The driving forces of the current Greek great depression

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Abstract: This paper provides a quantitative study of the main determinants of the Greek great depression since 2010. We use a medium-scale DSGE model calibrated to the Greek economy between 2000 and 2009 (the euphoria years that followed the adoption of the euro). Then, departing from 2010, our simulations show that the fiscal policy mix adopted, jointly with the deterioration in institutional quality and, specifically, in the degree of protection of property rights, can explain essentially all the total loss in GDP between 2010 and 2015 (around 26%). In particular, the fiscal policy mix accounts for 14% of the total output loss, while the deterioration in property rights accounts for another 8%. It thus naturally follows that a less distorting fiscal policy mix and a stronger protection of property rights are necessary conditions for economic recovery in this country.

Key words: Growth, public debt, institutions.
JEL classification numbers: O4, H6, E02.

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1. Introduction

Following the world financial crisis in 2008, most European Union countries have managed to pull out of recession since 2014. A distinct exception is Greece which has not yet entered a recovery mode (see European Commission, 2016, and CESifo, 2016). The Greek economy has been shrinking since 2009 and Greece has lost around 26% of its GDP over 2010-2015. Thus, the episode seems to satisfy all conditions of a “great depression” (see Kehoe and Prescott, 2002). Actually, and making it worse, the country is in a multiple crisis; public debt is around 177% of GDP, foreign debt is around 142% of GDP, unemployment is around 25% and there is still an environment of political uncertainty and polarization.

Despite three bailout packages of around 300 billion euros so far (financed by the European Union, the European Central Bank and the IMF), several structural reforms and the recent improvement in the international economic environment, Greece has not yet shown any sign of real recovery. Paradoxically, most of policymakers, both in Greece and the EU, have been searching for engines of economic growth, without having first studied the determinants of the continuing depression. The present paper tries to fill this gap. Identifying the barriers to growth is a prerequisite for credibly suggesting potential engines of growth.\footnote{2}

In particular, the aim of the current paper is to decompose the above loss in output into its main drivers. Our main results are as follows. Using a medium-scale DSGE model carefully calibrated to the Greek economy, our simulations show that the fiscal policy mix adopted, jointly with developments in institutional quality, and specifically in the degree of protection of property rights, can explain around 85% of the total loss in GDP between 2010 and 2015. In particular, when we use the tax-spending mix as it has been in the data since 2010, and we also assume that the observed deterioration in an index of property rights manifests itself into a decline in total factor productivity, our model can explain around 22% fall in GDP since 2010 (as said, the total loss in the data has been around 26%). We also show that the portion due to the fiscal policy mix is 14%, while the portion due to the deterioration in property rights is another 8%.
Two clarifications are necessary from the outset. The first is about fiscal consolidation. Our results should not be interpreted as saying that most of the Greek crisis is a consequence of fiscal austerity. A kind of fiscal austerity was necessary, given the imbalances inherited from the past; once sovereign risk premia emerged in 2010, Greek governments could not choose but undertake severe fiscal consolidation measures. Actually, as perhaps should be expected, when we simulate our model under the counter-factual scenario that fiscal policy had remained unchanged as in 2010, the model cannot deliver a dynamically stable solution implying an unsustainable fiscal situation, which, in simple words, means that the continuation of the status quo was not possible anymore and that some kind of fiscal stabilization was necessary. What our results do hint, however, is that the recessionary effects of fiscal stabilization could perhaps have been milder, had the policy mix been different from that actually adopted; Greece’s fiscal stabilization has been based on both spending cuts and tax rises but the increase in taxes has been particularly high (see subsection 3.1 below).³

The second clarification is about institutional quality. The importance of institutional quality, and especially of property rights, for economic growth is well known in the growth literature (see e.g. Acemoglu, 2009, chapter 4, for a review). It should be stressed that property rights may be affected by tax policy, but they are also affected by the quality of public order and safety, where the sharp deterioration of the latter is clearly documented in the Greek data since 2004 and, especially, after 2008 (see subsection 3.2 below). Thus, it should not come as a surprise, at least qualitatively, that this institutional deterioration is a driver of the Greek depression; on the other hand, our simulations show that its quantitative importance for the output loss is striking.

The way we work is as follows. We employ a medium-scale new-Keynesian DSGE model of a small open economy enriched with a number of real and nominal frictions so as to capture the main empirical features of the Greek economy.⁴ The model is calibrated to data up to and including the year 2009. We take 2009 as the pre-depression benchmark year because the first memorandum with the Troika (EU, ECB and IMF) was agreed in 2010. This first memorandum, as well as the next two in 2012 and 2015, have provided financial assistance and have offered credit to the Greek
economy at much more favourable terms than markets would have provided, but they have been “conditioned on” fiscal austerity measures (namely, measures to improve debt dynamics) and structural reforms that have been highly criticized and have led to political polarization and social unrest. Then, departing from 2010 and assuming an initial unanticipated shock to public debt as observed in the data during that year, we simulate the effects of the tax-spending mix, as it has been in the actual data during 2010-2015, so as to quantify the portion of the output loss caused by this particular policy mix. In turn, we repeat the same exercise by adding the effects of the deterioration in the property rights index, again as it has been in the actual data up to 2015, by assuming that this deterioration affects the efficiency, or productivity, with which factor inputs are used (namely, it affects the so-called TFP). Quoting Acemoglu (2009, p. 105), “when countries have large drops in their income, due to political instability, etc., these drops are associated with corresponding declines in TFP”.

A paper close to ours is Gourinchas et al. (2016), who also use a micro-founded DSGE model to analyze the Greek crisis. In their paper, the crisis is driven by a large menu of shocks, including shocks to default rates, banks’ funding costs, etc. We however believe that such variables can hardly be considered as (extrinsic) shocks. Here, by contrast, we try to identify the primitive sources of “shocks”. We show that the particular fiscal policy mix adopted and the deterioration in institutional quality, both as documented in the actual time-series data, can explain most of the drop in output since 2010.

The rest of the paper is organized as follows. Section 2 introduces the model, explains its calibration and presents the steady state solution. Section 3 presents simulations. Section 4 closes the paper.

2. A DSGE model

In this section, we describe the model used and provide its numerical steady state solution. The latter will serve as a point of departure for the simulations in the next section.
2.1 Description of the model

Our quantitative results will be based on a medium-scale DSGE model of a small open economy calibrated to Greek data. The model is a variant of the model used by the Bank of Greece (see Papageorgiou, 2014). We choose to work with this particular model because it is used by an official institution, like the Bank of Greece, and also because it is relatively detailed and hence can capture the main features of the Greek macro economy.

The model exhibits a number of real and nominal frictions so as to capture the key features of the Greek economy and thus provide a parameterized general equilibrium model suitable for policy simulations. These frictions include imperfectly competitive labor and product markets, the distinction between Ricardian and non-Ricardian households, real wage rigidity, Calvo-type short-term nominal fixities, habit persistence, various adjustment costs, a variety of firms so as to capture tradable and non-tradable goods, a relatively rich public sector including the production of public goods/services by the use of public employees, loss of monetary policy independence since Greece is part of the euro zone and also an imperfect world capital market where the interest rate at which domestic agents borrow from the world capital market rises with public debt.

The building blocks of the model and its final equilibrium system are presented in detail in the Appendix (see Appendix A). The final equilibrium system consists of 89 equations in 89 endogenous variables. This is given the exogenously set policy instruments, initial conditions for the state variables and total factor productivity (TFP) in the two sectors, tradables and non-tradables.

2.2 Numerical solution of the model

The above model is calibrated to data from the Greek economy. This means that (most of) its parameter values match average data values and that the exogenously set policy instruments are set as in the data. The data source is Eurostat, unless otherwise stated. The data are at annual frequency and cover the period 2000-2015, although the period used for this calibration stage is up to and
including 2009 (as explained in the Introduction, we use pre-crisis euro period data). Table A1 in the Appendix reports the calibrated parameter values and the average values of fiscal policy variables in the data.

Using these numerical values, the system is then solved using a Newton-type non-linear method as implemented in DYNARE (see below for specification of transition dynamics). Its steady state solution (at least for the key variables) is reported in Table 1. In this solution, we have exogenously set the debt-to-GDP ratio equal to the threshold level $d = 126\%$, which was the value of the public debt-to-GDP in 2009 (that was the year that risk premia emerged in Greece), so that one of the remaining fiscal policy instruments needs to be determined residually to satisfy the within period government budget constraint; we assume that it is lump-sum taxes that play this role.

As Table 1 shows, the solution is in line with data averages over 2000-2009 and can thus provide a reasonable departure point for the changes that have been taking place since 2010 and are described in the next sections. In particular, the solution does a relatively good job at mimicking the position of the country (and its different sectors) in the international capital market, as well as the consumption-investment behavior of the private sector over the euro pre-crisis years.

Table 1: Steady state solution and data averages 2000-09

<table>
<thead>
<tr>
<th>Variable</th>
<th>data</th>
<th>solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total private consumption-to-GDP</td>
<td>0.65</td>
<td>0.59</td>
</tr>
<tr>
<td>Private investment-to-GDP</td>
<td>0.18</td>
<td>0.17</td>
</tr>
<tr>
<td>Total work hours</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>Work hours in private sector</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Total public debt-to-GDP</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>Lump-sum taxes/transfers</td>
<td>-</td>
<td>0.045</td>
</tr>
<tr>
<td>Economy's net foreign liabilities-to-GDP</td>
<td>0.77</td>
<td>0.66</td>
</tr>
<tr>
<td>Private net foreign liabilities-to-GDP</td>
<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td>Exports-to-GDP</td>
<td>0.23</td>
<td>0.27</td>
</tr>
<tr>
<td>Total imports-to-GDP</td>
<td>0.34</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Note: (i) Average data over the euro period 2000-2009, with the exception of foreign liabilities which are over the period 2003-2009 and the public debt-to-GDP ratio which is set at its 2009 data value. The data source is Eurostat and the Bank of Greece. (ii) A positive value of the net foreign liabilities-to-GDP ratio means that the domestic country is a net borrower.
3. **Simulations**

As said above, departing from the “steady state” solution in Table 2, we will now simulate the above economy when fiscal policy and institutional quality change as observed in the data after 2010. To understand how the model works, we will start by assuming that only fiscal policy has changed and then we will add changes in institutional quality. That is, we study one dynamic driver at a time.

3.1 **Effects of the fiscal austerity mix as adopted in practice**

In this subsection, we will examine, other things equal, the impact of fiscal consolidation policies as adopted in Greece since 2010.

We work as follows. We assume that in 2010 there was an initial shock/increase in the public debt-to-GDP ratio by 20 pp (as observed in the data). We then set all exogenous fiscal (tax-spending) instruments as they have actually been in the data during 2010-15 (to isolate the impact of actual fiscal policy, we switch-off the extra feedback reaction to public debt during this sub-period). Besides, in order to mimic the memorandum package, we set the interest rate, at which the government borrows from abroad, as a weighted average of the risk-free world interest rate and the world interest rate that the economy would face if it had to borrow from the international capital market (the latter includes the country risk-premium as in the data). The private sector, on the other hand, continues to face the full world interest rate (that includes the country risk-premium) when it borrows from the international market. Recall that this premium is a function of the public debt gap, where, in this gap, the public debt threshold above which premia emerge is 126%.

We will assume that all the above features continue until the year 2015 (this is the year that this paper is being written in terms of data availability). Then, after 2015, the fiscal instruments are assumed to gradually return to their pre-crisis 2009 values. In particular, we assume that they follow an autoregressive process using as initial values the 2015 values and an autoregressive coefficient equal to 0.9. We allow one fiscal instrument to react to the public debt gap (see equation 27), where,
in this gap, the public debt target in the policy rules is the pre-shock value of 126%. The interest rate at which the government borrows from abroad is now allowed to react fully to the degree of government’s indebtedness.

Thus, in our first simulations, transition dynamics is driven by the above changes in fiscal policy. We solve the model under perfect foresight (as said above, we use a Newton-type non-linear method as implemented in DYNARE).

The simulated impulse response functions are plotted in Figure 1, while Table 2 summarizes the associated changes in the main macro variables vis-à-vis their values in the data. Inspection of the simulated results in the third column of Table 2, and comparison to the actual data in the second column, implies that the GDP decreases by around 14% between 2009 and 2015. In the data, the actual decrease has been 26% during the same time interval. That is, the particular fiscal austerity package, which has been adopted between 2010 and 2015, can account for more than half of the big fall in output observed in the data during this period.

**Figure 1: Impulse response functions driven by the fiscal austerity package**

![Graphs of various economic variables over time](image.png)

*Note: All variables are expressed as percentage deviations from the steady-state, with the exception of the CPI inflation, the interest rate, foreign assets and the public debt-to-GDP ratio that are expressed as percentage point deviations.*
Table 2: Changes in the main macro variables 2015-2009 (%)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Simulated model with the fiscal package</th>
<th>Simulated model with the fiscal package plus institutional shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>-26</td>
<td>-13.7</td>
<td>-22</td>
</tr>
<tr>
<td>Real private consumption</td>
<td>-27.7</td>
<td>-4.6</td>
<td>-9.1</td>
</tr>
<tr>
<td>Real private investment</td>
<td>-60</td>
<td>-19.1</td>
<td>-40.6</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>8</td>
<td>2.8</td>
<td>-2.5</td>
</tr>
<tr>
<td>Real exports</td>
<td>18.2</td>
<td>2.6</td>
<td>-6</td>
</tr>
</tbody>
</table>

Note: (i) The changes in the actual time series are computed as log deviations between their 2015 and 2009 values, with the exception of real private investment that is computed as \((I_{2015} - I_{2009}) / I_{2009}\). The data source is Eurostat. Changes in the simulated series correspond to log deviations from the initial steady state. (ii) A positive change for the real effective exchange rate means a real depreciation, i.e. an improvement in the country’s competitiveness.

Figure 2 depicts the dynamic paths of fiscal policy instruments under this scenario. It thus confirms the well-recognized feature that the Greek fiscal consolidation program has been based on both spending cuts and tax rises (see e.g. European Commission, 2015), although the clear rise in all effective tax rates is particularly striking for a country suffering from a deep recession.

Figure 2: Dynamic paths of fiscal policy instruments

Note: Government intermediate consumption, investment and the public sector wage bill are expressed as shares of the 2009 GDP. The effective tax rates are computed following the approach in Papageorgiou et al. (2012). The data source is Eurostat.
Finally, we close by reporting that the model would be dynamically unstable (meaning that there is no solution) if we had assumed that the independently set fiscal policy instruments remained as they were in the pre-2010 period. In other words, as said in the Introduction, the fiscal situation was not sustainable and hence some kind of fiscal policy adjustment was unavoidable in the aftermath of the 2008 world crisis.

3.2 Effects of the deterioration in institutional quality

We will now add another driver of transition dynamics, namely, changes in institutional quality and, in particular, an index that measures the protection of property rights.

As said in the Introduction, we assume that developments in this index manifest themselves as shocks to TFP. This is a short cut and is similar to the methodology of Chari et al. (2007). In other words, as a short cut, we construct an “effective” TFP series, where the degree of effectiveness is shaped by changes in the degree of property rights protection. On the other hand, it should be stressed that it is straightforward to enrich our model so as, in the presence of weak property rights, atomistic agents find it to optimal to allocate effort to conflict and extraction, and, in equilibrium, this leads to resource misallocation that eventually reduces the effective TFP; in Appendix B, we provide a simple version of our full-fledged DSGE model that shows this equivalence formally. Chari et al. (2007) also work with a prototype economy with wedges, or adverse shocks, and then show that micro-founded frictions in a more detailed economy manifest themselves as such wedges, or adverse shocks, in the prototype economy.

We therefore proceed as follows. First, we construct a series of institutional quality. Then, using this, we will construct a corresponding series for the effective TFP and, finally, will feed this resulting TFP series into our theoretical model in section 2. That is, now the model’s dynamics will be driven both by the fiscal austerity package and the effective TFP series.

To construct a measure of the quality of institutions that protect property rights, we use the World Bank’s “Worldwide Governance Indicators” dataset, which has been widely used in many
empirical studies (see e.g. Akitoby and Stratmann (2008) and Baldacci et al. (2011)). The institutional quality index is the sum of the following three indicators: “rule of law”, “regulatory quality” and “political stability and absence of violence/terrorism”. These indicators are all closely related to issues concerning the protection of property rights. Figure 3 shows the evolution of this composite index over the period 2002-2015. Notice the remarkable decline of institutional quality after 2008, which was a year of intense social and political turmoil in Greece. It should be stressed that these indicators are not linked (at least directly) to public finances.

Figure 3: Deterioration in property rights in Greece (2002-2015)

Note: The index is computed as the sum of the following three indicators: “rule of law”, “regulatory quality” and “political stability and absence of violence/terrorism”. The data source is Worldwide Governance Indicators, World DataBank.

In turn, as said above, we assume that changes in the TFP level are “shaped” by changes in the above index of institutional quality. In the model, there are two specific TFP levels, namely, in the tradable and the non-tradable sector. We allocate the changes over time in this index to the respective TFPs according to the relative size of the tradable and non-tradable sectors in the data (the ratio of the gross value added in the tradable sector to the non-tradable sector is 0.8). Thus, we obtain time series for the TFP levels in the two sectors and we normalize them so that their values
in 2009 to be consistent with the calibrated values of the TFPs (equal to 1 for the non-tradable and equal to 0.9241 for the tradable sector). Figure 4 shows the two constructed effective TFP series.

**Figure 4: TFP in tradable and non-tradable sectors**

“shaped” by the deterioration in property rights

<table>
<thead>
<tr>
<th>Year</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP: Tradable Sector</td>
<td>0.85</td>
<td>0.90</td>
<td>0.95</td>
<td>1.00</td>
<td>1.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP: Non-Tradable Sector</td>
<td>1.00</td>
<td>0.95</td>
<td>0.90</td>
<td>0.85</td>
<td>0.80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The path of the TFP levels is “shaped” by the changes in the institutional quality index according to the relative size of the respective sectors in the data. Source: Authors’ calculations.

Finally, using all the above, we repeat the same experiment as in section 3.1 by setting the TFP levels of tradables and non-tradables over 2010-2015 equal to the constructed series. The new impulse response functions are plotted in Figure 5, while the last column in Table 2, which was presented above, summarizes the associated changes in the main macro variables. Notice that now the reduction in GDP in column 3 of Table 2 is 22%, as compared to only 14% without the TFP/institutional shock in column 2.
4. Conclusions, discussion and extensions

In this paper, we studied the quantitative importance of the fiscal austerity program and the deterioration of institutional quality, both as observed in the recent data, for the Greek great depression since 2010. The main result is that the adopted fiscal policy mix and the deterioration in property rights are the main explanatory variables of the Greek great depression.

We close with acknowledging two caveats. First, here we did not explain why the specific fiscal policy mix has been chosen (which proves to be particularly distorting) or why the society has chosen to have weak and deteriorating property rights (which leads to misallocation of resources and hence to a relatively low TFP). In general, it is well recognized that the policies chosen and/or the way resources are (mis)allocated are an equilibrium outcome of a political process interacting with institutions and distribution (see e.g. Acemoglu (2009, chapter 4) and Jones (2011)). In the case of Greece, there is no shortage of conjectures about the root causes of such choices which go back to the post-world war II history of the country. Second, our analysis here was only positive. One could search, for instance, for alternative fiscal policy mixes and/or institutional regimes that could perhaps mitigate the recessionary effects of debt consolidation. We leave these extensions for future research.
References


1 Namely, the drop in output is large, occurred rapidly and is sustained; this is defined as a “great depression”. See Gogos et al. (2014) for an application of this methodology to the Greek economy before the euro period.

2 There is a growing literature on the current Greek crisis. For instance, Bortz (2015) discusses where the financial assistance has gone offering a different view from that of Sinn (2015); Arellano and Bai (2016) study the Greek default; Papageorgiou and Vourvachaki (2016) study the implications of structural reforms in light of the crisis; Gourinchas et al. (2016) search for shocks that can account for the Greek crisis. See below for further details.

3 See e.g. Philippopoulos et al. (2016) for the different implications of different fiscal policy mixes used for debt consolidation in Italy.

4 Alternatively, we could, for instance, use a VAR approach which requires a limited amount of theory to structure the data (see e.g. Canova, 2007, for methodology). We prefer to follow the DSGE approach so as to have well-defined micro-foundations that allow us to understand the behavioral channels through which exogenous changes affect macroeconomic outcomes.

5 There is a large literature that shows how weak institutions affect the efficiency with which factor inputs are used and, in particular, how weak property rights lead to distortive individual incentives, resource misallocation and eventually a lower level of total factor productivity. See e.g. Jones (2008, chapter 4, and 2011) and Acemoglu (2009, chapter 4) for reviews of the literature, while see below for further details and references. Here, working as in Chari et al. (2007), we will take a short cut by assuming that changes in property rights directly show up as shocks to TFP; nevertheless, as argued in subsection 3.2 below, this is equivalent to a richer model where the adverse effect of weak property rights on TFP works via the distortion of individual incentives.

6 See e.g. Chari et al. (2007) for a methodology paper on business cycle accounting.

7 We focus on the period during which Greece is part of the euro area but before the debt crisis erupted in early 2010.

8 In particular, we assume that \( \hat{R}_t^G = m\hat{R}_t^* + (1 - m)\hat{R}_t^H \), where we set the value of \( m \) equal to 0.5.

9 In the same spirit, Economides et al. (2007, 2008) and Angelopoulos et al. (2009, 2012) also provide micro-founded dynamic general equilibrium models, where the presence of weak property rights distorts private incentives and, in equilibrium, this leads to resource misallocation which, in turn, maps into reductions in the effective TFP. All this belongs to a rich and still growing literature that endogenizes the TFP and hence endogenizes long-term growth.

10 The rule of law indicator captures perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police and the courts, as well as the likelihood of crime and violence. The regulatory quality index captures perceptions of the ability of the government to formulate and implement sound policies and regulations, and the credibility of government’s commitment to such policies. The political stability and absence of violence/terrorism indicator captures perceptions of the likelihood that the government will be destabilized or overthrown by unconstitutional or violence means, including politically-motivate violence and terrorism. For further details see Kaufman et al. (2010). We report that each one of these three sub-indexes is highly correlated with key macroeconomic variables, such as real GDP and real investment, in the Greek data.
Appendix A: A DSGE model and calibration

This appendix presents the model used. It is similar to that in Papageorgiou (2014).

1. Households

The economy is populated by a continuum of households of mass one, indexed by $h \in [0,1]$, of which a fraction indexed by $i \in [0,1-\lambda]$ are referred as “Ricardian” or “optimizing households”, and a fraction indexed by $j \in (1-\lambda,1]$ are referred as “non-Ricardian” or “liquidity constrained households”. Optimizing households have access to capital and financial markets, where they can invest in the form of physical capital, government bonds and internationally traded assets. Liquidity constrained households, on the other hand, are not able to lend or borrow, so that they consume their disposable labor income in each time period. Both households supply differentiated labor services and act as wage-setters in monopolistically competitive markets.

1.1 Ricardian households

Ricardian households, indexed by $i$, have preferences over consumption and leisure. The inter-temporal utility function of each $i$ is:

$$U_i = \sum_{t=0}^{\infty} \beta^t u^\prime \left( C_{i,t} - \xi^c C_{i,t-1}^R, H_{i,t} \right)$$

where $\beta \in (0,1)$ is the discount factor, $C_{i,t}$ is $i$’s effective consumption (defined below) at $t$, $H_{i,t}$ is $i$’s total work hours at $t$, $\xi^c \in [0,1)$ is a parameter that measures the degree of external habit formation in consumption and $C_{i,t-1}^R$ denotes average (per household $i$) lagged-once effective consumption. Effective consumption is in turn defined to be a linear combination of private consumption, $C_{i,p}$, and public goods and services (education, health, etc) provided by the state sector, $Y^g$:

$$C_{i,t} = C_{i,p}^t + \vartheta Y^g_t$$

where $\vartheta \in [-1,1]$ is the degree of substitutability between private and public consumption.

The instantaneous utility function is assumed to be of the form:

$$u^\prime \left( C_{i,t} - \xi^c C_{i,t-1}^R, H_{i,t} \right) = \log \left( C_{i,t} - \xi^c C_{i,t-1}^R \right) - \kappa \frac{H_{i,t}^{1+\gamma}}{1+\gamma}$$

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1 See e.g. Christiano and Eichenbaum (1992), Forni et al. (2010a) and Economides et al. (2013).
where $\gamma$ is the inverse of Frisch labour supply elasticity and $\kappa > 0$ is a preference parameter related to work effort. Each household $i$ supplies work hours in the private sector, $H^p_{it}$, and the public sector, $H^g_{it}$. As in e.g. Ardagna (2001) and Forni et al. (2009), hours of work can be moved across the two sectors and are perfect substitutes in terms of (dis)utility, so that $H_{it} = H^p_{it} + H^g_{it}$ in each period $t$.

The Ricardian household can save in the form of physical capital, $I^p_{it}$, domestic government bonds, $B_{it}$, and foreign assets, $F^p_{it}$. It receives labour income from working in the private sector, $w^p_{it}H^p_{it}$, and the public sector, $w^g_{it}H^g_{it}$, where $w^p_{it}$ and $w^g_{it}$ are the real wage rates in the private and public sector respectively. The household rents out capital to firms and receives capital income, $r^k_{it}u_{it}K^p_{it}$, where $r^k_{it}$ is the real return to the effective amount of private capital, $K^p_{it}$ is the physical private capital stock and $u_{it} > 0$ is the utilization rate of capital. The household also earns interest income from domestic government bonds and internationally traded assets that pay a gross nominal interest $R^c_t \geq 0$ and $R^H_t \geq 0$ at $t+1$ respectively. In addition, households own all domestic firms, so that they receive their profits as dividends, $Div_{it}$. Finally, each Ricardian household receives a lump-sum government transfer, $G^p_{it}$. The household pays taxes on consumption, $0 < \tau^c_t < 1$, on labour income, $0 < \tau^l_t < 1$, on capital earnings and dividends, $0 < \tau^k_t < 1$, and lump-sum taxes, $T^c_t$.

Hence, the budget constraint of each Ricardian household $i$ is:

\[
(1 + \tau^c_t)C^p_{it} + \frac{P^c_t}{P^e_t}I^p_{it} + \frac{B_{it+1}}{P^e_t} + \frac{S^e_t F^p_{it+1}}{P^e_t} = \\
= (1 - \tau^c_t)(w^p_{it}H^p_{it} + w^g_{it}H^g_{it}) + (1 - \tau^l_t)(r^k_{it}u_{it}K^p_{it} + Div_{it}) + \\
+ R^c_{t-1} \frac{B_{it}}{P^c_t} + R^H_{t-1} \frac{S^e_t F^p_{it+1}}{P^c_t} + G^p_{it} - T^c_t - \Gamma^h_{it} \tag{4}
\]

where $P^c_t$ and $P^e_t$ are the prices of a unit of the private consumption final good and the investment final good respectively, and $S^e_t$ is the nominal exchange rate (expressed in terms of domestic currency per unit of foreign currency). The household faces costs when it adjusts its private foreign asset holdings, $\Gamma^h_{it}$, whenever the private foreign assets-to-GDP ratio, $\frac{S^e_t F^p_{it+1}}{P^c_t Y^{GDP}_t}$, deviates from its long-run target level, $\overline{f}$. In particular,

\[
\Gamma^h_{it} \left( S^e_t, F^p_{it+1}, P^e_t, Y^{GDP}_t, P^c_t \right) = \frac{\varepsilon_f^2}{2} \frac{P^c_t Y^{GDP}_t}{P^e_t} \left( \frac{S^e_t F^p_{it+1}}{P^c_t Y^{GDP}_t} - \overline{f} \right)^2 \tag{5}
\]
where \( Y_{t}^{GDP} \) is the economy’s real GDP, \( P_{t}^{Y} \) is the GDP deflator and \( \xi^{f} \geq 0 \) is an adjustment cost parameter.\(^2\)

The private capital stock evolves over time according to the following law of motion:
\[
K_{it+1}^{p} = \left(1 - \delta^{p} \left(u_{ij}ight)\right) K_{it}^{p} + \left[1 - \Psi^{I} \left(\frac{I_{it}^{p}}{I_{it-1}^{p}}\right)\right] I_{it}^{p}
\]
(6)

where \( \Psi^{I} \) is a convex adjustment cost function for investment, as in e.g. Christiano et al. (2005):
\[
\Psi^{I} \left(\frac{I_{it}^{p}}{I_{it-1}^{p}}\right) = \frac{\xi^{k}}{2} \left(\frac{I_{it}^{p}}{I_{it-1}^{p}} - 1\right)^{2}
\]
(7)

where \( \Psi^{I}(1) = \Psi^{I}(1) = 0 \) and \( \xi^{k} \geq 0 \) is an adjustment cost parameter. We assume that the depreciation rate of private capital depends on the rate of capacity utilization and is a convex function that satisfies \( \delta''^{p} > 0, \delta''^{p} > 0 \), so that \( \delta^{p} \left(u_{ij}\right) = \delta' u_{ij}^{\phi} \), where \( \delta^{p} \in (0,1) \) and \( \phi > 0 \) are respectively the average rate of depreciation of private capital and the elasticity of marginal depreciation cost.

The first-order conditions of this problem are written below when we present the final equilibrium system.

1.2 Non-Ricardian households

Liquidity constrained households, indexed by \( j \), have the same preferences as Ricardian households. They receive labour income from working in the private and public sectors, but have no access to capital or financial markets, so that, in each period, their consumption spending equals their after-tax wage income plus lump-sum government transfers. The period-by-period budget constraint of each household \( j \) is:
\[
\left(1 + \tau_{i}^{r}\right) C_{j,t} = \left(1 - \tau_{i}^{r}\right) \left( w_{j,t}^{p} H_{j,t}^{p} + w_{j,t}^{g} H_{j,t}^{g} \right) + G_{j,t}^{G}
\]
(8)

where \( H_{j,t}^{p} \) and \( H_{j,t}^{g} \) are respectively hours worked in the private and public sector by household \( j \) and \( G_{j,t}^{G} \) is a lump-sum government transfer to each \( j \). Thus, as in e.g. Coenen et al. (2013), we allow for a potentially uneven distribution of government transfers across Ricardian and non-Ricardian households.

The first-order conditions of this problem are written below when we present the final equilibrium system.

---

\(^2\) This specification ensures that foreign private assets are stationary; see e.g. Schmitt-Grohe and Uribe (2003).
2. Wage setting and the evolution of wages in the private sector

We assume that wages in the private sector are set by monopolistic unions, as in e.g. Forni et al. (2009) and Gali et al. (2007). More specifically, households supply differentiated labour varieties to a continuum of unions \( h \in [0,1] \), each of which represents a specific labour variety. Every variety is uniformly distributed across households, so that each union ultimately represents \( 1-\lambda \) fraction of Ricardian households and \( \lambda \) of non-Ricardian households. In every period, each union sets the wage rate for its own workers by trading off the utility derived from private sector labour income and the disutility of total work effort by taking into account the demand for the differentiated labour variety \( h \). At the same time, private and public sector firms allocate their labour demand uniformly across the \( h \) labour varieties independently of the type of households, which implies that hours worked by each type of household are equal, \( H_{h,t}^{p} = H_{h,t}^{g} \equiv H_{h,t}^{p} \) and \( H_{h,t}^{g} = H_{h,t}^{g} \equiv H_{h,t}^{g} \).3

Therefore, in each period, a typical union \( h \) chooses the wage rate, \( w_{h,t}^{p} \), to maximize:

\[
L_{w} = \lambda \left[ \Lambda_{h,t}^{NR} \left( 1-\tau^{\prime} \right) w_{h,t}^{p} H_{h,t}^{p} \right] + \left( 1-\lambda \right) \left[ \Lambda_{h,t}^{R} \left( 1-\tau^{\prime} \right) w_{h,t}^{p} H_{h,t}^{p} \right] - \kappa \frac{H_{h,t}^{1+\gamma}}{1+\gamma} \tag{9a}
\]

subject to

\[
H_{h,t}^{p} = \left( \frac{w_{h,t}^{p}}{w_{t}^{p}} \right)^{\frac{\mu_{w}^{p}}{\mu_{i}^{p}-1}} H_{t}^{p} \tag{9b}
\]

\[
H_{h,t} = H_{h,t}^{p} + H_{h,t}^{g} \tag{9c}
\]

where Eq. (9b) is the demand for the differentiated labour input \( h \), \( H_{t}^{p} \) is total labour demand in the private sector, \( w_{t}^{p} \) is the aggregate wage rate in the private sector, and \( \Lambda_{h,t}^{NR} \), \( \Lambda_{h,t}^{R} \) are the marginal utilities of consumption of non-Ricardian and Ricardian households of labour variety \( h \) respectively, used as weights. Finally, \( \mu_{w}^{p} / (\mu_{i}^{p} - 1) > 1 \) is the elasticity of substitution across the differentiated labour services, where \( \mu_{i}^{p} > 1 \) is the wage markup in the private labour market.

Focusing on a symmetric equilibrium in which all unions choose the same wage rate ex post, the first-order condition of the above problem is:

\[
w_{t}^{p} \left( \frac{\lambda}{MRS_{t}^{NR}} + \frac{1-\lambda}{MRS_{t}^{R}} \right) = \mu_{i}^{w} \tag{10}
\]

---

3 Total public sector labour demand for the differentiated labour input \( h \) is exogenous and is defined as \( H_{h,t}^{g} = \frac{1}{0} H_{h,t}^{g} dh \).
where $w^*_p$ is the optimal wage rate chosen by unions, and $MRS^{NR}_t$ and $MRS^R_t$ are the marginal rates of substitution between consumption and leisure of non-Ricardian and Ricardian households respectively.4

Following e.g. Hall (2005) and Blanchard and Gali (2007), we introduce further rigidities in the labour market by assuming that real wages respond sluggishly to labour market conditions. In particular, the real wage rate in the private sector is modeled as a weighted average of the lagged-once real wage rate and the optimal real wage rate chosen by unions:

$$w^*_p = (w^*_{p-1})^n (w^*_p)^{1-n}$$  

where $0 \leq n \leq 1$ denotes the degree of real wage rigidities and $w^*_p$ is given by (10).5 This formulation aims to capture the rigidities found in the Greek labour market (see e.g. the discussion in European Commission, 2010).6

3. Production in the private sector

There are two types of domestic firms. The first type consists of monopolistically competitive firms that produce intermediate goods, tradable and non-tradable. The continuum of firms producing differentiated varieties of tradables, indexed by $f^T \in [0,1]$, sell their output domestically or abroad (the latter are recorded as exports). The continuum of firms producing differentiated varieties of non-tradables, indexed by $f^N \in [0,1]$, sell their output domestically only. There is also a continuum of monopolistically competitive firms importing intermediate goods, indexed by $f^M \in [0,1]$. The second type of firms consists of four perfectly competitive firms that produce final goods. These firms combine purchases of intermediate goods to produce four non-tradable goods: a private consumption good, a private investment good, a public consumption good and a public investment good. Finally, there is a foreign final goods firm that combines purchases of the exported domestic intermediate goods.

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4 Note that when $\lambda = 0$, i.e. when all households are Ricardian, $\mu^p$ reduces to a markup of the optimally chosen real wage rate over the marginal rate of substitution between consumption and leisure of Ricardian households.

5 See also e.g. Uhlig (2007), Malley et al. (2009) and Kliem and Uhlig (2013) for a similar specification. Microfoundations for Eq. (11) can be found in e.g. Hall (2003), Petrongolo and Pissarides (2001) and Christoffel and Linzert (2010).

6 Papageorgiou (2014) finds that this specification can capture rather well the aggregate dynamics of work hours and real wages in Greece.
3.1 Final goods firms

As said above, there are four representative final goods firms that combine purchases of tradable intermediate goods with non-tradable goods to produce a private consumption good, \( C_t^p \), a private investment good, \( I_t^I \), a public consumption good, \( G_t^{pc} \), and a public investment good, \( G_t^{gi} \).

Private consumption goods producer

The representative producer of the private final consumption good combines a bundle of tradable consumption intermediate goods, \( C_t^C \), with a bundle of non-tradable intermediate goods, \( C_t^N \), according to a constant elasticity of substitution (CES) production function:

\[
C_t^p = \left[ \omega_c \gamma_{C} \left( C_t^C \right)^{\gamma_{C}-1} + \left( 1 - \omega_c \right) \gamma_{C} \left( C_t^N \right)^{\gamma_{C}-1} \right]^{\frac{1}{\gamma_{C}-1}}
\]

(12)

where \( \omega_c \in [0,1] \) measures the weight of tradable goods in the production of the final private consumption good and \( \gamma_{C} > 0 \) is the elasticity of substitution between tradable and non-tradable consumption goods.

In turn, the tradable intermediate consumption good bundle is a CES function of the domestically produced bundle of tradable intermediate consumption goods, \( C_t^D \), and the bundle of imported intermediate consumption goods, \( C_t^M \):

\[
C_t^C = \left[ \omega_{TC} \gamma_{TC} \left( C_t^D \right)^{\gamma_{TC}-1} + \left( 1 - \omega_{TC} \right) \gamma_{TC} \left( C_t^M \right)^{\gamma_{TC}-1} \right]^{\frac{1}{\gamma_{TC}-1}}
\]

(13)

where \( \omega_{TC} \in [0,1] \) measures the home bias in the production of the tradable intermediate consumption good, and \( \gamma_{TC} > 0 \) is the elasticity of substitution between domestic and imported intermediate consumption goods.

The intermediate consumption good bundles that are used as inputs combine differentiated varieties supplied by intermediate goods firms. Specifically, the varieties supplied by each tradable intermediate goods firm \( f^T \), \( C_{jT}^D \), each non-tradable intermediate-goods firm, \( f^N \), \( C_{jN}^N \), and each importing firm \( f^M \), \( C_{jM}^M \), are respectively combined using a CES technology into:

\[
C_t^D = \left( \int_0^1 \left( C_{jT}^D \right)^{\frac{1}{\gamma_{T}}} df^T \right)^{\frac{\gamma_{T}}{\gamma_{T}-1}}
\]

(14a)
where $\mu_t^T, \mu_t^N, \mu_t^M > 1$ are the intra-temporal elasticities of substitution between different varieties within each type of intermediate consumption good. As we show below, $\mu_t^T, \mu_t^N, \mu_t^M$ represent markups in the markets of domestic and imported intermediate goods.

Given the above technology, the producer of the final private consumption good solves a three stage problem. In the first stage, it takes as given the prices of domestic tradable, $P_{f^D,t}$, non-tradable, $P_{f^N,t}$, and imported intermediate goods, $P_{f^M,t}$, and chooses the amounts of the differentiated goods, $C_{f^D,t}, C_{f^N,t}, C_{f^M,t}$, in order to minimize total expenditures for the bundles of the differentiated goods, $\int_0^1 P_{f^D,t} C_{f^D,t} df^T$, $\int_0^1 P_{f^N,t} C_{f^N,t} df^N$, $\int_0^1 P_{f^M,t} C_{f^M,t} df^M$, subject to the aggregation constraints in (14a)-(14c). The solution of the cost minimization problem gives the demand functions for these intermediate goods $f^T$, $f^N$ and $f^M$ respectively:

\begin{align}
C_{f^T,t} &= \left( \frac{P_{f^T,t}}{P_t^T} \right)^{\frac{\mu_t^T}{\mu_t^T - 1}} C_t^T \\
C_{f^N,t} &= \left( \frac{P_{f^N,t}}{P_t^N} \right)^{\frac{\mu_t^N}{\mu_t^N - 1}} C_t^N \\
C_{f^M,t} &= \left( \frac{P_{f^M,t}}{P_t^M} \right)^{\frac{\mu_t^M}{\mu_t^M - 1}} C_t^M
\end{align}

where $P_t^T, P_t^N, P_t^M$ are the aggregate price indices of domestic tradable, non-tradable and imported intermediate consumption goods, respectively.

In the second stage, the firm chooses the bundles $C_t^D$ and $C_t^M$ in order to maximize its profits, $\Pi_t = P_t^{TC} C_t^T - P_t^D C_t^D - P_t^M C_t^M$ subject to the technology constraint (13) and by taking as given the price indexes of domestic tradables, $P_t^{TC}$ non-tradables, $P_t^D$ and imported intermediate consumption goods, $P_t^M$. Thus, it solves:
\[
\max_{C^T_i, C^N_i} P^T_i C^T_i - P^D_i C^D_i - P^M_i C^M_i
\]
subject to
\[
C^T_i = \left[ \omega_{TC} \left( C^P_i \right)^{\frac{1}{\varepsilon_{TC}}} + (1 - \omega_{TC}) \left( C^M_i \right)^{\frac{1}{\varepsilon_{TC}}} \right]^{\frac{\varepsilon_{TC}}{\varepsilon - 1}}
\]

In the third stage, the firm chooses the demand for \( C^T_i \) and \( C^N_i \) to maximize profits, \( \Pi_i = P^C_i C^C_i - P^{TC}_i C^{TC}_i - P^N_i C^N_i \), subject to the technology constraint (12) and by taking the input prices \( P^C_i, P^{TC}_i \) and \( P^N_i \) as given. Thus it solves:
\[
\max_{C^T_i, C^N_i} P^C_i C^C_i - P^{TC}_i C^{TC}_i - P^N_i C^N_i
\]
subject to
\[
C^P_i = \left[ \omega_{C} \left( C^P_i \right)^{\frac{1}{\varepsilon_{C}}} + (1 - \omega_{C}) \left( C^N_i \right)^{\frac{1}{\varepsilon_{C}}} \right]^{\frac{\varepsilon_{C}}{\varepsilon - 1}}
\]

The demand functions for domestic tradable and imported consumption goods, as well as for tradable and non-tradable intermediate consumption goods resulting from the optimization problem of the final consumption good firm, are:
\[
\frac{C^D_i}{C^T_i} = \omega_{TC} \left( \frac{P^D_i}{P^{TC}_i} \right)^{\varepsilon_{TC}} \tag{16a}
\]
\[
\frac{C^M_i}{C^T_i} = (1 - \omega_{TC}) \left( \frac{P^M_i}{P^{TC}_i} \right)^{\varepsilon_{TC}} \tag{16b}
\]
\[
\frac{C^T_i}{C^i} = \omega_{C} \left( \frac{P^{TC}_i}{P^C_i} \right)^{\varepsilon_{C}} \tag{16c}
\]
\[
\frac{C^N_i}{C^i} = (1 - \omega_{C}) \left( \frac{P^N_i}{P^i} \right)^{\varepsilon_{C}} \tag{16d}
\]

From the zero profit condition, we get the price index for tradable consumption goods
\[
P^{TC}_i = \left[ \omega_{TC} \left( P^D_i \right)^{1 - \varepsilon_{TC}} + (1 - \omega_{TC}) \left( P^M_i \right)^{1 - \varepsilon_{TC}} \right]^{\frac{1}{1 - \varepsilon_{TC}}}
\]
and the price index of a unit of the final consumption good (i.e. the Consumption Price Index) \( P^C_i = \left[ \omega_{C} \left( P^{TC}_i \right)^{1 - \varepsilon_{C}} + (1 - \omega_{C}) \left( P^N_i \right)^{1 - \varepsilon_{C}} \right]^{\frac{1}{1 - \varepsilon_{C}}} \), where \( P^D_i, P^N_i, P^M_i \) are the prices of domestic tradable intermediates, non-tradable intermediates and imported intermediate goods, respectively.
Private investment goods producer

Optimal decisions regarding the production of the final private investment good are derived in an analogous manner as above. The representative producer of the private investment good combines a composite bundle of tradable intermediate goods, $I^T_t$, with a bundle of non-tradable intermediate goods, $I^N_t$, to generate a composite final private investment good, $I'_t$, by using a constant elasticity of substitution (CES) production function:

$$I'_t = \left[ \frac{1}{\omega_t} I^T_t \left( \frac{\epsilon_{tI}}{\epsilon_{tI}} \right) + \left(1 - \frac{1}{\omega_t} \right) I^N_t \left( \frac{\epsilon_{tI}}{\epsilon_{tI}} \right) \right]^{\frac{\epsilon_{tI}}{\epsilon_{tI} - 1}}$$

where $\omega_t \in [0, 1]$ measures the weight of tradable goods in the production of the final private investment good, and $\epsilon_{tI} > 0$ is the elasticity of substitution between tradable and non-tradable investment goods.

In turn, the composite bundle of the tradable intermediate investment good that is used in the production of final investment goods is a CES function of domestically produced tradable intermediate investment goods, $I^D_t$, and imported intermediate investment goods, $I^M_t$:

$$I^T_t = \left[ \frac{1}{\omega_{tT}} \left( I^D_t \right) \left( \frac{\epsilon_{tT}}{\epsilon_{tT}} \right) + \left(1 - \frac{1}{\omega_{tT}} \right) I^M_t \left( \frac{\epsilon_{tT}}{\epsilon_{tT}} \right) \right]^{\frac{\epsilon_{tT}}{\epsilon_{tT} - 1}}$$

where $\omega_{tT} \in [0, 1]$ measures the home bias in the production of the tradable intermediate consumption good, and $\epsilon_{tT} > 0$ is the elasticity of substitution between domestic and imported investment goods.

The demand functions for domestic tradable and imported investment goods, as well as for tradable and non-tradable investment goods, are:

$$\frac{I^D_t}{I^T_t} = \omega_{tT} \left( \frac{P^D_t}{P^T_t} \right)^{-\epsilon_{tT}}$$

$$\frac{I^M_t}{I^T_t} = \left(1 - \omega_{tT} \right) \left( \frac{P^M_t}{P^T_t} \right)^{-\epsilon_{tT}}$$

$$\frac{I^T_t}{I'_t} = \omega_t \left( \frac{P^T_t}{P'_t} \right)^{-\epsilon_{tI}}$$

$$\frac{I^N_t}{I'_t} = \left(1 - \omega_t \right) \left( \frac{P^N_t}{P'_t} \right)^{-\epsilon_{tI}}$$
where \( P_i^{TI} = \left[ \omega_{ti} \left( P_i^{D} \right)^1 - \epsilon_{ti} \right] + (1 - \omega_{ti}) \left( P_i^{M} \right)^1 - \epsilon_{ti} \) and \( P_i' = \left[ \omega_{ti} \left( P_i^{D} \right)^1 - \epsilon_{ti} \right] + (1 - \omega_{ti}) \left( P_i^{N} \right)^1 - \epsilon_{ti} \) are respectively the price indices for tradable intermediate investment goods and final investment goods.

Public consumption and investment goods production

Regarding the final public consumption and investment goods, \( G_{t}^{GC} \) and \( G_{t}^{GI} \), we assume they are produced using only non-tradable intermediate goods. Hence, \( G_{t}^{GC} = GC_{t}^{N} \) and \( G_{t}^{GI} = GI_{t}^{N} \) where

\[
GC_{t}^{N} = \left( \int_{0}^{f^{N}} \left( GC_{f}^{N} \right)^{1} \frac{h^{N}}{f^{N}} df^{N} \right)^{1} \quad \text{and} \quad GI_{t}^{N} = \left( \int_{0}^{f^{N}} \left( GI_{f}^{N} \right)^{1} \frac{h^{N}}{f^{N}} df^{N} \right)^{1}
\]

The optimal demand functions are:

\[
GC_{f}^{N} = \left( \frac{P_{f}^{N}}{P_{t}^{N}} \right)^{\frac{\mu^{N}}{\mu^{N} - 1}} GC_{i}^{N} \quad \text{and} \quad GI_{f}^{N} = \left( \frac{P_{f}^{N}}{P_{t}^{N}} \right)^{\frac{\mu^{N}}{\mu^{N} - 1}} GI_{i}^{N}
\]

Aggregating across final good producing firms, we get the respective aggregate domestic demand functions for non-tradable, domestic tradable and imported intermediate goods, \( f^{T}, f^{N} \) and \( f^{M} \):

\[
Y_{f}^{NT} = C_{f}^{N} + I_{f}^{N} + GC_{f}^{N} + GI_{f}^{N} = \left( \frac{P_{f}^{N}}{P_{t}^{N}} \right)^{\frac{\mu^{N}}{\mu^{N} - 1}} Y_{t}^{NT}
\]

\[
Y_{f}^{D} = C_{f}^{D} + I_{f}^{D} = \left( \frac{P_{f}^{D}}{P_{t}^{D}} \right)^{\frac{\mu^{D}}{\mu^{D} - 1}} Y_{t}^{D}
\]

\[
Y_{f}^{M} = C_{f}^{M} + I_{f}^{M} = \left( \frac{P_{f}^{M}}{P_{t}^{M}} \right)^{\frac{\mu^{M}}{\mu^{M} - 1}} Y_{t}^{M}
\]

where \( Y_{t}^{NT} = C_{t} + I_{t}^{N} + GC_{t}^{N} + GI_{t}^{N} \), \( Y_{t}^{D} = C_{t} + I_{t}^{D} \) and \( Y_{t}^{M} = C_{t} + I_{t}^{M} \) are respectively the total demand for non-tradable goods, total domestic demand for domestically produced tradable goods and total demand for imports.

3.2 Intermediate goods firms

Each tradable and non-tradable intermediate good, \( Y_{f}^{T} \) and \( Y_{f}^{N} \), is produced by a continuum of monopolistically competitive intermediate goods firms indexed by \( f^{T} \in [0,1] \) and \( f^{N} \in [0,1] \) respectively, according to the production technologies:
\[ Y_{f^T,t} = A^T \left( K_{f^T,t} \right)^{a^T} \left( H_{f^T,t} \right)^{1-a^T} \left( \bar{K}^g_t \right)^{a^G} - \Phi_T \]  
\[ Y_{f^N,t} = A^N \left( K_{f^N,t} \right)^{a^N} \left( H_{f^N,t} \right)^{1-a^N} \left( \bar{K}^g_t \right)^{a^G} - \Phi_N \]  

where \( K_{f^T,t} \) is private capital, \( H_{f^T,t} \) is work hours in the private sector, \( \bar{K}^g_t \) is public capital, \( a_T, a_N > 0 \) are the output elasticities of capital services in the tradable and non-tradable sectors respectively, and \( a_G > 0 \) is the output elasticity of public capital. \(^7\) Finally, \( \Phi_T, \Phi_N \geq 0 \) are fixed costs of production, and \( A^T, A^N \) are sector-specific total factor productivity levels.

** Tradable sector**

In what follows, we present the problem of intermediate goods firms in the tradable sector. Domestic intermediate goods firms in the tradable sector solve a two-stage problem. In the first stage, each firm takes as given factor prices, \( r^T_t \) and \( w^p_t \), and chooses capital and labour inputs, \( K_{f^T,t} \) and \( H_{f^T,t} \), in order to minimize total real input cost. We also introduce a working capital channel in the form of a “cash-in-advance” constraint in the spirit of e.g. Mendoza (2010). In particular, at the beginning of each period, each firm borrows from international lenders in order to cover a fraction \( v_t \in (0,1) \) of their total labour costs in advance of revenues’ receipt. The working capital loan is repaid by the end of the period at the domestic country gross interest rate, \( R^H_t \). Thus, the intratemporal problem of each firm involves the minimization of their costs, inclusive of the costs of serving their intra-period working capital loan. In other words,

\[
\min_{K_{f^T,t},H_{f^T,t}} r^T_t K_{f^T,t} + w^p_t H_{f^T,t} + \left( R^H_t - 1 \right) v_t w^p_t H_{f^T,t} 
\]

subject to (17a).

The first-order conditions are:

\[
\left( 1 + v_t \left( R^H_t - 1 \right) \right) w^p_t = \left( 1 - a_T \right) \frac{Y_{f^T,t} + \Phi_T}{H_{f^T,t}} m_{f^T,t} 
\]

\[
r^T_t = a_T \left( 1 - \frac{Y_{f^T,t} + \Phi_T}{H_{f^T,t}} m_{f^T,t} \right) 
\]

where \( m_{f^T,t} \) is the Lagrange multiplier associated with the technology constraint, that is, the real marginal cost in terms of the consumer prices, \( P^C_t \). Because firms borrow to cover part of their

\(^7\) These production functions have increasing returns to scale with respect to all inputs and constant returns to scale with respect to private inputs (see also e.g. Baxter and King (1993) and Leeper et al. (2010)).
labour costs, the marginal cost of labour is higher than the wage rate in the private sector. As a result, increases in either the share of labour costs that are financed through working capital loans, or in the domestic interest rate, directly increase the cost of labour and thereby reduce labour demand.

The labour input, \( H_{j^T, t} \), is a composite aggregate of household-specific varieties, \( H^h_{j^T, t} \),

\[
H_{j^T, t} = \left( \int_0^1 \left( H^h_{j^T, t} \right)^{\frac{1}{\mu^W}} \right) \mu^W .
\]

Optimal demand is

\[
H^h_{j^T, t} = \left( \frac{w^p_{j^T, t}}{w^p_{j^T, t}} \right)^{\frac{1}{\mu^W}} H_{j^T, t} \text{ where } \mu^W / (\mu^W - 1) > 1 \text{ is the elasticity of substitution across differentiated labour services and } w^p_{j^T, t} \text{ is the aggregate real wage index in the private sector that is given by }
\]

\[
w^p_{j^T, t} = \left( \int_0^1 \left( w^p_{j^T, t} \right)^{1-\mu^W} \right)^{\frac{1}{1-\mu^W}} .
\]

In the second stage, intermediate good firms in the tradable sector choose the price that maximizes discounted real profits. As in Christoffel et al. (2008), firms charge different prices at home and abroad, setting prices in producer currency. In both domestic and foreign markets, we assume that prices are sticky á la Calvo (1983). In particular, each period \( t \), the firm \( f^T \) optimally resets prices with a constant probability \( 1 - \theta_p^D \) when it sells its differentiated product in the domestic market, and with probability \( 1 - \theta_p^X \) when it sells its product abroad. The firms that cannot optimize, partially index their prices to aggregate past inflation according to the price indexation schemes,

\[
P^D_{j^T, t} = P^D_{j^T, t-1} \left( \Pi^D_{t-1} \right)^{\gamma_0} \text{ and } P^X_{j^T, t} = P^X_{j^T, t-1} \left( \Pi^X_{t-1} \right)^{\gamma_X} , \text{ where } P^D_{j^T, t} \text{ denote the domestic price of good } f^T, P^X_{j^T, t} \text{ its foreign price, and } \Pi^D_t = P^D_t / P^D_{t-1}, \Pi^X_t = P^X_t / P^X_{t-1} \text{ where } P^D_t, P^X_t \text{ are the aggregate domestic and export price indices (defined below), respectively. The indexation parameters } \gamma_D, \gamma_X \in [0,1] \text{ determine the weights given to past inflation.}
\]

Each firm \( f^T \), which can optimally reset its price in period \( t \), knows, with probability \( \theta_p^D \), that this price will continue to be in effect \( \tau \) periods ahead, and so chooses the optimal price \( P^{*D}_{j^T, t} \) to maximize the discounted sum of expected real profits (in terms of consumer prices \( P^C_t \)), by taking aggregate domestic demand, \( Y^D_t \), and the aggregate price index in the domestic market, \( P^D_t \), as given. Thus, each firm \( f^T \) maximizes:

\[
\max_{P^{*D}_{j^T, t}} E_t \sum_{r=0}^{\infty} \left( \beta P^{D, \tau}_{j^T, t} \right) R^{\tau}_{t+\tau} \left( \frac{Y^D_t}{P^D_t} - m^D_{t+\tau} \right) P^{D, \tau}_{j^T, t} \right) \right)
\]

(21)
subject to

\[
Y_{f,t}^D = \left( \prod_{s=1}^T \left( \frac{(\Pi_{t+s-1}^D)^{\nu}}{\Pi_{t+s}^D} \right)^{\frac{\mu_s}{\mu_{s-1}}} \right) Y_{t+\tau}^D
\]

where \( mc_i^D = P_i^C mc_i^T / P_i^D \) is the real marginal cost in terms of the domestic price index and \( \Lambda_{t+\tau}^R / \Lambda_t^R \) is the ratio of the marginal utilities of consumption of Ricardian households - that are the owners of the firms - according to which firms value future profits. Notice that since all firms face the same marginal cost and take aggregate variables as given, any firm that optimizes will set the same price, \( P_{f,t}^{*D} = P_t^{*D} \).

Thus, the first-order condition of the above problem is:

\[
E_t \left( \beta \theta_P^D \right)^{\tau} \frac{\Lambda_{t+\tau}^R}{\Lambda_t^R} \left( \prod_{s=1}^T \left( \frac{(\Pi_{t+s-1}^D)^{\nu}}{\Pi_{t+s}^D} \right)^{\frac{\mu_s}{\mu_{s-1}}} \right) P_{f,t}^D \left( \prod_{s=1}^T \left( \frac{(\Pi_{t+s-1}^D)^{\nu}}{\Pi_{t+s}^D} \right)^{\frac{\mu_s}{\mu_{s-1}}} \right) Y_{t+\tau}^D P_{f,t}^{*D} \left( \prod_{s=1}^T \left( \frac{(\Pi_{t+s-1}^D)^{\nu}}{\Pi_{t+s}^D} \right)^{\frac{\mu_s}{\mu_{s-1}}} \right) \left( P_{t+\tau}^D \right)^{-1} \left( P_{t}^{*D} \right)^{-1} \left( \Pi_{t+\tau}^D \right) \left( \Pi_t^{*D} \right)^{-1} \mu_{t+\tau} mc_{t+\tau}^D = 0
\]

(23)

According to the above expression firms set nominal prices so as to equate the average future expected marginal revenues to average future expected marginal costs. The aggregate domestic index evolves according to

\[
P_t^D = \left( 1 - \theta_P^D \left( P_t^{*D} \right)^{\frac{1}{\nu_p^D}} + \theta_P^D \left( P_{t-1}^{*D} \left( \Pi_{t-1}^D \right)^{\nu} \right)^{\frac{1}{\nu_p^D}} \right)^{-\frac{1}{\nu_p^D}}
\]

Similarly, the associated first-order condition of each firm \( f^T \) that chooses its price in the foreign market in period \( t \) is:

\[
E_t \left( \beta \theta_P^X \right)^{\tau} \frac{\Lambda_{t+\tau}^R}{\Lambda_t^R} \left( \prod_{s=1}^T \left( \frac{(\Pi_{t+s-1}^X)^{\nu}}{\Pi_{t+s}^X} \right)^{\frac{\mu_s}{\mu_{s-1}}} \right) P_{f,t}^X \left( \prod_{s=1}^T \left( \frac{(\Pi_{t+s-1}^X)^{\nu}}{\Pi_{t+s}^X} \right)^{\frac{\mu_s}{\mu_{s-1}}} \right) Y_{t+\tau}^X P_{f,t}^{*X} \left( \prod_{s=1}^T \left( \frac{(\Pi_{t+s-1}^X)^{\nu}}{\Pi_{t+s}^X} \right)^{\frac{\mu_s}{\mu_{s-1}}} \right) \left( P_{t+\tau}^X \right)^{-1} \left( P_{t}^{*X} \right)^{-1} \left( \Pi_{t+\tau}^X \right) \left( \Pi_t^{*X} \right)^{-1} \mu_{t+\tau} mc_{t+\tau}^X = 0
\]

(24)

where \( mc_i^X = P_i^C mc_i^T / P_i^X \) is the real marginal cost in terms of the aggregate export price index, and the aggregate export price index is

\[
P_t^X = \left( 1 - \theta_P^X \left( P_t^{*X} \right)^{\frac{1}{\nu_p^X}} + \theta_P^X \left( P_{t-1}^{*X} \left( \Pi_{t-1}^X \right)^{\nu} \right)^{\frac{1}{\nu_p^X}} \right)^{-\frac{1}{\nu_p^X}}
\]

\[8\] In equilibrium, the marginal utility of consumption is common across Ricardian households, \( \Lambda_t^R = \Lambda_t^R \).

\[9\] In the case of fully flexible prices, \( \theta_P^D = 0 \), the above condition reduces to the static relation, \( P_t^{*D} = \mu_{t+\tau} mc_{t+\tau}^D \), which states that the price is equal to a markup over the nominal marginal cost.
Non-tradable sector

The optimal demand by each firm \( f^N \) for labour of type \( h \) is

\[
H_{f^N,t}^h = \left( \frac{w_{h,t}^p}{w_t^p} \right)^{-\mu_{HH}^N} H_{f^N,t}^h
\]

and the aggregate demand for labour of type \( h \) is

\[
H_{h,t}^N = \int_0^1 H_{f^N,t}^h df = \left( \frac{w_{h,t}^p}{w_t^p} \right)^{-\mu_{HH}^N} H_t^N,
\]

where \( H_t^N \) is total labour demand in the non-tradable intermediate good sector. As in the case of the tradable good firms, non-tradable intermediate good firms take short-term loans from international lenders at the home country’s gross interest rate \( R_t^H \) in order to finance a fraction \( v_t \) of their total labour costs.

To minimize costs, each firm takes as given the factor prices, \( r_t^k \) and \( w_t^p \) and chooses \( K_{f^N,t}, H_{f^N,t} \) in order to minimize total real input cost

\[
\left( 1 + v_t (R_t^H - 1) \right) w_t^p = \left( 1 - a_N \right) \frac{Y_{f^N,t} + \Phi_N}{H_{f^N,t}} mc_{f^N,t}
\]

\[
r_t^k = a_N \frac{Y_{f^N,t} + \Phi_T}{K_{f^N,t}} mc_{f^N,t},
\]

where due to symmetry \( mc_{f^N,t} = mc_t^N \).

Non-tradable intermediate good firms face price stickiness à la Calvo, with \( 1 - \theta_t^N \) being the probability that a firm \( f^N \) can optimally reset its price in any given period \( t \geq 0 \). The optimal pricing is characterized by the following conditions:

\[
E_t \left( \beta^{N} \right)^t \frac{\Lambda_{i+1}^N}{\Lambda^N} \left[ \prod_{s=1}^t \left( \frac{\Pi_{i+1}^N}{\Pi_{i,s}^N} \right)^{x_t} \frac{P_t^N}{P_i^N} \right]^{\frac{\mu_{i,t}^N}{\mu_{i,t}^N - 1}} Y_{i+1}^N \frac{P_t^N}{P_i^C} \left[ \prod_{s=1}^t \left( \frac{\Pi_{i+1}^N}{\Pi_{i,s}^N} \right)^{x_t} \right]^{\frac{P_i^N}{P_t^N} - \mu_{i,t}^N mc_{i+1}^N} = 0
\]

and the aggregate domestic index evolves according to:

\[
P_t^N = \left( 1 - \theta_t^N \right) P_t^N \left( 1 - \mu_{i,t}^N \right) + \theta_t^N \left( P_i^N \left( \Pi_{i+1}^N \right)^{x_t} \left( 1 - \mu_{i,t}^N \right) \right)^{1 - \mu_{i,t}^N}
\]

Importing firms

There is a continuum of importing firms \( f^M \in [0,1] \), each of which imports a single differentiated intermediate good, \( Y_{f^M,t}^M \). These firms operate under monopolistic competition, so that they have
pricing power. This creates a wedge between the price at which the importing firms buy the foreign differentiated goods in the world markets, \( S \cdot P^* \), and the price at which they sell these goods to domestic households, \( P^M \). Importing firms face price stickiness à la Calvo, with \( 1 - \theta^M \) being the probability that a firm can optimally reset its price in the domestic market in any given period \( t \geq 0 \), so that optimal pricing follows:

\[
E_t \left( \beta_p^n \right) \Lambda_t^R \left( \prod_{s=1}^{t} \left( \frac{\Pi^M_{t+s-1}}{\Pi^M_{t+s}} \right)^{\theta_p} \right)^{\frac{\mu^M_t}{\mu^{-1}}} Y^M_{t+\tau} \frac{P^M_{t+\tau}}{P^M_t} \left[ \prod_{s=1}^{t} \left( \frac{\Pi^M_{t+s-1}}{\Pi^M_{t+s}} \right)^{\theta_p} \right] P^*_M - \mu^M_{t+\tau} m_{t+\tau}^M = 0
\]

and the aggregate import price index evolves according to

\[
P^M_t = \left( 1 - \theta^m \right) \left( P^*_M \right)^{\frac{1}{1-\mu^M}} + \theta^m \left( P^M_{t-1} \left( \Pi^M_{t-1} \right)^{\theta_p} \right)^{\frac{1}{1-\mu^M}}.
\]

### 3.3 Demand from foreign final goods firms

We now model the demand coming from foreign firms or, equivalently, specify the domestic country’s exports. Recall that the domestic economy produces intermediate tradable goods that are also exported and so we need to model a final goods firm that transforms them into final goods. There is a representative foreign final good firm that purchases the differentiated exported goods, \( Y^X_{j,t} \), produced by the domestic tradable intermediate good firms \( f^T \), and transforms them into a homogeneous final good \( Y^X_t \) via the CES technology:

\[
Y^X_t = \left( \int_0^1 \left( Y^X_{f^X,t} \right)^{\frac{1}{\mu^X}} df^T \right)^{\frac{1}{\mu^X}}
\]

where \( \mu^X > 1 \) is related to the intratemporal elasticity of substitution between the differentiated outputs supplied by domestic intermediate good firms, \( \mu^X / (\mu^X - 1) > 1 \).

The foreign firm takes the prices of exported differentiated goods \( P^X_{j^X,t} / S \) (expressed in terms of the foreign currency) as given, and chooses the optimal amounts of differentiated inputs to minimize total input costs, \( \int_0^1 \left( P^X_{j^X,t} / S \right) Y^X_{j^X,t} df^T \), subject to (25), so that the optimal demand function for each input \( Y^X_{j^X,t} \) is \( Y^X_{j^X,t} = \left( P^X_{f^X,t} / P^X_t \right)^{\frac{\mu^X}{\mu^X - 1}} Y^X_t \), where \( P^X_t = \left( \int_0^1 \left( P^X_{j^X,t} \right)^{\frac{1}{1-\mu^X}} df^T \right)^{1-\mu^X} \) is the aggregate price index of exported domestic intermediate goods and \( Y^X_t \) is total foreign demand for domestic
intermediate goods. The latter is \( Y^x_t = \left( \frac{P^x_t}{P^x_{t-1}} \right) Y^x_t \), where \( P^x_t \) is the price of foreign competitors in the export markets, \( Y^x_t \) is foreign economy output and \( \varepsilon^x > 0 \) is the price elasticity of export demand.

4. The public sector
We now model the public sector.

4.1 Government budget constraint and fiscal policy instruments
The government levies taxes on consumption, labour income, capital earnings and dividends. We also assume lump-sum taxes/transfers. The government also sells one-period government bonds to the domestic bond market, \( B^g_{t+1} \), and the international market, \( F^g_{t+1} \). Total tax revenues plus the issue of new government bonds are used to finance government consumption, \( C^g_t \), investment, \( I^g_t \), transfers, \( T^g_t \), and the wage bill of public sector employees, \( W^g_t \). Moreover, the interest rates that the government pays on inherited domestic public debt and on foreign public debt are \( R^d_t \) and \( R^g_t \) respectively. We assume that \( R^g_t \) is a weighted average of the market interest rate that the country faces when it borrows from abroad and the risk-free world interest rate (see below for details). Thus, the within-period government budget constraint in per-capita terms is:

\[
\begin{align*}
B^g_t + S^g_t F^g_t &+ \tau^c_t C^g_t + \tau^l_t \left( w^p_t H^p_t + w^g_t H^g_t \right) + \tau^k_t \left( r^s_t u_t K^p_t + Div_t \right) + T_t = \\
&= \frac{P^N_t}{P^c_t} C^g_t + \frac{P^N_t}{P^c_t} I^g_t + \frac{w^g_t H^g_t}{P^c_t} + R^d_{t-1} B^g_{t-1} + R^g_{t-1} S^g_t F^g_t \\
&\quad = \frac{P^N_t}{P^c_t} C^g_t + \frac{P^N_t}{P^c_t} I^g_t + \frac{w^g_t H^g_t}{P^c_t} + R^d_{t-1} B^g_{t-1} + R^g_{t-1} S^g_t F^g_t
\end{align*}
\]

Therefore, the government has eleven policy instruments, \( X_t \in \{ \xi^c_t, \xi^l_t, \xi^k_t, T_t, w^g_t, H^g_t, G, G^d_t, G^g_t, B^g_t, F^g_t \} \), out of which ten can be exogenously set.

To ensure dynamic stability, we need to assume that one of the exogenously set fiscal policy instruments follows a state-contingent rule reacting systematically to deviations of the public debt-to-GDP ratio from a target level and the rate of change of debt:

\[
X_t = X_{t-1} + \phi^d \left( D^d_t - \bar{d} \right) + \phi^d \left( D^d_t - D^d_{t-1} \right)
\]

where \( D^d_t = \frac{D_t}{P^G_t Y^G_{t+1}} \) denotes the debt-to-GDP ratio in the beginning of period \( t \), \( \bar{d} \) is a target value of the public debt-to-GDP ratio (see below for details) and \( \phi^d, \phi^d > 0 \) are feedback policy reaction coefficients (as in a Taylor rule for the nominal interest rate).
For notational convenience, we will define the share of total public debt held by domestic agents at the end of period $t$ as $\zeta_t = \frac{B_{t+1}^g}{D_{t+1}}$, where $0 \leq \zeta_t \leq 1$, so that $S_t^g = (1 - \zeta_t)D_{t+1}$ and $D_{t+1} = B_{t+1}^g + S_t^g$.

### 4.2 Production of public goods/services

On the production side, following e.g. Forni et al. (2010a) and Economides et al. (2013, 2016), we assume that the government combines public spending on goods and services, $G_t^c$, and public employment, $H_t^c$, to produce public goods/services, $Y_t^g$, according to the production function:

$$Y_t^g = A_t \left( G_t^c \right)^{\chi} \left( H_t^c \right)^{1-\chi}$$  \hspace{1cm} (28)

where $0 \leq \chi \leq 1$ is a technology parameter.

The law of motion of public capital in per-capita terms is:

$$K_{t+1}^g = \left( 1 - \delta^g \right) K_t^g + G_t^c$$  \hspace{1cm} (29)

where $\delta^g \in (0,1)$ is the depreciation rate of public capital stock and $K_0^g > 0$ is given.

Regarding the inputs used in the above production function, $G_t^c$ is produced by final good firms that use only non-tradable intermediate goods as inputs (see above) while $H_t^c$ is total public sector demand for the differentiated labour variety $h$ that is exogenous (see above).

### 5. World capital markets and sovereign spreads

Following most of the literature on small open economies, we assume that, when domestic households and the government participate in the world capital market, they face an interest rate that is public debt elastic (see e.g. Schmitt-Grohe and Uribe, 2003).

In particular, the nominal interest rate at which the country borrows from the international market, $R_t^H$, bears a risk-premium term, $\tilde{\psi}_t \geq 0$, that introduces a wedge between the interest rate that the home country faces and the risk-free world nominal interest rate, $R^*_t$:

$$R_t^H = \max \left\{ R_t^* + \tilde{\psi}_t, R^*_t \right\}$$  \hspace{1cm} (30)

where, as in e.g. Christiano et al. (2011), Garcia-Cicco et al. (2010) and Philippopoulos et al. (2016), the risk-premium is assumed to be an increasing function of public debt imbalances:

$$\tilde{\psi}_t = \psi^d \left( \exp \left( D_{t+1} - \bar{d} \right) - 1 \right)$$  \hspace{1cm} (31)

where $\psi^d \geq 0$ is a risk parameter and $\bar{d} > 0$ is an exogenous threshold above which premia emerge. Thus, when the public debt-to-GDP ratio is above a threshold level, an interest rate spread
arises, which is consistent with empirical evidence (see e.g. Ardagna et al., 2008; Roeger and in’t Veld, 2013; Schuknecht et al., 2009; Corsetti et al., 2013). Conceptually, the risk premium component on the right hand side of (31) reflects the risk of sovereign or country default and thus provides a channel through which such a risk affects directly the real economy.

6. Monetary-exchange rate policy regime
We model the domestic economy as a member of a currency union in the sense that the nominal exchange rate, \( S_t \), is exogenously set, and at the same time, there is no monetary policy independence. The latter means that the domestic nominal interest rate on government bonds, \( R_t \), is determined endogenously (see e.g. Philippopoulos et al. (2016) for details).

7. Aggregation and market-clearing conditions
7.1 Aggregation
The aggregate quantity, expressed in per-capita terms, of any household specific variable \( X_{h,t} \), is given by \( X_t = \int_{0}^{1} X_{h,t} dh = (1-\lambda) X_{i,t} + \lambda X_{j,t} \). Thus, per capita private consumption is given by \( C_{i}^{p} = (1-\lambda) C_{i,t}^{p} + \lambda C_{j,t}^{p} \). Since only optimizing households have access to the capital, bond, dividend and international markets, the per capita quantities for private capital, private investment, domestic government bonds, foreign private assets and profits are respectively: \( K_{i}^{p} = (1-\lambda) K_{i,t}^{p} \), \( I_{i} = (1-\lambda) I_{i,t} \), \( B_{i} = (1-\lambda) B_{i,t} \), \( F_{i}^{p} = (1-\lambda) F_{i,t}^{p} \), and \( Div_{i} = (1-\lambda) Div_{i,t} \). Per capita government transfers are \( G_{i}^{r} = (1-\lambda) G_{i,t}^{r} + \lambda G_{j,t}^{r} \), where total transfers are allocated between optimizing and liquidity constraint households according to the following rules: \( G_{i}^{NR,pr} = \lambda G_{i,t}^{pr} = \bar{\lambda} G_{i}^{pr} \) and \( G_{i}^{r,pr} = (1-\lambda) G_{i,t}^{r} = (1-\bar{\lambda}) G_{i}^{r} \), with \( 0 \leq \bar{\lambda} \leq 1 \).

7.2 Market-clearing conditions
In the labor market, total labor supply needs to equal the amount of labor employed by the private and the public sectors:

\[
H_t = \int_{0}^{1} H_{w,t} dh = \int_{0}^{1} H_{w,t}^{p} dh + \int_{0}^{1} H_{w,t}^{g} dh = \left( \frac{w_{h,t}^{p}}{w_{h,t}^{g}} \right)^{\frac{\mu^{w}}{\nu^{w} - 1}} H_{w,t}^{p} dh + H_{w,t}^{g} = H_{w,t}^{p} + H_{w,t}^{g}
\] (32)
where \( H_t \) is total labour supply, \( H^P_t = H^N_t + H^T_t \) is total private sector demand and
\[
H^T_t = \int_0^1 \left( \frac{w^P_{ht}}{w^P_t} \right)^{\frac{\mu^P}{\mu^P - 1}} H^{T,P}_{f,t} df^T, \quad H^N_t = \int_0^1 \left( \frac{w^N_{ht}}{w^N_t} \right)^{\frac{\mu^N}{\mu^N - 1}} H^{N,P}_{f,t} df^N. \quad 10
\]

In the market for capital services, the supply of utilized private capital stock from households satisfies the demand for private capital services by intermediate good firms:
\[
K^T_t = \int_0^1 K^{T,P}_{f,t} df^T \quad (33)
\]
\[
K^N_t = \int_0^1 K^{N,P}_{f,t} df^N \quad (34)
\]
\[
K_t = u_t \int_0^1 K^{p,P}_{ht} dh = u_t K^P_t = K^T_t + K^N_t \quad (35)
\]

In the final goods markets, we have:
\[
C^C_t = C^P_t \quad (36)
\]
\[
I^I_t = I^P_t \quad (37)
\]
\[
G^C_t = G^C_t = G^C_t \quad (38)
\]
\[
G^I_t = G^I_t = G^I_t \quad (39)
\]

In the tradable sector, the supply of each differentiated good \( f^T \) needs to meet domestic and foreign demand:
\[
Y^{f,D}_{j,t} = Y^{D,D}_{j,t} + Y^{X,D}_{j,t} \quad (40)
\]

Aggregating over the continuum of intermediate good firms we get:
\[
Y^T_t = \int_0^1 Y^{T,D}_{f,t} df + \int_0^1 Y^{X,D}_{f,t} df = \int_0^1 \left( \frac{P^{D,P}_{f,t} C^D_t}{P^D_t} \right)^{\frac{\mu^D}{\mu^D - 1}} Y^{D,D}_{f,t} df + \int_0^1 \left( \frac{P^{X,P}_{f,t} C^X_t}{P^X_t} \right)^{\frac{\mu^X}{\mu^X - 1}} Y^{X,D}_{f,t} df \quad (41)
\]

or
\[
Y^T_t = u^D_t Y^D_t + u^X_t Y^X_t \quad (41)
\]

where \( Y^D_t = C^D_t + I^D_t \), and \( u^D_t = \int_0^1 \left( \frac{P^{D,P}_{f,t}}{P^D_t} \right)^{-\frac{\mu^D}{\mu^D - 1}} df^T \) and \( u^X_t = \int_0^1 \left( \frac{P^{X,P}_{f,t}}{P^X_t} \right)^{-\frac{\mu^X}{\mu^X - 1}} df^T \) measure the degree of price dispersion across the differentiated goods that are sold in the domestic and foreign markets, respectively. The two measures of price dispersion evolve according to:

---

10 We have used the fact that in a symmetric equilibrium, \( w^P_{ht} / w^P_t = 1 \).
\[ u_i^d = (1 - \theta_p^d) \left( \Pi_i^{*d} \right)^{\frac{\mu_i^d}{\mu_i^{d-1}}} + \theta_p^d \left( \frac{\Pi_i^{D-1}}{\Pi_i^{D}} \right) u_{i-1}^{d} \]  

(42)

\[ u_i^x = (1 - \theta_p^x) \left( \Pi_i^{*x} \right)^{\frac{\mu_i^x}{\mu_i^{x-1}}} + \theta_p^x \left( \frac{\Pi_i^{X-1}}{\Pi_i^{X}} \right) u_{i-1}^{x} \]  

(43)

where \( \Pi_i^{*d} = P_t^d / P_i, \) \( \Pi_i^{*x} = P_t^x / P_i, \) \( \Pi_i^D = P_t^D / P_{i-1} \) and \( \Pi_i^X = P_t^X / P_{i-1} \).

Also, by making use of the market clearing conditions in the labor and capital markets, the production function written in per capita terms is:

\[ Y_i^T = A_i^T \left( K_i^T \right)^{a_T} \left( H_i^T \right)^{1-a_T} \left( K_i^T \right)^{a_T} - \Phi_T \]  

(44)

Market clearing implies that the supply of each differentiated non-tradable good \( f^N \) needs to meet domestic demand:

\[ Y_{f_i}^N = C_{f_i}^N + I_{f_i}^N + GC_{f_i}^N + GI_{f_i}^N \]  

(45)

Aggregating over the continuum of intermediate good firms we get:

\[ Y_i^N = \int_0^1 \left( \frac{P_{f_i}^N}{P_i^N} \right)^{-\frac{\mu_i^N}{\mu_i^{N-1}}} Y_{i}^{NT} df^N = u_i^N Y_{i}^{NT} \]  

(46)

where \( Y_{i}^{NT} = C_i^N + I_i^N + GC_i^N + GI_i^N, \) and \( u_i^N = \int_0^1 \left( \frac{P_{f_i}^N}{P_i^N} \right)^{-\frac{\mu_i^N}{\mu_i^{N-1}}} df^N \) measures the degree of price dispersion across the differentiated non-tradable goods that evolves according to:

\[ u_i^N = (1 - \theta_p^N) \left( \Pi_i^{*N} \right)^{\frac{\mu_i^N}{\mu_i^{N-1}}} + \theta_p^N \left( \frac{\Pi_i^{N-1}}{\Pi_i^{N}} \right) u_{i-1}^{N} \]  

(47)

where \( \Pi_i^{*N} = P_t^N / P_i \) and \( \Pi_i^N = P_t^N / P_{i-1} \).

Also, by making use of the market clearing conditions in the labor and capital markets, the production function written in per capita terms is:

\[ Y_i^N = A_i^N \left( K_i^N \right)^{a_N} \left( H_i^N \right)^{1-a_N} \left( K_i^N \right)^{a_N} - \Phi_N \]  

(48)

In the market of imported intermediate goods, the supply of each differentiated importing good \( f^m \) needs to meet domestic demand:
\[ M_t = \int_0^1 Y^M_t \, df^M_t = \int_0^1 \left( \frac{P^m_{f^M, t}}{P^M_t} \right) \frac{\mu^M_t}{\mu^M_{t-1}} \, Y^M_t \, df^M_t = \mu^M_t Y^M_t \] (49)

where \( Y^M_t = C^M_t + I^M_t \) and \( u^m_t = \int_0^1 \left( \frac{P^M_{f^M, t}}{P^M_t} \right) \frac{\mu^M_t}{\mu^M_{t-1}} \, df^M_t \) measures the degree of price dispersion across the differentiated imported goods \( f^m \) that are sold in the domestic market that evolves according to:

\[
u^m_t = \left( 1 - \theta^M_p \right) \left( \Pi^*_t \right)^{\frac{\mu^M_t}{\mu^M_{t-1}}} + \theta^M_p \left( \frac{\Pi^M_t}{\Pi^M_{t-1}} \right)^{\frac{\mu^M_t}{\mu^M_{t-1}}} u^m_{t-1} \tag{50} \]

where \( \Pi^*_t = P^*_t / P^M_t \) and \( \Pi^M_t = P^M_t / P^M_{t-1} \).

In the dividend market, real profits, \( \text{Div}^{D}_{f^T} \), of the intermediate good \( f^T \), expressed in terms of the price of the final consumption good \( P^C_t \), can be written as:

\[	ext{Div}^{D}_{f^T} = \text{Div}^{D}_{f^T} + \text{Div}^{X}_{f^T} = \frac{P^D}{P^C} Y^D_{f^T} + \frac{P^X}{P^C} Y^X_{f^T} - w^p H_{f^T} - r^k K_{f^T} - (R^H_t - 1)w^p H^I_t \tag{51} \]

Aggregating over the continuum of intermediate tradable good firms, and using the corresponding demand functions for the intermediate good \( f^T \), and the market clearing conditions in the labour market, we get the per capita real profits of the intermediate goods sector:

\[	ext{Div}^{T}_t = \int_0^1 \text{Div}^{T}_{f^T} \, df^T = \frac{P^D}{P^C} Y^D_{f^T} + \frac{P^X}{P^C} Y^X_{f^T} - mc^T_t \left( Y^T_t + \Phi^T_t \right) - (R^H_t - 1)w^p H^I_t \tag{52} \]

Profits in the non-tradable sector are defined in an analogous manner:

\[	ext{Div}^{N}_t = \int_0^1 \text{Div}^{N}_{f^N} \, df^N = \frac{P^N}{P^C} Y^N_t - mc^N_t \left( Y^N_t + \Phi^N_t \right) - (R^H_t - 1)w^p H^N_t \tag{53} \]

Profits of the importing firm \( f^m \) (in terms of the price of the final consumption good, \( P^C_t \), are written as:

\[	ext{Div}^{M}_{f^M} = \frac{P^M}{P^C} \left( \frac{P^M_{f^M, t}}{P^M_t} \right) Y^M_{f^M} = \frac{P^M}{P^C} \left( \frac{P^M_{f^M, t}}{P^M_t} \right) Y^M_{f^M} - q^M_t \frac{P^Y_{f^M, t}}{P^M_t} Y^M_{f^M} \tag{54} \]
where \( q_t = \frac{S_t P_t^Y}{P_t} = q_{t-1} \frac{\Pi_t^Y}{\Pi_{t-1}^Y} \), is the real effective exchange rate, \( \Pi_t^Y = P_t^Y / P_{t-1}^Y \), and \( \Pi_t^Y = P_t^Y / P_{t-1}^Y \),

\( s_t = \frac{S_t}{S_{t-1}} \) is the gross growth rate of the nominal exchange rate and \( P_t^Y \) is the implicit price deflator in the foreign country.

Aggregating over the continuum of importing firms, we obtain the real per capita profits of importing firms:

\[
Div_t^M = \int_0^1 Div_t^{M, f} d f = P_t^M \left( Y_t^M - \frac{q_t P_t^Y}{P_t^M} u_t Y_t^M \right)
\]

(55)

Therefore, real aggregate profits in period \( t \) are \( Div_t^f = Div_t^Y + Div_t^{f, w} + Div_t^M \) and market clearing in the dividends market requires that all profits are paid out as dividends: \( Div_t^f = Div_t^f \).

Regarding the aggregate resource constraint and the evolution of net foreign assets, this is derived by the optimizing households’ budget constraint, after imposing the budget constraint of the liquidity constraint households, the government budget constraint, the definition of profits of intermediate good and importing firms, and by making use of the zero profit conditions of the final good firms:

\[
\frac{S_t F_{t+1}^p}{P_t^c} - \frac{S_t F_{t+1}^g}{P_t^c} v_t w_t H_t^p = R_{t+1}^{H_t} S_t F_{t+1}^p - R_{t+1}^{G_t} S_t F_{t+1}^g + P_t^X Y_t^X - q_t \frac{P_t^Y}{P_t^M} u_t Y_t^M - R_{t+1}^{H_t} v_t w_t H_t^p
\]

(56)

Note that the net interest payments on the firms’ working capital intra-period loans enter the economy’s aggregate financial flows as a liability of the home country, and thereby implicitly constitute a transfer of domestic production units to the international lenders.

### 7.3 Definition of Gross Domestic Product

Combining the market clearing conditions in the intermediate goods and the final goods sectors, we obtain the following expression for the nominal private sector GDP, \( Y_t^p \), that determines the implicit price index of domestic output (i.e. the GDP deflator), \( P_t^Y \):

\[
P_t^Y Y_t^p = P_t^C c_t^i + P_t^I I_t^i + P_t^N G_t^i + P_t^X Y_t^X - q_t P_t^Y M_t
\]

where \( P_t^Y Y_t^p \equiv P_t^D Y_t^{D, i} + P_t^X Y_t^X + P_t^N Y_t^N \) and \( P_t^Y Y_t^{GDP} \equiv P_t^Y Y_t^p + w_t^H H_t^p \), where \( Y_t^{GDP} \) is defined as the aggregate GDP, in a consistent way with National Accounts definitions.
7.4 Definition of real variables

The real net private foreign assets, the total real public debt, the foreign real public debt and the
domestic real public debt at the end of period \( t \) are defined as: 
\[
f_{t+1}^p = \frac{SF_{t+1}^p}{P_t^c}, \quad d_{t+1} = \frac{D_{t+1}}{P_t^c},
\]
\[
f_{t+1}^g = \frac{SF_{t+1}^g}{P_t^c} \quad \text{and} \quad b_{t+1}^g = \frac{B_{t+1}^g}{P_t^c},\]
respectively. In addition, we divide all price indices by the price index of the consumption good, \( P_t^C \). For instance, the relative price of non-tradable consumption goods is defined as \( p_t^N = P_t^N / P_t^C \). The price of foreign competitors in the export markets, \( P_t^x \), is also divided by the foreign GDP deflator \( P_t^y \).

8. Final equilibrium system

We can now collect all the above to present the final stationary system that is put in the computer.

\textit{Ricardian households}

\[
\Lambda_t^R = \left[ \frac{C_t^R - \xi^R C_{t+1}^R}{1 + \tau_t^P} \right]^{-1}
\]

(A1)

\[
\Lambda_t^R = \beta E_t \left[ \Lambda_{t+1}^R \frac{R_t}{\Pi_{t+1}^c} \right]
\]

(A2)

\[
\Lambda_t^R \left[ 1 + \xi^f \left( \frac{f_{t+1}^p}{P_t^C Y_t^GDP} - \bar{f} \right) \right] = \beta E_t \left[ \Lambda_{t+1}^R \frac{R_t^H}{\Pi_{t+1}^c} s_{t+1} \right]
\]

(A3)

\[
(1 - \tau_t^k) r_t^k = q_t \phi \delta u_{t+1}^\phi
\]

(A4)

\[
p_t^c = q_t \left[ 1 - \frac{\xi^k}{2} \left( \frac{I_t^p}{I_{t-1}^p} - 1 \right) \right] \left[ 1 + \xi^k \left( \frac{I_t^p}{I_{t-1}^p} - 1 \right) \right] + \beta E_t \left[ q_{t+1} \Lambda_{t+1}^R \frac{\Lambda_t^R}{\Lambda_t^R} \left( \frac{I_t^p}{I_t^p} - 1 \right) \left( \frac{I_t^p}{I_{t-1}^p} \right)^2 \right]
\]

(A5)

\[
q_t = \beta E_t \frac{\Lambda_{t+1}^R}{\Lambda_t^R} \left[ (1 - \tau_t^k) r_t^k u_{t+1} + q_{t+1} (1 - \delta^\phi u_{t+1}^\phi) \right]
\]

(A6)

\[
K_{t+1}^p = (1 - \delta^\phi u_t^\phi) K_t^p + \left[ 1 - \frac{\xi^k}{2} \left( \frac{I_t^p}{I_{t-1}^p} - 1 \right) \right] I_t^p
\]

(A7)

\[
C_t^R = C_t^{p,R} + \psi Y_t^R
\]

(A8)
Non-Ricardian households

\[
(1 + \tau_i^c)C_{i,t}^{p, NR} = (1 - \tau_i^c)\left( w_i^p H_i^{r} + w_i^g H_i^{g} \right) + \bar{\lambda} G_i^{gr}
\]
(A9)

\[
\Lambda_i^{NR} = \left[ C_i^{NR} - \bar{g} C_i^{NR} \right] \Lambda_i^{r} \quad \text{(A10)}
\]

\[
C_i^{NR} = C_i^{p, NR} + \bar{g} Y_t^{g}
\]
(A11)

Aggregate private consumption

\[
C_i^p = \lambda C_i^{p, NR} + (1 - \lambda) C_i^{p, R}
\]
(A12)

Optimal labour supply

\[
W_i^p \frac{(1 - \tau_i^c)}{\kappa H_i^{r}} \left[ \lambda \Lambda_i^{NR} + (1 - \lambda) \Lambda_i^{r} \right] = \mu_i^W
\]
(A13)

Real wage rate in the private sector

\[
w_i^p = \left( w_{i-1}^p \right)^n \left( w_i^{* p} \right)^{1-n}
\]
(A14)

Intermediate good firms – domestic tradable

\[
Y_i^T = A^T \left( K_i^T \right)^{\alpha_T} \left( H_i^T \right)^{1-\alpha_T} \left( K_i^T \right)^{\alpha_T} - \Phi_T
\]
(A15)

\[
(1 + v_i(R_i^r - 1)) W_i^p = mc_i^T (1 - a_T) \frac{Y_i^T + \Phi_T}{H_i^T}
\]
(A16)

\[
r_i^r = mc_i^T a_T \frac{Y_i^T + \Phi_T}{K_i^T}
\]
(A17)

\[
g_{i, t}^{D_i} = mc_i^{D_i} Y_i^{D_i} \mu_i^{D_i} \lambda_i^{D_i} p_i^{D_i} + \left( \beta \theta_i^{D_i} \right) E_i \left[ \frac{\left( \Pi_{i+1}^D \right)^{\eta_D}}{\Pi_{i+1}^{D_D}} \right] \frac{\kappa^{D_i}}{\rho^{D_i - 1}} g_{i, t+1}^{D_i}
\]
(A18)

\[
g_{i, t}^{D_i} = \Pi_i^{D_i} Y_i^{D_i} \lambda_i^{D_i} p_i^{D_i} + \left( \beta \theta_i^{D_i} \right) E_i \left[ \frac{\left( \Pi_{i+1}^D \right)^{\eta_D}}{\Pi_{i+1}^{D_D}} \right] \frac{\Pi_i^{D_D}}{\Pi_{i+1}^{D_D}} b_{i, t+1}^{D_i}
\]
(A19)

\[
g_{i, t}^{D_i} = g_{i, t}^{D_i}
\]
(A20)
1 = \left(1 - \theta_p^D\right) \left(\Pi_i^{*D}\right)^{\frac{1}{1-\mu^D}} + \theta_p^D \left[\frac{\left(\Pi_i^D\right)^{\frac{1}{1-\mu^p}}}{\Pi_i^D}\right]^{-\frac{\mu^X}{\mu^D-1}} \tag{A21}

mc_i^D = mc_i^T / p_i^D \tag{A22}

\begin{align*}
g_t^{N_i} &= mc_i^N Y_t^N \mu_i^N \Lambda_i^N p_i^N + \left(\beta \theta_p^N\right) E_t \left[\frac{\left(\Pi_i^{N}\right)^{\frac{1}{1-\mu^N}}}{\Pi_i^{N}}\right]^{-\frac{\mu^N}{\mu^N-1}} g_{t+1}^{N} \tag{A23}

g_t^{N_x} &= \Pi_i^{N_x} Y_t^N X_t^N + \left(\beta \theta_p^N\right) E_t \left[\frac{\left(\Pi_i^{N_x}\right)^{\frac{1}{1-\mu^N}}}{\Pi_i^{N_x}}\right]^{-\frac{\mu^N}{\mu^N-1}} \frac{\Pi_t^{N_x} g_{t+1}^{N_x}}{\Pi_t^{N_x}} \tag{A24}

g_t^{N_x} &= g_t^{N_x} \tag{A25}
\end{align*}

1 = \left(1 - \theta_p^N\right) \left(\Pi_i^{*N}\right)^{\frac{1}{1-\mu^N}} + \theta_p^N \left[\frac{\left(\Pi_i^{N}\right)^{\frac{1}{1-\mu^N}}}{\Pi_i^{N}}\right]^{-\frac{\mu^N}{\mu^N-1}} \tag{A26}

mc_i^T = mc_i^N \frac{p_i^N}{p_i^D} \tag{A27}

**Intermediate good firms – domestic non-tradable**

\begin{align*}
Y_t^N &= A_t^N \left(K_t^N \right)^{\alpha_n} \left(H_t^N \right)^{1-\alpha_n} \left(R_t^g \right)^{\alpha_n} - \Phi_N \tag{A28}

\left(1 + v_t \left(R_t^H - 1\right)\right) w_t^p &= mc_i^N \left(1-a_n\right) \frac{Y_t^N + \Phi_N}{H_t^N} \tag{A29}

r_t^N &= mc_i^N a_N \frac{Y_t^N + \Phi_N}{K_t^N} \tag{A30}
\end{align*}

\begin{align*}
g_t^{N_i} &= mc_i^N Y_t^N \mu_i^N \Lambda_i^N p_i^N + \left(\beta \theta_p^N\right) E_t \left[\frac{\left(\Pi_i^{N}\right)^{\frac{1}{1-\mu^N}}}{\Pi_i^{N}}\right]^{-\frac{\mu^N}{\mu^N-1}} g_{t+1}^{N_i} \tag{A31}

g_t^{N_x} &= \Pi_i^{N_x} Y_t^N X_t^N + \left(\beta \theta_p^N\right) E_t \left[\frac{\left(\Pi_i^{N_x}\right)^{\frac{1}{1-\mu^N}}}{\Pi_i^{N_x}}\right]^{-\frac{\mu^N}{\mu^N-1}} \frac{\Pi_t^{N_x} g_{t+1}^{N_x}}{\Pi_t^{N_x}} \tag{A32}

g_t^{N_x} &= g_t^{N_x} \tag{A33}
\end{align*}

1 = \left(1 - \theta_p^N\right) \left(\Pi_i^{*N}\right)^{\frac{1}{1-\mu^N}} + \theta_p^N \left[\frac{\left(\Pi_i^{N}\right)^{\frac{1}{1-\mu^N}}}{\Pi_i^{N}}\right]^{-\frac{\mu^N}{\mu^N-1}} \tag{A34}
mc^*_i = mc^N_i / p^N_i \quad (A35)

\[ g^M_{i} = mc^M_i Y^M_i \mu^M_i \Lambda^R_i p^M_i + (\beta \theta^M_p) E_i \left( \frac{\left( \Pi^M_{i} \right)^{\epsilon_M}}{\Pi^M_{i+1}} \right)^{\frac{\mu^M_i}{\mu^M_{i+1}} - 1} g^M_{i+1} \quad (A36) \]

\[ g^{M_z}_{i} = \Pi^M_{i} Y^M_i \Lambda^R_i p^M_i + (\beta \theta^M_p) E_i \left( \frac{\left( \Pi^M_{i} \right)^{\epsilon_M}}{\Pi^M_{i+1}} \right)^{1} - \frac{\mu^M_i}{\mu^M_{i+1}} \Pi^M_{i+1} g^{M_z}_{i+1} \quad (A37) \]

\[ g^{M_{t1}} = g^{M_z} \quad (A38) \]

\[ 1 = (1 - \theta^M_p) (\Pi^M_{i+1})^{1 - \epsilon_M} + \theta^M_p \left[ \left( \frac{\Pi^M_{i+1}}{\Pi^M_{i}} \right)^{\epsilon_M} \right]^{1 - \epsilon_M} \quad (A39) \]

\[ mc^M_i = q^*_{i} p^Y_i / p^M_i \quad (A40) \]

**Domestic consumption final good firms**

\[ C^P_i = \left[ \omega^P C^T_i \left( \frac{\epsilon_{C_i}}{\epsilon_{C}} \right) \epsilon_{C_i} - 1 + (1 - \omega^C) \epsilon_{C_i} \left( C^N_i \right) \epsilon_{C_i} - 1 \right]^{\epsilon_{C_i}} \quad (A41) \]

\[ C^T_i = \left[ \omega^T C_i \left( \frac{\epsilon_{C_i}}{\epsilon_{C}} \right) \epsilon_{C_i} - 1 + (1 - \omega^T) \epsilon_{C_i} \left( C^M_i \right) \epsilon_{C_i} - 1 \right]^{\epsilon_{C_i}} \quad (A42) \]

\[ \frac{C^D_i}{C^T_i} = \omega_i \left( \frac{p^D_i}{p^T_i} \right)^{-\epsilon_{C_i}} \quad (A43) \]

\[ \frac{C^N_i}{C^T_i} = (1 - \omega_i) \left( \frac{p^N_i}{p^T_i} \right)^{-\epsilon_{C_i}} \quad (A44) \]

\[ p^T_i = \left[ \omega_i \left( p^D_i \right)^{1 - \epsilon_{C_i}} + (1 - \omega_i) \left( p^M_i \right)^{1 - \epsilon_{C_i}} \right]^{\epsilon_{C_i}} \quad (A45) \]

\[ 1 = \left[ \omega_i \left( p^T_i \right)^{1 - \epsilon_{C_i}} + (1 - \omega_i) \left( p^N_i \right)^{1 - \epsilon_{C_i}} \right]^{\epsilon_{C_i}} \quad (A46) \]

**Domestic investment final good firms**

\[ I^P_i = \left[ \omega_i \left( I^T_i \right)^{\epsilon_{I_i}} - 1 + (1 - \omega_i) \left( I^N_i \right)^{\epsilon_{I_i}} - 1 \right]^{\epsilon_{I_i}} \quad (A47) \]
\[ I_t^D = \left[ \frac{1}{\omega_{T_t}^\varepsilon} \left( I_t^D \right)^{\frac{1}{\varepsilon_{T_t}}} + \frac{1}{\omega_{M_t}^\varepsilon} \left( I_t^M \right)^{\frac{1}{\varepsilon_{M_t}}} \right]^{\frac{1}{\varepsilon_{M_t}}} \]  

(A48)

\[ \frac{I_t^D}{I_t^N} =\omega_{T_t} \left( \frac{p_t^D}{p_t^N} \right) \]  

(A49)

\[ \frac{I_t^N}{I_t^M} = (1 - \omega_{T_t}) \left( \frac{p_t^N}{p_t^M} \right)^{-\varepsilon_t} \]  

(A50)

\[ p_t^{Nt} = \left[ \omega_{T_t} \left( p_t^N \right)^{1-\varepsilon_t} + (1 - \omega_{T_t}) \left( p_t^M \right)^{1-\varepsilon_t} \right]^{\frac{1}{1-\varepsilon_t}} \]  

(A51)

\[ p_t^{Mt} = \left[ \omega_{T_t} \left( p_t^M \right)^{1-\varepsilon_t} + (1 - \omega_{T_t}) \left( p_t^N \right)^{1-\varepsilon_t} \right]^{\frac{1}{1-\varepsilon_t}} \]  

(A52)

\[ \text{Real exports} \]  

\[ Y_t^* = \left( \frac{p_t^M / p_t^N}{q_t^M / p_t^N} \right)^{-\varepsilon_t} Y_t^* \]  

(A53)

\[ \text{Real imports} \]  

\[ M_t = u_t^M Y_t^M \]  

(A54)

\[ Y_t^m = C_t^m + I_t^m \]  

(A55)

\[ \text{Government} \]  

\[ d_{t+1} + \tau_i C_t^p + \tau_i^l \left( w_{t}^p H_{t}^p + w_{t}^g H_{t}^g \right) + \tau_i \left( \kappa_{i}^k K_{i} + \text{div}_{i} \right) + \tau_{ii} = \]  

\[ = p_t^N G_t^C + p_t^N G_t^G + G_t^p + w_t^g H_{t}^g + R_{t+1} \frac{\xi d_{t+1}}{\Pi_{t}} + R_{t+1} \frac{(1 - \xi)}{\Pi_{t}} s d_{t+1} \]  

(A56)

\[ \tau_t = \tau_{t+1} + \phi_{d} \left( d_{t} - \bar{d} \right) + \phi_{B} \left( d_{t} - d_{t+1} \right) \]  

(A57)

\[ d_t^b = \frac{d_t}{p_t^Y Y_{t}^{GDP}} \]  

(A58)

\[ Y_t^{g} = \left( G_t^C \right)^{\alpha} \left( H_t^g \right)^{1-\alpha} \]  

(A59)

\[ K_{t+1}^{g} = \left(1 - \delta^g \right) K_{t}^{g} + G_t^i \]  

(A60)

\[ R_t^G = m R_t^* + (1 - m) R_t^{H} \]  

(A61)
World capital markets

\[ R_i^H = \max \{ R_i^*, \tilde{\psi}_t, R_i^* \} \]  \hspace{1cm} (A62)

\[ \tilde{\psi}_t = \psi^d \left( \exp \left( \frac{d_{t+1}}{p_t^* Y_{GDP}^t} - \tilde{d} \right) - 1 \right) \]  \hspace{1cm} (A63)

Market-clearing conditions

\[ H_t = H_t^p + H_t^e \]  \hspace{1cm} (A64)

\[ K_t = u_t K_t^p \]  \hspace{1cm} (A65)

\[ K_t^p = K_t^{pT} + K_t^N \]  \hspace{1cm} (A66)

\[ Y_t^T = u_t^{D_T} Y_t^D + u_t^{X_T} Y_t^X \]  \hspace{1cm} (A67)

\[ Y_t^D = C_t^D + I_t^D \]  \hspace{1cm} (A68)

\[ Y_t^N = u_t^{N} \left( C_t^N + I_t^N + G_t^c + G_t^i \right) \]  \hspace{1cm} (A69)

\[ Y_t^p = Y_t^T + Y_t^N \]  \hspace{1cm} (A70)

\[ \text{div}_t = \text{div}_t^N + \text{div}_t^T + \text{div}_t^M \]  \hspace{1cm} (A71)

\[ \text{div}_t^N = p_t^N Y_t^N - mc_t^N \left( Y_t^N + \Phi_N \right) \]  \hspace{1cm} (A72)

\[ \text{div}_t^T = p_t^{D_T} Y_t^D + p_t^{X_T} Y_t^X - mc_t^T \left( Y_t^T + \Phi_T \right) \]  \hspace{1cm} (A73)

\[ \text{div}_t^M = p_t^M Y_t^M - q_t^{x_t} p_t^M M_t \]  \hspace{1cm} (A74)

GDP deflator

\[ p_t^Y Y_t^p = p_t^{D_Y} Y_t^D + p_t^{X_Y} Y_t^X + p_t^{N_Y} Y_t^N \]  \hspace{1cm} (A75)

Real GDP

\[ Y_t^{GDP} = Y_t^p + \left( p_t^Y \right)^{-1} W_t^g H_t^g \]  \hspace{1cm} (A76)

Evolution of net foreign assets

\[ f_{t+1}^p -(1 - \zeta_t) d_{t-1} - v_t w_t^p H_t^p = R_{t-1}^{H} \frac{S_t f_t^p}{ \Pi_t^c } - R_{t-1}^{C} \frac{S_t \left( 1 - \zeta_t \right) d_t}{ \Pi_t^c } + p_t^X Y_t^X - q_t^{x_t} p_t^Y u_t^M Y_t^M - R_{t-1}^{H} v_t w_t^p H_t^p \]  \hspace{1cm} (A77)
Trade balance-to-GDP ratio
The trade balance is defined as the value of exports minus the value of imports. We express the trade balance as a share of GDP:

\[
\text{tby}_t = \frac{p_t^X Y_t^X - q_t^p p_t^Y u_t^M Y_t^M}{p_t^Y Y_t^{GDP}}
\]  
(A78)

Current account balance-to-GDP ratio
The current account balance is defined as the change in net total foreign assets. We express the current account as share of GDP:

\[
cay_t = \text{tby}_t + \left( \frac{R_t^{H_t} - 1}{p_t^Y Y_t^{GDP}} \right) \left( \frac{s_t f_t^p}{\Pi_t^c} - \frac{s_t (1 - \zeta_t) d_t}{\Pi_t^l} \right)
\]  
(A79)

Real effective exchange rate

\[
q_t^{e^r} = q_{t-1}^{e^r} \frac{s_t \Pi_t^{Y}}{\Pi_t^l}
\]  
(A80)

Terms of trade

\[
tot_t = \frac{p_t^M}{p_t^X}
\]  
(A81)

Price dispersion

\[
u_t^d = \left(1 - \theta_p^d\right) \left(\Pi_t^{x^d}\right)^{-\mu_p^d / \mu_p^{d-1}} + \theta_p^d \left(\frac{\Pi_t^{D}}{\Pi_t^{D-1}}\right)^{-\mu_p^d / \mu_p^{d-1}} u_{t-1}^d
\]  
(A82)

\[
u_t^s = \left(1 - \theta_p^s\right) \left(\Pi_t^{x^s}\right)^{-\mu_p^s / \mu_p^{s-1}} + \theta_p^s \left(\frac{\Pi_t^{X}}{\Pi_t^{X-1}}\right)^{-\mu_p^s / \mu_p^{s-1}} u_{t-1}^s
\]  
(A83)

\[
u_t^m = \left(1 - \theta_p^m\right) \left(\Pi_t^{x^m}\right)^{-\mu_p^m / \mu_p^{m-1}} + \theta_p^m \left(\frac{\Pi_t^{M}}{\Pi_t^{M-1}}\right)^{-\mu_p^m / \mu_p^{m-1}} u_{t-1}^m
\]  
(A84)
**Inflation rates**

\[
\Pi^D_t = \frac{P^D_t}{P^D_{t-1}} \Pi^C_t \quad (A85)
\]

\[
\Pi^X_t = \frac{P^X_t}{P^X_{t-1}} \Pi^C_t \quad (A86)
\]

\[
\Pi^M_t = \frac{P^M_t}{P^M_{t-1}} \Pi^C_t \quad (A87)
\]

\[
\Pi^Y_t = \frac{P^Y_t}{P^Y_{t-1}} \Pi^C_t \quad (A88)
\]

\[
\Pi^I_t = \frac{P^I_t}{P^I_{t-1}} \Pi^C_t \quad (A89)
\]

Therefore, the final equilibrium system consists of 89 equations in 89 endogenous variables. This is given the exogenous policy instruments, initial conditions for the state variables and the TFP in the two sectors. The model is solved using a Newton-type non-linear method as implemented in DYNARE.

9. **Calibration to the Greek economy**

The above model is calibrated to the Greek economy at an annual frequency. The data source is Eurostat, unless otherwise stated. The data cover the period 2000-2015, although the period used for calibration purposes is up to and including 2009 (see the discussion in the main text). Table A1 reports the calibrated parameter values and the average values of the fiscal policy variables in the data.

Regarding fiscal policy variables, as said, we use pre-crisis data for calibration. We set the share of total public debt held by domestic agents equal to 0.48, which is the average value in the data over the 2003-2009 period. The threshold level of the public debt-to-GDP ratio above which premia emerge, \(\bar{d}\), is set equal to 126%, that corresponds to the value of the public debt-to-GDP ratio in 2009. The feedback parameters on the public debt-to-GDP ratio gap are chosen so as the debt ratio to converge to its steady state value after around forty years following a shock.

Regarding parameter values, most of them are as in Papageorgiou (2014) and Papageorgiou and Vourvachaki (2016) who provide a detailed discussion of the calibration step. We can therefore omit details here and just discuss some main points.

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11 For calibration, we focus on the period during which Greece is part of the euro but before the debt crisis in 2010.
We define as tradable sector the sum of agricultural, industry (excluding construction), and tourism related (transportation, hotels and restaurants) activities. Non-tradable sector includes the remaining business sector activities.\textsuperscript{12} We calibrate the price markups for the tradable and non-tradable sector to match the respective sector-specific net profit margins, which are calculated using national accounts at industry level.\textsuperscript{13} The resulting values are $\mu_t^T = 1.35$ for tradables and $\mu_t^N = 1.46$ for non-tradables. These values are within the range typically used by the related literature on other euro-area countries, see e.g. Forni et al. (2010b). We assume that the markup for importing activities is the same as the markup for the domestic tradable sector. The markup of the private sector wages and the export sector are set as in Papageorgiou (2014), equal to $\mu_t^W = 1.15$ and $\mu_t^X = 1.11$, respectively. We normalize the level of long-run aggregate productivity in the non-tradable sector $A_t^N$, equal to one and calibrate the long-run aggregate productivity in the tradable sector $A_t^T$ so as to be consistent with the different mark-ups across the two sectors. We assume symmetry, across the tradable and non-tradable sectors, regarding labour shares, the Calvo parameters and the inflation indexation parameters. The Calvo parameters $\theta_p^D, \theta_p^N, \theta_p^W$ are set equal to 0.5 and the Calvo parameter $\theta_p^X$ is set equal to 0.35. As shown in Christoffel et al. 2008, these values are in the range of estimates for the euro area countries. The economy-wide labour share is calibrated in a consistent way with our definition of tradables and non-tradables and following the methodology described in Papageorgiou (2012) by assuming that the self-employed earn an imputed labour income. The share of labour costs in the private sector financed by working capital loans, $\nu_t$, is set equal to the fraction of working capital loans in all new loans of non-financial firms.\textsuperscript{14} Finally, we set the fixed costs in either sector to ensure that dividends would be non-negative and close to zero across the policy experiments. As concerns the parameters of the CES consumption and investment technologies, $\omega_c, \omega_i$, that measure the weight of tradable goods in the production of the final good, we calibrate them to match the share of tradables (domestic tradables and imports) in aggregate consumption and investment, respectively. The home bias parameters, $\omega_T, \omega_I$, are respectively calibrated to match the share of imported consumption goods in total

\textsuperscript{12} We exclude real estate activities from business activities as they include imputed owner rents. Also, we exclude public administration and defense and compulsory social security contributions as these categories refer to the public sector.

\textsuperscript{13} Specifically, the net profit margin (NPM) is defined as the share of the net operating surplus in gross value added. The net operating surplus excludes depreciation costs and is adjusted to exclude the imputed labour income of the self-employed in each sector. The imputed labour income of the self-employed for each sector, tradable or non-tradable, is proxied by assuming that each self-employed person earns a wage rate equal to the average compensation per employee. The gross markup is then computed as $1/(1-NPM)$.

\textsuperscript{14} This information is taken from the European Commission’s Survey on Access to Finance of Enterprises.
private consumption and the share of imported investment goods in total private investment. The elasticities of substitution between tradable and non-tradable consumption and investment goods, $\varepsilon_c, \varepsilon_i$, are both set equal to 0.5.\textsuperscript{15} Given the value of the discount factor, $\beta$, which is calibrated by $\beta = 1/ R^*$ and by assuming a foreign nominal interest rate equal to 4.15% annually, it follows that the value of the private net foreign asset position is pinned down by the parameter $\overline{f}$. As is common in similar studies, the parameter $\overline{f}$ is set equal to zero, which implies a zero net foreign asset position for the private sector. Finally, we normalize to one the values of the utilization rate of capital, the prices of imported intermediate goods, and the inflation rates for all types of intermediates. Also, we set the adjustment cost parameter for foreign asset holdings, $\xi'$, to the lowest possible value so as to ensure that the equilibrium solution for foreign assets is stationary.

Table A1: Parameterization

<table>
<thead>
<tr>
<th>Parameter or Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Inverse of the Frisch elasticity of labour supply</td>
<td>1</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>Substitutability/complementarity between private and public goods</td>
<td>0.05</td>
</tr>
<tr>
<td>$\xi^c$</td>
<td>Habit persistence</td>
<td>0.60</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Time discount factor</td>
<td>0.9602</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Preference parameter</td>
<td>17.76</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Fraction of liquidity constrained households</td>
<td>0.35</td>
</tr>
<tr>
<td>$1 - a_r, 1 - a_N$</td>
<td>Labour elasticity in production - Tradables, Non-Tradables</td>
<td>0.58</td>
</tr>
<tr>
<td>$a_r, a_N$</td>
<td>Gross capital elasticity in production</td>
<td>0.42</td>
</tr>
<tr>
<td>$a_G$</td>
<td>Public capital elasticity in production</td>
<td>0.0538</td>
</tr>
<tr>
<td>$A^N$</td>
<td>Long-run aggregate productivity - Non-Tradables</td>
<td>1</td>
</tr>
<tr>
<td>$A^T$</td>
<td>Long-run aggregate productivity - Tradables</td>
<td>0.9241</td>
</tr>
<tr>
<td>$\delta^p$</td>
<td>Private capital quarterly depreciation rate</td>
<td>0.0688</td>
</tr>
<tr>
<td>$\delta^e$</td>
<td>Public capital quarterly depreciation rate</td>
<td>0.0428</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Elasticity of marginal depreciation costs</td>
<td>1.6032</td>
</tr>
<tr>
<td>$\mu_i^W$</td>
<td>Markup on private sector wages</td>
<td>1.15</td>
</tr>
<tr>
<td>$\mu_i^D$</td>
<td>Markup - domestic Tradables</td>
<td>1.352</td>
</tr>
<tr>
<td>$\mu_i^N$</td>
<td>Markup - domestic Non-Tradables</td>
<td>1.463</td>
</tr>
<tr>
<td>$\mu_i^X$</td>
<td>Markup - foreign markets</td>
<td>1.1</td>
</tr>
</tbody>
</table>

\textsuperscript{15} Gomes et al. (2013) use the same value. Also, the average value added share of tradable activities is around 40%.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_i^M$</td>
<td>Markup - importing firms</td>
<td>1.352</td>
</tr>
<tr>
<td>$n$</td>
<td>Degree of real wage rigidity</td>
<td>0.6491</td>
</tr>
<tr>
<td>$\theta_p^D, \theta_p^N, \theta_p^M$</td>
<td>Calvo parameters</td>
<td>0.5</td>
</tr>
<tr>
<td>$\theta_p^X$</td>
<td>Calvo parameter - foreign markets</td>
<td>0.35</td>
</tr>
<tr>
<td>$x_D, x_N, x_X, x_M$</td>
<td>Indexation parameters</td>
<td>0.26</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Share of wages financed by working capital loans</td>
<td>0.4</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Productivity of public spending on goods and services</td>
<td>0.3045</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Bias towards tradables in the production of consumption goods</td>
<td>0.66</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>Bias towards tradables in the production of investment goods</td>
<td>0.44</td>
</tr>
<tr>
<td>$\omega_{TC}$</td>
<td>Home bias in the production of tradable consumption goods</td>
<td>0.328</td>
</tr>
<tr>
<td>$\omega_{TI}$</td>
<td>Home bias in the production of tradable investment goods</td>
<td>0.2</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Elasticity of substitution between tradable and non-tradable consumption goods</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varepsilon_i$</td>
<td>Elasticity of substitution between tradable and non-tradable investment goods</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varepsilon_{TC}$</td>
<td>Elasticity of substitution between imported and domestic tradable consumption goods</td>
<td>3.351</td>
</tr>
<tr>
<td>$\varepsilon_{TI}$</td>
<td>Elasticity of substitution between imported and domestic tradable investment goods</td>
<td>6.352</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>Elasticity of exports</td>
<td>1.4</td>
</tr>
<tr>
<td>$\Phi_T$</td>
<td>Fixed cost parameter – Tradables</td>
<td>0.0115</td>
</tr>
<tr>
<td>$\Phi_N$</td>
<td>Fixed cost parameter - Non-Tradables</td>
<td>0.0458</td>
</tr>
<tr>
<td>$\frac{P^N Y}{P^G Y}$</td>
<td>Government purchases of goods and services-to-GDP ratio</td>
<td>0.1025</td>
</tr>
<tr>
<td>$\frac{P^N i}{P^G Y}$</td>
<td>Government investment-to-GDP ratio</td>
<td>0.057</td>
</tr>
<tr>
<td>$\frac{P^C i^r}{P^G Y}$</td>
<td>Government transfers-to-GDP ratio</td>
<td>0.2062</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>Tax rate on consumption</td>
<td>0.15</td>
</tr>
<tr>
<td>$\tau^l$</td>
<td>Tax rate on labor income</td>
<td>0.34</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>Tax rate on capital income</td>
<td>0.21</td>
</tr>
<tr>
<td>$H^g$</td>
<td>Hours worked in the public sector</td>
<td>0.048</td>
</tr>
<tr>
<td>$\xi^k$</td>
<td>Private capital adjustment cost parameter</td>
<td>0.9</td>
</tr>
<tr>
<td>$\xi^f$</td>
<td>Adjustment costs for net private foreign assets-to-GDP ratio</td>
<td>0.05</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Share of domestic public debt</td>
<td>0.48</td>
</tr>
<tr>
<td>$\psi^d$</td>
<td>Risk-premium coefficient on total public debt-to-output ratio</td>
<td>0.04</td>
</tr>
<tr>
<td>$\widetilde{d}$</td>
<td>Target level of total public debt-to-GDP ratio</td>
<td>1.26</td>
</tr>
<tr>
<td>$\widetilde{f}$</td>
<td>Target level of net private foreign assets-to-GDP ratio</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Share of total government transfers allocated to liquidity constrained households</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Appendix B: Weak property rights and effective TFP

In this appendix, we present a simple and static general equilibrium model to show the implications of weak property rights for resource (mis)allocation and how the latter shapes the “effective” TFP. Although the simple model presented here is only a stylized version of the DSGE model used in the paper, its qualitative results carry through to the full model.

**Households**

Say that there are \( i = 1, 2, \ldots, N \) agents/households. Each agent/household has one unit of effort time and can allocate it between productive work, \( 0 < u' \leq 1 \), and violence or extraction or conflict, \( 0 \leq 1 - u' < 1 \). Each agent \( i \) maximizes:

\[
U_i = \ln c_i
\]

subject to the budget constraint:

\[
c_i = \gamma (w_i + \pi') + (1 - \gamma) \frac{(1 - u')}{\sum_{j \neq i} (1 - u')} \sum_{j \neq i} (w_j + \pi')
\]

where \( 0 < \gamma \leq 1 \) is the degree of property rights in the aggregate economy and \( \sum_{j \neq i} (w_j + \pi') \) is the income of “the others”, or the contestable pie, from \( i \)’s point of view.

The first-order condition for \( 0 < u' \leq 1 \) is:

\[
\gamma w = \frac{(1 - \gamma)}{\sum_{j \neq i} (1 - u')} \sum_{j \neq i} (w_j + \pi')
\]

**Firms**

Firms maximize:

\[
\pi' = y'_i - w_i u'_i
\]

subject to:

\[
y'_i = A(u'_i)^\alpha
\]

The standard first-order condition is:

\[
w_i = \alpha \frac{y'_i}{u'_i}
\]
Equilibrium (symmetric) with resource misallocation

In a symmetric equilibrium, we have:

\[ 0 < u = \frac{\alpha \gamma}{1 - \gamma + \alpha \gamma} < 1 \]  \hspace{1cm} (7)

and thus

\[ y_i = A(u_i)^\alpha = A \left( \frac{\alpha \gamma}{1 - \gamma + \alpha \gamma} \right)^\alpha < A \]  \hspace{1cm} (8)

so that we get an effective TFP which is less than \( A \) (where \( A \) would be the case without resource misallocation). The effective TFP, \( A \left( \frac{\alpha \gamma}{1 - \gamma + \alpha \gamma} \right)^\alpha \), increases with \( \gamma \) (when \( \gamma = 1 \), all effort goes to productive work, \( u = 1 \), and thus the TFP equals \( A \)).

Notice that we get the above solution - even if we remain in the prototype economy without frictions in the form of weak property rights and thus without effort time allocated to extraction (that is, even if \( u = 1 \)) – if we use the effective TFP, defined as \( A \left( \frac{\alpha \gamma}{1 - \gamma + \alpha \gamma} \right)^\alpha \), instead of the nominal TFP, \( A \). In other words, a detailed economy, in which the technology is constant but property rights frictions vary over time affecting agents’ decisions, is equivalent to the prototype economy with effective time-varying productivity shocks. This is like in Chari et al. (2007), who show that an economy with various types of frictions, that distort the decisions of agents, is equivalent to a prototype economy with various types of time-varying shocks or what they label wedges. Thus, frictions, which lead to misallocation of resources, map into simple TFP distortions in the prototype economy.