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Stochastic dominance and bank liquidity risk: Evidence from the Lehman crisis*

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Abstract

We propose a new framework for analysing bank liquidity risk using stochastic dominance methods. We consider liquidity-risk diversification across asset and liability classes of U.S. commercial banks and provide evidence of banks' heterogeneous response to the Lehman crisis, depending on size. Small banks relied extensively on securities to meet liquidity needs and undo the ensuing tightening in funding conditions. Large banks appeared unconstrained by loan illiquidity and able to scale down loan exposures more aggressively than smaller banks. Non-transaction deposits played a key role in boosting liquidity of larger banks, but transaction deposits supported small banks' funding after the crisis.

Keywords: banking, liquidity risk, stochastic dominance efficiency

JEL classification codes: C14, C44, G11, G18, G21

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1 Introduction

The global financial crisis demonstrated that even solvent banks may fail when liquidity risk crystallizes. This is when banks are unable to meet their obligations on time. Following the Lehman default in September 15, 2008 and the ensuing liquidity crisis that affected the global financial system, the Basel Committee on Banking Supervision produced a new set of rules (Basel III) requiring, among other things, international banks to hold liquid assets as a buffer in case of crisis.

Under Basel III, banks will need to hold liquidity buffers of highly-rated, easy-to-sell assets against a potential 30-day market crisis similar to that caused by the Lehman collapse. At the time of writing this paper, such liquidity buffers where not supposed to include equities, securitizations and other types of assets. This is despite market participants and commentators arguing for the benefits of liquidity-risk diversification that could be achieved through an expanded pool of eligible assets.

In this paper we use a new approach to gauge bank liquidity risk and offer some guidance regarding the construction of regulatory liquidity buffers for banks. Using stochastic dominance (SD) methods and Call Report data for U.S. commercial banks, we analyse liquidity shocks to bank assets and liabilities during the global financial crisis and especially in the period following the Lehman default. Then we try to shed some light on the issue of liquidity-risk diversification and how various types of assets (and liabilities) could work together to minimize bank liquidity risk in stressed conditions.

Bank liquidity risk stems from the special structure of the banking firm's balance-sheet. Banks transform short-term liabilities (deposits) into long-term assets (loans), which makes them susceptible to a bank-run problem à la Diamond and Dybvig. Incentives to run-on-the-bank may stem from asymmetric information and the opaqueness of bank assets, especially during periods of crisis (Flannery et al. 2004, 2010). A perceived deterioration in asset quality could precipitate foreclosures of interbank credit lines, withdrawals of borrowed funds and bank failure (Rochet and Vives 2004).

Notwithstanding the stability of a bank's sources of funding, bank liquidity risk also depends on the composition of bank assets. Banks may hoard marketable securities that can easily liquidate to protect their loan books when they are hit by a funding shock. Kashyap and Stein (2000) show that securities holdings allow banks to undo liquidity shocks that result from contractionary monetary policy, to protect their loan books. Therefore, monetary policy shocks mostly affect the lending behaviour of smaller banks with illiquid asset portfolios.

In addition to funding shocks and the composition of assets, a bank's liquidity position may be compromised by contingent credit exposures, such as loan commitments and backstop liquidity facilities to off-balance sheet vehicles. Loan commitments are off-balance sheet contractual arrangements to extend a loan upon customer request, typically within a period of one year and in return for certain fees. In the run up to the global financial crisis, U.S. commercial banks experienced robust growth in such commitments. That was partly due to a search for fee income in a period of low risk premia, as well as the proliferation of off-balance sheet conduits and other vehicles used by sponsoring banks to raise wholesale funding in money markets, such as asset-backed commercial paper (ABCP). Berger and Bouwman (2009) find evidence that almost half of credit extension by all U.S. commercial banks between 1993-2003 takes place through loan commitments.

Prior to the crisis, banks used to extend back-up lines of credit to off-balance sheet entities to support ABCP issuance. That was an important source of short-term funding for banks, backed by customer loans that were shifted off-balance sheet. But since the onset of the crisis, money markets siezed-up and demand for ABCP from investors disappeared as loan impairments started to rise and some banks failed to provide liquidity support to off-balance sheet vehicles. As a result, sponsoring banks were forced to extend liquidity lifelines to off-balance sheet entities and, in addition, to take on-balance sheet the backlog of customer loans that were originated for the purpose of transfer off-balance sheet. That led to involuntary credit extension by banks at a time they would rather prefer scaling down their loan portfolios to preserve liquidity and boost their capital ratios.

Recent work on bank liquidity risk has focused primarily on deposit-loan synergies, using panel data regressions. Gatev et al. (2009) focus on the positive correlation that arises between loan demand and bank funding inflows in periods of market stress. They show that transaction deposits allow banks to hedge against liquidity risk that arises from borrowers drawing down of unused credit lines. Gatev and Strahan (2006) and Gatev et al. (2006) find evidence of a flight of investor funds into (insured) transaction deposits when market liquidity falls. Such a flight to safety tends to coincide with an increase in drawdown of loan commitments by borrowers, leading to a natural hedge of bank liquidity risk. Gozzi and Goetz (2010) examine the effect of negative funding-liquidity shocks on business lending by U.S. commercial banks below the 90th percentile of asset distribution, during the 2007-2009 crisis. They find evidence that banks with higher reliance on (short-term) wholesale funding, as a proportion of liabilities, cut down on lending more aggressively than banks with higher reliance on retail deposits.

In this paper we propose a new framework for analysing bank liquidity risk using stochastic dominance methods. We consider the balance-sheet of a bank as a portfolio of various assets and liabilities and we recognise possible diversification benefits of liquidity risk among them. Liquidity demand (supply) by various asset (liability) classes is approximated by the quarterly growth in the stock of asset and liability classes. A key quarter in our analysis is 2008Q3, which includes September 15, 2008, the date Lehman filed for Chapter 11 bankruptcy protection.

SD methods are nonparametric and consider the full spectrum of non-linear models that affect the balance-sheet structure. SD is also immune to endogeneity and multicolinearity problems in financial variables inherent to the balance-sheet constraint, where total assets must be equal to total liabilities by construction. In fact, our analysis builds on the balance-sheet constraint to generate appropriate benchmarks for optimal asset and liability portfolios. Last but not least, SD is immune to inherent non-stationarity of financial ratios, such as the asset and liability weights considered here (Ioannides and Peel 2003).

To the best of our knowledge, this is the first paper that applies SD methods in the banking literature. Stochastic dominance is a central theme in a wide variety of applications in economics and finance. Stochastic orderings are binary relations defined on classes of probability distributions. They translate mathematically intuitive ideas like “being larger” or “being more variable” for random quantities. SD is more general than mean-variance analysis by considering the full distribution, and not only its mean and variance. In an similar application to optimal portfolio construction in finance, Scaillet and Topaloglou (2010) use SD efficiency tests that can compare a given portfolio with an optimal diversified portfolio constructed from a set of assets. In this paper we follow a similar methodology using a set of attributes to construct the optimal portfolios of asset and liability classes.

We derive *optimal* portfolios of asset or liability classes that minimize a bank's liquidity risk, in a first-order stochastic dominance sense. The optimal portfolio obtained is the one that maximizes a bank's available liquidity, while it achieves the minimum variability among its set of competitors. Optimal weights are calculated relative to a relevant peer-group *benchmark* distribution. It turns out that such a benchmark for calculating optimal asset (liability) weights depends on the peer-group distribution of growth in equity and total liabilities (assets), adjusted for last period's leverage. In other words, the SD approach maximizes the distributional distance between the growth in equity and total liabilities (assets) and any possible portfolio of asset (liability) classes, taking into account last period's leverage. Similarly, the SD approach maximizing the distributional distance between the growth in total assets minus the growth in equity and any possible portfolio of liability classes, taking into acount last period's leverage.

From an analytical perspective, such peer group distributions can either be the preferred theoretical, or the empirical ones, or a combination of both. In our analysis, we consider the empirical distributions where a bank's peer-group is defined in terms of size, by total assets.

Optimal weights are calculated under the constraint that loans and non-transaction deposits take the highest weight among all asset and liability classes, which is consistent with the maturity-transformation role of banks. A low (high) optimal asset weight indicates high (low) liquidity intensity of the respective asset class. Similarly, a high (low) optimal liability weight captures an increased (reduced) importance of the respective liability class in supporting bank funding.

We provide evidence of liquidity-risk diversification benefits among all types of bank assets, including loans, let alone stocks and other securities. Even in the hypothetical situation where banks would be totally unconstrained by their maturity-transformation role and able to hold only securities in their books, they would still hold some loans, Fed funds and other assets in anticipation of a Lehman type shock. Especially large banks would assign to customer loans the largest asset weight (46%) among other types of assets.

That indicates there is scope for improvement in the current Basel proposals (Basel III) that consider only top quality securities in the definition of *liquidity coverage ratio*. Moreover, the results reveal some stark differences in how large and smaller banks responded to the Lehman crisis. That suggests some discrimination of Basel III liquidity standards may be warranted in terms of bank size, offering larger banks more flexibility in defining the composition of their liquidity buffers.

On the asset side, small banks relied extensively on securities holdings to meet their liquidity needs at the onset of the crisis in 2007, which is consistent with Kashyap and Stein (2000). Larger banks also appear to free-up liquidity in 2007 by cutting significantly on Fed funds sold and reverse repo. They also relied on securities holdings to meet their liquidity demand during the Lehman crisis in 2008.

Having drawn down on securities holdings in 2008, larger banks have subsequently replenished them in 2009 and 2010. Such an increase in securities holdings coincides with significant loan deleveraging. During the deleveraging process in 2009-2010, larger banks appeared relatively unconstrained by loan illiquidity and were able to scale down loan exposures more aggressively than smaller banks. Consistent with Loutskina (2010), that could be a result of large banks having better access to financial technology and state backstop facilities that allow them to liquefy their loan books more easily than smaller banks.

On the liability side, non-transaction deposits played a key role in boosting funding liquidity of large and medium-sized banks during the Lehman crisis, and remained the primary source of bank funding in the post-crisis period. In contrast, transaction deposits seem to support the funding position of small banks only after the crisis. Especially in 2010, transaction deposits emerged as an important source of funding for small and medium-sized banks.

The remainder of the paper is organized as follows. In section 2 we discuss the data and changes in the balance-sheet structure of U.S. commercial banks following market developments and policy response to the Lehman crisis. Section 3 defines the relevant benchmarks for constructing optimal asset and liability portfolios and section 3 elaborate on the mechanics of SD analysis. Section 4 present the empirical results and section 5 concludes. Proofs and detailed mathematical programming formulations are gathered in an appendix.

2 Data description

The analysis draws on a sample of individual bank data from the Reports of Condition and Income (Call Reports), for the period 2000Q1-2010Q4. For every quarter, we categorize banks in six size categories: banks of size 1 are below the 25th percentile of the distribution of total assets at the given quarter, size 2 between the 25th and 50th percentile, size 3 between the 50th and 75th, size 4 between the 75th and 90th, size 5 between the 90th and 98th and size 6 above the 98th percentile.

On the asset side, bank-specific variables include security holdings, customer loans, Fed funds sold and reverse repos, and other tangible assets.[1] On the liability side, we consider transaction deposits, non-transaction deposits, Fed funds purchased and repos, other lia-

[1] We focus on tangible assets because intangible assets, such as goodwill, do not have obvious implications for funding-liquidity risk, therefore excluded from our analysis.

bilities, and tangible equity.[2] We also consider off-balance sheet exposures in the form of unused loan commitments. The original sample consists of 369,221 quarterly bank observations. By dropping observations with missing values in any bank-specific variable, the sample is reduced to 354,190 quarterly observations from 10,512 banks.

We calculate growth rates in variables using log differences and we drop outlier observations. Outlier growth rates are defined here as observations below the 0.1th percentile, or above the 99.9th percentile of the sample distribution, at a given quarter. In addition, for variables that often report with zero value – for example, Fed funds and repos – growth rate is defined as zero if both the current and lagged level is zero. Otherwise, growth rate is reported missing if only of the current or lagged level is zero. By calculating growth rates and dropping outlier and missing values, the sample is reduced to 251,390 quarterly observations.

The sample is targeted to include only banks with a significant exposure to unused loan commitments. This is in order to examine liquidity shocks that arise not only on-balance-sheet, but also from off-balance-sheet exposures to such commitments. Therefore, for every quarter during the sample period, we consider only banks with a ratio of commitments to loans that is above the 75th percentile of the sample distribution. As a result, the sample generating process leads to a final sample that includes 62,558 quarterly observations from 5,741 banks. The maximum number of banks appears in 2001Q2 (1,698) and the minimum number in 2008Q3 (923).

A first look at the data shows that despite deterioration in credit conditions around the Lehman crisis, year-on-year (y-o-y) growth in customer lending remained strong at around 15% in 2008Q3, before falling steeply in 2009. Exercising of unused commitment may have contributed to such an unexpected growth in customer loans, as indicated by the commensurate decline in commitment growth (Figure 1).[3] Such a high growth in possibly involuntary

[2]Tangible equity is defined here as total assets minus total liabilities minus intangible assets.

[3]In an environment of heightened uncertainty about credit losses and severe stress in funding markets, is hard to imagine banks voluntarily expanding their loan book that aggressively. Therefore, a significant proportion of customer lending in the period following the Lehman collapse was possibly involuntarily due to exercising of loan commitments.

lending may have exerted substantial liquidity pressure on bank balance-sheets during the Lehman crisis. Yet, the steep decline in loan commitment growth could also be due to foreclosure of commitments by banks, either unconditionally, or due to breaches of covenants in commitment contracts.[4]

[Figures 1-6]

At the same time, growth rate in transaction deposits accelerated supporting funding-liquidity position of U.S. commercial banks (Figure 2). Such a natural liquidity hedge is consistent with empirical evidence by Gatev et al. (2006). A similar pattern arises with non-transaction deposits during the global financial crisis: when commitment growth started to fall, growth in non-transaction deposits remained strong above 10% y-o-y (Figure 3).

Other sources of bank funding-liquidity were withdrawn in the aftermath of the Lehman collapse, notably Federal funds purchased and repos (figure 4). This is despite the fact that Fed funds & repo funding grew very strongly between 2007Q2 and 2008Q2, mainly as a result of Fed interventions and large liquidity injections in money markets. But Fed funds & repo funding seemed to be withdrawn in 2008Q3 following the Lehman collapse, possibly due to concerns about counterparty risk in the interbank market. The other liabilities category provided significant liquidity support to banks during the Lehman crisis, when they grew strongly, but fell sharply since 2009Q1, when is thought to be the end of the Lehman crisis (Figure 5). Although we were not able to identify from the data the exact composition of our other-liabilities category in order to explain their behaviour during the crisis, we suspect they include government guaranteed paper issued by banks in the context of the Troubled Asset Relief Program (TARP).

Finally, the data show growth in loans and securities holdings moving in a countercyclical

[4]Data limitations do not allow us to identify the proportion of commitments that were foreclosed by banks due to breaches of covenants and those that were actually called down by customers.

fashion (Figure 6). Following a steep decline in securities growth around the Lehman collapse, loans and securities show a high degree of substitutability.

As a result of the global financial crisis and the Lehman default, in particular, was the balance-sheet structure of the U.S. commercial banks to change significantly between 2006 and 2010. Table I shows that during that period small banks reduced significantly their customer loans as a proportion of total tangible assets, as well as Fed funds sold and reverse repos.

[Table I]

But security asset holdings and other tangible assets increased during the same period. On the liability side, transaction deposits increased for all size categories, while non-transaction deposits increased significantly for all but the small banks. Fed funds purchased and repos also fell across all size categories, while other liabilities increased during the pick of the crisis in 2008 and subsided afterwards. Although some of these trends in the composition of assets and liabilities have subsequently reversed, a natural question that arises is whether it could be an *optimal* portfolio of asset or liability classes such that following a Lehman-type shock the funding-liquidity risk of a bank would be minimal.

2.1 TARP and balance-sheet structure

In analysing changes in bank balance-sheets during the Lehman crisis requires us also to recognize the response of the U.S. authorities to the crisis and how that response might have subsequently affected the balance-sheet structure of U.S. banks.

Given the categorization of assets and liabilities in our selected groups, official sector support during the crisis may have affected asset and liability classes to a lesser or greater extend, depending on the nature of the support and how it affected sentiment in funding markets. For example, implementation of the FDIC's Transaction Account Guarantee

Program in November 2008 introduced full deposit insurance in certain noninterest-bearing transaction deposits, in addition to the \$250,000 limit under the FDIC's general rules. That may have led to strong growth in transaction deposits observed in the data. Also, the FDIC's Debt Guarantee Program for certain newly-issued wholesale funding may have led to strong growth in other liabilities in 2008. On the asset side, liquidity support though collateral swaps may have affected other assets, which increased significantly since mid-2007 and especially following the Lehman default.

Since the beginning of the global financial crisis in August 2007, the Federal Reserve has added reserves in the banking system through cash-for-bonds operations. That included the Term Auction Facility (TAF), offering term funding to depository institutions, and term repos with the primary dealers. According to the Bank of England's Financial Stability Report, until June 2009 official sector support to financial institutions was approximately US\$10.4 trillion, or 73% of GDP. Although not all the official sector support lines where called upon by banks, such a policy response inevitably affected the balance-sheet structure of U.S. banks during the crisis, over and above any changes inflicted by purely market forces. Therefore, in our analysis we consider policy response as an integral part of funding conditions U.S. banks have faced during the crisis. Given such a policy response and banks' ex-ante choice of balance-sheet structure, we examine which combination of assets and liabilities worked best for banks in mitigating the ex-post liquidity shock due to the Lehman collapse.

3 Basic framework and portfolio benchmark

We denote by A_t, L_t and E_t the bank's total assets, liabilities and equity, respectively, at any period t. We denote by lev_t the leverage of the bank at time t, which is defined as the ratio of total assets to equity (i.e. $lev_t = \frac{A_t}{E_t}$). Let also g_t^A, g_t^L and g_t^E be the growth rate in total assets, liabilities and equity between periods $t-1$ and t.

Assets, liabilities and equity are all subject to random shocks. Such shocks may stem from *involuntary* asset growth (e.g. drawdown of loan commitments), unanticipated foreclosures of funding lines, write-offs and other factors. Therefore, we may think of A_t, L_t and E_t as random variables that should satisfy the budget constraint at all times

$$\widetilde{A}_t = \widetilde{L}_t + \widetilde{E}_t \tag{1}$$

If a bank were not able to intervene and *manufacture* a balancing of its balance sheet then, constraint 1 would only be satisfied with probability 0. Thus, the following assumptions

> Assumption 1: A bank is able to engage in *voluntary* asset growth if it has spare liquidity to employ.[5]
>
> Assumption 2: A bank is able to *refuse* to take on more funding in order to avoid creating spare liquidity.[6]

We express the balance sheet constraint 1 in terms of variable growth rates and last period's balance sheet

$$\left(1+\widetilde{g}_t^A\right) A_{t-1} = \left(1+\widetilde{g}_t^L\right) L_{t-1} + \left(1+\widetilde{g}_t^E\right) E_{t-1} \tag{2}$$

Dividing both sides by E_{t-1} and substituting L_{t-1} with $A_{t-1} - E_{t-1}$, equation 2 becomes

$$\left(1+\widetilde{g}_t^A\right) lev_{t-1} = \left(1+\widetilde{g}_t^L\right) \left(lev_{t-1}-1\right) + \left(1+\widetilde{g}_t^E\right) \tag{3}$$

[5]Voluntary asset growth could involve extending new loans to customers, lending in the Fed funds market to other banks, or purchasing securities and other assets.

[6]A bank could avoid spare liquidity by applying certain charges on deposits. For example, Bank of New York Mellon said in a letter to clients that as of August 8, 2011 it would start charging a 13 basis points annual fee, plus other charges, on deposit accounts of more than $50 million (FT August 4, 2011). As explained in the letter, such a move was necessary for preserving the "quality and strength" of its balance sheet on the wake of a "sudden and significant" flight of customer funds into the safe haven of bank deposits following tensions between the U.S. Congress and the White House in reaching an agreement over the government debt ceiling. Interest rates may also go negative in other liability classes, such as repo, following extraordinary inflow of funds in such markets due to flight to safety.

or

$$\left(\widetilde{g}_t^A - \widetilde{g}_t^L\right) lev_{t-1} = \widetilde{g}_t^E - \widetilde{g}_t^L \tag{4}$$

Consistently with our dataset, we consider four asset classes: customer loans $\widetilde{A}_t^{cl}$, securities $\widetilde{A}_t^{s}$, Fed fund sold and reverse repo $\widetilde{A}_t^{ff}$ and a residual category of other assets $\widetilde{A}_t^{oth}$. The balance sheet weights of asset classes (expressed as proportions of total assets) are $w_{A,t}^{cl}$, $w_{A,t}^{s}$, $w_{A,t}^{ff}$ and $w_{A,t}^{oth}$, respectively. Let also $\widetilde{g}_{A,t}^{cl}$, $\widetilde{g}_{A,t}^{s}$, $\widetilde{g}_{A,t}^{ff}$ and $\widetilde{g}_{A,t}^{oth}$ be the growth rates in customer loans, securities, Fed funds sold and reverse repo, and other assets between period $t-1$ and t. We may express the growth rate in total assets as the weighted sum of the growth rates in asset classes

$$\widetilde{g}_t^A = \sum_{i=cl,s,ff,oth} w_{A,t}^i \widetilde{g}_{A,t}^i \tag{5}$$

We also consider four liability classes: transaction deposits $\widetilde{L}_t^{td}$, non-transaction deposits $\widetilde{L}_t^{nd}$, Federal funds purchased and repo $\widetilde{L}_t^{ff}$ and a residual category of other liabilities $\widetilde{L}_t^{oth}$. The balance sheet weights of liability classes (as proportions of total liabilities) are $w_{L,t}^{td}$, $w_{L,t}^{nd}$, $w_{L,t}^{ff}$ and $w_{L,t}^{oth}$, respectively. Let also $\widetilde{g}_{L,t}^{td}$, $\widetilde{g}_{L,t}^{nd}$, $\widetilde{g}_{L,t}^{ff}$ and $\widetilde{g}_{L,t}^{oth}$ be the growth rates in transaction deposits, non-transaction deposits, Fed funds purchased and repo, and other liabilities, respectively, between period $t-1$ and t. As a result, the growth rate in total liabilities can be expressed as the weighted sum of growth rates in liability classes

$$\widetilde{g}_t^L = \sum_{i=td,nd,ff,oth} w_{L,t}^i \widetilde{g}_{L,t}^i \tag{6}$$

Next, we define our asset and liability bechmarks in order to derive optimal weights for asset and liability classes using stochastic dominance techniques.

3.1 Asset benchmark

In order to define an appropriate benchmark for asset growth we recast equation 4 as follows

$$\widetilde{g}_t^A = \frac{1}{lev_{t-1}}\widetilde{g}_t^E + \left(1 - \frac{1}{lev_{t-1}}\right)\widetilde{g}_t^L \tag{7}$$

Given that a bank can always engage in voluntary asset growth to employ any spare liquidity (assumption 1), equation 7 implies our first principle of prudent liquidity risk management of bank assets.

> Principle 1: In the presence of *involuntary* asset growth, the weighted sum of growth in equity and growth in total liabilities must stochastically dominate – in a first order stochastic dominance ($FOSD$) sense – the growth rate in total assets. That is
>
> $$\frac{1}{lev_{t-1}}\widetilde{g}_t^E + \left(1 - \frac{1}{lev_{t-1}}\right)\widetilde{g}_t^L \underset{FOSD}{\succ} \widetilde{g}_t^A \tag{8}$$

According to principle 1, minus weighted sum of growth rate in equity and total liabilities is defined as an appropriate benchmark for calculating optimal portfolio weights for bank assets. In other words, given the sample distribution of growth rate in asset class, equity and total liabilities, we search for optimal portfolio weights $w_{A,t}^{cl}$, $w_{A,t}^{s}$, $w_{A,t}^{ff}$ and $w_{A,t}^{oth}$ so as the following condition 9 be satisfied

$$\sum_{i=cl,s,ff,oth} w_{A,t}^i \left(-\widetilde{g}_{A,t}^i\right) \underset{FOSD}{\succ} -\frac{1}{lev_{t-1}}\widetilde{g}_t^E - \left(1 - \frac{1}{lev_{t-1}}\right)\widetilde{g}_t^L \tag{9}$$

3.2 Liability benchmark

We now turn to define an appropriate benchmark for liability growth. We start by recasting equation 4 as follows

$$\widetilde{g}_t^L = \frac{lev_{t-1}}{(lev_{t-1} - 1)}\widetilde{g}_t^A - \frac{1}{(lev_{t-1} - 1)}\widetilde{g}_t^E \tag{10}$$

Given that a bank can always refuse to take on more funding if that would lead to spare liquidity (assumption 2), relation 10 implies our second principle of prudent liquidity risk management of bank liabilities.

> Principle 2: In the presence of unanticipated *foreclosures* of funding lines, the growth rate in total liabilities must stochastically dominate – in a first order stochastic dominance ($FOSD$) sense – the weighted difference between the growth rate in total assets and the growth in equity. That is
>
> $$\widetilde{g}_t^L \underset{FOSD}{\succ} \frac{lev_{t-1}}{(lev_{t-1}-1)}\widetilde{g}_t^A - \frac{1}{(lev_{t-1}-1)}\widetilde{g}_t^E \tag{11}$$

Principle 2 implies that an appropriate benchmark for bank liabilities is the growth rate in total assets minus the growth rate in equity, adjusted for last period's leverage. In other words, given the sample distribution of growth rate in liability classes, equity and total assets, we search for optimal portfolio weights $w_{L,t}^{td}$, $w_{L,t}^{nd}$, $w_{L,t}^{ff}$ and $w_{L,t}^{oth}$, so that the following condition 12 be satisfied

$$\sum_{i=td,nd,ff,oth} w_{L,t}^{i}\widetilde{g}_{L,t}^{i} \underset{FOSD}{\succ} \frac{lev_{t-1}}{(lev_{t-1}-1)}\widetilde{g}_t^A - \frac{1}{(lev_{t-1}-1)}\widetilde{g}_t^E \tag{12}$$

3.3 Portfolio constraints

In order to compare calculated optimal weights with the actual asset and liability weights observed in the data, we apply certain (natural) constraints to the model.

Commercial banks are mainly in the business of taking deposits and giving out loans to customers. Therefore, we could assume that customer loans take *a priori* the highest weight among all other asset classes. Similarly, we could assume that deposits take the highest portfolio weight among all other liability classes. Given that transaction deposits are for transaction purposes and pay low, or no interest, we could further specify that non-transaction (time) deposits should take the highest weight among bank liabilities. Thus the following assumptions

> Assumption 3: A bank's optimal weights for customer loans $w^{cl}_{A,t}$ satisfies the following constraint at any time t: $w^{cl}_{A,t} \geq w^{j}_{A,t}$, where j is either securities, Fed funds sold and reverse repo, or other assets.
>
> Assumption 4: A bank's optimal weights for non-transaction deposits $w^{nd}_{A,t}$ satisfies the following constraint at any time t: $w^{nd}_{L,t} \geq w^{j}_{L,t}$, where j is either transaction deposits, Fed funds purchased and repo, or other liabilities.

Next we elaborate on the mechanics of SD analysis for calculating optimal asset and liability weights, given the constraints of assumptions 3 and 4.

4 SD Efficiency

We consider a strictly stationary process $\{Y_s;\, s \in Z\}$ taking values in R^n. The observations consist of a realization of $\{Y_s;\, s = 1, ..., S\}$. These data correspond to observed log-returns of various asset and liability classes. We denote by $F(y)$, the continuous cdf of $Y = (Y_1, ..., Y_n)'$ at point $y = (y_1, ..., y_n)'$.

Let us consider a portfolio of asset or liability classes $\mathbf{w} \in L$ where $L := \{\mathbf{w} \in R^n_+ : \mathbf{1}'\mathbf{w} = 1\}$ with $\mathbf{1}$ being a vector of ones. This means that all the different classes have positive weights and that these weights sum up to one. Let us denote by $G(z, \mathbf{w}; F)$ the cdf of the portfolio return $\mathbf{w}'Y$ at point z given by $G(z, \mathbf{w}; F) := \int_{\mathbb{R}^n} \mathbb{I}\{\mathbf{w}'u \leq z\} dF(u)$.

SD is a term which refers to a set of relations that may hold between distributions. SD efficiency is a direct extension of SD to the case where full diversification is allowed.

The distribution of the portfolio $\mathbf{w}$ dominates the distribution of the benchmark τ stochastically at first-order (SD1) if, for any argument z, $G(z, \tau; F) \geq G(z, \mathbf{w}; F)$. If z denotes a log-return, then this inequality means that the proportion of banks in distribution

$\mathbf{w}$ with log-returns smaller than z is not larger than the proportion of such banks in τ. If the portfolio $\mathbf{w}$ dominates the benchmark τ at first order, then log-returns in τ are always lower that $\mathbf{w}$, so that $\mathbf{w}$ is preferable.

We maximize the following objective function:

$$\underset{z,\mathbf{w}}{\texttt{Max}}[G(z, \boldsymbol{\tau}; F) - G(z, \mathbf{w}; F)] \tag{13}$$

The above maximization results in the optimal $\mathbf{w}$ constructed from the set of asset or liability classes in the sense that it reaches the greatest log-return for a given probability, implying that more banks have return above a given argument z.

It is worth mentioning that SD is considerably more general than mean-variance analysis which only looks at the first two moments of the two distributions under comparison. The latter only looks into a dominant relation with a higher mean and lower variance, whereas the former considers all possible moments of the asset or liability classes. Only in the case where we compare two normal distributions SD reduces to mean-variance analysis. However, the assumption of normality for each different class is difficult to support empirically. In contrast, SD analysis takes into account the whole distribution, not only the mean and the variance. SD is attractive because it is effectively non-parametric as no explicit specification of probability distribution functional form is required. In addition, the entire probability density function is taken into account rather than a finite number of moments so it can be considered less restrictive and more robust.

The SD1 criterion corresponds to all types of utility functions as long as they are non-decreasing. SD1 only relies on the fact that people are rational in the sense that they prefer more return rather than less (also known as the monotonicity axiom).

When there is no portfolio $\mathbf{w}$ that dominates the benchmark τ at first-order, we move to the SD2 criterion. The objective function that we use is the following:

$$\underset{z,\mathbf{w}}{\texttt{Max}} \int_{-\infty}^{z} G(u,\tau;F)du - \int_{-\infty}^{z} G(u,\mathbf{w};F)du \tag{14}$$

This maximization results in the optimal portfolio $\mathbf{w}$ constructed from the set of asset (liability) classes in the sense that it also gives the greatest return for a given probability.

We can further define for $z \in R$:

$$\begin{aligned}
\mathcal{J}_1(z,\mathbf{w};F) &:= G(z,\mathbf{w};F),\\
\mathcal{J}_2(z,\mathbf{w};F) &:= \int_{-\infty}^{z} G(u,\mathbf{w};F)du = \int_{-\infty}^{z} \mathcal{J}_1(u,\mathbf{w};F)du,\\
\mathcal{J}_3(z,\mathbf{w};F) &:= \int_{-\infty}^{z}\int_{a}^{u} G(v,\mathbf{w};F)dvdu = \int_{-\infty}^{z} \mathcal{J}_2(u,\mathbf{w};F)du,
\end{aligned} \tag{15}$$

and so on.

From Davidson and Duclos (2000) Equation (2), we know that

$$\mathcal{J}_j(z,\mathbf{w};F) = \int_{-\infty}^{z} \frac{1}{(j-1)!}(z-u)^{j-1} dG(u,\mathbf{w},F), \tag{16}$$

which can be rewritten as

$$\mathcal{J}_j(z,\mathbf{w};F) = \int_{\mathbb{R}^n} \frac{1}{(j-1)!}(z-\mathbf{w}'\boldsymbol{u})^{j-1}\mathbb{I}\{\mathbf{w}'\boldsymbol{u} \le z\} dF(\boldsymbol{u}). \tag{17}$$

In particular we obtain SD1 and SD2 when $j=1$ and $j=2$, respectively.

5 Mathematical formulation for SD1

The full formulation of the problem is given below:

$$\underset{z,\mathbf{w}}{\texttt{Max}} F = \sqrt{S}\frac{1}{S}\sum_{s=1}^{S}(L_s - Q_s) \tag{18a}$$

$$\text{s.t.} M(L_s - 1) \leq z - \boldsymbol{\tau}'\boldsymbol{Y}_s \leq ML_s, \qquad \forall s \tag{18b}$$

$$M(Q_s - 1) \leq z - \mathbf{w}'\boldsymbol{Y}_s \leq MQ_s, \qquad \forall s \tag{18c}$$

$$\mathbf{1}'\mathbf{w} = 1, \tag{18d}$$

$$\mathbf{w} \geq 0, \tag{18e}$$

$$Q_s \in \{0,1\},\ L_s \in \{0,1\}, \qquad \forall s \tag{18f}$$

with M being a large constant.

The model is a mixed integer program maximizing the distance between the sum over all scenarios of two binary variables, $\frac{1}{S}\sum_{s=1}^{S} L_s$ and $\frac{1}{S}\sum_{s=1}^{S} Q_s$ which represent $G(z,\boldsymbol{\tau};\hat{F})$ and $G(z,\mathbf{w};\hat{F})$, respectively (the empirical cdf of $\boldsymbol{\tau}$ and $\mathbf{w}$ at point z). According to inequalities (18b), L_s equals 1 for each scenario s for which $z \geq \boldsymbol{\tau}'\boldsymbol{Y}_s$, and 0 otherwise. Analogously, inequalities (18c) ensure that Q_s equals 1 for each scenario for which $z \geq \mathbf{w}'\boldsymbol{Y}_s$. Equation (18d) defines the sum of all weights to be unity, while inequality (18e) disallows negative weights. This formulation allows for a test of the dominance of the $\boldsymbol{\tau}$ over any potential linear combination $\mathbf{w}$ of the asset or liability classes.

When some of the variables are binary, corresponding to mixed integer programming, the problem becomes NP-complete (non-polynomial, i.e. formally intractable). We can see that there is a set of at most S values, say $\mathcal{R} = \{r_1, r_2, ..., r_S\}$, containing the optimal value of the variable z. A direct consequence is that we can solve the original problem by solving the smaller problems $P(r)$, $r \in \mathcal{R}$, in which z is fixed to r. Then we take the value for z that yields the best total result. The advantage is that the optimal values of the L_s variables are known in $P(r)$.

The reduced form of the problem is as follows[7]

$$\begin{aligned}
&\min \sum_{s=1}^{S} Q_s \\
&\text{s.t.} \mathbf{w}'\boldsymbol{Y}_s \geq r - (r - M_s)Q_s, \quad \forall s \\
&\quad \mathbf{1}'\mathbf{w} = 1, \\
&\quad \mathbf{w} \geq 0, \\
&\quad Q_s \in \{0, 1\}, \quad \forall s
\end{aligned} \tag{19a}$$

A similar procedure is used in the formulation of the test statistic for SD2. The exact formulation for testing for SD2 is presented in appendix E2.

6 Empirical results

In this section we discuss calculated optimal weights for assets and liabilities. Based on the comparison of optimal weights with the actual ones observed in the data, we examine liquidity tensions on bank balance-sheets and how U.S. commercial banks responded to the Lehman default in September 2008. In order to avoid seasonality problems and facilitate weight comparison across time, we focus on a single quarter for each calendar year, namely quarter three (Q3). Focusing on the period: 2000-2010, we split it in four subperiods: the pre-crisis period (2000Q3 to 2006Q3), the first quarter of the global financial crisis (2007Q3), the quarter of Lehman default(2008Q3), and the two post-Lehman quarters (2009Q3 and 2010Q3).

We are interested in the comparison of optimal weights across banks by size, with size 1 be the smallest and size 6 the largest, using the asset and liability split that we discussed in

[7]See Appendix for the derivation of this formulation and details on practical implementation.

section 2. We call *large banks* those of size 6, *medium-sized banks* those of sizes 4 and 5, and *small banks* of sizes 1 to 3. Optimal weights are calculated on the basis of quarterly growth rates in the stock of asset and liability classes and are shown for each quarter in Table II.

[Table II]

All optimal weights are calculated under the constrained model that requires loans and non-transaction deposits to take the highest weight among all asset and liability classes. This is consistent with the basic bank business-model of taking on deposits and giving out loans. Especially for 2008Q3, we also compute unconstrained optimal weights by imposing no restrictions on asset and liability weights. Optimal weights under the unconstrained model are shown in Table III. That allows us to better understand bank liquidity pressures during the Lehman crisis and which banks were better placed to deal with a Lehman-type liquidity shock.

[Table III]

6.1 Optimal asset weights

Calculated optimal asset weights are depicted with dots in Figures 7 to 12, by bank size. For each asset class, optimal weights are shown with the same color as the cross-sectional average of actual weights. That is, blue for loans, green for securities, orange for Fed funds sold and reverse repo, and purple for other assets. A fall (increase) in optimal weights indicates high (low) liquidity intensity of the respective asset classes, taking into account correlations and higher order moments across asset classes.

[Figures 7-12]

Since the onset of the global financial crisis in 2007, optimal loan weights have increased significantly as a result of loan deleveraging. But such an increase was temporarily reversed in 2008Q3, especially for large and medium-sized banks. Lower optimal loan weights in 2008Q3 are indicative of involuntary lending, possibly as a result of commitment draw-downs. Also the large increase in optimal loan weights in 2009Q3 and 2010Q3 indicates substantial deleveraging in loan books of U.S. commercial banks, relative to other asset classes.

In the post-Lehman period, optimal loan weights for large banks are higher that medium-sized banks, which are also significantly higher than for small banks. In 2009Q3, optimal loan weights for large banks (87%) largely overshoot the observed cross-sectional average, they are higher than the optimal loan weights for medium-sized banks (below 83%) and significantly higher than for small banks (below 73%). In 2010Q3, optimal loan weight for large banks (81%) are higher than for medium-sized (below 75%) and small banks (below 69%). Similarly, under the unconstrained model for 2008Q3, the optimal loan weight for large banks (46%) is higher than for medium-sized (below 30%) and small banks (below 18%).

The results indicate that large and medium-sized banks scaled down their loan books more aggressively to release liquidity than smaller banks in 2009-2010, similar to the previous downturn in 2001. That may be due to large banks having better access to financial technology, such as outright loan sales and securitization, which allow them to liquefy their loan books more easily than smaller banks in view of adverse shocks. Moreover, large banks may get access to state backstop facilities to deal with liquidity shocks, such as the Term Asset-Backed Securities Loan Facility (TALF) and the Capital Purchase Program (CPP) of TARP that were introduced in the aftermath of Lehman default.

Optimal weights for 2008Q3 also indicate that large and medium-sized banks also drew-down significantly on their security holdings during the Lehman crisis. In 2008Q3, securities have the second largest optimal weight (after loans) of around 21%-26% for small banks, 24%-33% for medium-sized banks and 22% for large banks. This is evidence that, during the

Lehman crisis, especially small and medium-sized banks drew-down significantly on their securities holding to meet their liquidity needs. In addition, actual securities holdings by small banks during the Lehman crisis were actually between 21%-26% of their tangible total assets (Table I), very close to their optimal weight.[8] Moreover, under the unconstrained model in 2008Q3, securities generally attract the higher optimal weight for small banks (below 60%), followed by medium-sized banks (below 58%) and large banks (31%). This coincides with a deteriorating conditions in funding markets, especially for transaction deposits, as we discuss in the next section.

Small banks also drew-down significantly on their security holdings to meet their liquidity needs at the onset of the crisis in 2007. Compared also to large and medium-sized banks, small banks relied more on securities to release liquidity at the onset of the crisis in 2007 to undo the ensuing tightening in funding conditions and to protect their loan books.

These results are consistent with Kashyap and Stein (2000) that show securities play an important role in allowing small banks to *undo* contractionary monetary shocks – and the ensuing tightening in funding conditions – to protect their loan books. Having drawn down on security holdings in 2008Q3, banks have subsequently replenished their security holdings in 2009 and 2010. This is captured by the significant fall in optimal weights for securities in 2009 and 2010. Especially for medium-sized banks, optimal securities weights fell, from 24%-33% in 2008Q3, to 2% in 2009Q3 and 3% 2010Q3. Similarly, the optimal securities weights for large banks fell, from 22% in 2008Q3, to 5% in 2009Q3 and 3% in 2010Q3.

Large and medium-sized banks free-up liquidity by reducing significantly Fed funds and reverse repo exposures. This is also reflected in the increase of Fed funds rate during in 2008Q3, that declined subsequently following TARP and aggressive monetary easing by the Fed.

Finally, we do not find any significant impact of the Legacy Securities Program on the

[8] In 2008Q3, actual securities holdings were below 20% of tangible total assets for medium-sized banks and 15% for large banks, which are significantly lower than their optimal levels.

optimal weights of securities, or other assets.[9] This is especially the case for the large U.S. banks, which would possibly gain the most from liquidating legacy assets. Moreover, high optimal weights for securities in 2008Q3 indicate that banks have unloaded a significant part of their securities portfolio before Legacy Securities Program was put in place.

6.2 Optimal liability weights

Optimal liability weights are shown in Figures 13-18 with dots of same color as the actual (average) weights of the corresponding liability class. That is, blue for non-transaction deposits, green for transaction deposits, orange for Fed funds purchased and repo, and purple for other liabilities. Optimal liability weights for the constrained models are depicted with square dots and for the unconstrained model (for 2008Q3) with diamond dots. Table II shows the actual values of optimal liability weights.

[Figures 13-18]

Non-transaction deposits played a very significant role in boosting funding liquidity of large and medium-sized banks in 2008Q3. This is captured by the high optimal weights of 69% on average for large, 76% for medium-sized and 58% for small banks.[10] Non-transaction deposits remained the primary source of funding liquidity in 2009, with optimal weight at around 86% for large banks, 78% for medium-sized and 67% for small banks, which is even higher that their pre-crisis levels in 2005 and 2006. That compares with optimal weights between 50% and 60% at the onset of the global financial crisis in 2007.

Transaction deposits played a less significant role in boosting bank liquidity in 2008Q3, although they received a higher optimal weight compared to previous years. This is captured by the low optimal weight of 14% for large banks, less than 5% for medium-sized, and between

[9] The Legacy Securities Program was a special TARP facility that was created in 2009Q2.

[10] Under the unconstrained model, non-transaction deposits still attract a high optimal weight in 2008Q3. That is 60% for large banks and 57%-64% for medium-sized banks (Table III).

5%-11% for small banks.[11] For large banks, transaction deposit also played a minor role in 2009 and 2010 with optimal weights of around 3%. But in 2010 transaction deposits emerged as a relatively important source of funding for small and medium-sized banks, with average optimal weights of 33% for small banks (size 2) and up to 26% for medium-sized banks (size 4).

Fed funds purchased and repo offered limited support to funding liquidity, although their importance increased at the onset of the crisis in 2007. For large banks, the optimal weight for Fed funds purchased and repo peaked in 2006Q3 at 23%, and for medium-sized banks in 2007Q3, at 16%. That is from approximately 14%, on average, in 2005. But, especially for large banks, it fell below 5% in 2008 and 2009, before peaking up again in 2010 at around 17%. Other liabilities had consistently received the second highest optimal weight in 2007Q3 and 2008Q3, after non-transaction deposits. But they have reduced in funding importance since 2009.

6.3 Capital implications of optimal weights

In this section we consider capital implications of optimal asset and liability weights for 2008Q3. An adjustment in actual weights towards their optimal level would possibly require a commensurate adjustment in capital. Therefore we would be interested to gauge the potential capital intensity (henceforth, *hedging cost*) of changing the asset and liability mix towards a stochastically dominating portfolio.

For each bank size, we estimate the cross-sectional regressions of growth in equity on the product of actual asset and liability weights at the beginning of 2008Q3 with the respective

[11] Transaction deposits also receive a very low optimal weight in 2008Q3 under the unconstrained model (Table III): 13% for large banks, less than 4% for medium-sized, and between 3%-12% for small banks.

growth rates in asset and liability classes during that quarter.

$$\begin{aligned} g_x^E &= b_0 + b_1 x_A^{cl} g_A^{cl} + b_2 x_A^s g_A^s + b_3 x_A^{ff} g_A^{ff} + b_4 x_A^{oth} g_A^{oth} + \\ & \quad b_5 x_L^{td} g_L^{td} + b_6 x_L^{nd} g_L^{nd} + b_7 x_L^{ff} g_L^{ff} + b_8 x_L^{oth} g_L^{oth} + \varepsilon \end{aligned} \tag{20}$$

Table IV shows the estimated regression coefficients, by bank size.

[Table IV]

With the estimated coefficients in hand, we replace the actual asset and liability weights x_A and x_L in (20) with their calculated optimal weights w_A and w_L.

$$\begin{aligned} g_w^E &= b_0 + b_1 w_A^{cl} g_A^{cl} + b_2 w_A^s g_A^s + b_3 w_A^{ff} g_A^{ff} + b_4 w_A^{oth} g_A^{oth} + \\ & \quad b_5 w_L^{td} g_L^{td} + b_6 w_L^{nd} g_L^{nd} + b_7 w_L^{ff} g_L^{ff} + b_8 w_L^{oth} g_L^{oth} \end{aligned} \tag{21}$$

Then we compare the cross-sectional distributions of estimated growth rates in equity under the actual and optimal weights ($\widetilde{g}_x^E$ and $\widetilde{g}_w^E$) according to the SD1 criterion. As described in section 4, the objective function that we maximise is as follows:

$$\underset{z}{\texttt{Max}}[G(z, g_w^E) - G(z, g_x^E)] \tag{22}$$

We find that growth rate in equity under the actual weights $\widetilde{g}_x^E$ stochastically dominates that under the optimal weights $\widetilde{g}_w^E$, for all bank sizes. This result indicates that implementing such optimal asset and liability weights would result in lower demand for equity in the aftermath of the Lehman default, compared to actual equity growth. This is important given the US$700 billion equity injection by the U.S. Treasury to financial institutions under the TARP Capital Purchase Program. Our optimal liquidity rule would not only reduce funding-liquidity risk of U.S. commercial banks, but could also result in lower capital injections by the official sector.

7 Conclusions

We used a novel approach to gauge bank liquidity risk and offer some guidance about regulatory liquidity buffers. Using stochastic dominance methods, we analysed liquidity shocks to bank assets and liabilities during the global financial crisis and especially around Lehman default. We shed some light on the issue of liquidity-risk diversification across asset and liability classes and provided evidence of banks' heterogeneous response to liquidity shocks, depending on size, in line with Kashyap and Stein (2000).

The results revealed some stark differences in the way large and smaller banks responded to the Lehman crisis. That suggests some discrimination of Basel III liquidity standards may be warranted in terms of bank size, offering larger banks more flexibility in defining the composition of their liquidity buffers. Since the onset of the global financial crisis, small banks relied extensively on securities holdings to meet their liquidity needs and undo the ensuing tightening in funding conditions. During the deleveraging process that followed, large banks appeared relatively unconstrained by loan illiquidity and were able to scale down loan exposures more aggressively than smaller banks. Non-transaction deposits played a key role in boosting funding liquidity of larger banks during the Lehman crisis. In contrast, transaction deposits supported the funding position of small banks only after the crisis.

Going forward, the analysis could be extended to consider a more granular categorisation of asset and liability classes and the construction of a bank illiquidity index based on the difference between actual and optimal weights. We could also focus on a key group of U.S. commercial banks, such as the largest and most systemically important institutions, or those that received liquidity support and equity injections from the official sector under TARP. Finally, we could consider alternative asset and liability benchmarks for constructing optimal portfolios, based on market-based measures of implied probability densities, as opposed to sample distributions we used in this paper.

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APPENDIX

A Mathematical formulation of SD1 Efficiency

The initial formulation for first order stochastic dominance efficiency is shown in model (18a).

We reformulate the problem in order to reduce the solving time and to obtain a tractable formulation. The steps are the following:

1) The factor $\sqrt{S}/S$ can be left out in the objective function, since S is fixed.

2) We can see that there is a set of at most S values, say $\mathcal{R} = \{r_1, r_2, ..., r_S\}$, containing the optimal value of the variable z.

Proof: Vectors $\boldsymbol{\tau}$ and $\boldsymbol{Y}_s$, $s = 1, ..., S$ being given, we can rank the values of $\boldsymbol{\tau}'\boldsymbol{Y}_s$, $s = 1, ..., S$, by increasing order. Let us call $r_1, ..., r_S$ the possible different values of $\boldsymbol{\tau}'\boldsymbol{Y}_s$, with $r_1 < r_2 < ... < r_S$ (actually there may be less than S different values). Now, for any z such that $r_i \leq z \leq r_i + 1$, $\sum_{s=1,...,S} L_s$ is constant (i.e. is equal to the number of s such that $\boldsymbol{\tau}'\boldsymbol{Y}_s \leq r_i$). Further, when $r_i \leq z \leq r_i + 1$, the maximum value of $-\sum_{t=1,...,S} Q_t$ is reached for $z = r_i$. Hence, we can restrict z to belong to the set $\mathcal{R}$.

3) A direct consequence is that we can solve the original problem by solving the smaller problems $P(r)$, $r \in \mathcal{R}$, in which z is fixed to r. Then we take take the value for z that yields the best total result. The advantage is that the optimal values of the L_t variables are known in $P(r)$. Precisely, $\sum_{s=1,...,S} L_s$ is equal to the number of s such that $\boldsymbol{\tau}'\boldsymbol{Y}_s \leq r$.

Hence problem $P(r)$ boils down to:

$$\min \sum_{s=1}^{S} Q_s$$
$$\text{s.t.} M(Q_s - 1) \leq r - \mathbf{w}'\boldsymbol{Y}_s \leq MQ_s, \quad \forall s$$
$$\mathbf{1}'\mathbf{w} = 1,$$
$$\mathbf{w} \geq 0,$$
$$Q_s \in \{0, 1\}, \quad \forall s \tag{23a}$$

Note that this becomes a minimization problem. Problem $P(r)$ amounts to find the largest set of constraints $\mathbf{w}'\boldsymbol{Y}_s \geq r$ consistent with $\mathbf{1}'\mathbf{w} = 1$ and $\mathbf{w} \geq 0$. Let $M_s = \min$ $\boldsymbol{Y}_{s,i}, i = 1, ..., n$, i.e. the smallest entry of vector $\boldsymbol{Y}_s$. Clearly, for all $\mathbf{w} \geq 0$ such that $\mathbf{1}'\mathbf{w} = 1$, we have that $\mathbf{w}'\boldsymbol{Y}_s \geq M_s$. Hence, Problem $P(r)$ can be rewritten in an even better reduced form:

$$\min \sum_{s=1}^{S} Q_s$$
$$\text{s.t.} \mathbf{w}'\boldsymbol{Y}_s \geq r - (r - M_s)Q_s, \quad \forall s$$
$$\mathbf{1}'\mathbf{w} = 1,$$
$$\mathbf{w} \geq 0,$$
$$Q_s \in \{0, 1\}, \quad \forall s \tag{23b}$$

We further simplify $P(r)$ by fixing the following variables:

- for all s such that $r \leq M_s$, the optimal value of Q_s is equal to 0 since the half space defined by the s-th inequality contains the simplex.

- for all t such that $r \geq M_s$, the optimal value of Q_s is equal to 1 since the half space defined by the s-th inequality has an empty intersection with the simplex.

The computational time for this mixed integer programming formulation is significantly reduced. For the optimal solution (which involves at most 235 mixed integer optimization programs, one for each discrete value of z) it takes less than five hours. The problems are optimized with IBM's OSL solver on an Sony Vaio computer with a 4*2.8 GHz Power, 4Gb of RAM. We note the almost exponential increase in solution time with the increasing number of observations. We stress here the computational burden that is managed for these tests. The optimization problems are modelled using the General Algebraic Modeling System (GAMS). The GAMS is a high-level modeling system for mathematical programming and optimization. It consists of a language compiler and a stable of integrated high-performance solvers. GAMS is tailored for complex, large scale modeling applications. The OSL solver uses the branch and bound technique to solve the MIP program.[12]

[12] For a typical run, the computational time for this formulation of the problem is less than six hours.

TABLE I – BALANCE SHEETS OF U.S. COMMERCIAL BANKS, BY SIZE

PANEL I: BALANCE SHEET COMPOSITION, AS OF 2006 Q4	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	139	224	312	285	259	94
Mean tang. assets (2009 $millions)	38	93	205	474	1758	77600
Median tang. assets (2009 $millions)	40	92	197	451	1404	14500
Proportion of tang. assets in the sample	0.1%	0.3%	0.8%	1.7%	5.7%	91.5%
Proportion of tangible assets						
Securities	23%	20%	19%	20%	19%	20%
Loans to customers	58%	64%	67%	68%	69%	63%
Fed funds sold & reverse repo	9%	7%	5%	3%	3%	3%
Other tangible assets	10%	10%	8%	9%	9%	14%
Total Deposits	82%	84%	84%	81%	78%	67%
Transaction deposits	25%	26%	22%	13%	10%	7%
Non-transaction deposits	57%	58%	62%	68%	68%	60%
Fed funds purchased & repo	1%	2%	2%	4%	6%	8%
Other liabilities	2%	3%	4%	6%	7%	16%
Tangible equity	14%	11%	10%	9%	9%	8%
Unused loan commitments	28%	21%	24%	25%	30%	55%

PANEL II: BALANCE SHEET COMPOSITION, AS OF 2008 Q4	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	173	204	237	229	204	83
Mean tang. assets (2008 $millions)	42	97	214	496	1692	105000
Median tang. assets (2008 $millions)	42	95	214	482	1291	14600
Proportion of tang. assets in the sample	0.1%	0.2%	0.5%	1.2%	3.7%	94.2%
Proportion of tangible assets						
Securities	26%	23%	21%	20%	17%	15%
Loans to customers	55%	62%	67%	68%	72%	65%
Fed funds sold & reverse repo	5%	4%	2%	2%	2%	2%
Other tangible assets	14%	11%	10%	10%	10%	18%
Total Deposits	83%	83%	82%	79%	76%	64%
Transaction deposits	28%	26%	20%	11%	9%	7%
Non-transaction deposits	55%	57%	63%	68%	68%	57%
Fed funds purchased & repo	1%	1%	2%	3%	5%	7%
Other liabilities	3%	4%	6%	8%	10%	20%
Tangible equity	13%	12%	10%	9%	9%	8%
Unused loan commitments	18%	18%	18%	20%	25%	59%

PANEL III: BALANCE SHEET COMPOSITION, AS OF 2010 Q4	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	191	211	287	258	217	74
Mean tang. assets (2010 $millions)	44	109	232	526	1880	110000
Median tang. assets (2010 $millions)	44	106	228	513	1470	15200
Proportion of tang. assets in the sample	0.1%	0.3%	0.8%	1.5%	4.6%	92.7%
Proportion of tangible assets						
Securities	26%	24%	22%	22%	22%	21%
Loans to customers	53%	60%	62%	64%	63%	59%
Fed funds sold & reverse repo	5%	3%	2%	1%	1%	2%
Other tangible assets	15%	14%	14%	13%	13%	18%
Total Deposits	84%	84%	84%	82%	81%	71%
Transaction deposits	29%	28%	22%	14%	10%	9%
Non-transaction deposits	55%	56%	62%	68%	71%	62%
Fed funds purchased & repo	1%	1%	2%	3%	4%	6%
Other liabilities	3%	4%	4%	6%	6%	13%
Tangible equity	12%	11%	10%	10%	9%	9%
Unused loan commitments	15%	15%	17%	18%	18%	33%

TABLE II – OPTIMAL BALANCE SHEETS ACCORDING TO FIRST ORDER STOCHASTIC DOMINANCE CRITERION FOR LIQUIDITY RISK OF U.S. COMMERCIAL BANKS, BY SIZE

PANEL I: OPTIMAL BALANCE SHEET COMPOSITION, FOR 2005 Q3	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	98	149	218	207	226	90
Mean tang. assets (2005 $millions)	36	84	180	418	1465	64700
Median tang. assets (2005 $millions)	39	83	171	409	1198	13100
Proportion of tang. assets in the sample	0.1%	0.2%	0.6%	1.4%	5.3%	92.5%
Proportion of tangible assets						
Securities	33%	25%	35%	29%	31%	39%
Loans to customers	39%	49%	49%	54%	52%	53%
Fed funds sold & reverse repo	17%	23%	12%	10%	7%	2%
Other tangible assets	11%	2%	4%	7%	10%	6%
Total Deposits	68%	69%	64%	73%	85%	83%
Transaction deposits	15%	15%	5%	3%	2%	1%
Non-transaction deposits	53%	54%	60%	70%	83%	82%
Fed funds purchased & repo	5%	3%	7%	11%	14%	14%
Other liabilities	27%	28%	28%	16%	1%	2%

PANEL II: OPTIMAL BALANCE SHEET COMPOSITION, FOR 2006 Q3	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	96	161	209	235	225	89
Mean tang. assets (2006 $millions)	37	84	191	441	1655	73000
Median tang. assets (2006 $millions)	39	83	181	426	1295	14100
Proportion of tang. assets in the sample	0.0%	0.2%	0.6%	1.5%	5.3%	92.4%
Proportion of tangible assets						
Securities	21%	18%	17%	6%	18%	17%
Loans to customers	35%	49%	51%	53%	45%	48%
Fed funds sold & reverse repo	15%	11%	3%	10%	8%	3%
Other tangible assets	29%	23%	28%	30%	29%	32%
Total Deposits	55%	68%	66%	74%	82%	73%
Transaction deposits	4%	9%	3%	2%	1%	1%
Non-transaction deposits	51%	59%	63%	72%	81%	73%
Fed funds purchased & repo	4%	4%	5%	8%	15%	23%
Other liabilities	41%	28%	30%	18%	3%	4%

PANEL III: OPTIMAL BALANCE SHEET COMPOSITION, FOR 2007 Q3	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	123	189	215	198	213	87
Mean tang. assets (2007 $millions)	37	89	194	466	1687	84500
Median tang. assets (2007 $millions)	40	89	187	452	1366	12800
Proportion of tang. assets in the sample	0.1%	0.2%	0.5%	1.2%	4.6%	93.5%
Proportion of tangible assets						
Securities	31%	23%	10%	5%	5%	7%
Loans to customers	41%	50%	59%	58%	57%	60%
Fed funds sold & reverse repo	20%	23%	21%	20%	22%	21%
Other tangible assets	8%	4%	9%	17%	17%	11%
Total Deposits	58%	66%	66%	59%	61%	50%
Transaction deposits	6%	4%	4%	1%	0%	1%
Non-transaction deposits	52%	62%	62%	59%	60%	49%
Fed funds purchased & repo	14%	4%	6%	16%	16%	15%
Other liabilities	28%	31%	29%	24%	23%	33%

TABLE II *(CONTINUED)* – OPTIMAL BALANCE SHEETS ACCORDING TO FIRST ORDER STOCHASTIC DOMINANCE CRITERION FOR LIQUIDITY RISK OF U.S. COMMERCIAL BANKS, BY SIZE

PANEL IV: OPTIMAL BALANCE SHEET COMPOSITION, FOR 2008 Q3	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	131	183	178	164	189	78
Mean tang. assets (2008 $millions)	38	98	213	501	1744	104000
Median tang. assets (2008 $millions)	40	99	216	489	1436	15800
Proportion of tang. assets in the sample	0.1%	0.2%	0.4%	1.0%	3.8%	94.5%
Proportion of tangible assets						
Securities	25%	21%	26%	33%	24%	22%
Loans to customers	39%	50%	50%	51%	53%	46%
Fed funds sold & reverse repo	21%	17%	10%	4%	7%	10%
Other tangible assets	15%	12%	14%	12%	16%	22%
Total Deposits	63%	72%	65%	80%	75%	82%
Transaction deposits	11%	10%	5%	5%	2%	14%
Non-transaction deposits	52%	62%	60%	75%	73%	68%
Fed funds purchased & repo	18%	9%	11%	11%	5%	4%
Other liabilities	19%	19%	24%	9%	20%	14%

PANEL V: OPTIMAL BALANCE SHEET COMPOSITION, FOR 2009 Q3	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	138	156	216	182	177	78
Mean tang. assets (2009 $millions)	42	104	223	511	1783	98600
Median tang. assets (2009 $millions)	42	103	216	503	1428	12200
Proportion of tang. assets in the sample	0.1%	0.2%	0.6%	1.1%	3.9%	94.1%
Proportion of tangible assets						
Securities	16%	15%	2%	2%	2%	5%
Loans to customers	44%	55%	73%	71%	83%	87%
Fed funds sold & reverse repo	23%	19%	21%	17%	10%	6%
Other tangible assets	17%	11%	3%	11%	5%	2%
Total Deposits	59%	88%	80%	79%	86%	87%
Transaction deposits	6%	10%	10%	4%	3%	3%
Non-transaction deposits	52%	78%	70%	75%	83%	84%
Fed funds purchased & repo	19%	4%	11%	14%	11%	5%
Other liabilities	23%	8%	10%	7%	3%	8%

PANEL VI: OPTIMAL BALANCE SHEET COMPOSITION, FOR 2010 Q3	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	166	184	230	233	205	72
Mean tang. assets (2010 $millions)	44	109	232	525	1803	113000
Median tang. assets (2010 $millions)	44	106	227	491	1403	17100
Proportion of tang. assets in the sample	0.1%	0.2%	0.6%	1.4%	4.3%	93.4%
Proportion of tangible assets						
Securities	31%	15%	6%	3%	3%	3%
Loans to customers	49%	65%	69%	70%	75%	81%
Fed funds sold & reverse repo	12%	13%	12%	17%	12%	3%
Other tangible assets	8%	7%	13%	10%	10%	13%
Total Deposits	61%	83%	74%	81%	78%	74%
Transaction deposits	19%	33%	26%	26%	7%	2%
Non-transaction deposits	42%	49%	48%	55%	70%	72%
Fed funds purchased & repo	14%	3%	7%	13%	13%	17%
Other liabilities	25%	14%	19%	6%	10%	9%

TABLE III – OPTIMAL BALANCE SHEETS UNDER THE UNCONSTRAINED MODEL, ACCORDING TO FIRST ORDER STOCHASTIC DOMINANCE CRITERION FOR LIQUIDITY RISK OF U.S. COMMERCIAL BANKS, BY SIZE

PANEL IV: OPTIMAL BALANCE SHEET COMPOSITION, FOR 2008 Q3	Below 25th percentile	Between 25th and 50th percentile	Between 50th and 75th percentile	Between 75th and 90th percentile	Between 90th and 98th percentile	Above 98th percentile
Number of banks	131	183	178	164	189	78
Mean tang. assets (2008 $millions)	38	98	213	501	1744	104000
Median tang. assets (2008 $millions)	40	99	216	489	1436	15800
Proportion of tang. assets in the sample	0.1%	0.2%	0.4%	1.0%	3.8%	94.5%
Proportion of tangible assets						
Securities	25%	43%	60%	58%	41%	31%
Loans to customers	16%	14%	18%	27%	30%	46%
Fed funds sold & reverse repo	38%	27%	4%	1%	11%	8%
Other tangible assets	22%	17%	18%	14%	19%	14%
Total Deposits	37%	28%	36%	69%	59%	73%
Transaction deposits	10%	12%	3%	4%	2%	13%
Non-transaction deposits	27%	16%	33%	64%	57%	60%
Fed funds purchased & repo	34%	29%	28%	9%	6%	4%
Other liabilities	29%	43%	36%	22%	35%	23%

TABLE IV – CROSS-SECTIONAL REGRESSIONS OF GROWTH IN EQUITY ON ACTUAL ASSET AND LIABILITY WEIGHTS MULTIPLIED BY THE RESPECTIVE GROWTH RATES IN ASSET AND LIABILITY CLASSES IN 2008Q3

A cross-sectional regression is estimated for each bank size, by total assets. Size 1 refers to the smallest banks (i.e. below the 25th percentile) while size 6 refers to the largest banks (i.e. above the 98th percentile). The regression for size 1 is estimated on 131 observations, for size 2 on 183, for size 3 on 178, for size 4 on 164, for size 5 on 189 and for size 6 on 78 observations. The adjusted R-squared of the regression for size 1 is 3%, 13% for size 2, 2% for size 3, 12% for size 4, 27% for size 5, and 38% for size 6.

	Below 25th percentile		Between 25th and 50th percentile		Between 50th and 75th percentile		Between 75th and 90th percentile		Between 90th and 98th percentile		Above 98th percentile	
DEPENDENT VARIABLE: *Quarterly growth in tangible equity in 2008Q3*	Coeff.	Std. Error	Coeff.	Std. Error	Coeff.	Std. Error	Coeff.	Std. Error	Coeff.	Std. Error	Coeff.	Std. Error
INDEPENDENT VARIABLES:												
Quarterly growth × Actual weight in 2008Q3												
Tangible assets												
Securities	0.342***	0.118	0.687***	0.147	0.448**	0.222	1.874***	0.402	2.498***	0.372	3.861***	0.616
Loans to customers	0.399**	0.178	0.382***	0.135	0.340**	0.171	1.205***	0.366	2.100***	0.360	1.334**	0.646
Fed funds sold & reverse repo	0.223***	0.077	0.125**	0.064	0.180	0.127	0.197	0.315	0.881***	0.198	2.380***	0.522
Other tangible assets	0.181	0.170	0.477***	0.152	0.314	0.211	0.261	0.496	1.635***	0.478	1.943***	0.614
Liabilities												
Transaction deposits	-0.193	0.120	-0.185	0.120	-0.123	0.085	-0.369**	0.186	-0.463	0.283	-2.059**	0.785
Non-transaction deposits	-0.265**	0.103	-0.386***	0.095	-0.294***	0.115	-0.688**	0.279	-1.494***	0.309	-1.751***	0.512
Fed funds purchased & repo	0.010	0.298	-0.243	0.177	-0.258	0.215	-0.372	0.352	-1.251***	0.273	-1.530***	0.405
Other liabilities	-0.315	0.324	-0.130	0.219	0.109	0.187	-0.827**	0.336	-1.477***	0.254	-1.069*	0.578
Constant	0.017***	0.006	0.009**	0.004	0.011**	0.005	-0.001	0.009	-0.007	0.007	0.009	0.011

Note. *significant at 10%; **significant at 5%; ***significant at 1%. For each size category, the left column reports the estimated coefficients of explanatory variables in the regression and the right column reports robust standard errors.

Figures 1-6 – Growth in balance-sheet items and commitments (year-on-year) of U.S. commercial banks

Figure 1: Customer loans and committments

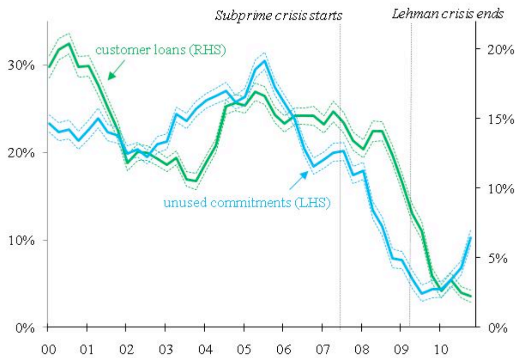


Figure 2: Loan committments and trans. deposits

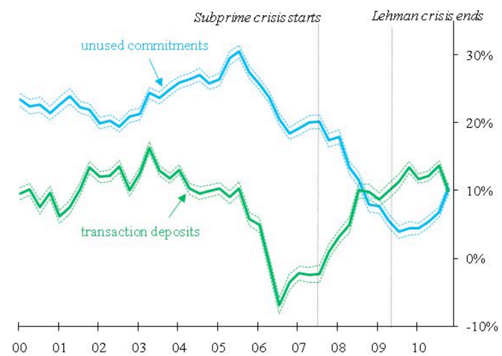


Figure 3: Loan committments and non trans. deposits

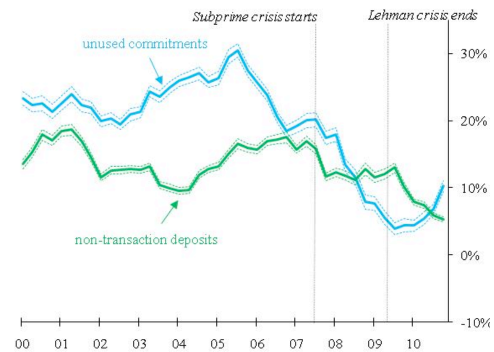


Figure 4: Customer loans and Fed funds & repo

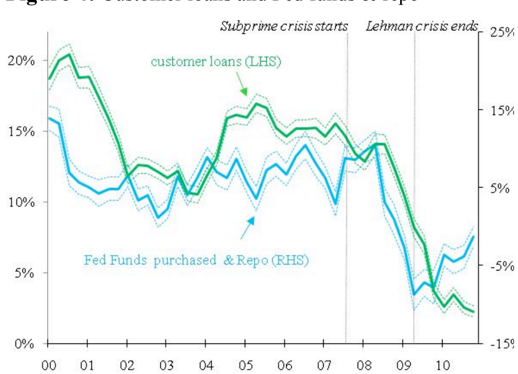


Figure 5: Customer loans and other liabilities

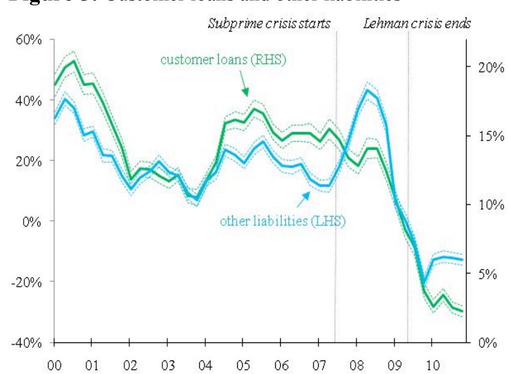
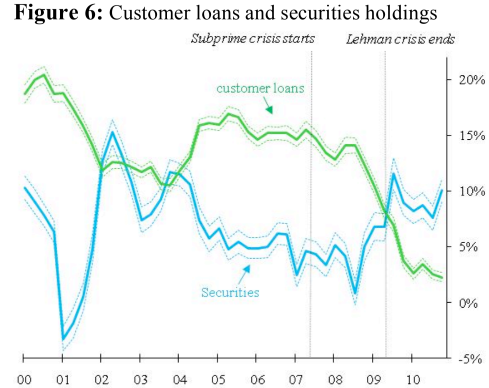


Figure 6: Customer loans and securities holdings



FIGURES 7-12 – AVERAGE ASSET COMPOSITION (LINES) AND OPTIMAL WEIGHTS (DOTS) OF U.S. COMMERCIAL BANKS, BY SIZE

Figure 7: Below 25th percentile

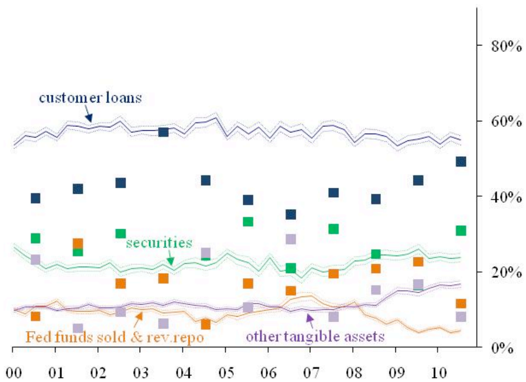


Figure 8: Between 25th and 50th percentile

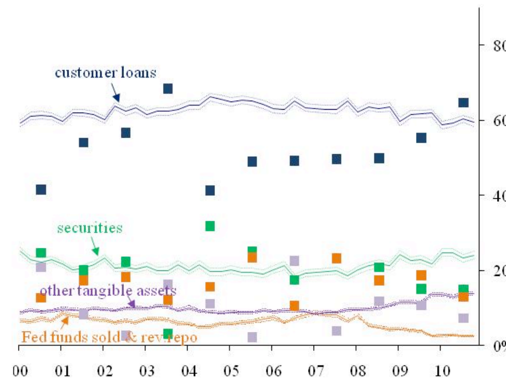


Figure 9: Between 50th and 75th percentile

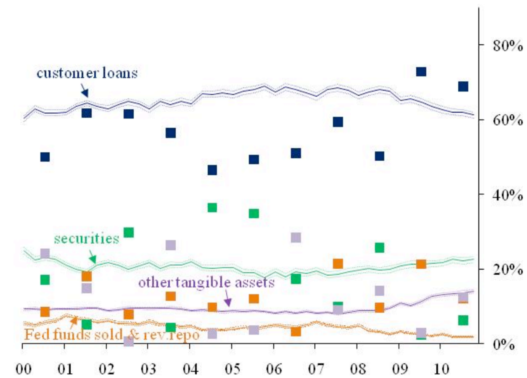


Figure 10: Between 75th and 90th percentile

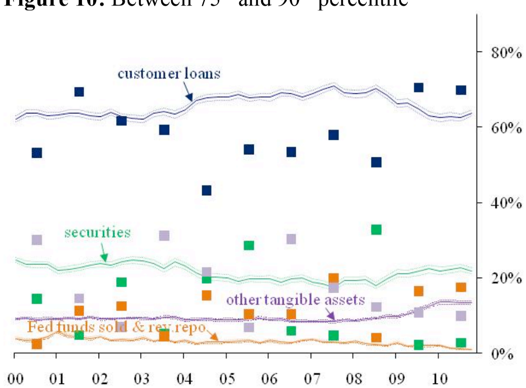


Figure 11: Between 90th and 98th percentile

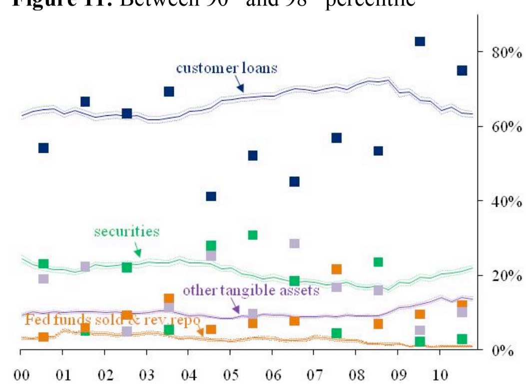
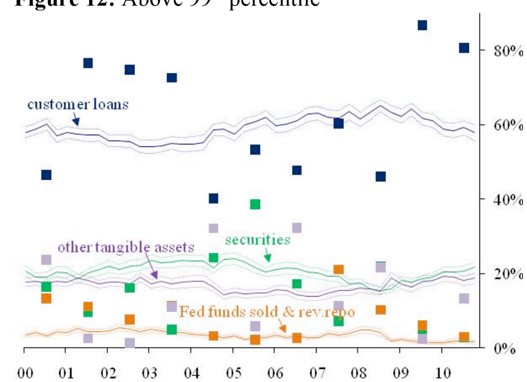


Figure 12: Above 99th percentile



Figures 13-18 – Average liability composition (lines) and optimal weights (dots) of U.S. commercial banks, by size

Figure 13: Below 25th percentile

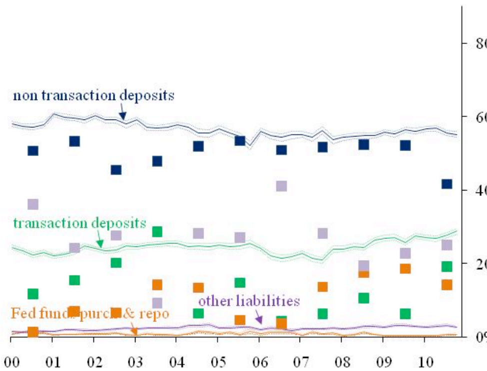


Figure 14: Between 25th and 50th percentile

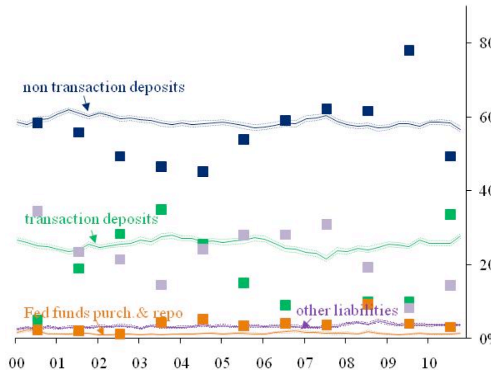


Figure 15: Between 50th and 75th percentile

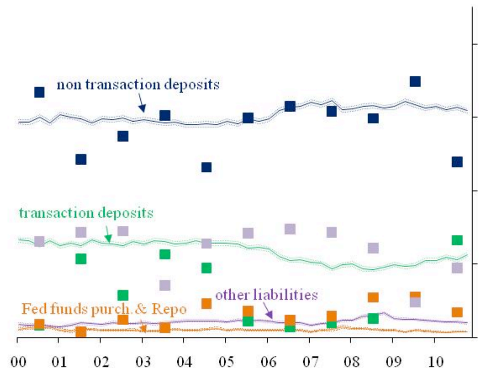


Figure 16: Between 75th and 90th percentile

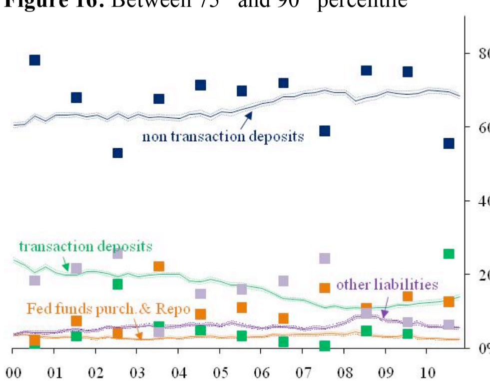


Figure 17: Between 90th and 98th percentile

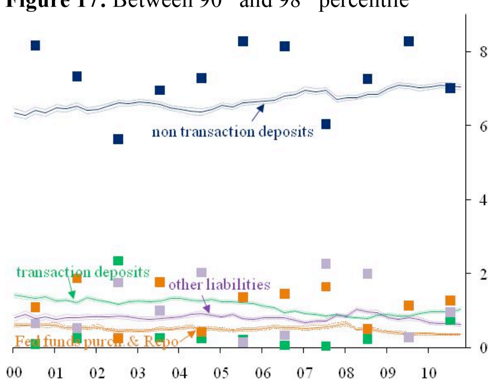


Figure 18: Above 99th percentile