

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS EXCOAH OIKONOMIKΩN EΠΙΣΤΗΜΩΝ SCHOOL OF ECONOMIC SCIENCES

TMHMA OIKONOMIKHZ ERIIZTHMHZ DEPARTMENT OF ECONOMICS

WORKING PAPER SERIES 04-2016

Unemployment Persistence, Inflation and Monetary Policy in A Dynamic Stochastic Model of the Phillips Curve

George Alogoskoufis

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 8203303-5 / Fax: 210 8238249

76, Patission Street, Athens 104 34 Greece. Tel.: (+30) 210 8203303-5 / Fax: (+30) 210 8238249 E-mail: econ@aueb.gr / <u>www.aueb.gr</u>

Unemployment Persistence, Inflation and Monetary Policy in A Dynamic Stochastic Model of the Phillips Curve

by

George Alogoskoufis*

March 2016

Abstract

This paper puts forward an alternative "new Keynesian" dynamic stochastic general equilibrium model of aggregate fluctuations. The model is characterized by one period nominal wage contracts and endogenous persistence of deviations of unemployment from its natural rate. Aggregate fluctuations are analyzed under both a Taylor nominal interest rate rule and under the assumption of optimal discretionary monetary policy. Under both types of monetary policy, the persistence of unemployment results in persistent inflation as the central bank responds to deviations of unemployment from its natural rate. Econometric evidence from the United States since the 1890s cannot reject the main predictions of the model.

Keywords: aggregate fluctuations, unemployment persistence, inflation, monetary policy, insiders outsiders, natural rate

JEL Classification: E3, E4, E5

* Department of Economics, Athens University of Economics and Business, 76, Patission street, GR-10434, Athens, Greece. The author would like to acknowledge financial support from a departmental research grant of the Athens University of Economics and Business and comments from the Economics Research Workshop at the AUEB.

Email: alogoskoufis@me.com Web Page: alogoskoufisg.com .

The typical "new Keynesian" model of aggregate fluctuations emphasizes imperfect competition in product and labor markets, and staggered price and nominal wage setting. Because of imperfect competition, the "natural" rates of output and employment are sub-optimally low, compared to competitive product and labor markets. In addition, because of staggered price and wage setting, deviations of real variables from their "natural" levels depend on both real and nominal shocks, and display persistence related to the persistence of such shocks. Monetary policy is usually modeled assuming a Taylor (1993) feedback rule, according to which the central bank sets nominal interest rates by responding to changes in the "natural" rate of interest, deviations of inflation from target, and deviations of output or unemployment from their "natural" rate.¹

This paper puts forward an alternative dynamic stochastic general equilibrium "new keynesian" model. This model is characterized by endogenous unemployment persistence, due to the dependence of the number of labor market "insiders" on past employment, but is otherwise a competitive model of the "natural" rate.

The main distinguishing characteristic of the present model is a dynamic "insider outsider" version of the "Phillips Curve", accounting for the persistence of unemployment and output following even non persistent nominal and real shocks. This model of the "Phillips curve" differs from the typical "new keynesian" Phillips curve, in that it satisfies the "natural" rate property, and in that the propagation mechanism of shocks is not the staggered setting of prices and wages, but the gradual adjustment of employment towards its "natural" rate.²

The present model combines and extends two strands of the literature on the determination of wages and unemployment. The Gray-Fischer model of periodic nominal wage setting, and the insideroutsider model of wage determination of Blanchard and Summers (1986), Lindbeck and Snower (1986) and Gottfries (1992).

According to the Gray-Fischer model of periodic nominal wage contracts, nominal wages are set at the beginning of each period and remain fixed for the period. Because shocks to inflation and

¹ See Taylor (1999) and Clarida, Gali and Gertler (2000) for how the Taylor rule describes the monetary policy of the Federal Reserve Board. See also Clarida, Gali and Gertler (1999) for the properties of the Taylor rule in "new" keynesian models with staggered price setting. More recent analyses and surveys can be found in, among others, Gali (2008, 2011a,b), Taylor and Williams (2011) and Woodford (2003, 2011).

² See McCallum (1994) and McCallum and Nelson (1999) for a discussion of how the "new keynesian" Phillips curve does not necessarily satisfy the "natural rate" property.

productivity are not known when nominal wages are set, unanticipated inflation reduces real wages and causes employment to increase along a downward sloping labor demand curve. In addition, shocks to productivity shift the demand for labor, and, thus, also cause fluctuations in employment, output, unemployment and real wages.³

According to the insider-outsider model of wage determination of Lindbeck and Snower (1986), Blanchard and Summers (1986) and Gottfries (1992), there is an asymmetry in the wage setting process between "insiders", who already have jobs, and "outsiders" who are seeking employment. "Outsiders" are disenfranchised from the labor market, and wages are set by "insiders", who seek to maximize the expected real wage consistent with the employment of "insiders".

The two strands were first combined by Blanchard and Summers (1986), who alluded to the dynamics of "insider" membership to explain the gradual adjustment of employment and the persistence of unemployment. Their argument was that "shocks that lead to reduced employment change the number of insiders and thereby change the subsequent equilibrium wage rate, giving rise to hysteresis". Thus, in their model unemployment displays persistence following nominal and real shocks. In the original Gray (1976) and Fischer (1977) models, there are only short lived deviations of unemployment from its "natural" rate. They only last for one period in the one period contract model of Gray (1976), and for two periods in the two period contract model of Fischer (1977).

This paper extends the Blanchard and Summers model, to a linear quadratic model of full intertemporal optimization on the part of labor market insiders, and embeds it in a dynamic stochastic model of aggregate fluctuations, where monetary policy responds to inflation and deviations of unemployment from its "natural" rate, via a Taylor (1993) interest rate rule.

We first derive the dynamic model of the "Phillips Curve", in which unanticipated shocks to inflation and productivity have persistent effects on unemployment. These persistent effects are compatible with full inter-temporal optimization on the part of labor market "insiders". The current number of "insiders" in each period depends on an exogenous number of "core" insiders, but also on those recently employed, who also influence the setting of contract wages at the firm level. Current "insiders" realize that shocks to employment will affect the future number of "insiders",

³ In the Gray (1976) model, the one we adopt, nominal wages are fixed in the beginning of each period, whereas in the Fischer (1977) model it is also allowed for nominal wages to be fixed in the beginning of alternate periods.

and thus future wage contacts, something which they take into account in their periodic wage setting decisions. Current wages thus depend not only on the current number of "insiders", which depends on past employment and the number of "core" insiders, but also on the expected future number of insiders, which depends on current and expected future employment. Thus, the persistence of employment and unemployment depends on expectations about the future evolution of employment and unemployment.

The dynamic "Phillips Curve" that we deduce is an alternative to the "new keynesian Phillips Curve", and this model provides an alternative source of unemployment persistence, compared to the typical "new keynesian" model, based on imperfect competition and staggered price and wage contracts.⁴

The distortions that matter for the fluctuations of unemployment and other real variables in our model arise in the labor market, through one period nominal wage contracts, and the wage setting behavior of "insiders". The product market is assumed competitive, although the model could in principle be extended to allow for the introduction of product market imperfections as well.⁵

On the demand side we assume that aggregate consumption and money demand are determined by a representative household, which maximizes its inter-temporal utility, and which can borrow and lend freely in a competitive financial market, at the market interest rate. Money enters the utility function of the representative household, and the demand for real money balances is proportional to consumption, and inversely related to the nominal interest rate. The Euler equation for consumption determines the evolution of private consumption and aggregate demand. The preferences of the representative household for consumption and real money balances are subject to persistent stochastic shocks, which shift both the Euler equation for consumption and the demand for money function.

Product market equilibrium is achieved through adjustments of the real interest rate, which is the relative price which shifts in order to equate aggregate demand with aggregate supply. Thus, the equilibrium real interest rate depends on both demand and supply shocks.

⁴ See Gali (2008) and Gali (2011b) for an extensive analysis of the typical "new keynesian" model.

⁵ This extension is not attempted here, in the interests of analytical simplicity. Gali (2016) has attempted this extension of the original Blanchard and Summers (1986) model.

The demand for real money balances turns out to be proportional to real output and inversely related to the nominal interest rate. If the central bank follows an interest rate rule, as we assume in this paper, the money supply adjusts endogenously to equilibrate the money market. If the central bank follows a money supply rule, nominal interest rates would be determined endogenously by the equilibrium condition in the money market.

We solve the model under the assumption that the central bank follows a feedback Taylor (1993) rule, adjusting nominal interest rates in response to changes in the "natural" real interest rate, deviations of inflation from a fixed inflation target, and deviations of unemployment from its "natural" rate. In addition we assume that the interest rate rule is subject to a white noise monetary policy shock.

This monetary policy shock captures potential errors in the implementation of monetary policy. There are three potential sources of such errors. First, an inaccurate estimate of the current "natural" rate of interest and/or the "natural" rate of unemployment, on behalf of the central bank. Second, an inaccurate estimate of current inflation and unemployment, and, third, non systematic policy errors in the implementation of the monetary policy rule. In any case, this is a non-systematic nominal shock.

We demonstrate that under a Taylor rule, the only shocks that cannot be completely neutralized by monetary policy are productivity shocks and shocks to monetary policy. Fluctuations of deviations of unemployment and output from their "natural" rates display persistence and are driven by these two types of disturbances.

Since productivity shocks are supply shocks, and their real effects can only partially be offset through unanticipated inflation, they cause a tradeoff between deviations of inflation from target, and unemployment from its "natural" rate. This is not the case for aggregate demand shocks, which, with the exception of monetary policy errors, can be fully neutralized by monetary policy, through appropriate changes in the nominal interest rate. It is for this reason that the only shocks that cause fluctuations in deviations of unemployment and output from their "natural" rates are productivity and monetary policy shocks. Because of the endogenous persistence of deviations of unemployment from its "natural" rate, the equilibrium inflation rate also displays persistence around the fixed inflation target of the central bank. The persistence of inflation arises from the fact that the central bank responds to deviations of unemployment from its "natural" rate. Since deviations of unemployment from its "natural" rate display persistence, due to the wage setting behavior of insiders, equilibrium inflation also displays persistence. This is the second main theoretical prediction of this paper. Unemployment persistence results in policy induced inflation persistence. The reason is that wage setters base their inflationary expectations on past deviations of unemployment from its "natural" rate, and therefore neutralize the attempts of the monetary authorities to smooth out these deviations. It is worth noting that the persistence in the fluctuations of the inflation rate do not affect the path of unemployment. It is only the unanticipated part of the inflation rate that can affect unemployment fluctuations in our model.

The persistence of inflation in the presence of endogenous unemployment persistence arises for the same reasons that there is an inflationary bias in the Kydland and Prescott (1977) and Barro and Gordon (1983) models of the "natural" rate, when the central bank systematically seeks to reduce unemployment below its "natural" rate. If the central bank responds to deviations of unemployment from its "natural rate", and wage setters have a different employment objective than the central bank, the only way for wage setters to ensure that the monetary authorities will follow the expected policy in the absence of shocks, is to adjust their expectations of inflation to the level which ensures that the central bank has no incentive to deviate from the expected inflation policy. It is exactly this mechanism which is responsible for the persistence of inflation when there is endogenous unemployment persistence as in the present model.

Contrary to the typical "new Keynesian" model, it is not the persistence of inflation and other shocks that causes unemployment persistence in our model, but the other way round. The persistence of unemployment causes inflation persistence through monetary policy, whether the monetary authorities follow a Taylor rule or the optimal discretionary monetary policy.

In the final part of the paper we present evidence from the United States, from the period of the gold standard till 2014. The evidence presented suggests that one cannot reject the prediction of the model that the persistence of deviations of unemployment and output from their "natural" rates induces persistence of inflation around the target of the monetary authorities.

The rest of the paper is as follows: In section 1 we present our basic dynamic model of the "Phillips curve", based on the distinction between "insiders" and "outsiders" in the labor market. In section 2 we derive the evolution of aggregate consumption and money demand, from the behavior of a representative household with access to a competitive financial market. In section 3 we analyze how the real interest rate adjusts to bring about equilibrium between aggregate demand and aggregate supply in the product market. In section 4 solve the model under the assumption that the central bank follows a Taylor (1993) rule, and derive our two main results, regarding the persistence of unemployment and inflation. In section 5 we show that the persistence of inflation is the same as the persistence from the United States economy, which suggests that one cannot reject the prediction of the model that the degree of persistence of inflation is the same as the degree of persistence of output and unemployment around their "natural" rates. The last section sums up our conclusions.

1. "Insiders and Outsiders" in a Dynamic Model of the "Phillips Curve"

Consider an economy consisting of competitive firms, indexed by *i*, where $i \in [0, 1]$.

The production function of firm *i* is given by,

$$Y(i)_t = A_t L(i)_t^{1-\alpha} \tag{1}$$

where Y(i) is output, A is exogenous productivity, and L(i) is employment. t is a time index, where t=0, 1,

Employment is determined by firms, who maximize profits, by equating the marginal product of labor to the real wage. Thus, employment is determined by the condition that,

$$(1-\alpha)A_t L(i)_t^{-\alpha} = \frac{W(i)_t}{P_t}$$
(2)

where W(i) is the nominal wage of firm *i*, and *P* is the price for the product of firm *i*. Since the product market is assumed to be competitive, all firms face the same price, and P(i)=P for all firms.

In log-linear form, (1) and (2) can be written as,

$$y(i)_{t} = a_{t} + (1 - \alpha)l(i)_{t}$$
(3)

$$l(i)_{t} = \bar{l} - \frac{1}{\alpha} (w(i)_{t} - p_{t} - a_{t})$$
(4)

where $\bar{l} = \frac{\ln(1-\alpha)}{\alpha}$

Lowercase letters denote the logarithms of the corresponding uppercase variables. (3) determines output as a positive function of employment, and (4) determines employment as a negative function of the deviation of real wages from productivity.

1.1 Wage Setting and Employment in a Linear Quadratic Insider Outsider Model

Nominal wages are set by "insiders" in each firm at the beginning of each period, before variables, such as current productivity and the current price level are known. Nominal wages remain constant for one period, and they are reset at the beginning of the following period. Thus, this model is characterized by nominal wage stickiness of the Gray (1976), Fischer (1977) variety. Employment is determined ex post by the firm, given the contract wage, the price level and productivity.

Following Blanchard and Summers (1986), we assume that the number of "insiders", who at the beginning of each period determine the contract wage, consists of an exogenous number of "core insiders", and the employees of the previous period. The key objective of "insiders" is to set the nominal wage which, given their rational expectations about the price level and productivity, will minimize deviations of expected employment from the number of "insiders". In our specification of the objective function of wage setters we allow for "core" insiders to have a different impact on the contract wage than other employees. The impact of recent employees on the wage setting process

depends on the structure of the labor market, and in particular the protection afforded by legislation to short turn employment.

Thus, the employment objective which determines the nominal wage in the contract depends on both the exogenous number of "core insiders" in each firm, but also those who were employed in period t-1. The expectations on the basis of which wages are set depend on information available until the end of period t-1, but not on information about prices and productivity in period t.

On the basis of the above, we assume that the objective of "insiders" is to make expected employment satisfy a path that minimizes the following quadratic inter-temporal loss function,

$$\min E_{t-1} \sum_{s=0}^{\infty} \beta^{s} \left[\frac{1}{2} \left(l(i)_{t+s} - \bar{n}(i) \right)^{2} + \frac{\omega}{2} \left(l(i)_{t+s} - l(i)_{t+s-1} \right)^{2} \right]$$
(5)

(5) is minimized subject to the labor demand equation (4), as employment in each period is determined ex post by the firm.

 \overline{n} is the logarithm of the number of core "insiders". $\beta = 1/(1+\rho) < 1$ is the discount factor, with ρ being the pure rate of time preference. ω is the weight of recent employees relative to "core insiders" in the wage setting process. As can be seen from (5), "outsiders", i.e the unemployed of the previous period, have no influence on the wage setting process.⁶

We shall assume that the total number of core "insiders" in the economy is always strictly smaller than the labor force. We shall thus assume that,

$$\int_{i=0}^{1} \bar{n}(i)di = \bar{n} < n , \ \forall t \tag{6}$$

where *n* is the log of the labor force.

⁶ An alternative interpretation of (5) would be in terms of "adjustment costs". "Insiders" seek to minimize deviations of expected employment from their number, but there is a cost to adjusting employment from period to period. Then, ω can be interpreted as the relative importance of adjustment costs relative to costs of deviations from the employment of core "insiders".

From the first order conditions for a minimum of (5), wages are set so that expected employment for each firm satisfies,

$$(1 + \omega(1 + \beta))E_{t-1}l(i)_{t} - \beta\omega E_{t-1}l(i)_{t+1} - \omega l(i)_{t-1} = \bar{n}(i)$$
(7)

Integrating over the number of firms *i*, expected aggregate employment must then satisfy,

$$(1 + \omega(1 + \beta))E_{t-1}l_t - \beta\omega E_{t-1}l_{t+1} - \omega l_{t-1} = n$$
(8)

(8) is the same as (7) without the i index.

(8) helps explain the differences of our wage setting model from Gray-Fischer contracts and Blanchard-Summers contracts.

With Gray-Fischer contracts, $\omega = 0$, as recent employees do not exert any separate influence in the wage setting process. Only "core insiders" would matter in Gray-Fischer type contracts. Setting $\omega = 0$ in (8), nominal wages in Gray-Fischer contracts would be set in order to ensure that,

$$E_{t-1}l_t = n \tag{8a}$$

On the other hand, with Blanchard-Summers contracts, there is no consideration of the effects of current contracts on expected future employment. This is equivalent to setting $\beta = 0$ in (8). This would imply from (8) that, with Blanchard-Summers contracts, nominal wages would be set in order to ensure that,

$$E_{t-1}l_{t} = \frac{1}{1+\omega}\bar{n} + \frac{\omega}{1+\omega}l_{t-1}$$
(8b)

(8b) is identical to equation (3.2) in Blanchard and Summers (1986). Nominal wages with Blanchard-Summers contracts would be set to make expected employment equal to a weighted

average of core "insiders" and those recently employed. The weight of those recently employed depends positively on ω , the relative weight of recent employees in the wage setting process.

In our more general forward looking dynamic model, expected employment is given by,

$$E_{t-1}l_{t} = \frac{1}{1+\omega(1+\beta)}\bar{n} + \frac{\omega}{1+\omega(1+\beta)}l_{t-1} + \frac{\beta\omega}{1+\omega(1+\beta)}E_{t-1}l_{t+1}$$
(8c)

Thus, in our model, "insiders" set nominal wages in order to achieve an employment target which depends on core "insiders", those previously employed, but also on expected future employment, as expected future employment will affect future wage setting behavior.

1.2 Wage Determination, Unemployment Persistence and the Phillips Curve

Subtracting (8) from the log of the labor force *n*, after some rearrangement, we get,

$$(1 + \omega(1 + \beta))E_{t-1}u_t - \beta\omega E_{t-1}u_{t+1} - \omega u_{t-1} = u$$
(9)

where, $u_t \simeq n_t - l_t$ is the unemployment rate, and $\bar{u} \simeq n - \bar{n} > 0$ is the "natural" unemployment rate. The "natural rate" of unemployment in this model is defined in terms of the difference between the labor force and the number of core "insiders". This is the equilibrium rate towards which the economy would converge in the absence of shocks.

To solve (9) for expected unemployment, define the operator F, as,

$$F^{s}u_{t} = E_{t-1}u_{t+s} \tag{10}$$

We can then rewrite (9) as,

$$\left(\left(1+\omega(1+\beta)\right)F^{0}-\beta\omega F-\omega F^{-1}\right)u_{t}=\bar{u}$$
(11)

(11) can be rearranged as,

$$-\beta\omega F^{-1}\left(F^2 - \frac{1+\omega(1+\beta)}{\beta\omega}F + \frac{1}{\beta}\right)u_t = \bar{u}$$
(12)

It is straightforward to show that if $0 < \beta < 1$ and $\omega > 0$ and finite, the characteristic equation of the quadratic in the forward shift operator (in brackets) has two distinct real roots, which lie on either side of unity. The two roots satisfy,

$$\lambda_1 + \lambda_2 = \frac{1 + \omega(1 + \beta)}{\beta \omega} \text{ and } \lambda_1 \lambda_2 = \frac{1}{\beta}$$
 (13)

Using (13) we can rewrite (12), as,

$$(F - \lambda_1)(F - \lambda_2)u_t = -\frac{1}{\beta\omega}\bar{u}$$
(14)

Assuming λ_1 is the smaller root, we can solve (14) as,

$$E_{t-1}u_{t} = \lambda_{1}u_{t-1} + \frac{\lambda_{1}}{\omega(1-\beta\lambda_{1})}\bar{u} = \lambda_{1}u_{t-1} + (1-\lambda_{1})\bar{u}$$
(15)

(15), which is the rational expectations solution of (9), determines the path of expected unemployment implied by the wage setting behavior of "insiders".

Actual unemployment, is determined from the employment decisions of firms, after information about prices, productivity and other shocks has been revealed.

Integrating the labour demand function over the number of firms *i*, aggregate employment is given by,

$$l_{t} = \bar{l} - \frac{1}{\alpha} (w_{t} - p_{t} - a_{t})$$
(16)

Subtracting the aggregate employment equation (16) from the log of the labor force n, actual unemployment is determined by,

$$u_{t} = n - \bar{l} + \frac{1}{\alpha} (w_{t} - p_{t} - a_{t})$$
(17)

Taking expectations on the basis of information available at the end of period t-1, the wage is set in order to make expected unemployment equal to the expression in (15), which defines the rate of unemployment consistent with the wage setting behavior of "insiders".

From (17), the wage is thus set in order to satisfy,

$$w_{t} = E_{t-1}p_{t} + E_{t-1}a_{t} + \alpha \left(E_{t-1}u_{t} - n + \bar{l}\right)$$
(18)

where $E_{t-1}u_t$ is determined by (15).

Substituting for the nominal wage in (17), using (18), then the unemployment rate evolves according to,

$$u_{t} = E_{t-1}u_{t} - \frac{1}{\alpha}(p_{t} - E_{t-1}p_{t} + a_{t} - E_{t-1}a_{t})$$
(19)

Substituting (15) in (19) thus gives us the solution for the unemployment rate.

$$u_{t} = \lambda_{1} u_{t-1} + (1 - \lambda_{1}) \bar{u} - \frac{1}{\alpha} (p_{t} - E_{t-1} p_{t} + a_{t} - E_{t-1} a_{t})$$
(20)

From (20), the unemployment rate is equal to the expected unemployment rate, as determined by the behavior of "insiders" in the labor market, and depends negatively on unanticipated shocks to

inflation and productivity. Unanticipated shocks to inflation reduce unemployment by a factor which depends on the elasticity of labor demand with respect to the real wage, as unanticipated inflation reduces real wages. Unanticipated shocks to productivity also reduce unemployment, as they reduce the difference between real wages and productivity and increase labor demand.

It is straightforward to show that an increase in ω , the relative weight of recent employees in the wage setting process, results in an increase in λ_1 , the coefficient that determines the persistence of unemployment. From the conditions (13), which define the two roots, it follows that,

$$\frac{\partial \lambda_1}{\partial \omega} = \left(\frac{\lambda_1}{\omega}\right)^2 > 0$$

Thus, the higher the weight of recent employees relative to core "insiders" in the wage setting process, the higher the persistence of unemployment. In addition, as ω tends to infinity, λ_1 tends to unity, and the model can account for "hysteresis" in unemployment.⁷

We can express (20) in terms of inflation, by adding and subtracting the lagged log of the price level in the last parenthesis. Thus, (20) takes the form of a *dynamic, expectations augmented "Phillips Curve"*.

$$u_{t} = \lambda_{1} u_{t-1} + (1 - \lambda_{1}) \bar{u} - \frac{1}{\alpha} (\pi_{t} - E_{t-1} \pi_{t} + a_{t} - E_{t-1} a_{t})$$
(21)

where π is the inflation rate.

(21) can be expressed in terms of deviations of unemployment from its "natural" rate, as,

$$u_{t} - \bar{u} = \lambda_{1}(u_{t-1} - \bar{u}) - \frac{1}{\alpha}(\pi_{t} - E_{t-1}\pi_{t} + a_{t} - E_{t-1}a_{t})$$
(22)

⁷ For example, assuming $\beta = 0.99$, with $\omega = 1$, $\lambda_1 = 0.38$. With $\omega = 2$, $\lambda_1 = 0.50$, with $\omega = 10$, $\lambda_1 = 0.73$ and with $\omega = 100$, $\lambda_1 = 0.91$. Thus, the higher the weight of recent employees in the wage setting process, the higher the persistence of unemployment. As ω tends to infinity, λ_1 tends to unity and there is no "natural" rate of unemployment.

From (22), deviations of unemployment from its "natural" level depend negatively on unanticipated shocks to inflation and productivity, as these cause a discrepancy between real wages and productivity, due to the fact that nominal wages are predetermined. Unanticipated shocks to inflation reduce real wages and induce firms to increase labor demand and employment beyond their "natural" level. Thus, unemployment falls relative to its "natural" rate. Unanticipated shocks to productivity, given inflation, cause an increase in productivity relative to real wages, and also cause firms to increase labor demand, employment and output, beyond their "natural" levels, which reduces unemployment beyond its "natural" rate.

It can easily be confirmed from (22) that following a shock to inflation or productivity, unemployment will converge gradually back to its "natural" rate, with the speed of adjustment being $(1-\lambda_I)$ per period.

1.3 The Relation between Output and Unemployment

The persistence of employment and unemployment, will also be translated into persistent output fluctuations.

Aggregating the firm production functions (3), the aggregate production function can be written as,

$$y_t = a_t + (1 - \alpha)l_t \tag{23}$$

Adding and subtracting $(1-\alpha)(n-\bar{n})$, the production function can be written as,

$$y_t = \bar{y}_t - (1 - \alpha)(u_t - \bar{u})$$
 (24)

where,

$$\bar{y}_t = (1 - \alpha)\bar{n} + a_t \tag{25}$$

is the log of the "natural" level of output.

(24) is an Okun (1962) type of relation, which suggests that fluctuations of output around its "natural" level will be negatively related to fluctuations of the unemployment rate around its own "natural" rate.

From (23) and (22), deviations of output from its "natural" level will be determined by,

$$y_{t} - \bar{y}_{t} = \lambda_{1}(y_{t-1} - \bar{y}_{t-1}) + \frac{1 - \alpha}{\alpha}(\pi_{t} - E_{t-1}\pi_{t} + a_{t} - E_{t-1}a_{t})$$
(26)

(26) shows that deviations of output from its "natural" level, also display persistence, because of the persistence of employment and unemployment.

(26) is a *dynamic output supply function*. Deviations of output from its "natural" level depend positively on unanticipated shocks to inflation and productivity, as these cause a discrepancy between real wages and productivity, due to the fact that nominal wages are predetermined. Unanticipated shocks to inflation reduce real wages and induce firms to increase labor demand, employment and output. Unanticipated shocks to productivity, given inflation, cause an increase in productivity relative to real wages, and also cause firms to increase labor demand, employment and output, beyond their "natural" levels. On the other hand, anticipated shocks to productivity increase both output and its "natural" level by the same proportion.

2. The Determination of Aggregate Consumption and Money Demand

We next turn to the determination of aggregate demand. We assume that the economy consists of a large number of identical households j, where $j \in [0, 1]$. Each household member supplies one unit of labor, and unemployment impacts all households in the same manner. Thus, if H is the number of households and N is the aggregate labor force, each household has N/H members. Of those, some are "insiders" in the labor market, and the rest are "outsiders". The proportion of insiders is the same for all households. In addition, the proportion of the unemployed is also assumed to be the same for all households.

The representative household chooses (aggregate) consumption and real money balances in order to maximize,

$$E_{t} \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho} \right)^{s} \left(\frac{1}{1-\theta} \left(V_{t+s}^{C} C_{t+s}^{1-\theta} + V_{t+s}^{M} \left(\frac{M}{P} \right)_{t+s}^{1-\theta} \right) \right)$$
(27)

subject to the sequence of expected budget constraints,

$$E_{t}\left(A_{t+s+1} - (1+i_{t+s})\left(A_{t+s} - \frac{i_{t+s}}{1+i_{t+s}}M_{t+s} + P_{t+s}\left(Y_{t+s} - C_{t+s} - T_{t+s}\right)\right)\right) = 0$$
(28)

where $A_t = B_t + M_t$.

 ρ denotes the pure rate of time preference, θ is the inverse of the elasticity of inter-temporal substitution, *i* the nominal interest rate, *A* the current value of household financial assets (one period nominal bonds *B* and money *M*), *Y* real non interest income and *T* real taxes net of transfers. *V*^C and *V*^M denote exogenous stochastic shocks in the utility from consumption and real money balances respectively.

From the first order conditions for a maximum,

$$V_t^C C_t^{-\theta} = \lambda_t (1+i_t) P_t \tag{29}$$

$$V_t^M \left(\frac{M}{P}\right)_t^{-\theta} = \lambda_t i_t P_t$$
(30)

$$E_t \lambda_{t+1} = E_t \left(\frac{1+\rho}{1+i_{t+1}} \right) \lambda_t$$
(31)

where λ_t is the Lagrange multiplier in period *t*.

(29)-(31) have the standard interpretations. (29) suggests that at the optimum the household equates the marginal utility of consumption to the value of savings. (30) suggests that the household equates the marginal utility of real money balances to the opportunity cost of money. Finally, (31) suggests that at the optimum, the real interest rate, adjusted for the expected change in the marginal utility of consumption, is equal to the pure rate of time preference.

From (29), (30) and (31), eliminating λ ,

$$\left(\frac{M}{P}\right)_{t} = C_{t} \left(\frac{V_{t}^{C}}{V_{t}^{M}} \frac{i_{t}}{1+i_{t}}\right)^{-\frac{1}{\theta}}$$
(32)

$$E_{t}\left(\frac{V_{t+1}^{C}\left(C_{t+1}\right)^{-\theta}}{P_{t+1}}\right) = \left(\frac{1+\rho}{1+i_{t}}\right)\left(\frac{V_{t}^{C}\left(C_{t}\right)^{-\theta}}{P_{t}}\right)$$
(33)

(31) is the money demand function, which is proportional to consumption and a negative function of the nominal interest rate, and (32) is the familiar Euler equation for consumption.

Log-linearizing (32) and (33),

$$m_t - p_t = c_t - \frac{1}{\theta} \ln\left(\frac{i_t}{1 + i_t}\right) + \frac{1}{\theta} \left(v_t^M - v_t^C\right)$$
(34)

$$c_{t} = E_{t}c_{t+1} - \frac{1}{\theta} (i_{t} - E_{t}\pi_{t+1} - \rho) + \frac{1}{\theta} (v_{t}^{C} - E_{t}v_{t+1}^{C})$$
(35)

where lowercase letters denote natural logarithms, and, $\pi_t = p_t - p_{t-1}$ is the rate of inflation.

We then turn to the determination of equilibrium in the product and money markets.

3. Equilibrium in the Product and Money Markets and the Real Interest Rate

Since there is no capital and investment in this model, product market equilibrium implies that output is equal to consumption.

$$Y_t = C_t \tag{36}$$

Substituting (36) in (34) and (35), we get the money and product market equilibrium conditions,

$$m_t - p_t = y_t - \frac{1}{\theta} \ln\left(\frac{i_t}{1 + i_t}\right) + \frac{1}{\theta} \left(v_t^M - v_t^C\right)$$
(37)

$$y_{t} = E_{t} y_{t+1} - \frac{1}{\theta} (i_{t} - E_{t} \pi_{t+1} - \rho) + \frac{1}{\theta} (v_{t}^{C} - E_{t} v_{t+1}^{C})$$
(38)

(37) is the money market equilibrium condition, the model equivalent of the *LM Curve*, and (38) is the product market equilibrium condition, the equivalent of a dynamic *IS Curve*.

Since output demand depends on deviations of the real interest from the pure rate of time preference, the real interest rate is the relative price that adjusts to equilibrate output (consumption) demand with output supply. No other relative price can play this role, as the real wage is determined in order to make expected labor demand equal to the number of "insiders" in the labor market.

3.1 The "Natural" Real Interest Rate and the Current Equilibrium Real Interest Rate

The real interest rate is defined by the Fisher (1896) equation,⁸

$$r_t = i_t - E_t \pi_{t+1}$$
(39)

The "natural" real interest rate is determined by the product market equilibrium condition, when output is at its "natural" rate. From (25) and (38), the "natural" real interest rate is given by,

⁸ To quote from Fisher (1896), "When prices are rising or falling, money is depreciating or appreciating *relative* to commodities. Our theory would therefore require high or low interest according as prices are rising or falling, provided we assume that the rate of interest in the commodity standard should not vary." (p. 58). The rate of interest in the commodity standard is the real interest rate, and rising or falling prices are expected inflation. The Fisher equation was further elaborated in Fisher (1930), where it was made even clearer that Fisher referred to *expected* inflation.

$$\bar{r}_{t} = \rho - \theta \left(a_{t} - E_{t} a_{t+1} \right) + \left(v_{t}^{C} - E_{t} v_{t+1}^{C} \right)$$
(40)

The "natural" real interest rate is equal to the pure rate of time preference, but also depends positively on deviations of current shocks to consumption from anticipated future shocks, and negatively on deviations of current productivity shocks from anticipated future shocks. Thus, real shocks, such as productivity shocks, that cause a temporary increase in the "natural" level of output reduce the "natural" real rate of interest, in order to bring about a corresponding increase in consumption and maintain product market equilibrium. On the other hand, real consumption preference shocks that cause a temporary increase in consumption, require an increase in the "natural" level of output, and maintain product market equilibrium.⁹

Because of the nominal rigidity of wages for one period, the current equilibrium real interest deviates from its "natural" rate. The current real interest rate is determined by the equation of the output demand function (38) with the output supply function (26). It is thus determined by,

$$r_t = r_t - \theta (1 - \lambda_1) (y_t - y_t) \tag{41}$$

Deviations of the current real interest rate from its "natural" rate depend negatively on deviations of output from its "natural" level. Since deviations of output from its "natural" level tend to persist, deviations of the real interest rate from its "natural" rate will tend to persist as well.

Unanticipated shocks to inflation or productivity, which cause a temporary rise in current output relative to its "natural" level, will reduce the current real interest rate relative to its "natural" rate. This is the well known "Wicksellian" mechanism, emphasized for the first time by Wicksell (1898).

3.2 Equilibrium Fluctuations with Exogenous Preference, Productivity and Labor Market Shocks

⁹ The concept of a "natural" rate of interest was introduced by Wicksell (1898). To quote, "There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the natural rate of interest ..." (p. 102).

We shall assume in what follows that the logarithms of the exogenous shocks to preferences and productivity follow stationary AR(1) processes.

$$v_t^C = \eta_C v_{t-1}^C + \varepsilon_t^C \tag{42}$$

$$v_t^M = \eta_M v_{t-1}^M + \varepsilon_t^M \tag{43}$$

$$a_t = \eta_A a_{t-1} + \varepsilon_t^A \tag{44}$$

where the autoregressive parameters satisfy, $0 < \eta_C, \eta_M, \eta_A < 1$, and $\varepsilon^C, \varepsilon^M, \varepsilon^A$, are white noise processes.

With these assumptions, current employment, unemployment, output, real wages and the real interest rate, as functions of the exogenous shocks and shocks to inflation, evolve according to,

$$l_{t} = (1 - \lambda_{1})\bar{n} + \lambda_{1}l_{t-1} + \frac{1}{\alpha} \left(\pi_{t} - E_{t-1}\pi_{t} + \varepsilon_{t}^{A}\right)$$
(45)

where \bar{n} is the "natural" level of employment.

$$u_{t} = (1 - \lambda_{1})\bar{u} + \lambda_{1}u_{t-1} - \frac{1}{\alpha} \left(\pi_{t} - E_{t-1}\pi_{t} + \varepsilon_{t}^{A} \right)$$
(46)

where \bar{u} is the "natural" rate of unemployment.

$$y_{t} = \bar{y}_{t} + \lambda_{1}(y_{t-1} - \bar{y}_{t-1}) + \frac{1 - \alpha}{\alpha} \left(\pi_{t} - E_{t-1}\pi_{t} + \varepsilon_{t}^{A} \right)$$
(47)

where, $\bar{y}_t = (1 - \alpha)\bar{n} + a_t$.

$$w_{t} - p_{t} = (\bar{w} - p)_{t} + \alpha \lambda_{1} (u_{t-1} - \bar{u}) - (\pi_{t} - E_{t-1} \pi_{t} + \varepsilon_{t}^{A})$$
(48)
where, $(\bar{w} - p)_{t} = a_{t} - \alpha (\bar{n} - \bar{l})$.

$$r_t = \bar{r}_t + \theta(1 - \alpha)(1 - \lambda_1)(u_t - \bar{u})$$
(49)

where $\bar{r}_t = \rho - \theta (1 - \eta_A) a_t + (1 - \eta_C) v_t^C$.

The "natural" rates (or levels) of real variables evolve as functions of the exogenous real shocks. However, unanticipated inflation, and innovations in productivity, by reducing real wages relative to their "natural" level, cause persistent increases in employment and output above their "natural" level, and persistent reductions in unemployment, real wages and the real interest rate, below their "natural" rates.

4. Fluctuations in Unemployment and Inflation under a Taylor Rule

We now turn to the determination of inflation and unemployment, under the assumption that the central bank follows a Taylor (1993) rule for nominal interest rates. We assume that the nominal interest rate is determined by the central bank, after the realization of current shocks to productivity. Thus, we assume that the central bank has an informational advantage over wage setters.

The principle of the Taylor rule requires the monetary authorities to set the nominal interest rate as a function of the "natural" real rate of interest, plus their inflation target, and adjust it on the basis of deviations of current inflation from target, and deviations of unemployment from its "natural" rate. We shall thus assume a Taylor rule of the form,

$$i_{t} = \bar{r}_{t} + \pi^{*} + \phi_{1} (\pi_{t} - \pi^{*}) - \phi_{2} (u_{t} - \bar{u}) + \varepsilon_{t}^{i}$$
(50)

where $\phi_1, \phi_2 > 0$ and ε_t^i is a white noise interest rate policy shock.¹⁰

¹⁰ Taylor (1993) proposed a rule is which the "natural" rate of interest was constant at 2%. In our analysis, since the "natural" real rate of interest depends on stochastic demand and supply shocks, we treat it as an endogenous variable. In addition, as our focus is on unemployment, we have expressed the Taylor rule in terms of deviations of the unemployment rate from its "natural" rate, instead of deviations of output from its "natural" level. One could use the Okun type equation (24) and conduct the analysis in terms of deviations of output from its "natural" level.

The monetary policy shock reflects either errors in estimating the current "natural" rate of interest, or errors in estimating the "natural" rate of unemployment, on behalf of the central bank, or noise in observing current inflation and unemployment, or simply non systematic errors in the implementation of the monetary policy rule.

4.1 Equilibrium Inflation under a Taylor Rule

Substituting (50) in the Fisher equation (39), after using the real interest equation (49) and the dynamic Phillips curve (46), we get the following process for inflation,

$$\boldsymbol{\pi}_{t} = \boldsymbol{\gamma}_{1} \boldsymbol{E}_{t} \boldsymbol{\pi}_{t+1} + \boldsymbol{\gamma}_{2} \boldsymbol{E}_{t-1} \boldsymbol{\pi}_{t} + \boldsymbol{\gamma}_{3} \boldsymbol{\pi}_{t-1} + \boldsymbol{\gamma}_{4} \boldsymbol{\pi}^{*} + \boldsymbol{\gamma}_{5} \boldsymbol{\varepsilon}_{t}^{A} + \boldsymbol{\gamma}_{6} \boldsymbol{\varepsilon}_{t}^{i} + \boldsymbol{\gamma}_{7} \boldsymbol{\varepsilon}_{t-1}^{i}$$
(51)

where,

$$\gamma_{1} = \frac{\alpha}{\phi_{1}\alpha + \phi_{2} + \theta(1 - \lambda_{1})(1 - \alpha) + \lambda_{1}\alpha}$$

$$\gamma_{2} = \frac{\phi_{2} + \theta(1 - \lambda_{1})(1 - \alpha)}{\phi_{1}\alpha + \phi_{2} + \theta(1 - \lambda_{1})(1 - \alpha) + \lambda_{1}\alpha}$$

$$\gamma_{3} = \frac{\lambda_{1}\phi_{1}\alpha}{\phi_{1}\alpha + \phi_{2} + \theta(1 - \lambda_{1})(1 - \alpha) + \lambda_{1}\alpha}$$

$$\gamma_{4} = \frac{(\phi_{1} - 1)(1 - \lambda_{1})\alpha}{\phi_{1}\alpha + \phi_{2} + \theta(1 - \lambda_{1})(1 - \alpha) + \lambda_{1}\alpha}$$

$$\gamma_{5} = -\gamma_{2}$$

$$\gamma_{6} = -\gamma_{1}$$

$$\gamma_{7} = \lambda_{1}\gamma_{1}$$

Note that, because of the persistence of unemployment, the inflationary process also displays persistence. It also depends on the current expectations about future inflation, through the definition of the real interest rate and on both parameters of the Taylor rule, as unanticipated inflation causes the unemployment rate to deviate from its natural rate. Finally, because of the persistence in unemployment both current and past nominal interest rate shocks affect the inflationary process.

The effects of productivity and nominal interest rate shocks on inflation also depend on the parameters of the Taylor rule.¹¹

In order to solve for inflation, we first take expectations of (51) conditional on information available up to the end of period t-1. This yields,

$$E_{t-1}\pi_{t} = \frac{1}{\phi_{1} + \lambda_{1}}E_{t-1}\pi_{t+1} + \frac{\phi_{1}\lambda_{1}}{\phi_{1} + \lambda_{1}}\pi_{t-1} + \frac{(\phi_{1} - 1)(1 - \lambda_{1})}{\phi_{1} + \lambda_{1}}\pi^{*} + \frac{\lambda_{1}}{\phi_{1} + \lambda_{1}}\varepsilon_{t-1}^{i}$$
(52)

The process (52) has two roots, λ_1 and ϕ_1 and will be stable if the two roots lie on either side of unity. Since $\lambda_1 < 1$, the expected inflation process will be stable if,

$$\phi_1 > 1 \tag{53}$$

Condition (53), is the well known *Taylor principle*. It requires that nominal interest rates over-react to deviations of current inflation from target inflation, in order to affect expected real rates. This is a sufficient condition for a stable and determinate process for expected (and actual) inflation.¹²

If (53) is satisfied, then the solution for the expected inflation process (52) is given by,

$$E_{t-1}\pi_{t} = (1-\lambda_{1})\pi^{*} + \lambda_{1}\pi_{t-1} + \frac{\lambda_{1}}{\phi_{1}}\varepsilon_{t-1}^{i}$$
(54)

From (54), it follows that,

$$E_t \pi_{t+1} = (1 - \lambda_1) \pi^* + \lambda_1 \pi_t + \frac{\lambda_1}{\phi_1} \varepsilon_t^i$$
(55)

¹¹ (51) being the inflationary process from a dynamic stochastic general equilibrium, in which the policy rule of the monetary authorities is taken into account, it does not suffer from the Lucas (1976) critique. Changing the parameters of the policy rule, would also change the parameters of the inflationary process.

¹² Woodford (2003), among others, contains a detailed discussion of the Taylor principle, and its significance for the resolution of the price level and inflation indeterminacy problem highlighted by Sargent and Wallace (1975) for non contingent interest rate rules.

Substituting (54) and (55) in the inflation process (51), the rational expectations solution for inflation is given by,

$$\pi_{t} = (1 - \lambda_{1})\pi^{*} + \lambda_{1}\pi_{t-1} - \psi_{1}\varepsilon_{t}^{A} - \psi_{2}\varepsilon_{t}^{i} + \psi_{3}\varepsilon_{t-1}^{i}$$
(56)

where,

$$\psi_1 = \frac{\phi_2 + \theta(1 - \lambda_1)(1 - \alpha)}{\phi_1 \alpha + \phi_2 + \theta(1 - \lambda_1)(1 - \alpha)} < 1$$

$$\psi_2 = \frac{\left(\phi_1 - \lambda_1\right)}{\phi_1} \frac{\alpha}{\phi_1 \alpha + \phi_2 + \theta(1 - \lambda_1)(1 - \alpha)} > 0$$

$$\psi_3 = \frac{\lambda_1}{\phi_1} > 0$$

From (56), the fluctuations of inflation around the target of the monetary authorities π^* are persistent, and depend on the current innovation in productivity and current and past interest rate shocks. Furthermore, the persistence of inflation is equal to the persistence of deviations of unemployment and other real variables, such as output, from their "natural" level.

The fluctuations of unemployment and output around their "natural" level are driven by unanticipated inflation and innovations in productivity. From (56), unanticipated inflation is determined by,

$$\pi_t - E_{t-1}\pi_t = -\psi_1 \varepsilon_t^A - \psi_2 \varepsilon_t^i \tag{57}$$

Substituting (57) in the "dynamic" Phillips curve (46) and the output supply function (47), deviations of unemployment and output from their "natural" rates are determined by,

$$(u_t - \bar{u}) = \lambda_1 (u_{t-1} - \bar{u}) - \frac{1}{\alpha} \left((1 - \psi_1) \varepsilon_t^A - \psi_2 \varepsilon_t^i \right)$$
(58)

$$(y_{t} - \bar{y}_{t}) = \lambda_{1}(y_{t-1} - \bar{y}_{t-1}) + \frac{1 - \alpha}{\alpha} ((1 - \psi_{1})\varepsilon_{t}^{A} - \psi_{2}\varepsilon_{t}^{i})$$
(59)

Thus, under the Taylor rule (50), only innovations in productivity and nominal interest rate shocks induce fluctuations of deviations of unemployment and output from their "natural" rates. Other demand shocks, such as shocks to consumption preferences, are fully neutralized by monetary policy, since the nominal interest rate is assumed to fully accommodate changes in the "natural" rate of interest.

5. Unemployment and Inflation under Optimal Monetary Policy

Up to now we have assumed that monetary policy is determined by a version of the Taylor (1993) rule. We shall now consider the implications of optimal monetary policy, under the assumption that the central bank uses its monetary policy instruments in order to minimize an inter temporal quadratic loss function, which depends on deviations of inflation from target, and deviations of unemployment from its "natural" rate.

Thus, the central bank is assumed to choose inflation in order to minimize,

$$\Lambda_t^D = E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{1}{2} (\pi_{t+s} - \pi^*)^2 + \frac{\zeta}{2} (u_{t+s} - \bar{u})^2 \right)$$
(60)

subject to the dynamic expectational Phillips curve (21). Superscript *D* denotes "discretion", β is the discount factor $\beta = 1/(1+\rho)$, where ρ is the pure rate of time preference, and ζ is the relative weight attached by the central bank to deviations of unemployment from its "natural" rate, relative to deviations of inflation from target.¹³

We term the policy that results from the minimization of (29) subject to (28), as *discretionary*, because the central bank target for unemployment differs from the current unemployment target of

¹³ We assume here that the central bank does not seek to reduce unemployment below its "natural" rate. Thus, we abstract from the systematic inflation bias that would result in case the central bank also sought to reduce unemployment below its "natural rate", as in Kydland and Prescott (1977) and Barro and Gordon (1983).

"insiders" in the labor market. Thus, under this policy, there is a conflict between the objectives of the monetary authorities and the objectives of wage setting "insiders". The central bank seeks to minimize deviations of unemployment from its "natural" rate, whereas wage setters seek to minimize deviations of unemployment from a weighted average of the "natural" rate and past unemployment.

From the first order conditions for a minimum of (60) subject to (21), we get,

$$\pi_{t} = \pi^{*} + \frac{\zeta}{\alpha} \left(u_{t} - \bar{u} \right) + \frac{\zeta \beta \lambda_{1}}{\alpha} E_{t} \left(u_{t+1} - \bar{u} \right) + \varepsilon_{t}^{d}$$
(61)

where ε^m is a white noise shock to monetary policy, satisfying,

$$\varepsilon_t^m \sim N(0, \sigma_m^2)$$

We allow for a monetary policy shock, for the same reasons that we introduced a monetary policy in the Taylor interest rate rule.

Using (21) to substitute for current and expected future deviations of the unemployment rate from its "natural" rate, after some rearrangement, we get,

$$\pi_{t} = \lambda_{1}\pi_{t-1} + (1-\lambda_{1})\pi^{*} - \frac{\zeta(1+\beta\lambda_{1}^{2})}{\alpha^{2}} \left(\pi_{t} - E_{t-1}\pi_{t} + \varepsilon_{t}^{A}\right) + \varepsilon_{t}^{m}$$

$$\tag{62}$$

The rational expectations solution of (62) is given by,

$$\pi_{t} = \lambda_{1}\pi_{t-1} + (1-\lambda_{1})\pi^{*} - \frac{\zeta(1+\beta\lambda_{1}^{2})}{\alpha^{2} + \zeta(1+\beta\lambda_{1}^{2})}\varepsilon_{t}^{A} + \frac{\alpha^{2}}{\alpha^{2} + \zeta(1+\beta\lambda_{1}^{2})}\left(\varepsilon_{t}^{M} - \lambda_{1}\varepsilon_{t-1}^{M}\right)$$
(63)

From (63), deviations of the optimal discretionary inflation rate from the inflation target π^* display the same degree of persistence, as the persistence of deviations of unemployment from its "natural" rate. The reason is that the central bank seeks to use inflation in order to minimize deviations of

unemployment from its "natural" rate. Since these deviations display persistence, deviations of inflation from target also display persistence under the optimal discretionary policy.¹⁴

The persistence of inflation under the optimal discretionary monetary policy does not affect the persistence of unemployment. The reason is that wage setters can anticipate the persistent part of the inflation process, incorporate it in their expectations, and neutralize the effects of persistent inflation on unemployment. The only part of monetary policy that matters for unemployment is the unanticipated part, which is a function of the current productivity shock and the current shock to monetary policy.

Note from (63) that anticipated inflation is given by,

$$E_{t-1}\pi_{t} = \lambda_{1}\pi_{t-1} + (1-\lambda_{1})\pi^{*} - \frac{\alpha^{2}}{\alpha^{2} + \zeta(1+\beta\lambda_{1}^{2})}\lambda_{1}\varepsilon_{t-1}^{m}$$
(64)

Thus, from (63) and (64), unanticipated inflation is given by,

$$\pi_t - E_{t-1}\pi_t = -\frac{\zeta(1+\beta\lambda_1^2)}{\alpha^2 + \zeta(1+\beta\lambda_1^2)}\varepsilon_t^A + \frac{\alpha^2}{\alpha^2 + \zeta(1+\beta\lambda_1^2)}\varepsilon_t^m$$
(65)

Substituting (65) in the dynamic expectational Phillips curve (21), we get,

$$u_{t} = \lambda_{1} u_{t-1} + (1 - \lambda_{1}) \bar{u} - \frac{\alpha}{\alpha^{2} + \zeta(1 + \beta \lambda_{1}^{2})} \left(\varepsilon_{t}^{A} + \varepsilon_{t}^{m}\right)$$
(66)

Optimal discretionary monetary policy also results in persistent inflation in the presence of unemployment persistence. Moreover, the degree of persistence of inflation is the same as the degree of persistence of unemployment. Yet, it is only the unanticipated part of inflation that helps mitigate the impact of productivity shocks on unemployment. The anticipated persistent part of

¹⁴ Note from (63) that if deviations of unemployment from its "natural" rate did not persist, i.e in the case $\lambda_I = 0$, the optimal discretionary monetary policy would not result in persistent deviations of inflation from target. There would be deviations of inflation from π^* only in response to unanticipated shocks to productivity.

inflation cannot affect unemployment, as it is neutralized by the adjustment of the expectations of the wage setting "insiders".

The theoretical predictions of this model suggest that even if the central bank follows the optimal discretionary policy, deviations of inflation from target should display the same degree of persistence as deviations of unemployment from its "natural" rate.¹⁵

6. Evidence from the United States

The main prediction of the model is that inflation should display the same degree of persistence as deviations of unemployment, and output, from their "natural" rates. This prediction is empirically testable. If deviations of inflation from target display the same degree of persistence as deviations of unemployment and output from their "natural" rates, then we shall take it as evidence that the Federal Reserve has been following a discretionary monetary policy. If deviations do not, then we shall take it as evidence that the Federal Reserve has been following rates display persistence but inflation deviations do not, then we shall take it as evidence that the Federal Reserve has been following a policy of full commitment to its inflation target.

The problem in implementing this test is that one has to make assumptions about the evolution of latent variables such as the "natural" rate of unemployment and the targets of the Fed with regard to inflation.

In the tests that we present below, the "natural" rate of unemployment and output is approximated by a Hodrik Prescott (1997) filter. With regard to inflation targets, we allow these to differ between the Gold Standard (1890-1913), World War I (1914-1918), the interwar period (1919-1938), World War II (1939-1945), the Bretton Woods period, 1946-1971, the first period of flexible exchange rates 1972-1982, and the post-Volcker period, 1983-2014, by constructing appropriate zero-one dummy variables. It turns out, that there is no difference in steady state inflation between the Bretton Woods period and the post-Volcker period, but that there is a significant difference between the first ten years of flexible exchange rates 1972-1982, which is characterized by high average inflation, and the other two sub-periods.

¹⁵ It is worth noting that by comparing coefficients between (63) and (56) and between (66) and (58) one can determine the parameters of the Taylor rule that correspond to the optimal discretionary monetary policy.

Our estimates for the US economy, using annual data for the 1890-2014 period are presented in Table 1. Table 1 presents estimates for deviations of unemployment and output from their "natural" rate, modeling the "natural" rate through a Hodrik Prescott (1997) filter. Table 1 also presents the results for deviations of inflation from fixed targets, allowing for different targets between different time periods. These different targets relate to the different monetary regimes we listed above. In Table 2, we present comparable estimates only for the post-Korean war period of 1954-2014.¹⁶

A number of observations are worth making concerning the estimates.

First, deviations of unemployment from its "natural" rate, output from its "natural" level and inflation from target appear to be 2nd order and not 1st order autoregressive processes. In order to summarize the degree of persistence of these variables, we report the sum of the estimated parameters on the two lags of the dependent variables, with their appropriate standard errors.

Second, neither deviations of unemployment and output from their Hodrik Prescott "natural" rate, nor deviations of inflation from target appear to be characterized by a unit root. The relevant Augmented Dickey Fuller (ADF) statistics do not indicate the presence of a unit root at conventional levels of significance for any of the variables, or the sub-periods concerned.

Third, the *degree of persistence*, i.e the sum of the two autoregressive parameters, is positive and statistically significant for all variables and sub-periods. The degree of persistence of deviations of unemployment from its "natural" rate ranges from 0.465 in the full sample, to 0.288 for the post Korean war period. The degree of persistence of deviations of output from its "natural" level ranges from 0.544 in the full sample, to 0.402 in the post Korean war period. The degree of persistence is statistically significant even at 1%. The post Korean war period. In all cases, the degree of persistence is statistically significant even at 1%. Thus, on the basis of these estimates, there does not seem to be evidence of significant differences in the degree of persistence of inflation sof unemployment and output from their "natural" rates and deviations of inflation from the targets of the Federal Reserve. This is confirmed by joint estimation of the unemployment, output and inflation equations by the method of seemingly unrelated regressions.

¹⁶ We describe the data used in a Data Appendix at the end of the paper. The data used is available upon request. 30

The joint hypothesis of the same degree of persistence of deviations of unemployment, output and inflation cannot be rejected for any of the two sub-periods. The relevant Wald test for the appropriate linear restrictions in the full sample is equal to 1.552, while for the 1954-2014 sample it is equal to 1.205. This test is distributed as $\chi^2(2)$, and the relevant critical values at 5% and 1% are equal to 5.99 and 9.21 respectively. Thus, the hypothesis of the same degree of persistence for all variables, including inflation, cannot be rejected.

Fourth, from the estimates of the coefficients of the dummy variables, there appears to have been a significant upward shift in the inflation target, during World War I, during World War II, and also in the post World War II period. A further temporary upward shifts appears significant following the collapse of the Bretton Woods system and the initial shift to flexible exchange rates in 1972. However, the inflation target has been roughly constant in the rest of the post war period, although higher than during the Gold Standard and the interwar period. According to the estimates of Table 1, the inflation target was not significantly different from zero during the Gold Standard period and the interwar period. During World War I it rose to 7.6% per annum. In the post World War II period it rose to 2.9% per annum. During the 1972-1982 period, after the collapse of Bretton Woods and before the Volcker disinflation, it rose temporarily to 8.2% per annum.¹⁷

To summarize, the evidence from the US is not incompatible with the main predictions of the model presented in this paper. Persistence of unemployment around its "natural" rate seems to result in the same degree of persistence of output around its "natural" level, and inflation around the target of the Federal Reserve. Thus, on the basis of the evidence, one cannot reject the predictions of the model for the US economy.

The evidence also suggests that monetary policy in the US has been discretionary throughout the period and has been characterized by different targets for inflation. Had monetary policy not been discretionary, inflation would not be characterized by the same degree of persistence as output and unemployment. The target for inflation in the US has shifted upwards in the post World War II period, where average inflation has been around 3%, and even higher in the 1972-1982 period, when it reached almost 8% on average.

¹⁷ It is worth reporting that the dummy variables for the different monetary regimes were also introduced in the regressions for unemployment and output, and were not statistically significant either jointly or individually.

7. Conclusions

This paper has put forward an alternative "new Keynesian" dynamic stochastic general equilibrium model of aggregate fluctuations, characterized by endogenous unemployment, output and inflation persistence.

The main distinguishing characteristic of the model is a dynamic "insider outsider" version of the "Phillips Curve", which accounts for unemployment persistence following nominal and real shocks.

The model differs from the typical "new Keynesian" model based on staggered contracts, in that employment and output dynamics do not result from staggered price and wage setting, but from one period nominal wage contracts and the dynamic adjustment of the pool of labor market "insiders". Thus, this model emphasizes a different propagation mechanism compared to the typical "new Keynesian" model.

We have analyzed the nature of aggregate fluctuations both under a Taylor nominal interest rate rule and under an optimal monetary policy rule, according to which the central bank minimizes an intertemporal quadratic loss function, that depends on deviations of inflation from target, and unemployment from its "natural" rate. Both real and nominal shocks affect aggregate fluctuations in this model, causing persistent deviations of output, employment, unemployment, real wages and the real interest rate from their "natural" rates. In addition, there is persistence in deviations of inflation from target, as, under both a Taylor rule and the optimal discretionary monetary policy, the nominal interest rate and inflation respond to persistent deviations of unemployment from its "natural" rate. These predictions do not seem to be rejected by evidence from the US economy for the 1890-2014 period.

Data Appendix

The data set used in this study is as follows:

u is the civilian unemployment rate, from *Economic Report of the President* and *Historical Statistics of the United States, From Colonial Times to 1970.* For the pre World War II period, the data used are as proposed by Darby (1976) and Romer (1986).

y is the log of real GDP, again from *Economic Report of the President* and *Historical Statistics of the United States, From Colonial Times to 1970.*

 π is the rate of change of the Consumer Price Index, from *Economic Report of the President* and *Historical Statistics of the United States*, From Colonial Times to 1970.

Table 1

The Persistence of Unemployment, Output and Inflation in the USA

Annual Data, 1890-2014

	$(u-\bar{u})_t$		$(y-\overline{y})_t$		π_t
Constant	0.000		0.000		0.017
	(0.001)		(0.004)		(0.006)
$(u-\bar{u})_{t-1}$	0.944	$(y-\overline{y})_{t-1}$	0.895	π_{t-1}	0.639
	(0.080)		(0.085)		(0.087)
$(u-\bar{u})_{t-2}$	-0.480	$(y-\bar{y})_{t-2}$	-0.351	π_{t-2}	-0.226
	(0.080)		(0.085)		(0.084)
Persistence	0.465		0.544		0.413
	(0.068)		(0.070)		(0.078)
Gold Standard					-0.014
					(0.009)
WW I					0.045
					(0.016)
Interwar					-0.020
					(0.009)
WW II					0.008
					(0.0.14)
Bretton Woods					0.002
					(0.009)
1972-1982					0.031
					(0.012)
R ²	0.542		0.508		0.550
S	0.014		0.039		0.033
DW	2.032		2.121		2.100
ADF	-7.861		-6.545		-7.582

OLS Estimates

Note: \bar{u} and \bar{y} are approximated by a Hodrick-Prescott filter. Persistence is the sum of the coefficients of the lagged dependent variables. Standard errors in parentheses. ADF is the Augmented Dickey-Fuller test.

Table 2

The Persistence of Unemployment, Output and Inflation in the USA

Annual Data, 1954-2014

OLS Estimates

	(-)				
	$(u-u)_t$		$(y-\overline{y})_t$		π_t
Constant	0.000		0.000		0.015
	(0.001)		(0.002)		(0.003)
$(u-\bar{u})_{t-1}$	0.696	$(y-\bar{y})_{t-1}$	0.674	π_{t-1}	0.805
	(0.123)		(0.125)		(0.122)
$(u-\bar{u})_{t-2}$	-0.409	$(y-\bar{y})_{t-2}$	-0.272	π_{t-2}	-0.370
	(0.122)		(0.125)		(0.110)
Persistence	0.288		0.402		0.435
	(0.123)		(0.120)		(0.099)
1972-1982					0.030
					(0.007)
R ²	0.360		0.339		0.776
S	0.008		0.017		0.014
DW	1.966		1.996		1.947
ADF	-5.794		-4.967		-5.709

Note: \bar{u} and \bar{y} are approximated by a Hodrick-Prescott filter. Persistence is the sum of the coefficients of the lagged dependent variables. Standard errors in parentheses. ADF is the Augmented Dickey-Fuller test.

References

- Barro R.J. and D. Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Setting", *Journal of Political Economy*, 91, pp. 589-610.
- Blanchard O.J. and Summers L.H. (1986), "Hysteresis and the European Unemployment Problem", *NBER Macroeconomics Annual*, 1, pp.15-78.
- Clarida R., Gali J. and Gertler M. (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature*, 37, pp. 1661-1707.
- Clarida R., Gali J. and Gertler M. (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory", *Quarterly Journal of Economics*, 105, pp. 147-180.
- Darby M. (1976), "Three and a Half Million US Employees have been Mislaid, or, An Explanation of Unemployment, 1934-1941.", *Journal of Political Economy*, 84, pp. 1-16.
- Fischer S. (1977), "Long Term Contracts, Rational Expectations and the Optimal Money Supply Rule", *Journal of Political Economy*, 85, pp. 191-205.
- Fisher I. (1896), *Appreciation and Interest*, Publications of the American Economic Association, 11, pp. 1-98.
- Fisher I. (1930), The Theory of Interest, New York, Macmillan.
- Gali J. (2008), *Monetary Policy, Inflation and the Business Cycle,* Princeton N.J., Princeton University Press.
- Gali J. (2011a), "Monetary Policy and Unemployment," in Friedman B. and Woodford M. (eds.) *Handbook of Monetary Economics*, vol. 3A, Amsterdam, Elsevier, pp. 487-546.
- Gali J. (2011b), Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective, Cambridge Mass., The MIT Press.
- Gali J. (2016), "Insider-Outsider Labor Markets, Hysteresis and Monetary Policy" mimeo, University Pompeu Fabra.
- Gottfries N. (1992), "Insiders, Outsiders and Nominal Wage Contracts", *Journal of Political Economy*, 100, pp. 252-270.
- Gray J. (1976), "Wage Indexation: A Macroeconomic Approach", *Journal of Monetary Economics*, 2, pp. 221-235.
- Hodrik R.J. and Prescott E.C. (1997), "Postwar U.S. Business Cycles: An Empirical Investigation", *Journal of Money Credit and Banking*, 29, pp. 1-16.
- Kydland F.E. and Prescott E.C. (1977), "Rules Rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy*, 85, pp. 473-492.

- Lindbeck A. and Snower D. (1986), "Wage Setting, Unemployment and Insider-Outsider Relations", *American Economic Review*, 76, pp. 235-239.
- Lucas R.E. Jr (1976), "Econometric Policy Evaluation: A Critique", *Carnegie Rochester Conference Series on Public Policy*, 1, pp. 19-46.
- McCallum B.T. (1994), "A Semi-Classical Model of Price Level Adjustment", *Carnegie Rochester Conference Series on Public Policy*, 41, pp. 251-284.
- McCallum B.T. and Nelson E. (1999), "Performance of Operational Policy Rules in an Estimated Semi-Classical Structural Model", in Taylor J.B. (ed), *Monetary Policy Rules*, Chicago III., University of Chicago Press and NBER.
- Okun A.M. (1962), "Potential GNP: Its Measurement and Significance", Cowles Foundation Paper no. 190, New Haven, CT, Yale University.
- Rogoff K. (1985), "The Optimal Degree of Commitment to an Intermediate Monetary Target", *Quarterly Journal of Economics*, 100, pp. 1169-1189.
- Romer C. (1986), "Spurious Volatility in Historical Unemployment Data", *Journal of Political Economy*, 94, pp. 1-37.
- Sargent T.J. and Wallace N. (1975), "Rational Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule", *Journal of Political Economy*, 83, pp. 241-254.
- Taylor J.B. (1993), "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy*, 39, pp. 195-214.
- Taylor J.B. (1999), "A Historical Analysis of Monetary Policy Rules", in Taylor J.B. (ed), *Monetary Policy Rules*, Chicago Ill., University of Chicago Press and NBER.
- Taylor J.B. and Williams J.C. (2011), "Simple and Robust Rules for Monetary Policy", in Friedman B. and Woodford M. (eds.) *Handbook of Monetary Economics*, vol. 3B, Amsterdam, Elsevier. pp. 829-859.
- Wicksell K. (1898), Interest and Prices, (English translation, Kahn R.F. 1936), London, Macmillan.
- Woodford M. (2003), Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton N.J., Princeton University Press.
- Woodford M. (2011), "Optimal Monetary Stabilization Policy", in Friedman B. and Woodford M. (eds.) *Handbook of Monetary Economics*, vol. 3B, Amsterdam, Elsevier, pp. 723-828.