - \boldsymbol{\lambda}^\top v(\exp^*(\mathbf{X}_t) - \mathbf{1}) \right)_+ - \frac{1}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \left( z - \boldsymbol{\lambda}^\top v(\exp^*(\varepsilon_t) - \mathbf{1}) \right)_+ \right| \right] \\ & \leq \frac{1}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \mathbb{E} \left[ \left| \boldsymbol{\lambda}^\top (v(\exp^*(\mathbf{X}_t) - \mathbf{1}) - v(\exp^*(\varepsilon_t) - \mathbf{1})) \right| \right] \tag{1} \\ & \stackrel{\text{CS in.}}{\leq} \frac{1}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \mathbb{E} \left[ \| (v(\exp^*(\mathbf{X}_t) - \mathbf{1}) - v(\exp^*(\varepsilon_t) - \mathbf{1})) \| \right] \\ & \stackrel{c}{\leq} \frac{1}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \sum_{i=1}^d \left[ \mathbb{E} \left[ \left| \exp(\mathbf{X}_{t(i)}) - \exp(\varepsilon_{t(i)}) \right|^{\delta_i} \right] + \mathbb{E} \left[ \left| \exp(\varepsilon_{t(i)}) - \mathbf{1} \right|^{\delta_i} \right] \right] \\ & \stackrel{\text{norms+tr.+J in.}}{\leq} \frac{c}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \mathbb{E} \left[ \left\| \exp^*(\mathbf{X}_t) - \exp^*(\varepsilon_t) \right\|^\delta \right] + \frac{c}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \left( \sum_{i=1}^d \mathbb{E} \left[ \exp(\delta \varepsilon_{0(i)}) \right] + d \right) \\ & \stackrel{\text{CS in.}}{\leq} \frac{1}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \mathbb{E} \left[ \left\| \left( \exp^* \left( \delta \sum_{k=1}^K \frac{C_k}{M(T,k)} \mathbb{I}\{t \in B_k\} \mathbf{X}_{t-1} \right) - \mathbf{1} \right) \odot \exp^{\delta^*}(\varepsilon_t) \right\| \right] \\ & \quad + \frac{c}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \left( \sum_{i=1}^d \mathbb{E} \left[ \exp(\delta \varepsilon_{0(i)}) \right] + d \right). \tag{2} \end{aligned}$$

Notice that due to Assumptions ME-AN,

$$\begin{aligned} & \frac{c}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} \left( \sum_{i=1}^d \mathbb{E} \left[ \exp(\delta \varepsilon_{0(i)}) \right] + d \right) \leq \frac{cd}{\sqrt{T}} \sum_{t \in \cup_{k=1}^K B_k} (\max_i \mathbb{E} \left[ \exp(\delta \varepsilon_{0(i)}) \right] + 1) \\ & \leq c \frac{\max_{t,k} B_k}{\sqrt{T}} = o(1). \end{aligned}$$

The asymptotic negligibility of the first and remaining term in the final bound of the previous display follows from the proof of Proposition 3 of AAT21.  $\square$

**Proof of Theorem 1.** (1) and (3) follow exactly as in the proofs of Propositions 4 and B.2 respectively in Arvanitis et al. (2019) (see their Online Appendix) given Proposition 4. For (2) notice that if  $\text{Var}(\mathcal{L}_v(z^*, \boldsymbol{\kappa}^*, \boldsymbol{\lambda}^*)) > 0$  and whenever  $\mathcal{L}_v(z^*, \boldsymbol{\kappa}^*, \boldsymbol{\lambda}^*) > 0$ , then  $\sup_{\Lambda} \inf_K \sup_{\mathcal{X}} \mathcal{L}_v(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}) \geq \mathcal{L}_v(z^*, \boldsymbol{\kappa}^*, \boldsymbol{\lambda}^*)$ . Due to zero mean Gaussianity this occurs with probability at least 0.5. The rest follows as in the proof of Proposition B.2 in Arvanitis et al. (2019) (see their Online Appendix).  $\square$

## Auxiliary Results

**Proposition 2.** *Suppose that Assumptions ME and AN hold.*

*Then  $D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})) = D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1}))$ ,  $\forall (z, \boldsymbol{\kappa}, \boldsymbol{\lambda})$ .*

*Proof.* Stationarity for the  $(\exp^*(\boldsymbol{\varepsilon}_t) - \mathbf{1})_t$  process implies stationarity for  $(v(\exp^*(\boldsymbol{\varepsilon}_t) - \mathbf{1}))_t$ , and by Proposition 1: for any  $z, \boldsymbol{\lambda}$ ,

$$\begin{aligned} & \left| \frac{1}{T} \sum_{t=1}^T \mathbb{E} \left[ \left( z - \boldsymbol{\lambda}^T v^*(\mathbf{R}_t) \right)_+ \right] - \mathbb{E} \left[ \left( z - \boldsymbol{\lambda}^T v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1}) \right)_+ \right] \right| \\ \leq & \mathbb{E} \left[ \left| \frac{1}{T} \sum_{t \in \cup_{k=1}^K B_k} \left( z - \boldsymbol{\lambda}^T v(\exp^*(\mathbf{X}_t) - \mathbf{1}) \right)_+ - \frac{1}{T} \sum_{t \in \cup_{k=1}^K B_k} \left( z - \boldsymbol{\lambda}^T v(\exp^*(\boldsymbol{\varepsilon}_t) - \mathbf{1}) \right)_+ \right| \right] = o\left(\frac{1}{\sqrt{T}}\right). \end{aligned}$$

$\square$

**Proposition 3.** *Suppose that Assumptions ME and AN hold. Then as  $T \rightarrow \infty$ ,*

$$\sup_{\Lambda, K, \mathcal{X}} \sqrt{T} |D_T(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})) - D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1}))| = o_p(1).$$

*Furthermore,*

$$\left| \sup_{\Lambda} \inf_K \sup_{\mathcal{X}} \sqrt{T} D_T(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})) - \sup_{\Lambda} \inf_K \sup_{\mathcal{X}} \sqrt{T} D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1})) \right| = o(1).$$

*Proof.* The first result follows from Proposition 1 and the fact that  $L_1$  convergence implies convergence in probability. The second from that the processes involved have almost surely bounded paths, from the first result and the CMT.  $\square$

**Proposition 4.** *Suppose that Assumptions ME and AN hold. Then as  $T \rightarrow \infty$ ,*

$$\begin{aligned} \sqrt{T} (D_T(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\mathbf{R})) - D^*(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}, v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1}))) & \rightsquigarrow \mathcal{L}_v(z, \boldsymbol{\kappa}, \boldsymbol{\lambda}) \\ & := \int_{\mathbb{R}} \left( (z - \boldsymbol{\kappa}^T \mathbf{x})_+ - (z - \boldsymbol{\lambda}^T \mathbf{x})_+ \right) d\mathcal{G}_{\mathbf{F}}(\mathbf{x}), \end{aligned}$$

*in the space of  $\mathbb{R}$  bounded functions on  $K \times \Lambda \times \mathcal{X}$  equipped with the sup norm, where  $\mathcal{G}$  is a centered Gaussian process with covariance kernel given by*

$$\text{Cov}(\mathcal{G}_{\mathbf{F}}(\mathbf{x}), \mathcal{G}_{\mathbf{F}}(\mathbf{y})) = \sum_{t \in \mathbb{Z}} \text{Cov}(\mathbb{I}\{v(\exp^*(\boldsymbol{\varepsilon}_0) - \mathbf{1}) \leq \mathbf{x}\}, \mathbb{I}\{v(\exp^*(\boldsymbol{\varepsilon}_t) - \mathbf{1}) \leq \mathbf{y}\})$$

and uniformly continuous sample paths on  $\mathbb{R}^d$ .

*Proof.* The result follows by the first part of Proposition 3, and Lemma A.1 in (the Technical Appendix of Arvanitis et. al., 2020).  $\square$

## Appendix 2: Tables

Table 4: Descriptive statistics until January, 31<sup>st</sup> 2020 (Before) & After February, 1<sup>st</sup> 2020 Covid-19 outbreak business days returns.

	Before				After			
	Mean	S.D.	Skewness	Kurtosis	Mean	S.D.	Skewness	Kurtosis
SPX Index	0,00043	0,00856	-0,49505	4,08696	0,00101	0,01794	-0,65274	11,89428
LBUSTRUU Index	0,00015	0,00195	-0,17162	1,12255	0,00012	0,00269	-1,68553	13,97304
US0001M Index	0,00005	0,00003	-0,00681	-1,45861	0,00001	0,00001	2,64115	6,06023
RTY Index	0,00031	0,01047	-0,31897	1,61806	0,00113	0,02314	-0,96251	7,45360
VIVAX US Equity	0,00033	0,00831	-0,55647	3,72566	0,00066	0,01844	-0,51655	10,40964
NAESX US Equity	0,00032	0,00963	-0,38123	1,97140	0,00109	0,02242	-0,18522	13,85714
SMB	-0,00010	0,00510	0,21541	1,01738	0,00040	0,00917	0,29294	3,39069
HML	-0,00016	0,00567	0,51288	1,75685	-0,00032	0,01506	0,24795	1,43266
BTC	0,00417	0,04606	0,36825	4,57600	0,00535	0,04972	-0,89238	9,46065
ETH	0,00737	0,08478	0,96834	13,75556	0,00981	0,06852	-0,16602	7,65338
XRP	0,00617	0,08661	4,25706	39,49960	0,00816	0,09498	2,42453	22,76583
LTC	0,00468	0,06981	2,76739	21,80853	0,00466	0,06678	-0,73435	5,56691
GOLD	0,01078	0,48903	-0,00662	-0,80269	0,06310	0,47865	-0,18183	-0,71974

Entries report the descriptive statistics on business daily returns from from mid-August 2015 to end-of-August 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. For GOLD we use the 1M Future closing price. Bitcoin, Ethereum, Ripple and Litecoin US dollar closing prices are used to assess the cryptocurrency market. The average return, the standard deviation (S.D.), the skewness, as well as the kurtosis are reported.

Table 5: Before January, 31<sup>st</sup> 2020 & After February, 1<sup>st</sup> 2020 Covid-19 outbreak out-of-sample performance: Parametric portfolio measures

	Before	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
Sharpe ratio		0.02221	0.09499	0.03774	0.07066	0.07066	0.09191
Downside Sharpe ratio		0.01601	0.06591	0.02718	0.04740	0.02011	0.06176
UP ratio		0.45575	0.61761	0.48436	0.53781	0.47946	0.58598
Portfolio Turnover		0.01145	0.01297	0.01227	0.02073	0.01673	0.01684
Return Loss		0.00093	-	0.00043	-	0.00083	-
Opportunity cost		0.00087	-	0.00028	-	0.00089	-
	After						
Sharpe ratio		0.04934	0.08388	0.03148	0.06636	0.04879	0.09097
Downside Sharpe ratio		0.03190	0.05451	0.04034	0.04115	0.03163	0.05962
UP ratio		0.59384	0.66036	0.54171	0.63514	0.62364	0.65929
Portfolio Turnover		0.09985	0.01250	0.01085	0.01663	0.01407	0.01579
Return Loss		0.00015	-	0.00065	-	0.00045	-
Opportunity cost		0.00005	-	0.00006	-	0.00014	-

Entries report the performance measures (Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss and Opportunity Cost) for the traditional and the augmented optimal portfolios. The realised business daily returns cover the period from mid-August 2015 to end-of-August 2021, separated on end of January 2020. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML, as well as Gold. The augmented portfolio includes additionally Bitcoin, Ethereum, Ripple and Litecoin. All values are rounded to the fifth decimal.

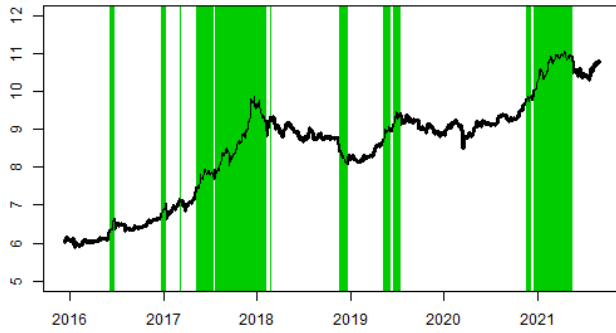
Table 6: Before January, 31<sup>st</sup> 2020 & After February, 1<sup>st</sup> 2020 Covid-19 outbreak out-of-sample performance: average portfolio composition

Before	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
SPX Index	0,06254	0,11056	0,07937	0,12934	0,14895	0,17696
LBSTRUU Index	0,09729	0,10519	0,14230	0,39097	0,02097	0,00230
US0001M Index	0,48099	0,48314	0,54337	0,30633	0,38133	0,46900
RTY Index	0,04227	0,02323	0,01603	0,04058	0,06533	0,04589
VIVAX US Equity	0,19409	0,09034	0,14638	0,04732	0,22189	0,07642
NAESX US Equity	0,07784	0,09630	0,03272	0,02740	0,08647	0,11249
SMB	0,03832	0,01069	0,03470	0,02272	0,05897	0,01449
HML	0,00561	0,00032	0,00514	0,01342	0,01521	0,00312
GOLD	0,00103	0,00102	0,00000	0,00001	0,00087	0,00072
BTC		0,02879		0,01004		0,03797
ETH		0,01083		0,00026		0,01136
XRP		0,01208		0,00301		0,01453
LTC		0,02751		0,00859		0,03476
After						
SPX Index	0,22204	0,13618	0,20035	0,18171	0,24823	0,23407
LBSTRUU Index	0,09837	0,00024	0,12286	0,14652	0,04661	0,00116
US0001M Index	0,40143	0,65399	0,57581	0,54330	0,39020	0,54326
RTY Index	0,02305	0,00131	0,00821	0,01318	0,04374	0,02076
VIVAX US Equity	0,08733	0,08689	0,05431	0,08132	0,12052	0,06235
NAESX US Equity	0,12973	0,09767	0,02148	0,01548	0,10375	0,08804
SMB	0,02331	0,00000	0,00885	0,01001	0,02666	0,00447
HML	0,01417	0,00006	0,00814	0,00521	0,02006	0,01164
GOLD	0,00056	0,00020	0,00000	0,00000	0,00022	0,00009
BTC		0,00766		0,00046		0,00989
ETH		0,00295		0,00025		0,00393
XRP		0,00474		0,00035		0,00565
LTC		0,00811		0,00219		0,01469

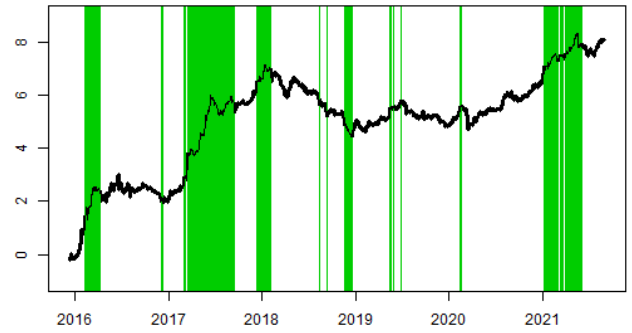
Entries report the average portfolio compositions for the full period from mid-August 2015 to end-of-August 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, the dynamic trading strategies SMB and HML as well as Gold. The augmented portfolio includes additionally Bitcoin, Ethereum, Ripple and Litecoin.

### Appendix 3: Timestamping Cryptocurrencies' Bubbles

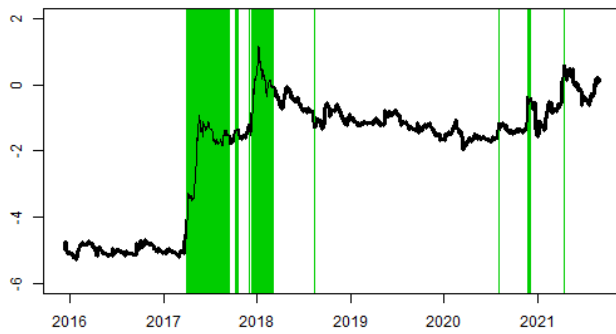
The shaded green areas in the following figure are the identified multiple bubble periods, obtained by using the the 95% bootstrap critical values for the cryptocurrencies involved.



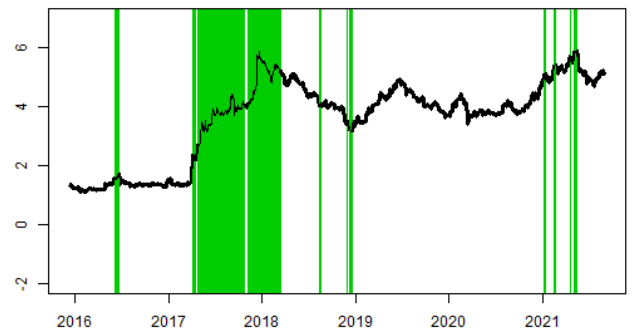
(a) Bitcoin



(b) Ethereum



(c) Ripple



(d) Litecoin



# Online Appendix to: Investors' Behavior in Cryptoassets Market

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## 1 Introduction

This Online Appendix contains the results of extra experiments regarding different variations of the traditional as well as the augmented choice sets. In all cases we employ the Linear Programming (LP) formulations as in Arvanitis et. al (2019), on the distorted dataset depending on the various investor types. We run all applications on Python 3.7 (Jupyter Notebook and PyCharm environments) with the Gurobi solver on a standard laptop with an Intel 8th Gen i7 processor and 16GB of RAM. The subsampling distributions of the test statistics in the in-sample analyses are derived for subsample sizes  $b_T \in [T^{0.6}, T^{0.7}, T^{0.8}, T^{0.9}]$ .

## 2 Experiment 1

### 2.1 In-sample

We test whether the traditional portfolio universe, consisting of stock and bond indices (without Gold),  $v$ -MESSD spans the augmented, with added cryptoassets, for each investor persona, under the framework of mild explosivity developed previously.  $\mathbf{A}$  denotes the augmented portfolio universe having as vertices the crypto-assets in addition to the traditional assets.  $\mathbf{K}$  is the one has only the traditional assets as vertices.

Table 1 reports the test statistics  $n_T^*$ , the associated critical values  $q_T^{BC}$  together with the relevant decisions, for the three investor personas. To mitigate specification error of the subsample length and correct for bias in finite samples, we employ the bias-correction method of Arvanitis et. al. (2019), where the regression estimates  $q_T^{BC}$  are given for significance level  $\alpha = 0.05$ .

The null hypothesis is rejected for the CPT and Loss Averse investor types. For the Markowitz type, the null hypothesis cannot be rejected. Thus, we can argue that for the chosen significance level, the performance of traditional portfolios can be improved with the inclusion of the four crypto assets for some CPT and Loss Averse investors; this does not seem to hold for the Markowitz type.

Table 1: In-sample performance with  $v$ -MESSD spanning test

	$n_T^*$	$q_T^{BC}$	Decision
CPT	0.00673	0.00173	Reject
Markowitz	0.00007	0.00017	Spanning
Loss Aversion	0.00802	0.00777	Reject

Entries report test statistics and critical values for stochastic spanning test of the augmented portfolios with respect to the traditional portfolio. The dataset spans the period from mid-of August 2015 to end-of August 2021, for a total of 1527 business days returns.

### 2.1.1 Out-of-sample

Now that the traditional portfolio contains "traditional" assets and the augmented annexes also cryptoassets, the realized returns time series for all cases are presented in Figure 1. Despite the differences in aggregate returns, the traditional under-performs the augmented for the CPT and LA cases, while for the Markowitz case their performance is more or less the same, on average, except from a breakout in the performance of the augmented in the final year. For the CPT and LA, the augmented portfolio performs at the end about two times higher than the traditional, while for Markowitz the spread between the augmented and the traditional is about 0.15%. Another interesting feature is that for all investor types, the augmented portfolios include relatively small weights for the cryptoassets (Table 6).



(a) CPT



(b) Markowitz



(c) Loss Averse

Figure 1: Experiment 1

For all three personae, anticipated gains and losses play a tremendous role in the formation of

portfolios, out-of-sample. Interestingly, we observe in Table 6 that all investor types prefer to invest lightly on cryptoassets, even though their average returns are higher compared to all other asset classes (Table 4). Thus, we could say that the main drivers of portfolio choice are not the high returns per-se but rather low dispersion (S.D.), which is the highest in the case of cryptoassets. Hence, large standard deviations indicate abrupt gains and losses. Although all three types exhibit different behaviors in gains and losses, they all depart from averting risk. That is why we observe high fund concentration on safer asset classes. These asset classes demonstrate low S.D. and relative high mean. In this case, these are: the S&P 500 Index, the Barclays Bond Index and the 1M T-Bill. They attract an average capital of 71% in the traditional portfolios for the three personae. Subsequently, when cryptoassets come on the foreground this percentage climbs to an average of 76%. This 5% difference is a strong indication of skepticism towards the crypto-market. Investors do want to benefit from the inclusion of cryptoassets but at the same time they are aware of the risk inherent in these exotic products. More specifically, the CPT and LA investors place a 6.6% and 7.8%, in total, on the cryptoassets group, while Markowitz investors stay at 1.6%.

The LA investor selects a different approach and reduces significantly her funding on the two Vanguard indices (more than CPT and Markowitz combined), and remains the most exposed to cryptoassets. A possible explanation for that may be the fact that the two Vanguard indices exhibit negative skewness and high kurtosis, combined with high S.D., indicating extreme returns in the data. It is apparent that the LA is focused in avoiding extreme situations, especially when it comes to losses and data extremity can be reasonably perceived as signal for potential losses. In an analogous manner, the Russel 2000 index attracts very low weight percentages for all investor types.

CPT and Markowitz investors invest heavily on the 1M T-Bill, while LA prefers the S&P 500 Index. Note that, when cryptoassets are introduced, CPT and LA accumulate more funds on the 1M T-Bill, while Markowitz acts in the opposite direction. It seems that they are interested in taking advantage of the turbulence in the crypto-market but at the same time reduce their exposure in the anticipation of certain losses. The certainty of losses stems from the fact that the specific market holds a record of numerous dire straits. Regarding skewness and kurtosis, only Ripple (XRP) differs significantly and subsequently the relevant weights are the lowest. On the other hand, the Sharpe-ratios of the cryptoassets are the highest but we need to consider its drawback of the assumption of normal distribution. Finally, the SMB and HML indices do not attract any type of investors because of the very low returns (HML exhibits a negative average return) and negative Sharpe ratios.

The out-of-sample exercise involves a rolling window of 1 year (the most usual time period for assessing performance and rebalancing, Barberis and Thaler 2003) and covers the dates from August 2016 till end of August 2021. During this period, the assets we employ, in this experiment, experienced two major expansions and two major contractions, with the cryptoassets exhibiting tremendous price deviations and subsequently performance. Thus, we can observe two main periods where gains are followed by losses (Figure 2). The first period starts from July 2017 till the end of November 2018 (Period 1), while the second starts from end of February of 2020 till end of June

of 2021 (Period 2). We are primarily interested in observing how, during these two characteristic periods, the three investor personae allocate weights in their optimal portfolios, on average. Hence, one can observe how aggregate gains and losses affect their decision making process.

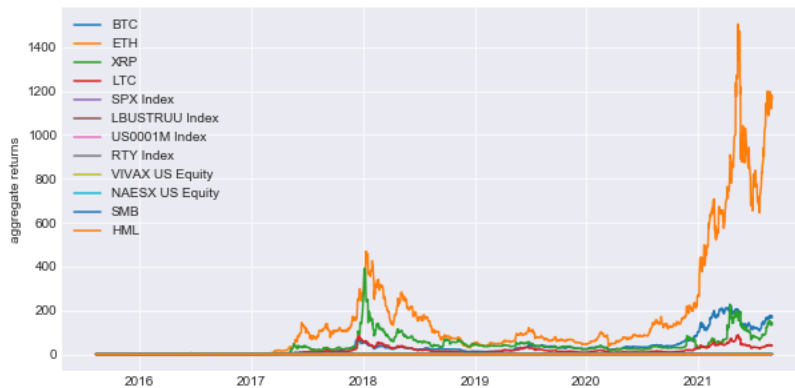


Figure 2: Aggregate performance

In order to dive deeper into the analysis, we focus on the two aforementioned subperiods and produce the average portfolios' composition, as well as the relevant descriptive statistics. Thus, there are two main periods with two sub-periods each, namely; period-1-gains, period-1-losses, period-2-gains and finally period-2-losses. Tables 7 to 14 demonstrate the average optimal weights and the assets' descriptive statics, involving the mean, standard deviation (S.D.), skewness and kurtosis.

One interesting result from period 1, by moving from gains to losses, is while the S&P 500 and 1M T-Bill are leading the weights' accumulation, Vanguard Small-Cap and cryptoassets increase their total weight for all investor types. We argue that this is most likely caused by the *disposition effect*. It is a fact that abrupt rebounds is something quite usual for the crypto-market and hence in the anticipation of such a rebound, all three investor types raise their bets on cryptoassets in order to compensate realized losses. Moreover, while transitioning from the gains period to the losses period, skewness moves to the negative domain for BTC and ETH while kurtosis decreases its magnitude, for all cryptoassets. Thus, extremities seem to leave site and there is room for safety and this can be an ideal environment for investors experiencing any form of loss aversion (i.e. CPT and LA). For the Markowitz type, the increment in the cryptoassets' weights is negligible and is not able to induce any further commenting. It seems that the crypto-market is not attractive for the Markowitz type, most probably due to its high risk of losses.

Starting from the gains season of the second period (period 2), cryptoassets present significantly higher weights, respectively to the ones of period 1. This can be primarily justified on the grounds of a relative stable period (Figure 2), between periods 1 and 2, where cryptoassets still exhibit high prices. Now, as investors experience transition from euphoria to depression, one can observe a contraction for all three investor types. We argue that loss aversion is the main force that drives this kind of decision making, especially for CPT and LA. Given severe losses in the past, it is

most likely that they both want to avoid losses of an even greater and certainly upcoming downfall. Analogously, no matter that the Markowitz type is risk-averting in the losses domain, the magnitude of price deviations in the crypto-market are such, that drives her to strongly reduce her exposure on cryptoassets.

### 3 Experiment 2

#### 3.1 In-sample

In this experiment we investigate a rather reasonable question. What if the three investor types would choose an alternative investing strategy totally focused on investments' safe havens. Now they are interested solely on cryptoassets, thus constructing optimal portfolios that contain only cryptoassets, and another were they choose to include the commodity of Gold under the form of 1M Future, as well (Feng et.al.2018; Klein et.al. 2018). Traditionally, Gold has attracted investors's interest and after the introduction of fractional reserves, Gold has served as a particular international benchmark, representing an investing safe haven because of its low-to-moderate long-run volatility, value stability and its detachment from the global economic development (Hoang et.al. 2016; Shahzad et.al. 2018). Hence, in the same spirit of our work above we ask whether the three different investor types would be better off if they buy-and-hold only cryptoassets or the addition of Gold in their portfolios will provide additional benefits (Hoang et.al. 2015a, 2015b)? We perform our analysis both in- and out-of-sample. Now, the "traditional" portfolio is the one containing only cryptoassets. One interesting feature in the technical background of the cryptoassets is that they are designed in a way that the units (tokens or coins) circulating have limited or in the case of ETH steady supply. Thus, they mimic real natural resources, as Gold, in terms of scarcity.

In this second experiment (Table 2), the null hypothesis cannot be rejected for all investor types. Thus, we can argue that for the chosen significance level, the performance of traditional portfolios cannot be improved with the inclusion of Gold, for all investors. More specifically, the traditional portfolio consisting solely of cryptoassets universally  $v$ -MESSD the augmented one that includes additionally Gold. Thus, we can argue that a Loss Averse, a CPT and a Markowitz type of investor will not be better off if s/he includes gold in her/his strategy, in-sample.

Table 2: Experiment 2: In-sample performance with  $v$ -MESSD spanning test

	$n_T^*$	$q_T^{BC}$	Decision
CPT	0.00888	0.01733	Spanning
Markowitz	0.01033	0.05959	Spanning
Loss Aversion	0.00107	0.00213	Spanning

Entries report test statistics and critical values for stochastic spanning test of the augmented portfolios with respect to the traditional portfolio. The dataset spans the period from mid-of August 2015 to end-of August 2021, for a total of 1527 business days returns.

### 3.2 Out-of-sample

The realized returns time series are presented in Figure 3. In experiment 2, despite the differences in aggregate returns, the traditional under-performs the augmented for the CPT case, eventually. For LA we can observe a better performance of the Traditional throughout the entire period and more particularly a breakout in the performance of the Traditional in the final year. While for the Markowitz, the portfolios' performance is more or less the same, on average, leaving a vague picture that needs further analysis. However, the interesting outcome is that compared to the previous experiments, now, the aggregate performance is extremely high for both portfolios and for all three investors. This can be justified on the grounds that because the cryptoassets' weights constitute either the 100% or 99% of portfolios (Table 16), in fact they drive performance. Thus, these unusual high aggregate returns are more or less anticipated in hindsight given the cryptoassets' exponential price increment. An interesting outcome is that for all investor types, the augmented portfolios place particularly small weights on Gold ( $< 1\%$ ), with Markowitz type placing the smallest.

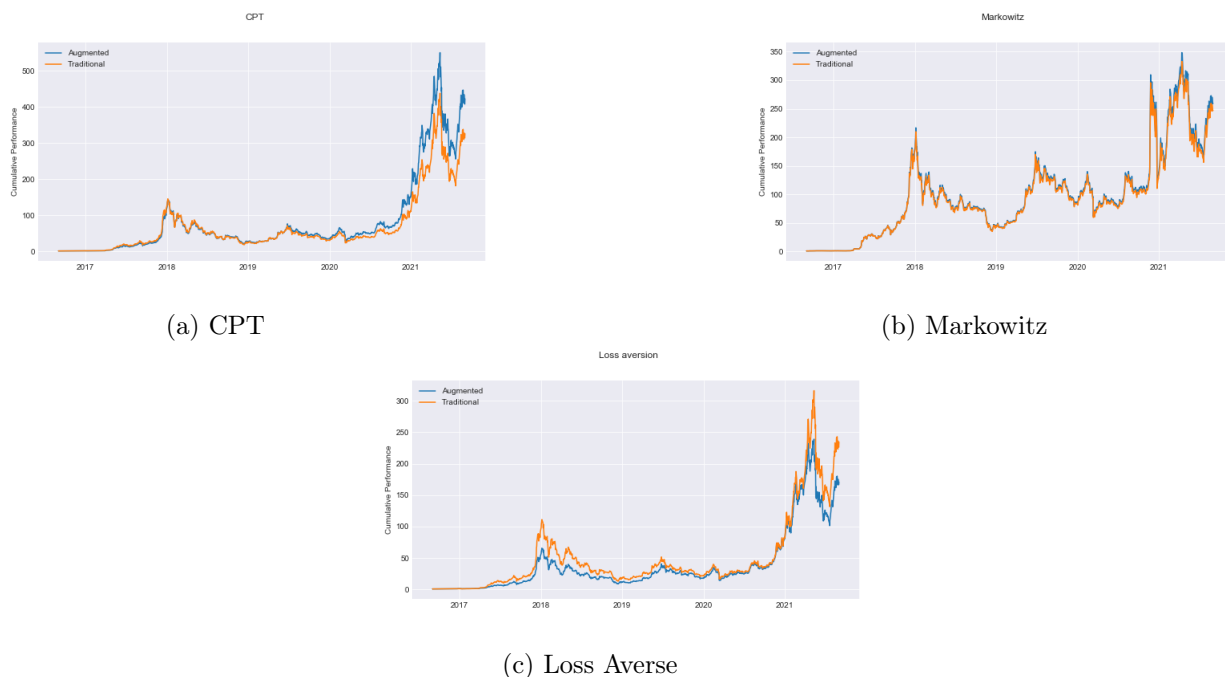


Figure 3: Experiment 2

For all three personae, anticipated gains and losses play a tremendous role in the formation of portfolios, out-of-sample. Interestingly, we observe (Table 16) that all investor types prefer to invest extremely lightly on GOLD, even though its average return is higher compared to all four cryptoassets (Table 4). Thus, we could say that the main drivers of portfolio choice are not the high returns but low dispersion (S.D.). Hence, higher standard deviations indicate more abrupt ups-and-downs, or simply abrupt gains and losses. Although all three types exhibit different behaviors in gains and losses, they all depart from averting risk. That is why we observe high fund concentration on cryptoassets, no matter than both cryptoassets as well as Gold are considered safe investing

havens due to non linkage with economies, governments and/or authorities. Moreover, the Sharpe-ratios of the cryptoassets are higher but we need to consider its drawback of the assumption of normal distribution. More particularly, we can observe a strong preference of the LA and Markowitz types towards BTC, most probably because of its "steady" price increase in the long-run.

### 3.3 A conservative test for pairwise (non-) dominance

We perform a non-parametric out-of-sample performance comparative assessment of the optimal portfolios, using the Davidson and Duclos (2013) pairwise (non-) dominance test, as modified by AAT21 to allow for the mildly explosive framework. The modified test retains as null the less logically strict hypothesis of non-dominance, yet in a composite form concerning the comparison between every possible cluster point of the empirically optimal portfolio weights sequences.

The procedure allows for processes that appear in the context of Assumptions ME-AN, and for stochastic portfolio weights that may not be consistent or convergent at all. This is generally expected to be the case for the empirically optimal portfolio emerging from the stochastic spanning criterion; it need not have unique optimisers.

The test is applied exactly in its AAT21 form, on the returns of the empirically optimal portfolios, and not on their DVTs. Thereby, for brevity, we do not present the details on the hypothesis structure, the form of the test statistic and the rejection region, and its limiting properties. Those are discussed in AAT21 and the interested reader is referred there. We however point out the following:

The null hypothesis is that for any pair of cluster points between the associated portfolio, the cluster point of the first, does not MESSD the second; MESSD is now the dominance relation defined before for  $v$  equal to the identity. The alternative posits the existence of a pair for which the first dominates the second.

The test statistic is essentially a supremum of easily computable  $t$  statistics over  $\mathcal{X}$ , and the rejection region is based on the standard normal distribution. Thereby, the procedure is independent of the choice of numerical approximation parameters (like the subsampling length) as resampling based approximations of the limiting rejection region are avoided.

Under assumptions involving properties of the associated cluster points and the LPMs around them, and if a consistent estimator for long run covariances is used, the test is shown to be asymptotically conservative and consistent.

### 3.4 Out of sample empirical results: pairwise (non-) dominance

We apply our modification of the Davidson and Duclos (2013) pairwise (non-) dominance test on the two optimal portfolios derived in the previous rolling analysis for experiment 1.

Table 3 reports the quartile  $p$ -values from the distribution of daily portfolio returns, for the null hypothesis that the augmented portfolio does not stochastically dominate the traditional one by second order (see Davidson and Duclos, 2013). The results entail  $T - 1$  (1157) overlapping periods for the in-sample fitting of the two portfolios with corresponding out-of-sample comparisons. The

$T - 1$   $p$ -values are considered from September 1, 2016 to August 31, 2021, using overlapping periods of 100 daily returns in all three cases. The quartile  $p$ -values from the distribution of the  $T - 1$  modified t-test statistics are computed.

We observe that, for the 25% and 50% (in 5%) quartile  $p$ -values, the null hypothesis that the augmented optimal portfolio does not stochastically dominate the traditional one by second order is rejected in all cases. Hence, the out-of-sample performance of optimal portfolios constructed by every investor persona, that include cryptoassets seems dominant to the performance of the corresponding benchmark.

Table 3: Out-of-sample performance: Non-parametric stochastic dominance test

Traditional vs Augmented	
CPT	
Quartile	
25% Rejection rate	47.13%
50% Rejection rate	65.31%
75% Rejection rate	84.19%
Markowitz	
Quartile	
25% Rejection rate	44.27%
50% Rejection rate	63.42%
75% Rejection rate	81.94%
Loss aversion	
Quartile	
25% Rejection rate	38.28%
50% Rejection rate	57.84%
75% Rejection rate	76.87%

Entries report quartile rejection rates from the distribution rejection rates across out-of-sample periods under the null hypothesis that the augmented cryptocurrencies optimal portfolio does not second order stochastically dominate the optimal traditional portfolio using a modification of the Davidson and Duclos (2013) test statistic, over the period from September 1, 2016 to August 31, 2021.

### 3.5 Further out-of-sample empirical results: parametric tests

We also apply a set of commonly used parametric performance measures: the Sharpe ratio, the downside Sharpe ratio (DS) (Ziembra, 2005), the upside potential (UP) and downside risk ratio (Sortino and van den Meer, 1991), the opportunity cost (Simaan, 1993), the portfolio turnover (P.T.) and a measure of the portfolio risk-adjusted returns net of transaction costs (RL). Since assets' returns exhibit asymmetric return distributions, the downside Sharpe and UP ratios are more appropriate measures than the typical Sharpe ratio. For the compatibility of those with Assumptions ME-AN see AAT21.

Table 5 reports the parametric performance measures for experiment 1. All cases enrich the evidence obtained from the non-parametric SD measures. The higher the value of each one of these measures, the greater the investment opportunity for including cryptoassets.



The results show that the inclusion of cryptoassets into the opportunity set increases all performance measures. We observe that since in all cases the UP ratio increases, all investors exhibit a benefit from the inclusion of cryptoassets. Furthermore, we observe that portfolios with only traditional assets induce less portfolio turnover than the ones with cryptoassets, which analogously creates more portfolio turnover for all types. Additionally, we can see that the return-loss measure is similarly positive for all types. Thus, the traditional portfolio has to increase its return in order to perform equally "well" with the augmented. Finally, in all cases we find positive opportunity costs  $\theta$  for CPT, LA and Markowitz investors. Hence, one needs to give a positive return equal to  $\theta$  to a CPT, LA and Markowitz investor who optimizes in the augmented opportunity set, so that she becomes willing not to include cryptoassets.

Table 15 analogously reports the parametric performance measures for experiment 2 . The results show that the inclusion of Gold into the opportunity set decreases all performance measures for Markowitz and LA types except for Portfolio Turnover without inducing any significant effects in their financial behaviors, while for CPT we obtain the opposite situation. We observe that since for the CPT case the UP ratio increases, the specific investor exhibits a benefit from the inclusion of Gold. Furthermore, we observe that portfolios with only cryptoassets induce less portfolio turnover for Markowitz and LA. Additionally, we see that the return-loss measure is negative for LA. Thus, the traditional portfolio has to decrease its return in order to perform similarly with the augmented. Finally, we find a positive opportunity cost  $\theta$  for CPT and Markowitz investors. Hence, one needs to eventually give a positive return equal to  $\theta$  to a CPT and Markowitz investor, who optimizes in the augmented opportunity set, so that she becomes willing not to include Gold, and contrariwise for the LA type.

## 4 Description of Performance Measures

For the DS ratio, we first evaluate the downside risk (downside variance) which is given by the formula:

$$\sigma_{P-}^2 = \frac{\sum_{t=1}^T (\min(x_t, 0))^2}{T - 1}$$

where,  $x_t$  are those returns of portfolio  $P$  at day  $t$  below 0, i.e. those days with losses. To get the total variance we use:  $2\sigma_{P-}^2$ , thus the DS ratio is,

$$S_P = \frac{\bar{R}_P - \bar{R}_f}{\sqrt{2\sigma_{P-}^2}}$$

where,  $\bar{R}_P$  is the average period return of portfolio  $P$  and  $\bar{R}_f$  is the average risk free rate.

The DS ratio removes any effects of upward price movement on the standard deviation in order to focus on the distribution of the returns that are below a predefined threshold/target that is set

by an investor (or fund) as a minimum required return. Its deviation from the Sharpe ratio lies in that it replaces the risk-free rate with the required return. In our experiments we assume that this required return is the average risk-free return of the whole period under examination.

The UP ratio compares the upside potential to the shortfall risk over a benchmark and is computed as follows. For  $R_t$  the realized daily return of portfolio  $P$  for  $t = 1, \dots, T$  of the backtesting period, where  $T$  is the number of experiments performed, and  $p_t$  the return of the benchmark (risk free rate), which in our case is the one month T-bill riskless asset for the same period:

$$\text{UP ratio} = \frac{\frac{1}{T_1} \sum_{t=1}^{T_1} \max(R_t - p_t, 0)}{\sqrt{\frac{1}{T_2} \sum_{t=1}^{T_2} (\max(p_t - R_t, 0))^2}}, T = T_1 + T_2$$

The numerator of the above ratio is the average excess return over the benchmark and thus it reflects the upside potential. In the same sense, the denominator measures downside risk, i.e. shortfall risk over the benchmark.

Next, we evaluate the P.T. to get a feeling of the degree of rebalancing required to implement each one of the investment strategies under examination. For any portfolio strategy  $P$ , the portfolio turnover is defined as the average of the absolute change of weights over the  $T$  rebalancing points in time and across the  $d$  available assets, i.e.

$$\text{P.T.} = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^M (|w_{P_i,t+1} - w_{P_i,t}|)$$

where  $w_{P_i,t+1}, w_{P_i,t}$  are the optimal weights of asset  $i$  under strategy  $P$  (Traditional or Augmented) at time  $t$  and  $t+1$ , respectively.

We also evaluate the performance of the portfolios under the risk-adjusted returns measure, which is net of transaction costs, proposed by DeMiguel et al. (2009). It indicates the way that the proportional transaction cost, generated by the P.T., affects the portfolio returns. Let  $\text{TrC}$  be the proportional transaction cost, and  $R_{P,t+1}$  the realized return of portfolio  $P$  at time  $t+1$ . The change in the net of transaction cost wealth  $NW_P$  of portfolio  $P$  through time is,

$$NW_{P,t+1} = NW_{P,t}(1 + R_{P,t+1}) \left(1 - \text{TrC} \times \sum_{i=1}^M (|w_{P_i,t+1} - w_{P_i,t}|)\right)$$

The portfolio return, net of transaction cost, is defined as,

$$RTC_{P,t+1} = \frac{NW_{P,t+1}}{NW_{P,t}} - 1$$

Let  $\mu_{Tr}, \mu_{Aug}$  be the out-of-sample mean of monthly  $RTC$  with the traditional and augmented opportunity set, respectively, and  $\sigma_{Tr}, \sigma_{Aug}$  be the corresponding standard deviations. Then, the

return-loss measure is,

$$R_{Loss} = \frac{\mu_{Aug}}{\sigma_{Aug}} \times \sigma_{Tr} - \mu_{Tr}$$

It evaluates the additional return needed so that the Traditional performs equally well with the Augmented. We follow the literature and use 35 basis points (bps), i.e. 0.35% , for the proportional transaction cost of stocks and bonds.

Finally, we use the concept of opportunity cost presented in Simaan (1993) to analyze the economic significance of the performance difference of the two optimal portfolios, in both experiments and for all three perspectives (i.e. investor types). Let  $R_{Tr}^i$  and  $R_{Aug}^i$  be the realized returns of the optimal Traditional and Augmented portfolio for every investor  $i$ . Then, the opportunity cost  $\theta$  is defined as the return that needs to be added to (or subtracted from)  $R_{Tr}^i$ , so that the investor is indifferent (in utility terms) between the strategies imposed by the two different investment opportunity classes:

$$\mathbb{E} \left[ U(1 + R_{Tr}^i + \theta) \right] = \mathbb{E} \left[ U(1 + R_{Aug}^i) \right] \quad ; i = \text{CPT, M, LA},$$

where now  $\mathbb{E}$  now denotes empirical expectation. A positive opportunity cost implies that an investor is better off if she includes additional assets in her portfolio, while a negative one implies that she would be worse off with the aforementioned inclusion. It is important to mention that the opportunity cost takes into account the entire probability distribution of portfolio returns and hence it is suitable to evaluate strategies even when the distribution is not normal. For the calculation of the opportunity cost we follow the literature and use the relevant *S-shaped* for CPT, *inverse S-shaped* for Markowitz (M), as well as the linear value function for the LA type.

## 5 Tables

Table 4: Descriptive statistics of business days returns

Asset	Mean	S.D.	Skewness	Kurtosis	Sharpe ratio
S&P 500 Index	0.000579	0.011754	-0.692944	19.285854	0.045633
Barclays bond Index	0.000139	0.002168	-0.934352	9.4163	0.044550
1M T-Bill	0.000043	0.000033	0.345543	-1.390497	-
Russel 2000	0.000526	0.014855	-0.966213	13.602642	0.032559
Vanguard Value	0.000418	0.01182	-0.622055	18.710689	0.031756
Vanguard Small-Cap	0.000523	0.014132	-0.217019	25.347711	0.033968
SMB	0.000032	0.006416	0.365433	5.124661	-0.001638
HML	-0.000204	0.009102	0.359444	6.115774	-0.027071
BTC	0.004476	0.047029	-0.016428	6.136154	0.094275
ETH	0.008007	0.080832	0.791753	13.348979	0.098535
XRP	0.00669	0.088847	3.683376	33.976011	0.074815
LTC	0.004677	0.069013	1.941846	18.103941	0.067149
GOLD	0.024448	0.486727	-0.052520	-0.793544	0.039148

Entries report the descriptive statistics on business daily returns from from mid-August 2015 to end-of-August 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. For GOLD we use the 1M Future closing price. Bitcoin, Ethereum, Ripple and Litecoin US dollar closing prices are used to assess the cryptocurrency market. The average return, the standard deviation (S.D.), the skewness, the kurtosis, as well as the Sharpe ratio are reported.

Table 5: Out-of-sample performance: Parametric portfolio measures

Performance measures	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
Sharpe ratio	0.04169	0.10011	0.03595	0.08161	0.04153	0.09388
Downside Sharpe ratio	0.02722	0.06652	0.03145	0.05312	0.02702	0.06098
UP ratio	0.52837	0.60139	0.51309	0.57967	0.51930	0.57745
Portfolio Turnover	0.00926	0.01250	0.01212	0.02107	0.01819	0.02432
Return Loss	0.00050	-	0.00015	-	0.00031	-
Opportunity cost	0.00036	-	0.00005	-	0.00044	-

Entries report the performance measures (Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss and Opportunity Cost) for the traditional and the augmented optimal portfolios. The realised business daily returns cover the period from mid-August 2015 to end-of-August 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. The augmented portfolio includes additionally Bitcoin, Ethereum, Ripple and Litecoin. All values are rounded to the fifth decimal.

Table 6: Out-of-sample analysis: average portfolio composition

Asset	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
S&P 500 Index	0.167	0.149	0.168	0.189	0.267	0.293
Barclays bond Index	0.142	0.098	0.098	0.242	0.122	0.162
1M T-Bill	0.336	0.523	0.515	0.434	0.177	0.223
Russel 2000	0.058	0.013	0.020	0.015	0.040	0.021
Vanguard Value	0.096	0.079	0.079	0.059	0.138	0.125
Vanguard Small-Cap	0.125	0.116	0.088	0.041	0.155	0.114
SMB	0.041	0.014	0.019	0.012	0.058	0.040
HML	0.036	0.008	0.014	0.009	0.043	0.023
BTC		0.026		0.008		0.039
ETH		0.020		0.005		0.025
XRP		0.008		0.001		0.007
LTC		0.012		0.002		0.007

Entries report the average portfolio compositions for the full period from mid-August 2015 to end-of-August 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. The augmented portfolio includes additionally Bitcoin, Ethereum, Ripple and Litecoin.

Table 7: Period 1 GAINS: average portfolio composition

Asset	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
S&P 500 Index	0.291	0.196	0.286	0.391	0.391	0.523
Barclays bond Index	0.135	0.000	0.129	0.049	0.167	0.092
1M T-Bill	0.322	0.615	0.449	0.520	0.095	0.214
Russel 2000	0.060	0.003	0.020	0.004	0.061	0.010
Vanguard Value	0.064	0.086	0.065	0.003	0.137	0.005
Vanguard Small-Cap	0.067	0.073	0.043	0.005	0.068	0.053
SMB	0.035	0.000	0.003	0.023	0.044	0.033
HML	0.027	0.000	0.006	0.002	0.038	0.043
BTC		0.009		0.001		0.011
ETH		0.010		0.001		0.009
XRP		0.003		0.000		0.003
LTC		0.005		0.000		0.004

Entries report the average portfolio compositions for the gains season of period 1 starting from July 2017 till the end of November 2018. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. The augmented portfolio includes additionally Bitcoin, Ethereum, Ripple and Litecoin.

Table 8: Period 1 GAINS: descriptive statistics

Asset	mean	S.D	Skewness	Kurtosis
S&P 500 Index	0,000981	0,003997	-0,502022	3,349851
Barclays bond Index	0,000103	0,001552	-0,145699	0,056810
1M T-Bill	0,000051	0,000004	1,691656	1,371581
Russel 2000	0,000854	0,006508	-0,000723	0,724665
Vanguard Value	0,000898	0,004157	-0,272151	2,522744
Vanguard Small-Cap	0,000876	0,005490	-0,222015	0,735263
SMB	-0,000171	0,004185	0,009047	1,085245
HML	-0,000009	0,004265	0,176893	0,731145
BTC	0,016469	0,067664	0,830312	2,275369
ETH	0,013353	0,079685	0,425172	1,686298
XRP	0,027000	0,130365	3,269689	15,121142
LTC	0,017599	0,109819	2,584362	14,892373

Entries report the descriptive statistics for the gains season of period 1 starting from July 2017 till the end of November 2018. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. Bitcoin, Ethereum, Ripple and Litecoin US dollar closing prices are used to assess the cryptocurrency market. The average return, the standard deviation (S.D.), the skewness as well as the kurtosis are reported.

Table 9: Period 1 LOSSES: average portfolio composition

Asset	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
S&P 500 Index	0.153	0.110	0.127	0.274	0.171	0.285
Barclays bond Index	0.100	0.108	0.026	0.272	0.130	0.138
1M T-Bill	0.387	0.476	0.659	0.334	0.252	0.258
Russel 2000	0.047	0.000	0.010	0.015	0.054	0.017
Vanguard Value	0.098	0.058	0.064	0.029	0.175	0.119
Vanguard Small-Cap	0.153	0.211	0.106	0.046	0.168	0.147
SMB	0.042	0.000	0.007	0.027	0.034	0.004
HML	0.021	0.001	0.001	0.001	0.014	0.002
BTC		0.010		0.002		0.008
ETH		0.014		0.002		0.011
XRP		0.004		0.000		0.004
LTC		0.008		0.000		0.007

Entries report the average portfolio compositions for the losses season of period 1 starting from July 2017 till the end of November 2018. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. The augmented portfolio includes additionally Bitcoin, Ethereum, Ripple and Litecoin.

Table 10: Period 1 LOSSES: descriptive statistics

Asset	mean	S.D	Skewness	Kurtosis
S&P 500 Index	-0,000018	0,010033	-0,900484	2,550777
Barclays bond Index	-0,000041	0,001773	0,155037	1,769589
1M T-Bill	0,000079	0,000009	-0,447212	-0,743529
Russel 2000	-0,000169	0,010700	-0,778012	1,953796
Vanguard Value	-0,000036	0,009331	-1,048358	3,234638
Vanguard Small-Cap	-0,000098	0,009973	-0,790156	1,885055
SMB	-0,000020	0,005395	-0,044345	0,074723
HML	-0,000412	0,005672	0,550917	1,621158
BTC	-0,005274	0,047388	-0,609905	2,450611
ETH	-0,007563	0,062139	-0,230146	1,068331
XRP	-0,006089	0,079855	0,197929	5,687594
LTC	-0,007294	0,058697	0,659990	4,945932

Entries report the descriptive statistics for the losses season of period 1 starting from July 2017 till the end of November 2018. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. Bitcoin, Ethereum, Ripple and Litecoin US dollar closing prices are used to assess the cryptocurrency market. The average return, the standard deviation (S.D.), the skewness as well as the kurtosis are reported.

Table 11: Period 2 GAINS: average portfolio composition

Asset	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
S&P 500 Index	0.097	0.125	0.127	0.150	0.222	0.227
Barclays bond Index	0.089	0.177	0.134	0.327	0.070	0.218
1M T-Bill	0.395	0.324	0.444	0.376	0.196	0.079
Russel 2000	0.067	0.038	0.029	0.050	0.061	0.072
Vanguard Value	0.147	0.080	0.124	0.002	0.193	0.086
Vanguard Small-Cap	0.109	0.075	0.084	0.018	0.122	0.084
SMB	0.045	0.043	0.042	0.030	0.082	0.083
HML	0.050	0.017	0.017	0.011	0.053	0.011
BTC		0.043		0.018		0.066
ETH		0.035		0.013		0.057
XRP		0.015		0.001		0.009
LTC		0.026		0.005		0.008

Entries report the average portfolio compositions for the gains season of period 2 starting from end of February 2020 till end of June 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. The augmented portfolio includes additionally Bitcoin, Ethereum, Ripple and Litecoin.

Table 12: Period 2 GAINS: descriptive statistics

Asset	mean	S.D	Skewness	Kurtosis
S&P 500 Index	0,001282	0,020621	-0,626652	9,346553
Barclays bond Index	0,000004	0,002963	-1,811787	13,382846
1M T-Bill	0,000010	0,000009	2,243107	3,515679
Russel 2000	0,001885	0,026549	-0,994990	5,900027
Vanguard Value	0,001000	0,021250	-0,508964	8,040876
Vanguard Small-Cap	0,001720	0,025962	-0,221773	10,742625
SMB	0,000838	0,009998	0,237764	3,261864
HML	-0,000175	0,016986	0,254357	0,845853
BTC	0,008348	0,050547	-1,164352	13,014931
ETH	0,010400	0,068614	0,002803	10,105078
XRP	0,006967	0,089630	3,066151	34,698571
LTC	0,006616	0,065442	-0,323127	5,611942

Entries report the descriptive statistics for the gains season of period 2 starting from end of February of 2020 till end of June of 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. Bitcoin, Ethereum, Ripple and Litecoin US dollar closing prices are used to assess the cryptocurrency market. The average return, the standard deviation (S.D.), the skewness as well as the kurtosis are reported.

Table 13: Period 2 LOSSES: average portfolio composition

Asset	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
S&P 500 Index	0.104	0.138	0.082	0.187	0.217	0.273
Barclays bond Index	0.237	0.336	0.108	0.326	0.110	0.178
1M T-Bill	0.251	0.142	0.491	0.220	0.136	0.000
Russel 2000	0.064	0.009	0.000	0.000	0.034	0.000
Vanguard Value	0.108	0.107	0.113	0.153	0.139	0.190
Vanguard Small-Cap	0.146	0.141	0.158	0.070	0.214	0.167
SMB	0.059	0.014	0.040	0.005	0.108	0.065
HML	0.032	0.016	0.008	0.018	0.041	0.018
BTC		0.038		0.006		0.055
ETH		0.032		0.012		0.044
XRP		0.013		0.001		0.005
LTC		0.013		0.003		0.007

Entries report the average portfolio compositions for the losses season of period 2 starting from end of February of 2020 till end of June of 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. The augmented portfolio includes additionally Bitcoin, Ethereum, Ripple and Litecoin.



Table 14: Period 2 LOSSES: descriptive statistics

Asset	mean	S.D	Skewness	Kurtosis
S&P 500 Index	0,001057	0,006754	-0,469061	1,060790
Barclays bond Index	0,000277	0,001731	0,045289	-0,667664
1M T-Bill	0,000004	0,000000	0,000000	0,000000
Russel 2000	0,000350	0,011553	-0,244609	0,440044
Vanguard Value	0,000674	0,007011	-0,333315	0,686296
Vanguard Small-Cap	0,000594	0,009661	-0,299237	1,009255
SMB	-0,000179	0,007435	0,159338	-0,266705
HML	-0,000170	0,010833	-0,556043	0,465216
BTC	-0,007185	0,056193	-0,301235	0,065995
ETH	0,003913	0,083947	-0,496116	1,630351
XRP	0,001377	0,117444	0,511463	2,708483
LTC	-0,003485	0,086336	-1,462925	3,806653

Entries report the descriptive statistics for the losses season of period 2 starting from end of February of 2020 till end of June of 2021. The traditional set includes the: S&P 500 Index, the Barclays US Bond Index, the 1 Month T-Bill, the Russell 2000 Index, the Vanguard Value Index, the Vanguard Small-Cap Index, and the dynamic trading strategies SMB and HML. Bitcoin, Ethereum, Ripple and Litecoin US dollar closing prices are used to assess the cryptocurrency market. The average return, the standard deviation (S.D.), the skewness as well as the kurtosis are reported.

Table 15: Experiment 2: Out-of-sample performance: Parametric portfolio measures

Performance measures	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
Sharpe ratio	0.11150	0.11470	0.11501	0.09685	0.10708	0.10405
Downside Sharpe ratio	0.00277	0.00268	0.00268	0.00267	0.00281	0.00277
UP ratio	0.74073	0.74670	0.78673	0.78413	0.72778	0.70857
Portfolio Turnover	0.00477	0.00942	0.01151	0.01159	0.00499	0.00978
Return Loss	0.00146	-	0.00009	-	-0.00199	-
Opportunity cost	0.00021	-	0.00003	-	-0.00032	-

Entries report the performance measures (Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss and Opportunity Cost) for the traditional and the augmented optimal portfolios. The realised business daily returns cover the period from mid-August 2015 to end-of-August 2021. The traditional set includes: Bitcoin, Ethereum, Ripple and Litecoin. The augmented portfolio includes additionally GOLD. All values are rounded to the fifth decimal.

Table 16: Experiment 2: Out-of-sample analysis: average portfolio composition

Asset	CPT (T)	CPT (A)	Mark. (T)	Mark. (A)	LA (T)	LA (A)
GOLD		0.006758		0.000902		0.002255
BTC	0.344225	0.373149	0.622619	0.621822	0.338593	0.450062
ETH	0.228963	0.213278	0.009345	0.009692	0.248666	0.211226
XRP	0.194906	0.211481	0.334618	0.333675	0.178747	0.121235
LTC	0.231906	0.195334	0.033418	0.033910	0.233994	0.215222

Entries report the average portfolio compositions for the full period from mid-August 2015 to end-of-August 2021. The traditional set includes : Bitcoin, Ethereum, Ripple and Litecoin. The augmented portfolio includes additionally GOLD.

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The Department is the oldest Department of Economics in Greece with a pioneering role in organising postgraduate studies in Economics since 1978. Its priority has always been to bring together highly qualified academics and top quality students. Faculty members specialize in a wide range of topics in economics, with teaching and research experience in world-class universities and publications in top academic journals.

The Department constantly strives to maintain its high level of research and teaching standards. It covers a wide range of economic studies in micro-and macroeconomic analysis, banking and finance, public and monetary economics, international and rural economics, labour economics, industrial organization and strategy, economics of the environment and natural resources, economic history and relevant quantitative tools of mathematics, statistics and econometrics.

Its undergraduate program attracts high quality students who, after successful completion of their studies, have excellent prospects for employment in the private and public sector, including areas such as business, banking, finance and advisory services. Also, graduates of the program have solid foundations in economics and related tools and are regularly admitted to top graduate programs internationally. Three specializations are offered: 1. Economic Theory and Policy, 2. Business Economics and Finance and 3. International and European Economics. The postgraduate programs of the Department (M.Sc and Ph.D) are highly regarded and attract a large number of quality candidates every year.

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