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# Optimal fiscal and monetary policy action in a closed economy

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# Optimal fiscal and monetary policy action in a closed economy*

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**Abstract**

We study optimized monetary and fiscal feedback policy rules. The setup is a New Keynesian DSGE model of a closed economy which is solved numerically using common parameter values and fiscal data from the euro area. Our aim is to welfare rank alternative tax-spending policy instruments used for shock stabilization and/or debt consolidation when, at the same time, the monetary authorities can follow a Taylor rule for the nominal interest rate.

Keywords: Feedback policy rules, New Keynesian.
JEL: E6, F3, H6.

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# 1 Introduction

Policymakers use their instruments to react to economic conditions. For instance, central banks can respond to inflation, the fiscal authorities to the state of public finances, and both of them to real economic activity. It is nevertheless believed that the use of fiscal policy is more complex and controversial than the use of monetary policy (see e.g. Leeper, 2010). This debate over the use of fiscal policy has been intensified since 2009 when most European governments embarked on the difficult task of reducing their public debts.

In this paper, we search for the best mix of monetary and fiscal policy actions when the policy role is twofold: to stabilize the economy against shocks and to improve resource allocation by gradually reducing the public debt burden over time. In order to do so, we welfare rank various fiscal policy instruments used jointly with interest rate policy.

Following most of the related literature (see below), we work with feedback policy rules. In particular, we specify feedback rules for public spending, the tax rate on labor income, the tax rate on capital income and the tax rate on consumption, when these fiscal policy instruments are allowed to respond to a number of macroeconomic variables used as indicators, while, at the same time, monetary policy can be used in a standard Taylor-type fashion. We optimally choose the indicators that the fiscal and monetary authorities react to, as well as the magnitude of feedback policy reaction to those indicators. The welfare criterion is household's expected discounted lifetime utility. This type of policy is known as "optimized" feeback policy rules (see Schmitt-Grohé and Uribe, 2005, 2007, and many others). This enables us to welfare rank alternative policies in a stochastic setup, without our results - and, in particular, our welfare ranking of alternative policies - being driven by ad hoc differences in feedback policy coefficients, as it happens in most of the related literature on debt consolidation (see below). We work within two policy environments. In the first, used as a benchmark, the authorities just stabilize the economy from shocks. In the second, the fiscal authorities also aim at gradually reducing the output share of public debt over time, which means that now we combine shock stabilization with resource allocation policy.

The setup is a rather standard New Keynesian DSGE model of a closed economy featuring imperfect competition, Calvo-type price fixities and nominal wage rigidities. The model is calibrated to match fiscal data from the euro area over 1995-2010. To solve the model and, in particular, to solve for welfare-maximizing policy rules, we adopt the methodology of Schmitt-Grohé and Uribe (2004, 2007), in the sense that we take a second-order approximation to both the equilbrium conditions and the welfare criterion. In turn, we compute the welfare-maximizing values of various feedback policy rules and the associated social welfare.

Our results are as follows. First, in all cases studied, the monetary authorities should aggressively react to price inflation and the fiscal authorities should react to public debt. Also, in most cases, interest rate reaction to the output gap is not recommended. On the other hand, the desirability of fiscal reaction to the output gap, the so-called fiscal activism,[1] depends crucially on the distorting effects of each fiscal instrument and the degree of rigidities in the labor market. Our results show that the more distorting a fiscal policy instrument is, the less it should be used for debt consolidation and the more it should be used to counter the economic downturn. This applies in particular to labor taxes all the time and to capital taxes in the mid and long run. Rigidities in the labor market provide further arguments for fiscal activism. All this means that, under optimized rules, the final or net change in fiscal instruments is determined by the reconciliation of two typically conflicting aims: to reduce public debt and to stimulate the economy. The final or net effect is a quantitative matter (see below).

Second, the arguments for debt consolidation appear to be weak. Debt consolidation is welfare-enhancing, other things equal, only when we care about long time horizons and, even if this is the case, the welfare gain is very small quantitatively. This should be contrasted to results in an open economy facing sovereign risk premia and a non-zero probability of default, where debt consolidation is clearly welfare-enhancing, except from the early phase of fiscal pain (see e.g. Philippopoulos et al., 2013, and the references therein).

Third, in the case of debt consolidation, the choice of the fiscal policy instrument matters for how quickly debt should be brought down. For instance, in our baseline experiments, public debt reduction from 85%, which is its average value in the recent euro data, to the 60% target level, which is the reference level of the Maastricht Treaty, should be achieved within 5 to 12.5 years depending on how distorting the fiscal instrument is (5 years if we use public spending, 6.5 years if we use consumption taxes, and 12.5 years if we use capital taxes). This pace should be slower if there are labor market rigidities since, in the presence of such rigidities, fiscal policy should be seriously concerned about the real economy. On the other hand, if we use labor taxes, which are a particularly distorting instrument at any time, the pace of debt reduction should be very slow, following an almost unit root process, and this is irrespectively of the degree of labour market rigidities.

Fourth, the choice of the fiscal policy instrument also matters for welfare. If there are no rigidities in the labor market, it is better to use public spending along with interest rate policy. This implies that initially public spending should be cut to bring public debt down.

[1] For a discussion of fiscal activism, see e.g. Feldstein (2009), Taylor (2009) and Wren-Lewis (2010).

In other words, since the fiscal instrument used is not so distorting, the concern for debt dominates the concern for output. On the other hand, if there are rigidities in the labor market, it is better to use labor or capital taxes on the side of fiscal policy. Now this implies that labor or capital taxes should be cut initially to stimulate the economy and only then be raised to bring public debt down gradually over time. In other words, since the instrument used is relatively distorting, the concern for output dominates the concern for debt. Again these results should be contrasted to those in an open economy facing sovereign risk premia and a non-zero probability of default, where the choice of the policy mix is not so important for welfare and where all policy instruments should be earmarked to debt consolidation (see Philippopoulos et al., 2013).

How does our work differ? Although there has been a rich literature on the interaction between fiscal and monetary policy,[2] as well as on debt consolidation,[3] there has not been a welfare comparison of the main tax-spending policy instruments in a unified framework, and how this comparison depends on policy goals, namely, shock stabilization only or shock stabilization plus debt consolidation. Also, as said above, here we study optimized policy rules.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the data, calibration and long-run solution. Section 4 explains how we work. Results are in Section 5. Section 6 closes the paper. Details are in an Appendix.

# 2 Model

The model is a standard New Keynesian model featuring imperfect competition and Calvo-type nominal rigidities, which is extended to include a rich menu of state-contingent policy rules.[4]

## 2.1 Households

There are $i = 1, 2, ..., .N$ households. The objective of each $i$ is to maximize expected discounted lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left( c_{i,t}, n_{i,t}, m_{i,t}, g_t \right) \tag{1}$$

[2] See e.g. Leeper (1991), Schmitt-Grohé and Uribe (2005 and 2007), Leith and Wren-Lewis (2008) and Leeper et al. (2009). For reviews, see e.g. Kirsanova et al. (2009), Wren-Lewis (2010) and Leeper (2010).

[3] See e.g. Coenen et al. (2008), Forni et al. (2010), Bi et al. (2012), Cantore et al. (2012), Cogan et al. (2013), Erceg and Lindé (2012, 2013) and Philippopoulos et al. (2013, 2014).

[4] For the New Keynesian model, see e.g. Gali (2008) and Wickens (2008).

where $c_{i,t}$ is $i$'s consumption bundle (defined below), $n_{i,t}$ is $i$'s hours of work, $m_{i,t}$ is $i$'s real money balances, $g_t$ is per capita public spending, $0 < \beta < 1$ is the time discount rate, and $E_0$ is the rational expectations operator conditional on the current period information set.

In our numerical solutions, we use the period utility function (see also e.g. Gali, 2008):

$$u_{i,t}\left(c_{i,t}, n_{i,t}, m_{i,t}, g_t\right) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} \tag{2}$$

where $\chi_n$, $\chi_m$, $\chi_g$, $\sigma$, $\eta$, $\mu$, $\zeta$ are preference parameters.

The budget constraint of each household $i$ (written in real terms) is:

$$\begin{gathered}(1+\tau_t^c)\, c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} = \left(1-\tau_t^k\right)(r_t^k k_{i,t-1} + d_{i,t}) + \\ +(1-\tau_t^n)\, w_t n_{i,t} + R_{t-1}\tfrac{P_{t-1}}{P_t} b_{i,t-1} + \tfrac{P_{t-1}}{P_t} m_{i,t-1} - \tau_{i,t}^l\end{gathered} \tag{3}$$

where $P_t$ is the general price index and small letters denote real variables, i.e. $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$, $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$, $w_t \equiv \frac{W_t}{P_t}$, $d_{i,t} \equiv \frac{D_{i,t}}{P_t}$, $\tau_{i,t}^l \equiv \frac{T_{i,t}^l}{P_t}$. Here, $x_{i,t}$ is $i$'s real investment, $B_{i,t}$ is $i$'s end-of-period nominal government bonds, $M_{i,t}$ is $i$'s end-of period nominal money holdings, $r_t^k$ is the real return to inherited capital denoted as $k_{i,t-1}$, $D_{i,t}$ is $i$'s nominal dividends paid by firms, $W_t$ is the nominal wage rate, $R_{t-1} \geq 1$ is the gross nominal return to government bonds between $t-1$ and $t$, $T_{i,t}^l$ is nominal lump-sum taxes/transfers made to each $i$ from the government, and $\tau_t^c$, $\tau_t^k$, $\tau_t^n$ are respectively tax rates on consumption, capital income and labour income.

The motion of physical capital for each household $i$ is:

$$k_{i,t} = (1-\delta)k_{i,t-1} + x_{i,t} \tag{4}$$

where $0 < \delta < 1$ is the depreciation rate of capital.

The quantity of variety $h$, produced monopolistically by firm $h$, and consumed by household $i$, is denoted as $c_{i,t}(h)$. Using a Dixit-Stiglitz aggregator, the composite of goods consumed by household $i$ is given by:[5]

$$c_{i,t} = \left[\sum_{h=1}^{N} \lambda [c_{i,t}(h)]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{5}$$

where $\phi > 0$ is the elasticity of substitution across goods produced and $\sum\limits_{h=1}^{N} \lambda = 1$ are weights (to avoid scale effects, we set $\lambda = 1/N$ in equilibrium). Household $i$'s total consumption expenditure is:

[5] As in e.g. Blanchard and Giavazzi (2003), we find it more convenient to work with summations rather than with integrals.

$$P_t c_{i,t} = \sum_{h=1}^{N} \lambda P_t(h) c_{i,t}(h) \tag{6}$$

where $P_t(h)$ is the price of variety $h$.

Each household $h$ acts competitively taking prices and policy variables as given. Details and first-order conditions are in Appendix 1.

## 2.2 Firms

There are $h = 1, 2, ..., .N$ firms. Each firm $h$ produces a differentiated good of variety $h$ under monopolistic competition facing Calvo-type nominal fixities. The nominal profit of firm $h$ is defined as:

$$D_t(h) = P_t(h) y_t(h) - P_t r_t^k k_{t-1}(h) - W_t n_t(h) \tag{7}$$

All firms use the same technology represented by the production function:

$$y_t(h) = A_t [k_{t-1}(h)]^{\alpha} [n_t(h)]^{1-\alpha} \tag{8}$$

where $A_t$ is an exogenous stochastic TFP process whose motion is defined below.

Profit maximization by firm $h$ is also subject to the demand for its product (see Appendix 2):

$$y_t(h) = c_t(h) + x_t(h) + g_t\,(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} y_t \tag{9}$$

so that demand for firm $h'$s product, $y_t\,(h)$, comes from households' consumption and investment, $c_t(h)$ and $x_t(h)$, where $c_t(h) \equiv \sum_{i=1}^{N} c_{i,t}(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} c_t$ and $x_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} x_t$, and from the government, $g_t\,(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} g_t$.

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price. A firm $h$, which is able to reset its price, chooses its price $P_t^{\#}\,(h)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed. This objective is:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k}\,(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^{\#}\,(h)\, y_{t+k}\,(h) - \Psi_{t+k}\,(y_{t+k}\,(h)) \right\} \tag{10}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm, $y_{t+k}(h) = \left[\frac{P_t^{\#}(h)}{P_{t+k}}\right]^{-\phi} y_{t+k}$ and $\Psi_t(.)$ denotes the minimum nominal cost function for producing $y_t^H(h)$ at $t$ so that $\Psi_t'(.)$ is the associated nominal marginal cost.

Details and first-order conditions are in Appendix 2.

## 2.3 Government budget constraint

The budget constraint of the consolidated government sector is in real terms (in aggregate quantities):

$$\begin{aligned} & b_t + m_t = R_{t-1}\tfrac{P_{t-1}}{P_t}b_{t-1} + \tfrac{P_{t-1}}{P_t}m_{t-1} + g_t - \\ & -\tau_t^c c_t - \tau_t^k(r_t^k k_{t-1} + d_t) - \tau_t^n w_t n_t - \tau_t^l \end{aligned} \tag{11}$$

where $b_t$ is the end-of-period total real public debt, $m_t$ is the end-of-period total stock of real money balances. We also use $c_t \equiv \sum_{i=1}^N c_{i,t}$, $k_{t-1} \equiv \sum_{i=1}^N k_{i,t-1}$, $D_t \equiv \sum_{i=1}^N D_{i,t}$, $n_t \equiv \sum_{i=1}^N n_{i,t}$, $B_{t-1} \equiv \sum_{i=1}^N B_{i,t-1}$ and $T_t^l \equiv \sum_{i=1}^N T_{i,t}^l$, and all other variables have been defined above. Notice that, as above, small letters denote real variables, namely, $b_t \equiv \frac{B_t}{P_t}$, $m_t \equiv \frac{M_t}{P_t}$, $d_t \equiv \frac{D_t}{P_t}$, $w_t \equiv \frac{W_t}{P_t}$ and $\tau_t^l \equiv \frac{T_t^l}{P_t}$.

In each period, one of the fiscal policy instruments $\tau_t^c$, $\tau_t^k$, $\tau_t^n$, $g_t$, $\tau_t^l$, $b_t$ has to follow residually to satisfy the government budget constraint (see below).

## 2.4 Decentralized equilibrium (given policy)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) all households maximize utility; (ii) a fraction $(1-\theta)$ of firms maximize profits by choosing an identical price $P_t^{\#}$, while the rest, $\theta$, set their previous period prices; (iii) all constraints, including the government budget constraint, are satisfied and (iv) all markets clear.

To proceed with the solution, we need to define the policy regime. Regarding monetary policy, we assume that the nominal interest rate, $R_t$, is used as a policy instrument, while, regarding fiscal policy, we assume that the residually determined public financing policy instrument is the end-of-period public debt, $b_t$ (see below for other cases).

Appendix 3 presents the DE system. It consists of 14 equations in 14 variables $\{y_t$, $c_t$, $n_t$, $x_t$, $k_t$, $m_t$, $b_t$, $P_t$, $P_t^{\#}$, $\widetilde{P}_t$, $w_t$, $mc_t$, $d_t$, $r_t^k\}_{t=0}^{\infty}$. This is given the independently set policy instruments, $\{R_t$, $\tau_t^c$, $\tau_t^k$, $\tau_t^n$, $g_t$, $\tau_t^l\}_{t=0}^{\infty}$, technology $\{A_t\}_{t=0}^{\infty}$, and initial conditions for the state variables. All these variables have been defined above, except from $\widetilde{P}_t$ and $mc_t$, where

$\widetilde{P}_t \equiv \left(\sum_{h=1}^{N} [P_t(h)]^{-\phi}\right)^{-\frac{1}{\phi}}$ and $mc_t$ is the firm's marginal cost as defined in Appendix 2.

Before we specify the processes of policy instruments and exogenous variables in the next two subsections, and by following the related literature, we transform the above equilibrium conditions. In particular, we express price levels in inflation rates, rewrite the firm's optimality condition in recursive form and finally introduce a new equation that helps us to compute expected discounted lifetime utility. Appendix 4 presents details and the resulting transformed DE system consisting of 17 equations in 17 variables.

### 2.5 Policy rules

Following the related literature, we focus on simple rules meaning that the monetary and fiscal authorities react to a small number of easily observable macroeconomic indicators. In particular, we allow the nominal interest rate, $R_t$, to follow a standard Taylor rule, meaning that it can react to inflation and output as deviations from a target, while we allow the distorting fiscal policy instruments, namely, government spending as a share of output, $s_t^g \equiv \frac{g_t}{y_t}$, and the tax rates on consumption, capital income and labor income, $\tau_t^c$, $\tau_t^k$ and $\tau_t^n$, to react to public debt and output, again as deviations from a target. The target values are defined below.

In particular, following e.g. Schmitt-Grohé and Uribe (2007), we use policy rules of the functional form:

$$\log\left(\frac{R_t}{R}\right) = \phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \phi_y \log\left(\frac{y_t}{y}\right) \tag{12}$$

$$s_t^g - s^g = -\gamma_l^g\left(l_{t-1} - l\right) - \gamma_y^g\left(y_t - y\right) \tag{13}$$

$$\tau_t^c - \tau^c = \gamma_l^c\left(l_{t-1} - l\right) + \gamma_y^c\left(y_t - y\right) \tag{14}$$

$$\tau_t^k - \tau^k = \gamma_l^k\left(l_{t-1} - l\right) + \gamma_y^k\left(y_t - y\right) \tag{15}$$

$$\tau_t^n - \tau^n = \gamma_l^n\left(l_{t-1} - l\right) + \gamma_y^n\left(y_t - y\right) \tag{16}$$

where variables without time subscripts denote target values, and $\phi_\pi$, $\phi_y$, $\gamma_l^q$, $\gamma_y^q \geq 0$, for $q \equiv (g, c, k, n)$, are feedback policy coefficients, and:

$$l_{t-1} \equiv \frac{R_{t-1}b_{t-1}}{y_{t-1}} \tag{17}$$

denotes the beginning-of-period public debt burden as share of GDP.

### 2.6 Exogenous stochastic variables

We now define the processes of exogenous stochastic variables. For notational simplicity, we include shocks to TFP only (we report that the main results do not change if we add other shocks, like demand and policy shocks). In particular, we assume that the TFP follows an $AR(1)$ process:

$$\log A_t = \left(1 - \rho^A\right) \log\left(A\right) + \rho^A \log A_{t-1} + \varepsilon_t^A \tag{18}$$

where $0 \leq \rho^A \leq 1$ is a persistence parameter and $\varepsilon_t^A \sim N\left(0,\ \sigma_A^2\right)$.

### 2.7 Final equilibrium system (given feedback policy coefficients)

The full equilibrium system consists of the 17 equations of the transformed DE presented at the end of Appendix 4, and the 5 feedback policy rules as well as the definition of $l_t$ presented in subsection 2.5. We thus end up with 23 equations in 23 variables $\{y_t,\ c_t,\ n_t,\ x_t,\ k_t,\ m_t,\ b_t,\ \Pi_t,\ \Theta_t,\ \Delta_t,\ w_t,\ mc_t,\ d_t,\ r_t^k,\ z_t^1,\ z_t^2,\ V_t,\ R_t,\ s_t^g,\ \tau_t^c,\ \tau_t^k,\ \tau_t^n,\ l_t\}_{t=0}^{\infty}$. Among them, there are 17 non-predetermined or jump variables, $\{y_t,\ c_t,\ n_t,\ x_t,\ \Pi_t,\ \Theta_t,\ w_t,\ mc_t,\ d_t,\ r_t^k,\ z_t^1,\ z_t^2,\ V_t,\ s_t^g,\ \tau_t^c,\ \tau_t^k,\ \tau_t^n\}_{t=0}^{\infty}$, and 6 predetermined or state variables, $\{R_t,\ k_t,\ b_t,\ m_t,\ \Delta_t,\ l_t\}_{t=0}^{\infty}$. This is given technology, $\{A_t\}_{t=0}^{\infty}$, initial conditions for the state variables, and the values of feedback policy coefficients.

To solve this first-order non-linear difference equation system, we will take a second-order approximation around its long-run solution. We therefore first solve for the long run in the next section. In turn, we will study transition dynamics and the optimal choice of feedback policy coefficients along this transition.

## 3 Data, parameterization and long-run solution

This section solves numerically for the long run of the above economy by using conventional parameter values and data from the euro zone. Recall that since money is neutral in the long-run, interest rate policy does not matter to the real economy in the long run. Also, since fiscal policy instruments react to deviations of macroeconomic indicators from their long-run target values, feedback fiscal policy coefficients also do not play any role in the long run.

### 3.1 Data and calibration

The fiscal data are from OECD Economic Outlook no. 89. The time unit is meant to be a quarter. Our baseline parameter values are summarized in Table 1.

Table 1: Parameter values

| Parameter | Value | Description |
|---|---|---|
| $a$ | 0.33 | share of capital |
| $\beta$ | 0.9926 | time preference rate |
| $\mu$ | 3.42 | parameter related to money demand elasticity |
| $\delta$ | 0.021 | capital depreciation rate (quarterly) |
| $\phi$ | 6 | price elasticity of demand |
| $\eta$ | 1 | the inverse of Frisch labour supply elasticity |
| $\sigma$ | 1 | elasticity of intertemporal substitution |
| $\zeta$ | 1 | elasticity of public consumption in utility |
| $\theta$ | 2/3 | price rigidity parameter |
| $\chi_m$ | 0.05 | preference parameter related to real money balances |
| $\chi_n$ | 6 | preference parameter related to work effort |
| $\chi_g$ | 0.1 | preference parameter related to public spending |
| $\rho^A$ | 0.8 | serial correlation of TFP shock |
| $\sigma_A$ | 0.017 | standard deviation of innovation to TFP shock |
| $R$ | 1.0075 | long-run nominal interest rate |
| $\tau^c$ | 0.19 | consumption tax rate |
| $\tau^k$ | 0.28 | capital tax rate |
| $\tau^n$ | 0.38 | labour tax rate |
| $s^g$ | 0.23 | government spending as share of output |
| $s^l$ | -0.2 | lump-sum transfers as share of output |

The value of the rate of time preference, $\beta$, follows from $R = 1.0075$, which is the average gross nominal interest rate in the data, and from setting $\Pi = 1$ for the long-run gross inflation rate. The real money balances elasticity, $\mu$, is taken from Pappa and Neiss (2005). The elasticity of intertemporal substitution, $\sigma$, the inverse of Frisch labour elasticity, $\eta$, and the price elasticity of demand, $\phi$, are as in Andrès and Doménech (2006) and Gali (2008). Regarding the preference parameters in the utility function, $\chi_m$ is chosen so as to obtain a value of real money balances as ratio of output equal to 1.97 (0.5) quarterly (annually), $\chi_n$ is chosen so as to obtain steady-state labour hours equal to 0.28, while $\chi_g$ is set at 0.1 which is a common valuation of public goods in related utility functions.

Concerning the exogenous stochastic variables, we start by setting $\rho^A = 0.8$ and $\sigma_A = 0.017$ for the persistence parameter and the standard deviation respectively of TFP in equation (18) (the value of $\rho^A$ is similar to that in Andrès and Domenéch, 2006, while the value of $\sigma_a$ is close to that in Bi, 2010, and Bi and Kumhof, 2009).

The long-run values of the exogenous policy instruments, $\tau_t^c$, $\tau_t^k$, $\tau_t^n$, $s_t^g$, $s_t^l$, $b_t$, are either set at their data averages, or are calibrated to deliver data-consistent long-run values for the endogenous variables. In particular, $\tau^c$, $\tau^k$, $\tau^n$ are the averages of the effective tax rates in the data. We set lump-sum taxes, $s^l$, so as to get a value of 0.43 for the sum $-s^l + s^g$, when the public debt-to-output ratio is 3.4 quarterly (or 0.85 annually) as in the average data over 2008-2011. The long-run values of policy instruments are summarized in Table 2.

## 3.2 Long-run solution or the status quo

Table 2 reports the long-run solution of the model economy when we use the parameter values and the policy instruments in Table 1. The solution makes sense and the resulting great ratios are close to their values in the data (recall that, since the time unit is meant to be a quarter, stock variables should be divided by 4 to give the annual value). In what follows, we will depart from this status quo long-run solution to study various policy experiments.

Table 2: Long-run solution or "the status quo"

| Variables | Long-run solution | Variables | Long-run solution | Data |
|---|---|---|---|---|
| $y$ | 0.74 | $d$ | 0.12 | - |
| $c$ | 0.46 | $r^k$ | 0.04 | - |
| $n$ | 0.28 | $z^1$ | 1.82 | - |
| $x$ | 0.11 | $z^2$ | 2.18 | - |
| $k$ | 5.19 | $V$ | 108.655 | - |
| $m$ | 1.46 | $u$ | 0.8040 | - |
| $b$ | 2.52 | $l$ | 3.43 | - |
| $\Pi$ | 1 | $\frac{c}{y}$ | 0.62 | 0.57 |
| $\Theta$ | 1 | $\frac{b}{y}$ | 3.4 | 3.4 |
| $\Delta$ | 1 | $\frac{x}{y}$ | 0.15 | 0.18 |
| $w$ | 1.47 | $\frac{m}{y}$ | 1.97 | - |
| $mc$ | 0.83 | $\frac{k}{y}$ | 7 | - |

In this long-run status quo solution, a lower public debt to output ratio leads to higher output, capital and welfare (except when we use government spending in which welfare is lower). This rationalizes the debt consolidation policies studied below.

# 4 How we model policy

In this section, we explain the policy experiments we focus on, how we model debt consolidation and how we compute optimized monetary and fiscal policy rules. Recall that, along the transition path, nominal rigidities imply that money is not neutral so that interest rate policy matter to the real economy. Also, recall that, along the transition path, different counter-cyclical fiscal policy rules can have different implications.

## 4.1 Policy experiments

We will study two environments regarding policy action. In the first, used as a benchmark, the role of policy is only to stabilize the economy against temporary shocks. In particular, we assume that the economy is hit by an adverse temporary TFP shock, as defined in equation (18) above, which produces a contraction in output and a rise in the public debt to output ratio. Then, the policy questions are which policy instrument to use, and how strong the reaction of policy instruments to deviations from targets should be, where these targets are given by the status quo long-run solution. Technically speaking, in this case, we depart from, and end up, at the same steady state, which is the status quo in subsection 3.2 above, while transition dynamics are driven by temporary shocks only.

The second environment is richer. Now the role of policy is twofold: to stabilize the economy against the same TFP shock as above and, at the same time, to improve resource allocation by gradually reducing the public debt ratio over time. The policy questions are as above except that now the policy targets are given by the long-run solution of the reformed economy. Technically speaking, in this case, we depart from the status quo solution, but we end up at a reformed long-run with lower public debt. Thus, now there are two sources of transition dynamics: temporary shocks and the difference between the initial and the new reformed steady state (see also Cantore et al., 2012).

Although our interest is in the latter reformed economy, the former serves as a natural welfare benchmark. The next subsection provides the definition of debt consolidation adopted here.

## 4.2 How we model debt consolidation

We assume that the government reduces the share of public debt from 85% (which is its average value in the data over the sample period and is also the status quo solution) to 60%. We choose the target value of 60% simply because it has been the reference rate of the Maastricht Treaty (we report however that our main results are not sensitive to the value of the debt target assumed). Debt reductions are accommodated by adjustments in the tax-spending policy instruments, namely, the output share of public spending, and the tax rates on capital income, labour income and consumption.

It is widely recognized that the implications of debt consolidation depend heavily on the public financing policy instrument used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see e.g. Leeper et al., 2010, and Leeper, 2010). To understand the logic of our results, we will use one fiscal instrument at a time. This means that, along the early costly phase, we allow one of the fiscal policy instruments to react to public debt imbalances, so as to stabilize debt around its new target value of 0.6 and, at the same time, the same fiscal policy instrument adjusts residually in the long-run to close the government budget. Thus, we assume that the same policy instrument bears the cost of, and reaps the benefit from, debt consolidation. The policy rules for these instruments are as in subsection 2.5 above except that now the targetted values are those of the reformed long-run equilibrium. All other fiscal policy instruments, except the one used for stabilization, remain unchanged and equal to their pre-reform status quo values.

In particular, we work as follows. We first solve and compare the long-run equilibria with and without debt consolidation. In turn, setting, as initial conditions for the state variables, their long-run values from the solution of the economy without debt consolidation (this is the status quo in subsection 3.2), we compute the equilibrium transition path of each reformed economy under optimized policy rules and in turn compute the associated conditional expected discounted lifetime utility of the household. This is for each method of public financing used. Thus, the feedback policy coefficients of the policy instrument used for stabilization along the transition path are chosen optimally. This is explained in the next subsection.

## 4.3 How we compute optimized policy rules

Irrespectively of the policy experiments studied, to make the comparison of different policies meaningful, we compute optimized policy rules, so that results do not depend on ad hoc differences in feedback policy coefficients across different policy rules. The welfare criterion is household's expected lifetime utility as defined in equation (67) in the Appendix.

To do so, we will work in two steps. In the first preliminary step, we search for the ranges of feedback policy coefficients, as defined in equations (12-16), which allow us to get a locally determinate equilibrium (this is what Schmitt-Grohé and Uribe, 2007, call implementable rules). If necessary, these ranges will be further restricted so as to give economically meaningful solutions for the policy instruments (e.g. tax rates less than one). In our search for local determinacy, we experiment with one, or more, policy instruments and one, or more, operating targets at a time.

In the second step, within the determinacy ranges found above, we compute the welfare-maximizing values of feedback policy coefficients (this is what Schmitt-Grohé and Uribe, 2005 and 2007, call optimized policy rules). The welfare criterion is to maximize conditional welfare, $E_0V_0$, as defined in (67), where conditionality refers to the initial conditions chosen; the latter are given by the status quo long-run solution. To this end, following e.g. Schmitt-Grohé and Uribe (2004), we take a second-order approximation to both the equilibrium conditions and the welfare criterion. As is well known, this is consistent with risk-averse behavior on the part of economic agents and can also help us to avoid possible spurious welfare results that may arise when one takes a second-order approximation to the welfare criterion combined with a first-order approximation to the equilibrium conditions (see e.g. Gali, 2008, Malley et al., 2009, and, for a recent review, Benigno and Woodford, 2012).

In other words, we first compute a second-order accurate approximation of conditional welfare, and the associated decentralized equilbrium, as functions of feedback policy coefficients by using the perturbation method of Schmitt-Grohé and Uribe (2004) and, in turn, we use a matlab function (fminsearch.m or fminsearchbnd.m) to compute the values of the feedback policy coefficients that maximize the second-order accurate approximation of conditional welfare (our matlab routines are available upon request). In this exercise, as said above, the feedback policy coefficients are restricted to be within some prespecified ranges delivering determinacy as well as meaningful values for policy instruments.

# 5 Results

This section presents numerical results. We start by defining the region of feeback policy coefficients that can give local determinacy.

## 5.1 Determinacy areas

We first check for local determinacy. As is well known, the latter can depend crucially on the values of feedback policy coefficiencts. We report that economic policy guarantees determinacy when the nominal interest rate reacts aggressively to inflation with $\phi_\pi > 1$, that is, when the Taylor principle is satisfied, and, at the same time, the fiscal policy instruments, $s_t^g$, $\tau_t^c$, $\tau_t^k$, $\tau_t^n$ react to public liabilities above a critical minimum value, $\gamma_l^q > \underline{\gamma}_l^q > 0$, where critical minimum values differ across different policy instruments. By contrast, the values of $\phi_y$ and $\gamma_y^q$, measuring respectively the reaction of interest rate policy and fiscal policy to the output gap, are not found to be critical to determinacy.[6]

Nevertheless, sometimes, the feedback fiscal policy coefficients on public debt, $\gamma_l^q$, where $q \equiv (g, c, k, n)$, need to be further restricted in order to get economically meaningful solutions for the fiscal instruments used, i.e. in order to get $0 \leq s_t^g, \tau_t^c, \tau_t^k, \tau_t^n < 1$. In particular, our computations imply that we need to work within the ranges $\gamma_l^k \in (0.05, 0.15)$ for the capital tax rate and $\gamma_l^c \in (0.05, 0.2)$ for the consumption tax rate, which are narrower than those required for determinacy only. This makes sense. When debt consolidation is among the policy aims, the fiscal authorities may find it optimal to increase tax rates, and/or reduce public spending, beyond meaningful or historical ranges. Our simulations imply that this applies in particular to the capital tax rate, $\tau_t^k$, which, if it is left free, it can easily rise above 100% in the short run due to the high value of $\gamma_l^k$ chosen (this is consistent with the Ramsey-Chamley result that, since capital is inelastic in the very short, the fiscal authorities may find it optimal to confiscate it). To avoid such problems, we restrict ourselves within the above ranges for $\gamma_l^k$ and $\gamma_l^c$. Similarly, since in some experiments monetary policy finds it optimal to increase the feedback policy coefficient on inflation, $\phi_\pi$, to very high values (see also Schmitt-Grohé and Uribe, 2007), we restrict $\phi_\pi$ within the range $1 < \phi_\pi \leq 3$. We report that our main results regarding the welfare ranking of policy instruments do not depend on those restrictions (results are available upon request). Also, note that such practice is usual both in the policy literature (see e.g. Cantore et al., 2012), as well as in the theoretical literature on optimal taxation (see e.g. Chamley, 1986). We finally report that the resulting equilibrium nominal interest rate is above the zero bound in all solutions reported below.

Thus, the general message is that monetary and fiscal policy need to interact with each other in a specific way for policy to guarantee determinacy or, as Leeper (2010) puts it, there is a "dirty little secret": for monetary policy to control inflation, fiscal policy must behave in a particular manner.

[6] Actually, we can distinguish two regions of determinacy. In addition to the one discussed above, there is another region in which fiscal policy does not react to public liabilities, i.e. $\gamma_l^q = 0$ for all fiscal instruments, while monetary policy reacts to inflation mildly with $\phi_\pi < 1$. This region is welfare inferior to the region discussed above. It also contains some sub-areas where determinacy breaks down. Several other papers have distinguished betwen the same two areas of determinacy (e.g. Leeper, 1991, and Schmitt-Grohé and Uribe, 2007).

## 5.2 Optimized policy rules and welfare with debt consolidation

Within the above determinacy ranges, we compute optimized policy rules. Results for the case with debt consolidation are reported in Table 3. The first colunm lists the pair of policy instruments used (one monetary and one fiscal), the second column reports the optimal reaction of the interest rate to inflation and output, and the third column reports the optimal reaction of each fiscal policy instrument to debt and output. Expected discounted lifetime utility, $E_0 V_0$, is reported in the last column.[7]

There are two messages from Table 3. First, if we rank policy instruments according to expected discounted lifetime utility, $E_0 V_0$, the best possible mix is $R_t$ and $s_t^g$. On the other hand, to the extent that the feedback policy coefficients are chosen optimally, the welfare differences across different policy mixes are small. Keep in mind however that these results are in terms of lifetime utility because shorter time horizons may imply different things (see below). Second, in all cases in Table 3, the interest rate should react aggressively to inflation, while interest rate reaction to the output gap is negligible. Notice that, in most cases, fiscal reaction to the output gap is smaller in magnitude than fiscal reaction to debt. As a result, public spending should fall, while consumption taxes and capital taxes should rise, to reduce the public debt. The exception is when we use the labor tax rate; in this case, the reaction to ouput is stronger than the reaction to debt meaning that a distorting policy instrument, like labor taxes, should be used to stabilize output rather than to reduce the public debt burden. Actually, the labor tax rate is reduced at impact to help the real economy. All this is confirmed by impulse response functions shown below.

[7] To compare regimes, we could also use a flat consumption subsidy that makes the agent indifferent between two regimes (see e.g. Lucas, 1990). The policy messages will be the same.

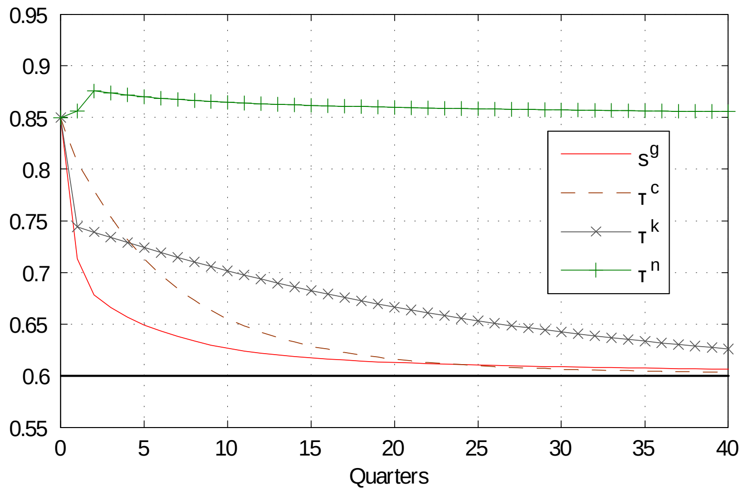
Table 3 : Optimal monetary reaction to inflation and output
and optimal fiscal reaction to debt and output

| Policy instruments | Optimal interest-rate reaction to inflation and output | Optimal fiscal reaction to debt and output | Lifetime utility $E_0 V_0$ |
|---|---|---|---|
| $R_t \quad s_t^g$ | $\phi_\pi = 1.24$<br>$\phi_y = 0.0011$ | $\gamma_l^g = 0.1899$<br>$\gamma_y^g = 0.1044$ | 109.1222 |
| $R_t \quad \tau_t^c$ | $\phi_\pi = 3$<br>$\phi_y = 0.0096$ | $\gamma_l^c = 0.2$<br>$\gamma_y^c = 0.0036$ | 109.0829 |
| $R_t \quad \tau_t^k$ | $\phi_\pi = 3$<br>$\phi_y = 0.0036$ | $\gamma_l^k = 0.15$<br>$\gamma_y^k = 0.0007$ | 109.0619 |
| $R_t \quad \tau_t^n$ | $\phi_\pi = 3$<br>$\phi_y = 0.0723$ | $\gamma_l^n = 0.013$<br>$\gamma_y^n = 0.1074$ | 108.7633 |

Notes: We restrict $\gamma_l^c \in [0.05, 0.2]$ in the case we use consumption taxes and $\gamma_l^k \in [0.05, 0.15]$ in the case we use capital taxes.

The optimized policy rules shape the motion of public debt over time. Figure 1 shows the resulting path of the public debt-to-GDP ratio. The duration of the debt consolidation phase, or equivalently the speed of debt reduction, depend heavily on the fiscal policy instrument used. In particular, more than 95% of debt consolidation should be achieved within 20 time periods or quarters (5 years), if we use the public spending ratio, $s_t^g$; within 25 time periods or quarters (6.25 years), if we use the consumption tax rate, $\tau_t^c$; and within 50 time periods or quarters (12.5 years), if we use the capital tax rate, $\tau_t^k$. On the other hand, if we use the labor tax rate, $\tau_t^n$, the debt-to-output ratio should converge very slowly to its 60% target looking like a unit-root process. The general idea is that the more distorting the policy instrument is, the slower the debt adjustment should be.

Figure 1: The path of public debt as share of output




### 5.3 Welfare over various time horizons with and without debt consolidation

We now study what happens to welfare over various time horizons. This is important because, for several (political-economy) reasons, economic agents' behavior can be short sighted. Setting the feedback policy coefficients as in Table 3 above, the expected discounted utility at various time horizons is reported in Table 4. In the same Table, we also report results without debt consolidation other things equal (these are the numbers in parentheses). As said, without debt consolidation, we again compute optimized policy rules but now the economy starts from, and returns to, the status quo solution.

There are two messages from Table 4. First, it is not clear that debt consolidation is welfare-enhancing. It is, only when we care about long time horizons and, even if this is the case, the welfare superiority of debt consolidation is small quantitatively (see the numbers for lifetime utility in Table 4). If we care about the short run, debt consolidation is costly; the exception is when we use the capital tax rate in which case debt consolidation is clearly superior. The latter is consistent with the Chamley-Judd result. Namely, in the very short run, capital taxes act as a capital levy on predetermined wealth so that the best fiscal policy is to bring public debt down by higher capital taxes.

Second, without debt consolidation, and to the extent that feedback policy coefficients are optimally chosen, the choice of the fiscal policy instrument used for cyclical stabilization is trivial. Welfare differences appear after the third decimal point across all time horizons (these are the numbers in parentheses). With debt consolidation, the choice of the policy instrument matters more (these are the numbers without parentheses). Now, except from the case in which we care only about the short run, the best policy mix is $R_t - s_t^g$ across all time horizons. In particular, this is the best mix to use after the first 15 periods across all time windows. In the short run, by contrast, the best mix is $R_t - \tau_t^k$ for the reasons explained above.

Table 4: Welfare at different time horizons
with, and without, debt consolidation

| | 2 periods | 4 periods | 10 periods | 15 periods | Lifetime utility $E_0 V_0$ | Long-run utility $u$ |
|---|---|---|---|---|---|---|
| $R_t$ $s_t^g$ | 1.5529 | 3.1702 | 7.9133 | 11.6991 | 109.1222 | 0.799788 |
| | (1.6022) | (3.1806) | (7.7758) | (11.4525) | (108.7461) | (0.804046) |
| $R_t$ $\tau_t^c$ | 1.4093 | 2.8611 | 7.3188 | 11.0164 | 109.0829 | 0.807216 |
| | (1.6024) | (3.1808) | (7.7765) | (11.4538) | (108.7376) | (0.804046) |
| $R_t$ $\tau_t^k$ | 1.6678 | 3.2975 | 7.9778 | 11.6670 | 109.0619 | 0.81227 |
| | (1.6023) | (3.1808) | (7.7764) | (11.4536) | (108.7357) | (0.804046) |
| $R_t$ $\tau_t^n$ | 1.6021 | 3.1804 | 7.7767 | 11.4549 | 108.7633 | 0.810852 |
| | (1.6024) | (3.1809) | (7.7767) | (11.4539) | (108.7361) | (0.804046) |

Notes: Results without debt consolidation in parentheses. Periods denote quarters.

### 5.4 Results with rigidities in the labor market

The previous analysis has assumed away any rigidities in the labor market. This is questionable since it is widely believed that labor market rigidities are a key feature of the European economy (see e.g. Blanchard, 2004). In this subsection, we extend the model to allow for such rigidities.

To avoid further complicating the model, but to also help it replicate the stylized facts in Europe regarding inertia in wage adjustment, we follow the setup employed in Blanchard and Galí (2007), Malley et al. (2009) and many others. In particular, we assume that the nominal wage rate at time $t$ is a weighted average of the nominal wage in the previous period, $t-1$, and the nominal wage that would arise in case the labor market worked perfectly. Expressing variables in real term, this implies that the real wage at time $t$ follows:

$$w_t \equiv \left( w_{t-1} \frac{P_{t-1}}{P_t} \right)^{\gamma} (MRS_t)^{1-\gamma} \tag{19}$$

where $0 \leq \gamma \leq 1$ measures the degree of wage sluggishness and $MRS_t \equiv \frac{\chi_n (1+\tau_t^c) n_{i,t}^{\eta}}{(1-\tau_t^n) c_{i,t}^{-\sigma}}$ (see equation (30) in Appendix 1). The idea behind this partial adjustment model is that real wages respond only sluggishly to current conditions in the labor market. As pointed out by Blanchard and Galí (2007), "this is a parsimonious way of modeling the slow adjustment of wages to labor market conditions, as found in a variety of models of real wage rigidities, without taking a stand on what is the "right" model". In other words, although *ad hoc*, this specification can be consistent with a number of possible sources of rigidity in European labor markets, e.g. institutional, legal and socio-political rigidities and safety nets, etc. Finally, notice that this modeling has the following advantages: (i) if $\gamma = 0$, the standard neoclassical model obtains; (ii) in the steady-state, i.e. when $w_t = w_{t-1} = w$, it follows that again $w = MRS$. If $\gamma = 1$, we have full persistence in wage setting. In our numerical solutions below, we set $\gamma = 0.95$, which is the value also used by Malley et al. (2009) for a number of European economies.

The model is resolved using the new specification in the labor market. The new results are reported in Tables 5 and 6. Tables 5 and 6 are like Tables 3 and 4 respectively. The new messages from the new Tables are as follows. First, when there are also rigidities in the labor market, it is better to use income tax rates along with the interest rate. That is, under debt consolidation, the mixes $R_t - \tau_t^k$ and $R_t - \tau_t^n$ score better than $R_t - s_t^g$ and this is the case both in the very short run and in the long run. Intuitively, it is better to use instruments, like income taxes, that are relatively close to the heart of the labor market imperfection. This is a second-best policy argument. Second, fiscal reaction to the output gap is now much bigger than in the case without labor market rigidities. That is, now the fiscal authorities find it optimal to also react to the recession so that debt reduction is not their only concern. All this is confirmed by impulse response functions shown below.

Table 5 : Optimal monetary reaction to inflation and output
and optimal fiscal reaction to debt and output

| Policy instruments | Optimal interest-rate reaction to inflation and output | Optimal fiscal reaction to debt and output | Lifetime utility $E_0 V_0$ |
|---|---|---|---|
| $R_t \quad s_t^g$ | $\phi_\pi = 3$<br>$\phi_y = 0$ | $\gamma_l^g = 0.1333$<br>$\gamma_y^g = 0.1234$ | 108.9656 |
| $R_t \quad \tau_t^c$ | $\phi_\pi = 3$<br>$\phi_y = 0.0188$ | $\gamma_l^c = 0.2664$<br>$\gamma_y^c = 0.1494$ | 109.0155 |
| $R_t \quad \tau_t^k$ | $\phi_\pi = 1.1$<br>$\phi_y = 0.0767$ | $\gamma_l^k = 0.0119$<br>$\gamma_y^k = 0.2847$ | 109.4024 |
| $R_t \quad \tau_t^n$ | $\phi_\pi = 1.1$<br>$\phi_y = 0.0748$ | $\gamma_l^n = 0.0115$<br>$\gamma_y^n = 0.2342$ | 109.2807 |

Notes: We restrict $\gamma_l^k \in [0.05, 0.15]$ in the case we use capital taxes.

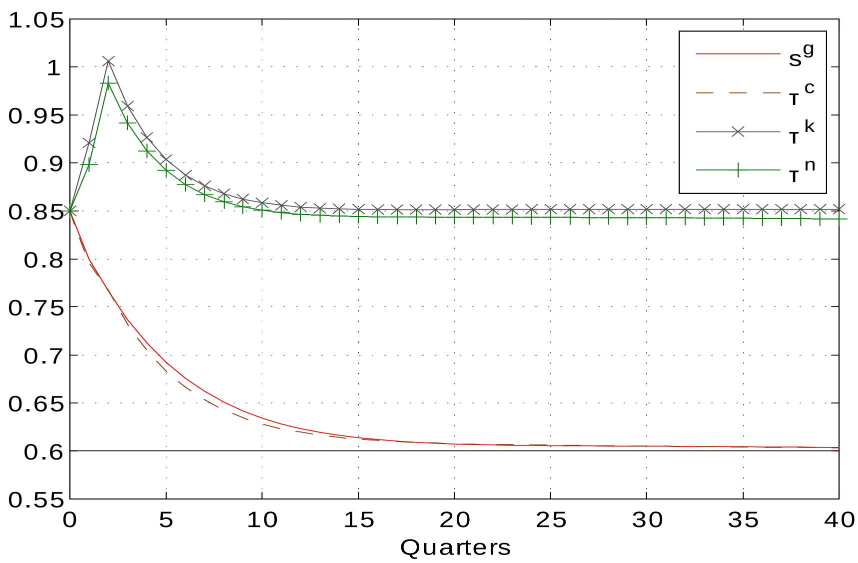
Table 6: Welfare at different time horizons
with, and without, debt consolidation

| | 2 periods | 4 periods | 10 periods | 15 periods | Lifetime utility $E_0 V_0$ | Long-run utility $u$ |
|---|---|---|---|---|---|---|
| $R_t \quad s_t^g$ | 1.5605<br>(1.6025) | 3.1329<br>(3.1802) | 7.8196<br>(7.7668) | 11.6202<br>(11.4357) | 108.9656<br>(108.6981) | 0.799788<br>(0.804046) |
| $R_t \quad \tau_t^c$ | 1.3157<br>(1.6031) | 2.7191<br>(3.1814) | 7.1902<br>(7.7712) | 10.9562<br>(11.4432) | 109.0155<br>(108.7298) | 0.807216<br>(0.804046) |
| $R_t \quad \tau_t^k$ | 1.6002<br>(1.6030) | 3.1746<br>(3.1811) | 7.7631<br>(7.7690) | 11.4463<br>(11.4386) | 109.4024<br>(108.7250) | 0.81227<br>(0.804046) |
| $R_t \quad \tau_t^n$ | 1.6025<br>(1.6030) | 3.1798<br>(3.1815) | 7.7750<br>(7.7719) | 11.4605<br>(11.4439) | 109.2807<br>(108.7248) | 0.810852<br>(0.804046) |

Notes: Results without debt consolidation in parentheses. Periods denote quarters.

The resulting public debt dynamics are shown in Figure 2. The general message is that now it is optimal to reduce public debt more gradually than before. In particular, when we use public spending or consumption taxes, debt adjustment should take place at a slower pace during the first 5-10 periods or quarters. But the difference from Figure 1 becomes more striking when we use capital and labour income taxes. Now, it is clearly optimal to let the debt-to-output ratio further rise in the very short run, so as to help the real economy recover first, and only then decrease the debt gradually by following an almost unit root process to its 60% target.

Figure 2 : The path of public debt as share of output

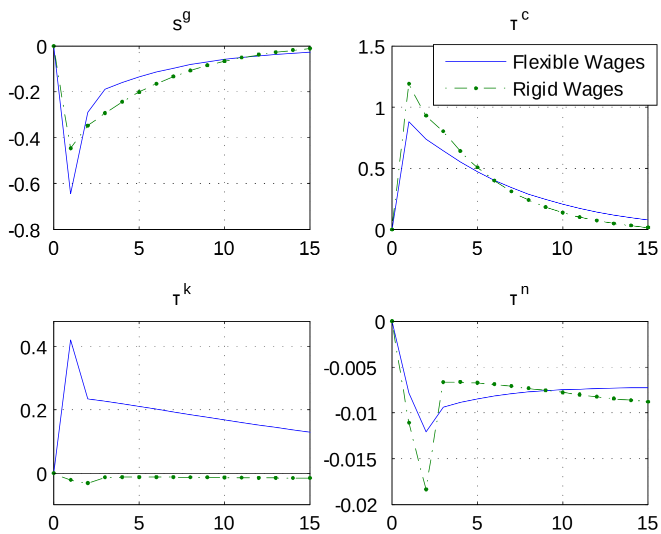



## 5.5 Impulse responce functions of optimized fiscal instruments

To make our results clearer, we also provide the impulse response functions of the optimized fiscal policy instruments studied above. This is in Figure 3. Impulse responce functions are shown as log-linear deviations from the status-quo solution. Solid lines correspond to the model without wage rigidities. Broken lines correspond to the model with wage rigidities. Recall that there are two driving forces of dynamics in our model: an adverse shock to TFP causing a recession and the debt consolidation reform.

As can be seen in Figure 3, public spending should fall, and consumption taxes should rise, with and without wage rigidities. Thus, the concern for public debt dominates the concern for the output gap, when we make use of a relatively non-distorting fiscal instrument, like public spending and consumption taxes. On the other hand, the degree of wage rigidities plays an importent role if we use capital taxes. If there are no labor market rigidities, capital taxes should rise to bring public debt down. But, if there are labor market rigidities, the change in capital taxes should be very mild (actually, as shown in Figure 3, the capital tax rate should be cut initially) to help the real economy recover first. The emphasis on real activity becomes more obvious if we use labor taxes which, as we have seen, are particularly distorting. Now, labor taxes should be reduced so as to counter the recession first and only later on should be raised to address the public debt problem. This is more obvious when there are also labor market rigidities.

Figure 3: Impulse responce functions of optimized fiscal instruments




### 5.6 Robustness checks

Our results are robust to several changes. For instance, they are robust to assuming a more volatile economy, as measured by the standard deviation of the TFP shock, or to adding demand and policy shocks. More interestingly, they are robust to changes in key parameter values. Among the latter, we have, in particular, experimented with changes in the values of the Calvo parameter in the firm's problem, $\theta$, and the preference parameter for public goods, $\chi_g$, whose values are relatively unknown. We report that that our main results do not change within the ranges $0.33 \leq \theta \leq 0.75$ and $0 \leq \chi_g \leq 0.09$. Finally, our main results continue to hold even when there are restrictions on the magnitude of changes in policy instruments used for stabilization. In particular, in the analysis above, all policy instruments were well-defined economically, in the sense that tax rates and output spending shares were between zero and one all the time. Also the nominal interest rate was positive all the time. Nevertheless, one could argue that, in addition, the values of policy instruments cannot differ substantially from those in the historical data (for various political economy reasons). We have therefore redone all the above computations restricting the feedback coefficients in the policy rules so as the policy instruments cannot change by more than 25% from their averages in the data. The welfare ranking and the main results do not change. Robustness checks are available upon request.

## 6 Concluding remarks and possible extensions

This paper studied the optimal mix of monetary and fiscal policy actions in a New Keynesian model of a closed economy. The aim has been to welfare rank different fiscal (tax and spending) policy instruments when the central bank follows a Taylor rule for the interest rate. We did so in two policy environments: first, when the policy task was to stabilize the economy against shocks and, second, when the government faced two tasks, shock stabilization and debt consolidation.

Since the results have been listed in the Introduction, we close with some extensions. First, it is interesting to check the robustness of our results when we move to an open economy and, in particular, a semi-small open economy facing sovereign premia or a multi-country model of a currency union, like the euroarea, with debtor and creditor country-members (see Philippopoulos et al., 2013 and 2014). Second, it would be interesting to study the implications of less conventional monetary policy instruments, like the case in which the central bank acts as a lender of last resort.

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# 7 Appendices

## 7.1 Households

This Appendix provides details for the household's problem. There are $i = 1, 2, ..., N$ households. Each household $i$ acts competitively to maximize expected lifetime utility.

### 7.1.1 Household's problem

Household $i$'s expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_{i,t}, n_{i,t}, m_{i,t}, g_t\right) \tag{20}$$

where $c_{i,t}$ is $i$'s consumption bundle (defined below), $n_{i,t}$ is $i$'s hours of work, $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$ is $i$'s real money balances, $g_t$ is per capita public spending, $0 < \beta < 1$ is the time discount rate, and $E_0$ is the rational expectations operator conditional on the current period information set.

In our numerical solutions, we use the period utility function (see also e.g. Gali, 2008):

$$u_{i,t}\left(c_{i,t}, n_{i,t}, m_{i,t}, g_t\right) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} \tag{21}$$

where $\chi_n$, $\chi_m$, $\chi_g$, $\sigma$, $\eta$, $\mu$, $\zeta$ are preference parameters.

The period budget constraint of each household $i$ is in nominal terms:

$$\begin{aligned}
&\left(1+\tau_t^c\right) P_t c_{i,t} + P_t x_{i,t} + B_{i,t} + M_{i,t} = \\
&\left(1-\tau_t^k\right)\left(r_t^k P_t k_{i,t-1} + D_{i,t}\right) + \left(1-\tau_t^n\right) W_t n_{i,t} + R_{t-1} B_{i,t-1} + M_{i,t-1} - T_{i,t}^l
\end{aligned} \tag{22}$$

where $P_t$ is the general price index, $x_{i,t}$ is $i$'s real investment, $B_{i,t}$ is $i$'s end-of-period nominal government bonds, $M_{i,t}$ is $i$'s end-of period nominal money holdings, $r_t^k$ is the real return to inherited capital, $k_{i,t-1}$, $D_{i,t}$ is $i$'s nominal dividends paid by firms, $W_t$ is the nominal wage rate, $R_{t-1}$ is the gross nominal return to government bonds between $t-1$ and $t$, $T_{i,t}^l$ is nominal lump-sum taxes/transfers to each $i$ from the government, and $\tau_t^c$, $\tau_t^k$, $\tau_t^n$ are respectively tax rates on private consumption, capital income and labour income.

Dividing by $P_t$, the budget constraint of each $i$ in real terms is:

$$\begin{aligned}
\left(1+\tau_t^c\right) c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} = \left(1-\tau_t^k\right)\left(r_t^k k_{i,t-1} + d_{i,t}\right) + \\
+ \left(1-\tau_t^n\right) w_t n_{i,t} + R_{t-1} \tfrac{P_{t-1}}{P_t} b_{i,t-1} + \tfrac{P_{t-1}}{P_t} m_{i,t-1} - \tau_{i,t}^l
\end{aligned} \tag{23}$$

where small letters denote real variables, i.e. $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$, $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$, $w_t \equiv \frac{W_t}{P_t}$, $d_{i,t} \equiv \frac{D_{i,t}}{P_t}$, $\tau^l_{i,t} \equiv \frac{T^l_{i,t}}{P_t}$, at individual level.

The motion of physical capital for each household $i$ is:

$$k_{i,t} = (1-\delta)k_{i,t-1} + x_{i,t} \tag{24}$$

where $0 < \delta < 1$ is the depreciation rate of capital.

Household $i$'s consumption bundle at $t$, $c_{i,t}$, is a composite of $h = 1, 2, ..., N$ varieties of goods, denoted as $c_{i,t}(h)$, where each variety $h$ is produced monopolistically by one firm $h$. Using a Dixit-Stiglitz aggregator, we define:

$$c_{i,t} = \left[ \sum_{h=1}^{N} \lambda [c_{i,t}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{25}$$

where $\phi > 0$ is the elasticity of substitution across goods produced and $\sum_{h=1}^{N} \lambda = 1$ are weights (to avoid scale effects, we assume $\lambda = 1/N$).

Household $i$'s total consumption expenditure is:

$$P_t c_{i,t} = \sum_{h=1}^{N} \lambda P_t(h) c_{i,t}(h) \tag{26}$$

where $P_t(h)$ is the price of variety $h$.

### 7.1.2 Household's optimality conditions

Each household $i$ acts competitively taking prices and policy as given. Following the literature, to solve the household's problem, we follow a two-step procedure. Thus, we first suppose that the household chooses its desired consumption of the composite good, $c_{i,t}$, and, in turn, chooses how to distribute its purchases of individual varieties, $c_{i,t}(h)$. Details are available upon request. Then, the first-order conditions include the budget constraint above and:

$$\frac{c_{i,t}^{-\sigma}}{(1+\tau_t^c)} = \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1+\tau_{t+1}^c)} \left[ \left(1 - \tau_{t+1}^k\right) r_{t+1}^k + (1-\delta) \right] \tag{27}$$

$$\frac{c_{i.t}^{-\sigma}}{(1+\tau_t^c)} = \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1+\tau_{t+1}^c)} R_t \frac{P_t}{P_{t+1}} \tag{28}$$

$$\chi_m m_{i,t}^{-\mu} - \frac{c_{i,t}^{-\sigma}}{(1+\tau_t^c)} + \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1+\tau_{t+1}^c)} \frac{P_t}{P_{t+1}} = 0 \tag{29}$$

$$\chi_n \frac{n_{i,t}^{\eta}}{c_{i,t}^{-\sigma}} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t \tag{30}$$

$$c_{i,t}(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} c_{i,t} \tag{31}$$

Equations (27) and (28) are respectively the Euler equations for capital and bonds, (29) is the optimality condition for money balances, (30) is the optimality condition for work hours and (31) shows the optimal demand for each variety of goods.

#### 7.1.3 Implications for price bundles

Equations (26) and (31) imply that the general price index is (see also e.g. Wickens, 2008, chapter 7):

$$P_t = \left[\sum_{h=1}^{N} \lambda [P_t(h)]^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{32}$$

### 7.2 Firms

This Appendix provides details for the firm's problem. There are $h = 1, 2, ..., N$ firms. Each firm $h$ produces a differentiated good of variety $h$ under monopolistic competition facing Calvo-type nominal fixities.hold $i$ acts competitively to maximize expected lifetime utility.

#### 7.2.1 Demand for firm's product

Each firm $h$ faces demand for its product, $y_t(h)$, coming from households' consumption and investment, $c_t(h)$ and $x_t(h)$, where $c_t(h) \equiv \sum_{i=1}^{N} c_{i,t}(h)$ and $x_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h)$, and from the government, $g_t(h)$. Thus, the demand for each firm's product is:

$$y_t(h) = c_t(h) + x_t(h) + g_t(h) \tag{33}$$

where from above:

$$c_t(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} c_t \tag{34}$$

and similarly:

$$x_t(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} x_t \tag{35}$$

$$g_t\left(h\right)=\left[\frac{P_t\left(h\right)}{P_t}\right]^{-\phi}g_t \tag{36}$$

where $c_t \equiv \sum_{i=1}^{N} c_{i,t}$, $x_t \equiv \sum_{i=1}^{N} x_{i,t}$ and $g_t$ is public spending.

Since, at the economy level:

$$y_t = c_t + x_t + g_t \tag{37}$$

the above equations imply that the demand for each firm's product is:

$$y_t\left(h\right)=c_t(h)+x_t(h)+g_t\left(h\right)=\left[\frac{P_t\left(h\right)}{P_t}\right]^{-\phi}y_t \tag{38}$$

### 7.2.2 Firm's problem

Each firm $h$ nominal profits in period $t$, $D_t(h)$, defined as:

$$D_t(h)=P_t(h)y_t(h)-P_t r_t^k k_{t-1}(h)-W_t n_t(h) \tag{39}$$

All firms use the same technology represented by the production function:

$$y_t(h)=A_t[k_{t-1}(h)]^{\alpha}[n_t(h)]^{1-\alpha} \tag{40}$$

where $A_t$ is an exogenous stochastic TFP process whose motion is defined below.

Under imperfect competition, profit maximization is subject to:

$$y_t(h)=\left[\frac{P_t(h)}{P_t}\right]^{-\phi}y_t \tag{41}$$

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price. A firm $h$, which is able to reset its price, chooses its price $P_t^{\#}\left(h\right)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed.

### 7.2.3 Firm's optimality conditions

Following the related literature, to solve the firm's problem above, we follow a two-step procedure. We first solve a cost minimization problem, where each firm $h$ minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price. Details are available upon request.

The solution to the cost minimization problem gives the input demand functions:

$$w_t = mc_t(1-a)A_t[k_{t-1}(h)]^{\alpha}[n_t(h)]^{-\alpha} \tag{42}$$

$$r_t^k = mc_t a A_t[k_{t-1}(h)]^{\alpha-1}[n_t(h)]^{1-\alpha} \tag{43}$$

where $mc_t = \Psi_t'(.)$ is the marginal nominal cost with $\Psi_t(.)$ denoting the associated minimum nominal cost function for producing $y_t\,(h)$ at $t$.

Then, the firm chooses its price, $P_t^{\#}\,(h)$, to maximize nominal profits written as:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k}\,(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^{\#}\,(h)\, y_{t+k}\,(h) - \Psi_{t+k}\,(y_{t+k}\,(h)) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm and where $y_{t+k}\,(h) = \left[\frac{P_t^{\#}(h)}{P_{t+k}}\right]^{-\phi} y_{t+k}$.

The first-order condition gives:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[\frac{P_t^{\#}\,(h)}{P_{t+k}}\right]^{-\phi} y_{t+k} \left\{ P_t^{\#}\,(h) - \frac{\phi}{\phi-1}\Psi'_{t+k} \right\} = 0 \tag{44}$$

We transform the above equation by dividing with the aggregate price index, $P_t$:

$$E_t \sum_{k=0}^{\infty} \theta^k [\Xi_{t,t+k} \left[\frac{P_t^{\#}\,(h)}{P_{t+k}}\right]^{-\phi} y_{t+k} \left\{ \frac{P_t^{\#}\,(h)}{P_t} - \frac{\phi}{\phi-1} mc_{t+k} \frac{P_{t+k}}{P_t} \right\}] = 0 \tag{45}$$

Therefore, the behaviour of each firm $h$ is summarized by the above three conditions (42), (43) and (45).

Each firm $h$ which can reset its price in period $t$ solves an identical problem, so $P_t^{\#}(h) = P_t^{\#}$ is independent of $h$, and each firm $h$ which cannot reset its price just sets its previous period price $P_t\,(h) = P_{t-1}\,(h)$. Then, it can be shown that the evolution of the aggregate price level is given by:

$$(P_t)^{1-\phi} = \theta\,(P_{t-1})^{1-\phi} + (1-\theta)\left(P_t^{\#}\right)^{1-\phi} \tag{46}$$

### 7.3 Decentralized equilibrium (given policy)

We now combine the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. In this DE, (i) all households maximize utility (ii) a fraction $(1-\theta)$ of firms maximize profits by choosing the identical price $P_t^{\#}$, while the rest, $\theta$, set their previous period prices (iii) all constraints are satisfied and (iv) all markets clear (details are available upon request).

The DE can be summarized by the following equilibrium conditions (quantities are in per capita terms):

$$\frac{c_t^{-\sigma}}{(1+\tau_t^c)} = \beta E_t \frac{c_{t+1}{}^{-\sigma}}{(1+\tau_{t+1}^c)}\left[\left(1-\tau_{t+1}^k\right)r_{t+1}^k + (1-\delta)\right] \tag{47}$$

$$c_t^{-\sigma}\frac{1}{(1+\tau_t^c)} = \beta E_t R_t \frac{c_{t+1}^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_t}{P_{t+1}} \tag{48}$$

$$\chi_m m_t^{-\mu} - \frac{c_t^{-\sigma}}{(1+\tau_t^c)} + \beta E_t \frac{c_{t+1}^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_t}{P_{t+1}} = 0 \tag{49}$$

$$\chi_n \frac{n_t^{\eta}}{c_t^{-\sigma}} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t \tag{50}$$

$$k_t = (1-\delta)\, k_{t-1} + x_t \tag{51}$$

$$E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Xi_{t,t+k}\left[\frac{P_t^{\#}}{P_{t+k}}\right]^{-\phi} y_{t+k}\left(\frac{P_t^{\#}}{P_t} - \frac{\phi}{\phi-1} mc_{t+k}\frac{P_{t+k}}{P_t}\right)\right\} = 0 \tag{52}$$

$$w_t = mc_t(1-a)\frac{y_t}{n_t} \tag{53}$$

$$r_t^k = mc_t a \frac{y_t}{k_t} \tag{54}$$

$$d_t = y_t - w_t n_t - r_t^k k_{t-1} \tag{55}$$

$$y_t = \frac{1}{\left(\frac{\tilde{P}_t}{P_t}\right)^{-\phi}} A_t k_{t-1}^a n_t^{1-a} \tag{56}$$

$$b_t + m_t = R_{t-1} b_{t-1}\frac{P_{t-1}}{P_t} + m_{t-1}\frac{P_{t-1}}{P_t} + g_t - \tau_t^c c_t - \tau_t^n w_t n_t - \tau_t^k\left(r_t^k k_{t-1} + d_t\right) - \tau_t^l \tag{57}$$

$$y_t = c_t + x_t + g_t \tag{58}$$

$$(P_t)^{1-\phi} = \theta(P_{t-1})^{1-\phi} + (1-\theta)\left(P_t^{\#}\right)^{1-\phi} \tag{59}$$

$$(\widetilde{P}_t)^{-\phi} = \theta(\widetilde{P}_{t-1})^{-\phi} + (1-\theta)\left(P_t^{\#}\right)^{-\phi} \tag{60}$$

where $\Xi_{t,t+k} \equiv \beta^k \frac{c_{t+k}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+k}} \frac{\tau_t^c}{\tau_{t+k}^c}$ and $\widetilde{P}_t^H \equiv \left(\sum_{h=1}^{N} \left[P_t\left(h\right)\right]^{-\phi}\right)^{-\frac{1}{\phi}}$. Thus, $\left(\frac{\widetilde{P}_t^H}{P_t^H}\right)^{-\phi}$ is a measure of price dispersion.

We thus have 14 equilibrium conditions for the DE. To solve the model, we need to specify the policy regime and thus classify policy instruments into endogenous and exogenous. Regarding the conduct of monetary policy, we assume that the nominal interest rate, $R_t$, is used as a policy instrument, while, regarding fiscal policy, we assume that the residually determined public financing policy instrument is the end-of-period public debt, $b_t$. Then, the 14 endogenous variables are $\{y_t,\ c_t,\ n_t,\ x_t,\ k_t,\ m_t,\ b_t,\ P_t,\ P_t^{\#},\ \widetilde{P}_t,\ w_t,\ mc_t,\ d_t,\ r_t^k\}_{t=0}^{\infty}$. This is given the independently set policy instruments, $\{R_t,\ \tau_t^c,\ \tau_t^k,\ \tau_t^n,\ g_t,\ \tau_t^l\}_{t=0}^{\infty}$, technology, $\{A_t\}_{t=0}^{\infty}$, and initial conditions for the state variables.

## 7.4 Decentralized equilibrium transformed (given policy)

Before we specify the motion of independently set policy instruments and exogenous stochastic variables, we rewrite the above equilibrium conditions, first, by using inflation rates rather than price levels, second, by writing the firm's optimality condition (35) in recursive form and, third, by introducing a new equation that helps us to compute expected discounted lifetime utility. Details for each step are available upon request.

### 7.4.1 Variables expressed in ratios

We define three new endogenous variables, which are the gross inflation rate $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, the auxiliary variable $\Theta_t \equiv \frac{P_t^{\#}}{P_t}$, and the price dispersion index $\Delta_t \equiv \left[\frac{\widetilde{P}_t}{P_t}\right]^{-\phi}$. We also find it convenient to express the two exogenous fiscal spending policy instruments as ratios of GDP, $s_t^g \equiv \frac{g_t}{y_t}$ and $s_t^l \equiv \frac{\tau_t^l}{y_t}$.

Thus, from now on, we use $\Pi_t$, $\Theta_t$, $\Delta_t$, $s_t^g$, $s_t^l$ instead of $P_t$, $P_t^{\#}$, $\widetilde{P}_t$, $g_t$, $\tau_t^l$ respectively.

### 7.4.2 Equation (52) expressed in recursive form

Following Schmitt-Grohé and Uribe (2007), we look for a recursive representation of (52):

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^{\#}}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ \frac{P_t^{\#}}{P_t} - \frac{\phi}{(\phi-1)} mc_{t+k} \frac{P_{t+k}}{P_t} \right\} = 0 \tag{61}$$

We define two auxiliary endogenous variables:

$$z_t^1 \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^{\#}}{P_{t+k}} \right]^{-\phi} y_{t+k} \frac{P_t^{\#}}{P_t} \tag{62}$$

$$z_t^2 \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^{\#}}{P_{t+k}} \right]^{-\phi} y_{t+k} mc_{t+k} \frac{P_{t+k}}{P_t} \tag{63}$$

Using these two auxiliary variables, $z_t^1$ and $z_t^2$, we come up with two new equations which enter the dynamic system and allow a recursive representation of (61). In particular, we can replace equilibrium equation (52) with:

$$z_t^1 = \frac{\phi}{(\phi-1)} z_t^2 \tag{64}$$

where:

$$z_t^1 = \Theta_t^{-\phi-1} y_t + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi-1} \left( \frac{1}{\Pi_{t+1}} \right)^{-\phi} z_{t+1}^1 \tag{65}$$

$$z_t^2 = \Theta_t^{-\phi} y_t mc_t + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi} \left( \frac{1}{\Pi_{t+1}} \right)^{1-\phi} z_{t+1}^2 \tag{66}$$

Thus, from now on, we use (64), (65) and (66) instead of (52).

#### 7.4.3 Lifetime utility written as a first-order dynamic equation

To compute social welfare, we follow Schmitt-Grohé and Uribe (2007) by defining a new endogenous variable, $V_t$, whose motion is:

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\phi}}{1+\phi} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} + \chi_g \frac{(s_t^g y_t)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \tag{67}$$

where $V_t$ is the expected discounted lifetime utility of the household at any $t$.

Thus, from now on, we add equation (67) and the new variable $V_t$ to the equilibrium system.

#### 7.4.4 Equations of transformed DE

Using all the above, the final non-linear stochastic equilbrium system is**:**

$$\frac{c_t{}^{-\sigma}}{(1+\tau_t^c)} = \beta E_t \frac{c_{t+1}{}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \left[\left(1-\tau_{t+1}^k\right) r_{t+1}^k + (1-\delta)\right] \tag{68}$$

$$\frac{c_t{}^{-\sigma}}{R_t} \frac{1}{(1+\tau_t^c)} = \beta E_t \frac{c_{t+1}{}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \frac{1}{\Pi_{t+1}} \tag{69}$$

$$\chi_m m_t^{-\mu} - \frac{c_t^{-\sigma}}{(1+\tau_t^c)} + \beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \frac{1}{\Pi_{t+1}} = 0 \tag{70}$$

$$\chi_n \frac{n_t^\eta}{c_t^{-\sigma}} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t \tag{71}$$

$$k_t = (1-\delta)\, k_{t-1} + x_t \tag{72}$$

$$z_t^1 = \frac{\phi - 1}{\phi} z_t^2 \tag{73}$$

$$w_t = mc_t(1-a)\frac{y_t}{n_t} \tag{74}$$

$$r_t^k = mc_t a \frac{y_t}{k_{t-1}} \tag{75}$$

$$d_t = y_t - w_t n_t - r_t^k k_{t-1} \tag{76}$$

$$y_t = \frac{1}{\Delta_t} A_t k_{t-1}^a n_t^{1-a} \tag{77}$$

$$b_t + m_t = R_{t-1} b_{t-1} \frac{1}{\Pi_t} + m_{t-1} \frac{1}{\Pi_t} + s_t^g y_t - \tau_t^c c_t - \tau_t^n w_t n_t - \tau_t^k \left[r_t^k k_{t-1} + d_t\right] - \tau^l \tag{78}$$

$$y_t = c_t + x_t + s_t^g y_t \tag{79}$$

$$\Pi_t^{1-\phi} = \theta + (1-\theta)\left[\Theta_t \Pi_t\right]^{1-\phi} \tag{80}$$

$$\Delta_t = (1-\theta)\, \Theta_t^{-\phi} + \theta \Pi_t^\phi \Delta_{t-1} \tag{81}$$

$$z_t^1 = y_t mc_t \Theta_t^{-\phi-1} + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi-1} \Pi_{t+1}^\phi z_{t+1}^1 \tag{82}$$

$$z_t^2 = \Theta_t^{-\phi} y_t + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi} \Pi_{t+1}^{\phi-1} z_{t+1}^2 \tag{83}$$

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} - \chi_n \frac{n_t^{1+\phi}}{1+\phi} + \chi_g \frac{(s_t^g y_t)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \tag{84}$$

There are 17 equations in 17 endogenous variables, $\{y_t, c_t, n_t, x_t, k_t, m_t, b_t, \Pi_t, \Theta_t, \Delta_t, w_t, mc_t, d_t, r_t^k, z_t^1, z_t^2, V_t\}_{t=0}^{\infty}$. This is given the independently set policy instruments, $\{R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n\}_{t=0}^{\infty}$, technology, $\{A_t\}_{t=0}^{\infty}$, and initial conditions for the state variables.