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Term spread regressions of the rational expectations hypothesis of the term structure allowing for risk premium efects

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Term spread regressions of the rational expectations hypothesis of the term structure allowing for risk premium effects

Efthymios Argyropoulos and Elias Tzavalis*

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Abstract

This paper suggests term spread regression based tests allowing for time-varying term premium effects, with the aim of explaining the empirical failures of the term spread to forecast future movements in interest rates. To capture the effects of a time-varying term premium on the term spread, the paper relies on a simple and empirically attractive arbitrage-free Gaussian dynamic term structure model, which assumes that the term structure of interest rates is spanned by three unobserved factors. To retrieve these factors from the data, the paper suggests a new empirically methodology which can overcome the effects of measurement (or pricing) errors on the estimates of the unobserved factors, and our tests. The results of the paper indicate that term spread regressions allowing for a time-varying term premium effects can provide unbiased estimates of future changes of long-term interest rates, as predicted by the rational expectations hypothesis of the term structure.

JEL classification: G12, E43

Keywords: Rational Expectations Hypothesis, Term Structure of Interest Rates, Time-Varying Term Premium, Dynamic Term Structure Models, Principal Component Analysis.

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1 Introduction

There is recently growing interest in explaining the empirical failure of the term spread between long and short-term interest rates to provide unbiased forecasts of future, one-period ahead changes of long-term rates, which is in contrast to the predictions of the expectations hypothesis of the term structure of interest rates (REHTS). See, e.g., Fama and Bliss (1987), Campbell and Shiller (1991), Hardouvelis (1994), Cuthbertson (1996), and Driffill et al (1998). This puzzling behavior of the term spread can be attributed to the existence of a time-varying risk (or term) premium which is required by the bond market investors as a compensation for holding a long term bond over a short period, e.g., one-month. To explain the effects of a time-varying term premium on term spread regression based tests of the REHTS, earlier studies in the literature consider ad hoc linear factor specifications of the term premium (see, e.g., Simon (1989), and Tzavalis and Wickens (1997)), while more recent studies rely on term premium models implied by affine dynamic term structure models (DTSMs) (see, e.g., Roberds and Whiteman (1999), Dai and Singleton (2002), Duffee (2002), or Hordahl et al (2006)). The last category of studies is mainly interested in investigating if affine DTSMs are able to explain the above failures of the term spread. To show this, they calibrate term spread regressions using estimates of DTSMs based on a few interest rates series.

In this paper, we suggest a new empirical methodology to test the predictions of the term spread about future changes of long-term interest rates allowing for term premium effects. The latter are modelled through a simple and empirically tractable affine Gaussian DTSM (GDTSM), which can be estimated based on a very rich data set of interest rates. Our methodology enables us to test not only the dynamic predictions of the term spread implied by the REHTS, but also the cross-section restrictions on the term structure of interest rates implied by non profitable arbitrage conditions in the bond market. This can be done using the same econometric framework. Testing these restrictions is critical in examining if a time-varying term premium can explain the puzzling behavior of the term spread to forecast future interest rates. If they are not satisfied, then DTSMs can not be thought of as the correct economic framework to capture the effects of term premium on tests of the REHTS.

The GDTSM employed by the paper to carry out the above tests, assumes that interest rates are spanned by three common unobserved factors (see, e.g., Ahn (2004) and Berardi (2009)). This number of factors is chosen based on principal components (PC) analysis, which shows that three factors can explain almost 99.50% of the levels, or first-differences, of interest rates across a very wide spectrum of maturity intervals (see, e.g., Litterman and Scheinkman (1991), and Ang and Piazzesi (2003)). To retrieve estimates of the unobserved factors spanning the term structure of interest rates, we rely on the approach of Pearson and Sun (1994), frequently used in practice (see, e.g. Brown and Schaefer (1993,1994), Duffee (2005)). According to this approach, a number of zero-coupon bond interest rates are used as instruments to obtain the unobserved interest rates factors. This is done by inverting the pricing equations of zero-coupon bonds implied by DTSMs models. However, this approach relies on the assumption that these zero-coupon bond instruments are priced without measurement errors, which may not be true in practice given that long-term zero-coupon bonds are often calculated based on approximation methods. To overcome this measurement errors problem, instead of observed values, we suggest employing projections (forecasts) of the above interest rate instruments based on the common factors spanning the whole term structure of interest rates. These common factors can be retrieved by PC analysis. Since it is based on a very large set of different maturity interest rates, this analysis can provide term structure factors which constitute well diversified portfolios of interest rates. By construction, the estimates of these factors will be orthogonal to interest rate measurement errors. Furthermore, this analysis can combine efficiently all the available information in the bond market.[1]

The paper provides a number of very useful results which have both academic and practical interest. It shows that adjusting term spread regressions for time-varying term premium effects can indeed provide unbiased forecasts of the future changes of long-term interest rates. These regressions can thus explain the forecasting failures of the term spread, mentioned before. The paper shows that these time-varying term premium effects can be sufficiently captured by the simple GDTSM suggested in the paper. The results of the paper show that this model satisfies the cross-section restrictions on interest rates implied by the no-arbitrage conditions of the bond market. Finally, the paper indicates that the time-varying term premium effects that are priced in the bond market are those associated with level and slope shifts of the term structure. The term premium effects which relate to curvature changes of the term structure are not found to be priced.

The paper is organized as follows. Section 2 presents the GDTSM and a term spread regression allowed for time-varying term premium effects. Section 3 estimates this model and carries out tests of its cross-section (no-arbitrage) restrictions. Then, it tests the REHTS based on the term spread regression adjusted for term premium effects. Finally, Section 4 concludes the paper.

2 Model set up

Consider an affine Gaussian Dynamic Term Structure Model (GDTSM) which assumes that the underlying unobserved common factors spanning the term structure of interest rates, denoted as x_{it} for $i = 1, 2, ..., K$, obey the following continuous-time stochastic processes:

$$dx_{it} = k_i(\theta_i - x_{it})dt + \sigma_i dW_{it}, \qquad i = 1, 2, ..., K \tag{1}$$

where θ_i is the long-run mean of x_{it}, k_i is its mean reversion parameter, σ_i is its volatility parameter and W_{it} is a Wiener process.[2] The pricing kernel, used to price all bonds in the economy, is given as

$$\frac{dM_t}{M_t} = -r_t dt - \Lambda_t' dW_t, \tag{2}$$

where r_t is the instantaneous interest rate, W_t is a $(KX1)$-dimension vector of Wiener processes W_{it} and $\Lambda_t = (\Lambda_{1t}, \Lambda_{2t}, ..., \Lambda_{Kt})'$ a $(KX1)$-dimension vector consisting of the market price of risk functions, given as

$$\Lambda_{it} = \sigma_i^{-1}(\lambda_i^{(0)} + \lambda_i^{(1)} x_{it}), \text{ for all } i. \tag{3}$$

[1] This is a very difficult computational task for other methods retrieving common interest rates factors from the data like the maximum likelihood or semiparametric latent variables methods (see, e.g., Chen and Scott (1993, 2003)). Thus, in practice, these methods rely on very small cross-section sets of interest rates.

[2] As our empirical analysis will show later on, this model fits satisfactorily into the data. It can be thought of as an extension of Vasicek's (1977) term structure model, which constitutes a special case of the general class of affine term structure (ATS) models of interest rates suggested by Duffie and Kan (1996). See also Dai and Singleton (2000), Ahn (2004), Kim and Orphanides (2005), and Berardi (2009)).

Ruling out profitable arbitrage conditions in the bond market, the above GDTSM predicts that the price of a zero-coupon bond at time t with maturity interval τ, denoted as $P_t(\tau)$, and its associated interest rate, denoted as $R_t(\tau)$, have the following K - factor representation:

$$P_t(\tau) = e^{-A(\tau)-B(\tau)'X_t} \tag{4}$$

$$\text{and} \qquad R_t(\tau) = (1/\tau)[A(\tau) + B(\tau)'X_t], \tag{5}$$

respectively, where $X_t = (x_{1t}, x_{2t}, ..., x_{Kt})'$ is a $(KX1)$-dimension vector which collects the unobserved factors x_{it}, $A(\tau)$ is a scalar function and $B(\tau)$ is a $(KX1)$-dimension vector of valued functions, i.e. $B(\tau) = (B_1(\tau), B_2(\tau), ..., B_K(\tau))'$. For $\tau = 0$, function (6) gives the instantaneous interest rate r_t as

$$r_t = A(0) + B(0)'X_t, \tag{6}$$

For $\tau = \tau_1$,$\tau_2, ..., \tau_N$ maturity intervals, functions $A(\tau)$ and $B(\tau)$ are given as the solutions to a set of ordinary differential equations (Duffie and Kan (1996)). Recursive relations of these functions can be found in Dai and Singleton (2002) or Kim and Orphanides (2005). Given our model assumptions, these relations become analytic, given as follows:

$$B_i(\tau) = (1 - e^{-\widetilde{k}_i\tau})(\widetilde{k}_i)^{-1}B_i(0), \text{ with } \widetilde{k}_i = k_i + \lambda_i^{(1)},$$

where $\widetilde{k}_i$ constitutes a risk-neutral measure of the mean reversion parameter k_i (see, e.g., Dai and Singleton (2002)).

The above GDTSM is quite flexible and allows for extra variation in the prices of risk functions Λ_{it}. As noted by Duffee (2002), or Duarte (2004), this variation is necessary in order to explain time-variability in the instantaneous expected excess holding period return of a τ-period bond over interest rate r_t, defined as

$$\Psi_t(\tau) \equiv E_t h_{t+1}(\tau) - r_t = -B(\tau)'\Sigma\Lambda_t, \tag{7}$$

where Σ is a (KXK)-dimension diagonal matrix which consists of the volatility parameters σ_i. This excess return is defined as the term premium for holding a τ-period bond over a short period. Substituting (3) into (7), term premium function $\Psi_t(\tau)$ can be analytically written as

$$\begin{aligned} \Psi_t(\tau) &= -\sum_{i=1}^{K} B_i(\tau)\sigma_i \left(\sigma_i^{-1}(\lambda_i^{(0)} + \lambda_i^{(1)}x_{it})\right) \\ &= -\sum_{i=1}^{K} B_i(\tau)\lambda_i^{(0)} - \sum_{i=1}^{K} B_i(\tau)\lambda_i^{(1)}x_{it}. \end{aligned} \tag{8}$$

Considering the instantaneous rate r_t as the one-period (e.g. a month) to maturity interest rate and assuming continuously compounded interest rates, implying $R_t(\tau) = -\frac{1}{\tau}\log P_t(\tau)$, equation (7) implies that the term premium, $\Psi_t(\tau)$, can be written in discrete time as follows:

$$\Psi_t(\tau) \equiv E_t\left[\log\left(\frac{P_{t+1}(\tau-1)}{P_t(\tau)}\right)\right] - r_t = -(\tau-1)E_t\left[R_{t+1}(\tau-1)\right] + \tau R_t(\tau) - r_t.$$

Rearranging terms, the last equation yields

$$(\tau-1)E_t[R_{t+1}(\tau-1)-R_t(\tau)]=[R_t(\tau)-r_t]-\Psi_t(\tau). \tag{9}$$

This equation clearly indicates that term spread $R_t(\tau)-r_t$ based tests of the REHTS, using the following regression model:

$$(\tau-1)[R_{t+1}(\tau-1)-R_t(\tau)]=a(\tau)+\beta(\tau)[R_t(\tau)-r_t]+u_{t+1}(\tau), \tag{10}$$

will lead to downward biased estimates of the slope coefficient $\beta(\tau)$ from unity, which is its predicted value by the theory. This can be attributed to the contemporaneous correlation between the term spread $R_t(\tau)-r_t$ and error term $u_{t+1}(\tau)$. As can be easily seen from relationships (3), (7) and (9), $u_{t+1}(\tau)$, in addition to expectation errors, defined as $v_{t+1}(\tau)=R_{t+1}(\tau)-E_t[R_{t+1}(\tau)]$, contains term premium $\Psi_t(\tau)$, i.e. $u_{t+1}(\tau)=-[\Psi_t(\tau)+v_{t+1}(\tau)]$. The latter has a common factor representation which is analogous to that of $R_t(\tau)-r_t$. The asymptotic bias of the least square (LS) estimator of $\beta(\tau)$, denoted as $\hat{\beta}(\tau)$, is given by the following formula:

$$Asybias\widehat{\beta}(\tau)=1-\frac{Cov[R_t(\tau)-r_t;\Psi_t(\tau)]}{Var[R_t(\tau)-r_t]}.$$

3 Empirical analysis

In this section, we estimate the affine GDTSM described by equations (1) -(8) based on a discrete-time econometric framework. Then, we carry out tests of the cross-section and time dimension predictions of this model to see if it is consistent with the data. The first category of these tests will examine if the no-arbitrage conditions of long-term interest rates $R_t(\tau)$ implied by the above GDTSM are consistent with the data, across different maturity intervals τ. The second category will investigate if the term spread $R_t(\tau)-r_t$ adjusted by time-varying term premium effects can predict future changes in long-term interest rates, as predicted by the REHTS.

The section is organized as follows. First, we present the data and the empirical methodology that we follow to retrieve unobserved factors x_{it} from the data. Second, we present unit root tests for interest rates $R_t(\tau)$, employed in our analysis. These are necessary before specifying the correct econometric framework for estimating and testing GDTSM (1)–(8). Evidence of unit roots in $R_t(\tau)$ immediately implies that there will be at least one common factor x_{it} which will contain a unit root in its autoregressive component. This will necessitate appropriate transformations of all series involved in the structural equations of the above model before conducting any inference about its estimates and testing its theoretical predictions. Third, we carry out principal component (PC) analysis to retrieve to determine determine the maximum number of factors spanning the term structure of interest rates and to study their futures. These should be analogous to those of unobserved factors x_{it}. Fourth, we estimate the GDTSM and test its cross-section restrictions. Finally, we carry out tests of the REHTS based on the term spread regressions allowing for time varying-premium effects.

3.1 *Data Description*

Our empirical analysis is based on US zero-coupon interest rates series calculated by zero coupon or coupon-bearing bonds . These series cover the period from 1997:7 to 2009:10. They are obtained from the data archive of J. Huston McCulloch (Economics Department, Ohio University).[3] They span a very large cross-section set of different maturity intervals τ, from one month to forty years. Figure 1 presents two and three dimension plots of the above all interest rates series, $R_t(\tau)$, over the whole sample. As can be seen from these plots, interest rates exhibit substantial volatility over both the time and cross-section (maturity) dimensions of our data. This implies a high volatility of term premium $\Psi_t(\tau)$, for all τ, which can substantially obscure bond market's expectations about future interest rates movements, and it thus makes the REHTS tests a very hard and challenging task.

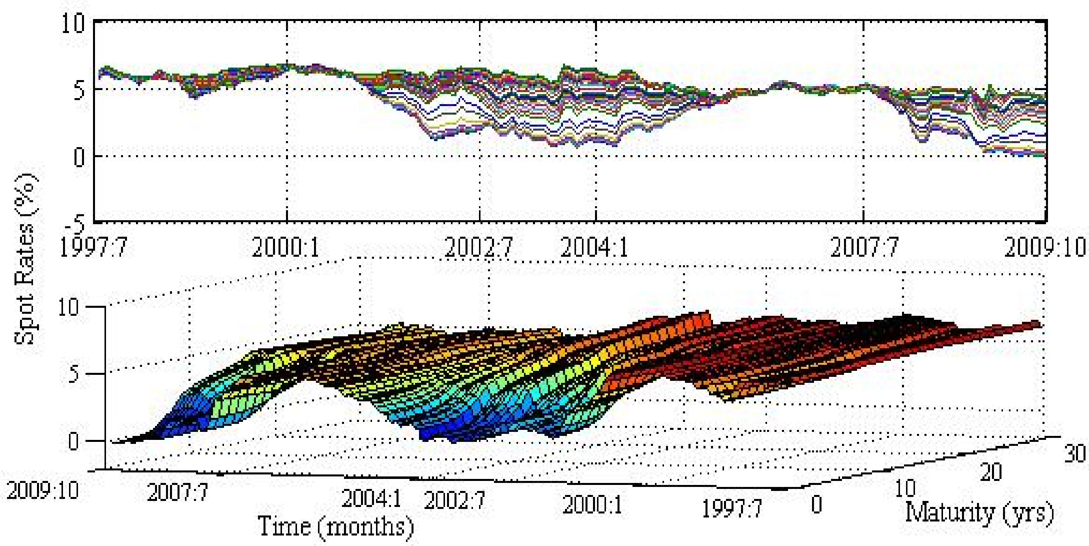

Figure 1. The US nominal term structure.

3.2 *Retrieving unobserved factors from interest rates data*

To estimate the GDTSM, given by equations (1)-(8), and test its cross-section and REHTS predictions, we will rely on estimates of the vector of unobserved K-factors X_t retrieved from the vector of observed interest rate series, denoted as $R_t = (R_t(\tau_1), R_t(\tau_2), ..., R_t(\tau_N))$, following Pearson's and Sun (1994) approach, denoted as P-S. This approach is modified to cope with the problem of measurement errors in interest rates (or bond) pricing equation (5), or (4). To explain this modification of the P-S approach, assume that equation (5) allows for measurement (or pricing) errors and write it in vector matrix format as follows:[4]

[3] http://www.econ.ohio-state.edu/jhm/ts/ts.html

[4] See, e.g., Diebold et al (2006).

$$R_t = A + BX_t + e_t, \tag{11}$$

where $A = ((1/\tau_1)A(\tau_1), (1/\tau_2)A(\tau_2), ..., (1/\tau_N)A(\tau_N))'$ is a $(NX1)$-dimension vector of constants $A(\tau)$, $B = [(1/\tau_1)B(\tau_1), (1/\tau_2)B(\tau_2),...,(1/\tau_N)B(\tau_N)]'$ is a (NXN)-dimension matrix consisting of the loading coefficients of the elements of the vector of unobserved factors X_t on those of vector R_t and $e_t = (e_t(\tau_1), e_t(\tau_2), ..., e_t(\tau_N))'$ is a $(NX1)$-dimension vector of $NIID(0, \Sigma_e)$ measurement errors, with variance-covariance matrix Σ_e. These errors are assumed to be independent of X_t.

P-S' approach assumes that there is $(KX1)$-vector of observed interest rates, or transformations of them (collected in vector Z_t, with elements z_{it}), which are measured with no errors, i.e.,

$$Z_t = A_K + B_K X_t,$$

where A_K and B_K are appropriately defined sub-arrays of vector A and matrix B. By inverting the last relationship, we can retrieve values of unobserved factors X_t based on vector Z_t, i.e., $X_t = B_K^{-1}(Z_t - A_K)$. Then, equation (11) can be written as

$$R_t = A + BB_K^{-1}(Z_t - A_K) + e_t = A - BB_K^{-1}A_K + BB_K^{-1}Z_t + e_t. \tag{12}$$

The last relationship enables us to estimate GDTSM (1)-(8) and test its cross-sectional or REHTS predictions based on GMM or NLLS estimation procedures, which can be easily applied. However, if Z_t is measured with errors as someone expects to happen in practice, then this approach will lead to biased estimates of matrix B or vector A. Thus, it will render tests of the GDTSM and REHTS biased. To see this more clearly, write vector Z_t as

$$Z_t = Z_t^* + e_{K,t}, \quad \text{where} \quad Z_t^* = A_K + B_K X_t$$

is the component of Z_t which is measured with no errors and $e_{K,t}$ is a $(KX1)$-dimension sub-vector of e_t. Then, equation (12) will be written as

$$R_t = A - BB_K^{-1}A_K + BB_K^{-1}Z_t + e_t - BB_K^{-1}e_{K,t}. \tag{13}$$

The last relationship clearly shows that estimating B or A based on a GMM or NLLS procedures will lead to biased estimates of them, since vectors of errors $e_{K,t}$ and e_t are correlated with Z_t.

One way of overcoming the above estimation problem is to rely on linear projections of vector Z_t on a set of instruments in (13), instead of using observed values of Z_t. These projections will be taken to be orthogonal to vector of errors $e_{K,t}$, or e_t. They can be obtained by regressing Z_t on well diversified portfolios of interest rates $R_t(\tau)$, which are net of measurement errors. A natural choice of such portfolios can be taken to be the K orthogonal principal components spanning the whole term structure of interest rates. These can be retrieved through principal component (PC) analysis, in first step.

Let us denote the $(KX1)$-dimension vector of the principal component factors spanning the term structure of interest rates $R_t(\tau)$, at time t as PC_t, with elements pc_{jt}, for j=1,2,...,K Then, it can be safely assumed that vector PC_t is net of measurement (or pricing) errors, i.e.

$$PC_t = WR_t = WA + WBX_t + We_t = WA + WBX_t, \quad \text{with } We_t = 0,$$

where W is a (KXN)-dimension matrix of the weights (loading coefficients) of interest rates $R_t(\tau)$ on principal component factors pc_{jt}. The assumption that We_t=0 means that vector PC_t does not suffer from measurement errors e_t (or $e_{K,t}$). This can be attributed to the fact that these errors are diversified away by forming principal component portfolios for a sufficiently large number of interest rates, N (see, e.g. Joslin et al (2011)).[5] The linear projections of Z_t on PC_t can be written as the following conditional mean:

$$E(Z_t|PC_t) = D_0 + DPC_t, \tag{14}$$

where $E(e_t|PC_t) = 0$ and $E(e_{K,t}|PC_t) = 0$. Rotation of the weights of PC_t's or the estimates of the term-structure loading coefficients, collected in matrix B, will not affect the estimates of conditional mean $E(Z_t|PC_t)$, which can provide estimates (or forecasts) of vector Z_t^* net of measurement error effects. These can be employed to retrieve estimates of the vector of unobserved factors X_t from interest rates data, by inverting the following relationship:

$$X_t = B_K^{-1}(E(Z_t|PC_t) - A_K).$$

3.3 *Unit root tests*

To test for unit root tests in interest rates $R_t(\tau)$, we will carry out a second generation of ADF tests, known as efficient ADF (E-ADF) test (see, e.g., Elliott et al. (1996), Elliott (1999), and Ng and Perron (2001)).[6] These tests are designed to have maximum power against stationary alternatives which are local to unity. Thus, they can improve the power performance of the standard ADF statistic, often used in practice to test for a unit root in $R_t(\tau)$.

Next, we carry out the E-ADF test suggested by Elliott et al. (1996). This requires to define first the quasi-differences of $R_t(\tau)$ as follows:

$$d(R_t(\tau)|\phi) = \begin{cases} R_t(\tau) & if \quad t = 1 \\ R_t(\tau) - \phi R_{t-1}(\tau) & if \quad t > 1, \end{cases}$$

where ϕ denotes the local parameter of the alternative hypothesis against which the unit root hypothesis is tested. Then, we will estimate the following regression of the quasi-differenced interest rates series $d(R_t(\tau)|\phi)$ on the quasi-differences of the vector of deterministic components $D_t = [1]$, or $D_t = [1, t]$, denoted as $d(D_t|\phi)$:

$$d(R_t(\tau)|\phi) = d(D_t|\phi)'\delta(\phi) + \nu_t,$$

based on the least squares (LS) estimation procedure. To estimate the last regression model, we need to fix a value for local parameter ϕ, i.e. $\phi = \overline{\phi}$. Depending on the specification of D_t, ϕ must be fixed to the following values which maximize the local power of the E-ADF test:[7]

$$\overline{\phi} = \begin{cases} 1 - 7/T & if \quad D_t = [1] \\ 1 - 13.5/T & if \quad D_t = [1, t], \end{cases}$$

[5] For large N, this means that We_t is zero.

[6] Evidence provided in the literature on unit root tests of interest rates series is mixed. Earlier studies of this literature based on single time series unit root tests, such as the standard ADF test, can not reject the null hypothesis of a unit root (see, e.g., Hall et al. (1992)). On the other hand, more recent studies based on panel data tests or Bayesian panel data methods tend to reject this hypothesis (see, e.g. Constantini and Lupi (2007) and Meligotsidou et al (2010)).

[7] See Elliott et al. (1996).

where T denotes the total number of time series observations of our sample. The estimates of slope coefficients of the last regression $\delta(\phi)$, denoted as $\widehat{\delta}(\phi)$, will be used to detrend interest rates series $R_t(\tau)$ as follows:

$$R_t(\tau)^d \equiv R_t(\tau) - D_t'\widehat{\delta}_{GLS}(\overline{\phi})$$

Then, the E-ADF unit root test can be carried out based on the following ADF auxiliary regression:

$$\Delta R_t(\tau)^d = (\phi - 1)R_{t-1}(\tau)^d + \sum_{l=1}^{p} \vartheta_l \Delta R_{t-1}(\tau)^d + \varepsilon_t, \tag{15}$$

based on detrended interest rates series $R_t(\tau)^d$. This model allows for serial correlation of $R_t(\tau)^d$ of maximum lag order p. In particular, the unit root hypothesis can be tested by examining if the following hypothesis is true: $1 - \phi = 0$ (or, $\phi = 1$). This can be done based on the t-ratio test statistic. When $D_t = [1]$, the asymptotic distribution of this statistic is the same to that of the standard ADF test. However, this is not true for the case that $D_t = [1, t]$. For this case, critical values of the distribution of this statistic are provided by Elliott et al. (1996).

In addition to the above efficient ADF unit root test, Elliott, Rothenberg and Stock (1996) have also proposed another efficient unit root test statistic known as point optimal test. This test statistic is defined as follows:

$$P_T = \frac{(SSR(\overline{\phi}) - \overline{\phi} SSR(1))}{\widehat{\omega}^2},$$

where $SSR(\phi) = \sum \widehat{\nu}_t^2(\phi)$ is the sum of squared residuals $\widehat{\nu}_t(\phi) = d(R_t(\tau)|\phi) - d(D_t|\phi)'\widehat{\delta}(\phi)$ and $\widehat{\omega}^2$ is an estimator of the residual spectrum at frequency zero. Note that test statistic P_T has the same asymptotic distribution as the E-ADF statistic, described before.

Table 1 reports the values of E-ADF and P_T unit root test statistics for a set of different maturity interest rates $R_t(\tau)$ used in our empirical analysis, i.e., $\tau = \{5, 10, 15, 20, 25, 30\}$ years. To capture a possible linear deterministic trend t in the levels of $R_t(\tau)$, occurring during our sample (see Figure 1), we assume that D_t contains a deterministic trend, i.e. $D_t = [1, t]$. The results of the table clearly indicate that, despite the fact that the values of the autoregressive coefficients ϕ are very close to unity, the unit root hypothesis is rejected against its stationary alternative, for all maturity intervals τ considered. This is true at 5%, or 1% significance levels. The estimates of the autoregressive coefficient ϕ reported in the table imply that interest rates $R_t(\tau)$ exhibit a very slow mean reversion towards their long run mean, especially these of the shorter maturity intervals of 5 or 10 years. As noted before, evidence of stationarity of interest rates $R_t(\tau)$ implies that the common factors spanning interest rates $R_t(\tau)$ must be stationary, too. This will be also confirmed in the next section, based on our PC analysis.

Table 1: Efficient Unit Root Tests for Interest Rates

$R_t(\tau)$, $\tau =$	5	10	15	20	25	30
$\phi - 1$	-0.07	-0.14	-0.21	-0.27	-0.22	-0.16
	(0.03)	(0.04)	(0.05)	(0.06)	(0.06)	(0.05)
ϕ	0.93	0.86	0.79	0.73	0.78	0.84
E-ADF	-2.35	-3.07*	-3.85**	-4.28**	-3.75**	-3.10*
P_T	5.11*	4.02*	2.37**	1.77**	2.57**	4.081*

Notes: Standard errors are in parentheses. D_t is defined as D_t=[1,t]. The lag order p of the auxiliary regressions used to carry out the unit root tests are chosen based on the AIC criterion. This lag found to be $p = 3$ for most maturity intervals τ considered (*) and (**) mean significance at 5% and 1% levels, respectively.

3.4 *Principal component (PC) analysis*

According to PC analysis, the common principal component factors spanning the term structure of interest rates (or their first differences $\Delta R_t(\tau)$), denoted as pc_{jt}, for $j = 1, 2, .., K$, can be obtained by the spectral decomposition of the variance-covariance matrix of vector of interest rates R_t. This can be done efficiently, based on a sufficiently large set of different maturity intervals N, where $N > K$.

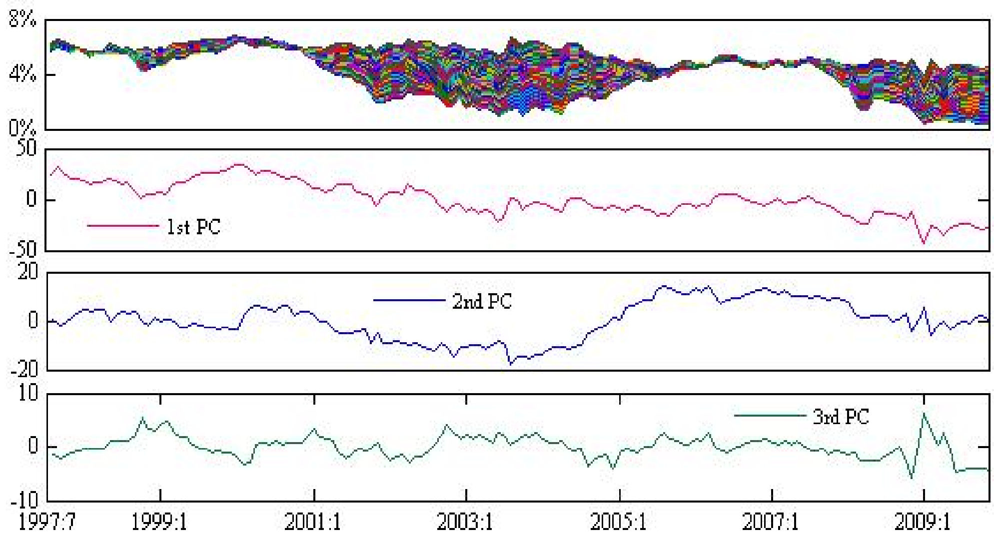

Figure 2. The first three principal components.

This matrix is defined as:

$$\Sigma_R = \Omega\Theta\Omega',$$

where Θ is a diagonal matrix of dimension $(N \times N)$ whose elements are the eigen values of matrix Σ_R and Ω is a $(N \times N)$-dimension orthogonal matrix whose columns are the eigen vectors corresponding to the eigen values of Σ_R. Given estimates of Ω and Θ, the $(K \times 1)$-dimension vector of principal component series $PC_t = (pc_{1t}, pc_{2t,}..., pc_{Kt})'$ can be retrieved from the $(N \times 1)$-dimension vector of interest rates series $R_t(\tau)$, denoted as R_t, as follows:

$$PC_t = \Omega'(R_t - \bar{R}),$$

where $\bar{R}$ is the sample mean of vector of interest rates series R_t.

Our PC analysis is based on a very large set of different maturity interest rates $R_t(\tau)$, i.e., $N = 468$. This covers maturity intervals τ from one one to forty years. This set also contains the one-month interest rate r_t, which is considered in our analysis as the short-term interest rate. This large cross-sectional set of different maturity interest rates guarantees that the retrieved pc_{jt} factors from the data will diversify away any measurement (or pricing) errors in interest rates $R_t(\tau)$, which is independent of unobserved factors x_{it}.[8] Figure 2 presents plots of the first three pc_{jt}, for $j = 1, 2, 3$, retrieved from our data. These correspond to the first three largest in magnitude eigen values of matrix Σ_R, which are found to explain 99.50% (or 97.81%) of the total variation of the levels (or first differences) of all interest rates $R_t(\tau)$, employed in our analysis. See the following table:[9]

Total Number of PCs	1	2	3
% variation explained in $\Delta R_t(\tau)$	82.94	94.56	97.81
% variation explained in $R_t(\tau)$	80.00	98.27	99.50

The above results of our PC analysis are consistent with those reported by other studies in the literature (see, e.g., Litterman and Scheinkman (1991), or Bliss (1997)). They show that three pc_{jt} factors explain almost all of the variation of the term structure of interest rates. The first factor, pc_{1t}, explains the largest part of this variation (e.g., 80% for the levels of $R_t(\tau)$). Together with the second factor, denoted as pc_{2t}, they explain the 98.27% of this variation. The remaining percentage, which is very small, is explained by the third factor, pc_{3t}.

To interpret pc_{jt} factors and study their stochastic properties, in Figure 3 we graphically present estimates of their loading coefficients on the first difference of interest rates series $\Delta R_t(\tau)$, while in Tables 2A and 2B we report some useful descriptive statistics of them, as well as the E-ADF and P_T unit root test statistics. Table 2A also reports estimates of the correlation coefficients of pc_{jt} with observed interest rate variables (or transformations of them), often used in the literature as proxies of unobserved interest rates factors x_{it} (see, e.g., Ang and Piazzesi (2003)). This set of variables include: the level interest rate with maturity $\tau = 40$ years, defined as $z_{1t} \equiv R_t(40)$, the term spread between the short and the long-term interest rates, defined as $z_{2t} \equiv r_t - R_t(40)$, and, finally, variable $z_{3t} \equiv r_t - 2R_t(6) + R_t(40)$. Variables z_{2t} and z_{3t} constitute linear transformations of short-term interest rate r_t and longer term interest rates. As can be seen from the results of Table 2A, there is almost one-to-one correspondence between variables z_{jt} and series pc_{jt}. Since variables z_{jt} are observed, in our analysis they will be taken to play the role of interest rate instruments. These will be projected on principal component factors pc_{jt} to obtain estimates of z_{jt} net of measurement errors.

[8] As shown in Bai and Ng (2002), and Bai (2003) consistent estimates of common factors spanning economic series can be obtained by PC analysis when the following condition holds: $\frac{\sqrt{T}}{N} \to 0$, where T is the total number of time series observations of each series and N is the cross-section dimension (here, maturity intervals τ).

[9] Note that these factors correspond to the first three largest in magnitude eigenvalues of variance-covariance matrix Σ. The relative variation of the three factors is calculated as

$$\frac{\sum_{j=1}^{i} \psi_i}{tr(\Psi)},$$

for $i = 1, 2, 3$, where ψ_i is the eigen value of matrix Σ and $tr(.)$ stands for the trace of a matrix.

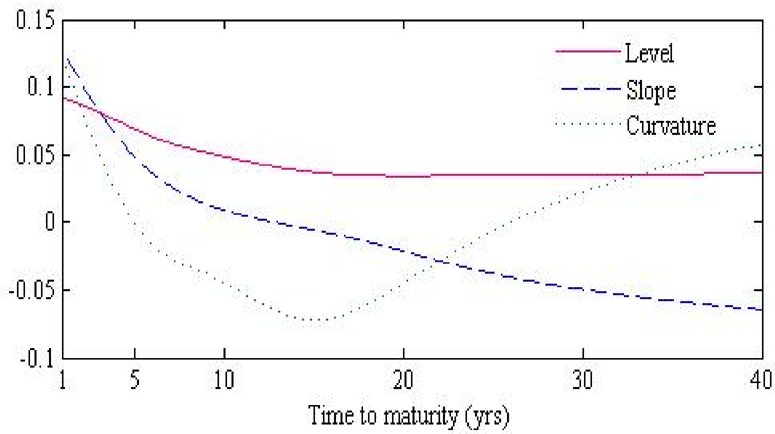

Figure 3. The loadings of the first three principal components.

Turning into a discussion on the interpretation of the three principle component factors pc_{jt}, the results of Tables 2A-2B and Figure 3 clearly indicate that these factors share similar features to those found in other empirical studies (see, e.g., Litterman and Scheinkman (1991)). In particular, pc_{1t} plays the role of a "level" factor, which causes almost parallel shifts to the whole maturity spectrum of interest rates. pc_{2t}, referred to as "slope" factor, determines the slope of the term structure, while pc_{3t} constitutes a "curvature" factor since its loading coefficients on interest rates have a U-shape. Note that, in contrast to pc_{1t} and pc_{3t}, the effect of pc_{2t} on interest rates $R_t(\tau)$ is always declining and increases in terms of magnitude with maturity interval τ. Since pc_{2t} is positively and highly correlated with term spread $r_t - R_t(40)$, the last result means that a positive value of term spread $R_t(40) - r_t$ (i.e. a negative of $r_t - R_t(40)$) will have a positive effect on the slope of the term structure. For interest rates of maturity interval $\tau > 15$ years, this effect will be offset by the "curvature" effects, captured by pc_{3t}. Finally note that, due to their high and positive correlation with principal component factors pc_{jt}, analogous interpretations to the above can be also given to observed variables z_{jt}, which are used as instruments to retrieve unobserved factors x_{it} in our analysis.

Table 2A: Summary Statistics of Interest Rates PCs

Factors	pc_{1t}	pc_{2t}	pc_{3t}
Mean	0.027	-0.029	0.000
Max. Value	33.71	14.39	6.01
Min. Value	-43.30	-17.91	-5.74
Variance	268.79	61.27	4.09
ρ_1	0.93	0.95	0.74
ρ_2	0.87	0.92	0.48
ρ_3	0.83	0.91	0.40
ρ_4	0.78	0.87	0.28
ρ_5	0.74	0.84	0.10
	Correlation Coefficients		
	z_{1t}	z_{2t}	z_{3t}
	0.98	0.84	0.77

Notes: Max stands for maximum, while Min. for minimum. ρ_s are the autocorrelations of PCs pc_{jt}, $j = 1, 2, 3$, of lag order $s = 1, 2, .., 5$.

Table 2B: Unit Root Tests for Interest Rates PCs

PCs:	pc_{1t}	pc_{2t}	pc_{3t}
$(\phi_i - 1)$	-0.15	-0.05	-0.20
	(0.04)	(0.02)	(0.05)
ϕ_i	0.85	0.95	0.80
p	0	7	3
E-ADF	-3.43*	-2.04*	-3.46*
P_T	4.41*	1.96**	3.44**

Notes: Standard errors are in parentheses, D_t is defined as D_t=[1,t]. The tests reported in the table are based on E-ADF autoregression: $\Delta x_{it}^d = (\phi_i - 1)x_{it-1}^d + \sum_{l=1}^{p} \vartheta_{il} \Delta x_{it}(\tau)^d + v_{it}$. The lag order p of the dynamic (first difference) terms of the E-ADF regressions chosen are based on the SIC criterion (except for x_{2t}). (*) and (**) mean significance at 5% and 1% levels.

Finally, the results of unit root tests reported in Table 2B indicate that all the three principal component factors pc_{jt} constitute stationary series. These results are consistent with those on interest rates series $R_t(\tau)$, reported in Table 1. From the econometric methodology point of view, they imply that the key equations of our term structure model (1), (5) and (8) can be estimated as a system of structural equations based on standard asymptotic estimation and inference procedures, as this system consists of stationary series. The estimates of the autoregressive coefficients ϕ reported in the table indicate that, among three factors pc_{jt}, the second (i.e. pc_{2t}), which is highly correlated with spread $r_t - R_t(40)$, is the most persistent one. This result can be also confirmed by the values of autocorrelation coefficients ρ_s, reported in Table 2A.

3.5 *Estimation of the GDTSM and Cross-Section restriction tests*

Having established good grounds to support that the three orthogonal and stationary factors explain almost all variation of the term structure of interest rates $R_t(\tau)$, in this section we estimate the structural equations of the GDTSM (1), (5) and (8). To retrieve estimates of the unobserved factors x_{it} from the data, we rely on our empirical methodology, presented in subsection 3.2. As vector of observed instruments Z_t, we will use the $(3X1)$-dimension vector $Z_t = (z_{1t}, z_{2t}, z_{3t})'$, where $z_{1t} \equiv R_t(40)$, $z_{2t} \equiv r_t - R_t(40)$ and $z_{3t} \equiv r_t - 2R_t(6) + R_t(40)$. As shown in the previous section, this vector consists of interest rate series or transformations of them which are highly correlated with the three principal component factors pc_{jt}, spanning the term structure of interest rates. Thus, the expectation of this vector conditional on that of three principal component series $PC_t = (pc_{1t}, pc_{2t}, pc_{3t})'$, i.e. $E(Z_t|PC_t) = D_0 + DPC_t$, which is employed to retrieve X_t from the data through relationship $X_t = B_K^{-1}(E(Z_{t+1}|PC_{t+1}) - A_K)$, will be accurately estimated and net of measurement errors.

More specifically, the system of equations that is employed to estimate the GDTSM is given as follows:

$$R_{t+1} = A + BX_{t+1} + e_{t+1}, \text{ with } X_{t+1} = B_3^{-1}(E(Z_{t+1}|PC_{t+1}) - A_3), \tag{16}$$

$$\Delta X_{t+1} = \Phi_0 + (\Phi - I)X_t + \omega_{t+1}, \tag{17}$$

$$Z_{t+1} = D_0 + DPC_{t+1} + \varepsilon_{t+1}, \text{ and} \tag{18}$$

$$EHR_{t+1} = \Gamma_0 + \Gamma X_t + \eta_{t+1}, \tag{19}$$

where EHR_{t+1} is a $(NX1)$-dimension vector of excess holding period returns $h_{t+1}(\tau) - r_t$, for $\tau = \tau_1, \tau_2, ..., \tau_N$, observed at time $t+1$. Equation (17) gives the vector of discretized continuous-time processes (1), for $i = 1, 2, 3$, where Φ is a $(3X3)$ diagonal matrix whose elements are given as $\Phi_{ii} = e^{-k_i \Delta}$. e_{t+1}, ω_{t+1}, ε_{t+1} and η_{+1} constitute vectors of zero-mean error terms.

The above system of equations can be employed to retrieve from the data estimates of the mean reversion and price of risk premium parameters k_i and $\lambda_i^{(1)}$, respectively. These are the key parameters of the GDTSM, which are important in forecasting future interest rate changes $\Delta R_{t+1}(\tau)$ and in capturing time-varying risk premium effects $\Psi_t(\tau)$, respectively. To retrieve their estimates, we will impose the following cross-section restrictions on the coefficients of the above system implied by the no-arbitrage conditions of the GDTSM (see Section 2):

$$B_i(\tau) = (1 - e^{-\widetilde{k}_i \tau})(\widetilde{k}_i \tau)^{-1} B_i(0), \text{ with } \widetilde{k}_i = k_i + \lambda_i^{(1)}, \text{ and } \Gamma_i(\tau) = -b_i(\tau)\lambda_i^{(1)}, \text{ for } \tau = \tau_1, \tau_2, ..., \tau_N, \tag{20}$$

where $B_i(\tau)$ and $\Gamma_i(\tau)$, $i = 1, 2, 3$, constitute elements of the row vectors of matrices B and Γ, for rows $\tau = \tau_1, \tau_2, ..., \tau_N$, and $B_i(0)$ are the load coefficients of factors x_{it} on short-term interest rate r_t, for $i = 1, 2, 3$. Note that the latter are the elements of the first row of matrix B.[10]

Compared to other econometric models used in the literature to estimate DTSMs (see, e.g., Dai and Singleton (2000,2002)), the above structural system of equations, in addition to equations (16) and (17)), also includes excess holding period returns equations (19). These equations provides useful information about the factors determining time variation of price of risk functions Λ_{it}. As can be seen from equation (8), this set of equations can help to identify the coefficients of price of risk functions Λ_{it} $\lambda_i^{(1)}$, under the physical probability measure. They can be also used to estimate term premium $\Psi_t(\tau)$, at time t, and, thus, to control its effects on term spread regressions (10).

The system of equations (16)-(19) is estimated subject to cross-section restrictions (20). This is done for the set of maturity intervals $\tau = \{0, 5, 10, 15, 20, 30, 25, 30\}$ years, where $\tau = 0$ stands for the short-term interest rate r_t maturity interval.[11] The estimation of this system is carried out based on the Generalized Method of Moments (GMM), using as set of instruments current and past time values of the vector of observed variables Z_t (see bottom of Table 3). GMM can provide asymptotically efficient estimates of the vector of structural parameters of the system which are robust to possible heteroscedasticity and/or missing serial correlation of error terms vectors e_{t+1}, ω_{t+1}, ε_{t+1} and η_{t+1}. Furthermore, since the number of orthogonality conditions used in the GMM estimation procedure (i.e., the number of instruments multiplied by the number of the individual equations) is bigger than the number of the parameters estimated, the set of overidentified conditions implied in the estimation procedure can be employed to test if our GDTSM constitutes a correct specification of the data. To this end, we will employ Sargan's overidentified restrictions test, denoted as J. This is distributed as χ^2, with degrees of freedom, which equal the number of overidentified restrictions (i.e. 7X18-33=93, in our case).

Table 3 presents the GMM estimation results of the key parameters of system (16)-(19) k_i and $\lambda_i^{(1)}$, as well as the elements of projection matrix D, denoted D_{ij}. A number of interesting conclusions emerge from the results of the table. First, the GDTSM is found to be consistent with the data, and satisfies the no-arbitrage restrictions (20). This can be justified by the probability values (p-values) of J test statistic, reported in the table. The values of the correlation coefficients of future interest rates $\Delta R_{t+1}(\tau)$ and excess returns $h_{t+1}(\tau) - r_t$ with their predicted values, denoted as $corr(\Delta R, \Delta \hat{R})$ and $corr(HR, \hat{H}R)$, indicate that the GDTSM has significant forecasting power on variables $\Delta R_{t+1}(\tau)$ and $h_{t+1}(\tau) - r_t$, for all τ. These variables are very difficult to forecast (see, e.g. Duffee (2002), Almeida and Vicente (2008), or Carriero et al (2012), more recently).

[10] Following empirical literature (see, e.g., Dai and Singleton (2002)), we do not impose any cross-section restriction on the intercepts of structural equations (16) and (19). These intercepts can also capture a fixed component of bond pricing errors or imperfections of the bond market.

[11] Note that, for this set of maturity intervals τ, the biggest failures of the expectations hypothesis of the term structure are reported in the literature (see, e.g., Dai and Singleton (2002)).

Table 3: GMM Estimates of system (16)-(19)

A. Parameter estimates			
	x_{1t}	x_{2t}	x_{3t}
k_i	0.32	0.22	0.37
	(0.02)	(0.07)	(0.05)
$\lambda_i^{(1)}$	-0.31	-0.12	0.02
	(0.02)	(0.06)	(0.05)
	Matrix $D \equiv [D_{ij}]$		
D_{1j}	0.03	-0.09	0.17
	(0.0009)	(0.002)	(0.006)
D_{2j}	-0.02	0.13	-0.68
	(0.002)	(0.005)	(0.01)
D_{3j}	0.11	0.09	0.80
	(0.001)	(0.004)	(0.01)
$J(93)$ =115.2 (p-value=0.06),			
Instruments: 1, z_{1t}, $z_{2t-1}, z_{2t-2}, z_{2t-3}, z_{2t-4}$ and z_{3t}			

B: Correlation coefficients						
τ	5	10	15	20	25	30
$corr(\Delta R, \Delta \hat{R})$	0.05	0.07	0.08	0.08	0.09	0.10
$corr(HR, \hat{H}R)$	0.13	0.13	0.10	0.10	0.10	0.11

Notes: Data are monthly from 1997:07 to 2009:10. The Newey - West heteroscedasticity and autocorrelation consistent standard errors shown in parentheses. J is Sargan's overidentified restrictions test statistic. This is chi-squared distributed with degrees of freedom which are equal to the number of orthogonality conditions employed in the estimation procedure ($7X18$) minus the number of the parameters estimated (33). The instruments used are lags and current values of the following variables: $z_{1t} = R_t(40)$, $z_{2t}= r_t - R_t(40)$ and $z_{3t}= r_t + R_t(40) - 2R_t(6)$. $corr(.)$ are the correlation coefficients of the fitted values of the first difference of interest rates $\Delta R_{t+1}(\tau)$ and excess holding period returns $h_{t+1}(\tau) - r_t$, respectively denoted as $\Delta\hat{R}$ and $\hat{H}R$, with their observed values, denoted as ΔR and h, respectively.

Second, the estimates of mean reversion parameters k_i are found to be significant, for all unobserved factors x_{it}. They imply very high persistency of all factor innovations, collected in vector ω_t, on the level of interest rates $R_t(\tau)$, for all τ. The implied by them estimates of autoregressive coefficients Φ_{ii} (i.e. the diagonal elements of matrix Φ) are as follows: $\Phi_{11} = 0.97$, $\Phi_{22} = 0.98$ and $\Phi_{33} = 0.96$. These estimates are more close to unity than those of the autoregressive coefficients of principal component factors pc_{jt}, reported in Table 2B. This result means that there is no one-to-one correspondence between factors x_{it} and pc_{jt}. This can be also confirmed by the inspection of plots of estimates of x_{it}, implied by the estimates of system (16)-(19). These are graphically presented in Figures 4A-C against those of factors pc_{it}. Their graphs indicate that the estimates of all three unobserved factors x_{it} are more volatile than pc_{jt}. In fact, the estimates of x_{it} seem to capture substantial and persistent shifts in the term structure of interest rates.[12] These seem to occur in years 1998, 2003, 2005 and 2008, and they are associated with world financial crises or US monetary policy regime changes**.** The estimates of pc_{jt} smooth out these shifts. These results imply that approximating unobserved factors x_{it} with principal component factors pc_{jt} may not provide accurate representations of the former, and thus may lead to inaccurate estimates of the GDTSM' parameters. An analogous conclusion can be also drawn for observed series z_{it}, used also in practice in place of x_{it}. The estimates of the elements of matrix D, D_{ij}, reported in the table, clearly show that there is no one-to-one correspondence between z_{it} and pc_{jt}, or x_{it}.

Regarding the price of risk function parameters $\lambda_i^{(1)}$, the results of Table 3 indicate that the estimates of them associated with the first two factors x_{1t} and x_{2t} are significant at 5%, or 1%, level. The estimate of $\lambda_3^{(1)}$, associated with factor x_{3t}, is not significant. These results imply that only time-varying effects of factors x_{1t} and x_{2t} are priced in the bond market. According to relationship (8), these two factors are responsible for the time-variation of term premium $\Psi_t(\tau)$. Thus, they can explain the bias of a time-varying term premium effects on the REHTS based on term spread $R_t(\tau) - r_t$. This result can be also confirmed by Figure 5, which graphically presents estimates of $\Psi_t(\tau)$ against the observed values of excess returns $h_{t+1}(\tau) - r_t$, for $\tau = 5$ years.[13] Inspection of this figure indicates that persistent shifts in term premium $\Psi_t(\tau)$, like those of financial crisis of years 1998 or 2008 seem to be mainly associated with shifts in term structure factors x_{1t} and x_{2t} in these years. It must be noted at this point that the insignificance of price of risk coefficient $\lambda_3^{(1)}$ does not mean that changes in factor x_{3t} do not determine the curvature of the term structure interest rates $R_t(\tau)$. As can be seen by equation (20), the curvature of $R_t(\tau)$ depends also the values of mean reversion parameters k_i, which is found to be different than zero for factor x_{3t}. Finally, note that both the sign and magnitude of the price of risk premium function coefficients $\lambda_1^{(1)}$ and $\lambda_2^{(1)}$ are consistent with that found by other studies (see, e.g., Duffie (2005)).

[12] See, e.g., Bansal and Zhou (2002), Dai et (2007), Cochrane and Piazzesi (2005).

[13] Analogous graphs of $\Psi_t(\tau)$ are taken for other maturity intervals τ.

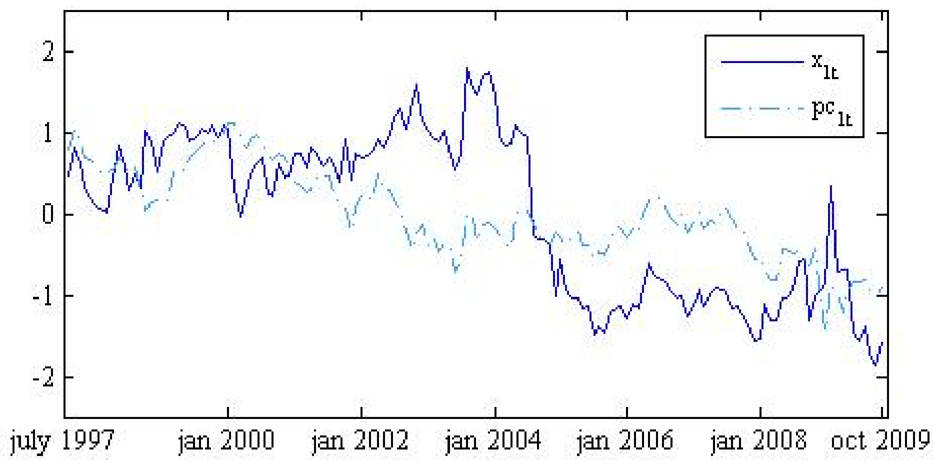

Figure 4A. Estimates of x_{1t} versus estimates of pc_{1t}.

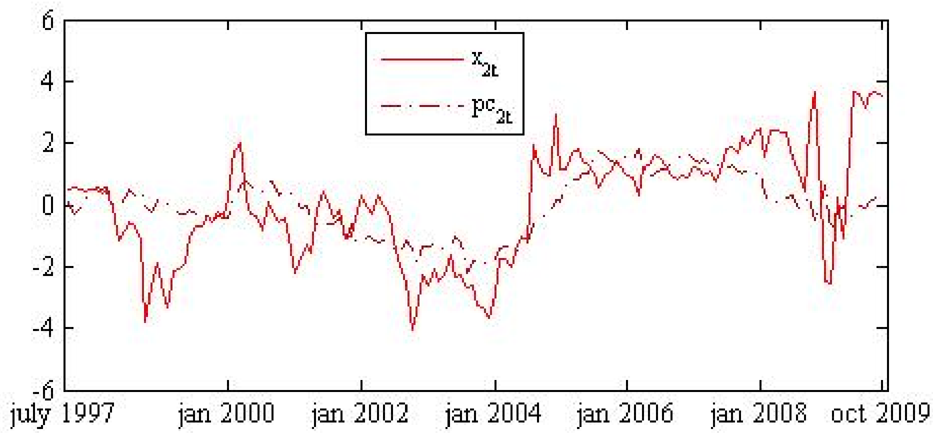

Figure 4B. Estimates of x_{2t} versus estimates of pc_{2t}.

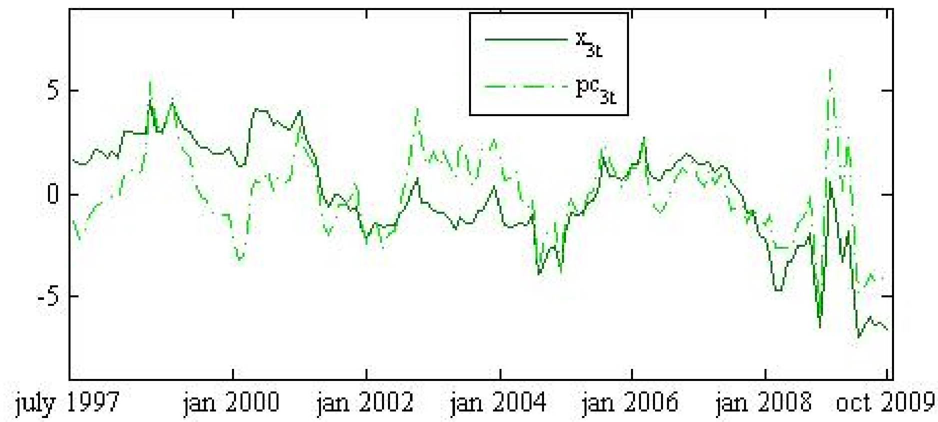

Figure 4C. Estimates of x_{3t} versus estimates of pc_{3t}.

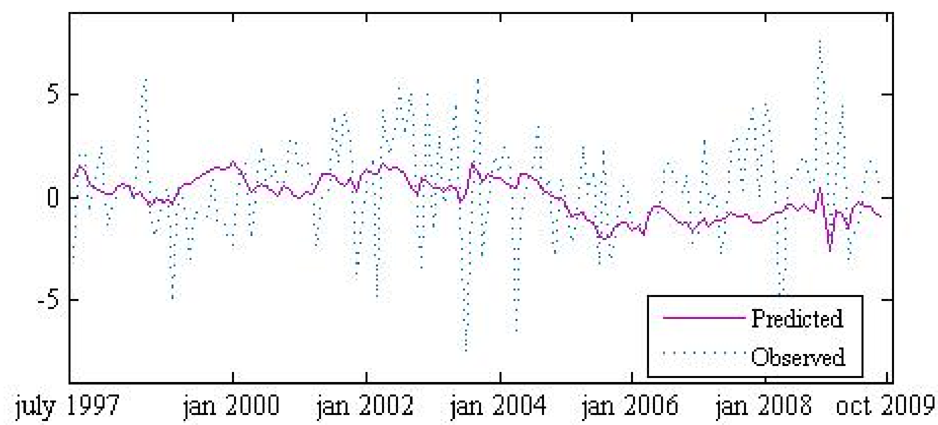

Figure 5. Predicted estimates versus observed excess holding returns $h_{t+1}(\tau) - r_t$, for $\tau = 5$ years.

3.6 *Tests of the REHTS allowing for term premium effects*

Having established good grounds to support that the GDTSM, given by equations (1)-(8), constitutes a correct specification of the data, in this section we formally examine if the term premium $\Psi_t(\tau)$ implied by this model can explain the failure of term spread $R_t(\tau) - r_t$ to forecast future changes in long-term interest rates $R_{t+1}(\tau-1) - R_t(\tau)$, which is against the predictions of the REHTS. To this end, we will estimate the following term spread regression model:

$$(\tau-1)[R_{t+1}(\tau-1) - R_t(\tau)] = a(\tau) + \beta(\tau)[R_t(\tau) - r_t] - \Psi_t(\tau) + v_{t+1}(\tau) \quad \text{or}$$

$$(\tau-1)[R_{t+1}(\tau-1) - R_t(\tau)] = a(\tau) + \beta(\tau)[R_t(\tau) - r_t] + \sum_{i=1}^{K} b_i(\tau)\lambda_i^{(1)} x_{it} + v_{t+1}(\tau) \qquad (21)$$

where $a^*(\tau) \equiv a(\tau) - \sum_{i=1}^{K} b_i(\tau)\lambda_i^{(0)}$. This model is adjusted by the term premium effects $\Psi_t(\tau)$, given by equation (8). By including the time-varying component of term premium $\Psi_t(\tau)$, i.e. $-\sum_{i=1}^{K} b_i(\tau)\lambda_i^{(1)} p_{it}$, in the right hand side (RHS) of regression model (21), we can capture bias effects of $\Psi_t(\tau)$ on the slope of the term spread $R_t(\tau) - r_t$, $\beta(\tau)$, about future movements in $R_{t+1}(\tau-1) - R_t(\tau)$.[14]

Regression model (21) is estimated jointly with the systems of equations (16)-(18), for all τ, based on the GMM procedure used before to obtain estimates of k_i and $\lambda_i^{(1)}$. This model has replaced that of excess return equations (19) in our previous system of equations, (16)-(19). Regression model (21) enables us to estimate freely from the data the term spread slope parameter $\beta(\tau)$ and, then, to examine if this coefficient is equal to unity, which is its theoretical value predicted by the REHTS. The above estimation is carried out under the set of no-arbitrage restrictions given by (20). This will help us to identify and estimate precisely from the data the structural parameters of the GDTSM k_i and $\lambda_i^{(1)}$, which determine time-variation of term premium $\Psi_t(\tau)$. Estimating these parameters based only on regression model (21) will run into multicollinearity problems due to the very high correlation between term spread $R_t(\tau) - r_t$ and the estimates of unobserved factors x_{it}, used as independent regressors in (21).

GMM estimates of model (21) together with the system of equations (16)-(18) are reported in Table 5, $\tau = \{5, 10, 15, 20, 25, 30\}$.[15] In Table 4, we report estimates of slope coefficient $\beta(\tau)$ which ignore term premium effects. These estimates are based on the least squares (LS) method.[16] They confirm the severity of the biases of the estimates of $\beta(\tau)$. They should be compared to those of Table 5, which allow for time-varying term premium effects.

[14] Analogous regressions have been suggested in the literature by Simon (1989), Tzavalis and Wickens (1997) and Tzavalis (2003), or Psaradakis et al (2005) for forward exchange rate. These regression models however use ad hoc specifications of the term premium effects. See Baillie (1989), for a survey.

[15] Note that in the estimation of regression model (10) the regressand, namely $R_{t+1}(\tau-1) - R_t(\tau)$, is not approximated by $R_{t+1}(\tau) - R_t(\tau)$, as often made in many studies due to the lack of monthly maturity interval (see, e.g., Campbell and Shiller (1991)). As was noted by Bekaert et al. (1997) and was confirmed by our empirical analysis, this approximation can cause further biases to the LS estimates of β than those implied by the time-varying term premium effects $\Phi_t(\tau)$.

[16] Note that GMM estimation of slope coefficients $\beta(\tau)$ based on the unadjusted for term premium effects term spread regression using as instruments values of variables z_{1t}, z_{2t} and z_{3t} can not save the REHTS since these variables are strongly correlated with the term premium $\Psi_t(\tau)$.

Table 4: Term spread regression model without term premium effects

Model: $(\tau-1)[R_{t+1}(\tau-1)-R_t(\tau)] = a(\tau)+\beta(\tau)[R_t(\tau)-r_t]+u_{t+1}(\tau)$

τ	5	10	15	20	25	30
$a(\tau)$	0.020	0.257	0.473	0.711	0.951	1.203
	(0.162)	(0.326)	(0.446)	(0.580)	(0.716)	(0.859)
$\beta(\tau)$	-2.726	-4.187	-4.884	-6.094	-7.766	-9.541
	(1.813)	(2.440)	(2.580)	(2.994)	(3.647)	(4.379)

Notes: Newey - West heteroscedasticity and autocorrelation consistent standard errors are shown in parenthesis

Comparison of the results of Tables 4 to those of Table 5 clearly indicates that allowing for a time-varying term premium in term spread regression based tests of the REHTS can indeed save this hypothesis. The estimates of intercepts $a(\tau)$ are very close to zero and slope coefficient $\beta(\tau)$ becomes very close to unity, for all τ , as predicted by the REHTS. With the exception of $\tau = 5$, the estimates of coefficients $a(\tau)$ and $\beta(\tau)$ reported in Table 5 can not reject the joint hypothesis that $a(\tau) = 0$ and $\beta(\tau) = 1$. Another interesting conclusion which can be drawn from the results of Table 5 is that the estimates of the mean reversion and price of risk parameters k_i and $\lambda_i^{(1)}$, respectively, estimated by this version of the system of equations the GDTSM, i.e., (16)-(18) and (21), hardy changes compared with those of its previous version (16)-(19), reported in Table 3.

Table 5: Estimates of the term spread model with term premium effects

$R_{t+1} = A + BX_{t+1} + e_{t+1}$, with $X_{t+1} = B_3^{-1}(E(Z_{t+1}|PC_{t+1}) - A_3)$,
$\Delta X_{t+1} = \Phi_0 + (\Phi - I)X_t + \omega_{t+1}$,
$Z_{t+1} = D_0 + DPC_{t+1} + \varepsilon_{t+1}$, and
$(\tau - 1)[R_{t+1}(\tau - 1) - R_t(\tau)] = a^*(\tau) + \beta^*(\tau)[R_t(\tau) - r_t] + \sum_{i=1}^{K} b_i(\tau)\lambda_i^{(1)} x_{it} + v_{t+1}(\tau)$
where $B_i(\tau) = (1 - e^{-\widetilde{k}_i\tau})(\widetilde{k}_i\tau)^{-1}B_i(0)$ and $\widetilde{k}_i = k_i + \lambda_i^{(1)}$.

	x_{1t}	x_{2t}	x_{3t}
k_i	0.30	0.24	0.38
	(0.02)	(0.07)	(0.15)
$\lambda_i^{(1)}$	-0.29	-0.15	0.04
	(0.02)	(0.07)	(0.15)
		Matrix $D \equiv [D_{ij}]$	
D_{1j}	0.03	-0.11	0.20
	(0.001)	(0.003)	(0.006)
D_{2j}	-0.02	0.15	-0.65
	(0.001)	(0.003)	(0.01)
D_{3j}	0.11	0.09	0.74
	(0.001)	(0.004)	(0.01)

Estimates of $a^*(\tau)$ and $\beta^*(\tau)$ coefficients

τ	5	10	15	20	25	30
$a^*(\tau)$	-0.25	-0.34	-0.36	-0.38	-0.44	-0.52
	(0.12)	(0.19)	(0.26)	(0.31)	(0.36)	(0.44)
$\beta^*(\tau)$	0.87	1.05	1.06	0.97	0.98	1.00
	(0.03)	(0.02)	(0.03)	(0.02)	(0.01)	(0.01)

$J(87) = 110.2$ ($p-$value =0.05), Instruments: 1, z_{1t}, $z_{2t-1}, z_{2t-2}, z_{2t-3}, z_{2t-4}$ and z_{3t}

Notes: Data are monthly from 1997:07 to 2009:10. The Newey - West heteroscedasticity and autocorrelation consistent standard errors shown in parentheses. J is Sargan's overidentified restrictions test statistic. This is chi-squared distributed with degrees of freedom which are equal to the number of orthogonality conditions employed in the estimation procedure (7×18) minus the number of the parameters estimated (39), i.e. 87. The instruments used are lags and current values of the following variables: $z_{1t} = R_t(40)$, $z_{2t}= r_t - R_t(40)$ and $z_{3t}= r_t + R_t(40) - 2R_t(6)$.

The above results together with the value of J test statistic reported in Table 5, which can not reject the overidentified restrictions implied by the GMM estimation procedure of structural system (16)-(18) augmented by term spread regression model (21), provide strong support of the view that our GDTSM can consistently explain both the cross-section (no arbitrage) and dynamic predictions of the rational expectations theory of the term structure. These results have important forecasting policy implications. They mean that adjusting term spread $R_t(\tau) - r_t$ with the term premium effects predicted by our GDTSM can provide forecasts of future changes of long term interest rates $R_{t+1}(\tau-1) - R_t(\tau)$ which are in the right direction.

To examine the stability and forecasting ability of regression model (21), in Table 6 we present values of some forecasting performance metrics and statistics for it, over different maturity intervals τ. This is done for an out-of-sample exercise which relies on a recursive estimation of the structural parameters of model (21) after period 2001:12, based on the system of equations (16)-(18) and (21), by adding to our chosen initial window of our sample 1997:07-2001:12 one observation at a time. The forecasting performance metrics include the mean square and mean absolute values of forecasting errors, denoted as MSE and MAE, respectively, while the test statistics include those of Diebold and Mariano (1995) (denoted as DM) and Giacomini and Rossi (2006) (denoted as GR). The DM test statistic compares the forecasting performance of model (21) to the random walk (RW) model of interest rates, which is often considered in the literature to describe movements in long-term interest rates $R_t(\tau)$ (see, e.g., Mankiw and Miron (1986), and Duffee (2002)).[17] The GR statistic examines the out-of-sample forecasting performance of model (21), by testing if its forecasts break down due to unforeseen structural breaks occurred during our sample.[18] In this case, the out-of-sample forecasts will not be consistent with the in-sample ones.

The results of Table 6 indicate that the forecasting performance of model (21) is more satisfactory than that of the RW. This can be supported by the values of both the MSE and MAE metrics reported in the table, for all τ. The values of DM statistic indicate that the superiority of model (21) relatively to the random walk can be also inferred from our data for most of the maturity intervals τ considered, i.e. $\tau = \{5, 10, 15, 20\}$ years. For τ=$\{25, 30\}$, the two models are found to performed equally, according to the DM statistic. This can be attributed to the fact that the forecasting ability of spread $R_t(\tau) - r_t$ cease at the very end of the term structure, due to mean reverting properties of the underlying term structure common factors x_{it}, especially those with slower mean reversion. Finally, the values of GR test statistic indicate that the forecasting performance of model (21) is stable over sample.

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[17] DM test statistic is based on the following loss difference: $d_t = L(\nu_{t+1}^{Model\ (21)}) - L(\nu_{t+\tau}^{RW})$. It is defined as $DM = \frac{\overline{d}}{(\widehat{\sigma_{\overline{d}}}/T)^{1/2}}$, where $\widehat{\sigma_{\overline{d}}}$ is a consistent estimate of the asymptotic (long-run) variance of $\sqrt{T}\overline{d}$.

[18] The GR statistic is based on the testing principle that, if the forecast performance of a model does not break down, then there should be no difference between its expected out -of-sample and in-sample performance. It is defined as $GR_{m,n,t} = \frac{\overline{SL}_{m,n}}{\widehat{\sigma}^2_{m,n}/\sqrt{n}}$, where $\overline{SL}_{m,n}$ is the average surprise loss given as $\overline{SL}_{m,n} = n^{-1} \sum_{t=m}^{t-\tau} \left\{ (L(Y_{t+\tau}) - \widehat{f}_t(\widehat{\beta}_t)) - m^{-1} \sum_{j=t-m+1}^{t} L(Y_j, \widehat{y}_j(\widehat{\beta}_t)) \right\}$, for $t = m, .., T-\tau$, where $Y_{t+\tau}$ is the forecasted variable and $n = T - \tau - m + 1$. $\widehat{\sigma}^2_{m,n}$ is given in Corollary 4 of Giacomini and Rossi (2005). $GR_{m,n,t}$ converges in distribution to a Standard Normal $N(0,1)$ as m, $n \to \infty$.

Table 6: Out-of-sample forecasting performance

τ :	5	10	15	20	25	30	5	10	15	20	25	30
	Random Walk						Regression model (21)					
MSE	0.10	0.10	0.08	0.07	0.07	0.07	0.09	0.09	0.07	0.05	0.05	0.05
MAE	0.24	0.23	0.21	0.19	0.18	0.19	0.23	0.21	0.19	0.18	0.17	0.18
DM							-1.87	-1.73	-1.76	-1.67	-0.94	-0.75
GR							0.28	0.43	0.06	0.11	0.16	0.25

Notes: The table presents values of metrics and statistic assessing the one period ahead (t+1) out-of-sample forecast performance of regression model (21) and the random walk (RW) model with drift. MSE and MAE denote the mean square and absolute error metrics, while DM and GR denote Diebold's and Mariano (1995) and Giacomini's and Rossi (2006) test statistics, respectively. These statistics follow the standard normal distribution. Our out-of-sample forecasts are based on a recursive estimation of model (21), jointly with the system of equations (16)-(18), and the random walk model. This is done after period after period 2002:01, by adding one observation at a time until the end of sample. The number of the out-of-sample observations used in our forecasting exercise is $n \equiv T - \tau - m + 1 = 93$, where m in our initial sample window of observations.

4 Conclusions

This paper suggests new term spread based regression tests allowing for time-varying premium effects with the aim of examining if a time-varying term premium can explain the puzzling behavior of the spread between the long and short-term interest rates to fail to forecast future movements in the former. This is against the predictions of the rational expectations hypothesis of the term structure of interest rates. To capture the time-varying term premium effects on the tests of this hypothesis, the paper employs a simple and empirically tractable Gaussian Dynamic Term Structure Model (GDTSM). To estimate this model, the paper suggests a new empirical methodology, which retrieves estimates of the unobserved factors spanning the term structure of interest rates net of measurement errors. This is done by projecting interest rates series (or transformations of them), used as instruments in inverting the interest rates relationships implied by affine term structure models, on well diversified portfolios of zero-coupon bond interest rates. The latter are net of measurement error effects and are estimated from a very large set of interest rates data based on principal component analysis.

The paper provides a number of interesting results, which have important policy implications. First, it shows that a three-factor and empirically tractable GDTSM can sufficiently explain the cross-section movements of the US term structure of interest rates implied by no-arbitrage conditions in the bond market. This model provides estimates of the unobserved factors of the term structure of interest rates which are persistent and can capture substantial movements in interest rates, observed during our sample. Second, the paper shows that adjusting term spread regressions by the term premium effects implied by the above GDTSM can explain the empirical failures of the term spread to forecast future movements in long-term interest rates. This result means that our model can be successfully employed to forecast the correct direction of future long-term interest rate changes, net of term premium effects, as predicted by the expectations hypothesis. Finally, the paper shows that the factors that are priced in the bond market and, thus, cause significant time-varying effects on the term spread regressions are those which are associated with "level" and "slope" shifts in the term structure of interest rates.

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