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Hardwiring of investment decisions to ratings and the information content of asset prices

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Abstract

We discuss how the mechanical response of investment decisions to rating changes, so called ratings hardwiring, could affect asset prices. We address this issue within a rational expectations framework, using an asset pricing model with asymmetrically informed traders that specialize in different driving factors of asset payoffs and uninformed (noise) traders that mechanically link their supply of the asset to ratings. We show that ratings hardwiring leads to predictable supply shocks that induce informed traders to overreact to new information. That creates a channel through which fundamental and non-fundamental shocks are amplified, leading to less informative and more volatile prices.

Key words: ratings, hardwiring, asymmetric information, price informativeness
JEL Classification: D82; D84; G12; G14

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1 Introduction

"We need to make sure that, as far as possible, we do not have a large pool of investors, whether they are pension funds or others, who have built into their portfolio decisions mechanical responses to changes in ratings, which you can see from time to time"
Sir Mervyn King, Governor of the Bank of England, appearing before the Treasury Select Committee, January 17, 2012.

The use of credit ratings in financial markets extends beyond the mere updating of investor beliefs in making informed decisions. Credit ratings are used to facilitate monitoring the risks of investments by regulated entities, such as SEC Rule 2a-7 that restricts money market funds from investing in commercial paper below a rating threshold.[1] Similar rules apply to insurance companies and pension funds. Ratings have also been used extensively in determining capital adequacy buffers for banks, insurance companies, broker-dealers and other regulated entities (e.g. Basel II Capital Accord, EU Solvency II Directive, SEC Rule 15c3-1) and to set collateral requirements by central banks for the provision of liquidity to the banking system. Many institutional investors are also forced by their own charter to sell securities whose rating has crossed some critical threshold.

Building mechanical responses of investment decisions to ratings (so called *ratings hardwiring*) could materially impact on demand and supply of rated securities, affecting information pooling and the signalling role of prices. That could be especially true when the market for rated securities is dominated by investors that are subject to ratings-based rules and regulatory restrictions.[2] Ratings hardwiring could also result from inclusion of a security in an index of highly rated securities (e.g. Barclays U.S. Corporate IG Index), leading to a mechanical increase in demand by index-trucking funds, while dropping the security from the index would lead to a mechanical increase in supply.

A number of policymakers and market participants, including the rating agencies themselves, have pointed to the fact that ratings hardwiring could destabilise markets, distort information discovery and impede the efficient allocation of financial resources. Empirical evidence also suggests that the regulatory use of ratings has a material impact on the market price of rated securities, which is distinct from the impact of information that is conveyed into ratings.

Bongaerts et al. (2012) find evidence that multiple ratings by issuers in the corporate bond market are motivated by regulatory certification, i.e. a bond to become eligible

[1] A notable dichotomy is between *investment grade* (IG) and *high yield* (HY) credits, defined by the BBB rating threshold in Standard & Poor's ratings scale, or the Baa rating by Moody's.

[2] On the basis of Federal Reserve Flow of Funds accounts, Campbell and Taksler (2003) calculate that more than 50% of the market for corporate bonds in the U.S. is dominated by institutional investors that face ratings-based restrictions.

for investment by regulated institutions, and not by information production. They also find evidence that regulatory certification has a very substantial price impact near the investment/sub-investment grade threshold. Ashcraft et al. (2011) find evidence of excessive correlation between prices of subprime mortgage-backed securities and their ratings in the period leading up to the global financial crisis. Deb et al. (2011) offer a comprehensive overview of the market for ratings and discuss possible financial stability and pricing implications of the use of ratings by market participants. Kisgen and Strahan (2010) show that the regulatory use of ratings has a material impact on a firm's debt cost of capital, which is distinct from the ratings impact due to their information content. There is also a considerable volume of empirical work focusing on the price impact of rating changes.[3]

However, asset pricing implications that arise from ratings hardwiring have received scant attention in theoretical literature, which focuses primarily on equilibrium bias in produced and in reported ratings. Bolton et al. (2011) show that, in the presence of ratings shopping by issuers, competition among rating agencies can reduce efficiency and inflate reported ratings in equilibrium, especially in boom periods when there are more trusting investors. That is consistent with Becker and Milbourn (2011), who provide evidence of less informative ratings when competition in the ratings market increases.

Skreta and Veldkamp (2009) show that, in an issuer-initiated market for ratings, ratings shopping may lead to bias in disclosed ratings of complex securities, even if the rating produced by each individual rating agency is an unbiased forecast. In contrast, simple assets tend to receive similar ratings by agencies, which eliminates the possibility for ratings shopping and bias in disclosed ratings disappears. That is consistent with empirical evidence by Ashcraft et al. (2010), showing that rating standards for (complex) mortgage-backed securities declined in the run up to the global financial crisis, while rating standards for (simple) corporate bonds remained conservative. Mathis et al. (2009) show that reputation considerations are not always sufficient to discipline a monopolist rating agency not to inflate the ratings of complex securities. Opp et al. (2012) show that rating standards could deteriorate as a result of ratings-based regulations, if the regulatory benefit to investors from holding highly rated securities exceeds an endogenous threshold. Such a threshold depends on the cost of gathering and processing information by the rating agency. For complex securities, whose evaluation is possibly costly, rating standards may be more lax than for simple securities for which the cost of acquiring information is possibly lower.

In this paper we examine the potential impact of ratings hardwiring on the information content and volatility of asset prices. We address this issue within a rational expectations framework, considering a model for trading a *simple* asset in a market with informed and uninformed agents. The asset is *simple* in the sense of Skreta and Veldkamp (2009) and

[3] Michaelides et al. (2012), Güntay and Hackbarth (2010), Hill and Faff (2010), Brooks et al. (2004), Kliger and Sarig (2000), Ederington and Goh (1998), Hand et al. (1992), Holthausen and Leftwich (1986).

Opp et al. (2012). Trading takes place intertemporally and a representative, non-strategic rating agency produces a public rating that is an unbiased estimate of the asset payoff next period.

In the model, informed traders specialize in different driving factors of asset payoffs. Yet, they rationally anticipate changes in all factors that affect future income and capital gains from holding the asset. There is also a residual class of uninformed (noise) traders that do not independently assess the risky asset, but rely exclusively on ratings and increase the supply of the asset when its rating falls, while reduce it when its rating increases. We use such a mechanical response of the supply of the asset to changes in ratings as a proxy for ratings hardwiring. In addition to ratings hardwiring, we assume that noise traders also trade for non-fundamental (liquidity) purposes à la Grossman and Stiglitz (1980).[4]

The distinction between informed traders and uninformed investors that hardwire their investments to ratings shares some parallels with the distinction between sophisticated and trusting investor clienteles in Bolton et al. (2011). They argue that the coexistence of the two types of agents may be due to different compensation schemes that lead to different incentives to carry out due diligence, or regulatory and internal charter restrictions that forces certain types of investors to hold assets with ratings only above a certain threshold. In Boot et al. (2005) such investors are simply called *institutional investors* and in Hirshleifer and Teoh (2003) investors of *limited attention and processing power*.

The analysis shows that hardwiring of investment decisions to ratings leads to less informative and more volatile asset prices. Our results also show that such an effect becomes more pronounced as traders' risk aversion increases. In order to ensure comparability of results for various levels of ratings hardwiring, we make sure that we do not induce additional volatility in the supply of the asset due to noise trading. Consequently, the extent of ratings hardwiring in the model translates into a proportion of noise-trading volatility that is due to hardwiring.

Ratings hardwiring results in less informative and more volatile prices because the noisy supply of the asset becomes correlated with fundamentals and, to a certain extent, predictable. As a result, informed traders react more aggressively to any item of information that could potentially be relevant to their trading decisions, creating a channel through which fundamental and non-fundamental shocks are amplified. More specifically, we show that hardwiring induces a stronger price reaction to fundamental innovations compared to the situation without hardwiring, overshooting even the hypothetical scenario with complete information. Hardwiring also leads to a larger misinterpretation and overreaction by traders to errors in their private information. The same effect obtains for any other non-fundamental and non ratings-related shock in the model, indicating that informed traders

[4]We abstract from rating shopping incentives by issuers and reputation considerations by the rating agency, focusing instead on pricing implications that arise from ratings hardwiring.

become more prone to misinterpret any item of news as information about fundamentals. That leads to prices becoming less informative and more volatile in equilibrium.

The analysis is in the line of literature initiated by Grossman and Stiglitz (1980) and Hellwig (1980). In the context of a discrete-time asset pricing model of infinite horizon, we consider a competitive asset market where asymmetrically informed traders place their orders with a Walrasian auctioneer. Informed traders receive private signals and also use the observed prices and ratings to infer as much as possible about the asset and then decide by maximizing their expected CARA utility of next period's wealth.

In such a market, we calculate noisy rational expectations equilibria (NREE) assuming that the true state variables are never perfectly revealed neither to traders, nor to the rating agency. In equilibrium, informed traders' beliefs have to be consistent with the actual law of motion that those beliefs generate. Thus, equilibrium in our model is calculated as a *fixed point* in the mapping from informed traders' perceived laws of motion to the actual law of motion that those perceptions generate. In line with Bacchetta and van Wincoop (2006) and Allen, Morris and Shin (2006), our modelling approach allows for higher order beliefs to have a material impact on asset prices.[5] However, in discrete-time models with asymmetric information, agents' rationality requires one to address the inferences that agents make from observable variables, knowing that others act in a similar fashion. Thus, higher order beliefs become hidden state variables and the dimension of the state vector, associated with agents' signal extraction problems, becomes unbounded.

We address the problem of infinite regress in expectations assuming that informed traders make their forecasts by fitting first-order vector autoregressive moving average (ARMA) models. As we know from Townsend (1983), Sargent (1991) and Hussman (1992), the equilibrium in first-order ARMA models is consistent with higher order beliefs and informed traders have no incentive to increase the order of either the AR or the MA component in order to improve their forecasts. In other words, equilibrium forecast errors are orthogonal to the Hilbert space that is generated by all past history of information.

The remainder of the paper is organised as follows: Section 2 presents the asset pricing model with imperfectly informed traders and ratings hardwiring. Section 3 introduces the complete information benchmark and our measure of price informativeness under incomplete information. Section 4 outlines the solution concept and NREE solution algorithm. Section 5 presents the results, comparative statics and impulse response analysis. Section 6 concludes. Technical details and figures are included in the appendix.

[5] Higher order beliefs is a basic feature of asset pricing under asymmetric information and it refers to the situation where opinions of other agents' opinions, and higher order than that, may have a material impact on asset prices. That is in line with Keynes' (1936) famous metaphor that the market is similar to a beauty contest, where an agent's subjective payoff from choosing the prettiest face from a list of contestants depends on how close her prediction were to the average opinion of other agents.

2 The model

We consider a competitive market for a risky asset, with informed and uninformed traders. The risky asset pays a dividend D_t in every period t, depending on the realisation of two fundamental factors θ_{1t} and θ_{2t} and a transitory component u_t

$$D_t = \theta_{1t} + \theta_{2t} + u_t \tag{1}$$

Factors θ_1 and θ_2 are orthogonal to each other and follow stationary autoregressive AR(1) processes with persistence parameters ρ_1 and ρ_2.

$$\theta_{1t} = \rho_1 \theta_{1t-1} + v_{1t} \quad , \quad \theta_{2t} = \rho_2 \theta_{2t-1} + v_{2t} \tag{2}$$

where, $\{u_t\}$, $\{v_{1t}\}$ and $\{v_{2t}\}$ are *i.i.d.* normal with mean zero and variance σ_u^2, σ_{1v}^2 and σ_{2v}^2, respectively. Let also θ_1 be more persistent than θ_2, i.e. $|\rho_2| < |\rho_1| < 1$.

There is a continuum of informed traders of total measure one that are infinitely lived and myopic in the sense that they only care about next period's wealth. They have CARA preferences over future wealth and trade conditionally on prices by submitting limit orders with a Walrasian auctioneer. As a result, they can infer information from the current price, at which their limit orders are settled.

In addition to prices, informed traders observe dividends and public ratings produced by a rating agency for the risky asset. They also have special price discovery skills and specialise in one fundamental factor by observing private information. Depending on the type of private information that they observe, traders are divided into two classes $j = 1, 2$. Proportion α of them belong to class 1 and proportion $1 - \alpha$ to class 2.[6]

There is also a residual set of uninformed traders, called noise traders, who trade both for non-fundamental (liquidity) purposes and for reasons related to ratings hardwiring. Non fundamental trading implies a random supply of the asset that is not forecastable. On the other hand, hardwiring of investment decisions to ratings implies the supply partly depends on ratings, becoming forecastable to a certain extent, as we discuss next.

2.1 Hardwiring investment decisions to ratings

Noise traders are represented by the random supply of the risky asset which may partly depend on ratings. We think of a situation where a set of investors may be forced by law, or by statute, to sell the asset if its rating fall below a certain threshold (e.g. investment grade). By anticipating such a possible crossover, a high/low rating one period could induce low/high supply of the asset. Similar effects could obtain as a result of analyst

[6]The information structure is given exogenously, without modelling explicitly the decision to acquire private information, focusing on informed traders' problem to filter information from observable variables.

recommendations on overall buying and selling depending on price levels, or due to tracking a benchmark index by certain types of investment funds.[7] In those case, a positive or negative recommendation, or the deletion of the asset from the benchmark index could trigger noise trading, affecting the supply of the asset in the market.

Let the supply S_t be equal to the sum of a deterministic component that depends (linearly) on the rating r_t and a random component ς_t due to non-fundamental trading

$$S_t = -\psi r_t + \varsigma_t \tag{3}$$

where, hardwiring parameter $\psi \geq 0$ implies that S_t correlates negatively with the asset rating and $\{\varsigma_t\}$ are *i.i.d.* normal with mean zero and variance σ_ς^2 and orthogonal to all other noise terms in the model.

From 3, the unconditional variance of the asset supply is $\sigma_S^2 = \psi^2\sigma_r^2 + \sigma_\varsigma^2$, where σ_r^2 is the unconditional variance of the rating process discussed in section 2.2. In order not to induce additional volatility in the supply because of hardwiring and to ensure comparability of results for various levels of hardwiring parameter ψ, we adjust the variance of ς_t appropriately. Thus, for any given level of the unconditional variance of the asset supply σ_S^2, the variance of ς_t is adjusted to be $\sigma_\varsigma^2 = \sigma_S^2 - \psi^2\sigma_r^2$, for $|\psi| < \frac{\sigma_S}{\sigma_r}$.

2.2 Private information and public ratings

Traders of class 1 specialise in the high-persistence factor θ_{1t} by observing signals s_t^1, while class 2 specialise in the low-persistence factor θ_{2t} by observing signals s_t^2

$$s_t^1 = \theta_{1t} + \eta_{1t} \quad , \quad s_t^2 = \theta_{2t} + \eta_{2t} \tag{4}$$

where idiosyncratic noise terms $\{\eta_{1t}\}$ and $\{\eta_{2t}\}$ are *i.i.d.* normal, orthogonal to each other and to all other noise terms in the model, with mean zero, variances $\sigma_{1\eta}^2$ and $\sigma_{2\eta}^2$.

We consider an exogenous non-trading and non-strategic agent that plays the role of a rating agency for the risky asset. In the model, the rating agency receives private noisy information about the asset and with a lag of one period makes public the updated rating. In this model, the rating reflects the best estimate of the fundamentals of the asset using as information only the history of the private information of the agency, neglecting the information reflected in the price and payoff of the asset.[8]

Let the rating r_t be a summary statistic, namely an unbiased estimator of the sum of

[7] Malmendier and Shanthikumar (2007) provide evidence on hardwiring on prices.

[8] Yet we abstract from the incentive structure in the rating industry, reputation considerations and other frictions that may impact on ratings and their information content.

the two fundamental factors, conditional on the history of the agency's private information[9]

$$r_t = E\left[\theta_{1t} + \theta_{2t} \mid s^r_{1s}, s^r_{2s}, s < t\right] \tag{5}$$

where s^r_{1t} and s^r_{2t} are signals about θ_{1t} and θ_{2t}, contaminated by idiosyncratic noise

$$s^r_{1t} = \theta_{1t} + e_{1t} \quad , \quad s^r_{2t} = \theta_{2t} + e_{2t} \tag{6}$$

and $\{e_{1t}\}$ and $\{e_{2t}\}$ are *i.i.d.* normal with mean zero and variance σ^2_{1e} and σ^2_{2e}, orthogonal to $\{u_t\}$, $\{v_{1t}\}$, $\{v_{2t}\}$, $\{\eta_{1t}\}$ and $\{\eta_{2t}\}$.

2.3 Trader forecasting rules

Given the linear specification of the model and the assumption that traders are myopic and care only about next period's wealth, Sargent (1991) and Hussman (1992) show that ARMA (1,1) forecasting rules are optimal in the sense that informed agents would have no incentive to increase the order of either the AR or the MA part to further improve their forecasts. Therefore, we assume that informed traders' perceptions about the law of motion of their observable variables are assumed to be of the general ARMA(1,1) form

$$z_{jt+1} = \mathbf{A}_j z_{jt} + \zeta_{jt+1} + \mathbf{C}_j \zeta_{jt} \quad , \quad j = 1, 2 \tag{7}$$

where $z'_{jt} \equiv \begin{bmatrix} p_t, & D_t, & r_t, & s^j_t \end{bmatrix}$, ζ_{jt+1} is the vector of conditional forecast errors and $\mathbf{A}_j$, $\mathbf{C}_j$ are matrices of ARMA coefficients.

Recasting 7 we get

$$x_{jt+1} = \mathbf{B}_j x_{jt} + \mathbf{v}_{jt+1} \quad , \quad j = 1, 2 \tag{8}$$

where $x_{jt} \equiv \begin{bmatrix} z_{jt} \\ \zeta_{jt} \end{bmatrix}$ is the vector of variables that informed traders observe in period t, including their realised forecast errors ζ_{jt}, $\mathbf{v}_{jt+1} = \begin{bmatrix} \zeta_{jt+1} \\ \zeta_{jt+1} \end{bmatrix}$, $\mathbf{B}_j \equiv \begin{bmatrix} \mathbf{A}_j & \mathbf{C}_j \\ \mathbf{0}_4 & \mathbf{0}_4 \end{bmatrix}$ and $\mathbf{0}_4$ is a 4×4 matrix of zeros.

Informed traders use 8 to forecast x_{jt+1} on the basis of observable x_{jt}

$$E\left[x_{jt+1} \mid x_{jt}\right] = \mathbf{B}_j x_{jt} \quad , \quad j = 1, 2 \tag{9}$$

[9] Alternatively, we could assume that the agency produces two public ratings: a long-term rating $r^L_t = E\left[\theta_{1t} \mid s^r_{1s}, s < t\right]$ about the persistent factor θ_1, and a short-term rating $r^S_t = E\left[\theta_{2t} \mid s^r_{2s}, s < t\right]$ about the less persistent factor θ_2.

2.4 Preferences and trader optimisation

Informed traders decide every period how much to invest in the risky asset, or in a safe bond. They reach their decisions by maximising their expected utility and inferring as much as possible about the asset from information I_{jt} they have observed up to period t. They choose their optimal demands q_t^j for the risky asset in order to maximise their expected CARA utility over next period's wealth w_{t+1}^j

$$q_t^j = Arg \max_{q_t^*} E\left[-\exp\left(-w_{t+1}^j/\phi_j\right) \mid I_{jt}\right] \quad , \quad j = 1, 2 \tag{3a}$$

subject to

$$w_{t+1}^j = R\left(w_t^j - q_t^* p_t\right) + q_t^*\left(p_{t+1} + D_{t+1}\right) \tag{3b}$$

where R is the constant gross interest rate on the safe bond.

The above maximisation problem gives the following optimal demands

$$q_t^j = \phi_j \frac{E\left[p_{t+1} + D_{t+1} \mid I_{jt}\right] - Rp_t}{Var\left[\zeta_{jt+1}^p + \zeta_{jt+1}^D\right]} \quad , \quad j = 1, 2 \tag{11}$$

where, ζ_{jt+1}^p and ζ_{jt+1}^D are the conditional forecast errors for p_{t+1} and D_{t+1}, respectively, that result from trader forecasting rules in 7.

2.5 Market clearing

We assume that traders' optimal demands are aggregated by a central auctioneer. The equilibrium price p_t is set to satisfy the market-clearing condition

$$\alpha q_t^1 + (1 - \alpha) q_t^2 = S_t \tag{12}$$

where q_t^1 and q_t^2 are agents' optimal demands for the risky asset, as given by 11, and S_t is the supply of the risky asset, as given by 3.

Substituting 11 into 12, the price process p_t becomes

$$p_t = \Lambda^{-1}\left[\alpha\sigma_2^2\phi_1 E_1[\cdot] + (1 - \alpha)\sigma_1^2\phi_2 E_2[\cdot] - \sigma_1^2\sigma_2^2 S_t\right] \tag{13}$$

where $E_j[\cdot] \equiv E\left[p_{t+1} + D_{t+1} \mid I_{jt}\right]$, $\sigma_j^2 = Var\left[\zeta_{jt+1}^p + \zeta_{jt+1}^D\right]$, for $j = 1, 2$, and parameter Λ is given by

$$\Lambda \equiv R\left[\sigma_2^2\alpha\phi_1 + \sigma_1^2(1 - \alpha)\phi_2\right] \tag{14}$$

Both, subjective beliefs $E_j[\cdot]$ and subjective measures of riskiness σ_j^2 are determined in equilibrium on the basis of investors' perceived laws of motion, as discussed in section 2.3.

3 Information content of asset prices

In this section we introduce our measure of price informativeness that allows us to gauge the impact of ratings hardwiring on the information content of asset prices. First we consider prices under complete information, as a benchmark for comparison with our baseline scenario of incomplete and asymmetric information.

3.1 Complete information benchmark

Suppose that informed traders observe perfectly, without idiosyncratic noise, the realisation of fundamental factors each period, but they still remain uncertain about their future realisations. Let also the net supply of the risky asset ς_t be deterministic in every period and equal $\overline{\varsigma}$. In that case, the price in every period becomes sufficient statistic with respect to both fundamental factors and the model is characterised by common knowledge of θ_{1t} and θ_{2t}. In that case, by solving 13 forward and by the law of iterated expectations, the full information price p_t^* becomes

$$p_t^* = E_t\left[\sum_{i=1}^{\infty} R^{-1} D_{t+i}\right] - \frac{\sigma^2 \overline{\varsigma}}{(\phi_1 + \phi_2)(R-1)} \tag{15}$$

where σ^2 is now the unconditional variance of $p_t^* + D_t$.

From 1 and 15, and substituting forward θ_{1t} and θ_{2t}, the full information price can be expressed in terms of the current realisation of fundamental factors

$$p_t^* = \frac{\rho_1}{R-\rho_1}\theta_{1t} + \frac{\rho_2}{R-\rho_2}\theta_{2t} - \frac{\sigma^2 \overline{\varsigma}}{(\phi_1 + \phi_2)(R-1)} \tag{16}$$

3.2 Information content of prices under signal extraction

Signal extraction problems imply that the amount of information about θ_{1t} and θ_{2t} conveyed in p_t may be different from the complete information benchmark p_t^*. The information content of prices is captured here by the expected squared difference of p_t minus p_t^*, conditional on θ_{1t} and θ_{2t}

$$V = E\left[\left.(p_t - p_t^*)^2\right| \theta_{1t}\ ,\ \theta_{2t}\right] \tag{17}$$

where, p_t^* is given by 16 and p_t is determined in equilibrium, as we discuss in section 4.

Equation 17 can be written as

$$V = Var\left(p_t \middle| \theta_{1t}\ ,\ \theta_{2t}\right) + \left[E\left(p_t \middle| \theta_{1t}\ ,\ \theta_{2t}\right) - p_t^*\right]^2 \tag{18}$$

Under incomplete and asymmetric information, the variable vector $\begin{bmatrix} p_t & \theta_{1t} & \theta_{2t} \end{bmatrix}$ follows a multivariate normal distribution with (unconditional) mean $\boldsymbol{\mu}$ and covariance matrix

$\boldsymbol{\Sigma}$. Given that all processes in the model have no drift and their disturbance terms have mean zero, it follows that $\boldsymbol{\mu}' = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$. Also the covariance matrix $\boldsymbol{\Sigma}$ is determined in equilibrium, as we discuss in section 4. Let σ_p^2 be the unconditional variance of p_t , $\boldsymbol{\Sigma}_{p\theta}$ the vector of covariances of p_t with $\begin{bmatrix} \theta_{1t} & \theta_{2t} \end{bmatrix}$ and $\boldsymbol{\Sigma}_{\theta\theta}$ the covariance matrix of $\begin{bmatrix} \theta_{1t} & \theta_{2t} \end{bmatrix}$. Conditionally on θ_{1t} and θ_{2t}, projection theorem implies that the distribution of p_t is normal with

$$(p_t | \, \theta_{1t} \, , \, \theta_{2t}) \sim N\left(\boldsymbol{\Sigma}_{p\theta} \boldsymbol{\Sigma}_{\theta\theta}^{-1} \begin{bmatrix} \theta_{1t} \\ \theta_{2t} \end{bmatrix}, \sigma_p^2 - \boldsymbol{\Sigma}_{p\theta} \boldsymbol{\Sigma}_{\theta\theta}^{-1} \boldsymbol{\Sigma}_{p\theta}' \right) \tag{19}$$

Orthogonality of θ_{1t} and θ_{2t} implies the covariance matrix $\boldsymbol{\Sigma}_{\theta\theta}$ is diagonal and from 18, 19 it follows

$$V = \sigma_p^2 - \left(\frac{\sigma_{p\theta_1}^2}{\sigma_{\theta_1}^2} + \frac{\sigma_{p\theta_2}^2}{\sigma_{\theta_2}^2} \right) + \left(\frac{\sigma_{p\theta_1}}{\sigma_{\theta_1}^2} \theta_1 + \frac{\sigma_{p\theta_2}}{\sigma_{\theta_2}^2} \theta_2 - p_t^* \right)^2 \tag{20}$$

or, by substituting p_t^* from 16

$$\begin{aligned} V \;\; = \;\; & \sigma_p^2 - \left(\frac{\sigma_{p\theta_1}^2}{\sigma_{\theta_1}^2} + \frac{\sigma_{p\theta_2}^2}{\sigma_{\theta_2}^2} \right) + \\ & \left[\left(\frac{\sigma_{p\theta_1}}{\sigma_{\theta_1}^2} - \frac{\rho_1}{R - \rho_1} \right) \theta_1 + \left(\frac{\sigma_{p\theta_2}}{\sigma_{\theta_2}^2} - \frac{\rho_2}{R - \rho_2} \right) \theta_2 - \Phi \right]^2 \end{aligned} \tag{21}$$

where, $\Phi = \frac{\sigma^2 \varsigma}{(\phi_1 + \phi_2)(R-1)}$ and $\sigma_{p\theta_j}$ is the covariance of price p_t with θ_{jt}, $j = 1, 2$.

Given that V depends on the realisation of fundamental factors, as a measure of the information content of prices we use its unconditional expectation

$$\overline{V} = \sigma_p^2 - \left(\frac{\sigma_{p\theta_1}^2}{\sigma_{\theta_1}^2} + \frac{\sigma_{p\theta_2}^2}{\sigma_{\theta_2}^2} \right) + \left(\frac{\sigma_{p\theta_1}}{\sigma_{\theta_1}^2} - \frac{\rho_1}{R - \rho_1} \right)^2 \sigma_{\theta_1}^2 + \left(\frac{\sigma_{p\theta_2}}{\sigma_{\theta_2}^2} - \frac{\rho_2}{R - \rho_2} \right)^2 \sigma_{\theta_2}^2 + \Phi^2 \tag{22}$$

where, $\sigma_{\theta_j}^2$, $j = 1, 2$, is the unconditional variance of θ_{jt} and is determined by.

Notice that $\sigma_{\theta_j}^2$ is exogenously determined by 2, i.e. $\sigma_{\theta_j}^2 = \frac{\sigma_{jv}^2}{1 - \rho_j^2}$, while the unconditional variance of price σ_p^2 and its covariance $\sigma_{p\theta_j}$ with θ_{jt} are determined in equilibrium, as we discuss next.

4 Outline of solution concept

Informed traders are characterised by information sets $I_{jt} = \{p_s, \; D_s, \; r_s, \; s_s^j; \; s \leq t\}, \;\; j = 1, 2$, which are records of data z_{jt} of the form

$$z_{jt}' = \begin{bmatrix} p_t, & D_t, & r_t, & s_t^j \end{bmatrix} \tag{23}$$

The state vector z_t that describes the market in period t includes all variables that are collectively observed by traders, the two latent factors θ_{1t} and θ_{2t}, the random supply

ς_t and the conditional forecast errors ζ_{jt} that depend on traders' information sets and forecasting rules.

$$z_t' = \begin{bmatrix} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t & \zeta_{1t} & \zeta_{2t} \end{bmatrix} \tag{24}$$

Let also vector ε_t specify all noise terms in the model in period t

$$\varepsilon_t' = \begin{bmatrix} u_t & \eta_{1t} & \eta_{2t} & v_{1t} & v_{2t} & e_{1t-1} & e_{2t-1} & \varsigma_t \end{bmatrix} \tag{25}$$

where, innovations u_t, v_{jt}, ς_t, η_{jt} and e_{jt} and are defined in 1, 2, 3, 4 and 6, respectively.

In every trading round t we consider the following timing of events and information:

1. Rating r_t is publicly announced based on information up to $t-1$.
2. Fundamentals are updated, traders observe private and public information and submit optimal demand schedules to a Walrasian auctioneer.
3. The rating agency receives information about the current level of fundamentals and, once again, makes a rating in period $t+1$.

The sequence of events captures the natural lag between ratings and prices. That is, innovations in θ_1 and θ_2 are reflected first into prices and then into ratings.[10] Also, the rating process in 5 is considered as an unbiased estimator of the sum of the two latent factors θ_1 and θ_2. It can be expressed in a recursive form as follows, using a Kalman filter representation.

Lemma 1 *The rating process $\{r_t\}$ exhibits positive autocorrelation and is generated by*

$$r_t = \lambda r_{t-1} + \lambda \left[\Sigma_1 s_{1t-1}^r + \Sigma_2 s_{2t-1}^r\right] \tag{26}$$

for $\lambda \equiv \rho_1 (\Sigma_1 + 1)^{-1} = \rho_2 (\Sigma_2 + 1)^{-1}$ and Σ_j is given by

$$\Sigma_j = \frac{1}{2}\left[\frac{\sigma_{jv}^2}{\sigma_{je}^2} - \left(1-\rho_j^2\right) + \sqrt{\left[\frac{\sigma_{jv}^2}{\sigma_{je}^2} - \left(1-\rho_j^2\right)\right]^2 + 4\frac{\sigma_{jv}^2}{\sigma_{je}^2}}\right] \quad , \quad j = 1,2 \tag{27}$$

Proof. *See appendix.* ■

We observe that the rating persistence λ depends on the precision of the agency's signals relative to fundamental innovations. Better quality signals – i.e. lower $\frac{\sigma_{jv}}{\sigma_{je}}$ – lead to a less persistent ratings and vice versa.[11]

[10] Such a natural lag between ratings and prices is justified by the fact that traders submit demand curves to the auctioneer, i.e. trading conditionally on prices, while the rating agency is not a trading party.

[11] That is a standard Kalman filter result, where better signal quality leads to higher weighting of new information in the recursive updating.

4.1 NREE definition

The fundamental requirement that a NREE must satisfy is that equilibrium prices have to be consistent with the presumption that traders know the actual law of motion of the asset market and choose their demands schedules accordingly. A competitive NREE for our asset market is defined as follows:

Definition 1

1. *Traders make conjectures about the law of motion of the variables they observe. Given their information sets, traders use statistically optimal ARMA(1,1) models to forecast their observable variables.*

2. *Given their ARMA(1,1) forecasting rules and information sets, traders choose their optimal demand schedules so as to maximise their expected utilities.*

3. *Given traders' optimal demands and the total supply of the risky asset, the market clearing price results from a Walrasian auction.*

4. *Traders' conjectures are correct, in the sense that ARMA(1,1,) forecasting rules are a fixed point in the correspondence that maps them to the actual law of motion they generate.*

Following Sargent (1991) and Hussman (1992), a market equilibrium with ARMA(1,1) forecasting rules is of *full-order*, meaning that traders would not be able to further improve their forecasts on the basis of available information. Therefore, conditioning traders' forecasts on an infinite history of data is equivalent to conditioning only on first-order lags and information sets $I_{jt} = \{p_s,\ D_s,\ r_s,\ s_s^j;\ s \leq t\}$ can be restated $I_{jt} = \left\{p_t,\ D_t,\ r_t,\ s_t^j\right\}$, $j = 1, 2$.

Trader forecasting rules are common knowledge in equilibrium. Therefore, conjecturing a law of motion about observable variables is equivalent to assume that traders conjecture an actual law of motion for the whole state vector z_t.[12] Thus, a market equilibrium here is characterised by the coefficient matrix $\mathbf{B} \equiv [\mathbf{B}_1\ \mathbf{B}_2]$ of traders' forecasting rules, as defined in 9. Let traders conjecture that z_t evolves according to the following law of motion

$$z_t = T(\mathbf{B})\, z_{t-1} + V(\mathbf{B})\, \varepsilon_t \tag{28}$$

where, $T(\mathbf{B})$, $V(\mathbf{B})$ are coefficient matrices.

[12] That follows from uniqueness of equilibrium, as we discuss at the end of this section.

Provided all eigenvalues of $T(\mathbf{B})$ lie inside the unit circle, equation 28 determines a unique covariance-stationary distribution for z_t, whose moment matrix $\mathbf{M}_z$ solves[13]

$$\mathbf{M}_z = T(\mathbf{B})\,\mathbf{M}_z T(\mathbf{B})' + V(\mathbf{B})\,\Omega V(\mathbf{B})' \tag{29}$$

where, $\mathbf{B} \equiv [\mathbf{B}_1\ \mathbf{B}_2]$ and Ω is the (diagonal) covariance matrix of noise vector ε_t.

Matrix $V(\mathbf{B})\,\Omega V(\mathbf{B})'$ is symmetric and, as a result, equation 29 defines a discrete-time Lyapunov equation. With all eigenvalues of $T(\mathbf{B})$ less than unity in modulus, there is a unique symmetric matrix M_z that solves 29.[14] Using appropriate selector matrices $\mathbf{u}_j$, $j = 1, 2$, we derive from $\mathbf{M}_z$ the covariance matrix $\mathbf{M}_{x_j}$ of traders' observable variables x_{jt}, as well as the covariance matrix $\mathbf{M}_{zx_j}$ of state vector z_t with x_{jt}

$$\mathbf{M}_{x_j} = \mathbf{u}_j \mathbf{M}_z \mathbf{u}_j' \quad , \quad j = 1, 2 \tag{30}$$

$$\mathbf{M}_{zx_j} = \mathbf{M}_z \mathbf{u}_j' \quad , \quad j = 1, 2 \tag{31}$$

where, $\mathbf{u}_j$ selects the subvector of observable variables x_{jt} from z_t.

Let us now consider the linear projection of vector x_{jt+1} on its previous realisation x_{jt}

$$E[x_{jt+1} \mid x_{jt}] = \mathbf{S}_j(\mathbf{B})\,x_{jt} \quad , \quad j = 1, 2 \tag{32}$$

With covariance matrices $\mathbf{M}_{x_j}$ and $\mathbf{M}_{zx_j}$ in hand, we evaluate the matrix $\mathbf{S}_j(\mathbf{B})$ of statistically optimal estimators in 32 as follows

$$\mathbf{S}_j(\mathbf{B}) = \mathbf{u}_j T(\mathbf{B})\,\mathbf{M}_{zx_j} \mathbf{M}_{x_j}^{-1} \quad , \quad j = 1, 2 \tag{33}$$

Setting $\mathbf{S}(\mathbf{B}) \equiv [\mathbf{S}_1(\mathbf{B})\ \ \mathbf{S}_2(\mathbf{B})]$, a rational expectations equilibrium is a fixed point in the correspondence that maps trader perceptions – as characterised by the coefficient matrix $\mathbf{B}$ – into statistically optimal projections $\mathbf{S}(\mathbf{B})$. In other words, an equilibrium is characterised by a coefficient matrix $\mathbf{B}$ such that $\mathbf{B} = \mathbf{S}(\mathbf{B})$.

Closing this section, we reiterate that trader conjectures about $\mathbf{B}$ are equivalent to making conjectures about the actual law of motion 28 of state vector z_t. Such an equivalence stems from the fact that, for a given coefficient matrix $\mathbf{B}$, equation 29 defines a *unique* moment matrix $\mathbf{M}_z$ for z_t, which in turn defines matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ of the actual coefficients. In other words, there is a one-to-one relationship between conjectures about coefficient matrix $\mathbf{B}$ and matrices $T(\mathbf{B})$, $V(\mathbf{B})$. In section 7.2 we discuss in detail the fixed-point solution algorithm and evaluation of matrices $T(\mathbf{B})$ and $V(\mathbf{B})$.

[13] All eigenvalues of matrix $T(\mathbf{B})$ lie inside the unit circle because both fundamental factors and noise terms in the model are stationary and, in addition, prices in 13 are linear in the lagged state vector, as we discuss in section 7.2 in the Appendix.

[14] From standard theory, there is a unique symmetric matrix $\mathbf{M}_z(\mathbf{B})$ that solves 29 *i.f.f.* no eigenvalue of $T(\mathbf{B})$ is the reciprocal of any other eigenvalue of $T(\mathbf{B})$. This is, *i.f.f.* $\mathbf{eig}[T(\mathbf{B})]\,\mathbf{eig}[T(\mathbf{B})]' - \mathbf{1} \neq \mathbf{0}$. Given that all eigenvalues of $T(\mathbf{B})$ lie inside the unit circle, none of them can be the reciprocal of another eigenvalue of $T(\mathbf{B})$.

5 Results

In this section we present equilibrium results about the price impact of ratings hardwiring, using numerical methods. The analysis is centered around the calculation of our measure $\overline{V}$ of price (non-)informativeness in equation 22. That is, the unconditional expectation of the squared difference of the equilibrium price p_t from its full information level p_t^*.

We also consider price informativeness vis à vis price volatility. In equilibrium, an increase in price volatility would not imply a cost if at the same time prices would become more informative, given noisy exogenous supply shocks. However, if combined with lower price informativeness, an increase in price volatility could be considered as a cost and, in that sense, excessive.

We gauge the impact of ratings hardwiring on asset prices by conducting comparative statics analysis of hardwiring parameter ψ and level of CARA. We also conduct impulse response analysis of prices to various shocks in the model. For all other parameters in the model, we consider the following parameterization, which is assumed to be common knowledge. The results are without lost of generality and hold for a wide range of parameterisations that we considered.

Gross interest rate	$R = 1.02$
Persistence of fundamentals	$\rho_1 = 0.8; \rho_2 = 0.4$
Informed trader proportions	$a = 0.5$
Variance of fundamental innovations	$\sigma_{1v}^2 = 0.1; \sigma_{2v}^2 = 0.1$
Variance of errors in traders' signals	$\sigma_{1\eta}^2 = 1; \sigma_{2\eta}^2 = 1$
Variance of errors in rating agency's signals	$\sigma_{1e}^2 = 0.1; \sigma_{2e}^2 = 0.6$
Variance of dividend innovations	$\sigma_u^2 = 1$
Random supply of the asset	$\sigma_\varsigma^2 = 0.01$

As an illustration, the equilibrium ARMA(1,1) coefficients $\mathbf{B}_j = \mathbf{S}_j(\mathbf{B})$, $j = 1, 2$, of observables p_t , D_t, r_t, s_t^j, ζ_t^j under no ratings hardwiring and $CARA = 2$, are calculated to be

$$\mathbf{B}_1 = \begin{bmatrix} 0.4508 & 0.0093 & 0.0133 & 1.2300 & -0.4154 & 0.3108 & 1.2612 & -1.0067 \\ 0.0000 & 0.4000 & -0.0000 & 0.4000 & 0.0007 & -0.2835 & 0.3473 & -0.3110 \\ 0.0000 & 0.2080 & 0.3376 & 0.2545 & 0.0004 & -0.1427 & 0.2001 & -0.2025 \\ -0.0000 & -0.0000 & -0.0000 & 0.8000 & 0.0004 & 0.0814 & 0.3368 & -0.7033 \\ 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \end{bmatrix}$$

$$\mathbf{B}_2 = \begin{bmatrix} 0.5580 & 0.8120 & 0.0465 & -0.8123 & -0.4605 & -0.4900 & 1.3148 & 0.8183 \\ 0.0000 & 0.8000 & -0.0000 & -0.4000 & 0.0253 & -0.6845 & 0.3796 & 0.4225 \\ 0.0000 & 0.4624 & 0.3376 & -0.2545 & 0.0147 & -0.3975 & 0.2192 & 0.2652 \\ 0.0000 & -0.0000 & -0.0000 & 0.4000 & -0.0022 & 0.0316 & 0.0038 & -0.3616 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \end{bmatrix}$$

The last four rows of $\mathbf{B}_j$ give the coefficients in the projection of forecast errors ζ_{t+1}^j on p_t, D_t, r_t, s_t^j and ζ_t^j. That these coefficients are zero is a necessary condition for ζ_{t+1}^j to be conditional vector *i.i.d.* normal with zero mean.

From the calculated moment matrix $\mathbf{M}_z$ in equilibrium, we select the upper-left covariance submatrix $\mathbf{M}$ for variables p_t, D_t, r_t, s_t^1, s_t^2, θ_{1t}, θ_{2t} and S_t

$$\mathbf{M} = \begin{bmatrix} 2.7476 & 1.0841 & 0.5166 & 0.7028 & 0.1004 & 0.5343 & 0.0856 & -0.0908 \\ 1.0841 & 1.3968 & 0.1522 & 0.2778 & 0.1190 & 0.2778 & 0.1190 & 0.0000 \\ 0.5166 & 0.1522 & 0.1789 & 0.1408 & 0.0115 & 0.1408 & 0.0115 & 0.0000 \\ 0.7028 & 0.2778 & 0.1408 & 1.2778 & 0.0000 & 0.2778 & 0.0000 & 0.0000 \\ 0.1004 & 0.1190 & 0.0115 & 0.0000 & 1.1190 & 0.0000 & 0.1190 & 0.0000 \\ 0.5343 & 0.2778 & 0.1408 & 0.2778 & 0.0000 & 0.2778 & 0.0000 & 0.0000 \\ 0.0856 & 0.1190 & 0.0115 & 0.0000 & 0.1190 & 0.0000 & 0.1190 & 0.0000 \\ -0.0908 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0100 \end{bmatrix}$$

With covariance matrix $\mathbf{M}$ in hand, we can easily get our measure $\overline{V} = 2.4742$ of price (non-)informativeness in equation 22 and price volatility $\sigma_p = 1.6576$ under no ratings hardwiring.

Assuming ratings hardwiring $\psi = 0.03$ and adjusting appropriately the random supply ς_t to ensure comparability with the no-hardwiring case, the covariance matrix $\mathbf{M}$ for variables p_t, D_t, r_t, s_t^1, s_t^2, θ_{1t}, θ_{2t} and S_t becomes

$$\mathbf{M} = \begin{bmatrix} 6.8896 & 1.5904 & 0.8511 & 1.0651 & 0.1425 & 0.8445 & 0.1235 & -0.1617 \\ 1.5904 & 1.3968 & 0.1522 & 0.2778 & 0.1190 & 0.2778 & 0.1190 & -0.0035 \\ 0.8511 & 0.1522 & 0.1789 & 0.1408 & 0.0115 & 0.1408 & 0.0115 & -0.0038 \\ 1.0651 & 0.2778 & 0.1408 & 1.2778 & 0.0000 & 0.2778 & 0.0000 & -0.0034 \\ 0.1425 & 0.1190 & 0.0115 & 0.0000 & 1.1190 & 0.0000 & 0.1190 & -0.0001 \\ 0.8445 & 0.2778 & 0.1408 & 0.2778 & 0.0000 & 0.2778 & 0.0000 & -0.0034 \\ 0.1235 & 0.1190 & 0.0115 & 0.0000 & 0.1190 & 0.0000 & 0.1190 & -0.0001 \\ -0.1617 & -0.0035 & -0.0038 & -0.0034 & -0.0001 & -0.0034 & -0.0001 & 0.0100 \end{bmatrix}$$

Evaluating our measure of price (non-)informativeness and volatility we get that ratings hardwiring results in lower price informativeness ($\overline{V} = 4.3110$) and higher price volatility

($\sigma_p = 2.6248$). This result is general and obtains for all alternative parameterisations that we considered. Next we present comparative statics of price informativeness and volatility with respect to different CARA coefficients and hardwiring parameter ψ.

5.1 Comparative statics

Rational agents in NREE models trade securities for two main reasons: (i) to share risk when they are endowed with different quantities of the risky asset; (ii) to exploit information when they have access to different information sources and possess different assessments of asset payoffs. Risk sharing and price exploitation motives interact in equilibrium affecting prices in various ways, depending on model parameters.

***Risk aversion*:** Higher risk aversion increases the risk-sharing motive. That tends to dominate the motive to exploit information, thus higher risk aversion leads to less informative prices. This result originates in Hellwig's (1980) static model and reproduced here by our multiperiod model, as shown in figure 1.

[Figures 1]

As far as price volatility is concerned, it depends both on the degree of price informativeness and serial correlation. On the one hand, prices may become more volatile simply because they become more informative, i.e. more responsive to fundamental innovations. On the other hand, the higher the serial correlation of prices the higher the unconditional variance of the price process. Prices may be serially correlated because of serial correlation in fundamentals, strong risk-sharing motives, filtering problems, or other externalities that may induce investors to trade with less confidence on private information and place more weight on publicly observed signals, such as prices.

[Figures 2]

Figure 2 reports the impact of risk aversion on price volatility. In line with Hellwig (1980), figure 2 illustrates that, in a market with higher risk aversion, rational traders are aware that risk sharing dominates information exploitation. As a result, prices become less informative, which leads to less accurate forecasts. Given that the long-run (unconditional) mean of the price process is common knowledge among traders, less accurate forecasts induce traders to respond more aggressively to temporary price deviations from the mean, in anticipation of a subsequent mean reversion.[15] Thus, prices are characterised in equilibrium by stronger mean reversion and, therefore, higher serial correlation and volatility.

[15] In other words, the less accurate trader forecasts become, the price tends to become a *focal point* that coordinates traders' beliefs. As a result, long-run (unconditional) mean reversion of prices becomes

Ratings hardwiring: Figure 1 and 2 show that the impact of risk aversion on the information content and volatility of prices also depends on the degree of ratings hardwiring. Other than the no-hardwiring case $\psi = 0$, we consider three alternative levels of hardwiring parameter ψ, i.e. 0.02, 0.03, 0.05. Given our basic parameterisation, those levels correspond to proportions 8%, 13% and 21%, respectively, of noise-trading volatility be attributable to ratings hardwiring.[16] The higher the ratings hardwiring the more quickly price informativeness falls and price volatility increases with risk aversion. Especially for $\psi = 0.05$, even relatively low levels of risk aversion lead to prices becoming almost totally uninformative, while price volatility explodes.

We also conduct comparative statics analysis of price informativeness and volatility with respect to a wider range of hardwiring parameter ψ, taking values in $[0, 0.12]$. That implies the proportion of noise-trading volatility attributable to ratings hardwiring takes values in the range $[0\%, 51\%]$. For such comparative statics we set CARA=1.

[Figures 3]

Figures 3 illustrates the decrease in the information content of prices ($\overline{V}$ increases exponentially) with respect to hardwiring parameter ψ. Despite the relative low level of risk aversion, price informativeness almost disappears as the proportion of noise-trading volatility attributable to ratings hardwiring approaches 51%, i.e. ψ approaches 0.12. For higher levels of risk aversion, price informativeness disappears at lower values of ψ. Figure 4 also shows that price volatility increases with ratings hardwiring.

[Figures 4]

Hardwiring of trading decisions to ratings may lead to lower price informativeness and higher price volatility because it leads to supply shocks to the asset become, to a certain extent, predictable and correlated with the fundamentals. As a result, informed traders tend to react more aggressively to new information and hardwiring creates a channel through which the shocks to fundamentals are amplified.

We verify this intuition by conducting impulse response analysis of prices to various shocks in the model. In particular, we consider the price impact of fundamental innovations v_{1t} and v_{2t}, trader signal errors η_{1t} and η_{2t}, rating agency signal errors e_{1t} and e_{2t}, random supply shocks ς_t and non-fundamental dividend shocks u_t.

self-fulfilled at earlier trading rounds. This is consistent with the results of Allen, Morris and Shin (2006) who solve a similar type of equilibrium but with three trading rounds, totally uninformative prices and a public signal about fundamentals that acts as a focal point and *skews* agents beliefs towards it.

[16]That follows from the unconditional variance of noise-trading supply $\sigma_S^2 = \psi^2\sigma_r^2 + \sigma_\varsigma^2$. The variance of the rating process $\sigma_r^2 = 0.1789$ and noise-trading supply $\sigma_S^2 = 0.0100$ correspond to the 3^{rd} and 8^{th} diagonal element of $\mathbf{M}$.

5.2 Impulse response

Coefficient matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ define the law of motion of state vector z_t in 28 and determine the impulse response of prices to various elements of the innovations vector ε_t. Given that the first element of z_t corresponds to price, its impulse response to a shock of one standard deviation σ_i in the i^{th} element of ε_t is given by the function

$$f(t) = \left[T(\mathbf{B})^{t-1} V(\mathbf{B})\right]^{(1,i)} \sigma_i \tag{34}$$

where, superscript $(1,i)$ refers to the i^{th} element in the first row of the matrix in brackets.

Under complete information, prices reflect instantly the exact realisation of θ_1 and θ_2. Therefore, fundamental shocks v_1 and v_2 would be the only elements of ε_t with a persistent impact on prices. However, when traders face signal extraction problems, all elements of ε_t, including non-fundamental innovations, may have a persistent price impact. That is because, signal extraction problems inhibit traders from accurately identifying whether changes in observable variables are due to fundamental, or non-fundamental shocks. As a result, traders may misinterpret non-fundamental noise as being fundamental information and that may last for sometime until they gradually filter out the actual realisation of past fundamental shocks.

In addition, if the market is characterised by a degree of ratings hardwiring then, shocks to the supply of the asset become correlated with the fundamentals and to a certain extent predictable. In that case, traders might become overly sensitive to any element of information about dividends and capital gains that are directly linked to supply shocks through market clearing.

Figures 5-12 illustrate the price impact of a one standard deviation shock to fundamental and non-fundamental factors, assuming $CARA = 2$. In addition to the no hardwiring case $\psi = 0$, we also consider $\psi = 0.02$ and $\psi = 0.03$. Under complete information, the impulse response of prices to fundamental shocks is shown with a dotted line.

The price impact of fundamentals shocks v_1 and v_2 is illustrated in figures 5 and 6. We notice that hardwiring induces a stronger reaction of prices to fundamental innovations compared to the case without hardwiring, overshooting even the complete information benchmark. The price impact of trader signal errors is reported in figures 7 and 8. In that case, hardwiring leads to larger misinterpretation and overreaction by traders to errors in their private signals, compared to the situation without hardwiring. Notice that informed traders are the only parties who observe their private signals. As a result, the induced price overreaction to private signal errors is exclusively due to more aggressive trading by informed traders, rather than stemming directly from noise trading.

Figures 9 and 10 show the impact of rating signal errors on prices. Also in this case, the undue price impact of rating signal errors is higher with than without hardwiring.

Figure 11 illustrates the impulse response of prices to the random supply ς of the risky asset. Price overreaction to such a shock increases with the extent of ratings hardwiring. As already mentioned, the variance of ς was properly adjusted for hardwiring to ensure comparability with the no-hardwiring case. Finally, figure 12 shows how prices respond to non-fundamental dividend shocks u. As with private signal errors, informed traders are the only parties who consider dividends among their observables. Therefore, any price overreaction to non-fundamental dividend shocks is exclusively due to trader overreaction to the shocks and to no other reason.

6 Conclusions

The recent Dodd-Frank Act in the U.S. provides for the elimination of ratings-based rules from financial regulatory. A growing volume of theoretical and empirical literature confirms that such a regulatory initiative is warranted given that, in an issuer-initiated market for ratings, the use of rating scores for regulatory purposes creates perverse incentives to issuers and rating agencies, with possible destabilising effects for financial markets. In this paper, we join those voices arguing for the detrimental implication of regulatory rules that promote the hardwiring of investment decisions to ratings, yet from a different perspective.

We consider a competitive market for a risky asset under conditions of asymmetric information. A non-strategic rating agency produces a public rating that is its best guess about asset payoffs, based on its private information. Asymmetrically informed traders rationally anticipate changes in the supply of the asset, which is due to a residual class of (noise) traders that mechanically link their supply of the asset to ratings. We show that ratings hardwiring leads to predictable supply shocks that induce informed traders to overreact to new information. That creates a channel through which fundamental and non-fundamental shocks are amplified, leading to less informative and more volatile prices.

The theoretical predictions of the model are consistent with empirical work regarding the price impact of forced sales as a result of the mechanical response of portfolio decisions to ratings. Ellul et al. (2011) argue that forced sales by insurance companies due to rating downgradings lead to price deviations from fundamentals for a significant period of time following the downgrading. They argue that such a persistent mispricing arises as a result of non-competitive markets and the lack of a sufficient number of counterparties to absorb large-scale sale volumes from insurance companies. In our model, such a mispricing could arise even in a competitive asset market due to signal extraction problems by trading parties that do not necessarily hardwire their investment decisions to ratings, but they rationally anticipate the hardwired supply of the asset by others.

The model could be extended to consider a market for a risky asset where informed traders act strategically à la Kyle (1985) and Foster and Viswanathan (1996). That would allow us to examine the effect of mechanical responses of investment decisions to ratings in relation to asset market characteristics, such as market depth, and draw implications for the order flow and bid-ask spreads.

Finally, the analysis has wider implications for the incorporation into investment decisions of mechanical responses to public signals more broadly, not necessarily ratings. In particular, hardwiring of certain investment decisions to ratings is related also to the effect of analyst recommendations on overall buying and selling, similar to the evidence provided by Malmendier and Shanthikumar (2007). However, analyst recommendations are in relation to the current price, which is an endogenous variable in the model. That would require modification of the model in order to associate noise trading with the endogenously specified price and not with the asset's rating that is exogenously specified. Yet, we leave these interesting extensions of the model for future research.

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7 Appendix

7.1 Proof of Lemma 1

Let $\mathbf{s}^r_{jt-1}$be the vector of signals that the rating agency receives up to $t-1$ about factor θ_j, $j = 1, 2$. Given normality of θ_{jt} and signal vector $\mathbf{s}^r_{jt-1}$, the conditional distribution of θ_{jt}, conditional on signal vector $\mathbf{s}^r_{jt-1}$, is also normal with conditional mean and variance

$$\overline{\theta}_{jt|t-1} \equiv E\left(\theta_{jt} \mid \mathbf{s}^r_{jt-1}\right) \tag{35}$$

$$\Sigma_{jt|t-1} \equiv Var\left(\theta_{jt} \mid \mathbf{s}^r_{jt-1}\right) \tag{36}$$

Let us suppose that the conditional mean $\overline{\theta}_{jt|t-1}$ and variance $\Sigma_{jt|t-1}$ have been calculated and with those in hand we are able to evaluate $\overline{\theta}_{jt+1|t}$ and $\Sigma_{jt+1|t}$. From 6, the conditional expectation of the signal received at t, conditional on the signal information up to $t-1$, is given by

$$E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right) = E\left(\theta_{jt} \mid \mathbf{s}^r_{jt-1}\right) = \overline{\theta}_{jt|t-1} \tag{37}$$

and the forecast error $s^r_{jt} - E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right)$ becomes

$$s^r_{jt} - E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right) = \left(\theta_{jt} - \overline{\theta}_{jt|t-1}\right) + e_{jt} \tag{38}$$

Since e_{jt} are independent over time and orthogonal to θ_{jt}, they are also independent of $\overline{\theta}_{jt|t-1}$. This implies that the conditional variance of the forecast error 38 is

$$Var\left[s^r_{jt} - E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right)\right] = \Sigma_{jt|t-1} + \sigma^2_{je} \tag{39}$$

where $\sigma^2_{je} \equiv Var\left[e_{jt}\right]$. Similarly, the conditional covariance between the forecast errors $s^r_{jt} - E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right)$ and $\theta_{jt} - E\left(\theta_{jt} \mid \mathbf{s}^r_{jt-1}\right)$ is

$$\begin{aligned} & Cov\left[s^r_{jt}, \theta_{jt} \mid \mathbf{s}^r_{jt-1}\right] \\ = \;& E\left[\left(\theta_{jt} - \overline{\theta}_{jt|t-1} + e_{jt}\right)\left(\theta_{jt} - \overline{\theta}_{jt|t-1}\right)\right] \\ = \;& \Sigma_{jt|t-1} \end{aligned} \tag{40}$$

From 35, 36, 37, 39 and 40 we get the conditional joint distribution of signal s^r_{jt} and fundamental factor θ_{jt}, conditional on signal information $\mathbf{s}^r_{jt-1}$ up to period $t-1$

$$\begin{bmatrix} s^r_{jt} \mid \mathbf{s}^r_{jt-1} \\ \theta_{jt} \mid \mathbf{s}^r_{jt-1} \end{bmatrix} \sim N\left(\begin{bmatrix} \overline{\theta}_{jt|t-1} \\ \overline{\theta}_{jt|t-1} \end{bmatrix}, \begin{bmatrix} \Sigma_{jt|t-1} + \sigma^2_{je} & \Sigma_{jt|t-1} \\ \Sigma_{jt|t-1} & \Sigma_{jt|t-1} \end{bmatrix}\right) \tag{41}$$

Let us now define $\overline{\theta}_{jt|t}$ as the conditional expectation of factor θ_{jt} conditional on signal vector $\mathbf{s}^r_{jt}$, namely, all signals s^r_j up to period t

$$\begin{aligned} \overline{\theta}_{jt|t} &\equiv E\left(\theta_{jt} \mid \mathbf{s}^r_{jt}\right) \\ &= E\left(\theta_{jt} \mid s^r_{jt} \middle| \mathbf{s}^r_{jt-1}\right) \end{aligned} \tag{42}$$

The conditional expectation $\overline{\theta}_{jt|t}$ and the conditional variance $\Sigma_{jt|t}$ of the forecast error can be evaluated by applying the projection theorem, using the join distribution in 41

$$\begin{aligned} \overline{\theta}_{jt|t} &= \overline{\theta}_{jt|t-1} + \Sigma_{jt|t-1}\left(\Sigma_{jt|t-1} + \sigma_{je}^2\right)^{-1}\left(s_{jt}^r - \overline{\theta}_{jt|t-1}\right) && (43) \\ \Sigma_{jt|t} &= \Sigma_{jt|t-1} - \Sigma_{jt|t-1}^2\left(\Sigma_{jt|t-1} + \sigma_{je}^2\right)^{-1} && (44) \end{aligned}$$

Moreover, from 2 and also the fact that v_{jt} are orthogonal to every element of the signal vector $\mathbf{s}_{jt}^r$, we get

$$\begin{aligned} \overline{\theta}_{jt+1|t} &\equiv E\left(\theta_{jt+1} \mid \mathbf{s}_{jt}^r\right) \\ &= E\left(\rho_j\theta_{jt} + v_{jt} \mid \mathbf{s}_{jt}^r\right) && (45) \\ &= \rho_j\overline{\theta}_{jt|t} \end{aligned}$$

$$\begin{aligned} \Sigma_{jt+1|t} &\equiv Var\left(\theta_{jt+1} \mid \mathbf{s}_{jt}^r\right) \\ &= Var\left(\rho_j\theta_{jt} + v_{jt} \mid \mathbf{s}_{jt}^r\right) && (46) \\ &= \rho_j^2\Sigma_{jt|t} + \sigma_{jv}^2 \end{aligned}$$

Combining 43 with 45, and 44 with 46 we derive the following Kalman filter representation that gives the one-period forecast $\overline{\theta}_{jt+1|t}$ as a function of $\overline{\theta}_{jt|t-1}$

$$\overline{\theta}_{jt+1|t} = \rho_j\overline{\theta}_{jt|t-1} + \rho_j\Sigma_{jt|t-1}\left(\Sigma_{jt|t-1} + \sigma_{je}^2\right)^{-1}\left(s_{jt}^r - \overline{\theta}_{jt|t-1}\right)$$

or

$$\overline{\theta}_{jt+1|t} = \rho_j\left[1 - \Sigma_{jt|t-1}\left(\Sigma_{jt|t-1} + \sigma_{je}^2\right)^{-1}\right]\overline{\theta}_{jt|t-1} + \rho_j\Sigma_{jt|t-1}\left(\Sigma_{jt|t-1} + \sigma_{je}^2\right)^{-1} s_{jt}^r \qquad (47)$$

where $\Sigma_{jt+1|t}$ solves

$$\Sigma_{jt+1|t} = \rho_j^2\Sigma_{jt|t-1} - \rho_j^2\Sigma_{jt|t-1}^2\left(\Sigma_{jt|t-1} + \sigma_{je}^2\right)^{-1} + \sigma_{jv}^2 \qquad (48)$$

Given that $\left|\rho_j\right| < 1$, $\sigma_{je}^2 > 0$ and $\sigma_{jv}^2 > 0$, the conditional variance $\Sigma_{jt|t-1}$ converges to a unique (positive) steady-state constant Σ^* that solves[17]

$$\Sigma_j^* = \rho_j^2\Sigma_j^*\left[1 - \Sigma_j^*\left(\Sigma_j^* + \sigma_{je}^2\right)^{-1}\right] + \sigma_{jv}^2 \qquad (49)$$

It is easy to show that the solution to 49 is

$$\Sigma_j^* = \frac{1}{2}\sigma_{je}^2\left[\frac{\sigma_{jv}^2}{\sigma_{je}^2} - \left(1-\rho_j^2\right) + \sqrt{\left[\frac{\sigma_{jv}^2}{\sigma_{je}^2} - \left(1-\rho_j^2\right)\right]^2 + 4\frac{\sigma_{jv}^2}{\sigma_{je}^2}}\right] \qquad (50)$$

[17]See, for example, Hamilton (1994), Proposition 13.1, page 390.

Independence between θ_1 and θ_2, s_1^r and s_2^r implies that the rating process r_t is given by

$$\begin{aligned} r_t &= E\left[\theta_{1,t}+\theta_{2,t} \mid s_{1,s}^r, s_{2,s}^r, s<t\right] \\ &= \overline{\theta}_{1,t|t-1}+\overline{\theta}_{2,t|t-1} \end{aligned}$$

or, from 47

$$r_t = \rho_1\left(\Sigma_1+1\right)^{-1}\overline{\theta}_{1,t-1|t-2}+\rho_2\left(\Sigma_2+1\right)^{-1}\overline{\theta}_{2,t-1|t-2}+\rho_1\Sigma_1\left(\Sigma_1+1\right)^{-1}s_{1t}^r+\rho_2\Sigma_2\left(\Sigma_2+1\right)^{-1}s_{2t}^r \tag{51}$$

For $\rho_1\left(\Sigma_1+1\right)^{-1}=\rho_2\left(\Sigma_2+1\right)^{-1}$, the rating process 51 simplifies as follows

$$r_t = \lambda r_{t-1}+\lambda\left[\Sigma_1 s_{1t-1}^r+\Sigma_2 s_{2t-1}^r\right] \tag{52}$$

where, $\lambda=\rho_j\left(\Sigma_j+1\right)^{-1}$ and $\Sigma_j=\frac{1}{2}\left[\frac{\sigma_{jv}^2}{\sigma_{je}^2}-\left(1-\rho_j^2\right)+\sqrt{\left[\frac{\sigma_{jv}^2}{\sigma_{je}^2}-\left(1-\rho_j^2\right)\right]^2+4\frac{\sigma_{jv}^2}{\sigma_{je}^2}}\right]$, $j=1,2$.

Q.E.D.

7.2 Fixed-point solution algorithm

Following Hussman (1992), we outline here the main steps we need to follow in order to calculate a linear REE equilibrium of our securities market. To derive such an equilibrium we need to evaluate matrices $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$ of the actual law of motion 28. We start by choosing arbitrary values for their first row, which corresponds to the price process, and for the conditional variances σ_j^2 and coefficient matrices $\mathbf{B}_j$, $j=1,2$. We also define selector matrices $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{u}_1$, $\mathbf{u}_2$ that satisfy the following set of equations

$$\begin{aligned} z_{1t} &= \mathbf{e}_1 z_t \qquad x_{1t}=\mathbf{u}_1 z_t \\ z_{2t} &= \mathbf{e}_2 z_t \qquad x_{2t}=\mathbf{u}_2 z_t \\ r_t &= \mathbf{e}_r z_t \qquad \varsigma_t=\mathbf{e}_s\varepsilon_t \end{aligned} \tag{53}$$

Let also matrix $\mathbf{c}$ be such that $p_{t+1}+D_{t+1}=\mathbf{c}x_{jt+1}$. Given 53, we can easily see that $E\left[p_{t+1}+D_{t+1}\mid I_{jt}\right]=\mathbf{cB}_j\mathbf{u}_j z_t$ and the equilibrium price 13 can be restated as

$$p_t=\Lambda^{-1}\left[\alpha\sigma_2^2N\phi_1\mathbf{cB}_1\mathbf{u}_1+\left(1-\alpha\right)\sigma_1^2N\phi_2\mathbf{cB}_2\mathbf{u}_2-M\mathbf{e}_r\right]z_t-\Lambda^{-1}\sigma_1^2\sigma_2^2\mathbf{e}_s\varepsilon_t \tag{54}$$

Substituting z_t from 28 into the price equation 54 we derive the following expression for the price process

$$p_t=\mathbf{d}_p z_{t-1}+\boldsymbol{\epsilon}_p\varepsilon_t$$

where row matrices $\mathbf{d}_p$ and $\boldsymbol{\epsilon}_p$ define the first row of $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$, respectively, and they are given by

$$\begin{aligned} \mathbf{d}_p &\equiv \Lambda^{-1}\left[\alpha\sigma_2^2N\phi_1\mathbf{cB}_1\mathbf{u}_1+\left(1-\alpha\right)\sigma_1^2N\phi_2\mathbf{cB}_2\mathbf{u}_2-M\mathbf{e}_r\right]T\left(\mathbf{B}\right) \\ \boldsymbol{\epsilon}_p &\equiv \Lambda^{-1}\left[\alpha\sigma_2^2N\phi_1\mathbf{cB}_1\mathbf{u}_1+\left(1-\alpha\right)\sigma_1^2N\phi_2\mathbf{cB}_2\mathbf{u}_2-M\mathbf{e}_r\right]V\left(\mathbf{B}\right)-\Lambda^{-1}\sigma_1^2\sigma_2^2\mathbf{e}_s \end{aligned}$$

The second row of $T(\mathbf{B})$ and $V(\mathbf{B})$, which corresponds to the payoff process D_t, is implied by 1, while the third row, which corresponds to the rating process, is implied by lemma 1. The fourth and fifth row of $T(\mathbf{B})$ and $V(\mathbf{B})$, which correspond to investors' private signals s_t^j are implied by 4, and the sixth and seventh row by 2. Row eight of $V(\mathbf{B})$ corresponds to supply of the risky asset and is set equal to

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

With respect to investors' forecast errors ζ_{jt} we define selector matrices $\mathbf{e}_{zj}$ such that

$$\zeta_{jt} = \mathbf{e}_{zj} z_t$$

From the actual law of motion 28, from investors' perceptions 7 and from selector matrices $\mathbf{e}_{zj}$ and $\mathbf{e}_j$, the forecast errors ζ_{jt} can be written as

$$\begin{aligned} \zeta_{1t} &= \left[\mathbf{e}_1 T(\mathbf{B}) - \mathbf{A}_1 \mathbf{e}_1 - \mathbf{C}_1 \mathbf{e}_{z1}\right] z_{t-1} + \mathbf{e}_1 V(\mathbf{B}) \varepsilon_t \\ \zeta_{2t} &= \left[\mathbf{e}_2 T(\mathbf{B}) - \mathbf{A}_2 \mathbf{e}_2 - \mathbf{C}_2 \mathbf{e}_{z2}\right] z_{t-1} + \mathbf{e}_2 V(\mathbf{B}) \varepsilon_t \end{aligned} \tag{55}$$

Equations in 55 define the following matrices $\mathbf{d}_\zeta$ and $\mathbf{e}_\zeta$

$$\mathbf{d}_\zeta \equiv \begin{bmatrix} \mathbf{e}_1 T(\mathbf{B}) - \mathbf{A}_1 \mathbf{e}_1 - \mathbf{C}_1 \mathbf{e}_{z1} \\ \mathbf{e}_2 T(\mathbf{B}) - \mathbf{A}_2 \mathbf{e}_2 - \mathbf{C}_2 \mathbf{e}_{z2} \end{bmatrix}$$
$$\mathbf{e}_\zeta \equiv \begin{bmatrix} \mathbf{e}_1 V(\mathbf{B}) \\ \mathbf{e}_2 V(\mathbf{B}) \end{bmatrix}$$

Matrix $\mathbf{d}_\zeta$ defines rows 9 to 16 of $T(\mathbf{B})$, while matrix $\mathbf{e}_\zeta$ defines rows 9 to 16 of $V(\mathbf{B})$. It is worth noting that in equations 55 selector matrices $\mathbf{e}_1$ and $\mathbf{e}_2$ select elements only from the first five rows of matrices $T(\mathbf{B})$ and $V(\mathbf{B})$. However, the rows of matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ that are relevant to ζ_{1t} are rows 9 to 12, while for ζ_{2t} rows 13 to 16. Consequently, $\mathbf{e}_1$ and $\mathbf{e}_2$ do not select any of the coefficients of matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ that are relevant to the evaluation of forecast errors ζ_{1t} and ζ_{2t}. Thus, there is no need to evaluate a fixed point for the rows of $T(\mathbf{B})$ and $V(\mathbf{B})$ that correspond to investors' forecast errors.

Figure 1: Price (non-)informativeness and risk aversion

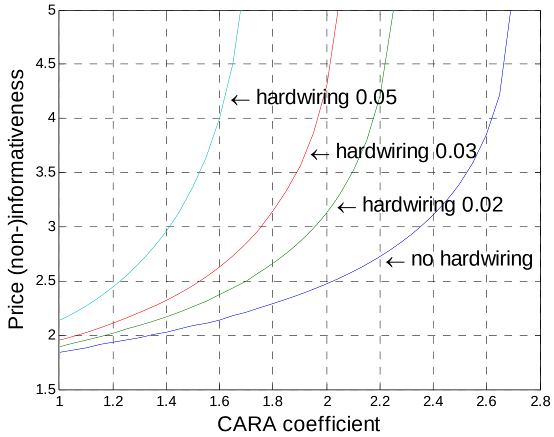

Figure 2: Price volatility and risk aversion

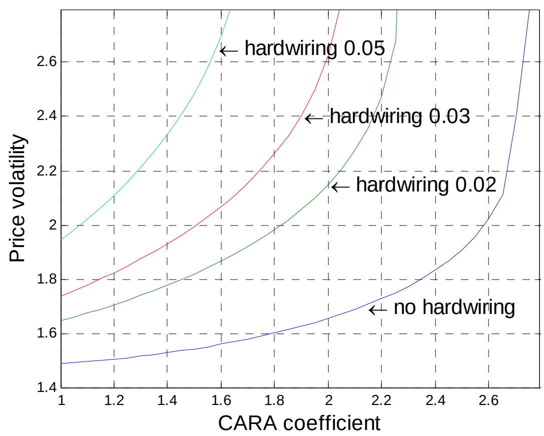

Figure 3: Price (non-)informativeness and ratings hardwiring

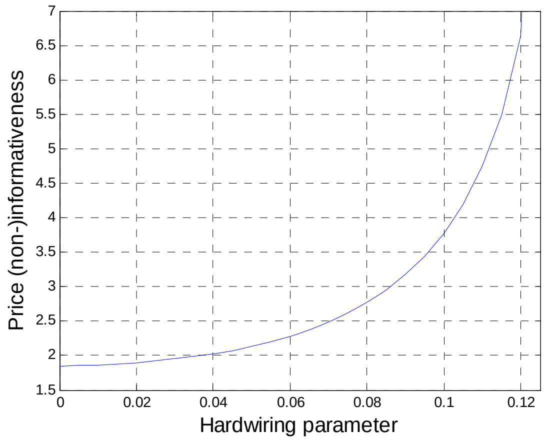

Figure 4: Price volatility and ratings hardwiring

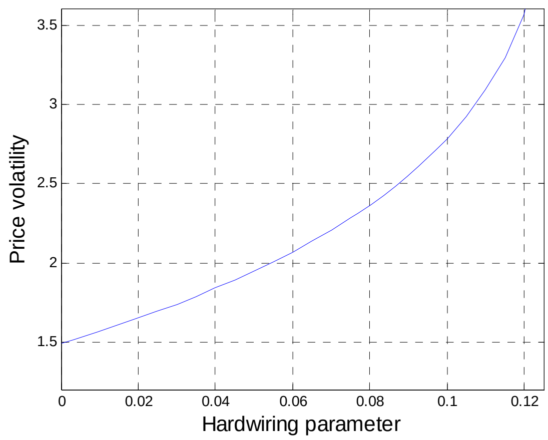

Figure 5: Price impact of shock to fundamental factor θ_1

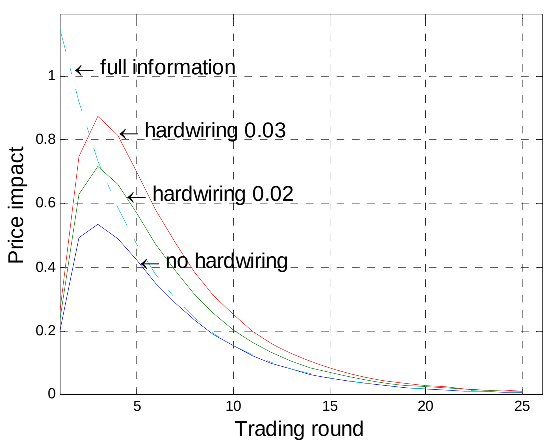

Figure 6: Price impact of shock to fundamental factor θ_2

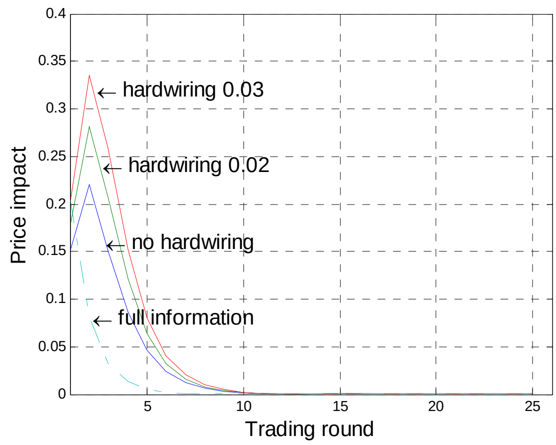

Figure 7: Price impact of trader signal error about θ_1

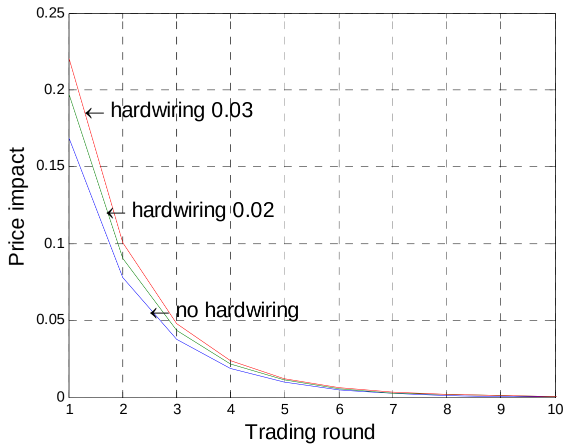

Figure 8: Price impact of trader signal error about θ_2

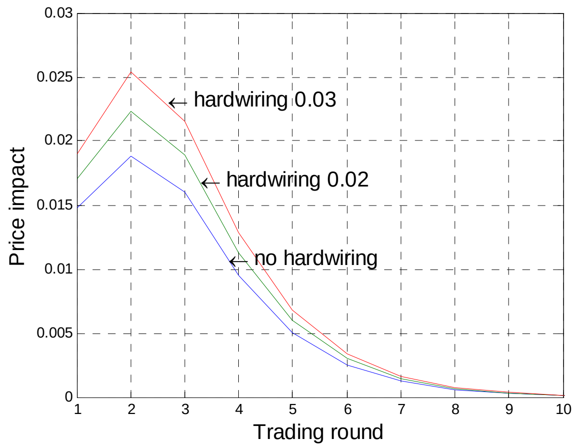

Figure 9: Price impact of rating agency signal error about θ_1

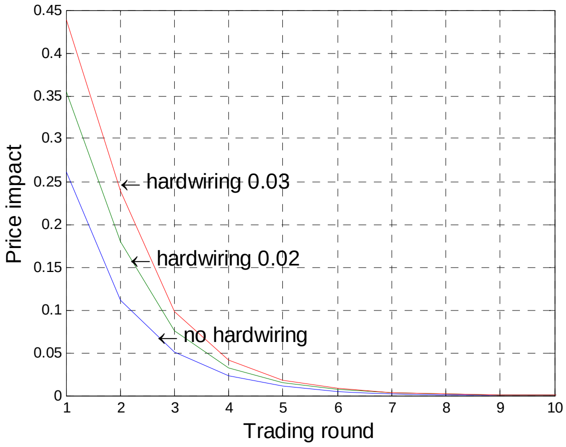

Figure 10: Price impact of rating agency signal error about θ_2

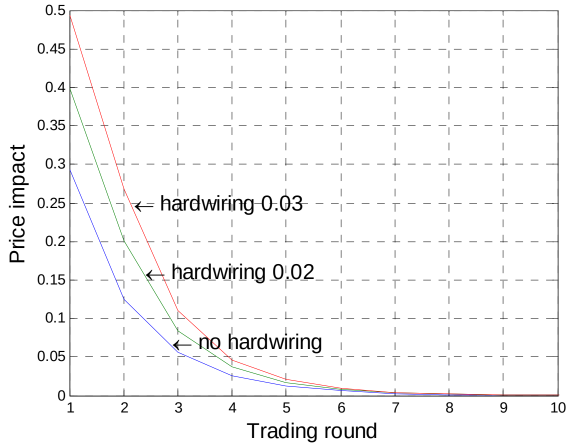

Figure 11: Price impact of random supply shock

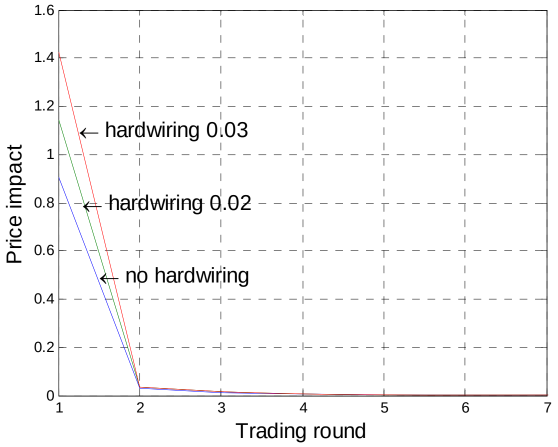

Figure 12: Price impact of non-fundamental dividend shock