Abstract: We study the effects of federal transfer programs on non-coordinated national environmental policies, economic growth and natural resources in a federal economy. Natural resources are a world public good. In each country, production degrades the environment, but clean-up policy can improve it. Clean-up policy is financed by taxes on polluting firms’ output and federal redistributive transfers. We solve for a symmetric Nash equilibrium among national governments. Federal transfer policies that lead to higher pollution taxes make existence harder, and are harmful not only to growth but also to the environment. The best way to improve federation-wide environmental quality is to implement a taxation system that stimulates growth and broadens tax bases to finance national clean-up policies.

Keywords: Environment. Fiscal federalism. Economic growth.

JEL classification: F43, O4, Q2, H4.

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I. INTRODUCTION

The open-access and public-good features of environmental quality imply that the costs of pollution are not fully internalized by private agents. Such externalities justify government intervention. Also, domestically generated pollution can cause damages to other countries too, i.e. pollution is transboundary. This can justify cooperation of national policies. A form of international cooperation is fiscal federalism.¹ In that case, the usual practice is between full decentralization and full federalism. For instance, national governments choose their own pollution taxes and clean-up policies to advance their own objectives, while federal authorities design institutions to control transboundary pollution and promote environmental convergence across federation.² This is especially true in the European Union (EU), where a key federal policy instrument is “financial support measures”. The latter are transfer programs which redistribute income across regions/countries to subsidize environmental infrastructure projects in regions/countries in need of funds (see e.g. Dewatripont et al. [1995], Silva and Caplan [1997] and Caplan et al. [2000]).

With few exceptions (see below), little theoretical attention has been given to the link between environmental policies at both federal and national level, economic growth and natural resources. Therefore, this paper studies how commonly used federal transfer programs affect the properties and dynamics of non-cooperative national environmental policies, economic growth and natural resources. The model is a general equilibrium growth model of a decentralized federalist public economy. We focus on positive issues.

The model is as follows. A federation consists of a federal government and a number of member-countries. In each country, households get utility from private

¹ The stronger the spillover effects, the stronger the arguments against decentralized (Nash) decisions. Then, the question is “Cooperation, at what level?”’. There is cooperation at inter-governmental level (national governments choose jointly their policy instruments) and at supra-national or federal level (e.g. creation of federal institutions, rules and law). In a second-best world, cooperation should take place at both levels (see e.g. Persson and Tabellini [1995] and Oates [1999]).

² For instance, in the European Union (EU), according to the Fifth Environmental Action Program, which was approved by the Council and the Representatives of the Governments of the Member States on February 1, 1993, Sustainable Development requires a wide range of instruments like: (i) legislation to set environmental standards; (ii) economic instruments to encourage environmentally friendly products and processes; (iii) horizontal support measures like education, information and research; (iv) financial support measures, i.e. funds. Concerns about the environment have been reconfirmed in a stronger way more recently. The Agenda 2000 emphasizes the need for environmental cooperation and integration.
consumption and federation-wide environmental quality, which is a world public good. Firms produce goods by using a linear, $AK$-type technology. In doing so, they pollute the environment, i.e. pollution is a by-product of production. In each country, clean-up policy is financed by taxes on polluting firms’ output and federal transfers. Federal transfers redistribute resources from above-average to below-average income countries, as well as from countries with above-average environmental quality to countries with below-average environmental quality. We do not claim that these are necessarily good criteria for, or the only criteria for, transferring resources across countries. However, they seem to reflect commonly used transfer programs, especially in the EU where redistributive transfers are based on the need for financial support and the status of national environmental standards.3

Domestic taxes on polluting firms’ output are chosen by benevolent national governments. When the latter choose their distortionary tax policy, they face the dual task of raising revenues to finance the maintenance of public good (environmental quality) on the one hand, and internalizing negative externalities (pollution) on the other hand. This is a second-best setup. National governments play non-cooperatively (Nash) vis-à-vis each other. As argued above, this is a typical situation in decentralized federalist economies. We focus on symmetric Nash equilibria in national policies. Within this framework, we first establish existence and uniqueness of an equilibrium.4 We focus on Sustainable Balanced Growth Paths (SBGPs), i.e. long-run equilibria in which the economy can grow without damaging the environment. We then study the dynamic properties of this SBGP, and in particular how it is affected by federal policies.

3 In its Agenda 2000, the European Commission proposes (among others): (a) the integration of the environment as an objective for assistance from the Structural Funds; (b) the adoption of a degraded environment as one of the criteria for defining eligible urban areas; (c) the introduction of a partnership involving environmental bodies for the preparation of Cohesion policy intervention programs. Thus, relatively low levels of environmental quality seem to be reasons for transfers in the EU. Also, along the same lines, the US and EU have recently promised to help Ukraine to clean up and shut down the Chernobyl nuclear power station, as well as to support the Ukrainian energy industry.

4 The problem of existence and uniqueness is no mere theoretical curiosity but also produces some real issues in the choice of policy. In particular, we will show that badly-targeted federal transfer policies, which give rise to distortions like moral hazard, make also existence of an equilibrium harder. But should we care about existence? Baumol and Oates [1988, p. 9] answer this question by saying: “The issue of existence is highly relevant. If no solution exists, theoretical discussion of policy is basically pointless. It is possible also that the necessary or sufficient conditions for existence will themselves turn out to have some direct policy implications”. Our results will show exactly this.
Our main results are as follows. When the federal transfer program is designed to reward those member-countries that pollute less than the average, all countries have an incentive to pollute less. This makes existence of a non-cooperative equilibrium easier. It also leads to lower tax rates on polluting firms’ output. Lower tax rates, in turn, lead to higher economic growth and improving environmental quality. This is because higher economic growth generates larger tax bases and hence higher clean-up policy. Eventually, there is better environmental quality, despite the adverse effect of higher growth and pollution. By contrast, when the federal transfer program is designed to help those member-countries whose environmental quality (measured by their stock of natural resources) is below the average, all countries have a disincentive to care about their environment. Now, there is a typical moral hazard behavior, which makes existence of a non-cooperative equilibrium harder. It also leads to higher tax rates on polluting firms’ output. Higher tax rates, in turn, lead to lower economic growth, smaller tax bases, lower clean-up policy and eventually deteriorating environmental quality.5 Therefore, only growing economies can afford the resources to combat environmental problems, and this ability is affected by whether federal transfer policies are well-targeted.

Therefore, there are two policy lessons (for details, see section IV below). First, the best way to improve environmental quality on an enduring basis is to implement a tax-and-transfer system that stimulates growth and generates tax bases broad enough to finance clean-up policies (see also John and Pecchenino [1994] and Philippopoulos and Economides [2001], while for empirical evidence see Grossman and Krueger [1995]). Second, federal transfer policies that give the wrong incentives and lead to high tax rates and low capital accumulation reduce social welfare, because they are harmful not only to economic growth but also to the environment.

Our paper combines two directions in the literature. The first direction is on environmental policy in federal economies. For instance, Wellisch [1995] and Silva and Caplan [1997] have used static models to study how alternative federal systems internalize spillover effects and control global pollution. The second direction is on economic growth and environmental policy. This is a very rich and still growing

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5 This is well known. That is, in moral hazard models, redistribution leads to higher taxes, and this is bad for efficiency. Here, we also show that redistribution (when it leads to high pollution tax rates) can be bad
literature (see e.g. Kolstad and Krautkraemer [1993] and Smulders [1995] for review papers). Our paper combines these two directions by constructing a growth model with environmental policy at both federal and national level. A paper close to ours is that of John and Pecchenino [1997], who present a two-country growth model with environmental externalities and inter-country transfers. However, they assume social planners, lump-sum transfers and no physical capital. By contrast, our paper studies how state-contingent federal transfers affect national policies, economic growth and natural resources in a second-best setup.

The rest of the paper is as follows. Section II presents the model and solves for a competitive equilibrium given policy. Section III solves for a Nash equilibrium in national policies. Section IV summarizes policy implications and closes the paper. Mathematical proofs are gathered in an Appendix.

II. A WORLD FEDERAL ECONOMY

Consider a world, federal economy composed of a finite number of member-countries, \( i = 1,2,\ldots,I \). Each country \( i \) is populated by private agents (a representative household and a representative firm) and a national government. Households purchase goods, work and save in the form of capital. Firms produce goods by using a linear technology. In doing so, they pollute the environment.\(^6\) In each country \( i \), clean-up policy is financed by domestic taxes on polluting firms’ output and federal transfers. For simplicity, all factors of production are internationally immobile.\(^7\) We assume continuous time, infinite horizons and perfect foresight. This section solves for a competitive equilibrium given economic policy. Note that we take the coexistence of federal and not only for growth but also for the environment.

\(^6\) Thus, pollution is a by-product of production. Our results do not change if pollution is also a by-product of consumption. On the other hand, modeling pollution as a by-product of economic activity (production and consumption) differs from the case in which natural resources are extracted from preserved natural environments to be used as inputs in private production. See Economides et al. [2002] for a formal comparison between pollution emissions and resource extraction.

\(^7\) Cremer and Pestieau [1999] also assume factor immobility in a model with federal redistribution. This means that there are no spillover effects arising from factor mobility. Thus, national governments compete with each other for federal transfers only. They do not compete for mobile tax bases, as in the literature on international capital taxation. Our main results are not affected by this.
national governments as given (see also e.g. Silva and Caplan [1997], Dixit and Londregan [1998] and Caplan et al. [2000]).

**Households’ behavior**

The environment is a global, public good that affects welfare in each country. Specifically, we assume that the average stock of natural resources across countries, 

$$\bar{N} \equiv \frac{1}{I} \sum_{i=1}^{I} N^i$$

provides direct utility to households in each country $i$ (where $N^i$ denotes the stock of natural resources in country $i$). The representative infinite-lived household in country $i$ maximizes intertemporal utility:

$$\int_{0}^{\infty} [u(c^i, \bar{N})] e^{-\rho t} \, dt$$

(1a)

where $c^i$ is private consumption in country $i$ and the parameter $\rho > 0$ is the rate of time preference. The instantaneous utility function $u(.)$ is increasing and concave in its two arguments, and also satisfies the Inada conditions. For simplicity, we assume that $u(.)$ is additively separable and logarithmic so that:

$$u(c^i, \bar{N}) = \log c^i + \nu \log \bar{N}$$

(1b)

where the parameter $\nu > 0$ is the weight given to environmental quality relative to private consumption.

Households save in the form of domestic capital, $k^i$. When the household in country $i$ rents out $k^i$ to domestic firms, it receives a rate of return, $r^i$. It also supplies inelastically one unit of labor services and gets a labor income $w^i$. Further, it receives profits $\pi^i$ from domestic firms. The budget constraint of the household in country $i$ is:

$$k^i + c^i = r^i k^i + w^i + \pi^i$$

(2)
where a dot over a variable denotes time derivative. Countries can differ in their initial capital stocks, $k^i(0)$.

The household acts competitively by taking prices and federation-average environmental quality, $\overline{N}$, as given. The latter is justified by the open-access and public-good features of the environment. The control variables are $c^i$ and $k^i$, so that the first-order conditions for a maximum are equation (2) as well as the familiar Euler condition:

$$c^i = (r^i - \rho)c^i$$  \hspace{1cm} (3)$$

*Firms’ behavior*

In each country $i$, output $y^i$ is linear in capital $k^i$, as in the well-known $AK$ model of endogenous growth. Thus, the production function takes the form:

$$y^i = A k^i$$  \hspace{1cm} (4)$$

where $A > 0$ is a parameter.

Let $0 \leq \theta^i < 1$ be the tax rate on polluting firms’ output. Then, the net profit of the representative firm in country $i$ is:

$$\pi^i = (1 - \theta^i)y^i - r^i k^i - w^i$$  \hspace{1cm} (5)$$

The firm acts competitively by taking prices and tax policy as given. This is a static problem. The control variable is $k^i$, so that the first-order condition for a maximum is simply:

$$r^i = (1 - \theta^i) A$$  \hspace{1cm} (6a)$$
which equates the rate of return to the after-tax marginal product of capital. In turn, using (4) and (6a) into (5), we get for zero profits:

\[ w' = (1 - \theta^i) y^i - r^i k^i = 0 \quad (6b) \]

that is, all realized income goes to capital. This is a known result in the \( AK \) model.\(^9\)

**Government Budget Constraint**

Each national government \( i \) finances its clean-up policy, \( g^i \), by taxes on domestic firms’ output, \( \theta^i y^i \), and transfers from the federal government, \( t^i \). Thus, at any point of time, the budget constraint of the national government in each country \( i \) is:

\[ g^i = \theta^i y^i + t^i \quad (7) \]

**Renewable Natural Resources and Pollution**

The stock of renewable natural resources in each country \( i \), \( N^i \), evolves over time according to:

\[ \dot{N}^i = \delta N^i - p^i + g^i \quad (8) \]

where the parameter \( \delta \geq 0 \) is the rate of regeneration of natural resources. That is, natural resources increase over time with natural regeneration, \( \delta N^i \), and clean-up policy, \( g^i \), but they decrease with pollution emission, \( p^i \).\(^{10}\) Countries can differ in their initial stocks of natural resources, \( N^i(0) \).

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8 Our main results do not change if taxes are imposed on households.
9 See e.g. Barro and Sala-i-Martin [1995, pp. 141-2].
10 John and Pecchenino [1994] use a more general specification of (8) of the form \( \dot{N}^i = -\delta N^i - \beta p^i + \sigma g^i \), so that \( \delta \) is now the rate of depreciation of natural resources, \( \beta \) is the rate of degradation of the environment as a result of economic activity and \( \sigma \) is a clean-up technology parameter.
Pollution, $p^i$, is modeled as a by-product of output produced, $y^i$. Specifically, we assume:

$$p^i = y^i$$

Equation (9) also implies that output taxes act as pollution taxes.

**World Competitive Equilibrium (WCE)**

We now characterize a World Competitive Equilibrium (WCE). This is for any feasible (national and federal) economic policy.

Using (6a)-(6b), (3) and (2) give respectively the private agents’ optimal rules for consumption and saving in each country $i$ in a WCE:

$$c^i = [(1 - \theta^i)A - \rho]c^i$$  \hspace{1cm} (10a)

$$k^i = (1 - \theta^i)Ak^i - c^i$$  \hspace{1cm} (10b)

Using (4), (7) and (9) into (8), the motion of natural resources in each country $i$ in a WCE is:

$$N^i = \delta N^i - (1 - \theta^i)Ak^i + t^i$$  \hspace{1cm} (10c)

To complete the world economy, we have to specify federal policy, $t^i$. We assume that the transfer $t^i$ from the federal government to each member-country $i$ follows the linear feedback rule:

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11 This assumption is consistent with empirical evidence that the world’s largest emitters are the rich countries, like the US, Japan and Germany. A clear example is $CO_2$ emissions. This does not include e.g. land-use and forestry, which have to do with resource extraction. In the case of resource extraction, most of the environmental damage takes place in the developing countries.
\[ t^i = \tau_1(\bar{y} - y^i) + \tau_2(\bar{N} - N^i) \]  

(11a)

where \( \tau_1 \geq 0 \) and \( \tau_2 \geq 0 \) are policy parameters, \( \bar{y} \equiv \frac{1}{I} \sum_{i=1}^{I} y^i \) is the federation-average output, and \( \bar{N} \equiv \frac{1}{I} \sum_{i=1}^{I} N^i \) is the federation-average stock of natural resources. According to the policy rule (11a), the federal government redistributes from above-average income countries to below-average income countries,\(^{14}\) as well as from countries with above-average environmental quality to countries with below-average environmental quality. As we have argued above, this rule reflects commonly used international transfer systems.

At any point of time, the federal government balances its budget. Thus, the sum of net transfers is zero:

\[ \sum_{i=1}^{I} t^i = 0 \]  

(11b)

To sum up, this section has solved for a World Competitive Equilibrium (WCE) in a world, federal economy with international transfers. In this equilibrium: (i) Private decisions maximize households’ utility and firms’ profits [this is summarized by (10a) and (10b)]. (ii) All constraints are satisfied and all markets clear [this is summarized by (10c), (11a) and (11b)]. The WCE holds for given initial conditions and any (national and federal) economic policy. The next section will endogenize national tax policies, \( \theta^i \).

### III. NON-COORDINATED NATIONAL POLICIES

This section solves for a non-cooperative (Nash) game among benevolent national governments. Each national government \( i \) maximizes the utility of its own representative

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\(^{12}\) That is, the emission residuals generated by economic activity are equal to the level of economic activity. Assuming a one-to-one link is only for simplicity.

\(^{13}\) See e.g. Park and Philippopoulos [2001] for similar redistribution policy rules.
household by taking as given the policies of other national governments, \( j \neq i \). In doing so, it plays Stackelberg vis-à-vis private agents; that is, each national government takes into account the WCE above.

Each national government \( i \) chooses \( \theta^i, c^i, k^i, N^i \) to maximize (1a)-(1b), subject to (10a), (10b) and (10c), and by taking as given \( \theta^j, c^j, k^j, N^j \) where \( j \neq i \). Using (4) and (11a) for \( y^i \) and \( t^i \) respectively into (10c), the current-value Hamiltonian of this problem, denoted by \( H^i \), is: \(^{15}\)

\[
H^i \equiv \log c^i + v \log \frac{\sum N^i}{I} + \lambda^i c^i [(1 - \theta^i) A - \rho] + \gamma^i (1 - \theta^i) A k^i - c^i] + \\
\quad + \mu^i \left[ \delta N^i - (1 - \theta^i) A k^i + \tau_i \left( \frac{\sum A k^i}{I} - A k^i \right) + \tau_2 \left( \frac{\sum N^i}{I} - N^i \right) \right]
\]

where \( \lambda^i \), \( \gamma^i \) and \( \mu^i \) are dynamic multipliers associated with the constraints (10a), (10b) and (10c) respectively. Thus, from the viewpoint of country \( i \), \( \lambda^i \) represents the social value of private valuation of assets, \( \gamma^i \) represents the social value of its physical capital, and \( \mu^i \) is the social value of the country’s natural resources.

**Symmetric Nash Equilibrium**

We will focus on Symmetric Nash Equilibria (SNE) in policy strategies. At a SNE, strategies are symmetric *ex-post*, so that \( x^i = x^j \equiv x \), where \( i \neq j \) and \( x \equiv (\theta, c, k, N, \gamma, \lambda, \mu) \). This means that no actual transfer payments take place in equilibrium. This is not very restrictive. What matters is the *anticipation* of net transfers as opposed to actual transfers. As Benabou [1996] points out, “the fight over the pie does not necessarily lead to higher transfers, just to higher distortions”. And this (i.e. incentives) can be captured by symmetric equilibria.

\(^{14}\) That is, if \( y^i > y^j \), country \( i \) is a receiver, while if \( y^i < y^j \), country \( i \) is a donor.

\(^{15}\) We assume commitment technologies on behalf of governments.
Invoking symmetry into the first-order conditions for \( \theta, c, \lambda, \gamma, k, \mu, N \), we get respectively (from now on, we omit the country-superscript \( i \)).\(^{16}\)

\[
\begin{align*}
\lambda c + \gamma k &= \mu k \quad (12a) \\
\lambda &= \rho \lambda - \frac{1}{c} - \lambda [(1 - \theta)A - \rho] + \gamma \quad (12b) \\
c &= [(1 - \theta)A - \rho]c \quad (12c) \\
k &= (1 - \theta)Ak - c \quad (12d) \\
\gamma &= \rho \gamma - (1 - \theta)A\gamma + (1 - \theta)A\mu + A\hat{\tau}, \mu \quad (12e) \\
N &= \delta N - (1 - \theta)Ak \quad (12f) \\
\mu &= \rho \mu - \frac{\hat{v}}{N} - \delta \mu + \hat{\tau}_2 \mu \quad (12g)
\end{align*}
\]

where \( \hat{v} \equiv \frac{v}{I} \) denotes the “effective” weight given to environmental quality relative to private consumption (recall that \( I \) is the exogenous number of member-countries), while \( \hat{\tau}_1 \equiv \tau_1 (1 - \frac{1}{I}) \geq 0 \) and \( \hat{\tau}_2 \equiv \tau_2 (1 - \frac{1}{I}) \geq 0 \) denote “effective” redistribution parameters.\(^{17}\)

These necessary conditions are completed with the addition of a transversality condition that guarantees utility is bounded. A sufficient condition for this to hold is:

\[
[(1 - \theta)A - \rho] + \delta < \rho \quad (12h)
\]

so that the growth rate of consumption, \([ (1 - \theta)A - \rho ] \), plus the rate of regeneration of natural resources, \( \delta \), is less than the rate of time preference, \( \rho \).

\(^{16}\) The world market-clearing condition \((11b)\) is also satisfied.

\(^{17}\) If there is only one country, i.e. \( I = 1 \), redistribution does not make sense and so \( \hat{\tau}_1 = \hat{\tau}_2 = 0 \).
Following usual practice in growth models, we transform the variables to facilitate analytical tractability. Let us define \( z \equiv \frac{c}{k} \), \( \psi \equiv \mu k \) and \( \phi \equiv \mu N \). Then, Appendix A shows that the dynamics of (12a)-(12g) are equivalent to the dynamics of (13a)-(13d) below:

\[
\begin{align*}
\dot{z} &= (z - \rho)z \\
\dot{\psi} &= \left[ (1 - \theta)A - z + \rho - \frac{\hat{\psi}}{\phi} - \delta + \hat{\tau}_2 \right] \psi \\
\dot{\phi} &= \left[ \rho - \frac{\hat{\psi}}{\phi} - \frac{(1 - \theta)A \psi}{\phi} + \hat{\tau}_2 \right] \phi \\
\left[ z + \delta + \frac{\hat{\psi}}{\phi} + A \hat{\tau}_1 - \hat{\tau}_2 \right] \psi &= 1
\end{align*}
\]

where (13a)-(13d) constitute a system in \( z, \psi, \phi, \theta \). Note that only (13a)-(13c) are dynamic, while (13d) is static. That is, the dynamics of \( \theta \) are the dynamics of \( z, \psi, \phi \).

To sum up, we have solved for a Symmetric Nash Equilibrium in pollution tax rates among national policymakers. This world equilibrium is summarized by equations (13a)-(13d) and the transversality condition (12h). We will now check its properties.

**Long-run Nash Equilibrium**

This subsection solves for a long-run equilibrium of (13a)-(13d). This is defined by \( \dot{z} = \dot{\psi} = \dot{\phi} = 0 \). Since \( z \equiv \frac{c}{k} \), \( \psi \equiv \mu k \) and \( \phi \equiv \mu N \), this steady state implies that all per capita quantities can grow at the same constant positive rate.\(^{18}\) This is typical of *AK*

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\(^{18}\) This is as follows: (i) Since \( z = \frac{c}{k} \), \( \dot{z} = 0 \) implies that consumption, \( c \), and the stock of capital, \( k \), grow at the same rate. (ii) Since \( \psi = \mu k \), \( \dot{\psi} = 0 \) implies that the social valuation of natural resources, \( \mu \), and \( k \) grow at equal, but opposite, rates. (iii) Since \( \phi = \mu N \), \( \dot{\phi} = 0 \) implies that \( \mu \) and the stock of natural resources, \( N \), grow at equal, but opposite, rates. Then, (i)-(iii) imply that \( c \), \( k \) and \( N \) grow at the same
type growth models. Thus, this is a Balanced Growth Path. It is also a Sustainable Balanced Growth Path (SBGP), because consumption and capital can grow positively without damaging the environment.

Let us denote the steady state values of \((z, \psi, \phi, \theta)\) by \((\tilde{z}, \tilde{\psi}, \tilde{\phi}, \tilde{\theta})\). Appendix B solves for \((\tilde{z}, \tilde{\psi}, \tilde{\phi}, \tilde{\theta})\) and proves the existence and uniqueness of a long-run equilibrium. The main results are summarized by the following proposition:

**PROPOSITION 1:** Focusing on symmetric non-cooperative (Nash) equilibria in national environmental policies, if the parameter values satisfy the conditions:

\[
A > \rho + \delta \quad (14a)
\]

\[
\rho > 2\delta \quad (14b)
\]

\[
\hat{\nu} \delta (A \hat{\tau}_1 + 2\rho) > \rho (\rho + \hat{\tau}_2 - \delta) \quad (14c)
\]

there exists a unique long-run pollution tax rate denoted by \(\tilde{\theta}^{nash}\). This tax rate lies in the region \(0 < 1 - \frac{\rho + \delta}{A} < \tilde{\theta}^{nash} < 1 - \frac{\rho}{A} < 1\) and is a solution to:

\[
\hat{\nu} \left[ \rho + \delta - (1 - \tilde{\theta})A \right][A \hat{\tau}_1 + \rho + (1 - \tilde{\theta})A] = (1 - \tilde{\theta})A [(1 - \tilde{\theta})A + \hat{\tau}_2 - \delta] \quad (15)
\]

This tax rate, in turn, supports a unique steady state in which consumption, capital and renewable natural resources grow at the same constant positive rate. Hence, the steady state is a Sustainable Balanced Growth Path (SBGP).

**Proof:** See Appendix B.

Conditions (14a)-(14c) are jointly sufficient for a well-defined and unique long-run equilibrium to exist. The algebra is in Appendix B. Here, we will just discuss condition (14c).\(^{19}\) This is required for existence. It implies that: (i) A high rate. Note that renewable natural resources are indeed capable of growth, especially if one takes into account human intervention and maintenance policy.

\(^{19}\) Also note that condition (14a) requires the productivity of private capital, \(A\), to be higher than the rate of time preference, \(\rho\), plus the regeneration rate of natural resources, \(\delta\). This is a familiar condition for long-term growth (see e.g. Barro and Sala-i-Martin [1995, p. 142]), but here we require a stronger condition than usually because the economy must also devote resources to clean-up policy. Condition (14b)
\[ \hat{\tau}_1 \equiv \tau_1 \left(1 - \frac{1}{I}\right) \geq 0 \] helps existence. This essentially means that the existence of a long-run sustainable equilibrium gets easier when the federal transfer program is designed to reward those countries that pollute less than the average. The intuition is as follows. Since pollution is a by-product of output produced (see (9) above), transfers to countries with output less than the average are essentially transfers to countries that pollute less than the average. The anticipation of such transfers gives an incentive to all countries to pollute less. In equilibrium, all countries do the same, and this is good for existence. (ii) A low \[ \hat{\tau}_2 \equiv \tau_2 \left(1 - \frac{1}{I}\right) \geq 0 \] helps existence. Thus, existence gets easier when the federal transfer program does not attempt to subsidize those countries whose environmental standards are below the average. Now, we have a typical moral hazard problem. The anticipation of such transfers gives all countries a disincentive to care about their environment. In equilibrium, all countries do the same, and this is bad for existence. (iii) Existence gets easier when the rate of regeneration of natural resources, \( \delta \), increases. (iv) Existence gets easier when the productivity of private capital, \( A \), is relatively high. (v) Existence gets easier when agents value environmental quality, i.e. \( \hat{\nu} \) is high. (vi) Existence gets easier when the size of population, \( I \), decreases (recall that \( \hat{\nu} = \frac{\nu}{I} \)). Point (vi) is a familiar result in the literature on public goods: as the size of population increases, any problems associated with decentralized (i.e. Nash) decision-making become worse.

**Properties of Long-run Nash Equilibrium**

By totally differentiating (15), we get:\(^{20}\)

\[
\tilde{\theta} = \theta(\delta, \rho, \hat{\tau}_1, \hat{\tau}_2, A, \hat{\nu})
\]  

(16)

We therefore have the following comparative static results for the long-run pollution tax rate, \( \tilde{\theta} \): (i) The easier natural resources regenerate themselves (i.e. the guarantees that (12h) holds and so the attainable utility is bounded (see e.g. Barro and Sala-i-Martin [1995, chapter 2]).
higher is $\delta$), the smaller the tax rate. (ii) The more we care about the future (i.e. the lower is $\rho$), the higher is the tax rate. The idea is that a low $\rho$ leads to high growth, so that a higher $\tilde{\theta}$ is needed to slow growth down and make utility bounded (see e.g. Barro and Sala-i-Martin [1995, chapter 2]). (iii) The higher the anticipation of federal transfers from high-income to low-income countries (i.e. the higher is $\hat{\tau}$), the lower the pollution tax rate. As we explained above, such transfers give an incentive to all countries to pollute less. In equilibrium, pollution is low and so the optimal tax rate falls. (iv) The effect of $\hat{\tau}_2$ is exactly opposite from that of $\hat{\tau}_1$. Now, moral hazard behavior leads to little natural resources and this requires tougher environmental policy. $^{21}$ (v) When the productivity of private capital is high (i.e. $A$ is high), we can afford higher tax rates and lower economic growth. (vi) The more economic agents value environmental quality relative to private consumption (i.e. the higher is $\hat{\nu}$), the lower should be the tax rate. As is explained in more detail below, the idea behind this seemingly counter-intuitive result is that, when agents value environmental quality, the government needs resources, or tax revenues, to finance its clean-up policy, and this can be achieved only by large tax bases.

In turn, the properties of the Sustainable Balanced Growth Path (SBGP) follow directly from the properties of the long-run pollution tax rate, $\tilde{\theta}$. Recall that along the SBGP, consumption, capital and natural resources can grow at the same positive rate, and that this common rate is decreasing in $\tilde{\theta}$. That is, a lower (resp. higher) pollution tax rate leads to higher (resp. lower) economic growth and improving (resp. deteriorating) environmental quality. Intuitively, lower tax rates lead to higher capital accumulation, higher economic growth and therefore larger tax bases, which lead to a greater ability to engage in clean-up policy. This improves environmental quality, despite the adverse effect of higher economic growth and pollution. $^{22}$ Also, note that better environmental

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$^{20}$ Signs above parameters give equilibrium properties.

$^{21}$ Therefore, competition for federal transfers of this type pushes the economy towards the opposite direction from that under competition for tax bases. Thus, although federal transfers from above-average endowed countries to below-average endowed countries cannot fully explain non-falling tax rates under factor mobility, they can nevertheless provide a route that mitigates the tax competition effect. This can add a possible route to the literature on non-falling tax rates in the presence of tax competition for mobile tax bases (see Persson and Tabellini [1995]).

$^{22}$ Recall that here we use an $AK$ technology, so that the return to capital and the rate of growth are independent of the beginning-of-period capital stock. This is why the growth rate rises, while the level of output is relatively low. Obviously, this is a special model. However, it allows us to get clear analytical
quality is expressed both by a higher rate of natural resources and a higher nature-to-capital ratio. In particular, \( \frac{\tilde{k}}{\tilde{N}} = \frac{\delta + \rho}{A(1 - \tilde{\theta})} - 1 \). That is, the tax rate, \( \tilde{\theta} \), and the nature-to-capital ratio, \( \frac{\tilde{N}}{\tilde{k}} \), move in opposite directions.

Therefore, the very same factors that determine the long-run pollution tax rate in (16) also determine - in a symmetrically opposite way - the SBGP. Focusing on the effects of federal policies on the SBGP and social welfare, the higher the anticipation of federal transfers to countries that pollute less than the average (see the effect of \( \hat{\tau}_1 \)), the lower the pollution tax rate and the higher the SBGP (i.e. the common rate at which consumption, capital and natural resources can grow). The higher rate of consumption, in combination with better environmental quality, increases welfare (recall that utility increases with consumption and environmental quality). By contrast, the higher the anticipation of federal transfers to those countries whose stock of natural resources is below the average (see the effect of \( \hat{\tau}_2 \)), the higher the pollution tax rate and the lower the SBGP. Now, the lower rate of consumption, combined with worse environmental quality, depresses welfare. Therefore, other things equal, \( \hat{\tau}_2 \)-type federal policies decrease social welfare, while \( \hat{\tau}_1 \)-type federal policies increase social welfare.

This completes the properties of the long-run equilibrium.\(^{24}\)

results, which show the importance of high growth and large tax bases for the final provision of public goods. In a more general model, we would also have a short-run effect that works in the opposite direction from our long-run effect. Specifically, in the short-run, capital tax bases are inelastic so that a lower tax rate would push tax revenues down. If the short-run effect dominates, then lower tax rates can lead to less clean-up policy and worse environmental quality.

\(^{23}\) Wellisch [1995] and Silva and Caplan [1997] also present federal systems that provide the right incentives and so improve efficiency when national policies are decentralized. However, they use different mechanisms. In Wellisch’s paper, this happens via household mobility. In Silva and Caplan’s paper, this happens when national policymakers act as Stackelberg leaders vis-à-vis the federal government.

\(^{24}\) What is the effect of the number of member-countries, \( I \), on equilibrium outcomes? Without loss of generality, consider the case without federal transfers, i.e. \( \tau_1 = \tau_2 = \hat{\tau}_1 = \hat{\tau}_2 \equiv 0 \). Then, since \( \hat{\nu} \equiv \frac{\nu}{I} \), equation (16) implies that an increase in \( I \) leads ceteris paribus to an increase in the long-run Nash tax rate, \( \tilde{\theta} \). Such a positive effect from the size of population on the Nash tax rate seems to be opposite from the standard one, which is negative (the standard idea is that, as the number of participants increases, their incentive to free ride on the supply provided by others becomes stronger and hence their willingness to pay taxes decreases). Here, we get a result opposite from the standard one: the long-run Nash tax rate increases with the size of population. This is because, in a dynamic growth economy like the one we have here, free
**Transitional Dynamics**

We will now check whether the above long-run equilibrium is dynamically stable. We study stability properties around steady state. Linearizing (13a), (13b) and (13c) around the unique steady state in Proposition 1 implies that the local dynamics are approximated by the linear system:

\[
\begin{bmatrix}
\dot{z} \\
\dot{\psi} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
J_{zz} & J_{z\psi} & J_{z\phi} \\
J_{\psi z} & J_{\psi\psi} & J_{\psi\phi} \\
J_{\phi z} & J_{\phi\psi} & J_{\phi\phi}
\end{bmatrix}
\begin{bmatrix}
z \\
\psi \\
\phi
\end{bmatrix}
\]

(17)

where the elements of the Jacobian evaluated at steady state are in Appendix C.

The determinant of the Jacobian matrix in (17), denoted by \( \det(J) \), is

\[
\det(J) = \rho \hat{\nu} (1 - \hat{\theta}) \frac{\psi}{\phi^z},
\]

which is positive. Hence, there are two possibilities: Either there are three positive eigenvalues, or one positive and two negative eigenvalues. Since all three variables \((z, \psi, \phi)\) are forward-looking or jump variables, the former possibility (i.e. three positive roots) would imply local determinacy (i.e. a unique trajectory), while the latter possibility (i.e. one positive and two negative roots) would imply local indeterminacy (i.e. multiple trajectories, each of which is consistent with the same initial condition and with convergence to the same steady state).\(^{25}\) By examining the characteristic equation of the Jacobian matrix and applying Descartes’ Theorem (see Appendix D), we show that all three roots are positive. Thus, there is local determinacy.

What does it mean? Without predetermined variables, determinacy means that the jump variables jump immediately and in a unique way to take their long-run values and stay there (until the system is disturbed in some way). This is as in the basic AK model. Thus, given the initial capital stock, the choice of consumption keeps the consumption-to-

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\(^{25}\) See e.g. Benhabib and Perli [1994].
capital ratio constant over time and the growth rate of consumption and capital are also constant over time (see e.g. Barro and Sala-i-Martin [1995]).

The above results are summarized by:

**PROPOSITION 2:** Under the conditions in Proposition 1, the long-run pollution tax rate and the associated SBGP are locally determinate.

### IV. POLICY IMPLICATIONS, CONCLUSIONS AND EXTENSIONS

This paper has investigated how commonly used federal transfer policies affect the dynamics of Nash national environmental policies, economic growth and natural resources in a decentralized federalist economy. Since the main results have been written in the Introduction, here we focus on policy issues and extensions.

First, policies that aim overtly to protect the environment may retard growth and eventually harm the environment. The best way to improve environmental quality, on an enduring basis, is to implement tax-and-transfer policies that stimulate growth and generate tax bases broad enough to finance clean-up policies. That is, large tax bases can improve environmental quality, despite the adverse effect of higher economic activity and pollution emissions. This result is consistent with e.g. John and Pecchenino [1994] and Philippopoulos and Economides [2001], who show that only rich economies can afford the resources to engage in environmental maintenance. Therefore, tax-and-transfer policies that stimulate growth, and hence lead to a higher rate of consumption and better environmental quality, increase social welfare.

Second, federal transfer policies should be well targeted. When federal transfer policies are designed to subsidize those countries whose stock of natural resources is below the average, they encourage moral hazard behavior. This leads to high tax rates on polluting firms’ output, low capital accumulation, small tax bases and low cleanup, and this is eventually bad for both economic growth and the environment. On the other hand, when federal transfer policies are designed to reward those member-countries whose economic activity generates less emission residuals than the average, the results are symmetrically opposite. That is, such transfers lead to low tax rates on polluting firms’ output, and this is eventually good for both economic growth and the environment. Therefore, well-targeted
federal transfer policies can, in the medium run, mitigate the tradeoff between redistribution and efficiency. Also, such policies provide the right incentives and work as a substitute for centralized, cooperative decision-making. Note that these results are consistent with social policies that rely more on growth to reduce poverty and improve equity rather than on active redistributive measures with only very short-run benefits (see the papers in the volume edited by Tanzi et al. [1999]). Our contribution has been to show how badly-targeted federal transfer policies, which lead to a high tax burden, impede growth and therefore lack the revenue to provide a better environment.

We will close with three possible extensions. First, here we have assumed that federal policy follows the command-and-control rule (11a). Although this is not unrealistic (it is believed that federal policies are typically not optimal), it would be interesting to choose federal policy (i.e. the policy parameters $\tau_1$ and $\tau_2$ in (11a)) endogenously, as in e.g. Silva and Caplan [1997] and Caplan et al. [2000]. Second, it is interesting to use a richer menu of policy instruments. Here we have focused on pollution taxes and clean-up policy. 

26 Nowadays, a big policy question is the debate on pollution taxes (i.e. tax-based policies) vs. pollution limits (i.e. quantity-based policies). It is interesting to study this debate within the context of a growth model with federal transfers. Third, there is the general question “Why redistribution?” Theoretically, the link between inequality, redistribution and growth is still an open issue (see e.g. Aghion and Howitt [1998]). In the EU, the official argument is that equality and reduction of social conflicts are necessary for the viability of European integration (see e.g. Dewatripont et al. [1995]). In this paper, we just took redistribution as given, and asked how it affects decentralized national policies, natural resources and growth.

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26 Environmental policy instruments are price-based (e.g. pollution taxes, output taxes and input taxes), quantity-based (e.g. pollution limits and regulation of an input) or other regulations (e.g. regulation of a dimension of technology which is correlated with pollution and forced shutdown). On the spending side, there is pollution abatement (or clean-up policy) which requires tax revenues. For a survey, see e.g. Bohm and Russell [1985].
APPENDICES

APPENDIX A: From equations (12a)-(12g) to equations (13a)-(13d)

(12a)-(12g) in the text constitute a seven-equation dynamic system in \( \theta, \lambda, c, k, \gamma, N, \mu \). Taking logarithms on both sides of (12a) and differentiating with respect to time, we get:

\[
\frac{\dot{\lambda}c + \dot{\lambda}c + \dot{\gamma}k + \gamma k}{\lambda c + \gamma k} = \frac{\mu k + \mu k}{\mu k}
\]

(A.1)

Substituting (12b), (12c), (12d), (12e) and (12g) for the rates of growth of \( \lambda, c, k, \gamma \) and \( \mu \) respectively into (A.1), we obtain:

\[-1 + A \dot{\gamma}, \mu k = -\mu c - \delta \mu k - \frac{\hat{v}k}{N} + \hat{\gamma}, \mu k
\]

(A.2)

Next, if we define \( z = \frac{c}{k} \), (12c) and (12d) give (13a) in the text. If we also define \( \psi = \mu k \) and \( \phi = \mu N \), (12d), (12g) and (12f) give (13b) and (13c) in the text. Finally, using these definitions into (A.2) above, we obtain (13d) in the text.

APPENDIX B: Proof of Proposition 1

Setting \( z = 0 \), equation (13a) in the text implies:

\[\ddot{z} = \rho\]

(B.1)

Setting \( \psi = 0 \) and using (B.1), equation (13b) in the text implies:

\[\ddot{\psi} = \frac{\dot{v}}{[(1-\overline{\theta})A + \hat{\gamma}, \delta]}
\]

(B.2)

Setting \( \phi = 0 \) and using (B.1)-(B.2), equation (13c) in the text implies:

\[\ddot{\psi} = \frac{\dot{v}[\rho + \delta - (1-\overline{\theta})A]}{(1-\overline{\theta})A[(1-\overline{\theta})A + \hat{\gamma}, \delta]}
\]

(B.3)

Then, using (B.1)-(B.3) into (13d) in the text, we get:

\[\dot{v}[\rho + \delta - (1-\overline{\theta})A][A \hat{\gamma}, \rho + (1-\overline{\theta})A] = (1-\overline{\theta})A[(1-\overline{\theta})A + \hat{\gamma}, \delta]]
\]

(B.4)

which is (15) in the text.
(B.4) is a quadratic equation in \( \tilde{\theta} \) only. If we solve (B.4) for \( \tilde{\theta} \), (B.2) and (B.3) respectively will give \( \tilde{\phi} \) and \( \tilde{\psi} \). So the main task is to solve (B.4). We choose to work as follows: In the first step, we specify the region in which a well-defined solution (if any) should lie. In the second step, we establish existence and uniqueness of such a solution.

Consider the first step. A well-defined solution requires: (i) \( (1 - \tilde{\theta})A - \rho > 0 \), i.e. \( \tilde{\theta} < 1 - \frac{\rho}{A} \). This is required for the economy to grow in the long-run. (ii) \( (1 - \tilde{\theta})A + \delta < 2 \rho \), i.e. \( 1 - \frac{2\rho - \delta}{A} < \tilde{\theta} \). This is required for the transversality condition (12h) to hold. (iii) \( \rho + \delta - (1 - \tilde{\theta})A > 0 \), i.e. \( 1 - \frac{\rho + \delta}{A} < \tilde{\theta} \). This follows from inspection of (B.2)-(B.4) above. (iv) \( (1 - \tilde{\theta})A + \hat{\tau}_2 - \delta > 0 \), i.e. \( \tilde{\theta} < 1 - \frac{\delta - \hat{\tau}_2}{A} \). Again this follows from inspection of (B.2)-(B.4) above. (iv) \( A \hat{\tau}_1 + 2(1 - \tilde{\theta})A - \delta > 0 \), i.e. \( \tilde{\theta} < 1 - \frac{\delta - A \hat{\tau}_1}{2A} \). This is required for the left-hand side of (B.4) to be monotonically increasing in \( \tilde{\theta} \) (see below why we need this). Now, if we combine (i)-(iv), and given the parameter restrictions in (14a) and (14b) in Proposition 1, it follows that the “binding” lower boundary for \( \tilde{\theta} \) is \( 0 < 1 - \frac{\rho + \delta}{A} \), while the “binding” upper boundary for \( \tilde{\theta} \) is \( 1 - \frac{\rho}{A} < 1 \).

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27 In particular, if \( \rho > 2\delta \) [which is condition (14b)], it follows \( 1 - \frac{2\rho - \delta}{A} < 1 - \frac{\rho + \delta}{A} < \tilde{\theta} \). That is, when \( 1 - \frac{\rho + \delta}{A} < \tilde{\theta} \), it also always holds \( 1 - \frac{2\rho - \delta}{A} < \tilde{\theta} \) so that the transversality condition is satisfied. In turn, we assume \( A > \rho + \delta \) [which is condition (14a)] so that \( 1 - \frac{\rho + \delta}{A} > 0 \). Therefore, the binding lower boundary for \( \tilde{\theta} \) is \( 1 - \frac{\rho + \delta}{A} \), which is positive.

28 In particular, \( \rho > 2\delta \) implies \( 1 - \frac{\rho}{A} < 1 - \frac{\delta - A \hat{\tau}_1}{2A} \) and also \( 1 - \frac{\rho}{A} < 1 - \frac{\delta - \hat{\tau}_2}{A} \). Therefore, the binding upper boundary for \( \tilde{\theta} \) is \( 1 - \frac{\rho}{A} \).
\[ 0 < 1 - \frac{\rho + \delta}{A} < \widetilde{\theta} < 1 - \frac{\rho}{A} < 1 \quad (B.5) \]

which gives the region in which a well-defined solution (if any) for \( \widetilde{\theta} \) should lie.

Consider now the second step. We study whether such a solution for \( \widetilde{\theta} \) actually exists and it is unique. Recall that \( \widetilde{\theta} \) solves (B.4). Define the left-hand side of (B.4) by \( L(\widetilde{\theta}) \) and the right-hand side by \( R(\widetilde{\theta}) \). Then, \( L_\theta(\widetilde{\theta}) > 0 \) (see condition (iv) above) and \( R_\theta(\widetilde{\theta}) < 0 \). Concerning the lower boundary, i.e. \( 1 - \frac{\rho + \delta}{A} \), we have \( L \left( 1 - \frac{\rho + \delta}{A} \right) = 0 \) which is always smaller than \( R \left( 1 - \frac{\rho + \delta}{A} \right) > 0 \). Concerning the upper boundary, i.e. \( 1 - \frac{\rho}{A} \), we have \( L \left( 1 - \frac{\rho}{A} \right) > R \left( 1 - \frac{\rho}{A} \right) > 0 \), if the parameter values satisfy condition (14c) in the text. Since \( L_\theta(\widetilde{\theta}) > 0 \) and \( R_\theta(\widetilde{\theta}) < 0 \) monotonically, these values of \( L(\widetilde{\theta}) \) and \( R(\widetilde{\theta}) \) at the lower and upper boundaries mean that \( L(\widetilde{\theta}) \) and \( R(\widetilde{\theta}) \) intersect only once, as shown in Figure 1 below.

Therefore, a unique, well-defined solution for \( \widetilde{\theta} \) exists. This in turn supports - via (B.2) and (B.3) - a unique well-defined solution for \( \widetilde{\phi} \) and \( \widetilde{\psi} \).

**APPENDIX C: The Jacobian in equation (17)**

The elements of the Jacobian matrix evaluated at the steady state are:

\[
\begin{align*}
J_{zz} &\equiv \frac{\partial z}{\partial z} = \rho > 0, \quad J_{zy} \equiv \frac{\partial z}{\partial \psi} = 0, \quad J_{z\phi} \equiv \frac{\partial z}{\partial \phi} = 0, \\
J_{yz} &\equiv \frac{\partial \psi}{\partial z} = -\tilde{\psi} < 0, \quad J_{zy} \equiv \frac{\partial \psi}{\partial \psi} = 0, \quad J_{y\phi} \equiv \frac{\partial \psi}{\partial \phi} = \frac{\dot{\psi}\ddot{\psi}}{\dot{\phi}^2} > 0, \\
J_{z\phi} &\equiv \frac{\partial \phi}{\partial z} = 0, \quad J_{\phi} \equiv \frac{\partial \phi}{\partial \psi} = -(1 - \widetilde{\theta}) \widetilde{A} < 0, \quad J_{\phi\phi} \equiv \frac{\partial \phi}{\partial \phi} = \rho + \dot{\tilde{\tau}} > 0.
\end{align*}
\]
APPENDIX D: Transitional Dynamics

The characteristic equation of the Jacobian in equation (17) being evaluated at the steady state is:

\[
\varepsilon^3 - [2\rho + \tilde{\tau}_2] \varepsilon^2 + \left[ \rho(\rho + \tilde{\tau}_2) + \frac{(1-\tilde{\theta})\tilde{\psi}\tilde{\psi}}{\phi^2} \right] \varepsilon - \frac{\rho(1-\tilde{\theta})\tilde{\psi}\tilde{\psi}}{\phi^2} = 0 \tag{D.1}
\]

where \(\varepsilon\) is an eigenvalue. Thus, the coefficient on \(\varepsilon^2\) is negative, the coefficient on \(\varepsilon\) is positive and the constant term is negative. That is, there are three sign alterations in (D.1). We can now use Descartes’ Theorem (see e.g. Azariadis [1993]), which states that the number of positive roots cannot be higher than the number of sign alterations. Thus, we cannot have more than three positive roots. Next define \(\varepsilon' \equiv -\varepsilon\). In this case, there are no sign alterations in (D.1). Thus, we cannot have a negative root. Combining results, it follows that there are three positive roots. Therefore, with three jump variables, there is local determinacy.
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