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Liberalization of product and labour markets: Winners and losers

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Liberalization of product and labour markets:

Winners and losers

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Abstract

This paper studies the implications of higher flexibility, or liberalization, in product and labour markets. We study both efficiency and distribution implications. The vehicle is a dynamic general equilibrium (DGE) model that incorporates heterogeneous agents (entrepreneurs and workers) and imperfectly competitive product and labour markets. The combination of market power in the product market with agents' heterogeneity influences the political feasibility of structural reforms. The model is solved numerically employing parameter values and fiscal data from the Euro Area. Regarding efficiency, our solutions show that per capita output, as well as per capita welfare, rises when we liberalize one or both markets. Moreover, the net income of both agents rises as we move to a more competitive economy, either in one or both markets. Thus, any form of liberalization, even incomplete, is Pareto superior. The crucial question is then who might gain more from this. Regarding distribution, inequality rises when liberalization of the labour market only takes place, while it falls when liberalization of the product market only takes place or when both markets become more flexible.

Keywords: structural reforms, labour market, bargaining …

JEL classifications:

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1 Introduction

The global financial crisis, which hit most countries in 2008, caused a severe and widespread economic downturn. Debt imbalances and excessive unemployment rates plagued economies in the periphery of Europe (Eichengreen, 2010 and Eggertsson *et al*,. 2014). There is also the belief that market imperfections are equally contributing for the current problems (Blanchard and Giavazzi, 2003 and Blanchard, 2004). As a result, Cyprus, Greece, Italy, Portugal and Spain have been repeatedly urged to undertake structural product and labour market reforms. Due to the existing structure in these markets, the aggregate and distributional effects of the attempted reforms have been rather complex and unpredictable (Domeij and Heathcote, 2004 and Economides *et al*., 2012). However, in the periphery the progress on implementing the reforms has been slow and these countries are still struggling both in terms of the real economy and public finances. This has generated questions about the existence and future of the Euro Area (EA). In light of the fact that the suggested policy reforms are mainly initiated from policy-makers in the core of EA, with relatively flexible economies, it is vital to provide an insight on the responsiveness of more rigid economic environments to the attempted reforms.

In particular important policy concerns arise: how efficient these reforms are in terms of output and employment for the periphery countries? Which are the distributional implications for the social groups? Do all agents profit equally or there are winners and losers? Does the public financing instrument accommodating the budget affect the efficiency of the attempted reforms? Are there different ways of public financing if a government prioritises the benefits for one social group vis a vis another?[1]

This paper provides a framework for evaluating structural reforms in the direction of a Pareto superior allocation of reforms relative to the status quo economy. The vehicle is a rather standard DGE model that incorporates heterogeneous agents (entrepreneurs and workers) and imperfectly competitive product and labour markets. The combination of the above imperfections generates a non-trivial conflict of interest that needs to be taken into account when one wishes to evaluate the political feasibility of reforms, even if these reforms are good for the economy, in general.

Starting from the status quo economy, market power in the product market arises because firms produce differentiated products under monopolistic competition; while, market power in the labour market arises because unionized workers can bargain over wages subject to demand for labour. Alesina and Giavazzi (2006) have argued that these two types of market imperfections play a key role in explaining the stagnation of European economies relative to the US. And, there is empirical evidence that lack of flexibility in product markets is closely related to the degree of wage bargaining in labour markets (Nicoletti and Scarpetta, 2005).

We solve our model numerically, employing parameter values commonly used in the related literature (Ardagna, 2007 and Economides *et al*., 2012) and fiscal data from EA over the recent years. This enable us to asses quantitatively the implications of structural reforms in product and/or labour markets in terms of efficiency and inequality both in the short as well as in the long run. Finally, the significance of a different instrument of public financing in implementing these structural reforms is explored.

Our main results are as follows. First, and not surprisingly we find that stronger competition in the product market, coupled with higher flexibility in the labour market, leads to the highest long-term social gains both in terms of per capita output and per capita welfare.[2] In addition, if one compares deregulation in the product market only to deregulation in the labour market only, the results are mainly driven by rent creation in the product market (Drazen, 2002).

Since there are two different groups of households in the society – workers and entrepreneurs – these income/welfare gains from each particular structural reform may be distributed unequally for each group in society. Thus we now turn to individual outcomes or, equivalently, to distribution. Since any reform entails benefits for both groups in society, we show that the benefits to one group are greater when we reduce the efficient losses generated by the market power of the other. Needless to say, that our results show that both social groups earn the maximum under full liberalization. A key question now is who gains more. Even if a policy reform produces a win-win outcome, relative outcomes can be also important. The political economics literature has pointed out several reasons for this, including political ideology, habit, envy, etc. Departing from the status quo, inequality falls, when there is an initiative towards the liberalization of the product markets. Thus, as one would expect, workers benefit more from product market liberalization than entrepreneurs. However, we often observe workers opposing to product market reforms (see Blanchard, 2004). This brings up another hot issue: which market's liberalization should be

[1] By public financing we mean the policy instruments that adjust to accommodate the exogenous changes in imperfections of product and labour markets.

given priority? In a more competitive product market workers may lose as rent extractors, but they gain significantly through increased investment, employment opportunities and finally output.[3] This is in accordance with IMF (2005) report arguing that after product market liberalization, opposition on labour market reforms would decline and prepare the ground for further reforms. This is also consistent with empirical evidence that either product market liberalization does not deteriorate income inequality (Nicoletti *et al.,* 2001) or even more the Gini coefficient falls (Cardullo, 2009).[4] However, inequality increases when we focus on liberalization of the labour market only. Workers cannot act as rent extractors any more due to the absence of bargaining power and their benefits are lower than those of entrepreneurs whose pie of rents is now increasing. This is an issue which has not been so far analyzed much and well quantified although has been acknowledged in recent IMF (2009) and World Bank (2013) reports "… labour market deregulation generally has an insignificant impact on growth but in most cases it increases income inequality…", This is in line with our findings that in case of reforms a product market deregulation should be a top priority.

It is worth investigating now, how the implications of exogenous structural reforms, like the above, depend on the public financing policy instrument used namely, which fiscal policy instrument takes advantage of the switch to a more efficient economy. Since each structural reform can be accompanied by a different policy instrument adjusting to ensure a balanced budget for the government, there is a secondary round of efficiency gains that likewise may be distributed unequally to workers and entrepreneurs. It is true that, as we move from the status quo to a more efficient economy, increased output and employment opportunities provide a larger tax base and thus, an extra fiscal space is created. The question is now, which category of public spending to increase or which type of taxes to cut. We examine what happens when, in this fully liberalized economy, the government also allows either social transfers, or public investment, or the capital tax rate or the consumption tax rate to take advantage of the fiscal space created. As one would expect, it is better to allow capital taxes to take advantage of the fiscal space. This cut engineers a second round of efficiency gains and a further enhancement of aggregate per capita output.[5] This is consistent with the well-known result that capital taxes are particularly distorting in the medium and the long run and thus enhancing aggregate efficiency (Chamley, 1986). The second best instrument is public spending which has a significant impact on output. Therefore, the best way of using the fiscal space generated by a switch to a more deregulated economy is to cut capital tax rate or to increase public investment spending. These are the policy instruments with relatively high multiplier. This confirms the importance of policy mixes in the conduct of economic policy (see Wren-Lewis, 2010). Regarding distributional effects, workers would prefer the case in which it is government transfers that take advantage of the fiscal space, while the interest of entrepreneurs coincides with the aggregate interest and they would prefer the capital tax rate to be cut. In other words, in the long run, a smaller public sector, meaning lower capital taxes combined with lower public spending, benefit the aggregate economy and the entrepreneurs, but it worsens the relative position of workers in the income ladder. This is in line with the often voiced opposition on behalf of trade unions regarding reduction in capital taxes, since income distribution is prime goal in their agenda (ITUC, 2012). Transition dynamics are not qualitatively different from the long run ones

[2] This is something generally acknowledged by the related literature. Higher aggregate efficiency in the case of deregulation in the product and labour market may arise through the classical advocacy that a more competitive economy leads to a better allocation of recourses.

[3] We have a lot of evidence that product market liberalization boosts long-run employment levels, wages and output (OECD, 2001 and Scarpetta, 2005).

[4] In particular, the inequality index in the status quo economy is not improved when the reform takes place only in the labour market.

[5] Using consumption tax rate or transfer payments we have a positive impact on output however, this is not as significant as in the case of capital tax and public investment (keeping in mind that this is a long run result only).

and it is worth mentioning here that in the first eight to nine years half of the benefits have been achieved.

The rest of the paper is organised as follows: section 2 presents the theoretical model, section 3 includes baseline parameterization, section 4 gives the long run solution and finally, the last section concludes the results. An Appendix includes technical details.

2 Model

We construct and solve a model economy with imperfect competition in both labour and product markets. The economy consists of households, firms, trade unions, and a government. There are two types of households: entrepreneurs and workers. Entrepreneurs can work and save in the form of physical capital and government bonds. They supply their one unit of labour services inelastically. They also own the firms and receive their profits. Workers, due to implicit prohibitive transactions costs, do not participate in financial markets and thus consume all their disposable income in each period. If unemployed, they receive unemployment benefits from the government. All workers (employed and unemployed) are represented by firm-level trade unions which bargain with firms over the wage rate with the aim of maximising the average labour income of their members (right-to-manage union model). On the production side, there are both final and intermediate good firms. Final good firms act competitively. In contrast, intermediate good producers have monopoly power in their own product market. Finally, the government issues new bonds and taxes consumption, labour income and interest income from physical capital and profits to finance its spending. The latter includes a uniform lump-sum transfer that increase households' income, public investment that augments public infrastructure, public consumption that provides direct utility to households and unemployment benefits. The time horizon is infinite and the time is discrete. For simplicity, there is no uncertainty.

2.1 Population

Total population, N, is exogenous and constant over time with entrepreneurs and workers, respectively, being denoted as N^k and N^w. We also define the population share of entrepreneurs, $N^k / N \equiv n^k$, and workers, $n^w = 1 - n^k$. We assume that each entrepreneur owns one of the N^i intermediate good firms; thus, the number of those firms equals the number of entrepreneurs, $N^i = N^k$.

2.2 Households

As said, there are two types of households, called capitalists and workers. Workers face unemployment and are represented by trade unions. Unions guarantee that their members have equal employment and wages.[6] Each worker is randomly allocated to a union which bargains with a firm to determine the wage rate.[7]

Moreover, households have unequal access to the financial markets. This can be motivated by imperfections in asset markets that require agents to pay transactions costs to participate.[8] Therefore we distinguish among entrepreneurs and workers and assume that they face, respectively, the minimum i.e. zero and maximum i.e. infinity participation costs in the financial markets. We focus on heterogeneity that is driven by differences in asset ownership and hence we will work with a symmetric equilibrium in the labour market.

The discounted sum of lifetime utility of each household $j = k, w$ is:

$$\sum_{t=0}^{\infty} \beta^t \; u\left(C_t^j + \psi \bar{G}_t^c\right) \tag{1}$$

where, $\beta \in (0,1)$ is the time discount factor, C_t^j is household j's private consumption at time t, and $\bar{G}_t^c$ is the average (per household) public consumption goods and services provided by the government at time t. Thus, public consumption influences the private utility through the value of the parameter $\psi \in [-1,1]$.[9]

The instantaneous utility function is assumed to be of the form:

$$u\left(C_t^j + \psi \bar{G}_t^c\right) = \frac{\left(C_t^j + \psi \bar{G}_t^c\right)^{1-\sigma}}{1-\sigma} \tag{2}$$

where, $\sigma > 1$ is the constant coefficient of relative risk aversion.

2.2.1 Entrepreneurs

[6] Under unionised labour markets, a common assumption in the literature is that the unions insure their members against potential idiosyncratic employment risk (see e.g. Maffezzoli, 2001 and Blanchard and Giavazzi, 2003). Effectively, unions act as a substitute for a competitive insurance market and issue actuarially fair insurance to their members.

[7] Given that employment and the wage rate will be the same for all households, the allocation of households to unions does not matter.

[8] These participation premia differ between the agents due to, for instance, past experience, socioeconomic background, networks, or firm ownership that gives an insider advantage in financial transactions (Ardagna, 2007).

[9] See also Chriastiano and Eichenbaum (1992).

A representative entrepreneur can save in the form of physical capital, I_t^k , and government bonds, D_t^k and receives gross income from working, w_t^k , capital income, $r_t^k K_t^k$, and interest income from government bonds, $r_t^b B_t^k$, r_t^k is the gross return to physical capital, K_t^k , and r_t^b is the gross return to government bonds, B_t^k .[10, 11] Two additional sources of income are the profits of an intermediate good firm that are distributed in the form of dividends, π_t^k , and average (per household) net lump-sum government transfers, $\bar{G}_t^t$. Thus, the budget constraint of each entrepreneur at time t is given by

$$\begin{aligned}&(1+\tau_t^c)C_t^k + I_t^k + D_t^k = \\ &= \left(1-\tau_t^l\right) w_t^k + r_t^k K_t^k - \tau_t^k \left(r_t^k - \delta^p\right) K_t^k + r_t^b B_t^k + \left(1-\tau_t^k\right)\pi_t^k + \bar{G}_t^t\end{aligned} \tag{3}$$

where, $0 \le \tau_t^c < 1$ is the tax rate on consumption, $0 \le \tau_t^l < 1$ is the tax rate on labour income, $0 \le \tau_t^k < 1$ is the tax rate on income from capital earnings and dividends, and $\delta^p \in (0,1)$ is the constant depreciation rate of private capital stock.[12, 13]

The law of motion of private capital and government bonds are:

$$K_{t+1}^k = (1-\delta^p)K_t^k + I_t^k,\ K_0^k > 0 \text{ given} \tag{4}$$

$$B_{t+1}^k = B_t^k + D_t^k,\ B_0^k > 0 \text{ given} \tag{5}$$

Therefore, the entrepreneur's problem is to choose $\left\{C_t^k, K_{t+1}^k, B_{t+1}^k\right\}_{t=0}^{\infty}$ to maximize (1) and (2) subject to the budget constraint, Eq. (3), and the law of motion of capital and bonds, Eqs. (4) and (5), taking market prices $\left\{r_t^b, r_t^k, w_t^k\right\}_{t=0}^{\infty}$, profits $\left\{\pi_t^k\right\}_{t=0}^{\infty}$, policy variables $\left\{\tau_t^c, \tau_t^k, \tau_t^l, \bar{G}_t^t\right\}_{t=0}^{\infty}$, and initial condition for K_0^k and B_0^k as given. The first order conditions include the constraints (3)-(5) and:

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[10] Capitalists do not face unemployment and supply one unit of labour services inelastically. They supply different labour services and hence receive a different wage rate from workers, $w_t^k \neq w_t^w$.

[11] For simplicity, we do not explicitly include taxes on unemployment benefits (see Ardagna, 2007).

[12] We assume capital taxes net of depreciation as Angelopoulos *et al.*, (2013) and that the fiscal authority cannot impose a separate tax rate on profits and on interest income from private capital, since it is difficult, in practice, to distinguish these two sources of capital income (see, Guo and Lansing, 1999). Also, we assume that returns on government bonds are not taxed.

[13] The entrepreneur pays taxes on consumption and on income from working and capital earnings.

$$\frac{1}{(1+\tau_t^c)}\frac{\partial u_t(.)}{\partial C_t^k}=\beta\left[\frac{1}{(1+\tau_{t+1}^c)}\frac{\partial u_{t+1}(.)}{\partial C_{t+1}^k}\left(\left(1-\tau_{t+1}^k\right)\left(r_{t+1}^k-\delta^p\right)+1\right)\right] \tag{6a}$$

$$\frac{1}{(1+\tau_t^c)}\frac{\partial u_t(.)}{\partial C_t^k}=\beta\left[\frac{1}{(1+\tau_{t+1}^c)}\frac{\partial u_{t+1}(.)}{\partial C_{t+1}^k}\left(1+r_{t+1}^b\right)\right] \tag{6b}$$

where, (6a) and (6b) are the Euler equations for K_{t+1}^k and B_{t+1}^k respectively. The optimality conditions are completed with the transversality conditions for the two assets, namely $\lim_{t\to\infty}\beta^t\frac{\partial u_t(.)}{\partial C_t^k}K_{t+1}^k=0$ and $\lim_{t\to\infty}\beta^t\frac{\partial u_t(.)}{\partial C_t^k}B_{t+1}^k=0$.

2.2.2 Workers

Since workers are excluded from financial markets, their within period budget constraint is simply:

$$\left(1+\tau_t^c\right)C_t^w=\left(1-\tau_t^l\right)w_t^w e_t^w+\bar{G}_t^u\left(1-e_t^w\right)+\bar{G}_t^t \tag{7}$$

Workers do not save and given that their work hours also depend on the outcome of the firm-union bargaining, consumption simply follows residually from the budget constraint in (7).

2.3 Firms

The production environment consists of two sectors: intermediate goods and final good. We follow e.g. Guo and Lansing, 1999, in allowing for monopolistic power in the intermediate goods market. Hence, these producers can earn positive economic profits even though the final good sector of the economy is perfectly competitive.

2.3.1 Final good producers

A unique final good, Y_t, is produced according to the following constant returns to scale technology:

$$Y_t = \left[\sum_{i=1}^{N^i} \lambda^i \left(Y_t^i \right)^{\theta} \right]^{\frac{1}{\theta}} \tag{8}$$

where, $\sum_{i=1}^{N^i} \lambda^i = 1$ are weights attached to intermediate good producers, i, and $\theta \in (0,1]$ implies the degree of monopoly power of intermediate good producers.[14] Final good producers behave competitively and choose intermediate inputs, Y_t^i, to maximize profits, Π_t, taking the relative prices of these inputs, P_t^i, as given:

$$\Pi_t = Y_t - \sum_{i=1}^{N^i} \lambda^i P_t^i Y_t^i \tag{9}$$

The first-order condition for this problem yields:

$$P_t^i = \left(\frac{Y_t}{Y_t^i} \right)^{1-\theta} \tag{10}$$

The above expression represents the demand function for the Y_t^i intermediate good.

2.3.2 Intermediate good producers

[14] When $\theta = 1$, intermediate goods are perfect substitutes in the production of the final goods implying that intermediate good producers have no power in the product market. In this case, prices are given for these producers and thus there is perfect competition.

Each intermediate firm produces a homogeneous product, Y_t^i, by choosing three private inputs, capital, K_t^i, and labour services from workers, $L_t^{i,w}$, and capitalists $L_t^{i,k}$ and by using average (per firm) public capital, $\frac{K_t^g}{N^i}$. Its production function is:[15]

$$Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1}\left(L_t^{i,w}\right)^{\alpha_2}\left(L_t^{i,k}\right)^{\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\alpha_4} \tag{11}$$

where, A is total productivity and $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in (0,1)$ denote the output elasticity of private capital, workers' labour services, capitalists' labour services and public capital, respectively. We assume constant returns to all three inputs and specifically $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$.

The profit earned by the intermediate good producer at time t is:

$$\pi_t^i = P_t^i Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \tag{12}$$

Taking factor prices, r_t^k, w_t^w and w_t^k, final output, Y_t, and average public capital, $\frac{K_t^g}{N^i}$, as given, the intermediate good firm chooses K_t^i, $L_t^{i,w}$ and $L_t^{i,k}$ to maximize profits, Eq. (12), subject to its production function, Eq. (11), and the demand function for its output, Eq. (10).

The first order conditions are:

$$\theta\alpha_1 \frac{\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}}{K_t^i} = r_t^k \tag{13a}$$

$$\theta\alpha_2 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^{\theta}}{L_t^{i,w}} = w_t^w \tag{13b}$$

[15] We include public investment, and hence public capital, because we wish to have as many fiscal policy instruments as possible and to be close to the data. See e.g. Lansing (1998), for a similar production function.

$$\theta\alpha_3 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^{\theta}}{L_t^{i,k}} = w_t^k \tag{13c}$$

the above conditions equate factor returns to marginal products. In turn, the profit of each intermediate good firm is:

$$\pi_t^i = \left(1-\theta\alpha_1-\theta\alpha_2-\theta\alpha_3\right)\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} \tag{14}$$

2.4 Trade Unions

We employ a standard right-to-manage setup where unions (represent workers only) and firms (intermediate good producers) bargain over the wage rate. For simplicity, we assume that each union bargains with one firm to determine the wage rate (Pissarides, 1998). Moreover, for tractability, and following e.g. Domeij, 2005 and Koskela and von Thadden, 2008, we make one simplifying assumption regarding this bargaining process. We assume that unions and firms do not internalize the effects of the bargained wage rate on capital accumulation and thus on future prices.[16]

The union and the intermediate good producer bargain over the wage rate to maximize a weighted average of workers' labour income and profits:

$$U_t^N = \left[\left(1-\tau_t^l\right)w_t^w \frac{n^k}{n^w} L_t^{i,w} + \bar{G}_t^u\left(1-\frac{n^k}{n^w}L_t^{i,w}\right) - \bar{G}_t^u\right]^{\phi}\left[\pi_t^i + r_t^k K_t^i + w_t^k L_t^{i,k}\right]^{1-\phi} \tag{15}$$

subject to the workers' labour demand function Eq. (13b) and the intermediate firm's product demand function, Eq. (10), taking the capital stock, K_t^i, entrepreneurs' labour services, $L_t^{i,k}$, final output, Y_t, and the fiscal policy variables, $\left\{\tau_t^c, \tau_t^k, \tau_t^l, \bar{G}_t^u, \bar{G}_t^t\right\}$, as given.

In the above setup, $\frac{n^k}{n^w}L_t^{i,w} \equiv e_t^w$ is the average employment rate, so that $\left(1-\frac{n^k}{n^w}L_t^{i,w}\right)$ is the unemployment rate and $\phi \in [0,1]$ describes the relative bargaining power of the union, with $\phi = 1$ representing the monopoly union case. The outside option for the union is the unemployment benefit, $\bar{G}_t^u$,

[16] The above assumption implies that unions and firms take capital as given when bargaining over the wage rate. This form of myopia allows for a technical simplification in that it effectively reduces the wage-bargaining problem to a series of static problems, as in e.g. Pissarides (1998).

while for the firm it is the sunk cost of capital, $-r_t^k K_t^i$, which is a consequence of the assumption that the representative firm takes the average capital accumulation and the capitalists' labour services as given.[17]

The first order condition is:

$$\left(1-\tau_t^l\right)\theta\alpha_2\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}=\frac{\left[\phi+(1-\phi)\theta\alpha_2\right]}{\theta\alpha_2}\bar{G}_t^u L_t^{i,w} \tag{16}$$

2.5 Government

The government issues new bonds, B_{t+1}, and taxes consumption, labour income and interest income from physical capital and profits, at the rates $0\leq\tau_t^c<1$, $0\leq\tau_t^l<1$ and $0\leq\tau_t^k<1$, respectively, to finance total unemployment benefits, $N^w\bar{G}_t^u\left(1-e_t^w\right)$, total lump-sum transfers, $N\bar{G}_t^t$, total public investment, $N\bar{G}_t^i$ (where we define $\bar{G}_t^i$ as the per capita public investment), and total public consumption, $N\bar{G}_t^c$.

The government budget constraint (GBC) is:

$$\begin{aligned}&N\bar{G}_t^c+N\bar{G}_t^t+N\bar{G}_t^i+N^w\bar{G}_t^u\left(1-e_t^w\right)+\left(1+r_t^b\right)B_t=\\&=N^k\tau_t^k\left(r_t^k-\delta^p\right)K_t^k+N^k\tau_t^k\pi_t^k+N^w\tau_t^l w_t^w e_t^w+N^k\tau_t^l w_t^k+N^w\tau_t^c C_t^w+N^k\tau_t^c C_t^k+B_{t+1}\end{aligned} \tag{17}$$

Public investment spending is used to augment public capital used by firms. If we define the per capita public capital as $\bar{k}_t^g\equiv\frac{K_t^g}{N}$, its law of motion is:

$$\bar{k}_{t+1}^g=(1-\delta^g)\bar{k}_t^g+\bar{G}_t^i \tag{18}$$

where, $\delta^g\in\left(0,1\right)$ is the depreciation rate of public capital.

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[17] $\pi_t^i=P_t^iY_t^i-r_t^kK_t^i-w_t^wL_t^{i,w}-w_t^kL_t^{i,k}\xrightarrow{Eq.(11)}\pi_t^i=P_t^i\mathrm{A}\left(K_t^i\right)^{\alpha_1}\left(L_t^{i,w}\right)^{\alpha_2}\left(L_t^{i,k}\right)^{\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\alpha_4}-r_t^kK_t^i-w_t^wL_t^{i,w}-w_t^kL_t^{i,k}$

if $L_t^{i,w}=0$, then $\left.\pi_t^i\right|_{L_t^{i,w}=0}=-r_t^kK_t^i-w_t^kL_t^{i,k}$ is the outside option of the firm.

If we divide the aggregate GBC, Eq. (17), by total population number, N, we have the per capita GBC:

$$\begin{aligned} &\bar{G}_t^c + \bar{G}_t^t + \bar{G}_t^i + n^w \bar{G}_t^u \left(1 - e_t^w\right) + \left(1 + r_t^b\right) \frac{B_t}{N} = \\ &= n^k \tau_t^k \left(r_t^k - \delta^p\right) K_t^k + n^k \tau_t^k \pi_t^k + n^w \tau_t^l w_t^w e_t^w + n^k \tau_t^l w_t^k + n^w \tau_t^c C_t^w + n^k \tau_t^c C_t^k + \frac{B_{t+1}}{N} \end{aligned} \tag{19}$$

Thus, in each period, there are eight policy instruments $\left(\tau_t^c, \tau_t^k, \tau_t^l, \bar{G}_t^u, \bar{G}_t^t, \bar{G}_t^c, \bar{G}_t^i, B_{t+1}\right)$ out of which only seven can be set independently, with the eighth following residually to satisfy the government budget constraint. Following most of the related literature, we assume that, along the transition path, the adjusting instrument is the end-of-period public debt, B_{t+1}, so that the rest can be set exogenously by the government. At the steady state, we instead set the debt to output ratio as in the data and allow the government transfers to be the residually determined instrument.[18]

For convenience, concerning spending policy instruments, we work in terms of their GDP shares, namely,

$$s_t^c = \frac{\bar{G}_t^c}{n^k Y_t^i},$$

$$s_t^i = \frac{\bar{G}_t^i}{n^k Y_t^i},$$

$$s_t^u = \frac{n^w \bar{G}_t^u (1 - e_t^w)}{n^k Y_t^i}$$

and $s_t^t = \dfrac{\bar{G}_t^t}{n^k Y_t^i}$.

2.6 Decentralized equilibrium (DE) of the status quo economy

[18] See also Mendoza and Tesar, 2005.

We solve for a symmetric decentralized equilibrium (DE) where $Y_t^i = Y_t$, and $P_t^i = 1$ for all *i*. Given the exogenously set policy instruments $\left\{\tau_t^l, \tau_t^c, \tau_t^k, s_t^u, s_t^t, s_t^c, s_t^i\right\}_{t=0}^{\infty}$, and initial conditions for the state variables, K_0^k and B_0^k, a decentralized equilibrium is defined to be an allocation $\left\{Y_t^i, C_t^k, K_{t+1}^k, C_t^w, e_t^w, \pi_t^k, \overline{k}_{t+1}^g, r_t^b, r_t^k, w_t^w, w_t^k, B_{t+1}^k\right\}_{t=0}^{\infty}$ such that (i) households, firms and unions undertake their respective optimization problems outlined above; (ii) all budget constraints are satisfied; and (iii) all markets clear, where in the labour market any deviation from full employment ($e_t^w = 1$) is voluntary. This equilibrium is for any feasible policy. The DE equilibrium is presented in Appendix 1.

3 Baseline parameterization and status-quo solution

3.1 Parameterization

Table 1 reports the baseline parameter values for technology and preference, as well as the values of exogenous policy variables, used to solve the above model economy. The time unit is meant to be a year. Regarding parameters for technology and preference, we use relatively standard values used by the business cycle literature. Public spending and tax rate values are those of data averages of the Eurozone economy over 1990-2008. The data are obtained from OECD, Economic Outlook, no. 90.

Table 1 around here

Baseline parameterization

Let us discuss, briefly, the values summarized in Table 1. The workers' and capitalists' labour share in the production function of the intermediate firm, α_2 and α_3, is set at 0.45 and 0.20, respectively. The public capital share, α_4, is set equal to 0.02, which is public investment as share of output in the data, see e.g. Baxter and King, 1993 for the US. Given the values of α_2, α_3 and α_4, the private capital share is $\alpha_1 = 1 - \alpha_2 - \alpha_3 - \alpha_4 = 0.33$. We normalise the productivity parameter, A, to 1. We also use common values from the literature for the intertemporal elasticity of substitution, $1/\sigma = 0.5$ or $\sigma = 2$ and the rate of time preference $\beta = 0.97$. We assume that the depreciation rate for physical capital is 10%, which is the value calculated by Angelopoulos *et al.*, (2009), and also set the same depreciation rate for public capital. Note that the depreciation rates matter for the long-run value of the investment share in GDP, but have little effect on near steady-state dynamics in this class of model (see, e.g. King and Rebelo, 1999, p.954). The parameter, ψ, which measures the degree of substitutability/complementary between private and public consumption in the utility function, is set equal to 0; as Christiano and Eichenbaum (1992) explain, this means that government consumption is equivalent to a resource drain in the macro-economy. We set the share of entrepreneurs, n^k, to 0.3. This is the share of households, as calculated by Angelopoulos *et al.*, (2013), who have savings above 10,000£. We choose a value for union power, $\phi = 0.5$, which is in the middle of the range (i.e. 0.4 to 0.6) of values typically used in the literature, and a value for market power in the product market, $\theta = 0.9$, implying that profits, in equilibrium, amount to 10% of GDP.[19,20]

The effective tax rates on consumption, capital and labour are respectively $\tau^c = 0.1936$ (Economides *et al.*, 2012), $\tau^k = 0.3209$ and $\tau^l = 0.3667$ (Ardagna, 2007). The data values of output share of public spending on consumption and unemployment benefits, are respectively $s_t^c = 0.20$, and $s_t^u = 0.024$. At steady state, the public debt to output ratio, B^k / Y^i, is set at 0.60 because it has been the reference value of the initial Maastricht Treaty. Total government transfers as a share of output, s_t^t, are allowed to follow residually in the long run of the status quo economy so as to match the above mentioned spending-tax data and the public debt to output ratio.

3.2 The status quo or benchmark equilibrium

Given the parameter and policy values in Table 1, the steady state solution of model economy is reported in the first column of Table 2.

Table 2 around here

Steady state solutions

Notice that the solution is well defined. For instance, the solution for the key ratios, like consumption and private investment as shares of output, as well as the replacement rate, are very close to those in the data. This is what we call the status quo solution. In the next sections, departing from this status quo solution, we study the implications of structural reforms in product and labour markets.

4 Structural reforms in product and labour markets

[19] See e.g. Domeij (2005) for a discussion of the relevant studies and empirical evidence.

[20] This value approximates the magnitude typically employed in New Keynesian models to capture the price mark-up over marginal costs. See e.g. Faccini *et al.*, (2011) for the estimated price mark-up for the UK.

Departing from this situation, or what we have called the status quo, we study the implications of higher flexibility meaning more competitive product and/or labour markets when we use the share of government transfers to output, s_t^t, as an instrument of public financing. Especially, we consider three reformed economies. First, we study product market liberalization, column (2) in Table 2. Thus, we have a perfectly competitive product market and a unionised labour market. To obtain the solution of this reformed economy, we use the base line parameterization (Table 1), except that now the parameter θ is set at one. Second, we study labour market liberalization, column (3). Thus, now we have an imperfectly competitive product market and a perfectly competitive labour market. This can follow from the status quo model if we simply assume that the exogenously set labour supply is equal to one. Third, we consider a scenario of full liberalization meaning perfect competition in both product and labour market, column (4). The solution for this economy can be obtained from the status quo economy if we set the degree of monopoly power, θ, and the labour supply equal to unity. Notice that, the relative bargaining power of the union, ϕ, and the net unemployment benefit, $\overline{G}_t^u$, are not relevant since the unemployment is zero.

4.1 How we work

We start with comparing the long-run equilibrium of the status quo economy to the corresponding long-run equilibria of the three reformed economies. We then study transition results. This implies two steps. In the first step we log-linearize around the steady-state solution of each model economy and check out its saddle path stability. In the second step, setting as initial conditions for the state variables their values in the long-run status quo solution, we compute the equilibrium transition path as we travel towards the steady state of each reformed economy. One should not forget that, since the model is deterministic, the only source of transitional dynamics is due to the policy reforms.

In all cases, we study both aggregate and distributional implications. Regarding aggregate outcomes, we look, for instance, at per capita output and per capita welfare.[21] Regarding distribution, we compute separately the income and welfare of the representative member in each social group i.e. entrepreneurs vis a vis workers. The above values are then compared to their respective values had we remain in the status quo economy permanently (see also e.g. Cooley and Hansen 1992, Economides *et al.*, 2012). Finally, we present the public financing effects. By public financing we mean the policy instruments that adjust to accommodate the exogenous changes in imperfections of product and labour markets.

4.2 Long-run results

We start with comparison of steady state solutions. Results for each case, the status quo and the three reformed economies, are reported in the first four columns of Table 2.

4.2.1 Aggregate effects

Regarding aggregate outcomes or, equivalently, efficiency, we find that stronger competition in the product market, coupled with higher flexibility in the labour market, leads to the highest long-term social gains both in terms of per capita output and per capita welfare. In other words, as it should be expected, full liberalization gives the most efficient outcome. This is something generally acknowledged by the related literature. Higher aggregate efficiency in the case of deregulation in the product and labour market may arise through the classical advocacy that a more competitive economy leads to a better allocation of recourses. This is in line with the classical view of complementarity in policies heading to both product and labour market reforms (Boeri et al., 2000 and Nicoletti et al., 2000) and "…the need to accompany product market reforms with appropriate labour market policies..."(OECD, 2001).

In addition, if one compares deregulation in the product market only to deregulation in the labour market only, the latter is more desirable socially. In particular, we get 2.3740 for per capita output, when we reform only the product market, against 2.4576 in the case of a flexible labour market only (see columns (2) and (3) of Table 2). This is similar to Angelopoulos et al., (2013). In other words, comparing the two market imperfections, trade union power on the part of workers is more distortive than product market power on the part of firms, in our model. This is because of the specification of the fall-back position of the firm, which implies that the potential rent is relatively small for the firm. As a result, in this model, the relative big problem is the pursuit of the rent rather than the rent itself. However, if the firm's outside option is instead zero (this would imply a higher rent for the firm), then the product market reforms would be more beneficial socially.

4.2.2 Distributional effects

Since there are two different groups of households in the society – workers and entrepreneurs – these income/welfare gains from each particular structural reform may be distributed unequally for each group in society. Thus, we now turn to individual outcomes or, equivalently, to distribution. We start with the net income of each agent, $netY^k$ and $netY^w$.[22] Our results show that both social groups earn the maximum under full liberalization. In other words, all types of agents should prefer fully liberalized product and labour markets in the long run.[23] Since any reform entails benefits for both groups in society, we show that the benefits to one group are greater when we reduce the efficient losses generated by the market power of the other.

A key question now is who gains more. Even if a policy reform produces a win-win outcome, in the sense that both $netY^k$ and $netY^w$ rise, relative outcomes can be also important. The political economics literature has pointed out several reasons for this, including political ideology, habit, envy, etc. In our model, relative outcomes will be measured by changes in the ratio of net incomes, $netY^k / netY^w$.

[21]Aggregate per capita welfare is defined as the weighted average of entrepreneurs' and workers' welfare.

[22] Any difference in the results between net per capita income and individual welfare is due to a positive depreciation rate of private capital in the entrepreneur's problem. If $\delta^k = 0$, we get similar results.

Departing from the status quo, this ratio falls, or equivalently inequality falls, when product markets are reformed (compare columns (1) and (2)).[24, 25] Thus, as one would expect, workers benefit more from product market liberalization than capitalists. However, we often observe workers opposing to product market reforms (see Blanchard, 2004).[26] This reminds us another hot issue: which market's liberalization should be given priority? In a more competitive product market workers may lose as rent extractors, but they gain significantly through increased investment, employment opportunities and finally output.[27] This is also in line with empirical evidence that either product market liberalization does not deteriorate income inequality (Nicoletti *et al.,* 2001) or even more the Gini coefficient falls (Cardullo, 2009). However, inequality raises vis a vis the status quo and a deregulated product market economy, when we focus on liberalization of the labour market only (compare columns (1), (2) and (3)).[28] Workers cannot act as rent extractors any more due to the absence of bargaining power and their benefits are lower than those of entrepreneurs whose pie of rents is now increasing. This issue which has not been so far fully developed and well quantified but has been acknowledged in recent IMF (2009) and World Bank's reports (2013) "… labour market deregulation generally has an insignificant impact on growth but in most cases it increases income inequality…". Thus according to our findings in case of reforms a product market deregulation should be a top priority.[29]

4.2.3 Public financing effects

It is worth investigating now, how the implications of exogenous structural reforms, like the above, depend on the public financing policy instrument used namely, which fiscal policy instrument takes advantage of the switch to a more efficient economy, and hence a larger tax base, made possible by the structural reforms.

It is true that, as we move from the status quo to a more efficient economy in Table 2, increased output and employment opportunities provide a larger tax base and thus, an extra fiscal space is created. The question is now, which category of public spending to increase or which type of taxes to cut.[30]

To save on space, we focus on the most efficient case, which is the case in column (4) of Table 2,

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[23] The results are similar when unions have all power in bargaining (monopoly union model).

[24] Starting with an index of 3.2541 in the status quo economy we move to a lower inequality index 2.9108 when the product market is getting more competitive.

[25] It is known that the positive impact of a more deregulated product market on employment is greater the more regulated is the labour market or the stronger the workers' bargaining power (Griffith *et al*., 2007 and Fiori, 2008).

[26] One should not neglect the natural tendency over the maintenance of the status quo and the role of lobbies and interest groups emphasizing the risks of any reform against the expected benefits (Boeri 2001, Cardullo 2009).

[27] We have a lot of evidence that product market liberalization boosts long-run employment levels, wages and output (OECD, 2001 and Scarpetta, 2005).

[28] In particular, the inequality index of 3.2541 in the status quo economy rises to 3.3643 when the reform takes place only in the labour market. We have similar results when we measure inequality by the share of consumption and by net income of entrepreneurs minus net income of workers over net total income too.

[29] It is worth mentioning here that, by reforming both markets, we cannot get an improvement in inequality vis a vis the case of a deregulated product market only (compare columns (1) and (4)).

[30] Starting with a share of government transfers of 13.12% in the status quo we move to 14.65% when both markets are more competitive.

where both markets are reformed. We examine what happens when, in this fully liberalized economy, the government also allows either social transfers, or public investment, or the capital tax rate or the consumption tax rate to take advantage of the fiscal space created, see columns (4),(5),(6) and (7) of Table 2. As one would expect, it is better to allow capital taxes to take advantage of the fiscal space. Actually, capital tax rate can be cut from 32.09% in the data to 22.75%.[31] This cut engineers a second round of efficiency gains and a further enhancement of aggregate per capita output.[32] This is consistent with the well-known result that capital taxes are particularly distorting in the medium and the long run and thus enhancing aggregate efficiency (Chamley, 1986). The second best instrument is public investment which has a significant impact on output, see column (5). Therefore, the best way of using the fiscal space generated by a switch to a more deregulated economy is to cut capital tax rate or to increase public investment spending. These are the policy instruments with relatively high multiplier. In other words, lower capital tax rates or higher public investment spending generates a second round beneficial effect on the aggregate economy. This confirms the importance of policy mixes in the conduct of economic policy (see Wren-Lewis, 2010).

Regarding distributional effects, however, workers would prefer the case in which it is government transfers that take advantage of the fiscal space, while the interest of entrepreneurs coincides with the aggregate interest. That is, entrepreneurs would prefer the capital tax rate to be cut rather than transfers to be increased.[33] Intuitively, transfers have a direct impact on workers income since they represent a larger part of their budget constraint, while capital taxes influence mainly the investment decision of entrepreneurs and consequently their income. In our results, when the capital tax rate is the residual instrument, entrepreneurs' and workers' net income is 1.3446 and 0.4270 respectively, while it is 1.2953 and 0.4306 in the case of transfers. In addition in the reformed economy by using capital tax inequality is higher.[34] In other words, in the long run, a smaller public sector, meaning lower capital taxes combined with lower public spending, benefit the aggregate economy and the entrepreneurs, but it worsens the relative position of workers in the income ladder. This is in line with the often voiced opposition on behalf of trade unions regarding reduction in capital taxes, since income distribution is prime goal in their agenda (ITUC 2012).

4.3 Transition results

We next study what happens when we depart from the status quo economy and travel towards a new long run according to the type of reform we perform. In Table 3, we calculate the post-reform welfare, conditional on the initial, status quo steady-state, for each type of reform and each type of agent, $U^{k}_{i=5,10,20}$

[31] Correspondingly public investment is increased from 2% of GDP to 3.28%, and consumption tax rate is decreased from 19.36% to 16.6%.

[32] Using consumption tax rate or transfer payments we have a positive impact on output however, this is not so significant as in the case of capital tax and public investment (keeping in mind that this is a long run result only).

[33] Workers' income is at the highest level of 0.4306 Table 2, column (4) when government transfers are used to finance the reforms, while entrepreneurs' income is at the highest level of 1.3446 column (6) when capital tax rate is used to finance the reforms.

[34] In particular, the inequality index is 3.1491 when capital tax rate is the residual instrument and becomes 3.0083 in the case of transfers.

for the entrepreneurs and $U^{w}_{i=5,10,20}$ for the workers, as well as for the aggregate economy, $U_{i=5,10,20}$ (see Table 3). In the same table, we report the corresponding values for lifetime and steady state utilities.

We also calculate the welfare gains/losses from the reform for each type of agent and at the aggregate level, by computing the consumption supplement required to make the agents in the status quo regime as well as in the reformed economy. We denote welfare gains or losses for entrepreneurs, workers and the aggregate economy as $\zeta^{k}_{i=5,10,20}$, $\zeta^{w}_{i=5,10,20}$ and $\zeta_{i=5,10,20}$ (see Appendix 2 and Table 3). When these gains or losses refer to the corresponding lifetime and steady-state values they are denoted by the subscripts *lt* and *ss*. In addition, we present the dynamic transition paths for the most important endogenous variables in Figures 1-6. We observe that the short run dynamics are not different from the long run results already presented. It is interesting to point out however that it takes around eight to nine years in order to achieve half the efficiency gains from each structural reform. In addition, we do not observe any efficiency differences by choosing either public investment spending or capital tax as public instrument.

4.3 Robustness

We now check the sensitivity of our results to changes in the assumed parameter values and, especially, the value of the population share of entrepreneurs, n^{k}. We report that all results above are robust to parameter values used. For instance, keeping everything else as in the baseline parameterization of Table 1, we now arbitrarily set $n^{k} = 0.25$. Results are reported in Tables 4.

Table 4 around here

5 Conclusions and possible extensions

This paper has analysed the efficiency and distributional implications of structural reforms in product and labour markets. Moreover, the significance of the instrument of public financing these reforms has been highlighted

- Regarding aggregate efficiency, we find that reforming both markets, lead to the highest long term gains both in per capita output and per capita welfare
- If one compares deregulation in the product market with reforms in the labour market, it is the latter that is found to be more efficient
- Starting from the status quo economy inequality falls when there is an initiative towards the liberalization of the product market.

- It is worth noting that by reforming only one market the gain for each agent is higher when the reform has to do with the reduction of the market power of the other.
- Regarding net incomes and benefits of social groups in the reformed economy one should clearly mention that in terms of individual efficiency and inequality workers prefer the case in which it is government transfers that take advantage of moving to a more efficient set up while entrepreneurs prefer the reduction of capital tax.

Extensions

- Endogenous mark-ups
- Public / private employees
- Richer modeling of labour market i.e. hiring-firing costs, social security contributions and flexicurity.

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APPENDICES

Appendix 1: Decentralized Equilibrium (DE)

The DE consists of the following equations:[35]

The entrepreneur's Euler equation with respect to capital:

$$\frac{(1+\tau_{t+1}^{c})\left(C_{t+1}^{k}+\psi s_{t+1}^{c} n^{k} Y_{t+1}^{i}\right)^{\sigma}}{(1+\tau_{t}^{c})\left(C_{t}^{k}+\psi s_{t}^{c} n^{k} Y_{t}^{i}\right)^{\sigma}}=\beta\left[1+\left(1-\tau_{t+1}^{k}\right)\left(r_{t+1}^{k}-\delta^{p}\right)\right] \tag{A.1a}$$

The entrepreneur's Euler equation with respect to bonds:

$$r_{t+1}^{b}=\left(1-\tau_{t+1}^{k}\right)\left(r_{t+1}^{k}-\delta^{p}\right) \tag{A.2b}$$

The worker's budget constraint:

$$(1+\tau_{t}^{c})C_{t}^{w}=\left(1-\tau_{t}^{l}\right)w_{t}^{w}e_{t}^{w}+\left(\frac{s_{t}^{u}}{n^{w}}+s_{t}^{t}\right)n^{k}Y_{t}^{i} \tag{A.2c}$$

[35] Note that relying on Walras's law, we drop the budget constraint of the entrepreneur from the DE.

The intermediate firm's optimality condition for $L_t^{i,w}$:

$$\theta\alpha_2 \frac{Y_t^i}{\frac{n^w}{n^k} e_t^w} = w_t^w \qquad \text{(A.2d)}$$

The intermediate firm's optimality condition for $L_t^{i,k}$:

$$\theta\alpha_3 Y_t^i = w_t^k \qquad \text{(A.2e)}$$

The intermediate firm's optimality condition for K_t^i :

$$\theta\alpha_1 \frac{Y_t^i}{K_t^k} = r_t^k \qquad \text{(A.2f)}$$

The intermediate firm's profit function:

$$\pi_t^k = \left(1 - \theta\alpha_1 - \theta\alpha_2 - \theta\alpha_3\right) Y_t^i \qquad \text{(A.2g)}$$

The intermediate firm's production function:

$$Y_t^i = \mathrm{A}\left(K_t^k\right)^{\alpha_1} \left(\frac{n^w}{n^k} e_t^w\right)^{\alpha_2} \left(\frac{\overline{k}_t^g}{n^k}\right)^{\alpha_4} \qquad \text{(A.2h)}$$

The union's optimality condition for the wage rare:

$$\left(1-\tau_t^l\right)\theta\alpha_2 Y_t^i = \frac{\phi+\left(1-\phi\right)\theta\alpha_2}{\theta\alpha_2}\frac{s_t^u Y_t^i}{(1-e_t^w)}e_t^w \tag{A.2i}$$

The Government's Budget Constraint:

$$\begin{aligned}&\left(s_t^c+s_t^t+s_t^i+s_t^u\right)n^k Y_t^i + n^k\left(1+r_t^b\right)B_t^k = \\ &= n^k\tau_t^k\left(r_t^k-\delta^p\right)K_t^k + n^k\tau_t^k\pi_t^k + n^k\tau_t^l w_t^k + n^w\tau_t^l w_t^w e_t^w + n^w\tau_t^c C_t^w + n^k\tau_t^c C_t^k + n^k B_{t+1}^k\end{aligned} \tag{A.2j}$$

The law of motion of public capital:

$$\overline{k}_{t+1}^g = (1-\delta^g)\overline{k}_t^g + s_t^i n^k Y_t^i \tag{A.2k}$$

The resource constraint:

$$\left(1-s_t^c-s_t^i\right)n^k Y_t^i = n^k C_t^k + n^w C_t^w + n^k\left[K_{t+1}^k-(1-\delta^p)K_t^k\right] \tag{A.2l}$$

Therefore, the DE is a system of twelve non-linear difference equations in the paths of $Y_t^i, C_t^k, K_{t+1}^k, C_t^w, e_t^w, \pi_t^k, \overline{k}_{t+1}^g, r_t^b, r_t^k, w_t^w, w_t^k$ and one of the eight policy instruments, $\tau_t^l, \tau_t^c, \tau_t^k, s_t^u, s_t^t, s_t^c, s_t^i, B_{t+1}^k$, that is residually determined. This equilibrium is given the paths of the other seven tax-spending policy instruments.

Appendix 2: Welfare Comparisons

The discounted lifetime utility in the status quo economy (the pre-reformed economy):

$$U_{sqe}^j = \sum_{t=0}^{T}\beta^t\frac{\left(C_{sqe}^j\right)^{1-\sigma}}{1-\sigma}$$

The discounted lifetime utility in the reformed economy (the post-reform economy):

$$U_{re}^{j}=\sum_{t=0}^{T}\beta^{t}\frac{\left(C_{re,t}^{j}\right)^{1-\sigma}}{1-\sigma}$$

We follow e.g. Lucas (1990) and compute the permanent percentage supplement in private consumption required to make agents in the status quo regime as well as in the reformed economy. This percentage supplement is defined as ζ. More specifically, we find the value of ζ that satisfies the following equation:

$$U_{re}^{j}-\sum_{t=0}^{T}\beta^{t}\frac{\left(\left(1+\zeta\right)C_{sqe}^{j}\right)^{1-\sigma}}{1-\sigma}=0\Rightarrow\zeta=\left(\frac{U_{re}^{j}}{U_{sqe}^{j}}\right)^{\frac{1}{1-\sigma}}-1$$

If ζ > 0 (respectively ζ < 0), there is a welfare gain (respectively loss) of moving from the initial steady state to the new reform one.

TABLES

Table 1: Baseline parameterization		
Parameters and policy instruments	**Definition**	**Value**
$0 \leq \beta \leq 1$	Rate of time preference	0.97
$0 \leq \alpha_1 \leq 1$	Private capital share in production	0.33
$0 \leq \alpha_2 \leq 1$	Workers' labour share in production	0.45
$0 \leq \alpha_3 \leq 1$	Capitalists' labour share in production	0.20
$0 \leq \alpha_4 \leq 1$	Public capital share in production	0.02
$0 \leq \delta^p \leq 1$	Depreciation rate on private capital	0.10
$0 \leq \delta^g \leq 1$	Depreciation rate on public capital	0.10
$0 \leq n^k \leq 1$	Population share of entrepreneurs	0.30
$\sigma > 1$	Relative risk aversion coefficient	2
A	TFP level	1
$-1 \leq \psi \leq 1$	Substitutability between private and public consumption in utility	0
$0 \leq \phi \leq 1$	Union power	0.50
$0 \leq \theta \leq 1$	Product market power	0.90
$0 \leq \tau^c \leq 1$	Consumption tax rate	0.1936
$0 \leq \tau^k \leq 1$	Tax rate on capital income	0.3209
$0 \leq \tau^l \leq 1$	Tax rate on labour income	0.3667
s_t^u	Unemployment benefits to output ratio	0.024
$\frac{B^k}{Y^i}$	Public debt to output ratio	0.60
s_t^c	Public Consumption to output ratio	0.20
s_t^i	Public Investment to output ratio	0.02

Μπορούμε να χρησιμοποιήσουμε τους πίνακες 2a (σελ.24) και 2b (σελ.25) ή τον πίνακα 2 (σελ.26).

Table 2a: Long-run solutions

Variable	Status Quo (1)	Liberalization in Product Market (2)	Liberalization in Labour Market (3)	Liberalization in Product and Labour Market (4)
Y	2.2146	2.3740	2.4576	2.5926
$netY^w$	0.3533	0.4016	0.3845	0.4306
$netY^k$	1.1498	1.1690	1.2937	1.2953
$netY^k/netY^w$	3.2541	2.9108	3.3643	3.0083
C^k / C^w	1.9751	1.5705	2.0601	1.6430
s^t	0.1312	0.1225	0.1552	0.1465

Note: The residual policy instrument is the share of government transfers to output, s^t .

Table 2b: Long-run solutions				
Variable	Liberalization in Product and Labour Market s^t policy instrument (1)	Liberalization in Product and Labour Market s^i policy instrument (2)	Liberalization in Product and Labour Market τ^k policy instrument (3)	Liberalization in Product and Labour Market τ^c policy instrument (4)
Y	2.5926	2.6324	2.6439	2.5926
$netY^w$	0.4306	0.4251	0.4270	0.4187
$netY^k$	1.2953	1.3031	1.3446	1.2834
$netY^k / netY^w$	3.0083	3.0652	3.1491	3.0652
C^k / C^w	1.6430	1.6612	1.6899	1.6612
s^t	**0.1465**	0.1312	0.1312	0.1312
s^i	0.0200	**0.0328**	0.0200	0.0200
τ^k	0.3209	0.3209	**0.2275**	0.3209
τ^c	0.1936	0.1936	0.1936	**0.1660**

Table 2: Long-run solutions

Variable	Status Quo s^t policy instrument (1)	Liberalization in Product Market s^t policy instrument (2)	Liberalization in Labour Market s^t policy instrument (3)	Liberalization in Product and Labour Market s^t policy instrument (4)	Liberalization in Product and Labour Market s^i policy instrument (5)	Liberalization in Product and Labour Market τ^k policy instrument (6)	Liberalization in Product and Labour Market τ^c policy instrument (7)
Y	2.2146	2.3740	2.4576	2.5926	2.6324	2.6439	2.5926
$netY^w$	0.3533	0.4016	0.3845	0.4306	0.4251	0.4270	0.4187
$netY^k$	1.1498	1.1690	1.2937	1.2953	1.3031	1.3446	1.2834
$netY^k / netY^w$	3.2541	2.9108	3.3643	3.0083	3.0652	3.1491	3.0652
C^k / C^w	1.9751	1.5705	2.0601	1.6430	1.6612	1.6899	1.6612
s^t	0.1312	0.1225	0.1552	0.1465	0.1312	0.1312	0.1312
s^i	0.0200	0.0200	0.0200	0.0200	0.0328	0.0200	0.0200
τ^k	0.3209	0.3209	0.3209	0.3209	0.3209	0.2275	0.3209
τ^c	0.1936	0.1936	0.1936	0.1936	0.1936	0.1936	0.1660

Table 3: Utilities and Gains/losses					
(1)	Status Quo (2)	(3)	Liberalization in Product Market (4)	Liberalization in Labour Market (5)	Liberalization in Product and Labour Market (4)
U_5^k	-9.7636	ζ_5^k	-0,125	0,125	-0,048
U_5^w	-19.2845	ζ_5^w	0,113	0,085	0,164
U_5	-16,4283	ζ_5	0,062	0,092	0,119
U_{10}^k	-16,6793	ζ_{10}^k	-0,116	0,133	-0,033
U_{10}^w	-32,9440	ζ_{10}^w	0,123	0,092	0,179
U_{10}	-28,0646	ζ_{10}	0,072	0,099	0,135
U_{20}^k	-27,7178	ζ_{20}^k	-0,105	0,142	-0,016
U_{20}^w	-54,7467	ζ_{20}^w	0,135	0,099	0,197
U_{20}	-46,6381	ζ_{20}	0,083	0,107	0,152
U_{lt}^k	-57,0084	ζ_{lt}^k	-0,096	0,135	0,014
U_{lt}^w	-112,5999	ζ_{lt}^w	0,137	0,088	0,219
U_{lt}	-95,9225	ζ_{lt}	0,087	0,096	0,176
U_{ss}^k	-1,7103	ζ_{ss}^k	-0,096	0,135	0,014
U_{ss}^w	-3,3780	ζ_{ss}^w	0,137	0,088	0,219
U_{ss}	-2,8777	ζ_{ss}	0,087	0,096	0,176

Table 4: Long-run solutions with a decrease in the share of entrepreneurs, n^k, from 0.30 to 0.25

Variable	Status Quo s^t policy instrument (1)	Liberalization in Product Market s^t policy instrument (2)	Liberalization in Labour Market s^t policy instrument (3)	Liberalization in Product and Labour Market s^t policy instrument (4)	Liberalization in Product and Labour Market s^i policy instrument (5)	Liberalization in Product and Labour Market τ^k policy instrument (6)	Liberalization in Product and Labour Market τ^c policy instrument (7)
Y	2.6354	2.8252	2.9247	3.0854	3.1326	3.1464	3.0854
$netY^w$	0.3328	0.3775	0.3635	0.4061	0.4003	0.4021	0.3943
$netY^k$	1.3511	1.3738	1.5169	1.5189	1.5302	1.5795	1.5071
$netY^k / netY^w$	4.0595	3.6395	4.1731	3.7404	3.8222	3.9282	3.8222
C^k / C^w	2.4436	1.9425	2.5312	2.0176	2.0480	2.0842	2.0480
s^t	0.1312	0.1225	0.1552	0.1465	0.1312	0.1312	0.1312
s^i	0.0200	0.0200	0.0200	0.0200	0.0328	0.0200	0.0200
τ^k	0.3209	0.3209	0.3209	0.3209	0.3209	0.2275	0.3209
τ^c	0.1936	0.1936	0.1936	0.1936	0.1936	0.1936	0.1660

FIGURES

Figure 1: From Status Quo to Liberalization in product market when the share of government transfers to GDP is the residually determined policy instrument

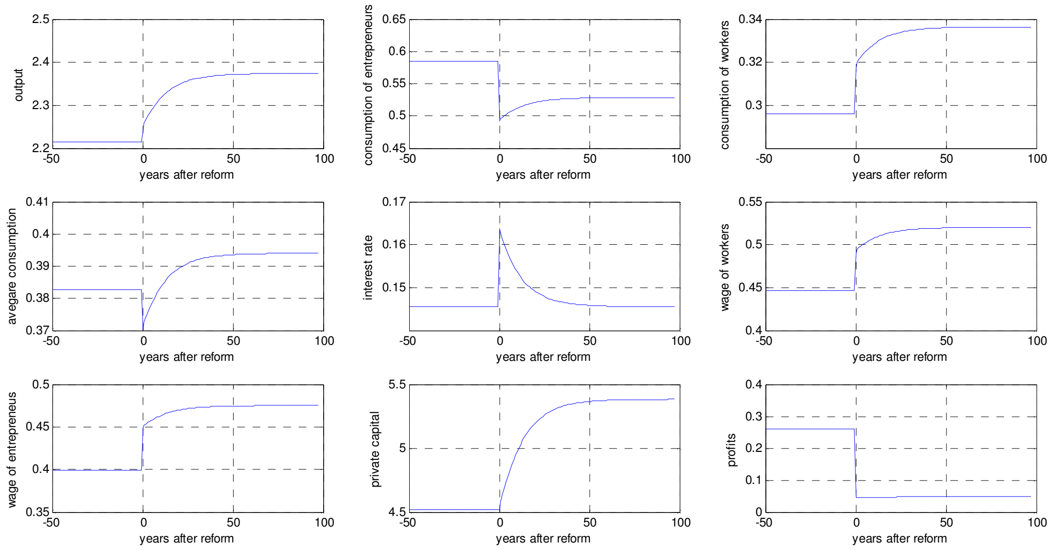


Figure 2: Status Quo to Liberalization in labour market when the share of government transfers to GDP is the residually determined policy instrument

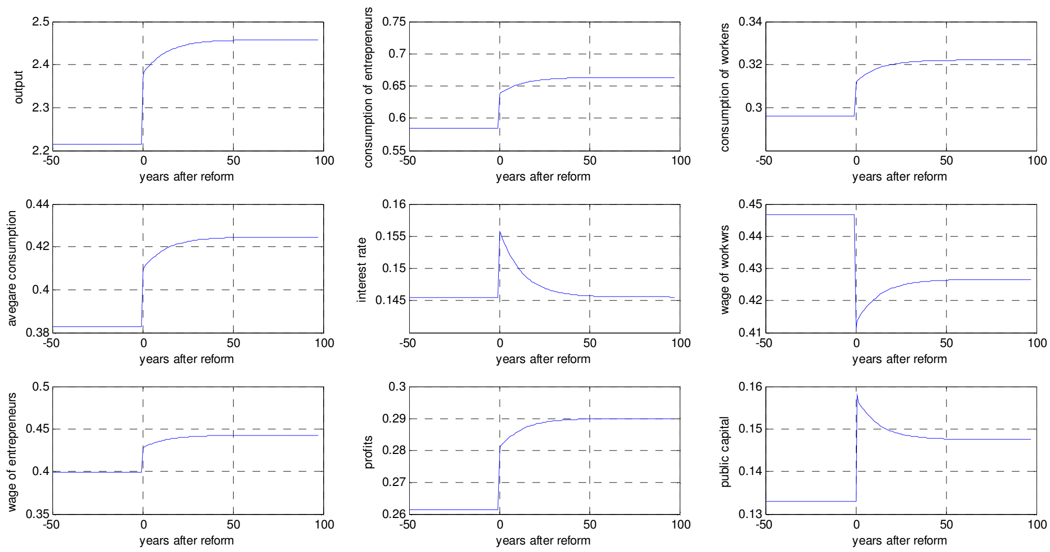


Figure 3: Status Quo to Liberalization in both markets when share of government transfers to GDP is the residually determined policy instrument

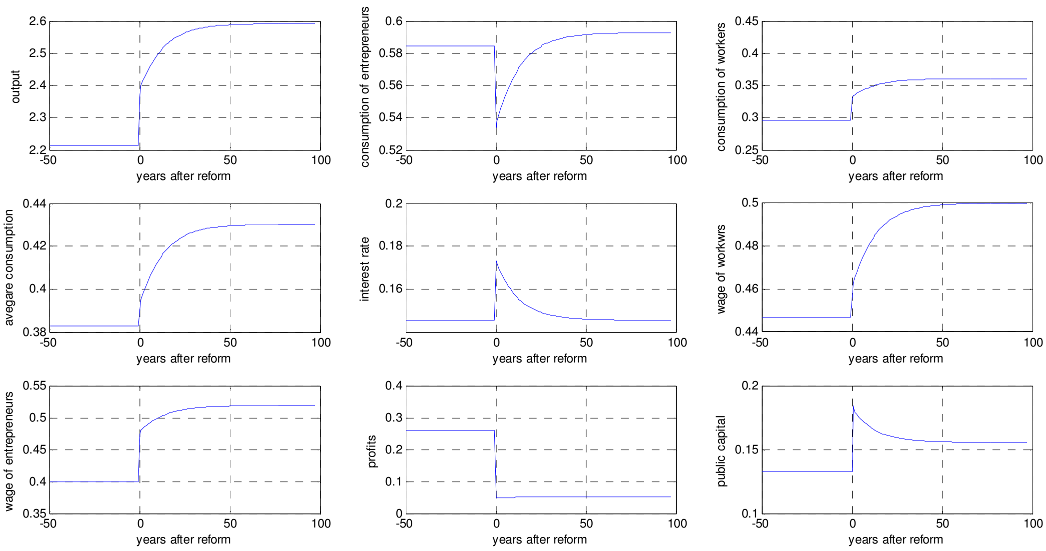


Figure 4: Status Quo to Liberalization in both markets when the share of public investment to GDP is the residually determined policy instrument

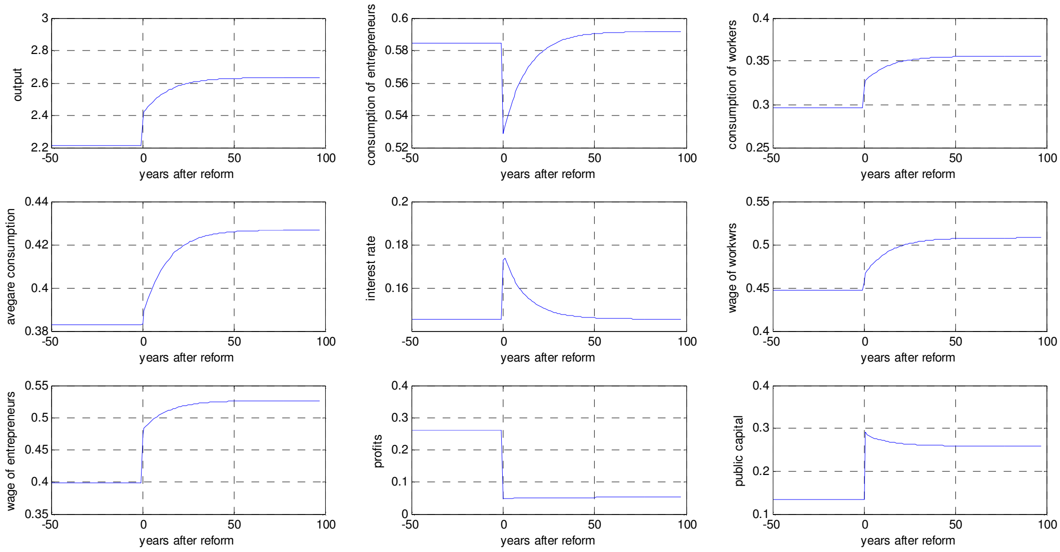


Figure 5: Status Quo to Liberalization in both markets when capital tax rate is the residually determined policy instrument

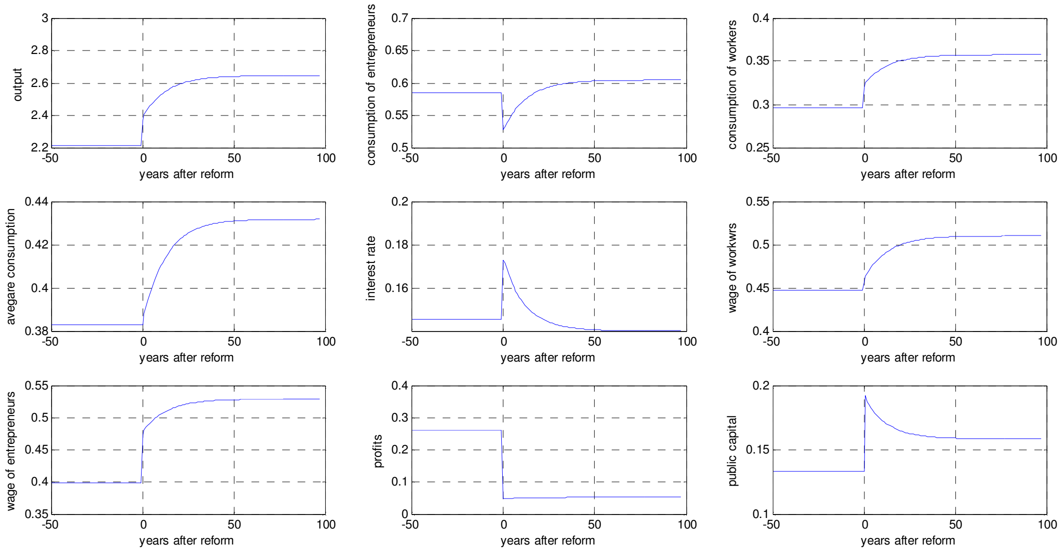
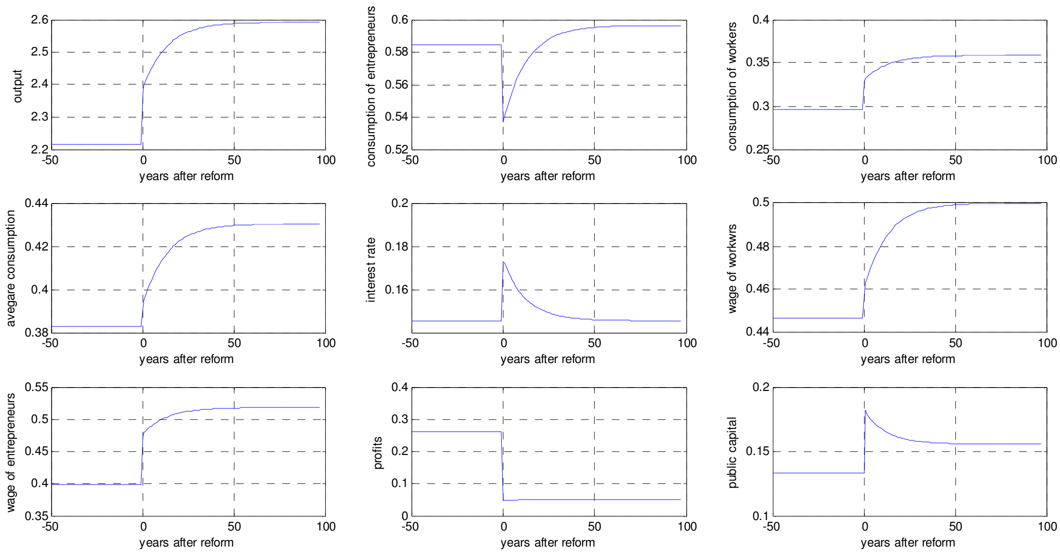


Figure 6: Status Quo to Liberalization in both markets when consumption tax rate is the residually determined policy instrument



APPENDIX (extended)

Appendix A: The entrepreneur's problem

$$\max_{\{C_t^k, K_{t+1}^k, B_{t+1}^k\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t^k + \psi \bar{G}_t^c\right)^{1-\sigma}}{1-\sigma}$$

$$\begin{aligned} s.t.\ \ & (1+\tau_t^c)C_t^k + K_{t+1}^k - (1-\delta^p)K_t^k + B_{t+1}^k = \\ & r_t^k K_t^k - \tau_t^k\left(r_t^k - \delta^p\right)K_t^k + (1+r_t^b)B_t^k + \left(1-\tau_t^k\right)\pi_t^k + \left(1-\tau_t^l\right)w_t^k + \bar{G}_t^t \end{aligned}$$

$$\text{given} \quad \left\{r_t^b, r_t^k, w_t^k\right\}_{t=0}^{\infty}, \left\{\tau_t^c, \tau_t^k, \tau_t^l, \bar{G}_t^t, \bar{G}_t^c\right\}_{t=0}^{\infty}, K_0^k, B_0^k$$

The Lagrangian function is:

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \dfrac{\left(C_t^k + \psi \bar{G}_t^c\right)^{1-\sigma}}{1-\sigma} + \\ +\lambda_t^k \left[\begin{array}{l} (1+\tau_t^c)C_t^k + K_{t+1}^k - (1-\delta^p)K_t^k + B_{t+1}^k - r_t^k K_t^k + \tau_t^k\left(r_t^k - \delta^p\right)K_t^k \\ -(1+r_t^b)B_t^k - \left(1-\tau_t^k\right)\pi_t^k - \left(1-\tau^l\right)w_t^k - \bar{G}_t^t \end{array} \right] \end{array} \right\}$$

$$\begin{aligned} &= \ldots + \frac{\left(C_t^k + \psi \bar{G}_t^c\right)^{1-\sigma}}{1-\sigma} + \lambda_t^k \left[\begin{array}{l} (1+\tau_t^c)C_t^k + K_{t+1}^k - (1-\delta^p)K_t^k + B_{t+1}^k - r_t^k K_t^k + \tau_t^k\left(r_t^k - \delta^p\right)K_t^k \\ -(1+r_t^b)B_t^k - \left(1-\tau_t^k\right)\pi_t^k - \left(1-\tau_t^l\right)w_t^k - \bar{G}_t^t \end{array} \right] \\ &+\beta \left\{ \frac{\left(C_{t+1}^k + \psi \bar{G}_{t+1}^c\right)^{1-\sigma}}{1-\sigma} + \lambda_{t+1}^k \left[\begin{array}{l} (1+\tau_{t+1}^c)C_{t+1}^k + K_{t+2}^k - (1-\delta^p)K_{t+1}^k + B_{t+2}^k - r_{t+1}^k K_{t+1}^k + \tau_{t+1}^k\left(r_{t+1}^k - \delta^p\right)K_{t+1}^k \\ -(1+r_{t+1}^b)B_{t+1}^k - \left(1-\tau_{t+1}^k\right)\pi_{t+1}^k - \left(1-\tau_{t+1}^l\right)w_{t+1}^k - \bar{G}_{t+1}^t \end{array} \right] \right\} + \ldots \end{aligned}$$

The first order conditions are:

$$\frac{\partial L}{\partial C_t^k}=0\Rightarrow(1-\sigma)\frac{\left(C_t^k+\psi\bar{G}_t^c\right)^{-\sigma}}{1-\sigma}+\lambda_t^k(1+\tau_t^c)=0\Rightarrow\lambda_t^k(1+\tau_t^c)=-\frac{1}{\left(C_t^k+\psi\bar{G}_t^c\right)^{\sigma}}\Rightarrow$$

$$\Rightarrow\lambda_t^k=-\frac{1}{(1+\tau_t^c)\left(C_t^k+\psi\bar{G}_t^c\right)^{\sigma}}\quad(1.\text{A})$$

From the Lagrangian function the period t multiplier is equal to the increase in the value of the objective function when the period t budget constraint is increased by one unit. This is equal to the marginal utility of wealth or income which in this model is equal to the marginal utility of consumption.

$$\frac{\partial L}{\partial K_{t+1}^k}=0\Rightarrow\lambda_t^k+\beta\lambda_{t+1}^k\left[-(1-\delta^p)-r_{t+1}^k+\tau_{t+1}^k\left(r_{t+1}^k-\delta^p\right)\right]=0\Rightarrow$$

$$\Rightarrow\lambda_t^k-\beta\lambda_{t+1}^k\left[(1-\delta^p)+r_{t+1}^k-\tau_{t+1}^k\left(r_{t+1}^k-\delta^p\right)\right]=0\Rightarrow$$

$$\Rightarrow\lambda_t^k=\beta\lambda_{t+1}^k\left[1-\delta^p+r_{t+1}^k-\tau_{t+1}^k\left(r_{t+1}^k-\delta^p\right)\right]\quad(2.\text{A})$$

$$\frac{\partial L}{\partial B_{t+1}^k}=0\Rightarrow\lambda_t^k+\beta\lambda_{t+1}^k\left[-\left(1+r_{t+1}^b\right)\right]=0\Rightarrow\lambda_t^k-\beta\lambda_{t+1}^k\left(1+r_{t+1}^b\right)=0\Rightarrow$$

$$\Rightarrow\lambda_t^k=\beta\lambda_{t+1}^k\left(1+r_{t+1}^b\right)\quad(3.\text{A})$$

$$\frac{\partial L}{\partial\lambda_t^k}=0\Rightarrow(1+\tau_t^c)C_t^k+K_{t+1}^k-(1-\delta^p)K_t^k+B_{t+1}^k=$$

$$=r_t^kK_t^k-\tau_t^k\left(r_t^k-\delta^p\right)K_t^k+(1+r_t^b)B_t^k+\left(1-\tau_t^k\right)\pi_t^k+\left(1-\tau_t^l\right)w_t^k+\bar{G}_t^t\quad(4.\text{A})$$

Substituting the multiplier the equations are:

$$\lambda_t^k=\beta\lambda_{t+1}^k\left[1-\delta^p+r_{t+1}^k-\tau_{t+1}^k\left(r_{t+1}^k-\delta^p\right)\right](2.\text{A})\xrightarrow{\lambda_t^k=-\frac{1}{(1+\tau_t^c)\left(C_t^k\right)^{\sigma}}(1.\text{A})}$$

$$-\frac{1}{(1+\tau_t^c)\left(C_t^k+\psi\bar{G}_t^c\right)^{\sigma}}=\beta\left[-\frac{1}{(1+\tau_{t+1}^c)\left(C_{t+1}^k+\psi\bar{G}_{t+1}^c\right)^{\sigma}}\right]\left[1+\left(r_{t+1}^k-\delta^p\right)-\tau_{t+1}^k\left(r_{t+1}^k-\delta^p\right)\right]\Rightarrow$$

$$\frac{(1+\tau_{t+1}^c)\left(C_{t+1}^k+\psi\bar{G}_{t+1}^c\right)^{\sigma}}{(1+\tau_t^c)\left(C_t^k+\psi\bar{G}_t^c\right)^{\sigma}}=\beta\left[1+\left(1-\tau_{t+1}^k\right)\left(r_{t+1}^k-\delta^p\right)\right](5.\text{A})$$

Equation (5.A) is the Euler with respect to capital.

$$\lambda_t^k = \beta\lambda_{t+1}^k\left[1-\delta^p + r_{t+1}^k - \tau_{t+1}^k\left(r_{t+1}^k - \delta^p\right)\right](3.\text{A}) \xrightarrow{\lambda_t^k = \beta\lambda_{t+1}^k\left(1+r_{t+1}^b\right)(2.\text{A})}$$
$$\beta\lambda_{t+1}^k\left(1+r_{t+1}^b\right) = \beta\lambda_{t+1}^k\left[1+\left(r_{t+1}^k-\delta^p\right)-\tau_{t+1}^k\left(r_{t+1}^k-\delta^p\right)\right] \Rightarrow$$
$$1+r_{t+1}^b = 1+\left(1-\tau_{t+1}^k\right)\left(r_{t+1}^k-\delta^p\right) \Rightarrow r_{t+1}^b = \left(1-\tau_{t+1}^k\right)\left(r_{t+1}^k-\delta^p\right)(6.\text{A})$$

Equation (6.A) gives a relation between the return to physical capital and the return to government bonds.

And the entrepreneurs' budget constraint:

$$(1+\tau_t^c)C_t^k + K_{t+1}^k - (1-\delta^p)K_t^k + B_{t+1}^k =$$
$$r_t^k K_t^k - \tau_t^k\left(r_t^k - \delta^p\right)K_t^k + (1+r_t^b)B_t^k + \left(1-\tau_t^k\right)\pi_t^k + \left(1-\tau_t^l\right)w_t^k + \bar{G}_t^t\,(4.\text{A})$$

A system of 3 equations (4.A), (5.A), (6.A), in 3 unknowns $\left\{C_t^k, K_{t+1}^k, B_{t+1}^k\right\}_{t=0}^{\infty}$

Appendix B: The final good producer's problem

$$\left.\begin{aligned} &\max_{Y_t^i} \Pi_t = Y_t - \sum_{i=1}^{N^i}\lambda^i P_t^i Y_t^i \\ &s.t.\ Y_t = \left[\sum_{i=1}^{N^i}\lambda^i\left(Y_t^i\right)^{\theta}\right]^{\frac{1}{\theta}} \end{aligned}\right\} \Rightarrow \max_{Y_t^i} \Pi_t = \left[\sum_{i=1}^{N^i}\lambda^i\left(Y_t^i\right)^{\theta}\right]^{\frac{1}{\theta}} - \sum_{i=1}^{N^i}\lambda^i P_t^i Y_t^i$$

The first order condition is:

$$\frac{\partial \Pi_t}{\partial Y_t^i} = 0 \Rightarrow \frac{1}{\theta}\left[\sum_{i=1}^{N^i} \lambda^i \left(Y_t^i\right)^{\theta}\right]^{\frac{1}{\theta}-1} \theta \lambda^i \left(Y_t^i\right)^{\theta-1} - \lambda^i P_t^i = 0 \Rightarrow$$

$$\frac{Y_t}{\sum_{i=1}^{N^i} \lambda^i \left(Y_t^i\right)^{\theta}} \left(Y_t^i\right)^{\theta-1} = P_t^i \Rightarrow \frac{Y_t}{\left(Y_t\right)^{\theta}} \left(Y_t^i\right)^{\theta-1} = P_t^i \Rightarrow P_t^i = \left(\frac{Y_t}{Y_t^i}\right)^{1-\theta} \quad (1.B)$$

Appendix C: The intermediate firm's problem

$$\left.\begin{aligned} &\max_{K_t^i, L_t^{i,w}, L_t^{i,k}} \quad \pi_t^i = P_t^i Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \\ &s.t.\ Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1} \left(L_t^{i,w}\right)^{\alpha_2} \left(L_t^{i,k}\right)^{\alpha_3} \left(\frac{K_t^g}{N^i}\right)^{\alpha_4 \alpha_3} \\ &\qquad P_t^i = \left(\frac{Y_t}{Y_t^i}\right)^{1-\theta} \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} &\max_{K_t^i, L_t^{i,w}, L_t^{i,k}} \quad \pi_t^i = \left(\frac{Y_t}{Y_t^i}\right)^{1-\theta} Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \\ &s.t.\ Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1} \left(L_t^{i,w}\right)^{\alpha_2} \left(L_t^{i,k}\right)^{\alpha_3} \left(\frac{K_t^g}{N^i}\right)^{\alpha_4} \end{aligned}\right\} \Rightarrow$$

$$\max_{K_t^i, L_t^{i,w}, L_t^{i,k}} \quad \pi_t^i = \left(Y_t\right)^{1-\theta} \left[\mathrm{A}\left(K_t^i\right)^{\alpha_1} \left(L_t^{i,w}\right)^{\alpha_2} \left(L_t^{i,k}\right)^{\alpha_3} \left(\frac{K_t^g}{N^i}\right)^{\alpha_4}\right]^{\theta} - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k}$$

The first order conditions are:

$$\frac{\partial \pi_t^i}{\partial L_t^{i,w}} = 0 \Rightarrow Y_t^{1-\theta} \mathrm{A}^{\theta} \left(K_t^i\right)^{\alpha_1 \theta} \theta \alpha_2 \left(L_t^{i,w}\right)^{\theta\alpha_2 - 1} \left(L_t^{i,k}\right)^{\theta\alpha_3} \left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4} = w_t^w \Rightarrow$$

$$\frac{\theta\alpha_2}{L_t^{i,w}} Y_t^{1-\theta} \left[\mathrm{A}\left(K_t^i\right)^{\alpha_1} \left(L_t^{i,w}\right)^{\alpha_2} \left(L_t^{i,k}\right)^{\alpha_3} \left(\frac{K_t^g}{N^i}\right)^{\alpha_4} \right]^{\theta} = w_t^w \Rightarrow$$

$$\theta\alpha_2 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^{\theta}}{L_t^{i,w}} = w_t^w \quad (1.C)$$

$$\frac{\partial \pi_t^i}{\partial L_t^{i,k}} = 0 \Rightarrow Y_t^{1-\theta} \mathrm{A}^{\theta} \left(K_t^i\right)^{\alpha_1 \theta} \left(L_t^{i,w}\right)^{\theta\alpha_2} \theta \alpha_3 \left(L_t^{i,k}\right)^{\theta\alpha_3 - 1} \left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4} = w_t^k \Rightarrow$$

$$\frac{\theta\alpha_3}{L_t^{i,k}} Y_t^{1-\theta} \left[\mathrm{A}\left(K_t^i\right)^{\alpha_1} \left(L_t^{i,w}\right)^{\alpha_2} \left(L_t^{i,k}\right)^{\alpha_3} \left(\frac{K_t^g}{N^i}\right)^{\alpha_4} \right]^{\theta} = w_t^k \Rightarrow$$

$$\theta\alpha_3 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^{\theta}}{L_t^{i,k}} = w_t^k \quad (2.C)$$

$$\frac{\partial \pi_t^i}{\partial K_t^i} = 0 \Rightarrow Y_t^{1-\theta} \mathrm{A}^{\theta} \theta\alpha_1 \left(K_t^i\right)^{\alpha_1 \theta - 1} \left(L_t^{i,w}\right)^{\theta\alpha_2} \left(L_t^{i,k}\right)^{\theta\alpha_3} \left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4} = r_t^k \Rightarrow$$

$$\theta\alpha_1 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^{\theta}}{K_t^i} = r_t^k \quad (3.C)$$

Then, economic profits are:

$$\pi_t^i = P_t^i Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \xrightarrow{(1.B),(1.C),(2.C),(3.C)}$$

$$\pi_t^i = \left(\frac{Y_t}{Y_t^i}\right)^{1-\theta} Y_t^i - \theta\alpha_1 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^{\theta}}{K_t^i} K_t^i - \theta\alpha_2 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^{\theta}}{L_t^{i,w}} L_t^{i,w} - \theta\alpha_3 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^{\theta}}{L_t^{i,k}} L_t^{i,k} \Rightarrow$$

$$\pi_t^i = \left(\frac{Y_t}{Y_t^i}\right)^{1-\theta} Y_t^i - \theta\alpha_1 Y_t^{1-\theta}\left(Y_t^i\right)^{\theta} - \theta\alpha_2 Y_t^{1-\theta}\left(Y_t^i\right)^{\theta} - \theta\alpha_3 Y_t^{1-\theta}\left(Y_t^i\right)^{\theta} \Rightarrow$$

$$\pi_t^i = \left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} - \theta\alpha_1 Y_t^{1-\theta}\left(Y_t^i\right)^{\theta} - \theta\alpha_2 Y_t^{1-\theta}\left(Y_t^i\right)^{\theta} - \theta\alpha_3 Y_t^{1-\theta}\left(Y_t^i\right)^{\theta} \Rightarrow$$

$$\pi_t^i = \left(1 - \theta\alpha_1 - \theta\alpha_2 - \theta\alpha_3\right)\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} \quad (3.C)$$

<u>Appendix D: The right-to-manage union model</u>

$$
\left.
\begin{array}{l}
\max_{w_t^w} \; U_t^N = \left[\left(1-\tau_t^l\right) w_t^w \dfrac{n^k}{n^w} L_t^{i,w} + \bar{G}_t^u \left(1 - \dfrac{n^k}{n^w} L_t^{i,w} \right) - \bar{G}_t^u \right]^{\phi} \left[\pi_t^i + r_t^k K_t^i + w_t^k L_t^{i,k} \right]^{1-\phi} \\
s.t. \quad \dfrac{\partial \pi_t^i}{\partial L_t^{i,w}} = 0 \Rightarrow Y_t^{1-\theta} \mathrm{A}^{\theta} \left(K_t^i\right)^{\alpha_1 \theta} \theta\alpha_2 \left(L_t^{i,w}\right)^{\theta\alpha_2 - 1} \left(L_t^{i,k}\right)^{\theta\alpha_3} \left(\dfrac{K_t^g}{N^i} \right)^{\theta\alpha_4} = w_t^w \; (1.C) \\
\quad P_t^i = \left(\dfrac{Y_t}{Y_t^i} \right)^{1-\theta} \; (1.B) \\
\quad Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1} \left(L_t^{i,w}\right)^{\alpha_2} \left(L_t^{i,k}\right)^{\alpha_3} \left(\dfrac{K_t^g}{N^i} \right)^{\alpha_4} \\
\text{Where } \pi_t^i = P_t^i Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k}
\end{array}
\right\} \Rightarrow
$$

$$
\left.
\begin{array}{l}
\max_{w_t^w} \; U_t^N = \left[\left(1-\tau_t^l\right) w_t^w \dfrac{n^k}{n^w} L_t^{i,w} + \bar{G}_t^u \left(1 - \dfrac{n^k}{n^w} L_t^{i,w} \right) - \bar{G}_t^u \right]^{\phi} \left[\pi_t^i + r_t^k K_t^i + w_t^k L_t^{i,k} \right]^{1-\phi} \\
s.t. \quad \dfrac{\partial \pi_t^i}{\partial L_t^{i,w}} = 0 \Rightarrow Y_t^{1-\theta} \mathrm{A}^{\theta} \left(K_t^i\right)^{\alpha_1 \theta} \theta\alpha_2 \left(L_t^{i,w}\right)^{\theta\alpha_2 - 1} \left(L_t^{i,k}\right)^{\theta\alpha_3} \left(\dfrac{K_t^g}{N^i} \right)^{\theta\alpha_4} = w_t^w \\
\text{Where } \; \pi_t^i = P_t^i Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \xrightarrow{(1.B)} \\
\pi_t^i = \left(\dfrac{Y_t}{Y_t^i} \right)^{1-\theta} Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \Rightarrow \\
\pi_t^i = \left(Y_t\right)^{1-\theta} \left(Y_t^i\right)^{\theta} - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \xrightarrow{Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1}\left(L_t^{i,w}\right)^{\alpha_2}\left(L_t^{i,k}\right)^{\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\alpha_4}} \\
\pi_t^i = \left(Y_t\right)^{1-\theta} \left[\mathrm{A}\left(K_t^i\right)^{\alpha_1} \left(L_t^{i,w}\right)^{\alpha_2} \left(L_t^{i,k}\right)^{\alpha_3} \left(\dfrac{K_t^g}{N^i} \right)^{\alpha_4} \right]^{\theta} - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k}
\end{array}
\right\} \Rightarrow
$$

$$\left.\begin{array}{l}\max\limits_{w_t^w}\ U_t^N=\left[\left(1-\tau_t^l\right)w_t^w\,\dfrac{n^k}{n^w}L_t^{i,w}-\bar{G}_t^u\,\dfrac{n^k}{n^w}L_t^{i,w}\right]^{\phi}*\\ *\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\theta\alpha_1}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}-w_t^wL_t^{i,w}\right]^{1-\phi}\\ s.t.\ \ \dfrac{\partial\pi_t^i}{\partial L_t^{i,w}}=0\Rightarrow\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\theta\alpha_2\left(L_t^{i,w}\right)^{\theta\alpha_2-1}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}=w_t^w\end{array}\right\}\Rightarrow$$

$$\begin{array}{l}\max\limits_{L_t^{i,w}}\ U_t^N=\left\{\left(1-\tau_t^l\right)\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\theta\alpha_2\left(L_t^{i,w}\right)^{\theta\alpha_2-1}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\dfrac{n^k}{n^w}L_t^{i,w}-\bar{G}_t^u\,\dfrac{n^k}{n^w}L_t^{i,w}\right\}^{\phi}*\\ *\left\{\begin{array}{l}\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}-\\ -\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\theta\alpha_2\left(L_t^{i,w}\right)^{\theta\alpha_2-1}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]L_t^{i,w}\end{array}\right\}^{1-\phi}\Rightarrow\end{array}$$

$$\begin{array}{l}\max\limits_{L_t^{i,w}}\ U_t^N=\left\{\left(1-\tau_t^l\right)\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\theta\alpha_2\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\dfrac{n^k}{n^w}-\bar{G}_t^u\,\dfrac{n^k}{n^w}L_t^{i,w}\right\}^{\phi}*\\ *\left\{\begin{array}{l}\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}-\\ -\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\theta\alpha_2\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\end{array}\right\}^{1-\phi}\Rightarrow\end{array}$$

$$\begin{array}{l}\max\limits_{L_t^{i,w}}\ U_t^N=\left\{\left(1-\tau_t^l\right)\theta\alpha_2\dfrac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]-\bar{G}_t^u\,\dfrac{n^k}{n^w}L_t^{i,w}\right\}^{\phi}*\\ *\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^{\theta}\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\dfrac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\right\}^{1-\phi}\end{array}$$

The first order condition is:

$$\frac{\partial U_t^N}{\partial L_t^{i,w}} = 0 \Rightarrow \phi\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi-1} *$$

$$*\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\theta\alpha_2\left(L_t^{i,w}\right)^{\theta\alpha_2-1}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]-\bar{G}_t^u\frac{n^k}{n^w}\right\}*$$

$$*\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\right\}^{1-\phi} +$$

$$\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi} *$$

$$*\left(1-\phi\right)\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\right\}^{-\phi} *$$

$$*\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\theta\alpha_2\left(L_t^{i,w}\right)^{\theta\alpha_2-1}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\right\} = 0 \Rightarrow$$

$$\phi\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi-1} *$$

$$*\left\{\left(1-\tau_t^l\right)\left(\theta\alpha_2\right)^2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\left(L_t^{i,w}\right)^{-1}-\bar{G}_t^u\frac{n^k}{n^w}\right\}*$$

$$*\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\right\}^{1-\phi} +$$

$$\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi} *$$

$$*\left(1-\phi\right)\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\right\}^{-\phi} *$$

$$*\left\{\left(1-\theta\alpha_2\right)\theta\alpha_2\left[\left(Y_t\right)^{1-\theta}\mathrm{A}^\theta\left(K_t^i\right)^{\alpha_1\theta}\left(L_t^{i,w}\right)^{\theta\alpha_2}\left(L_t^{i,k}\right)^{\theta\alpha_3}\left(\frac{K_t^g}{N^i}\right)^{\theta\alpha_4}\right]\left(L_t^{i,w}\right)^{-1}\right\} = 0 \Rightarrow$$

$$\phi\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi-1}*$$

$$*\left\{\left(1-\tau_t^l\right)\left(\theta\alpha_2\right)^2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\left(L_t^{i,w}\right)^{-1}-\bar{G}_t^u\frac{n^k}{n^w}\right\}*$$

$$*\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\right\}^{1-\phi}+$$

$$+\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi}*$$

$$*\left(1-\phi\right)\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\right\}^{-\phi}\left\{\left(1-\theta\alpha_2\right)\theta\alpha_2\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\left(L_t^{i,w}\right)^{-1}\right\}=0\xrightarrow{*L_t^{i,w}}$$

$$\phi\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi-1}*$$

$$*\left\{\left(1-\tau_t^l\right)\left(\theta\alpha_2\right)^2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}*$$

$$*\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\right\}^{1-\phi}+$$

$$+\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi}*$$

$$*\left(1-\phi\right)\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\right\}^{-\phi}*$$

$$*\left\{\left(1-\theta\alpha_2\right)\theta\alpha_2\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\right\}=0\xrightarrow{*\frac{\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\right\}^{\phi}}{\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}^{\phi-1}}}$$

$$\phi*\left\{\left(1-\tau_t^l\right)\left(\theta\alpha_2\right)^2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}*$$

$$*\left\{\left(1-\theta\alpha_2\right)\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]\right\}+$$

$$+\left\{\left(1-\tau_t^l\right)\theta\alpha_2\frac{n^k}{n^w}\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]-\bar{G}_t^u\frac{n^k}{n^w}L_t^{i,w}\right\}*$$

$$*\left(1-\phi\right)\left(1-\theta\alpha_2\right)\theta\alpha_2\left[\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]=0\Rightarrow$$

$$\phi\left[\left(1-\tau_t^l\right)\left(\theta\alpha_2\right)^2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}-\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w}\right]\left(1-\theta\alpha_2\right)\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}+$$

$$+\left[\left(1-\tau_t^l\right)\theta\alpha_2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}-\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w}\right]\left(1-\phi\right)\left(1-\theta\alpha_2\right)\theta\alpha_2\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}=0 \Rightarrow$$

$$\phi\left[\left(1-\tau_t^l\right)\left(\theta\alpha_2\right)^2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}-\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w}\right]+$$

$$+\left(1-\phi\right)\theta\alpha_2\left[\left(1-\tau_t^l\right)\theta\alpha_2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}-\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w}\right]=0 \Rightarrow$$

$$\left[\phi\left(1-\tau_t^l\right)\left(\theta\alpha_2\right)^2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}-\phi\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w}\right]+$$

$$+\left[\left(1-\phi\right)\theta\alpha_2\left(1-\tau_t^l\right)\theta\alpha_2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}-\left(1-\phi\right)\theta\alpha_2\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w}\right]=0 \Rightarrow$$

$$\left[\phi\left(1-\tau_t^l\right)\left(\theta\alpha_2\right)^2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]+\left[\left(1-\phi\right)\theta\alpha_2\left(1-\tau_t^l\right)\theta\alpha_2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}\right]=$$

$$=\phi\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w}+\left(1-\phi\right)\theta\alpha_2\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w} \Rightarrow$$

$$\theta\alpha_2\left(1-\tau_t^l\right)\theta\alpha_2 \frac{n^k}{n^w}\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}=\left[\phi+\left(1-\phi\right)\theta\alpha_2\right]\bar{G}_t^u \frac{n^k}{n^w} L_t^{i,w} \Rightarrow$$

$$\left(1-\tau_t^l\right)\theta\alpha_2\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta}=\frac{\phi+\left(1-\phi\right)\theta\alpha_2}{\theta\alpha_2}\bar{G}_t^u L_t^{i,w}$$

Appendix E: The resource constraint

The aggregate government budget constraint:

$$N\overline{G}_t^c + N\overline{G}_t^t + N\overline{G}_t^i + \left(N - N^k L_t^{i,w} - N^k L_t^{i,k}\right)\overline{G}_t^u + (1+r_t^b)B_t =$$

$$= N^k \tau_t^k \left(r_t^k - \delta\right) K_t^k + N^k \tau_t^k \pi_t^k + N^k \tau_t^l w_t^k + N^w \tau_t^l w_t^w e_t^w + N^w \tau_t^c C_t^w + N^k \tau_t^c C_t^k + B_{t+1} \xrightarrow{B_t = N^k B_t^k}$$

$$N\overline{G}_t^c + N\overline{G}_t^t + N\overline{G}_t^i + \left(N - N^k L_t^{i,w} - N^k L_t^{i,k}\right)\overline{G}_t^u + N^k(1+r_t^b)B_t^k =$$

$$= N^k \tau_t^k \left(r_t^k - \delta\right) K_t^k + N^k \tau_t^k \pi_t^k + N^k \tau_t^l w_t^k + N^w \tau_t^l w_t^w e_t^w + N^w \tau_t^c C_t^w + N^k \tau_t^c C_t^k + N^k B_{t+1}^k$$

If we divide the above equation with N, we have the per-capita government budget constraint:

$$\overline{G}_t^c + \overline{G}_t^t + \overline{G}_t^i + \left(1 - n^w e_t^w - n^k\right)\overline{G}_t^u + n^k(1+r_t^b)B_t^k =$$

$$= n^k \tau_t^k \left(r_t^k - \delta\right) K_t^k + n^k \tau_t^k \pi_t^k + n^k \tau_t^l w_t^k + n^w \tau_t^l w_t^w e_t^w + n^w \tau_t^c C_t^w + n^k \tau_t^c C_t^k + n^k B_{t+1}^k (2.E)$$

The entrepreneur's budget constraint:

$$(1+\tau_t^c)C_t^k + K_{t+1}^k - (1-\delta^p)K_t^k + B_{t+1}^k =$$

$$r_t^k K_t^k - \tau_t^k\left(r_t^k - \delta^p\right)K_t^k + (1+r_t^b)B_t^k + \left(1-\tau_t^k\right)\pi_t^k + \left(1-\tau_t^l\right)w_t^k + \bar{G}_t^t \xrightarrow{*n^k}$$

$$n^k(1+\tau_t^c)C_t^k + n^k K_{t+1}^k - n^k(1-\delta^p)K_t^k + n^k B_{t+1}^k =$$

$$n^k r_t^k K_t^k - n^k\tau_t^k\left(r_t^k - \delta^p\right)K_t^k + n^k(1+r_t^b)B_t^k + n^k\left(1-\tau_t^k\right)\pi_t^k + n^k\left(1-\tau_t^l\right)w_t^k + n^k\bar{G}_t^t \,(4.E)$$

The worker's budget constraint:

$$\left(1+\tau_t^c\right)C_t^w = \left(1-\tau_t^l\right)w_t^w e_t^w + \bar{G}_t^u\left(1-e_t^w\right) + \bar{G}_t^t \xrightarrow{*n^w}$$

$$n^w\left(1+\tau_t^c\right)C_t^w = n^w\left(1-\tau_t^l\right)w_t^w e_t^w + n^w\bar{G}_t^u\left(1-e_t^w\right) + n^w\bar{G}_t^t \,(6.E)$$

If we add the above equations, we finally have:

$$n^k\left(1+\tau_t^c\right)C_t^k + n^w\left(1+\tau_t^c\right)C_t^w + n^k\left[K_{t+1}^k - (1-\delta^p)K_t^k\right] + n^k B_{t+1}^k =$$

$$n^k\left[r_t K_t^k - \tau_t^k\left(r_t^k - \delta^p\right)K_t^k\right] + n^k(1+r_t^b)B_t^k + n^k\left(1-\tau_t^k\right)\pi_t^k + n^k\left(1-\tau_t^l\right)w_t^k + n^w\left(1-\tau_t^l\right)w_t^w e_t^w + n^w\bar{G}_t^u\left(1-e_t^w\right) + \bar{G}_t^t$$

$$\xrightarrow{\bar{G}_t^t = -\bar{G}_t^c - \bar{G}_t^i - \left(n^w - n^w e_t^w\right)\bar{G}_t^u - n^k(1+r_t^b)B_t^k + n^k\tau_t^k\left(r_t^k - \delta\right)K_t^k + n^k\tau_t^k\pi_t^k + n^k\tau_t^l w_t^k + n^w\tau_t^l w_t^w e_t^w + n^w\tau_t^c C_t^w + n^k\tau_t^c C_t^k + n^k B_{t+1}^k \,(2.E)}$$

$$n^k\left(1+\tau_t^c\right)C_t^k + n^w\left(1+\tau_t^c\right)C_t^w + n^k\left[K_{t+1}^k - (1-\delta^p)K_t^k\right] + n^k B_{t+1}^k =$$

$$n^k\left[r_t K_t^k - \tau_t^k\left(r_t^k - \delta^p\right)K_t^k\right] + n^k(1+r_t^b)B_t^k + n^k\left(1-\tau_t^k\right)\pi_t^k + n^k\left(1-\tau_t^l\right)w_t^k + n^w\left(1-\tau_t^l\right)w_t^w e_t^w + n^w\bar{G}_t^u\left(1-e_t^w\right) -$$

$$-\bar{G}_t^c - \bar{G}_t^i - n^w\bar{G}_t^u\left(1-e_t^w\right) - n^k(1+r_t^b)B_t^k + n^k\tau_t^k\left(r_t^k - \delta^p\right)K_t^k + n^k\tau_t^k\pi_t^k + \tau_t^l\left(n^k w_t^k + n^w w_t^w e_t^w\right) + \tau_t^c\left(n^w C_t^w + n^k C_t^k\right) + n^k B_{t+1}^k \Rightarrow$$

$$n^k C_t^k + n^k\tau_t^c C_t^k + n^w C_t^w + \tau_t^c n^w C_t^w + n^k K_{t+1}^k - n^k(1-\delta^p)K_t^k + n^k B_{t+1}^k =$$

$$n^k r_t K_t^k - n^k\tau_t^k\left(r_t^k - \delta^p\right)K_t^k + n^k\pi_t^k - \tau_t^k n^k\pi_t^k + n^k w_t^k - \tau_t^l n^k w_t^k + n^w w_t^w e_t^w - \tau_t^l n^w w_t^w e_t^w -$$

$$-\bar{G}_t^c - \bar{G}_t^i + n^k\tau_t^k\left(r_t^k - \delta^p\right)K_t^k + n^k\tau_t^k\pi_t^k + \tau_t^l\left(n^k w_t^k + n^w w_t^w e_t^w\right) + \tau_t^c\left(n^w C_t^w + n^k C_t^k\right) + n^k B_{t+1}^k \Rightarrow$$

$$n^k C_t^k + n^w C_t^w + n^k K_{t+1}^k - n^k(1-\delta^p)K_t^k =$$

$$n^k r_t K_t^k + n^k\pi_t^k + n^k w_t^k + n^w w_t^w e_t^w - \bar{G}_t^c - \bar{G}_t^i \Rightarrow$$

$$n^k C_t^k + n^w C_t^w + n^k K_{t+1}^k - n^k(1-\delta^p)K_t^k + \bar{G}_t^c + \bar{G}_t^i =$$

$$n^k r_t K_t^k + n^k\pi_t^k + n^k w_t^k + n^w w_t^w e_t^w \,(7.E)$$

The profits earned by the intermediate goods producer at time t are:

$$\pi_t^i = P_t^i Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k}\,(7) \xrightarrow{P_t^i = \left(\frac{Y_t}{Y_t^i}\right)^{1-\theta}}$$

$$\pi_t^i = \left(\frac{Y_t}{Y_t^i}\right)^{1-\theta} Y_t^i - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \Rightarrow$$

$$\pi_t^i = \left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} - r_t^k K_t^i - w_t^w L_t^{i,w} - w_t^k L_t^{i,k} \xrightarrow{*n^k}$$

$$n^k \pi_t^i = n^k \left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} - n^k r_t^k K_t^i - n^k w_t^w L_t^{i,w} - n^k w_t^k L_t^{i,k} \xrightarrow{\pi_t^i = \pi_t^k,\ n^k L_t^{i,w} = n^w e_t^w,\ L_t^{i,k} = 1}$$

$$n^k \pi_t^k = n^k \left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} - n^k r_t^k K_t^k - n^w w_t^w e_t^w - n^k w_t^k \Rightarrow$$

$$n^k \left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} = n^k \pi_t^k + n^k r_t^k K_t^k + n^w w_t^w e_t^w + n^k w_t^k\,(8.E)$$

Then, the resource constraint is:

$$(7.E) \xrightarrow{(8.E)} n^k \left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} = n^k C_t^k + n^w C_t^w + n^k\left[K_{t+1}^k - (1-\delta^p)K_t^k\right] + \bar{G}_t^c + \bar{G}_t^i \Rightarrow$$

$$n^k \left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} = n^k C_t^k + n^w C_t^w + n^k\left[K_{t+1}^k - (1-\delta^p)K_t^k\right] + \bar{G}_t^c + \bar{G}_t^i\,(9.E)$$

<u>Appendix F.1: The Decentralized Equilibrium</u>

Substituting $Y_t^i = Y_t$, $\bar{k}_t^g = \frac{K_t^g}{N}$, $P_t^i = 1$, $K_t^k = K_t^i$, $\pi_t^k = \pi_t^i$, $B_t = N^k B_t^k$, $n^w e_t^w = n^k L_t^{i,w}$, $1 = L_t^{i,k}$, $s_t^c \equiv \frac{N\bar{G}_t^c}{N^k Y_t^i} = \frac{\bar{G}_t^c}{n^k Y_t^i}$,

$$s_t^i \equiv \frac{N\bar{G}_t^i}{N^k Y_t^i} = \frac{\bar{G}_t^i}{n^k Y_t^i}$$,

$$s_t^u \equiv \frac{\bar{G}_t^u (N - N^k L_t^{i,w} - N^k L_t^{i,k})}{N^k Y_t^i} = \frac{\bar{G}_t^u \left(\frac{N}{N} - \frac{N^k}{N} L_t^{i,w} - \frac{N^k}{N} L_t^{i,k} \right)}{\frac{N^k}{N} Y_t^i} = \frac{\bar{G}_t^u (1 - n^k L_t^{i,w} - n^k L_t^{i,k})}{n^k Y_t^i} = \frac{\bar{G}_t^u (1 - n^k - n^w e_t^w)}{n^k Y_t^i}$$

or

$$s_t^u \equiv \frac{\bar{G}_t^u (N^w - N^k L_t^{i,w})}{N^k Y_t^i} = \frac{\bar{G}_t^u \left(\frac{N^w}{N} - \frac{N^k}{N} L_t^{i,w} \right)}{\frac{N^k}{N} Y_t^i} = \frac{\bar{G}_t^u (n^w - n^k L_t^{i,w})}{n^k Y_t^i} = \frac{\bar{G}_t^u (n^w - n^w e_t^w)}{n^k Y_t^i} = \frac{n^w \bar{G}_t^u (1 - e_t^w)}{n^k Y_t^i}$$

and $s_t^t \equiv \frac{N\bar{G}_t^t}{N^k Y_t^i} = \frac{\bar{G}_t^t}{n^k Y_t^i}$ the system is:

The entrepreneur's Euler equation with respect to capital:

$$\frac{(1+\tau_{t+1}^c)\left(C_{t+1}^k + \psi s_{t+1}^c n^k Y_{t+1}^i\right)^\sigma}{(1+\tau_t^c)\left(C_t^k + \psi s_t^c n^k Y_t^i\right)^\sigma} = \beta\left[1 + \left(1 - \tau_{t+1}^k\right)\left(r_{t+1}^k - \delta^p\right)\right] \quad (1.F.1)$$

The entrepreneur's Euler equation with respect to bonds:

$$r_{t+1}^b = \left(1 - \tau_{t+1}^k\right)\left(r_{t+1}^k - \delta^p\right) \quad (2.F.1)$$

The worker's budget constraint:

$$(1+\tau_t^c) C_t^w = \left(1 - \tau_t^l\right) w_t^w e_t^w + \bar{G}_t^u \left(1 - e_t^w\right) + \bar{G}_t^t \xrightarrow{s_t^u = \frac{n^w \bar{G}_t^u (1-e_t^w)}{n^k Y_t^i},\, s_t^t = \frac{\bar{G}_t^t}{n^k Y_t^i}}$$

$$(1+\tau_t^c) C_t^w = \left(1 - \tau_t^l\right) w_t^w e_t^w + \frac{s_t^u}{n^w} n^k Y_t^i + s_t^t n^k Y_t^i \Rightarrow \quad (3.F.1)$$

$$(1+\tau_t^c) C_t^w = \left(1 - \tau_t^l\right) w_t^w e_t^w + \left(\frac{s_t^u}{n^w} + s_t^t\right) n^k Y_t^i$$

The intermediate firm's optimality condition for $L_t^{i,w}$:

$$\theta\alpha_2 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^\theta}{L_t^{i,w}} = w_t^w \xrightarrow{\text{Symmetry}} \theta\alpha_2 \frac{Y_t^i}{L_t^{i,w}} = w_t^w \xrightarrow{n^w e_t^w = n^k L_t^{i,w}} \theta\alpha_2 \frac{Y_t^i}{\frac{n^w}{n^k} e_t^w} = w_t^w \quad (4.F.1)$$

The intermediate firm's optimality condition for $L_t^{i,k}$:

$$\theta\alpha_2 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^\theta}{L_t^{i,k}} = w_t^k \xrightarrow{\text{Symmetry}} \mathrm{A}^k \theta\alpha_2 \frac{Y_t^i}{L_t^{i,k}} = w_t^k \xrightarrow{1 = L_t^{i,w}} \theta\alpha_2 Y_t^i = w_t^k \quad (4.F.1)$$

The intermediate firm's optimality condition for K_t^i:

$$\theta\alpha_1 \frac{Y_t^{1-\theta}\left(Y_t^i\right)^\theta}{K_t^i} = r_t^k \xrightarrow{\text{Symmetry}, K_t^k = K_t^i} \theta\alpha_1 \frac{Y_t^i}{K_t^k} = r_t^k \quad (5.F.1)$$

The intermediate firm's profit function:

$$\pi_t^i = \left(1-\theta\alpha_1-\theta\alpha_2-\theta\alpha_3\right)\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^\theta \xrightarrow{symmetry, \pi_t^k = \pi_t^i} \pi_t^k = \left(1-\theta\alpha_1-\theta\alpha_2-\theta\alpha_3\right)Y_t^i \quad (6.F.1)$$

The intermediate firm's production function:

$$Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1}\left(L_t^{i,w}\right)^{\alpha_2}\left(L_t^{i,k}\right)^{\alpha_3}\left(\frac{K_t^g}{N^k}\right)^{\alpha_4} \Rightarrow Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1}\left(L_t^{i,w}\right)^{\alpha_2}\left(L_t^{i,k}\right)^{\alpha_3}\left(\frac{K_t^g}{N^k}\frac{N}{N}\right)^{\alpha_4} \Rightarrow$$

$$Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1}\left(L_t^{i,w}\right)^{\alpha_2}\left(L_t^{i,k}\right)^{\alpha_3}\left(\frac{K_t^g}{N}\frac{N}{N^k}\right)^{\alpha_4} \xrightarrow{\bar{k}_t^g \equiv \frac{K_t^g}{N}} Y_t^i = \mathrm{A}\left(K_t^i\right)^{\alpha_1}\left(L_t^{i,w}\right)^{\alpha_2}\left(L_t^{i,k}\right)^{\alpha_3}\left(\frac{\bar{k}_t^g}{n^k}\right)^{\alpha_4}$$

$$\xrightarrow{K_t^k = K_t^i,\, n^w e_t^w = n^k L_t^{i,k},\, 1 = L_t^{i,k}} Y_t^i = \mathrm{A}\left(K_t^k\right)^{\alpha_1}\left(\frac{n^w}{n^k}e_t^w\right)^{\alpha_2}\left(\frac{\bar{k}_t^g}{n^k}\right)^{\alpha_4} \tag{7.F.1}$$

$$\left(1-\tau_t^l\right)\theta\alpha_2\left(Y_t\right)^{1-\theta}\left(Y_t^i\right)^{\theta} = \frac{\left[\phi+\left(1-\phi\right)\theta\alpha_2\right]}{\theta\alpha_2}\bar{G}_t^u L_t^{i,w} \xrightarrow{\text{Symmetry}, \mathrm{n}^w e_t^w = n^k L_t^{i,w},\, 1 = L_t^{i,k}}$$

$$\left(1-\tau_t^l\right)\theta\alpha_2 Y_t^i = \frac{\left[\phi+\left(1-\phi\right)\theta\alpha_2\right]}{\theta\alpha_2}\bar{G}_t^u \frac{n^w}{n^k}e_t^w \xrightarrow{s_t^u = \frac{n^w \bar{G}_t^u (1-e_t^w)}{n^k Y_t^i} \Rightarrow \frac{s_t^u n^k Y_t^i}{n^w (1-e_t^w)} = \bar{G}_t^u} \tag{8.F.1}$$

$$\left(1-\tau_t^l\right)\theta\alpha_2 Y_t^i = \frac{\left[\phi+\left(1-\phi\right)\theta\alpha_2\right]}{\theta\alpha_2} s_t^u Y_t^i \frac{e_t^w}{1-e_t^w}$$

The Government's Budget Constraint:

$$\bar{G}_t^c + \bar{G}_t^t + \bar{G}_t^i + n^w \bar{G}_t^u\left(1-e_t^w\right) + n^k(1+r_t^b)B_t^k =$$
$$= n^k\tau_t^k\left(r_t^k-\delta^p\right)K_t^k + n^k\tau_t^k\pi_t^k + n^k\tau_t^l w_t^k + n^w\tau_t^l w_t^w e_t^w + n^w\tau_t^c C_t^w + n^k\tau_t^c C_t^k + n^k B_{t+1}^k$$

$$\xrightarrow{s_t^u = \frac{n^w\bar{G}_t^u(1-e_t^w)}{n^k Y_t^i},\, s_t^t = \frac{\bar{G}_t^t}{n^k Y_t^i},\, s_t^c = \frac{\bar{G}_t^c}{n^k Y_t^i},\, s_t^i = \frac{\bar{G}_t^i}{n^k Y_t^i}} \tag{9.F.1}$$

$$\left(s_t^c + s_t^t + s_t^i + s_t^u\right)n^k Y_t^i + n^k(1+r_t^b)B_t^k =$$
$$= n^k\tau_t^k\left(r_t^k-\delta^p\right)K_t^k + n^k\tau_t^k\pi_t^k + n^k\tau_t^l w_t^k + n^w\tau_t^l w_t^w e_t^w + n^w\tau_t^c C_t^w + n^k\tau_t^c C_t^k + n^k B_{t+1}^k$$

The law of motion of public capital:

$$\bar{k}_{t+1}^g = (1-\delta^g)\bar{k}_t^g + \bar{G}_t^i \xrightarrow{s_t^i = \frac{\bar{G}_t^i}{n^k Y_t^i}} \bar{k}_{t+1}^g = (1-\delta^g)\bar{k}_t^g + s_t^i n^k Y_t^i \tag{10.F.1}$$

The resource constraint:

$$n^k Y_t^{1-\theta}\left(Y_t^i\right)^{\theta} = n^k C_t^k + n^w C_t^w + n^k\left[K_{t+1}^k - (1-\delta^p)K_t^k\right] + \bar{G}_t^c + \bar{G}_t^i \xrightarrow{\text{Symmetry}}$$

$$n^k Y_t^i = n^k C_t^k + n^w C_t^w + n^k\left[K_{t+1}^k - (1-\delta^p)K_t^k\right] + \bar{G}_t^c + \bar{G}_t^i \xrightarrow{s_t^i = \frac{\bar{G}_t^i}{n^k Y_t^i},\, s_t^c = \frac{\bar{G}_t^c}{n^k Y_t^i}} \quad (11.F.1)$$

$$\left(1 - s_t^c - s_t^i\right) n^k Y_t^i = n^k C_t^k + n^w C_t^w + n^k\left[K_{t+1}^k - (1-\delta^p)K_t^k\right]$$

Appendix G: The Steady State

$$\frac{(1+\tau_{t+1}^c)\left(C_{t+1}^k\right)^{\sigma}}{(1+\tau_t^c)\left(C_t^k\right)^{\sigma}} = \beta\left[1+\left(1-\tau_{t+1}^k\right)\left(r_{t+1}^k - \delta^p\right)\right](18) \xrightarrow{\text{Long run}} 1 = \beta\left[1+\left(1-\tau^k\right)\left(r^k - \delta^p\right)\right](18*)$$

$$r_{t+1}^b = \left(1-\tau_{t+1}^k\right)\left(r_{t+1}^k - \delta^p\right)(19) \xrightarrow{\text{Long run}} r^b = \left(1-\tau^k\right)\left(r^k - \delta^p\right)(19*)$$

$$(1+\tau_t^c)C_t^w = \left(1-\tau_t^l\right)w_t^w e_t^w + \bar{G}_t^u\left(1-e_t^w\right) + \bar{G}_t^t(3) \xrightarrow{\text{Long run}} (1+\tau^c)C^w = \left(1-\tau^l\right)w^w e^w + \bar{G}^u\left(1-e^w\right) + \bar{G}^t(3*)$$

$$\frac{\theta\alpha_2 Y_t^i}{\frac{n^w}{n^k}e_t^w} = w_t^w(21artm") \xrightarrow{\text{Long run}} \frac{\theta\alpha_2 Y^i}{\frac{n^w}{n^k}e^w} = w^w(21artm*)$$

$$\theta\alpha_2 Y_t^i = w_t^k(21brtm") \xrightarrow{\text{Long run}} \theta\alpha_2 Y^i = w^k(21brtm*)$$

$$\theta\alpha_1\frac{Y_t^i}{K_t^k} = r_t^k \;\; (22rtm") \xrightarrow{\text{Long run}} \theta\alpha_1\frac{Y^i}{K^k} = r^k \;\; (22rtm*)$$

$$\pi_t^k = \left(1-\theta\alpha_1-\theta\alpha_2-\theta\alpha_3\right)Y_t^i(23rtm") \xrightarrow{\text{Long run}} \pi^k = \left(1-\theta\alpha_1-\theta\alpha_2-\theta\alpha_3\right)Y^i(23rtm*)$$

$$Y_t^i = \mathrm{A}\left(K_t^k\right)^{\alpha_1}\left(\frac{n^w}{n^k}e_t^i\right)^{\alpha_2}\left(\frac{\overline{k}_t^{\,g}}{n^k}\right)^{1-\alpha_1-\alpha_2-\alpha_3} \quad (6') \xrightarrow{\text{Long run}} Y^i = \mathrm{A}\left(K^k\right)^{\alpha_1}\left(\frac{n^w}{n^k}e^i\right)^{\alpha_2}\left(\frac{\overline{k}^{\,g}}{n^k}\right)^{1-\alpha_1-\alpha_2-\alpha_3} \quad (6^*)$$

$$\left(1-\tau_t^l\right)\theta\alpha_2 Y_t^i = \frac{\left[\phi+\left(1-\phi\right)\alpha_2\theta\right]}{\theta\alpha_2}\overline{G}_t^u\,\frac{n^w}{n^k}e_t^w\,(24rtm'') \xrightarrow{\text{Long run}}$$

$$\left(1-\tau^l\right)\theta\alpha_2 Y^i = \frac{\left[\phi+\left(1-\phi\right)\alpha_2\theta\right]}{\theta\alpha_2}\overline{G}^u\,\frac{n^w}{n^k}e^w\,(24rtm^*)$$

$$\overline{G}_t^c + \overline{G}_t^t + \overline{G}_t^i + n^w\overline{G}_t^u\left(1-e_t^w\right) + n^k(1+r_t^b)B_t^k =$$
$$= n^k\tau_t^k\left(r_t^k-\delta^p\right)K_t^k + n^k\tau_t^k\pi_t^k + n^k\tau_t^l w_t^k + n^w\tau_t^l w_t^w e_t^w + n^w\tau_t^c C_t^w + n^k\tau_t^c C_t^k + n^k B_{t+1}^k \xrightarrow{\text{Long run}}$$
$$\overline{G}^c + \overline{G}^t + \overline{G}^i + n^w\overline{G}^u\left(1-e^w\right) + n^k(1+r^b)B^k =$$
$$= n^k\tau^k\left(r^k-\delta^p\right)K^k + n^k\tau^k\pi^k + n^k\tau^l w^k + n^w\tau^l w^w e^w + n^w\tau^c C^w + n^k\tau^c C^k + n^k B^k\,(9^*)$$

$$\overline{k}_{t+1}^{\,g} = (1-\delta^g)\overline{k}_t^{\,g} + s_t^i n^k Y_t^i\,(25) \xrightarrow{\text{Long run}} \delta^g\overline{k}^{\,g} = s^i n^k Y^i\,(25^*)$$

$$n^k Y_t^i = n^k C_t^k + n^w C_t^w + n^k\left[K_{t+1}^k-(1-\delta^p)K_t^k\right] + \overline{G}_t^c + \overline{G}_t^i\,(14') \xrightarrow{\text{Long run}}$$
$$n^k Y^i = n^k C^k + n^w C^w + n^k\delta^p K^k + \overline{G}^c + \overline{G}^i \Rightarrow$$
$$\left(1-s^c-s^i\right)n^k Y^i = n^k C^k + n^w C^w + n^k\delta^p K^k\,(14^*)$$

12 equations in $K^k, \overline{k}^{\,g}, \pi^k, e^w, C^k, C^w, r^b, r^k, w^w, w^k, Y^i$ and one of the eight policy instruments $\tau^c, \tau^l, \tau^k, \overline{G}^u, \overline{G}^t, \overline{G}^c, \overline{G}^i, B^k$