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On the Taylor Rule and Optimal Monetary Policy in a "Natural Rate" Model

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On the Taylor Rule and Optimal Monetary Policy in a "Natural Rate" Model

by

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Abstract

This paper investigates the stabilizing role of monetary policy in a dynamic, stochastic general equilibrium model of the "natural rate", in which non indexed nominal wages are periodically set by labor market "insiders". This nominal distortion allows for nominal shocks to have temporary real effects, and thus, for monetary policy to be able to affect short run fluctuations in both inflation and real output. We derive and analyze optimal monetary policy in the presence of real and nominal shocks, and highlight the properties of the optimal monetary policy rule. The optimal policy rule is second best, as it cannot completely neutralize productivity shocks, and is associated with a tradeoff between the stabilization of inflation and output. We also demonstrate that the optimal policy can be replicated by a set of appropriately parametrized Taylor rules, according to which deviations of the current nominal interest rate from its "natural" rate, depend on deviations of inflation from target and output from its "natural" level. We prove that the optimal Taylor rule is not unique, as multiple sets of parameters are consistent with optimality. Provided that the monetary authorities attach a sufficiently low weight to deviations of output from its "natural" level, the optimal policy could also be replicated through a unique, appropriately parametrized Wicksell rule, according to which deviations of the nominal interest rate from its "natural" rate depend only on deviations of inflation from target. The optimal set of Taylor rules is a set of simple, but not too simple rules, as, the nominal interest rate must react to changes in the "natural" rate of interest, in addition to deviations of inflation from target, and output from its "natural"level.

Keywords: monetary policy, aggregate fluctuations, insiders and outsiders, natural rate
JEL Classification: E3, E4, E5

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The design of policies that the monetary authorities should follow in order to stabilize the economy has always been one of the most important concerns of monetary economics. There is wide agreement about the key objectives of monetary policy, which are none other than price stability and high and stable output and employment, but a number of other disagreements remain regarding the nature of appropriate policies.

The long standing arguments about rules versus discretion have been resolved in favor of rules. Very little disagreement exists today about the view that monetary policy should be determined on the basis of rules. Yet, there is still a very lively debate about the appropriate design of such rules.[1]

Another long standing debate has concerned the appropriate instrument of monetary policy. In recent years the debate has tilted in favor of interest rate rather than money supply rules, in view of the difficulties in controlling, or even defining, the appropriate measure of the money supply.[2]

The literature on interest rate rules in the past twenty years has focused on the analysis of one particular such rule, the Taylor (1993, 1999) rule. This rule has been shown to describe the monetary policy of the USA and other advanced economies relatively well, since at least the early 1980s. The Taylor rule, which is a generalized version of a rule proposed more than a century ago by Wicksell (1898), has mainly been analyzed in the context of *new keynesian models* of aggregate fluctuations. It has been compared to optimal monetary policy, and a number of proposals have been put forward for improvements in the choice of parameters of this particular rule.[3]

This paper contributes to the discussion of the pros and cons of the Taylor rule, by analyzing optimal monetary policy in the context of a "natural" rate model. We use a dynamic, stochastic general equilibrium model, in which nominal wages are set periodically by "insiders" in the labor market, and are non indexed. There are two distortions in the model of this paper. One is a real distortion, arising from the fact that "outsiders" are disenfranchised from the labor market, and the second is a nominal distortion, arising from the fact that nominal wage contracts are not indexed, and can only be reopened at the beginning of each period, but not during the period.

The structure of the labor market, as well as the assumption of competitive product markets, constitute the most important differences of this model from the standard "new keynesian" model, as exposed for example in Woodford (2003) and Gali (2008). On the other hand, the present model is more parsimonious that the standard "new keynesian" model, and is characterized by simpler

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[1] Among the first to discuss the dilemma of rules versus discretion was Simons (1936). The dilemma was analyzed more rigorously by Kydland and Prescott (1977) in the context of models with rational expectations. Fischer (1990) surveys the relevant literature.

[2] The classic analysis of the appropriate choice of monetary instruments was Poole (1970). In an important paper, Sargent and Wallace (1975) demonstrated that under rational expectations, a non-contingent interest rate target leads to price level indeterminacy and instability. However, it is now accepted that this problem does not arise in the case of contingent interest rate rules that make the nominal interest rate depend on the price level (McCallum 1981), or a sufficiently sensitive positive function of inflation. See Woodford (2003) Ch.1 for the relevant arguments. In addition, central banks, have in the last thirty years been consistently using interest rates as their main monetary policy instrument. As noted by Bernanke (2006), " In practice, the difficulty has been that, deregulation, financial innovation, and other factors have led to recurrent instability in the relationships between various monetary aggregates and other nominal variables.".

[3] See Clarida, Gali and Gertler (1999), Gali (2008, 2011a,b), Taylor and Williams (2011) and Woodford (2003, 2011) for surveys of the findings of this literature. Woodford (2003) and Orphanides (2008) survey the analysis of interest rate rules from Wicksell (1898) to Taylor (1993, 1999).

dynamics, thus allowing us to derive analytical solutions that help sharpen our understanding of the relevant issues.

The real distortion in our model makes the “natural” rate of unemployment inefficiently high, while the nominal distortion allows for nominal shocks to have temporary real effects, and thus, for monetary policy to be able to affect fluctuations in both inflation and real output and employment.

The model is characterized by an expectations augmented “Phillips curve”, in which deviations of output and employment from their “natural” level depend on unanticipated current inflation and unanticipated productivity shocks. Aggregate demand is determined by the optimal behavior of a representative household, with access to a competitive financial market, choosing the path of consumption and real money balances in order to maximize its inter-temporal utility function. The model is also characterized by exogenous shocks to productivity, preferences for consumption and money demand, as well as labor market shocks.[4]

We first characterize optimal monetary policy in the presence of these real and nominal shocks. We demonstrate that under the optimal monetary policy rule, the only shocks that cannot be completely neutralized by monetary policy are productivity shocks. Since productivity shocks are supply shocks, whose real effects can only be offset through unanticipated inflation, they cause a tradeoff between deviations of inflation from target, and output from its “natural” level, even under the optimal policy. This is not the case for demand shocks, which can be fully neutralized by monetary policy, through appropriate changes in interest rates. It is for this reason that, in the presence of productivity shocks, the optimal policy is second best.[5]

The optimal monetary policy can in principle be implemented either using the nominal interest rate, or the money supply as the monetary instrument.

Under an optimal interest rate rule, the nominal interest rate shadows the “natural” real interest rate, augmented by the inflation target of the monetary authorities, minus a fraction of the current innovation to productivity, which causes output and the real interest rate to deviate from their “natural” levels. By allowing nominal interest rates to respond to productivity shocks, the monetary authorities allow inflation to deviate from expected inflation, thus helping reduce deviations of output and employment from their “natural” level.

The informational requirements of such optimal interest rate rules are significant. The monetary authorities must react optimally to the current “natural” real rate of interest, plus deviations of inflation from target, and current output from its “natural” level. In the model itself, if the monetary authorities can observe current output and inflation, they can deduce the three relevant underlying shocks, i.e shocks to productivity, consumption demand and the labor market. Thus the optimal interest rate rule can in principle be implemented in the context of this model.

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[4] Labor market shocks take the form of shocks to the number of “insiders”, and are similar to labor supply shocks in a competitive labor market model.

[5] Productivity shocks disrupt the “divine coincidence” characterizing the typical “new keynesian” model. The *divine coincidence*, noted by Blanchard and Gali (2007) for the standard new keynesian model, means that stabilizing deviations of inflation from target also stabilizes deviations of output from its “natural” level. Blanchard and Gali (2007) introduced exogenous real wage rigidity to the new keynesian model to break this link.

The informational requirements of an optimal contingent money supply rule are higher than an optimal interest rate rule. It is shown, that an optimal contingent money supply rule would require the monetary authorities to also identify shocks to money demand, in addition to information on all the other shocks required to implement the optimal contingent interest rate rule. In the present model, if the authorities follow a money supply rule, they would be able to identify money demand shocks, through an observation of the nominal interest rate, output and inflation.[6]

Having characterized the optimal policy, we then proceed to investigate the properties of the Taylor (1993, 1999) rule, and compare it to the Wicksell (1898) rule and the optimal policy. The Wicksell rule requires the central bank to set the interest rate at a level different from its "natural" rate, only in response to deviations of inflation from target. The Taylor rule is more general, and requires the central bank to set the interest rate at a level different from its "natural rate" in response to two variables, deviations of inflation from its target, *and* deviations of output from its "natural" level.

We establish five main results regarding these rules: First, an appropriately parametrized Taylor rule can replicate the optimal policy. Second, the optimal Taylor rule is not unique. A optimal Taylor rule can be defined for more than one pair of parameters determining the response of nominal interest rates to deviations of inflation from target, and deviations of output from its "natural" level. Third, an appropriately parametrized Wicksell rule, is equivalent to an optimal Taylor rule, provided that the weight attached to output relative to inflation in the preferences of the monetary authorities is sufficiently low. Otherwise, the optimal Wicksell rule may not satisfy the Taylor principle, which guarantees the stability of the inflationary process. Fourth, the optimal Wicksell rule, if it exists and is stable, is unique. Finally, we demonstrate that a policy of absolute inflation targeting is only optimal if the central bank is only concerned with deviations of inflation from target, and not deviations of output from its "natural" level.

In a separate analysis, delegated to an appendix, we examine a simplified, but widely used, version of the Taylor rule, the rule with a constant intercept. This rule is shown to be sub-optimal in the context of our model, as it generates persistent, policy induced, fluctuations of inflation and high variability of deviations of output from its "natural" level. Given the constant intercept in the rule, persistent fluctuations of inflation are the only way for the real interest rate to equilibrate the product market in the face of persistent real shocks to the "natural" rate of interest. In addition, because all shocks affect inflation under this rule, deviations of output from its "natural" level depend on innovations in all real shocks, and not only productivity shocks, as under the optimal rule. Thus, a constant intercept Taylor rule could result in potentially significant costs for the monetary authorities, in terms of persistent fluctuations of inflation, and higher variability of deviations of output from its "natural" level than the optimal rule.

The rest of the paper is as follows:

In section 1 we present a wage setting model, based on "insiders" and "outsiders" in the labor market, and derive a positive relation between unanticipated inflation and deviations of output and employment from their "natural" levels.

The wage setting model introduced in this paper combines and extends two strands of the literature.

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[6] The requirement for money supply rules to respond optimally to money demand shocks, something that under an interest rate rule is achieved automatically, is one of the arguments in favor of interest rate rules, in addition to the difficulties of defining and controlling the appropriate monetary aggregate.

First, the Gray (1976)-Fischer (1977) model of predetermined nominal wages, according to which nominal wages are set at the beginning of each period, and remain fixed for one period. Because shocks to inflation are not known when nominal wage contracts are negotiated, unanticipated inflation reduces real wages and causes employment to increase along a downward sloping labor demand curve. Thus, the model produces a "Phillips Curve" type of tradeoff between unanticipated inflation and output and employment.

The second strand of the literature is the insider-outsider theory of wage determination of Lindbeck and Snower (1986), Blanchard and Summers (1986) and Gottfries (1992). According to this approach, there is an asymmetry in the wage setting process between "insiders", who already have jobs, and "outsiders" who are seeking employment. "Outsiders" are disenfranchised from the labor market, and wages are set by "insiders", who seek to maximize the real wage consistent with their own employment, and not with the employment of the full labor force. This causes the "natural rate" of unemployment to be inefficiently high. The total number of "insiders" in the economy is assumed to be subject to stochastic shocks, resulting in exogenous fluctuations in the "natural" rate of unemployment and the "natural" level of employment and output.

Employment and output are determined by competitive firms, which set employment in each period at the level which equates the real wage to the marginal product of labor. The marginal product of labor is subject to persistent productivity shocks, which affect both labor demand, and the output produced for given employment.

In section 2, we analyze the demand side. Consumption and money demand are determined by a representative household, able to borrow and lend freely in a competitive financial market, at the market interest rate. Money enters the utility function of the representative household, and the demand for real money balances is proportional to consumption, and inversely related to the nominal interest rate. The Euler equation for consumption determines the evolution of private consumption. The preferences of the representative household for consumption and real money balances are subject to persistent stochastic shocks, which shift both the Euler equation for consumption and the demand for money function.

In section 3 we analyze product and money market equilibrium. Product market equilibrium is achieved through adjustment of the real interest rate, which equates aggregate demand and aggregate supply. The money market equilibrium condition depends on whether the central bank follows a money supply rule, or an interest rate rule. The demand for real money balances is proportional to real output and inversely related to the nominal interest rate. If the central bank follows an interest rate rule, the money supply adjusts endogenously to equilibrate the money market, while if the central bank follows a money supply rule, nominal interest rates are endogenously determined by the equilibrium condition in the money market.

In section 4 we characterize optimal monetary policy. Monetary policy is decided by a central bank, which minimizes an inter-temporal quadratic loss function that depends on deviations of inflation from a fixed target, and output from its "natural" level. Since the central bank does not seek to achieve full employment, but seeks to stabilize output around its lower "natural" level, the optimal policy does not result in inflation bias relative to the central bank target. We derive the optimal policy, and show that, in the presence of supply shocks, it is second best, in that the central bank cannot perfectly stabilize either deviations of inflation from its target, or output from its "natural" level. However, such deviations are non persistent under the optimal policy, and only depend on productivity shocks. Demand shocks are fully neutralized through appropriate fluctuations in nominal and real interest rates.

In section 5 we analyze the properties of the Wicksell (1898) and Taylor (1993, 1999) rules. We show how an appropriately parametrized Taylor rule can replicate the optimal policy. We also show that the optimal Taylor rule is not unique, as it can replicate the optimal policy under alternative combinations of parameters defining the response of nominal interest rates to deviations of inflation from target, and deviations of output from its "natural" level. We also demonstrate that an appropriately parametrized Wicksell rule, with a sufficiently high optimal response of the nominal interest rate to deviations of inflation from target, is equally effective with an optimal Taylor rule and that, if it exists, it is unique. Finally, we prove that a policy of absolute inflation targeting is only optimal if the central bank is concerned only with deviations of inflation from target, and not deviations of output from its "natural" level. The sub-optimal simple Taylor rule, with a constant intercept, is analyzed in the appendix.

In the last section we sum up our conclusions.

1. "Insiders", Wage Setting and the "Phillips Curve"

Consider an economy consisting of competitive firms, indexed by *i,* where $i \in [0,1]$. Labor is the only variable factor of production, and firms determine employment by equating the marginal product of labor to the real wage.

1.1 Output, Employment and Labor Demand

The production function of firm *i* is given by,

$$Y(i)_t = A_t L(i)_t^{1-\alpha} \tag{1}$$

where *Y(i)* is output, *A* is exogenous productivity, and *L(i)* is employment. *t* is a time index, where $t=0,1,\ldots$. $0<1-\alpha<1$ is the elasticity of output with respect to employment.

Employment is determined by firms, which maximize profits by equating the marginal product of labor to the real wage. Thus, employment is determined by the condition that,

$$(1-\alpha)A_t L(i)_t^{-\alpha} = \frac{W(i)_t}{P_t} \tag{2}$$

where *W(i)* is the nominal wage of firm *i,* and *P* is the price for the product of firm *i.* Since the product market is assumed to be competitive, all firms face the same price, and *P(i)=P* for all firms.

In log-linear form, (1) and (2) can be written as,

$$y(i)_t = a_t + (1-\alpha)l(i)_t \tag{3}$$

$$l(i)_t = \bar{l} - \frac{1}{\alpha}(w(i)_t - p_t - a_t) \tag{4}$$

where $\bar{l} = \frac{\ln(1-\alpha)}{\alpha}$

Lowercase letters denote the logarithms of the corresponding uppercase variables. (3) determines output as a positive function of employment, and (4) determines employment as a negative function of deviations of real wages from productivity.

1.2 Wage Setting and Employment in an "Insider Outsider" Model

Nominal wages are set by "insiders" in each firm, at the beginning of each period, before variables, such as current productivity and the current price level are known. Thus, nominal wages are set on the basis of the rational expectations of "insiders" about these shocks. Nominal wages remain constant for one period, and they are reset at the beginning of the following period. Thus, this model is characterized by the real distortions emphasized by Lindbeck and Snower (1976), leading to an inefficiently high "natural" rate of unemployment, and by the nominal wage stickiness of the Gray (1976), Fischer (1977), Gottfries (1992) models. Employment is determined ex post by firms, given the contract wage, after the current price level and productivity have been revealed. This set up leads to nominal shocks and monetary policy having temporary real effects.

The number of "insiders", who at the beginning of each period determine the contract wage, is assumed exogenous. The key objective of "insiders" is to set a nominal wage which, given their rational expectations about the price level and productivity, will minimize deviations of expected employment from the pool of "insiders".

The expectations on the basis of which wages are set depend on information available until the end of period *t-1*, but not on information about prices and productivity in period *t.*

On the basis of the above, we assume that the objective of wage setters is to make expected employment satisfy a path that minimizes the following quadratic inter-temporal loss function,

$$\min E_{t-1} \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{2} \left(l(i)_{t+s} - \bar{n}(i)_{t+s} \right)^2 \right] \tag{5}$$

$\bar{n}$ is the logarithm of the number of "insiders". $\beta=1/(1+\rho)<1$ is the discount factor, with ρ being the pure rate of time preference. (5) is minimized subject to the labor demand equation (4).

We assume that the total number of " insiders" in the economy is always strictly smaller than the labor force. We thus assume that,

$$\int_{i=0}^{1} \bar{n}(i)_t \, di = \bar{n}_t < n_t \, , \ \forall t \tag{6}$$

where n is the log of the labor force.

From the first order conditions for a minimum of (5), wages are set so that expected employment for each firm satisfies,

$$E_{t-1}l(i)_t = \bar{n}(i)_t \tag{7}$$

Integrating over *i, expected aggregate employment* must then satisfy,

$$E_{t-1}l_t = \bar{n}_t \tag{8}$$

(8) is the same as (7) without the *i* index. (8) determines the “natural” level of employment, solely on the basis of the number of “insiders” in the wage setting process. Since the number of insiders is assumed to always be smaller than the labor force, the "natural" level of employment is inefficiently low.

Actual employment is determined by firms, after the nominal wage has been set, and after information about current prices, productivity and other shocks has been revealed.

Integrating the labour demand function over the number of firms *i,* aggregate employment is given by,

$$l_t = \bar{l} - \frac{1}{\alpha}(w_t - p_t - a_t) \tag{9}$$

From (8) and (9), the contract wage satisfies,

$$w_t = E_{t-1}p_t + E_{t-1}a_t - \alpha(\bar{n}_t - \bar{l}) \tag{10}$$

The wage is set so as to make expected employment equal to the number of insiders, and is based on one period ahead expectations about the price level and productivity.

1.3 An Expectations Augmented Phillips Curve

Substituting (10) in (9), actual employment evolves according to,

$$l_t = \bar{n}_t + \frac{1}{\alpha}\left(p_t - E_{t-1}p_t + a_t - E_{t-1}a_t\right) = \bar{n}_t + \frac{1}{\alpha}\left(\pi_t - E_{t-1}\pi_t + a_t - E_{t-1}a_t\right) \tag{11}$$

where, $\pi_t = p_t - p_{t-1}$ is the rate of inflation.

From (11), employment deviates from its “natural” level to the extent that there are unanticipated shocks to inflation and productivity. Unanticipated increases in inflation cause a reduction in real wages and increase labour demand and employment, while, unanticipated increases in productivity increase productivity relative to real wages, and thus increase labour demand and employment.

We can define the unemployment rate as,

$$u_t \simeq n_t - l_t \tag{12}$$

We can define the "natural rate" of unemployment as,[7]

$$\bar{u}_t \simeq n_t - \bar{n}_t \tag{13}$$

From (11), (12) and (13), it follows that,

$$u_t = \bar{u}_t - \frac{1}{\alpha}\left(\pi_t - E_{t-1}\pi_t + a_t - E_{t-1}a_t\right) \tag{14}$$

The unemployment rate deviates from its "natural" rate as a result of unexpected shocks to inflation and productivity, because both reduce real wages relative to productivity, compared with the prior expectations of wage setters. (14) has the form of an *expectations augmented Phillips curve*, which arises because nominal wages are set for one period and before current inflation and productivity are known.

We can also express this expectations augmented Phillips curve in terms of output. From the log-linear version of the firm production function in (3), aggregating over firms, we get an aggregate production function in log-linear form, as,

$$y_t = a_t + (1-\alpha)l_t \tag{15}$$

Substituting (11) in the log-linear version of the production function (15), output supply evolves according to,

$$y_t = \bar{y}_t + \frac{1-\alpha}{\alpha}\left(\pi_t - E_{t-1}\pi_t + a_t - E_{t-1}a_t\right) \tag{16}$$

where,

$$\bar{y}_t = (1-\alpha)\bar{n}_t + a_t \tag{17}$$

is the "natural" level of output.

Unexpected shocks to inflation and productivity cause output to be higher than its "natural" level, as they cause employment to be higher than its own "natural" level. (16) can be seen as the output version of the "expectations augmented Phillips curve", or as a short run "output supply function".

1.4 The "Natural" Rate of Unemployment and the "Natural" Level of Output

It is worth distinguishing between the "natural" level of output and the "full employment" level output. Full employment output is given by,

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[7] The concept of the "natural" rate of unemployment is due to Friedman (1968). To quote, "The "natural rate of unemployment", … is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the costs of mobility, and so on." (p. 8). It is worth noting that Friedman defined the "natural" rate of unemployment in an analogous way to the "natural" rate of interest of Wicksell (1898). He made a direct reference to Wicksell, and was fully aware of the analogy between the two concepts.

$$y_t^f = (1-\alpha)n_t + a_t \tag{18}$$

Full employment output is always higher than "natural" output in this model. The reason is that equilibrium employment is lower than full employment, since the pool of "insiders", who are ones who determine equilibrium employment through their wage setting power, is smaller than the labor force. Thus, because of this real distortion in the labor market, the "natural" level of output is inefficiently low, and the "natural rate" of unemployment is inefficiently high.

From (17) and (18), the relation between the "natural rate" of unemployment and deviations of output from "full employment" output is given by,

$$y_t^f - \bar{y}_t = (1-\alpha)(n_t - \bar{n}_t) = (1-\alpha)\bar{u}_t \tag{19}$$

This is the real distortion in this model, a distortion that cannot be addressed by monetary policy.

The effects of the nominal distortion, i.e the real effects of unanticipated changes in prices, can of course be partly offset by monetary policy.

2. The Determination of Aggregate Consumption and Money Demand

We next turn to the determination of aggregate demand. We assume that the economy consists of a large number of identical households *j*, where *j* ∈ *[0,1]*. Each household member supplies one unit of labor, and unemployment impacts all households in the same manner. Thus, if *H* is the number of households and *N* is the aggregate labor force, each household has *N/H* members. Of those, some are "insiders" in the labor market, and the rest are "outsiders". The proportion of insiders is the same for all households. In addition, the proportion of the unemployed is also assumed to be the same for all households.

The representative household chooses (aggregate) consumption and real money balances to maximize,

$$E_t \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho} \right)^s \left(\frac{1}{1-\theta} \left(V_{t+s}^C C_{t+s}^{1-\theta} + V_{t+s}^M \left(\frac{M}{P} \right)_{t+s}^{1-\theta} \right) \right) \tag{20}$$

subject to the sequence of expected budget constraints,

$$E_t \left(F_{t+s+1} - (1+i_{t+s}) \left(F_{t+s} - \frac{i_{t+s}}{1+i_{t+s}} M_{t+s} + P_{t+s} \left(Y_{t+s} - C_{t+s} - T_{t+s} \right) \right) \right) = 0 \tag{21}$$

where $F_t = B_t + M_t$.

ρ denotes the pure rate of time preference, θ is the inverse of the elasticity of inter-temporal substitution, i the nominal interest rate, F the current value of household financial assets (one period nominal bonds B and money M), Y real non interest income and T real taxes net of transfers. V^C and

V^M denote exogenous stochastic shocks in the utility from consumption and real money balances respectively.

From the first order conditions for a maximum,

$$V_t^C C_t^{-\theta} = \lambda_t (1+i_t) P_t \tag{22}$$

$$V_t^M \left(\frac{M}{P}\right)_t^{-\theta} = \lambda_t i_t P_t \tag{23}$$

$$E_t \lambda_{t+1} = E_t \left(\frac{1+\rho}{1+i_{t+1}}\right) \lambda_t \tag{24}$$

where λ_t is the Lagrange multiplier in period t.

(22)-(24) have the standard interpretations. (22) suggests that at the optimum the household equates the marginal utility of consumption to the value of savings. (23) suggests that the household equates the marginal utility of real money balances to the opportunity cost of money. Finally, (24) suggests that at the optimum, the real interest rate, adjusted for the expected increase in the marginal utility of consumption, is equal to the pure rate of time preference.

From (22), (23) and (24), eliminating λ,

$$\left(\frac{M}{P}\right)_t = C_t \left(\frac{V_t^C}{V_t^M} \frac{i_t}{1+i_t}\right)^{-\frac{1}{\theta}} \tag{25}$$

$$E_t \left(\frac{V_{t+1}^C (C_{t+1})^{-\theta}}{P_{t+1}}\right) = \left(\frac{1+\rho}{1+i_t}\right)\left(\frac{V_t^C (C_t)^{-\theta}}{P_t}\right) \tag{26}$$

(25) is the money demand function, which is proportional to consumption and a negative function of the nominal interest rate, and (26) is the familiar Euler equation for consumption.

Log-linearizing (25) and (26),

$$m_t - p_t = c_t - \frac{1}{\theta} \ln\left(\frac{i_t}{1+i_t}\right) + \frac{1}{\theta}\left(v_t^M - v_t^C\right) \tag{27}$$

$$c_t = E_t c_{t+1} - \frac{1}{\theta}\left(i_t - E_t \pi_{t+1} - \rho\right) + \frac{1}{\theta}(v_t^C - E_t v_{t+1}^C) \tag{28}$$

where lowercase letters denote natural logarithms, and, $\pi_t = p_t - p_{t-1}$ is the rate of inflation.

We then turn to the determination of equilibrium in the product and money markets.

3. Equilibrium in the Product and Money Markets

Since there is no capital and investment in this model, product market equilibrium implies that output is equal to consumption.

$$Y_t = C_t \tag{29}$$

Substituting (29) in (27) and (28), we get the money and product market equilibrium conditions,

$$m_t - p_t = y_t - \frac{1}{\theta}\ln\left(\frac{i_t}{1+i_t}\right) + \frac{1}{\theta}\left(v_t^M - v_t^C\right) \tag{30}$$

$$y_t = E_t y_{t+1} - \frac{1}{\theta}\left(i_t - E_t\pi_{t+1} - \rho\right) + \frac{1}{\theta}(v_t^C - E_t v_{t+1}^C) \tag{31}$$

(30) is the money market equilibrium condition, the equivalent of the *LM Curve*, and (31) is the product market equilibrium condition, the equivalent of the *IS Curve.* Since output demand depends on deviations of the real interest from the pure rate of time preference, the real interest rate is the relative price that adjusts to equilibrate output demand with output supply. No other relative price can play this role, as the real wage is determined in order to make expected labor demand equal to the number of "insiders" in the labor market.

3.1 The "Natural" Real Interest Rate and the Current Real Interest Rate

The real interest rate is defined by the Fisher (1896) equation,[8]

$$r_t = i_t - E_t\pi_{t+1} \tag{32}$$

The "natural" real interest rate is determined by the product market equilibrium condition, when output is at its "natural" rate. From (17) and (31), the "natural" real interest rate is given by,

$$\bar{r}_t = \rho - \theta\left((1-\alpha)\left(\bar{n}_t - E_t\,\bar{n}_{t+1}\right) + \left(a_t - E_t a_{t+1}\right)\right) + \left(v_t^C - E_t v_{t+1}^C\right) \tag{33}$$

The "natural" real interest rate is equal to the pure rate of time preference, but also depends positively on deviations of current shocks to consumption from anticipated future shocks, and negatively on deviations of current productivity shocks from anticipated future shocks, as well as deviations of the current "natural" level of employment from its anticipated future level. Thus, real shocks that cause a temporary increase in the "natural" level of output reduce the "natural" real rate of interest, in order to bring about an corresponding reduction in consumption and maintain product market equilibrium. On the other hand, real shocks that cause a temporary increase in consumption,

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[8] To quote from Fisher (1896), "When prices are rising or falling, money is depreciating or appreciating *relative* to commodities. Our theory would therefore require high or low interest according as prices are rising or falling, provided we assume that the rate of interest in the commodity standard should not vary." (p. 58). The rate of interest in the commodity standard is the real interest rate, and rising or falling prices are expected inflation.The Fisher equation was further elaborated in Fisher (1930), where it was made even clearer that Fisher referred to expected inflation.

require an increase in the "natural" real rate of interest, in order to induce lower consumption, and maintain product market equilibrium.[9]

Because of the nominal rigidity of wages for one period, the current equilibrium real interest deviates from its "natural" rate. The current real interest rate is determined by the equation of the output demand function (31) with the output supply function (16). It is thus determined by,

$$r_t = \bar{r}_t - \frac{\theta(1-\alpha)}{\alpha}\left(\pi_t - E_{t-1}\pi_t + a_t - E_{t-1}a_t\right) \tag{34}$$

Unanticipated shocks to inflation or productivity, which cause a temporary rise in current output relative to its "natural" level, also reduce the current real interest rate relative to its "natural" rate. This is the well known "Wicksellian" mechanism, emphasized for the first time by Wicksell (1898). We shall return to this mechanism when we discuss alternative interest rate rules in sections 4 and 5.

3.2 Equilibrium Fluctuations with Exogenous Preference, Productivity and Labor Market Shocks

We shall assume in what follows that the logarithms of the exogenous shocks to preferences and productivity follow stationary AR(1) processes.

$$v_t^C = \eta_C v_{t-1}^C + \varepsilon_t^C \tag{35}$$

$$v_t^M = \eta_M v_{t-1}^M + \varepsilon_t^M \tag{36}$$

$$a_t = \eta_A a_{t-1} + \varepsilon_t^A \tag{37}$$

where the autoregressive parameters satisfy, $0 < \eta_C, \eta_M, \eta_A < 1$, and ε^C, ε^M, ε^A, are white noise processes.

We shall further assume that the (log of the) labor force is fixed at *n,* and that the exogenous number of "insiders" also follows a stationary AR(1) process, of the form,

$$\bar{n}_t = (1-\eta_N)\bar{n} + \eta_N \bar{n}_{t-1} + \varepsilon_t^N \tag{38}$$

where $0 < \bar{n} < n$, is a constant, $0 < \eta_N < 1$, and ε^N is a white noise process. ε^N is a labor market shock, increasing the number of "insiders".

From (38) and the definition of the "natural" rate of unemployment in (13), we also have that,

$$\bar{u}_t = (1-\eta_N)\bar{u} + \eta_N \bar{u}_{t-1} - \varepsilon_t^N \tag{39}$$

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[9] The concept of a "natural" rate of interest was introduced by Wicksell (1898). To quote, "There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the natural rate of interest …" (p. 102).

Thus, the "natural" rate of unemployment converges to a constant, but it displays fluctuations around this constant, caused by persistent shocks to the number of insiders.

With these assumptions, current employment, unemployment, output, real wages and the real interest rate, as functions the exogenous shocks and shocks to inflation, evolve according to,

$$l_t = \bar{n}_t + \frac{1}{\alpha}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) \tag{40}$$

where $\bar{n}_t$ is given by (38).

$$u_t = \bar{u}_t - \frac{1}{\alpha}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) \tag{41}$$

where $\bar{u}_t$ is given by (39).

$$y_t = \bar{y}_t + \frac{1-\alpha}{\alpha}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) \tag{42}$$

where, $\bar{y}_t = (1-\alpha)\bar{n}_t + a_t$.

$$w_t - p_t = (\overline{w-p})_t - \left(\pi_t - E_{t-1}\pi_t\right) \tag{43}$$

where, $(\overline{w-p})_t = \eta_A a_{t-1} - \alpha(\bar{n}_t - \bar{l})$.

$$r_t = \bar{r}_t - \frac{\theta(1-\alpha)}{\alpha}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) \tag{44}$$

where $\bar{r}_t = \rho - \theta\left((1-\alpha)(1-\eta_N)(\bar{n}_t - \bar{n}) + (1-\eta_A)a_t\right) + (1-\eta_C)v_t^C$

The "natural" rates (or levels) of real variables evolve as functions of the exogenous real shocks. However, unanticipated inflation, and innovations in productivity, by reducing real wages relative to productivity, cause a temporary increase in employment and output above their "natural" level, and a temporary reduction in unemployment and the real interest rate below their "natural" rates.

We next turn to monetary policy and the determination of the inflation rate.

4. Optimal Monetary Policy

We approach the determination of the inflation rate, and other nominal variables, assuming that the monetary authorities determine monetary policy in order to minimize deviations of inflation from a constant inflation target π^* and deviations of output from its "natural" level. We thus assume, that, through the appropriate policy instrument, the monetary authorities choose the inflation rate in order to minimize,

$$\Lambda_t = E_t \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho}\right)^s \left(\frac{1}{2}\left(\pi_{t+s} - \pi^*\right)^2 + \frac{\psi}{2}\left(y_{t+s} - \bar{y}_{t+s}\right)^2\right) \tag{45}$$

where ψ is the relative weight of output relative to inflation in the preferences of the monetary authorities. In contrast to wage setters, it is assumed that the monetary authorities have full information about current shocks when they determine monetary policy.

4.1 Optimal Inflation Policy

From the first order conditions for a minimum of (45) subject to the expectations augmented output supply function (42), we get,

$$\pi_t = \pi^* - \xi\varepsilon_t^A \tag{46}$$

where $\xi = \dfrac{\psi(1-\alpha)^2}{\alpha^2 + \psi(1-\alpha)^2} < 1$.

From (46), under the optimal policy, inflation is equal to the constant inflation target of the monetary authorities, minus a fraction of the current shock to productivity, which tends to increase (labour demand, employment and) output above its “natural” level. By reducing inflation relative to expectations in the case of a positive shock to productivity, the monetary authorities reduce the deviation of output from its “natural” level, due to the innovation in productivity ε^A. Thus, under the optimal policy, the deviation of real output from its “natural” level is given by,[10]

$$y_t - \bar{y}_t = \frac{1-\alpha}{\alpha}\left(\pi_t - E_{t-1}\pi_t + \varepsilon_t^A\right) = \frac{1-\alpha}{\alpha}(1-\xi)\varepsilon_t^A \tag{47}$$

Under the optimal inflation policy, the deviations of output from its “natural” level depend only on the part of the innovation of productivity that is not neutralized by monetary policy.

Thus, the nominal distortion arising from the fact that nominal wages are set in advance and cannot be changed, can only partially be corrected through optimal monetary policy. The reason is that monetary policy operates through the demand side. It can neutralize demand side shocks completely, but it can only affect aggregate supply through unanticipated inflation. Thus, the *optimal policy is second best*, since the monetary authorities have one instrument, inflation, through which they seek to attain two linearly independent targets. The *divine coincidence,* noted by Blanchard and Gali (2007) for the standard *new keynesian model,* does not apply to this “natural rate” model. In the presence of supply shocks, inflation stabilization does not result in output stabilization, because of innovations in productivity. Even under the optimal policy, there is a meaningful tradeoff between inflation stabilization and output stabilization in this model.

Thus, the expected inter-temporal welfare loss of the monetary authorities under the optimal policy is positive. From (45), (46) and (47), it is given by,

$$\Lambda_t^* = \frac{1}{2}\frac{1+\rho}{\rho}\left(Var(\pi_t - \pi^*) + \psi Var(y_t - \bar{y}_t)\right) = \frac{1}{2}\frac{1+\rho}{\rho}\left(\left(\xi^2 + \psi\left(\frac{1-\alpha}{\alpha}\right)^2(1-\xi)^2\right)\sigma_A^2\right) \tag{48}$$

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[10] If the monetary authorities only cared about inflation, i.e ψ=0, then ξ=0 and the optimal policy would have been first best.

where σ_A^2 is the variance of the innovations in productivity ε^A.

4.2 The Case of Inflation Bias

Note that if the objective of the monetary authorities depended on deviations of output from *full employment output*, rather than the *natural level of output*, then the optimal inflation rate would have taken the form,

$$\pi_t = \pi^* + \alpha\xi \bar{u}_t - \xi\varepsilon_t^A \qquad (46')$$

In such a case, inflation would be systematically higher than π^* and would also depend positively on fluctuations in the "natural" rate of unemployment. Since the "natural" rate of unemployment is positive, there would be a positive inflation bias in such a case, as the inflationary expectations of "insiders" would adjust upwards, in order to neutralize the incentives of the monetary authorities to systematically try and increase employment and output above their "natural" levels.[11]

In what follows, we shall confine ourselves to the case in which the objective of the monetary authorities is described by (45), and the optimal policy of the monetary authorities does not suffer from the inflationary bias associated with trying to achieve full employment.

4.3 The Optimal Nominal Interest Rate Rule

Using the Fischer equation, equation (44) for the real interest rate, and equation (46) for inflation, the optimal nominal interest rate rule that delivers the inflation outcome in (46) is given by,

$$i_t = \bar{r}_t + \pi^* - \frac{\theta(1-\alpha)(1-\xi)}{\alpha}\varepsilon_t^A = \bar{r}_t + \pi^* - \theta(y_t - \bar{y}_t) = \bar{r}_t + \pi^* + \frac{\theta(1-\alpha)(1-\xi)}{\alpha\xi}\left(\pi_t - \pi^*\right) \qquad (49)$$

The "optimal" nominal rate of interest would have to be set equal to the equilibrium real interest rate, plus expected inflation π^*. Because from (44) the equilibrium real interest rate is equal to the "natural" real interest rate plus a term that depends on unanticipated inflation and innovations to productivity, the "optimal" interest rate rule can be written and interpreted in a number of ways, as in (49).

The "optimal" nominal interest rate must be equal to the "natural" real interest rate, plus the inflation target, minus a fraction of the current innovation to productivity, which increases output above its "natural" level. The rationale is that a positive innovation in productivity causes output to be higher than its "natural" level. Thus, the equilibrium real interest rate must be lower than its "natural" rate in the case of a positive productivity shock, so that there is a corresponding increase in aggregate consumption, to maintain product market equilibrium. The only way this can be done under the optimal policy is by reducing the current nominal interest rate, since expected future inflation under the optimal policy is always equal to π^*.

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[11] This is the well known Kydland and Prescott (1977) and Barro and Gordon (1983) inflation bias of time consistent monetary policy. Assuming that monetary policy is in the hands of an independent central banker, with an objective as in (45), addresses the inflation bias problem (Rogoff 1985).

The "optimal" nominal interest rate rule can also be expressed in terms of deviations of inflation from target. If inflation is above its expected value, signifying a negative innovation in productivity, the monetary authorities increase the nominal interest rate, which increases the real interest rate to the level that ensures equality between the lower output and consumption.

Note, that this interpretation of the optimal interest rate rule is "Wicksellian". The nominal interest rate is set to equal the "natural" real rate of interest, plus the inflation target of the authorities. However, the monetary authorities increase the nominal interest rate when inflation is higher than their target, and reduce the nominal interest rate when inflation is lower than their target.[12]

A remark on the informational requirements of the optimal interest rate rule is in order. There are three real shocks that affect inflation and real variables economy under the optimal nominal interest rate rule. Productivity shocks, employment shocks (shocks to the number of "insiders") and shocks to the preferences of consumers. Money demand shocks only affect real money balances.

If the authorities observe output and inflation, they can deduce all three real shocks. Inflation reveals information about productivity shocks. Given this information, output (or unemployment) reveal information about employment shocks, and, by the equality of output demand and supply, also information about shocks to consumer preferences. Knowing the shocks, the authorities can calculate the "natural" real interest rate, even if they do not directly observe it. Thus, in the context of this model, by observing output and inflation, the authorities can fully deduce the whole array of relevant shocks.

To further examine the informational requirements of the optimal interest rate rule, we can use (44) to substitute for the "natural" real interest rate in (49), in terms of the underlying parameters of the model and the various shocks. Then, one gets,

$$i_t = \rho + \pi^* - \theta\left((1-\alpha)(1-\eta_N)\bar{n}_t + (1-\eta^A)a_t \right) + (1-\eta_C)v_t^C - \frac{\theta(1-\alpha)(1-\xi)}{\alpha}\varepsilon_t^A \qquad (50)$$

The optimal rule requires exact knowledge of all the parameters of the model, in order for the authorities to calculate the optimal response to various shocks. Under rational expectations, the authorities can deduce the current realization of all the shocks, if they can observe current output and inflation. They deduce the current productivity shock from the output supply function, and the current consumption shock from the output demand function. Of course, the optimal rule is model specific and therefore not robust if the model used by the authorities is misspecified. However, we shall keep assuming rational expectations on the part of the central bank, meaning that the central bank knows the true model.

Even if the authorities have full information about the various shocks, as we have assumed, the real distortion, due to the high "natural rate" of unemployment, cannot be corrected by monetary policy. An attempt to do so would only result in inflationary bias, as we have already demonstrated in (46′).

[12] Wicksell (1898) was probably the earliest advocate of interest rate rules and price level targeting. He advocated the use of nominal interest rates by central banks to counteract deviations of the price level from a target price level. Under his rule, when inflation is higher than target inflation, nominal interest rates should rise above the "natural" interest rate to bring inflation back to target. When inflation is lower than target inflation, nominal interest rates should fall below the "natural" interest rate. See Section 5 for a fuller discussion of Wicksell's rule.

4.4 The Optimal Rule for the Rate of Growth of the Money Supply

The optimal monetary policy can also be expressed as an optimal rule for the rate of growth of the money supply. This requires the use of the money demand function (27). The money demand function (27) can be approximated around the long run equilibrium nominal interest rate $\rho+\pi^*$ as in,

$$m_t - p_t = y_t - \frac{1}{\theta}\ln\left(\frac{i_t}{1+i_t}\right) + \frac{1}{\theta}\left(v_t^M - v_t^C\right) \simeq m_0 + y_t - \zeta\left(r_t + \pi^*\right) + \frac{1}{\theta}\left(v_t^M - v_t^C\right) \tag{51}$$

where $m_0 = -\frac{1}{\theta}\left(\ln\left(\frac{\rho+\pi^*}{1+\rho+\pi^*}\right) - \frac{1}{1+\rho+\pi^*}\right)$, and $\zeta = \frac{1}{\theta(\rho+\pi^*)(1+\rho+\pi^*)}$.

ζ is the semi-elasticity of money demand with respect to the nominal interest rate.

Taking first differences in (51), and substituting for the optimal inflation rate from (46), we get the following optimal contingent rule for monetary growth,

$$\Delta m_t = \pi^* - \xi\varepsilon_t^A + \Delta y_t - \zeta\Delta r_t + \frac{1}{\theta}\left(\Delta v_t^M - \Delta v_t^C\right) \tag{52}$$

The optimal contingent rule for the money supply, requires the rate of change of the money supply to be equal to the inflation target of the central bank, but to also accommodate shocks to output, inflation and the real interest rate that shift the money demand function.

The optimal contingent rule for the money supply has a Friedman (1960) ring to it, although the famous Friedman *x* percent rule was a non contingent rule, and did not envisage the response of money growth to shocks in productivity, employment, consumption demand or money demand. The Friedman rule, for a constant rate of growth of the money supply, would only be optimal in the absence shocks that shift the money demand function.

Comparing (52) to (50) or (49), one cannot help noticing that the optimal contingent rule for the rate of growth of the money supply has higher informational requirements than the corresponding optimal contingent rule for the nominal interest rate. The optimal money growth rule requires that the monetary authorities observe and react to both real shocks, such as shocks to productivity and preferences for consumption, and monetary shocks, such as shocks to money demand.

The optimal nominal interest rate rule does not require knowledge of shocks to money demand, as, with an interest rate rule, the money supply automatically accommodates shocks to money demand. In all other respects, and under the assumption that the authorities have full information and can fully control the money supply, a money supply rule and an interest rate rule would lead to the same outcome.

5. Inflation and Output Fluctuations under the Taylor Rule

We next turn our attention from optimal policy to interest rate rules such as the Taylor (1993) and the Wicksell (1898) rule. We confine ourselves to interest rate rules, as, by revealed preference, the short run nominal interest rate is the instrument of choice of central banks. This does not necessarily mean that we ignore the implications of these rules for the money supply, as, through the money demand function, any interest rate rule can be transformed into a corresponding money supply rule.

5.1 Inflation Targeting Interest Rate Rules

As we have already mentioned, Wicksell (1898) was probably the earliest advocate of a stabilizing interest rate rule. He proposed that, "So long as prices remain unaltered, the banks' rate of interest is to remain unaltered. If prices rise, the rate of interest is to be raised; and if prices fall, the rate of interest is to be lowered; and the rate of interest is henceforth to be maintained at its new level until a further movement of prices calls for a further change in one direction or the other." (Wicksell 1898, p. 189).

Recall that Wicksell was also the one who introduced the distinction between the "natural" and the current interest rate, and that he was writing at a time when average (expected) inflation was zero, and before Fisher's (1896) distinction between nominal and real interest rates had become relevant.

In the context of our model, a Wicksell rule could be written as,

$$i_t = \bar{r}_t + \pi^* + \phi(\pi_t - \pi^*) \tag{53}$$

where $\bar{r}_t$ is the "natural" real rate of interest, and $\phi > 0$.

We have substituted inflation for the price level, but otherwise (53) is very close to the spirit of Wicksell. We have already demonstrated that the optimal interest rate rule in our model can be written in the form of a Wicksell rule, with,

$$\phi = \frac{\theta(1-\alpha)(1-\xi)}{\alpha\xi} \tag{54}$$

A special case of the Wicksell rule is the *absolute price level targeting* rule of Fisher (1919) and Simons (1936), who went further than Wicksell, by suggesting a policy of complete stabilization of the price level. In the present context, this could be expressed as *absolute inflation targeting*, i.e setting the nominal interest rate so that inflation is always equal to the target of the authorities π^*. This is the limit of the Wicksell rule (53), as the response of nominal interest rates to deviations of inflation from target tends to infinity. In this case, inflation would tend to be equal to π^* at all times.

Thus, the absolute inflation targeting rule could be expressed as the limit of (53), when $\phi \to \infty$.

5.2 The Taylor Rule and Optimal Policy

The Taylor (1993) rule is more general than the Wicksell rule. The principle of the Taylor rule requires the monetary authorities to set the nominal interest rate as a function of the "natural" real rate of interest, plus their inflation target, and adjust it on the basis of deviations of current inflation from target, and deviations of output from its "natural" level. We shall assume that this generalized Taylor rule takes the form,

$$i_t = \bar{r}_t + \pi^* + \phi_1(\pi_t - \pi^*) + \phi_2(y_t - \bar{y}_t) \quad (55)$$

where $\phi_1, \phi_2 > 0$.[13]

It is obvious that the Taylor rule (55) encompasses the previous rules as special cases. The Wicksell rule (53) is a special case of the Taylor rule, with $\phi_2=0$. The Fisher-Simons rule is the limiting case of (55) with $\phi_1 \to \infty$, and $\phi_2=0$. Thus, by analyzing the Taylor rule, we can also derive conclusions about both the Wicksell and the Fisher-Simons rules.

Substituting (55) in the Fisher equation (32), after using the real interest equation (44) and the output supply function (42), we get the following process for inflation,

$$\pi_t = \gamma_1 E_{t-1}\pi_t + \gamma_2 E_t \pi_{t+1} + (\phi_1 - 1)\gamma_2 \pi^* - \gamma_1 \varepsilon_t^A \quad (56)$$

where $\gamma_1 = \dfrac{(\phi_2+\theta)(1-\alpha)}{\phi_1\alpha + (\phi_2+\theta)(1-\alpha)} < 1$, $\gamma_2 = \dfrac{\alpha}{\phi_1\alpha + (\phi_2+\theta)(1-\alpha)}$.

Note that the inflationary process depends on both parameters of the Taylor rule, as unanticipated inflation causes output to deviate from its natural rate. In addition, the effects of productivity shocks on inflation, also depend on the parameters of the Taylor rule.[14]

One can show that the process (56) is stable, if, $\gamma_1+\gamma_2<1$. This requires,

$$\phi_1 > 1 \quad (57)$$

Condition (57), often referred to as the *Taylor principle*, requires that nominal interest rates react more than one to one deviations of current inflation from target inflation, in order to affect real rates. This is a sufficient condition for a stable and determinate inflation process.[15]

If (57) is satisfied, then the rational expectations solution of the inflation equation (56) is given by,

$$\pi_t = \pi^* - \gamma_1 \varepsilon_t^A \quad (58)$$

From (59), the fluctuations of inflation around the target of the monetary authorities π^* are not persistent, and only depend on the current innovation in productivity.[16]

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[13] Taylor (1993) proposed a rule is which the "natural" rate of interest was constant at 2%. We shall term this rule the *constant intercept Taylor rule*, and we analyze it in Appendix 2. In the body of the paper we stick to the spirit of the analysis of Taylor, and treat the "natural" real rate of interest as an endogenous variable.

[14] (56) being the inflationary process from a dynamic stochastic general equilibrium, in which the policy rule of the monetary authorities is taken into account, it does not suffer from the Lucas (1976) critique. Changing the parameters of the policy rule, would also change the parameters of the inflationary process.

[15] Woodford (2003), among others, contains a detailed discussion of the Taylor principle, and its significance for the resolution of the price level indeterminacy problem highlighted by Sargent and Wallace (1975) for non contingent interest rate rules.

[16] The solution of (56) under rational expectations is derived in Appendix 1.

The fluctuations of output around its "natural" level depend on unanticipated inflation and innovations in productivity. From (58), unanticipated inflation is determined by,

$$\pi_t - E_{t-1}\pi_t = \pi_t - \pi^* = -\gamma_1 \varepsilon_t^A \tag{59}$$

Substituting (59) in the output supply function (42), deviations of output from its natural rate under the Taylor rule are determined by,

$$y_t - \bar{y}_t = \frac{1-\alpha}{\alpha}(1-\gamma_1)\varepsilon_t^A \tag{60}$$

Like the case of the optimal policy, under the Taylor rule (55), only innovations in productivity induce deviations of inflation from target, and output from its "natural" level. Demand shocks, such as shocks to consumption preferences, are fully neutralized by monetary policy, as the nominal interest rate fully accommodates changes in the "natural" rate of interest. Only supply (productivity) shocks cause unanticipated inflation under a Taylor rule.

The expected loss of the monetary authorities under a Taylor rule would be given by,

$$\Lambda_t^{TR} = \frac{1}{2}\frac{1+\rho}{\rho}\left(Var(\pi_t - \pi^*) + \psi Var(y_t - \bar{y}_t)\right) = \frac{1}{2}\frac{1+\rho}{\rho}\left(\left((\mu_1)^2 + \psi\left(\frac{1-\alpha}{\alpha}\right)^2 \left(1-\gamma_1\right)^2\right)\sigma_A^2\right) \tag{61}$$

The Taylor rule (55) would be the optimal policy, if γ_1 was equal to the optimal response of unanticipated inflation to innovations in productivity ξ derived in the previous section. From the definition of ξ in (46), and the definition of γ_1 in (56), this requires that ϕ_1 and ϕ_2 must satisfy,

$$\frac{(\phi_2+\theta)(1-\alpha)}{\phi_1\alpha + (\phi_2+\theta)(1-\alpha)} = \frac{\psi(1-\alpha)^2}{\alpha^2 + \psi(1-\alpha)^2} \tag{62}$$

As long as the policy parameters are chosen to satisfy (62), a Taylor rule can fully replicate the optimal policy in this model.

Solving (62) for the optimal ϕ_1 we get,

$$\phi_1 = (\theta + \phi_2)\frac{\alpha}{\psi(1-\alpha)} \tag{63}$$

A number of results can be deduced about the optimal Taylor rule.

1. Existence. There exists a stable optimal Taylor rule of the form of (55), which minimizes the welfare loss of the monetary authorities (45), subject to the economy being described by equations (40) to (44).

Proof: The optimal Taylor rule exists, if there exists a pair of ϕ_1 and ϕ_2 which satisfy (63) and (57). Since $\theta>0$, $\psi>0$ and $1>\alpha>0$, there exists at least one pair of $\phi_1>0$ and $\phi_2>0$ which satisfy (63). For the Taylor principle (57) to be satisfied, the condition is that,

$$\phi_2 > 1 - \frac{\theta\alpha}{\psi(1-\alpha)} \qquad (64)$$

As long as the monetary authorities set ϕ_2 to satisfy (64), $\phi_1 > 1$ and the optimal Taylor rule is stable.

As can be seen from (64), there is a minimum response parameter of interest rates to deviations of output from its "natural" level ϕ_2 which guarantees the existence and stability of the optimal Taylor rule. This parameter is higher, the higher is the weight ψ attached by the monetary authorities to fluctuations in output relative to inflation. It is also higher, the higher is the inter-temporal elasticity of substitution in consumption $1/\theta$, and the higher is the elasticity of output with response to employment $1\text{-}\alpha$.

2. Non Uniqueness. There exists an infinite number of parameters satisfying both (63) and (57) for given θ, ψ and α. Thus, there exists an infinite set of optimal Taylor rules.

Proof: There is an infinite set of values of ϕ_2 which satisfy (64). This belongs to the continuous interval,

$$(1 - \frac{\theta\alpha}{\psi(1-\alpha)}, \infty)$$

Since ϕ_1 satisfies (63), which makes it a linear function of ϕ_2, there is also a infinite set of values of ϕ_1.

Thus, since (62) depends on two policy parameters, ϕ_1 and ϕ_2, there will be multiple optimal pairs of solutions, even for the same preferences of the monetary authorities and the other two structural parameters θ and α. However, the optimal pair must satisfy the extra restriction (57), that $\phi_1 > 1$. This is the condition for stability of the inflation process (56). Only a subset of the optimal parameters satisfying (62) will also satisfy (57). Yet the set of optimal Taylor rule parameters is infinite in this model as long as (64) and (63) are satisfied.

If we set $\phi_2 = 0$, as in the case of the Wicksell rule, we can still replicate the optimal policy by appropriate choice of ϕ_1. Thus, the optimal Taylor rule can in principle be replicated by an optimal Wicksell rule. Setting $\phi_2 = 0$ in (63), we get,

$$\phi_1 = \frac{\theta\alpha}{\psi(1-\alpha)} \qquad (65)$$

For stability, from (57), this ought to be higher that one. The condition for this is that,

$$\psi < \frac{\theta\alpha}{1-\alpha} \qquad (66)$$

Thus, for the optimal policy to be replicable by a Wicksell rule, according to which the interest rate does not respond to deviations of output from its "natural" level, but only to deviations of inflation from target, the weight of output relative to inflation in the preferences of the monetary authorities ought to be sufficiently low. Only if (66) is satisfied will a Wicksell rule be both optimal and stable.

Otherwise, a Taylor rule is the only stable interest rate rule. This proves property 3, which can be stated as,

3. Existence and Uniqueness of a Stable Wicksell Rule. As long as the weight ψ attached by the monetary authorities to deviations of output from its "natural" level, relative to deviations of inflation from target, is sufficiently low and satisfies (66), there exists a stable and unique optimal Wicksell rule, which is described by (65).

Finally, the Fisher-Simons absolute inflation targeting rule, which requires ϕ_1 to tend to infinity, will only be optimal if ψ, the relative weight attached by the monetary authorities to deviations of output from its "natural" level, relative to deviations of inflation from target, is equal to zero. This can be confirmed from both (63) and (65), for which it holds that,

$$\lim_{\psi \to 0} \phi_1 = \infty \tag{67}$$

This proves our last result, which can be stated as,

4. Optimality of Absolute Inflation Targeting. Absolute inflation targeting is optimal only if the weight ψ attached by the monetary authorities to deviations of output from its "natural" level, relative to deviations of inflation from target, is equal to zero.

One could use (63) and (64) to calculate the set of optimal Taylor rule parameters for different values of the structural parameters of the model and the preferences of the monetary authorities for output versus inflation. Assuming that $\theta=1$ and $\alpha=1/3$, the pair favored by Taylor, i.e $\phi_1=1.5$, $\phi_2=0.5$, suggests, in the context of our model, a relative weight on output relative to inflation ψ of about 0.5. However, the same outcome, under the same preferences of the monetary authorities, could be achieved by an infinite number of other parameters that satisfy (63) and (64).

On the other hand, for a Wicksell rule to be both optimal and stable, both (65) and (66) must be satisfied. Using the same values, $\theta=1$ and $\alpha=1/3$, (66) implies that ψ must be lower than 0.5. Thus, for the Wicksell rule to be applicable, ψ must be lower that 0.5 in the context of our model. For example, if $\psi=0.4$, the Wicksell rule would be optimal with $\phi_1=1.25$, which is uniquely determined from (65).

One final point is in order. In many discussions and applications of the Taylor rule, the current "natural" real rate of interest in (55) is replaced by the long run equilibrium real interest rate ρ. We call this *the constant intercept Taylor Rule*.[17]

In the context of our model, the constant intercept Taylor rule can be shown to be sub-optimal. It is demonstrated in Appendix 2, that this simpler version of the Taylor rule would result in policy

[17] Taylor (1993) is quite explicit that the intercept in the rule is a constant, equal to the long run equilibrium real interest rate (ρ in our model) and the inflation target π^*. He further specifies the sum of the two at 4%. He also specifies $\phi_1=1.5$ and $\phi_2=0.5$ on the basis of simulations with econometric models. Taylor and Williams (2011) defend such a rule on account of its simplicity. We have however adopted a broader interpretation of the Taylor rule in this paper, assuming that the intercept depends on the central bank's estimate of the current "natural" real interest rate. Note that the Taylor rule (55) requires that the authorities observe current inflation and current real output, and that they can also deduce the current deviation of output from its "natural" level from (42). If they have this information, they can also deduce the "natural" rate of interest in our model. Thus, the informational requirements of the constant intercept Taylor rule are the same as the more general rule (55).

induced inflation persistence, higher variability of both inflation and output and dependence of the variance of inflation and output on all real demand and supply shocks. The reason is that this version of the Taylor rule does not allow the nominal interest rate to react directly to fluctuations in the "natural" real interest rate, but only through deviations of current inflation from target, and output from its "natural" level.

6. Conclusions

In this paper we have analyzed the stabilizing role of monetary policy in a dynamic, stochastic general equilibrium model of the "natural rate", in which nominal wages are periodically set by labor market "insiders".

There are two distortions in this model, a real distortion, arising from the fact that "outsiders" are disenfranchised from the labor market, and a nominal distortion, arising from the fact that nominal wage contracts are not indexed, and can only be reopened at the beginning of each period. The nominal distortion allows for nominal shocks to have temporary real effects, and thus, for monetary policy to be able to affect short run fluctuations in both inflation and employment.

We have derived and discussed optimal monetary policy in the presence of a variety of real and nominal shocks, and highlighted the properties of the optimal monetary policy rule.

The optimal monetary policy rule is second best, as it cannot neutralize supply (productivity) shocks, and the authorities have to accept a tradeoff between inflation and output stabilization even under the optimal policy.

We have also demonstrated that the optimal policy can be replicated by an appropriately parametrized set of Taylor rules, according to which deviations of the current nominal interest rate from its "natural" rate, depend on deviations of inflation from target and output from its "natural" level. We have shown that the set of parameters of the optimal Taylor rule is not unique in the context of this model. We have also shown that, provided the monetary authorities attach a relatively low weight to output relative to inflation deviations, the optimal policy can also be replicated by an appropriately parametrized unique Wicksell rule, according to which deviations of the current nominal interest rate from its "natural" rate depend only on deviations of inflation from target.

We have thus established five main results regarding these rules: First, an appropriately parametrized Taylor rule can replicate the optimal policy. Second, that the set of parameters of the optimal Taylor rule is not unique. The Taylor rule can replicate the optimal policy for a number of pairs of parameters defining the response of nominal interest rates to deviations of inflation from target, and deviations of output from its "natural" level. Third, we have established that, provided the monetary authorities attach a relatively low weight to output relative to inflation, an appropriately parametrized Wicksell rule can also replicate the optimal policy. Fourth, in contrast to the optimal Taylor rule, the optimal Wicksell rule, if stable, is also unique. Finally, we have established that a policy of absolute inflation targeting is only optimal if the central bank is only concerned with deviations of inflation from target, and not deviations of output from its "natural" level.

In a separate analysis, we have examined the properties of a simplified, but widely used, version of the Taylor rule, the rule with a constant intercept. This rule is shown to be clearly sub-optimal, as it generates persistent fluctuations of inflation and high variability of deviations of output from its “natural” level. The persistent fluctuations of inflation are the only way for the real interest rate to equilibrate the product market in the presence of persistent demand and supply shocks under this simplified rule. In addition, because all shocks affect inflation under this rule, deviations of output from its “natural” level depend on innovations in all real shocks, and not only productivity shocks, as under the optimal rule. Thus, a constant intercept Taylor rule could impose potentially significant costs, in terms of persistent fluctuations of inflation, and higher variability of deviations of output from its “natural” level than the optimal rule.

To conclude, an optimal Taylor rule would require the central bank policy rate to react to the following three variables: 1. the current “natural” real rate of interest, 2. deviations of inflation from the central bank target, and, 3. deviations of output from its “natural” level. The policy rate should change point for point with the “natural” real rate of interest, but the optimal set of the other two reaction parameters is not unique. The reaction of the central bank policy rate to deviations of inflation from target should be more than one to one, satisfying the Taylor principle, but there is an infinite number of combinations of the inflation and output reaction parameters that are consistent with this, and would constitute an optimal Taylor rule. Thus, the Taylor rule is simple, but not too simple, in the sense that the monetary authorities cannot avoid seeking the best possible estimates of the current “natural” real rate of interest and the “natural” level of output or employment, and, they cannot avoid selecting a pair of reaction parameters to deviations of inflation from target and output from its “natural” level, from the wide range of options consistent with optimal policy.

Appendix 1
The Rational Expectations Solution for Inflation under the Taylor Rule

In this appendix we derive the rational expectations solution of the inflation equation (56) under the Taylor rule, and prove that this is given by equation (58). We thus start from,

$$\pi_t = \gamma_1 E_{t-1}\pi_t + \gamma_2 E_t \pi_{t+1} + (\phi_1 - 1)\gamma_2 \pi^* - \gamma_1 \varepsilon_t^A \tag{56}$$

where,

$$\gamma_1 = \frac{(\phi_2 + \theta)(1-\alpha)}{\phi_1 \alpha + (\phi_2 + \theta)(1-\alpha)}, \; \gamma_2 = \frac{\alpha}{\phi_1 \alpha + (\phi_2 + \theta)(1-\alpha)}. \tag{A.1.1}$$

(56) contains expectations about inflation conditioned on different information sets, *t-1* and *t*.

Note from (A.1.1) that,

$$\frac{\gamma_2}{1-\gamma_1} = \frac{1}{\phi_1} \tag{A.1.2}$$

$$\gamma_1 + (\phi_1 - 1)\gamma_2 = 1 - \gamma_2 \tag{A.1.3}$$

Taking the rational expectation of (56), based on information up to the end of period *t-1,* we get,

$$E_{t-1}\pi_t = \frac{(\phi_1 - 1)\gamma_2}{1-\gamma_1}\pi^* + \frac{\gamma_2}{1-\gamma_1}E_{t-1}\pi_{t+1} = \left(1 - \frac{1}{\phi_1}\right)\pi^* + \frac{1}{\phi_1}E_{t-1}\pi_{t+1} \tag{A.1.4}$$

The condition for (A.1.4) to be stable, is that $1/\phi_1<1$, which requires $\phi_1>1$. This proves condition (57), i.e. the Taylor principle.

Since inflationary expectations are a non predetermined variable, the fundamental solution of equation (A.1.4) is thus given by,

$$E_{t-1}\pi_t = \pi^* \tag{A.1.5}$$

Substituting (A.1.4) in (56), we get,

$$\pi_t = \left(\gamma_1 + (\phi_1 - 1)\gamma_2\right)\pi^* + \gamma_2 E_t \pi_{t+1} - \gamma_1 \varepsilon_t^A \tag{A.1.6}$$

If $\phi_1>1$, then $\gamma_2<1$ and the process (A.1.6) is stable too. Its fundamental solution, after we make use of (A.1.3) and the assumption that ε^A is a white noise process, takes the form of (58),

$$\pi_t = \frac{\gamma_1 + (\phi_1 - 1)\gamma_2}{1-\gamma_2}\pi^* - \gamma_1 E_t \sum_{s=0}^{\infty} (\gamma_2)^s \varepsilon_{t+s}^A = \pi^* - \gamma_1 \varepsilon_t^A \tag{58}$$

Appendix 2
Inflation and Output Fluctuations under a Constant Intercept Taylor Rule

In this appendix, we examine the properties of the constant intercept Taylor rule. Under the constant intercept Taylor rule, the interest rate is determined by,

$$i_t = \rho + \pi^* + \phi_1(\pi_t - \pi^*) + \phi_2(y_t - \bar{y}_t) \tag{A.2.1}$$

where $\phi_1, \phi_2 > 0$.

Substituting (A.2.1) in the Fisher equation (32), after using the real interest equation (44) and the output supply function (42), we get the following process for inflation,

$$\pi_t = \gamma_1 E_{t-1}\pi_t + \gamma_2 E_t \pi_{t+1} + (\phi_1 - 1)\gamma_2 \pi^* + \gamma_2(\bar{r}_t - \rho) - \gamma_1 \varepsilon_t^A \tag{A.2.2}$$

where $\gamma_1 = \dfrac{(\phi_2 + \theta)(1-\alpha)}{\phi_1 \alpha + (\phi_2 + \theta)(1-\alpha)} < 1$, $\gamma_2 = \dfrac{\alpha}{\phi_1 \alpha + (\phi_2 + \theta)(1-\alpha)}$.

One can show that the process (A.2.2) is stable, if, $\gamma_1 + \gamma_2 < 1$. This requires,

$$\phi_1 > 1 \tag{A.2.3}$$

The rational expectations solution of the inflation equation (A.2.2) is given by,[18]

$$\pi_t = \pi^* + \left(\frac{1-\eta_C}{\phi_1 - \eta_C} v_t^C - \frac{\theta(1-\alpha)(1-\eta_N)}{\phi_1 - \eta_N}(\bar{n}_t - \bar{n}) - \frac{\theta(1-\eta_A)}{\phi_1 - \eta_A} a_t \right) - \gamma_1 \varepsilon_t^A \tag{A.2.4}$$

From (A.2.4), inflation displays persistent fluctuations around the target of the monetary authorities π^*. It depends on the three autoregressive processes driving the shocks to consumption preferences, the labor market and productivity, as under the constant intercept Taylor rule the nominal interest rate does not react directly to the shocks affecting the "natural" real rate of interest. Thus, persistent fluctuations in inflation are generated under this rule, in order for the real interest rate to ensure equality between output and consumption. Yet, these persistent fluctuations in inflation are undesirable from the point of view of the monetary authorities, especially as they do not contribute to the stabilization of output around its "natural" level.

The fluctuations of output around its "natural" level depend on unanticipated inflation and innovations in productivity. From (A.2.4), unanticipated inflation is determined by,

$$\pi_t - E_{t-1}\pi_t = \left(\frac{1-\eta_c}{\phi_1 - \eta_c} \varepsilon_t^C - \frac{\theta(1-\alpha)(1-\eta_N)}{\phi_1 - \eta_N} \varepsilon_t^N - \left(\gamma_1 + \frac{\theta(1-\eta_A)}{\phi_1 - \eta_A} \right) \varepsilon_t^A \right) \tag{A.2.5}$$

[18] The rational expectations solution is arrived at by a process similar to the one used in Appendix 1.

Substituting (A.2.5) in the output supply function (42), deviations of output from its natural rate under the constant intercept Taylor rule are determined by,

$$y_t - \bar{y}_t = \frac{1-\alpha}{\alpha}\left(\frac{1-\eta_c}{\phi_1 - \eta_c}\varepsilon_t^C - \frac{\theta(1-\alpha)(1-\eta_N)}{\phi_1 - \eta_N}\varepsilon_t^N - \left(\gamma_1 + \frac{\theta(1-\eta_A)}{\phi_1 - \eta_A}\right)\varepsilon_t^A \right) \tag{A.2.6}$$

Unlike the case of the optimal policy, where only innovations in productivity induced deviations of output from its "natural" level, under the constant intercept Taylor rule, all real shocks, such as shocks to consumer demand, employment and productivity induce such deviations. The reason is that they cause unanticipated inflation through the interest rate rule.

To conclude, the simple Taylor rule with a constant intercept would result in policy induced persistent fluctuations of inflation, in order to ensure equilibrium fluctuations in the real interest rate. It would also cause unanticipated inflation and deviations of output from its natural level to depend on innovations in all real shocks, and not only productivity shocks, as under the optimal rule. This would result in higher losses for the monetary authorities compared to the optimal rule.[19]

The expected loss of the monetary authorities under the constant intercept Taylor rule (CITR) would be given by,

$$\Lambda_t^{CITR} = \frac{1}{2}\frac{1+\rho}{\rho}\left(Var(\pi_t - \pi^*) + \psi Var(y_t - \bar{y}_t) \right) \tag{A.2.7}$$

where,

$$Var(\pi_t - \pi^*) = \left(\left(\frac{1}{\phi_1 - \eta_C}\right)^2 \sigma_C^2 + \left(\frac{\theta(1-\alpha)}{\phi_1 - \eta_N}\right)^2 \sigma_N^2 + \left(\frac{\theta}{\phi_1 - \eta_A} + \gamma_1\right)^2 \sigma_A^2 \right)$$

$$Var(y_t - \bar{y}_t) = \left(\frac{1-\alpha}{\alpha}\right)^2 \left(\left(\frac{1-\eta_C}{\phi_1 - \eta_C}\right)^2 \sigma_C^2 + \left(\frac{\theta(1-\alpha)(1-\eta_N)}{\phi_1 - \eta_N}\right)^2 \sigma_N^2 + \left(\frac{\theta(1-\eta_A)}{\phi_1 - \eta_A} + \gamma_1\right)^2 \sigma_A^2 \right)$$

All the real shocks contribute to the variance of both deviations of inflation from target and output from its "natural" level.

To conclude, there are a number of differences of the constant intercept Taylor rule from the optimal monetary policy rule. First, under this rule, deviations of inflation from target display policy induced persistence, as nominal interest rates do not react to directly to anticipated fluctuations in the "natural" rate of interest, but only through their reaction to inflation. Second, under this rule, all shocks affect inflation, unanticipated inflation and deviations of output from its "natural" level, unlike the optimal policy, under which only productivity shocks cause such deviations.

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[19] A similar point has been made by Woodford (2001) in the context of a new keynesian model.

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