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Pollution Abatement as a Source of Stabilisation and Long-Run Growth

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Pollution Abatement as a Source of Stabilisation and Long-Run Growth*

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Abstract

In a two-period overlapping generations model with production, we consider the damaging impact of environmental degradation on health and, consequently, life expectancy. The government's involvement on policies of environmental preservation proves crucial for both the economy's short-term dynamics and its long-term prospects. Particularly, an active policy of pollution abatement emerges as an important engine of long-run economic growth. Furthermore, by eliminating the occurrence of limit cycles, pollution abatement is also a powerful source of stabilisation.

JEL classification: O41; Q56

Keywords: Growth; Long-run cycles; Environmental quality; Pollution abatement

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1 Introduction

In recent years, environmental issues have gained prominence in both academic and political discussions. At the same time, they have received considerable media attention. Problems such as the emission of greenhouse gases, the depletion of natural resources, and the presence of hazardous chemicals have become major issues of concern. This is of course not surprising, given their significant direct and indirect repercussions on our health and therefore our overall quality of life. According to the Environmental Protection Agency, water pollution caused by untreated wastewater (from toxic waste, sewage, etc.) is responsible for gastrointestinal and neurological conditions.[1] Soil is contaminated by carcinogenic chemical pollutants (e.g., pesticides) that affect humans either through direct contact or through the food chain (Pimentel *et al.* 1998). Air pollution (mainly from the burning of fossil fuels and the emission of industrial chemicals) is a major cause of respiratory diseases, such as asthma and chronic bronchitis, as well as various forms of cancer (Lave and Seskin 1970). Furthermore, the associated global warming can increase the incidence of vector-born diseases such as encephalitis and malaria (Khasnis and Nettleman 2005). It is not surprising that all these effects have a staggering impact in terms of loss of life. In fact, Pimentel *et al.* (1998) estimate that the direct and indirect impacts of environmental degradation, mainly caused by organic and chemical pollutants, can account for almost 40% of deaths worldwide.

Naturally, economic growth has been an indispensable aspect of all the discussion in relation to environmental problems; after all, environmental degradation is a by-product of economic activities such as production and consumption. One point of view focuses on this latter idea, as well as the economic importance of a prosperous natural environment, to suggest that societies, and their policy makers in particular, should shift their attention away from economic growth and towards policies and actions that preserve environmental quality (e.g., Arrow *et al.* 1995). Otherwise, the reckless and short-sighted quest for economic prosperity today will deteriorate severely the quality of the environment bestowed to future generations. Another point of view discards the aforementioned arguments. It is based on empirical analyses (e.g., Grossman and Krueger 1995; Millimet *et al.* 2003) showing that the relation between measures of pollution and per capita GDP is not monotonically positive; rather, it appears to be an inverse-U-shaped relation that is nowadays widely known as the

[1] See water.epa.gov/drink/contaminants/#List.

Environmental Kuznets Curve (EKC thereafter). The main implication of the EKC is that higher national income may not be an impediment to environmental quality after all.

It is worth noting that, rather than being generally accepted as a stylised fact, the EKC is probably one of the most contested issues in the environmental economics literature. A significant number of analyses have criticised both the methodological framework and the interpretation of the results supporting the EKC (Stern 2004), while others have failed to reproduce co-movements in measures of pollution and income that resemble EKCs (e.g., Dijkgraaf and Vollebergh 2005; Azomahou *et al.* 2006). Instead, these latter studies obtain either a monotonically positive relation between various pollutants and per capita GDP, or a relation that is better represented diagrammatically by an N-shaped curve.

All these issues generate the questions that motivate our analysis. What are the long-term economic prospects (in terms of capital accumulation) in an economy where environmental degradation has a negative effect on health and life expectancy? Can environmental policy alter these prospects, despite the fact that growth is still detrimental to environmental quality as the evidence on the relation between per capita GDP and pollution, being either positive or N-shaped, suggests?

To analyse these issues, we build a two-period overlapping generations model in which pollution affects a person's prospects of survival to the next period and labour productivity is enhanced by an aggregate learning-by-doing externality.[2] Despite the fact that this type of externality is the source of aggregate constant returns that could potentially allow for an equilibrium path with a positive growth rate in the long-run, when pollution is left unabated in our model, the economy cannot achieve such a path. Instead, as long as there is a sufficiently high initial endowment of capital stock, the economy will either converge to a positive stationary level of capital per worker or to a stable cycle in which capital per worker oscillates permanently. Nevertheless, when resources are devoted towards pollution abatement, the equilibrium outcomes change drastically. Specifically, by influencing longevity and saving behavior, public policy (in the form of pollution abatement) can put the economy on a sustainable growth path. Economic growth is environmentally sustainable because a sufficient level of environmental quality is maintained. These outcomes occur despite the fact that economic growth has a net damaging effect on environmental quality – with or

[2] There is a large number of existing theoretical analyses that incorporate endogenous longevity in dynamic general equilibrium models. See, among others, Boucekkine *et al.* (2002); Chakraborty (2004); and Bhattacharya and Qiao (2007).

without environmental policy – and even though the quality of the environment is essential for supporting longevity and, therefore, saving and capital accumulation. Furthermore, by eliminating long-run cycles, environmental policy smoothens income, thereby becoming a source of stabilisation, albeit one whose scope and implications are quite different from the more conventional counter-cyclical policies designed against short-term fluctuations.

In the last main section of our analysis, we endogenise the government's expenditure allocation. In particular, we consider the case where the public sector allocates optimally its spending between public health care and environmental activities. The first main outcome from this procedure is that the government finds it optimal to initiate any spending towards environmental support only after the economy's capital resources exceed a certain threshold. Casual empirical observation suggests that actual economies tend to engage in active environmental preservation only at later stages of their development process – hence, providing support for our theoretical result. We also show that, once the government supports pollution abatement activities optimally, the economy may sustain economic growth in the long-run while the dynamics do not converge to endogenous cycles.

Our model shares similarities with other studies that have introduced elements of environmental quality in OLG models. John and Pecchenino (1994) was the first study to take this approach, followed by others, such as Mariani *et al.* (2010) and Balestra and Dottori (2012). These papers account for the double causality between the economic activity and the environment. Nevertheless, closer to our paper are models of capital accumulation and environmental quality that have identified the possibility of endogenous fluctuations, such as Zhang (1999), Ono (2003) and Seegmuller and Verchère (2004). All of them employ the John and Pecchenino (1994) framework to introduce environmental quality; in particular, their mechanism of endogenous cycles differs from ours.[3] In our model, cycles may emerge because unbounded environmental degradation, and its impact on longevity, introduces non-monotonicity in the dynamics of capital accumulation. More importantly, in our framework, environmental policy is a source of stabilisation (i.e., it eliminates long-run cycles) and growth.

The link between pollution abatement and economic growth is also analyzed in two papers by Bovenberg and Smulders (1995, 1996). In the former paper, the authors develop a

[3] Ono (2003) introduces environmental quality in a model of cycles and growth. The implication from environmental policy in his framework is different from ours in that he obtains a critical level of tax above which higher growth and improved environmental quality may actually require a less stringent environmental policy, that is, a reduction in abatement efforts.

two-sector representative-agent model, which incorporates pollution-augmenting technical change, and derive technical conditions under which sustainable growth is both feasible and optimal. They then explore optimal environmental policies. In the latter paper, they also incorporate the public consumption element of environmental quality and provide an analytical description of the economy's transitional dynamics following a tightening of environmental policy. They find that the short-term and long-term effects of a tighter environmental policy are quite opposite: while the former are negative, in the long-term environmental policy may boost the rate of economic growth.

Closer to our setting is the analysis of Smulders and Gradus (1996). They use a one sector growth model in which pollution reduction has a direct benefit to the economy's productivity. They find that pollution abatement allows the economy to sustain growth in the long-run. Nevertheless, this is possible only when appropriate parameter restrictions allow abatement to grow at a faster rate compared to pollution, meaning that pollution declines along the balanced growth path. In our model, this type of environmental policy allows the economy to sustain long-run growth despite the fact that the environment is essential for survival and output growth has a monotonically negative effect on environmental quality, irrespective of whether pollution is abated or not.[4]

The rest of the paper is organised as follows: Section 2 sets-up the economic model. In Section 3 we analyse the different equilibrium outcomes of the model, according to whether pollution abatement is active or not. In Section 4, we discuss some important implications from our analysis and in Section 5 we consider the case where the government's expenditure towards pollution abatement is determined endogenously. Section 6 summarises and concludes.

2 The Economic Framework

We construct an overlapping generations economy in which time, indicated by $t = 0,1,2,\ldots$, is measured in discrete intervals that represent periods. The economy is populated by an infinite sequence of agents who face a potential lifetime of two periods. In particular, an agent will live during the period following her birth, i.e., her youth, but she may or may not survive to her old age. We assume that, before her survival prospect is realised, each agent

[4] In Smulders and Gradus (1996), pollution declines constantly along the balanced growth path because abatement is sufficiently strong. In our model, abatement can only reduce the rate of environmental degradation. As a result, pollution increases even in the presence of abatement efforts.

reproduces asexually and gives birth to an offspring. Thus, the prospect of untimely death does not have any repercussions for the population mass of newly-born agents, whose size we normalise to one.

During youth, each agent is endowed with one unit of labour. She supplies her labour to firms inelastically and receives the competitive salary, w_t. Even if she survives to maturity, nature does not bestow to her the ability to work when old; therefore, w_t is her only source of income during her lifetime. For this reason, and in order to satisfy her future consumption needs, she deposits an amount s_t, when young, to a financial intermediary that promises to repay it next period, augmented by the gross interest rate r_{t+1}.

As mentioned earlier, survival to maturity is not certain. Particularly, we assume that a young person will survive to maturity with probability $\beta_t \in [0,1)$ whereas with probability $1-\beta_t$ she dies prematurely. Furthermore, we assume that life expectancy is endogenous in the sense that the agent's survival prospect depends on her health characteristics (or health status), denoted as h_t, according to[5]

$$\beta_t = \mathrm{B}(h_t), \tag{1}$$

where $\mathrm{B}'(h_t) > 0$, $\mathrm{B}''(h_t) < 0$, $\mathrm{B}(0) = 0$, $\mathrm{B}(\infty) = \lambda$, $\lambda \in (0,1)$, $\mathrm{B}'(0) = \psi$, $\psi \in (0,1)$, and $\mathrm{B}'(\infty) = 0$. Thus, we employ essentially the same assumptions used by Chakraborty (2004) in his seminal analysis of endogenous lifetime and economic growth. See also Blackburn and Cipriani (2002), who also incorporate life expectancy in a similar manner.

We delve further into the determinants of life expectancy by assuming that an agent's health status depends positively on the extent to which the government supports the provision of health services g_t (e.g., public hospitals; the presence of a national health system; preventive measures; funding and support of medical research; the design and implementation of health and safety rules, etc.) and on the quality of the natural environment e_t (e.g., the cleanliness of air, soil and water; the relative abundance of natural

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[5] An agent's expected lifetime at birth is equal to $\beta_t 2 + 1 - \beta_t = 1 + \beta_t$ periods. For this reason, we shall be using such terms as 'life expectancy', 'longevity' and 'survival probability' interchangeably. In fact, an alternative interpretation is that in principle all agents survive to the second period, but are alive only a fraction $\beta = \mathrm{B}(h_t) \in [0,1]$ of the period as, for example, in Bhattacharya and Qiao (2007). Also, we could have additionally assumed that pollution affects the productivity of the young as well as the utility of the old, i.e., that the environment has a "productive" and an "amenity" value, respectively (see Smulders and Gradus 1996). These extensions will actually strengthen the main result of the paper, which is that pollution abatement can be a source of stabilisation and long-run growth.

resources such as forestry and other forms of plantation, etc.). Formally, these ideas are captured by[6]

$$h_t = g_t^{\varphi} e_t^{\chi}, \qquad (2)$$

where $0 < \varphi < 1$ and $0 < \chi < 1$.[7]

The assumption that the arguments affecting health status are introduced through a Cobb-Douglas specification is not new (for example, see van Zon and Muysken 2001). Obviously, one main reason is the tractability associated with its form. However, this tractability does not come at the cost of intuitive reasoning. The Cobb-Douglas form implies that the positive health impact of public spending is more pronounced in conducive, i.e., less polluted environments. Further support for this idea is provided by Ballestra and Dottori (2012). In their parametric restrictions for a health function with the same arguments, they argue that "*it seems reasonable to assume that health expenditure and environmental quality are only imperfect substitutes and exhibit some complementarity to effectively improve health status*" (2012; p. 1068).

All choices made by an agent during her lifetime are governed by her *ex ante* (i.e., expected) lifetime utility function

$$V^t = \ln c_t + \beta_t \ln d_{t+1}, \qquad (3)$$

where c_t and d_{t+1} denote the levels of consumption during youth and old age respectively.

There is a single, perishable commodity through which agents can satisfy their consumption needs. It is produced by perfectly competitive firms who combine physical capital, K_t (which they rent from financial intermediaries at a price of R_t per unit), and labour, L_t, so as to produce Y_t units of output according to

$$Y_t = K_t^{\gamma}(\mathrm{A}_t L_t)^{1-\gamma}, \quad 0 < \gamma < 1, \qquad (4)$$

where A_t is assumed to be positively related to the economy's average amount of capital, $\bar{K}_t$, as in Romer (1986). Thus, behind equation (4) lies the idea that workers gain knowledge

[6] Note that health status is a flow and not a stock variable. Although a health stock would be more appropriate in an environment where agents live for three or more periods, our current assumption seems more suitable in a setting where an agent's potential lifetime is divided in two broad periods. Of course, even under a two period setting, one can argue that a health stock may make sense once we consider the intergenerational transmission of genetic attributes. Such issues, however, go way beyond the scope of our paper; that is why we have decided to abscond from them.

[7] The limiting case for which $\varphi = 1$ and $\chi = 0$ is examined by Chakraborty (2004). In his paper, he does not consider issues pertaining to the natural environment.

and become more productive by handling more capital goods – knowledge that spreads costlessly over the whole economy in the manner of an externality. Formally,

$$\mathrm{A}_t = \mathrm{A}'\overline{K}_t,\ \mathrm{A}' > 0. \tag{5}$$

One unfortunate by-product from firms' activities is pollution. We assume that one unit of produced output generates $p > 0$ units of pollutant emissions; therefore, total pollution is

$$P_t = pY_t. \tag{6}$$

Although pollution is the major determinant of environmental degradation, D_t, the latter can be mitigated by government-funded activities that are designed and implemented so as to reduce the extent of environmental damage for given levels of pollutant emissions. We may think of recycling facilities; wastewater management facilities; installation and operation of renewable energy techniques that reduce the emission of greenhouse gases and toxic pollutants (e.g., wind turbines, hydroelectric plants and solar photovoltaics); clean-up operations, etc. For the purposes of our analysis, we shall refer to them as pollution abatement activities, and denote them by $a_t \geq 0$. Environmental degradation is, hence, formally given by

$$D_t = \frac{P_t}{1 + a_t}. \tag{7}$$

Our assumption inherent is (7) is that $\partial^2 D_t / \partial P_t \partial a_t < 0$, i.e., the positive effect of pollution on environmental degradation is mitigated by abatement activities. We view this as an improvement over the alternative scenario in which abatement does not impinge on the negative effect of pollution to the quality of the environment (additive separability). Actually, our model is not the first to consider non-separable effects. The papers by Pautrel (2009) and Clemens and Pittel (2011) use similar formulations by assuming that environmental degradation takes the form P_t / a_t. An inherent problem with this formulation is the implicit assumption that degradation is infinite when $a_t = 0$. Our functional form eliminates this possibility.

Given the aforementioned arguments, the quality of the natural environment, $e_t \geq 0$, depends on the extent of environmental degradation. We capture this idea through

$$e_t = \begin{cases} E - D_t & \text{if } D_t < E \\ 0 & \text{otherwise} \end{cases}, \tag{8}$$

where $E > 0$.

Note that, according to (7), the environmental impacts of pollution and abatement are not separable. Given that, in equilibrium, both of them are proportional to income, higher production will always entail environmental degradation and net environmental costs – irrespective on whether pollution is abated ($a_t > 0$) or not ($a_t = 0$). This is a deviation of our paper in comparison to the framework by John and Pecchenino (1994). While that framework is indubitably a useful methodological starting point for analysing many issues, it makes abatement so strong that higher income may lead to improvements in environmental quality. On the contrary, in our model, abatement can only reduce the rate of environmental degradation that results from economic activity.

As it is evident in (8), we abstract from the dynamics of environmental quality by assuming that e_t is a flow and not a stock variable. On the one hand, a similar assumption has been used in the analyses of Stokey (1998), Jones and Manuelli (2001) and Hartman and Kwon (2005) among others; on the other hand, it may not be so restrictive in a two-period overlapping generations setting. It is fair, however, to admit that this choice has been dictated by the need for analytical tractability. As it will become clear later, even in its current form the model is very complicated and any analytical solutions that allow the reader to understand the intuition and the mechanisms involved are made possible only when environmental quality is a flow variable. Nevertheless, in order to show that our results survive under more general settings where environmental quality takes the form of a stock, in Appendix A9 we solve the model for two different formulations regarding the dynamics of environmental quality.[8] Particularly, we consider the cases for which $e_t = e_{t-1}^{\eta}(E - D_t)^{1-\eta}$ and $e_t = \eta e_{t-1} + (1-\eta)E - D_t$, where $0 < \eta < 1$ and D_t is given in equation (7).[9] Despite the fact that it is impossible to trace the transitional dynamics analytically due to the added dimension and the ensuing mathematical complication, our numerical simulations suggest that our main results and their implications survive even under these more general settings.

We complete our description of the economy's structure with a discussion on the process under which the government finances its activities. We utilise the widely-used assumption that the government imposes a flat tax rate $\tau \in (0,1)$ on firms' production revenues. Assuming that the government abides by a balanced budget rule in each period, our previous

[8] We are grateful to two referees for suggesting to us this extension.

[9] In both cases, we can see that (8) is nothing else but the limiting case for which $\eta = 0$.

assumptions imply that $g_t + a_t = \tau Y_t$. If we denote the fixed fraction of revenues devoted towards pollution abatement by $\upsilon \in [0,1)$, it is straightforward to establish that

$$g_t = (1-\upsilon)\tau Y_t, \tag{9}$$

and

$$a_t = \upsilon\tau Y_t, \tag{10}$$

give the levels of public health spending and pollution abatement activities in relation to the economy's total output, respectively.

3 Temporary Equilibrium

We begin our analysis with a description of the economy's temporary equilibrium. This is provided in the form of

Definition 1. *The temporary equilibrium of the economy is a set of quantities* $\{c_t, d_t, d_{t+1}, s_t, L_t, Y_t, \mathrm{A}_t, \beta_t, h_t, e_t, D_t, L_t, P_t, a_t, g_t, K_t, K_{t+1}\}$ *and prices* $\{w_t, R_t, R_{t+1}, r_{t+1}\}$ *such that:*

- *(i)* *Given* w_t, r_{t+1} *and* β_t, *the quantities* c_t, d_{t+1} *and* s_t *solve the optimisation problem of a worker born at time* t;
- *(ii)* *Given* w_t *and* R_t, *all firms choose quantities for* L_t *and* K_t *in order to maximise profits;*
- *(iii)* *The labour market clears, i.e.,* $L_t = 1$;
- *(iv)* *The goods market clears, i.e.,* $Y_t = c_t + \beta_{t-1}d_t + s_t + g_t + a_t$;
- *(v)* *The financial market clears;*
- *(vi)* *The government's budget is balanced.*

The objective of a young agent is to choose the levels of consumption, in both periods, and saving so as to maximise V^t subject to $c_t = w_t - s_t$ and $d_{t+1} = r_{t+1}s_t$. Alternatively, given (3), the problem can be modified to $\max_{0\le s_t \le 1}\{\ln(w_t - s_t) + \beta_t \ln(r_{t+1}s_t)\}$. The solution to this problem is

$$s_t = \frac{\beta_t}{1+\beta_t} w_t. \tag{11}$$

Naturally, the prospect of premature death modifies an agent's saving behaviour. In terms of intuition, an increase in longevity raises the marginal utility of an agent's consumption when old; therefore, to restore the equilibrium, the marginal utility derived from her first period consumption must increase as well. She can achieve this by choosing to save more and consume less while she is young.

Profit maximisation by firms entails that each input's marginal product is equal to its respective price. Formally,

$$w_t = (1-\tau)(1-\gamma)K_t^{\gamma}L_t^{-\gamma}\mathrm{A}_t^{1-\gamma} = (1-\tau)(1-\gamma)k_t^{\gamma}\mathrm{A}_t^{1-\gamma}, \tag{12}$$

and

$$R_t = (1-\tau)\gamma K_t^{\gamma-1}L_t^{1-\gamma}\mathrm{A}_t^{1-\gamma} = (1-\tau)\gamma k_t^{\gamma-1}\mathrm{A}_t^{1-\gamma}, \tag{13}$$

where $k_t = K_t / L_t$ is the amount of capital per worker. In equilibrium, $L_t = 1$ and $k_t = K_t = \overline{K}_t$. Consequently, using the notation $\Gamma = \mathrm{A}'^{1-\gamma}$, we can write (12) and (13) as

$$w_t = (1-\tau)(1-\gamma)\Gamma k_t, \tag{14}$$

and

$$R_t = (1-\tau)\gamma\Gamma \equiv \hat{\mathrm{R}} \ , \tag{15}$$

respectively.

There are two conditions that describe the financial market equilibrium. We assume that perfectly competitive financial intermediaries undertake the task of channelling capital from depositors to firms. Specifically, they transform saving deposits into capital by accessing a technology that transforms time-t output into time-$t+1$ capital on a one-to-one basis. They, subsequently, supply this capital to firms that manufacture the economy's single commodity. Hence, $K_{t+1} = L_t s_t$ or, in intensive form,

$$k_{t+1} = s_t . \tag{16}$$

To resolve the issue of saving under an uncertain lifetime, we assume, following Chakraborty (2004), that financial intermediaries represent mutual funds that offer contingent annuities. Specifically, when accepting deposits, intermediaries promise to offer retirement income (in our case, $r_{t+1}s_t$) provided that the depositor survives to old age. Otherwise, the income of those who die is shared equally among surviving members of the mutual fund. Considering this assumption, and the fact that financial intermediaries operate under perfect competition, we have

$$\beta_t r_{t+1} = \mathrm{R}_{t+1} = \hat{\mathrm{R}}, \tag{17}$$

which translates into the equilibrium condition requiring costs (i.e., the total return to all surviving savers) to be equal to revenues (i.e., the revenues they receive from firms who rent capital) – the reason being that financial intermediaries make zero economic profits from their activities.

Next, we can use the labour market clearing condition, $L_t = 1$, together with (5), in equation (4) so as to obtain an expression for output per worker $y_t = Y_t / L_t$. That is,

$$y_t = \Gamma k_t . \tag{18}$$

If we combine the expression in (18) together with (1), (2), (6), (7), (8), (9) and (10), and substitute together with (11) and (14) in equation (16), we can eventually derive

$$k_{t+1} = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi} \left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right)^{\chi} \right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi} \left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right)^{\chi} \right)} k_t = z(k_t) . \tag{19}$$

Thus, we have reduced our model into a dynamical system of one first-order difference equation for capital per worker. The analysis of this equation will facilitate us in understanding the dynamics and the long-run equilibrium of the economy. This is the issue to which we now turn our attention.

3 Dynamic Equilibrium

The economy's dynamic equilibrium is formally described through

Definition 2. *For $k_0 > 0$, the dynamic equilibrium is a sequence of temporary equilibria that satisfy $k_{t+1} = z(k_t)$ for every t.*

We can facilitate our subsequent analysis by defining a new variable, θ_{t+1}, which denotes the growth rate of physical capital per worker. That is,

$$\theta_{t+1} = \frac{k_{t+1}}{k_t} - 1 . \tag{20}$$

Furthermore, our subsequent results will be further clarified with the use of

Definition 3. *Consider $k_0 > 0$. An equilibrium orbit $\{k_t\}$ is a 'no growth' equilibrium if there exists $M > 0$ such that $k_t < M$ $\forall t$. If $\lim_{t\to\infty} k_t = \hat{k}$ then we call $\hat{k}$ a 'no growth' steady state equilibrium. If, in addition, $\hat{k} = 0$ then the equilibrium is a 'poverty trap'. If there does not exist such an M, then the* *equilibrium orbit is called a 'long-run growth' equilibrium and satisfies* $\lim_{t\to\infty}\frac{k_{t+1}}{k_t}=\lim_{t\to\infty}(1+\theta_{t+1})=1+\hat{\theta}>1$.

Our purpose is to examine two scenarios that differ with respect to the government's provision of pollution abatement services. As we shall see, the public sector's stance on environmental protection has significant repercussions for both the economy's dynamics and its long-term prospects. Furthermore, the subsequent analysis will be utilising

Assumption 1. $(1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}(\Omega)}{1+\mathrm{B}(\Omega)}>1$, *where* $\Omega=\left(\frac{\varphi\tau}{p}\right)^{\varphi}\chi^{\chi}\left(\frac{E}{\varphi+\chi}\right)^{\varphi+\chi}$,

as well as

Assumption 2. $\chi\leq\varphi$.

The first assumption is essential for the existence of a meaningful long-run equilibrium. As we shall see later, the slope of the phase line at the origin is below unity – an outcome that raises the possibility that the only steady state equilibrium entails the corner solution of a zero capital stock. Assumption 1 eliminates this possibility and ensures the existence of an interior equilibrium. In terms of interpretation, we can think of it as necessitating that structural parameters conducive to the economy's capital formation (such as the productivity parameter Γ; the parameters of the health function $\mathrm{B}(\cdot)$; or the environmental parameter E) are sufficiently high to guarantee a positive rate of capital accumulation for at least some range on the capital stock's domain (see also Appendix A1).

The second assumption is not essential for our results and is employed purely for expositional purposes (see Footnote 13). It is actually relaxed in Appendix A5, where we show that our results still remain qualitatively similar.

3.1 Dynamic Equilibrium without Pollution Abatement

We begin our analysis with the case where $\upsilon=0$, a case which translates into a scenario where the government is not actively engaged in policies of environmental preservation. Given (19), we have

$$z(k_t) = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left((\tau\Gamma k_t)^{\varphi}(E-p\Gamma k_t)^{\chi}\right)}{1+\mathrm{B}\left((\tau\Gamma k_t)^{\varphi}(E-p\Gamma k_t)^{\chi}\right)} k_t . \tag{21}$$

First, we are interested in obtaining the model's steady-state equilibria. These are fixed points of the map $z(\cdot)$, i.e., values $\hat{k}$ of capital per worker that satisfy $\hat{k} = z\left(\hat{k}\right)$. A formal analysis of (21) allows us to derive

Lemma 1. *There exist three steady-state equilibria* $\hat{k}'$, $\hat{k}''$ *and* $\hat{k}'''$, *such that* $\hat{k}' = 0$ *and* $\hat{k}''' > \hat{k}'' > 0$. *The steady state* $\hat{k}'$ *is locally asymptotically stable,* $\hat{k}''$ *is an unstable steady state, while* $\hat{k}'''$ *may be either locally asymptotically stable or unstable.*

Note that all the proofs are relegated to the Appendix. The result from Lemma 1 facilitates us in tracing the economy's dynamic behaviour and transitional dynamics. We can formally present these ideas in the form of

Proposition 1. *Consider* $k_0 > 0$. *Then:*

- *(i)* *If* $k_0 < \hat{k}''$, *the economy will converge to the poverty trap* $\hat{k}' = 0$;
- *(ii)* *If* $k_0 > \hat{k}''$, *the economy will converge to a 'no growth' equilibrium. Particularly, if* $\hat{k}'''$ *is locally asymptotically stable, then it will also be the stationary equilibrium for the stock of capital per worker – otherwise, the economy will asymptotically converge to an equilibrium where capital per worker displays permanent cycles around* $\hat{k}'''$.

The different possible scenarios are depicted in Figures 1-3. In all three cases, we see that the point $\hat{k}''$ acts as a natural threshold which allows history (approximated by the initial capital endowment) to determine the long-term prospects of the economy. The model's ability to generate multiple steady-state equilibria rests on the beneficial effect of publicly provided health services on saving behaviour – an effect that lies on the idea that health services promote longevity. Specifically, for some levels of k_t, capital accumulation and saving complement each other. Thus, for relatively low levels of initial capital endowment, saving is not sufficient enough to guarantee a positive rate of capital accumulation: capital per worker declines constantly until it rests on an equilibrium which is, essentially, a poverty trap. If, however, the initial endowment is sufficient enough, the economy can escape the poverty trap because saving allows growth at positive (albeit declining) rates during the early stages of its transition.

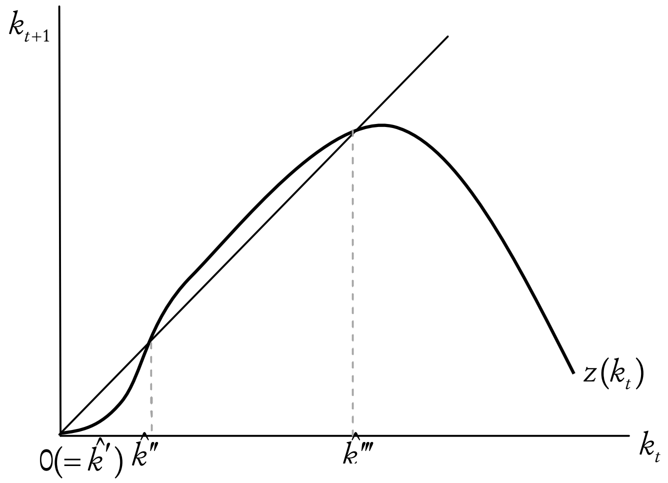

Figure 1. $\upsilon = 0$ and $0 < z'(\hat{k}''') < 1$

So far, the results and their intuition are similar to those discussed in Chakraborty (2004). Nevertheless, our model is able to generate richer implications for the dynamics of an economy whose history allows it to move on the right side of the natural threshold $\hat{k}''$. The reason for such implications is economic activity's contribution to environmental degradation and the corresponding repercussions for health status and longevity. Particularly, for sufficiently high values of k_t the negative effect of pollution on life expectancy and saving dominates the positive effect of publicly provided goods and services on health. Hence, the dynamics of capital accumulation are non-monotonic and $\hat{k}'''$ may actually lie on the downward sloping part of $z(k_t)$. Furthermore, as Figure 3 illustrates, when the slope of the graph at the steady state $\hat{k}'''$ is steep enough, the economy may converge to an equilibrium in which capital per worker oscillates permanently around $\hat{k}'''$ – i.e., an equilibrium with a permanent, endogenously determined cycle. In terms of intuition, a relatively high level of capital per worker implies relatively high pollution. The health status is affected negatively and, consequently, saving is reduced. Capital accumulation is mitigated, but this also implies that the extent of environmental degradation is mitigated as well. Next period's health status improves and so is saving which promotes capital accumulation. This sequence of events may ultimately become self-repeating, thus generating an equilibrium with persistent cycles.

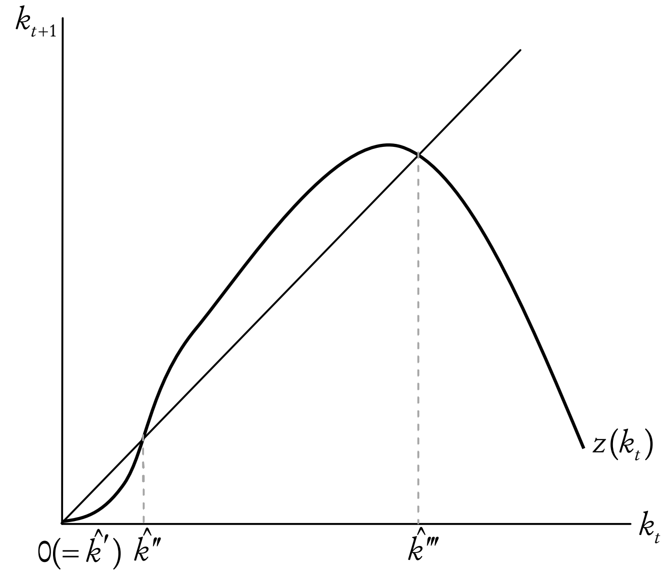

Figure 2. $\upsilon = 0$ and $-1 < z'(\hat{k}''') < 0$

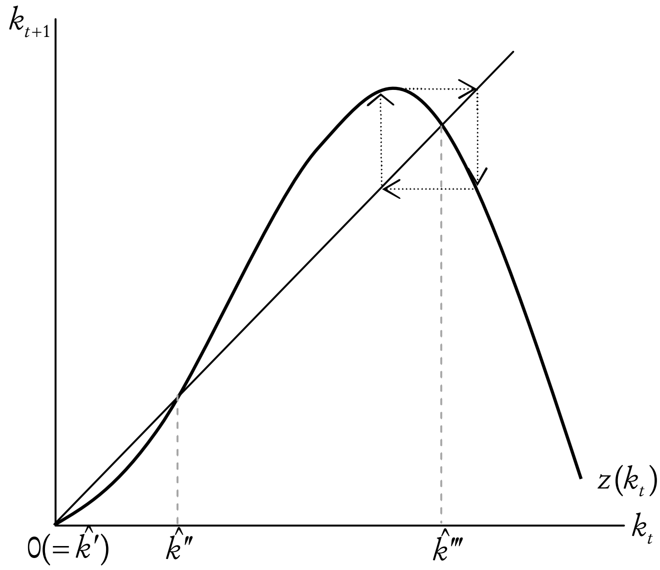

Figure 3. $\upsilon = 0$ and $z'(\hat{k}''') < -1$: an example with a period-2 cycle

In Appendix A3 we present a numerical example that illustrates the results of Proposition 1. We adopt the following functional form: $B(h_t) = \lambda h_t / (1 + h_t)$, $0 < \lambda < 1$, which has also been used in Chakraborty (2004) and Bunzel and Qiao (2005). Accordingly, for a low value of λ the origin is the only fixed point (Assumption 1 is not satisfied). As the value of λ rises the number of fixed points increases to two and then to three. When there are three steady-state equilibria, the intermediate one is repelling. On the other hand, the highest equilibrium is initially stable. Nevertheless, as we raise the value of λ, the stability of this equilibrium changes from stable to repelling. When this happens a cycle of period-2 emerges, which is stable. As we raise the value of λ further the period-2 cycle becomes an unstable one. Instead, there is a period-4 cycle now, which is stable. This process continues as λ increases.

Concerning the dynamic behaviour of environmental quality, it should be obvious that this will be dictated by the dynamics of the capital stock. More specifically, if the economy converges to a poverty trap, then environmental quality approaches its maximum level E given that economic activity is the ultimate cause of environmental deterioration; nevertheless, the severe limitation of resources towards public health means that agents cannot benefit from the improved environmental conditions and, hence, they live essentially for one period. If, on the other hand, the capital stock converges to a stationary (periodic) equilibrium then so does environmental quality.

Readers familiar with the concept of the EKC may feel uncomfortable with the fact that our equilibrium implies a monotonically negative relation between environmental quality and income. More recent evidence by Benos *et al.* (2012), however, corroborates with our theoretical prediction on the environmental characteristics of multiple income equilibria and does not offer support to the existence of poverty-environment traps. Specifically, they employ a distribution dynamics approach based on Markov chains to show that, in the long-run, two main groups of countries emerge: poor countries with low pollution and relatively rich countries with high pollution. Furthermore, recall that the EKC is far from being considered as being a stylised fact, as there are empirical analyses that fail to obtain similar co-movements in income and various indicators of environmental quality (Dijkgraaf and Vollebergh 2005; Azomahou *et al.* 2006)

These results, as well as the intuition behind them, merit some discussion in relation to their empirical relevance. As we can see, the equilibrium behaviour of all variables, including environmental quality and life expectancy/mortality, can be periodic under some circumstances. With respect to the former, there is evidence to show that indicators of environmental quality display such cyclical movements (e.g., Mayer, 1999). With respect to the latter, while conventional wisdom may be at odds with the model's mechanisms, there is evidence to actually support them. For example, the papers by Ruhm (2000) and Tapia Granados (2005) provide evidence on the procyclicality of mortality rates. In particular, they argue that that mortality rates appear to decline following episodes of recession whereas expansions are associated with an increase in mortality rates. Even more related to our mechanisms is the evidence brought forward by Chay and Greenstone (2003). Their empirical analysis provides support to the idea that the *periodically* positive link between mortality and economic activity is associated with variations of the pollutant emissions and their corresponding effects on health.[10]

In any case, the subsequent section of our analysis will show that in the presence of environmental policy, the economy's dynamics and the repercussions for life expectancy become drastically different. Thus, an additional implication will be the identification of the possible importance of environmental policy in preserving the positive, *on average*, link between longevity and per capita GDP that we observe in cross-section data.

3.2 Dynamic Equilibrium with Active Pollution Abatement

The scenario we analyse now allows the government to actively pursue a policy of environmental preservation – i.e., we assume $0 < \upsilon < 1$. Therefore, the dynamics of capital accumulation are represented by the difference equation we originally obtained in (19).

Once more, we shall begin our formal analysis with the derivation of the model's steady-state equilibrium. The steady-state implications are summarised in

Lemma 2. *Suppose that* $\tau > p/\upsilon E$ *holds. Then, there exist two steady-state equilibria* $\hat{k}_1$ *and* $\hat{k}_2$*, such than* $\hat{k}_1 = 0$ *and* $\hat{k}_2 > 0$*. The steady state* $\hat{k}_1$ *is locally asymptotically stable, while the steady state* $\hat{k}_2$ *is unstable.*

The condition $\tau > p/\upsilon E$ imposes a lower bound on the share of government expenditure that is allocated to abatement. Equivalently, it imposes an upper bound on the emission rate so that, even at very high levels of output, the effect of degradation due to pollution does not exceed the natural capacity of the environment (i.e., E). If this condition does not hold, the dynamic equilibrium of the economy resembles the one derived for $\upsilon = 0$; so the interested reader may resort to the analysis of that scenario in order to identify the possible

[10] Admittedly, the analysis of Chay and Greenstone (2003) focuses on infant mortality, while our paper deals with adult mortality. Nevertheless, their main message, which is the idea that pollution can be responsible for the periodic procyclicality of health-deterioration and mortality, corroborates with the implications of our theoretical set-up.

equilibrium scenarios of relaxing this restriction. Using Lemma 2, we can identify the economy's dynamic behaviour and transitional properties in the long-run. We do this through

Proposition 2. *Consider* $k_0 > 0$. *Then:*

(i) *If* $k_0 < \hat{k}_2$, *the economy will converge to the poverty trap* $\hat{k}_1 = 0$;

(ii) *If* $k_0 > \hat{k}_2$, *the economy will eventually converge to a 'long-run growth' equilibrium in which both capital per worker and output per worker grow at the rate* $\hat{\theta} = (1-\tau)(1-\gamma)\Gamma\frac{\lambda}{1+\lambda} - 1$.[11]

The dynamics of the economy are illustrated in Figure 4. Similarly to the previous scenario, the steady state $\hat{k}_2$ emerges as an endogenous threshold that determines long-term prospects according to the initial stock of capital per worker. Once more, an economy which is initially endowed with resources below this threshold will degenerate towards the poverty trap, where capital and output are very low – so low, in fact, that the reduced pollution cannot be translated into improvements in the health characteristics of the population. Naturally, the intuition behind this result is identical to the one provided in the case without pollution abatement.

What is particularly interesting, is the situation that occurs when the economy kick-starts its transition from a point that lies above the endogenous threshold $\hat{k}_2$. Contrary to the case where $\upsilon = 0$, in which capital per worker converges to an equilibrium without growth (that is, either a positive level for the stock of capital or a limit cycle), in this case the economy is able to sustain a positive rate of economic growth in the long-run. The reason is that pollution abatement limits the extent to which economic activity causes environmental damage. Thus, pollution abatement protects the population's health against the damage from environmental degradation and, therefore, the saving behaviour of workers is not impeded as the economy grows. Combined with the effect of the learning-by-doing externality in the production technology, a policy of environmental preservation allows the social marginal return of capital to be high enough so as to guarantee a positive rate of capital accumulation that, eventually, allows the economy to achieve balanced growth as an equilibrium outcome.

[11] Naturally, we assume that the value of Γ is sufficiently above unity so as to render the growth rate positive.

Moreover, as the economy grows without bound, environmental quality approaches from above a constant level that is equal to level $E-(p/\upsilon\tau)$;[12] for this to be positive it must be the case that $\tau > p/\upsilon E$, which we assumed in Lemma 2.[13]

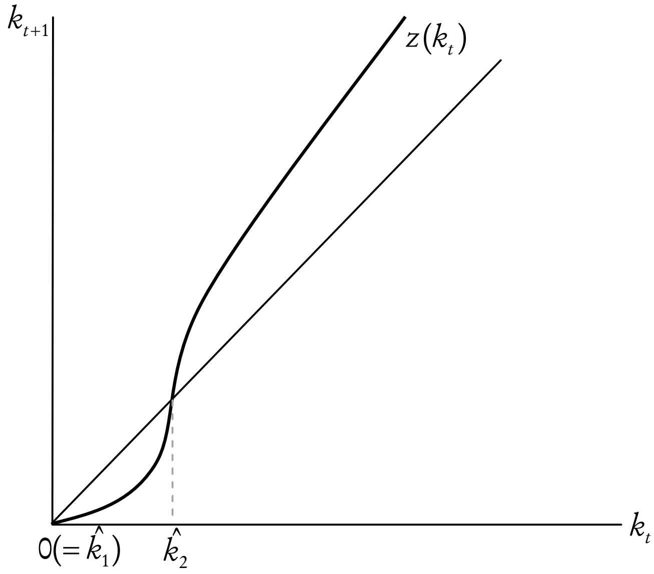

Figure 4. $0<\upsilon<1$

4 Some Important Implications

In the preceding sections of this paper, we have examined the transitional dynamics and the long-term equilibrium of an economy under two opposite scenarios concerning the government's engagement in policies that are designed to mitigate pollution and promote environmental quality. Apart from the common theme of multiple equilibria and the existence of poverty traps (an outcome related to the positive complementarities between saving and investment for some levels of the capital stock), the predictions from the two scenarios concerning the long-term prospects of economies that escape such poverty traps are strikingly different. The purpose of this section is to compare and contrast these

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12 Note that, as long as long as $\tau > p/\upsilon E$ (see Lemma 2), we can use previous results to obtain $\lim_{k_t\to\infty} e_t = \lim_{k_t\to\infty}\left(E-\frac{P_t}{1+a_t}\right) = \lim_{k_t\to\infty}\left(E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right) = E-\frac{p}{\upsilon\tau} > 0$.

13 The restriction imposed with Assumption 2 is sufficient but not necessary for the results of Lemma 2 and Proposition 2. Effectively, it ensures that only one endogenous threshold separates the two opposite convergence scenarios. In Appendix A6 we show that when this assumption is relaxed, it is possible that more equilibria emerge between the poverty trap and the long-run growth equilibrium. Nevertheless, the implication regarding the economy's ability to sustain a positive growth rate in the long-run remains intact.

predictions in order to derive important implications that arise as a result of the government's stance on activities of pollution abatement.

We begin with the implications concerning economic growth. As we have seen from equations (4) and (5), the labour's contribution to aggregate production is augmented by a productivity variable that is driven by the presence of an economy-wide, learning-by-doing externality similar to that used by Romer (1986). It is well known that, in standard dynamic general equilibrium models with production, such externalities allow the emergence of an equilibrium with ongoing output growth. In our framework, however, we have established that the learning-by-doing mechanism is not by itself sufficient to guarantee growth in the long-run. Indeed, such an equilibrium exists only when the government commits sufficient resources towards activities that abate pollution. Therefore, one significant implication from our analysis is given in

Corollary 1. *For an economy that avoids the poverty trap, pollution abatement is a complementary engine of long-run economic growth.*

This idea comes in stark contrast to previously held views concerning the macroeconomic repercussions of pollution. In her influential paper, Stokey (1998) argued that the prospects of long-run growth may be hampered as a result of the society's need to implement policies that support the quality of the natural environment – policies that are costly and, therefore, reduce the marginal product of capital to the extent that capital accumulation cannot be permanently sustained. By taking account of the well-documented effects of environmental quality to the overall health characteristics of the population, and their consequence for saving behaviour, our model has reached a different conclusion: policies that preserve some degree of environmental quality are, actually, essential for the existence of an equilibrium with ongoing output growth. Furthermore, notice that for environmental policy to achieve this outcome we do not require an equilibrium in which pollution declines constantly over time (see, for example, Smulders and Gradus 1996). In fact, pollution abatement supports long-run growth even though it is only capable of reducing the rate of environmental degradation, rather than eliminating it altogether.

Another important implication of our analysis is related to the existence of limit cycles. As we have seen, when pollution abatement is absent, it is possible for capital per worker to oscillate permanently around its positive steady state. Of course, such persistent fluctuations are different in nature from cycles whose impulse sources may be exogenous demand

and/or supply disturbances – the type of disturbances considered in the RBC and New-Keynesian literatures. In our model, both the impulse source and the propagation mechanism of cycles rest on the presence of non-monotonicity in the dynamics of capital accumulation. Thus, our framework shares more common features with the papers of Benhabib and Nishimura (1985); Comin and Gertler (2006); and Bambi and Licandro (2011) – all of whom discuss and derive cycles as endogenously determined phenomena whose existence depends on an economy's structural characteristics. That is, our model displays low frequency movements that are more closely associated with the movements that economists refer to as Juglar or Kondratieff cycles. Note that the analyses of Comin and Gertler (2006) and Bambi and Licandro (2011) provide some quantitative support for the empirical relevance of such movements.

Naturally, policies that could eradicate such fluctuations are policies that would address the source of non-monotonicities rather than counter-cyclical rules designed to mitigate temporary shifts from a given trend. With this in mind, a straightforward comparison between our two different scenarios allows us to infer

Corollary 2. *For an economy that avoids the poverty trap, pollution abatement is a source of stabilisation, in the sense that it eliminates the possibility of permanent cycles.*

Needless to say, we do not advocate the replacement of countercyclical (fiscal or monetary) policy by environmental policy. We make a reference to stabilisation in relation to medium- to long-term, rather than short-term cycles.

Given that environmental policy has an indirect positive effect on health and, consequently, life expectancy, our model derives implications which differ from those of Bhattacharya and Qiao (2007). In their model, the positive complementarities between private and public health spending implies that there is a trade-off between saving and private health expenditures. This trade-off generates non-monotonic capital dynamics, hence rendering health-enhancing public policy a source of endogenous fluctuations. In our model, a policy that facilitates health improvements (albeit indirectly through pollution abatement) actually eliminates such fluctuations.

Finally, by contrasting the results of our two different scenarios, it is possible to provide a novel explanation on the relationship between cycles and economic growth. We summarise this implication in

Corollary 3. *The government's stance on pollution abatement can generate a negative relationship between growth and cycles, in the sense that a policy supporting sustained long-run growth automatically eliminates the likelihood of persistent cycles.*

5 Endogenous Allocation of Government Expenditure

In this section we analyse the case where the government allocates its spending between public health services and pollution abatement endogenously. To simplify the algebra we restrict our attention to the case where $\varphi = \chi \leq 1$. Furthermore, we will also consider the case where the government allocates its spending to maximize the health status/lifespan of the citizens. That is,

$$\max_{0 \leq \upsilon_t \leq 1} \left\{ h_t = [(1-\upsilon_t)\tau\Gamma k_t]^{\varphi} \left(E - \frac{p\Gamma k_t}{1+\upsilon_t \tau \Gamma k_t} \right)^{\varphi} \right\}. \tag{22}$$

Note that under reasonable conditions, the scenario in (22) can be derived from a situation where the choice of υ_t maximises the welfare of a representative generation. For example, suppose that our original problem allows young agents to choose υ_t, in addition to saving, through a majority voting rule (A similar analysis is performed in Appendix B of Chakraborty (2004), where agents choose a tax rate). Then the problem would be to choose s_t and υ_t to maximise $V^t = \ln c_t + \beta_t \ln d_{t+1}$ subject to $c_t = w_t - s_t$, $d_{t+1} = r_{t+1} s_t$, (1), (2), and (6)-(10). It is straightforward to verify that the solution for saving remains the same as in (11), whereas the condition for υ_t can be written as $(\partial\beta_t / \partial\upsilon_t)\ln d_{t+1} = 0$. As long as $\ln d_{t+1} > 0$ holds, a condition for which we can appeal to appropriate restrictions in parameter values and the initial condition, then the optimal choice of υ_t can be derived from (22) because $\beta'(h_t) > 0$. We should note that the restriction $\ln d_{t+1} > 0$ is by no means an illogical one; failure of this condition would imply that agents prefer as limited a life span as possible – a possibility partially attributed to the technicalities of logarithmic utility. Naturally, one would prefer to assume away such an unreasonable scenario.

The solution to this maximisation problem in (22) is formally described in

Proposition 3. *Suppose that* $\tau E > p$. *Then, there exists a threshold* $\underset{\frown}{k} = \frac{1}{2\tau\Gamma}\left(\sqrt{\frac{4E\tau}{p} + 1} - 1 \right)$ *such that*

$$\upsilon_t = \begin{cases} \dfrac{-E + \left[E p \Gamma k_t (1 + \tau \Gamma k_t)\right]^{\frac{1}{2}}}{E \tau \Gamma k_t} \in (0,1) & \text{if } k_t > \underset{\frown}{k} \\ 0 & \text{if } k_t \leq \underset{\frown}{k} \end{cases}.$$

The result from Proposition 3 states that the government will find it optimal to initiate its efforts towards environmental preservation only at later stages of its development process. This becomes apparent in

Proposition 4. *Consider* $k_0 > 0$. *If* υ_t *is chosen endogenously, there is always a threshold level, say* $\overline{k}$*, such that, as long as* $k_0 > \overline{k}$*, the economy will eventually converge to a 'long-run growth' equilibrium in which both capital per worker and output per worker grow at a positive rate* $\hat{\theta} = (1-\tau)(1-\gamma)\Gamma \dfrac{\lambda}{1+\lambda} - 1$.

In Appendix A9 we show that there may be two cases leading to the result of Proposition 4. These two cases depend on whether the parameter values satisfy $\tau E > 2p$ or $2p > \tau E > p$. In the former case, there is only one non-trivial steady-state equilibrium, labelled as $\hat{k}_2$, which is unstable. Once again, this steady state emerges as an endogenous threshold that determines long-term prospects according to the initial stock of capital per worker (in terms of Proposition 4, it is $\hat{k}_2 = \overline{k}$). Countries that start with an initial capital stock below this threshold will decline monotonically towards a poverty trap where the (stable) steady state is $\hat{k}_1 = 0$. On the other hand, countries that start above this threshold level will experience smooth long-run growth. Diagrammatically, equilibrium outcomes resemble those presented in Figure 4.

In the latter case, however, outcomes may be slightly different in the sense that an additional (stable) steady-state equilibrium may emerge between the poverty trap and the long-run growth equilibrium. If this happens, then an economy for which $k_0 < \overline{k}$ need not necessarily fall into a poverty trap; instead, it may converge to a positive steady-state level of capital per worker. Still, however, this will be a stationary equilibrium with no long-run growth; achieving long-run growth requires that $k_0 > \overline{k}$. Diagrammatically, the equilibrium will either resemble the one presented in Figure 4 or the one presented in Figure 5.

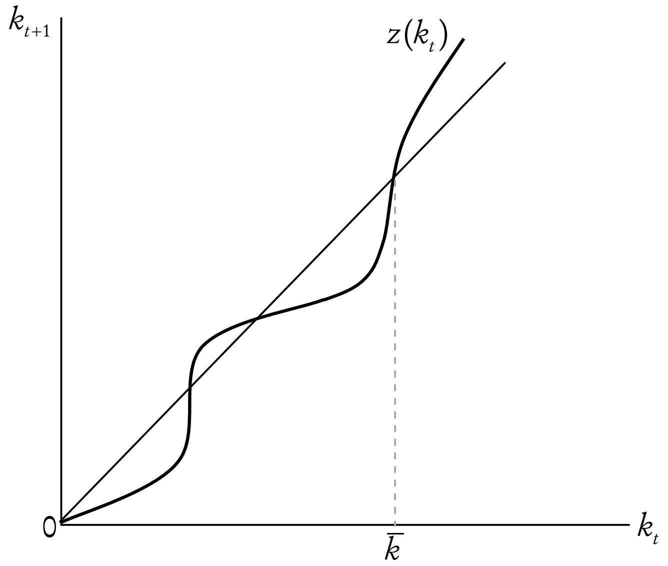

Figure 5

These details notwithstanding, we can conclude that, even with endogenous allocation of government resources, the commitment of some of these resources towards pollution abatement can allow some economies to achieve long-run growth. Furthermore, notice that, in comparison to the case where υ is set (permanently) equal to zero, the endogenous allocation of public spending eliminates the possibility of endogenous fluctuations. Hence, it verifies the role of pollution abatement as a tool for stabilisation in our framework.

6 Summary and Conclusion

We have constructed a two-period overlapping generations model where life expectancy is positively affected by the provision of public health services and by the quality of the natural environment. Environmental quality declines due to pollution – a by-product of economic activity. We showed that, despite the presence of an aggregate learning-by-doing externality, the economy cannot sustain a positive growth rate in the long-run if resources are not devoted towards environmental preservation. As the environment deteriorates without bound, the negative impact on life expectancy causes a reduction in saving and, therefore, the rate of capital formation: the economy's capital stock either converges to a stationary level or oscillates permanently. An equilibrium with ongoing output growth is possible only if the government commits a sufficient amount of resources towards pollution abatement.

Given that the possibility of cycles disappears in the latter scenario, we concluded that an active policy of environmental preservation is not only an important complementary engine of long-run growth, but a powerful tool of stabilisation as well.

We view our analysis and its results as pinpointing the importance of environmentally-oriented policies, as a means of not only supporting the environment, but also supporting the economy's prospects for sustained economic growth. This is an important consideration in light of the emerging acceptance of the EKC hypothesis and its misinterpretation as a proof that environmental problems will be (more or less) automatically resolved as economies achieve higher levels of GDP per capita – a view that combined with the perceived trade-off between growth-promoting and environment-promoting activities, may lead to the implementation of misguided policies. As long as environmental degradation entails the negative externalities that formed part of our theoretical framework – and indeed, there is ample evidence to suggest this – failure to take actions today will not only cause serious environmental problems in the future, but it will also impede the prospects of economic growth and may result in economic instability.

Obviously, our framework can be enriched with respect to several aspects that could broaden its scope and implications. For example, an obvious direction is to consider private resources in support of abatement activities, in addition to the public ones. Despite the fact that such an extension generates free-riding issues and requires a crucial assumption regarding the degree at which individuals internalise the effect of their own activities on an aggregate outcome such as environmental quality, it would allow us to examine the trade-off between saving and environmental spending. This trade-off would most probably allow an additional channel through which environmental factors impinge on saving and capital accumulation. Moreover, a similar trade-off exists between saving and individual health spending. As our focus in this paper has been on the public policy dimension of pollution abatement and health, we have decided to abstract from these issues. Nevertheless, we view them as important extensions that we plan to undertake in the near future.

Appendix

(A major portion of this Appendix is not intended for publication)

A1 Proof of Lemma 1

Using equation (21), we define the function

$$J(k_t) = \frac{z(k_t)}{k_t} = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left((\tau\Gamma k_t)^{\varphi}(E-p\Gamma k_t)^{\chi}\right)}{1+\mathrm{B}\left((\tau\Gamma k_t)^{\varphi}(E-p\Gamma k_t)^{\chi}\right)}. \tag{A1.1}$$

Clearly, any interior steady state must satisfy $J(\hat{k}) = 1 \Leftrightarrow \hat{k} = z(\hat{k})$. From (A1.1), we have $J(0) = 0$ and, by virtue of (8), $J(k_t) = 0 \ \forall k_t \geq E / p\Gamma$. Thus, for an interior steady state to exist, there must be at least one $\tilde{k}$ such that $J(\tilde{k}) \geq 1$. When this condition holds with strict inequality then there will be at least two interior steady states; otherwise, there will not be any interior equilibrium at all (see Figure A1).

Combining (A1.1) with (1), (2), (7), (8) and (9) allows us to derive

$$J'(k_t) = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2} \frac{dh_t}{dk_t}, \tag{A1.2}$$

where

$$\frac{\partial h_t}{\partial k_t} = \varphi\tau\Gamma(\tau\Gamma k_t)^{\varphi-1}(E-p\Gamma k_t)^{\chi} - p\Gamma\chi(\tau\Gamma k_t)^{\varphi}(E-p\Gamma k_t)^{\chi-1}. \tag{A1.3}$$

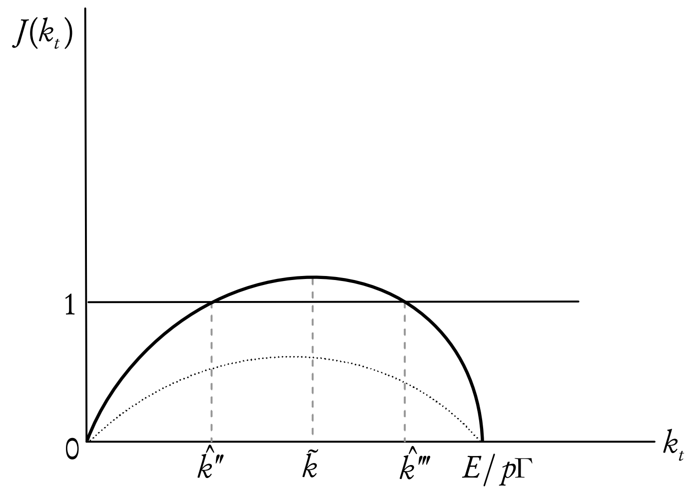

Figure A1. Interior solutions require $J(\tilde{k}) > 1$

For $0 \le k_t \le E / p\Gamma$, the sign of (A1.3) determines the sign of $J'(k_t)$. Straightforward factorisation allows us to write (A1.3) as

$$\frac{\partial h_t}{\partial k_t} = (\tau\Gamma k_t)^{\varphi}(E - p\Gamma k_t)^{\chi}\left(\frac{\varphi}{k_t} - \frac{\chi p\Gamma}{E - p\Gamma k_t}\right),$$

which means that $\frac{\partial h_t}{\partial k_t} \ge 0$ iff

$$\frac{\varphi}{k_t} \ge \frac{\chi p\Gamma}{E - p\Gamma k_t} \Rightarrow$$

$$\varphi E - \varphi p\Gamma k_t \ge \chi p\Gamma k_t \Rightarrow$$

$$k_t \le \frac{\varphi}{\varphi + \chi}\frac{E}{p\Gamma} \equiv \tilde{k}.$$

The preceding analysis implies that there exists a unique $\tilde{k} \in (0, E / p\Gamma)$ such that

$$J'(k_t)\begin{cases} > 0 & \text{for} \quad k_t < \tilde{k} \\ = 0 & \text{for} \quad k_t = \tilde{k}, \\ < 0 & \text{for} \quad k_t > \tilde{k} \end{cases}$$

i.e., $J(\tilde{k})$ is a global maximum. We can use this result to identify the parameter combination that allows the existence of interior equilibria. Particularly, we can solve $(\tau\Gamma\tilde{k})^{\varphi}(E - p\Gamma\tilde{k})^{\chi}$ using $\tilde{k} = \varphi E / (\varphi + \chi) p\Gamma$. Doing so, we derive $(\varphi\tau / p)^{\varphi}\chi^{\chi}[E / (\varphi + \chi)]^{\varphi+\chi} \equiv \Omega$. Hence, by the Intermediate Value Theorem, Assumption 1 is a sufficient condition for the existence of interior equilibria. Moreover, if this condition holds, then there exist two interior steady-state equilibria $\hat{k}'''$ and $\hat{k}''$ satisfying $\hat{k}''' > \tilde{k} > \hat{k}'' > 0$; thus, $J'(\hat{k}'') > 0$ and $J'(\hat{k}''') < 0$.

Using (A1.1) we can derive

$$J'(k_t) = \frac{z'(k_t)k_t - z(k_t)}{(k_t)^2}. \tag{A1.4}$$

Given (A1.4), $J'(\hat{k}'') > 0$ implies

$$z'(\hat{k}'') > \frac{z(\hat{k}'')}{\hat{k}''} \Rightarrow$$

$$z'(\hat{k}'') > J(\hat{k}'') \Rightarrow$$

$$z'(\hat{k}'') > 1,$$

because $J(\hat{k}'')=1$. Thus, $\hat{k}''$ is an unstable equilibrium.

Similarly, (A1.4) implies that $J'(\hat{k}''')<0$ is equivalent to $z'(\hat{k}''')<1$. In this case, however, we cannot make any definite conclusions concerning the stability of this equilibrium as we do not yet know whether the dynamics generated by equation (21) are monotonic. For this reason, let us return to the transition equation $k_{t+1}=z(k_t)$. Given (21), we can see that $z(0)=0$, $z(k_t)=0\ \forall k_t \geq E/p\Gamma$ and $z(k_t)>0$ for $k_t \in (0, E/p\Gamma)$. Thus, the dynamics of capital accumulation may not be non-monotonic which means that, indeed, the stability properties of $\hat{k}'''$ cannot be determined with certainty. Particularly, $\hat{k}'''$ is a stable long-run equilibrium if $z'(\hat{k}''')>-1$; otherwise, i.e., if $z'(\hat{k}''')<-1$, the equilibrium $\hat{k}'''$ is unstable.

In our preceding analysis, we have established that $z(0)=0$. Of course, this result indicates that $\hat{k}'=0$ is a steady state. Moreover,

$$z'(k_t)=J'(k_t)k_t+J(k_t),$$

and since, from equations (A1.2) and (A1.3),

$$\lim_{k\to 0}\left(\frac{dh_t}{dk_t}k_t\right)=0 \quad \text{and} \quad J'(k_t)k_t=0,$$

it follows that $z'(\hat{k}')=z'(0)=0$, i.e., $\hat{k}'=0$ is a super-stable equilibrium. ■

A2 Proof of Proposition 1

Part (i) follows from Lemma 1 in which we have shown that $\hat{k}'=0$ is an asymptotically stable equilibrium while $\hat{k}''>0$ is an unstable one. Hence, given $\hat{k}''>\hat{k}'$, we can safely conclude that, for any $k_0<\hat{k}''$, it is $k_{t+1}=z(k_t)<k_t$, i.e., the economy's capital per worker will constantly decline until it converges to the poverty trap $\hat{k}'=0$.

To prove part (ii), we can once more utilise Lemma 1. In particular, let us consider the case where $\hat{k}'''$ is an asymptotically stable equilibrium, i.e., the case for which $\left|z'(\hat{k}''')\right|<1$. Given $\hat{k}'''>\hat{k}''$, we may conclude that for $k_0>\hat{k}''$ the transitional dynamics imply that $\lim_{t\to\infty}k_t=\hat{k}'''$. Also, using (20), we have $\theta_{t+1}=(k_{t+1}/k_t)-1$ and, thus,

$$\lim_{t\to\infty}\theta_{t+1}=\lim_{t\to\infty}\left(\frac{k_{t+1}}{k_t}\right)-1=\lim_{t\to\infty}\left(\frac{z(k_t)}{k_t}\right)-1=\lim_{t\to\infty}J(k_t)-1=J(\hat{k}''')-1=0. \qquad \text{(A2.1)}$$

Therefore, the economy will converge (either monotonically or through damped oscillations) to a long-run equilibrium with a positive stock for capital per worker, but zero growth.

Now, let us consider the possibility that $z'(\hat{k}''') \leq -1$. Although $\hat{k}'''$ is an unstable steady-state equilibrium, it is well known that when the transition equation is non-monotonic and its slope at the steady state is negative and sufficiently steep (that is, below -1), then the dynamical system may exhibit periodic equilibria. In terms of our model, consider a sequence of n discrete points along the 45^0 line, denoted $\breve{k}_\eta$ for $\eta = \{1, 2, ..., i-1, i, i+1, ...n\}$, such that $\breve{k}_1 < ... < \breve{k}_{i-1} < \breve{k}_i < \hat{k}''' < \breve{k}_{i+1} < ... < \breve{k}_n$ and

$$z(k_t)\begin{cases} > k_t & \text{for} \quad \eta \in [1, i] \\ \\ < k_t & \text{for} \quad \eta (i, n] \end{cases}.$$

If, for $k_0 > \hat{k}''$, the capital stock passes repeatedly through the points $\breve{k}_\eta$ during its transition, then the economy converges to a period-n cycle, where the sequence $\breve{k}_\eta$ represents periodic (rather than stationary) equilibria. Indeed, as long as $z'(\hat{k}''') < -1$, the function $z(k_t)$ satisfies the following

Theorem (Azariadis, 1993, 86-88). *Suppose* 0 *and* $\hat{k} > 0$ *are fixed points of the scalar system* $k_{t+1} = z(k_t)$ *in which* $z : R_+ \supseteq X \to R_+$ *and* $z \in C^1$. *Suppose also that there exists a* $b > \hat{k}$ *such that* $b > z(b)$ *and* $b > z^2(b)$, *where* z^2 *is the second iterate of* z. *Then* $z'(\hat{k}) < -1$ *is a sufficient condition for the existence of a period-2 cycle* $\{\breve{k}_1, \breve{k}_2\}$ *that satisfies* $\breve{k}_1 < \hat{k} < \breve{k}_2 < b$.

Thus, the system $k_{t+1} = z(k_t)$ exhibits (at least) a period-2 cycle. To apply this Theorem to our case, let $\hat{k} = \hat{k}'''$ and $b = E / p\Gamma$. Naturally, the growth rate θ_{t+1} will be positive during phases of the transition for which $\eta \in [1, i]$ but negative during phases of the transition for which $\eta \in (i, n]$. Hence, a long-run equilibrium with a constantly positive growth rate does not exist. ■

A3 An Example of an Economy with Cycles

We can illustrate the results in Proposition 1 by means of a simple numerical example. Suppose that

$$\mathrm{B}(h_t)=\frac{\lambda h_t}{1+h_t},\ 0<\lambda<1.$$

This functional form satisfies the properties of $\mathrm{B}(\cdot)$. Let also $\tau=0.2$, $\gamma=0.3$, $p=0.3$, $\Gamma=10$, $E=1$, $\varphi=0.7$, $\chi=0.2$. Then at $\lambda=0.682$ a saddle-node bifurcation occurs; that is, the number of fixed points (steady states), except from the origin, is none for $\lambda<0.682$, one for $\lambda=0.682$ and two for values of $\lambda>0.682$. In particular, if $\lambda<0.682$ the origin is the only steady-state equilibrium (Assumption 1 is not satisfied). At $\lambda=0.682$ the function $z(k_t)$ is tangent to the 45^0 degree line and hence there is only one interior steady state. If $\lambda>0.682$ there are two interior steady-state equilibria, say $\hat{k}''$ and $\hat{k}'''$. The lower equilibrium, $\hat{k}''$, is repelling, whereas the stability of the higher equilibrium, $\hat{k}'''$, depends on the value of λ. For example, if $\lambda=0.7$ then any orbit that starts in the neighbourhood of $\hat{k}'''$ converges to it monotonically, since $0<z'(\hat{k}''')<1$. On the other hand, if we let $\lambda=0.75$, then the convergence to $\hat{k}'''$ occurs through damped oscillations since $0>z'(\hat{k}''')>-1$. Next, suppose that we let $\lambda=0.78$. Simple calculations show that the stability of the equilibrium $\hat{k}'''$ changes since $z'(\hat{k}''')<-1$; i.e., $\hat{k}'''$ becomes a repelling equilibrium. At the same time there is a period-2 cycle $\{0.306, 0.326\}$, which is stable since its multiplier is $z^{2\prime}(0.306)=\ z^{2\prime}(0.326)=z'(0.306)z'(0.326)=-0.452>-1$ (z^2 denotes the second iterate of z, i.e., $z^2(k_t)=z(z(k_t))$). Next, suppose that we raise λ to 0.8. Then again simple calculations reveal that, while $\hat{k}'''$ remains a repelling equilibrium, the period-2 cycle has become an unstable one (the value of its multiplier is lower than -1). Instead, there is a period-4 cycle now, which is stable. This process continues as λ increases. In other words, the system undergoes a sequence of period-doubling bifurcations; that is, there is an increasing sequence of bifurcation points, such that for values of λ between any two consecutive members of the sequence λ_n and λ_{n+1} the prime 2^n-period solution is stable, while the periodic solutions of all other periods $2,4,\ldots,2^{n-1}$ become unstable.

A4 Proof of Lemma 2

Consider again the function

$$J(k_t)=\frac{z(k_t)}{k_t}=(1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}. \tag{A4.1}$$

Given the properties of $\mathrm{B}(h_t)$ and the restriction $\tau > p/\upsilon E$, it can be easily established that $J(0)=0$ and $J(\infty)=(1-\tau)(1-\gamma)\Gamma\lambda/(1+\lambda)$. An interior steady state must satisfy $J(\hat{k})=1 \Rightarrow \hat{k}=z(\hat{k})$. Therefore, Assumption 1 represents a sufficient condition for the existence of an interior equilibrium. This is because $\mathrm{B}(\infty)=\lambda$ and $\mathrm{B}(h_t)/[1+\mathrm{B}(h_t)]$ is increasing in h_t; therefore $\lambda > \mathrm{B}(\Omega)$.

Differentiating (A4.1) yields

$$J'(k_t)=(1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2}\frac{dh_t}{dk_t},$$

where

$$\begin{aligned}\frac{dh_t}{dk_t}&=\varphi(1-\upsilon)\tau\Gamma[(1-\upsilon)\tau\Gamma k_t]^{\varphi-1}\left(E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\\&-\chi[(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi-1}\frac{p\Gamma}{(1+\upsilon\tau\Gamma k_t)^2}.\end{aligned} \tag{A4.2}$$

Substituting (A4.2) in the expression for $J'(k_t)$ gives us

$$J'(k_t)=\frac{(1-\tau)(1-\gamma)\Gamma\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2}[(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\Xi(k_t). \tag{A4.3}$$

where

$$\Xi(k_t)=\frac{\varphi}{k_t}-\frac{\chi p\Gamma}{(1+\upsilon\tau\Gamma k_t)^2}\frac{1}{E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}}. \tag{A4.4}$$

Obviously, the sign of $J'(k_t)$ depends on the sign of $\Xi(k_t)$ in (A4.4). Particularly, for this to be non-negative, it must be $\Xi(k_t)\geq 0$. After some algebraic manipulation, the inequality $\Xi(k_t)\geq 0$ is reduced to a quadratic expression

$$(k_t)^2+\frac{(\upsilon\tau E-p)+\left(\upsilon\tau E-\frac{p\chi}{\varphi}\right)}{(\upsilon\tau E-p)\upsilon\tau\Gamma}k_t+\frac{E}{(\upsilon\tau E-p)\upsilon\tau\Gamma^2}\geq 0. \tag{A4.5}$$

As long as $2\upsilon\tau E > p(\varphi+\chi)/\varphi$, which is true if $\tau > p/\upsilon E$ and $\chi \le \varphi$ (Assumption 2), the above expression holds with strict inequality and, by virtue of (A4.3) and (A4.4), $J'(k_t) > 0\ \forall k_t$. Hence, there is only one interior steady state $\hat{k}_2$ with $J'(\hat{k}_2) > 0$. Moreover, it can be easily checked that $J'(\hat{k}_2) > 0 \Rightarrow z'(\hat{k}_2) > 1$, i.e., the interior steady state is unstable.

Next, notice from equation (19) that $z(0) = 0$; hence, $\hat{k}_1 = 0$ is a steady state. Moreover,

$$z'(k_t) = J'(k_t)k_t + J(k_t),$$

and, since from equations (A4.3) and (A4.4)

$$\lim_{k\to 0} \Xi(k_t)k_t = 0 \quad \text{and} \quad J'(k_t)k_t = 0,$$

it follows that $z'(\hat{k}_1) = z'(0) = 0$, i.e., $\hat{k}_1 = 0$ is a super-stable equilibrium. ■

A5 Proof of Proposition 2

Part (i) follows from Lemma 2. Specifically, given that $\hat{k}_1 = 0$ is an asymptotically stable equilibrium and $\hat{k}_2 > 0$ is an unstable one, for any $k_0 < \hat{k}_2$, we have $k_{t+1} < k_t$ for all subsequent steps of the transition. Hence, the economy's stock of capital per worker will constantly decline until it converges to the poverty trap $\hat{k}_1 = 0$.

To prove part (ii), we can use (19) and (20) to write the gross growth rate as

$$\frac{k_{t+1}}{k_t} = 1+\theta_{t+1} = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E - \dfrac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E - \dfrac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}, \tag{A5.1}$$

for which Appendix A4 establishes that $k_{t+1} > k_t \Rightarrow 1+\theta_{t+1} > 1$ (as long as $k_0 > \hat{k}_2$), because the dynamics of capital accumulation are monotonic. Therefore, (A5.1) can be written as

$$k_t = \prod_{\varepsilon=0}^{t}(1+\theta_\varepsilon)k_0. \tag{A5.2}$$

From equation (A5.2) we can verify that $\lim_{t\to\infty} k_t = k_\infty \to \infty$. Therefore, we can use equation (A5.1) to establish that

$$\lim_{t\to\infty} \theta_{t+1} = \theta_\infty =$$

$$\lim_{t\to\infty}\left[(1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E-\frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}-1\right]=$$

$$(1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_\infty]^{\varphi}\left(E-\frac{p\Gamma k_\infty}{1+\upsilon\tau\Gamma k_\infty}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_\infty]^{\varphi}\left(E-\frac{p\Gamma k_\infty}{1+\upsilon\tau\Gamma k_\infty}\right)^{\chi}\right)}-1=$$

$$(1-\tau)(1-\gamma)\Gamma\frac{\lambda}{1+\lambda}-1=\hat{\theta}.$$

Since $(1-\tau)(1-\gamma)\Gamma\lambda/(1+\lambda)>1$ holds by assumption, $\hat{\theta}>0$: asymptotically, the economy will converge to a balanced growth path where capital per worker grows at a rate $\hat{\theta}$. ■

A6 Analysis of the Model when Assumption A2 is Relaxed

Our basic analysis utilised the restriction $\chi\le\varphi$. In this part of the Appendix, we shall demonstrate that all the main implications of our model survive even when this restriction is relaxed. To begin with, we can readily verify that this restriction has no bearing at all for the analysis and results of the case with no pollution abatement ($\upsilon=0$). Indeed, Assumption 2 was not used in the proofs of Lemma 1 and Proposition 1. For this reason, we shall focus on the case where policies of pollution abatement are active.

The main repercussion from relaxing $\chi\le\varphi$ relates to the possibility that we may have $2\upsilon\tau E<p(\varphi+\chi)/\varphi$. Therefore, the inequality $\Xi(k_t)\ge 0$ which we examined in equation (A4.4) (see Appendix A4, proof of Lemma 2) may not hold for every k_t. Using obvious definitions, we can rewrite the left-hand side of (A4.5) in the form $(k_t)^2+\zeta k_t+\delta\Rightarrow(k_t-k_-)(k_t-k_+)$, where

$$k_-=\frac{-\zeta-\sqrt{\zeta^2-4\delta}}{2}\quad\text{and}\quad k_+=\frac{-\zeta+\sqrt{\zeta^2-4\delta}}{2}. \tag{A6.1}$$

Notice that, for $2\upsilon\tau E<p(\varphi+\chi)/\varphi$, it is $\zeta<0$. Moreover, after some tedious but straightforward algebra it can be shown that $\zeta^2-4\delta>0$, i.e., both roots are real and positive. Therefore, we can use (A6.1) in (A4.4) so as to infer that, given (A4.3), we have

$$J'(k_t)\begin{cases} >0 & \text{for} \quad k_t < k_- \\ <0 & \text{for} \quad k_- < k_t < k_+ \\ >0 & \text{for} \quad k_t > k_+ \end{cases}.$$

Given $J(0)=0$ and $J(\infty)=(1-\tau)(1-\gamma)\Gamma\lambda/(1+\lambda)$, the preceding analysis shows that k_- corresponds to local maximum while k_+ corresponds to a local minimum. Consequently, there may be three interior steady-state solutions from which the lowest and the highest are unstable. Thus, the difference with the results of Section 3.2 is that we may have an additional, asymptotically stable steady state for the stock of capital per worker, separating the poverty trap and the long-run growth equilibrium. Furthermore, in this case we would have two endogenous thresholds – one separating the poverty trap and the no-growth equilibrium while the other separating the no-growth and the long-run growth equilibria. Figures A2 and A3, below, illustrate such outcomes.

Notice that, although the situation illustrated in Figures A2 and A3 is possible, under certain conditions the model's equilibrium may still be qualitatively identical to the one derived in Section 3.2. Particularly, this happens if either $J(k_-)<1$ or $J(k_+)>1$ (see Figures A4 and A5, respectively). In both cases, there can only be one steady state, $\hat{k}^2$, with $J'(\hat{k}^2)>0 \Rightarrow z'(\hat{k}^2)>1$, i.e., an unstable steady state. Therefore, the model's behaviour resembles the one described in the main part of the paper.

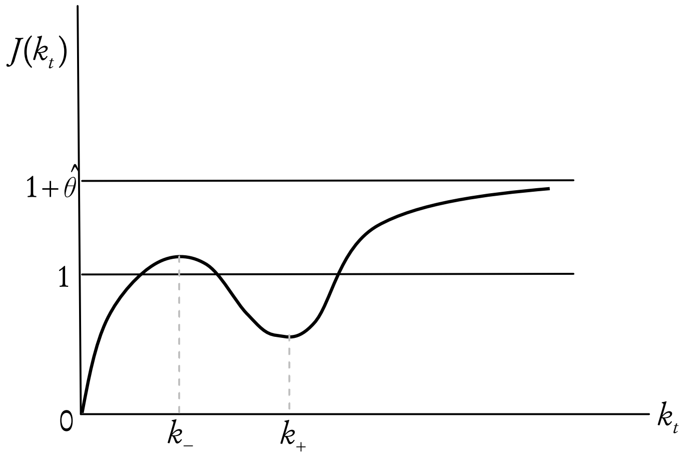

Figure A2. Three interior steady states

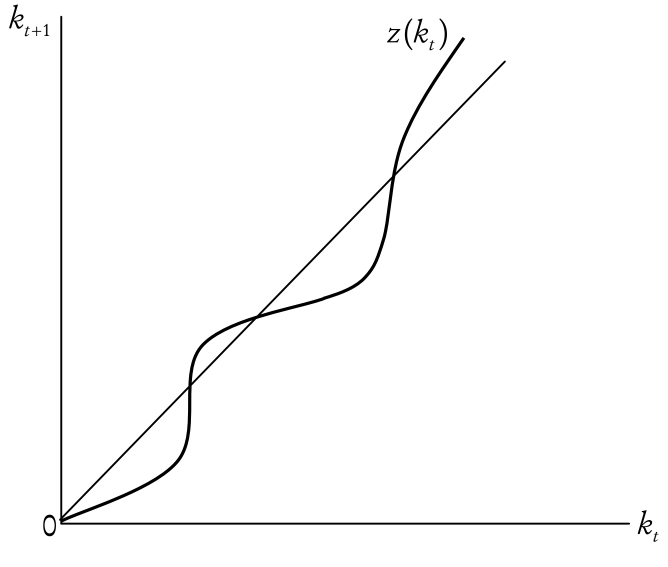

Figure A3. The dynamics of capital accumulation with three interior steady states

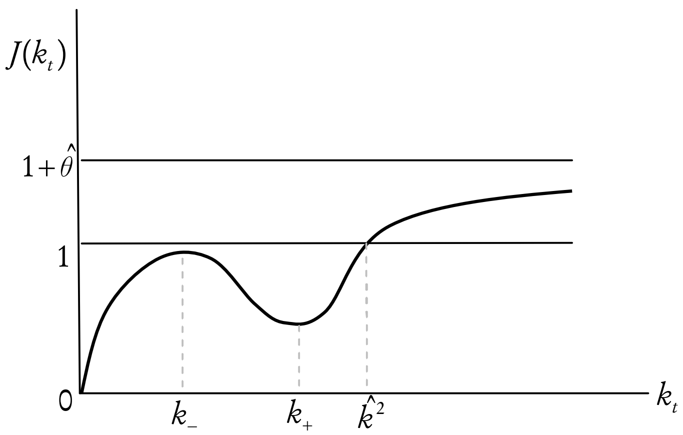

Figure A4. $J(k_-) < 1$

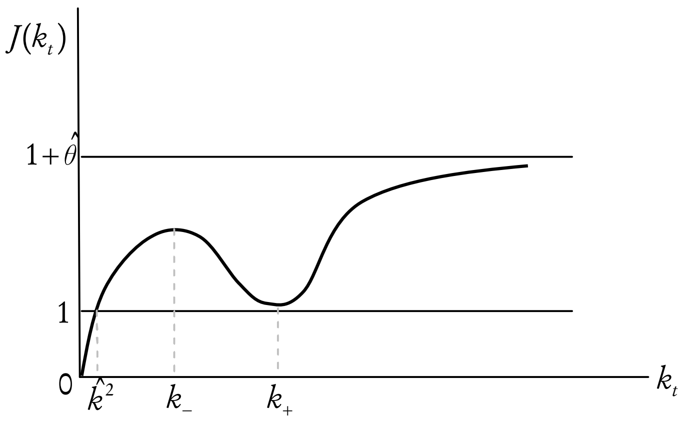

Figure A5. $J(k_+) > 1$

A7 Proof of Proposition 3

The maximisation problem in (22) leads to

$$\upsilon_t^* = \frac{-E + \left[Ep\Gamma k_t\left(1+\tau\Gamma k_t\right)\right]^{1/2}}{E\tau\Gamma k_t}. \tag{A7.1}$$

Note that a sufficient condition for $\upsilon_t^* < 1$ is $\tau E > p$. It is also straightforward to establish that the non-negativity constraint $\upsilon_t^* \geq 0$ is satisfied for $k_t \geq \underset{\frown}{k}$, where

$$\underset{\frown}{k} = \frac{1}{2\tau\Gamma}\left(\sqrt{\frac{4E\tau}{p}+1}-1\right). \tag{A7.2}$$

■

A8 Proof of Proposition 4

Using the result in Proposition 3 and substituting (A7.1), together with (8), (9) and (10), in (2) we derive

$$h_t = \begin{cases} \left[(\tau\Gamma k_t)(E - p\Gamma k_t)\right]^{\varphi} & \text{if } k_t \leq \underset{\frown}{k} \\ \left\{[E(1+\tau\Gamma k_t)]^{1/2} - (p\Gamma k_t)^{1/2}\right\}^{2\varphi} & \text{if } k_t > \underset{\frown}{k} \end{cases}. \tag{A8.1}$$

Appropriate substitution of (A7.2) in (A8.1) reveals that the function h_t is continuous; therefore, the function $z(k_t)$ is continuous as well. Also note that $\lim\limits_{k_t\to\infty} h_t = \lim\limits_{k_t\to\infty}(p\Gamma k_t)^{\varphi} \lim\limits_{k_t\to\infty}\left\{\left[\frac{E(1+\tau\Gamma k_t)}{p\Gamma k_t}\right]^{1/2} - 1\right\}^{2\varphi} = \infty$ since $\tau E > p$ implies $E(1+\tau\Gamma k_t)/p\Gamma k_t > 1$. Thus, $\lim\limits_{k_t\to\infty} \mathrm{B}(h_t) = \mathrm{B}(\infty) = \lambda$.

Consider

$$J(k_t) = \frac{k_{t+1}}{k_t} = \frac{z(k_t)}{k_t} = (1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}(h_t)}{1+\mathrm{B}(h_t)}.$$

Obviously, for $k_t \leq \underset{\frown}{k}$ the properties of this expression are identical to those analysed in Appendix A1. Now let us examine the properties for $k_t > \underset{\frown}{k}$. First of all, we can use the previous analysis to establish that $J(\infty) = (1-\tau)(1-\gamma)\Gamma\lambda/(1+\lambda) > 1$. Furthermore, it is

$$J'(k_t) = (1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2}\frac{dh_t}{dk_t},$$

where

$$\frac{dh_t}{dk_t} = 2\varphi h_t^{(2\varphi-1)/2\varphi}\left\{[E(1+\tau\Gamma k_t)]^{-1/2}E\tau\Gamma - (p\Gamma k_t)^{-1/2}p\Gamma\right\}.$$

Hence,

$$J'(k_t) > 0 \quad \text{iff} \quad k_t > \frac{p}{\tau\Gamma(E\tau - p)} \equiv \varkappa .$$

In Appendix A1 we showed that the expression $J(k_t)$ is increasing for $k_t < \tilde{k}$ where $\tilde{k} = \frac{\varphi}{\varphi + \chi}\frac{E}{p\Gamma}$. Now, since $\varphi = \chi$, the corresponding value is $\tilde{k} = \frac{E}{2p\Gamma}$. Of course, as long as $\tilde{k} > \underset{\frown}{k}$, the switch in regime from $\upsilon_t^* = 0$ to $\upsilon_t^* > 0$ occurs in the upward sloping part of $J(k_t)$. After some straightforward algebra, we can show that $\tilde{k} > \underset{\frown}{k} > \varkappa$ iff $\tau E > 2p$.

Assume for the moment that $\tau E > 2p$. Notice that if $k_t < \underset{\frown}{k}$, then the function $J(k_t)_{\upsilon=0}$ is monotonically increasing since $k_t < \underset{\frown}{k} < \tilde{k}$. Also, if $k_t > \underset{\frown}{k}$, then the function $J(k_t)_{\upsilon=\upsilon^*}$ is again monotonically increasing because $k_t > \underset{\frown}{k} > \varkappa$. Thus, as long as $\tau E > 2p$, it is $J'(k_t) > 0$ for every $k_t > 0$. Given that $J(0) = 0$ (recall that for $k_t \leq \underset{\frown}{k}$ it is $\upsilon_t^* = 0$) and $J(\infty) > 1$, there is only one steady-state equilibrium, say $\bar{k}$, which is clearly unstable. An analysis similar to that in Appendix A5 suffices to establish that for $k_0 > \bar{k}$, the economy can achieve long-run economic growth.

Next, we consider the case where $2p > \tau E > p$. In this case $\tilde{k} < \underset{\frown}{k} < \varkappa$ and the behaviour of the system may or may not be qualitatively identical to the one described above. Based on the previous results we can infer that the function $J(k_t)$ is increasing over the interval $(0,\tilde{k})$, decreasing over the interval $(\tilde{k},\varkappa)$ and increasing for values of k_t greater than $\varkappa$. Hence, if $J(\varkappa) > 1$, then there is again one unstable interior steady state, $\bar{k}$, and for $k_0 > \bar{k}$, the economy will achieve long-run economic growth. Nevertheless, if $J(\varkappa) < 1$, then it is easy to check that, in addition to the stable steady-state $\hat{k}_1 = 0$, there will be three interior steady-states $\hat{k}_2 < \hat{k}_3 < \bar{k}$ from which $\hat{k}_2 \in (0,\tilde{k})$ and $\bar{k} > \varkappa$ will be unstable (because $J'(\hat{k}_2), J'(\bar{k}) > 0$) but $\hat{k}_3 \in (\tilde{k},\varkappa)$ may be stable since $J'(\hat{k}_3) < 0$. Once more, for $k_0 > \bar{k}$ the economy will attain positive growth in the long-run. For $k_0 < \bar{k}$, however, the economy may converge to $\hat{k}_3 > 0$ instead of the poverty trap. ■

A9 Environmental Quality as a Stock Variable

In this section we test the robustness of our results when environmental quality is a stock instead of a flow variable. To that end, we replace equation (8) in the main text with the following equation:

$$e_t = \begin{cases} e_{t-1}^{\eta}(E - D_t)^{1-\eta} & \text{if } D_t < E \\ 0 & \text{otherwise} \end{cases}, \tag{8'}$$

where $0 < \eta < 1$ and D_t is given in equation (7). Now, environmental quality is a combination of the maximum long-run level of environmental quality E, the current environmental degradation D_t, and the environmental quality of last period e_{t-1}. Clearly, this formulation encompasses the case analysed in the main text. Indeed, if $\eta = 0$, then equation (8′) gives (8). Following the same steps as before, we find that the dynamics of the system are now described by equations

$$k_{t+1} = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(m_{t+1}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(m_{t+1}\right)^{\chi}\right)} k_t, \tag{A9.1}$$

$$m_{t+1} = m_t^{\eta}\left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{1-\eta}, \tag{A9.2}$$

where $m_{t+1} \equiv e_t$. This planar system of difference equations is quite complex. We therefore consider a parametric example. Specifically, we adopt the previously mentioned functional form for $\mathrm{B}(\cdot)$: $B(h_t) = \lambda h_t/(1+h_t)$, $0 < \lambda < 1$, which satisfies all the properties listed after equation (1). Moreover, we let $\varphi = \chi = 1$, so that equation (A9.1) becomes

$$k_{t+1} = \frac{(1-\tau)(1-\gamma)\lambda(1-\upsilon)\tau\Gamma^2 k_t^2 m_t^{\eta}\left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{1-\eta}}{1+(1+\lambda)(1-\upsilon)\tau\Gamma k_t m_t^{\eta}\left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{1-\eta}}. \tag{A9.3}$$

Whenever it is necessary, we also use the following parameter values: $\tau = 0.4$, $\gamma = 0.4$, $p = 0.3$, $\Gamma = 25$, $E = 10$, $\upsilon = 0.5$, and $\eta = 0.5$. Nevertheless, our results are robust to perturbations to these parameter values.

The steady-state loci $k_{t+1} = k_t = k$ and $m_{t+1} = m_t = m$ are given by

$$m=\left\{\frac{1}{\left[(1-\tau)(1-\gamma)\Gamma\lambda-(1+\lambda)\right]\Gamma\tau(1-\upsilon)k\left(E-\frac{p\Gamma k}{1+\upsilon\tau\Gamma k}\right)^{1-\eta}}\right\}^{\frac{1}{\eta}}, \qquad \text{(A9.4)}$$

and

$$m=E-\frac{p\Gamma k}{1+\upsilon\tau\Gamma k}, \qquad \text{(A9.5)}$$

respectively. To ensure that the steady state is well-defined we must impose throughout this section the condition

$$\lambda>\frac{1}{(1-\tau)(1-\gamma)\Gamma-1}, \qquad \text{(A9.6)}$$

which, with the above mentioned parameter values, requires that $\lambda>0.125$.

No Pollution Abatement. We consider first the case where there is no pollution abatement, that is $\upsilon=0$. The system of equations (A9.3) and (A9.2) simplifies to

$$k_{t+1}=\frac{(1-\tau)(1-\gamma)\lambda\tau\Gamma^2k_t^2m_t^\eta\left(E-p\Gamma k_t\right)^{1-\eta}}{1+(1+\lambda)\tau\Gamma k_t m_t^\eta\left(E-p\Gamma k_t\right)^{1-\eta}} \qquad \text{(A9.7)}$$

and

$$m_{t+1}=m_t^\eta\left(E-p\Gamma k_t\right)^{1-\eta}. \qquad \text{(A9.8)}$$

Moreover, the steady-state loci (A9.4) and (A9.5) become

$$m=\left\{\frac{1}{\left[(1-\tau)(1-\gamma)\Gamma\lambda-(1+\lambda)\right]\Gamma\tau k\left(E-p\Gamma k\right)^{1-\eta}}\right\}^{\frac{1}{\eta}}, \qquad \text{(A9.9)}$$

and

$$m=E-p\Gamma k, \qquad \text{(A9.10)}$$

respectively. These two loci are drawn in Figure A6.

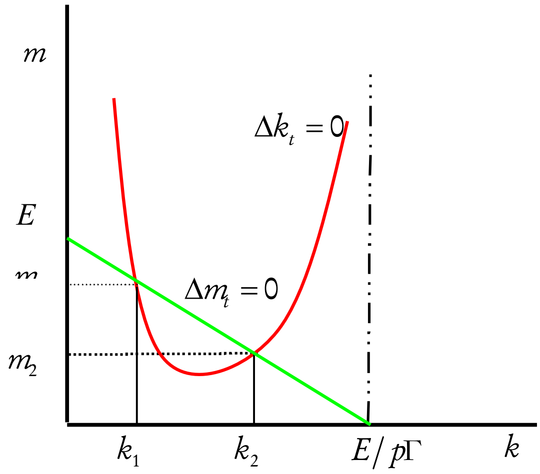

Figure A6. Multiple Equilibria with Environmental Quality as in Eq. $(8')$

There are two steady-state equilibria. For example, if $\lambda = 0.4$ then the two equilibria are $(k_1, m_1) = (0.005, 9.966)$ and $(k_2, m_2) = (1.329, 0.034)$. Moreover, when $\lambda = 0.4$, at both equilibria one eigenvalue has modulus greater and the other less one. Thus, both equilibria are saddle-path stable. Nevertheless, the stability properties of the equilibria change with λ. In particular, at $\lambda = 0.159183$ one of the two steady-state equilibria is (1.296, 0.282). At this equilibrium, one eigenvalue is real and falls on the boundary of the unit circle with value -1. Hence a flip (or period doubling) bifurcation occurs. Moreover, the following facts occur: a) the other eigenvalue has modulus less than 1 and b) the derivative of the modulus of the first eigenvalue evaluated at $\lambda = 0.159183$ is different from zero (422.343). It follows then from the Flip Bifurcation Theorem (see, for example, Azariadis p. 97, Theorem 8.4) that in a sufficiently small neighbourhood of the equilibrium the system has a periodic orbit of period 2. Using numerical methods, we can find that the cycle occurs for values of λ that are greater than 0.159183. For example, when $\lambda = 0.4$ the two-period cycle is (0.72, 0.0142) and (1.33, 0.256). Moreover, since the steady-state equilibrium $(k_2, m_2) = (1.329, 0.034)$ that the cycle surrounds is unstable (saddle-path stable), the cycle is attracting.

<u>Pollution Abatement.</u> If there is pollution abatement, that is $\upsilon > 0$, then the system is described by equations (A9.2) and (A9.3) and the steady-state loci $k_{t+1} = k_t = k$ and $m_{t+1} = m_t = m$ are given by the equations (A9.4) and (A9.5), respectively. It follows that if, as in Lemma 2, the percentage of tax revenue that the government allocates to abatement (υ) is high enough or, more specifically, if $\upsilon > p/(\tau(1-\eta)E)$, then there will be a unique steady-state equilibrium (see Figure A7). For example, if we adopt the parameter values specified above, then for every value $\lambda > 0.125$ there is a unique steady-state equilibrium, which is saddle-path stable (both eigenvalues are real; one has modulus greater and the other less than one). Thus, no cycles emerge. A country that starts with an initial capital stock that is greater than k_1 and not on the saddle path will be able to grow unboundedly.

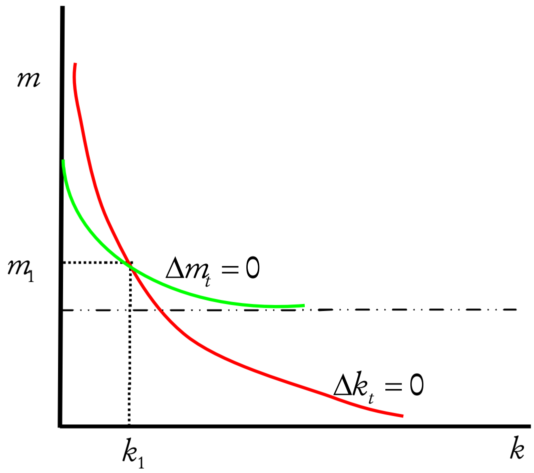

Figure A7. Pollution Abatement with Environmental Quality as in Eq. $(8')$

<u>An Alternative Specification.</u> Next, we analyze the stability of the system using a different specification from that in equation $(8')$. Let

$$e_t = \begin{cases} (1-\eta)E + \eta e_{t-1} - D_t & \text{if } D_t < (1-\eta)E + \eta e_{t-1} \\ 0 & \text{otherwise} \end{cases}, \tag{8''}$$

where D_t is again given in equation (7). Now, environmental quality is a convex combination of the maximum long-run level of environmental quality E and the environmental quality of last period. This formulation can also encompass the case analyzed in the main text. If $\eta = 0$, then equation $(8'')$ gives (8). Following the same steps as before the system becomes

$$k_{t+1} = \frac{(1-\tau)(1-\gamma)\lambda(1-\upsilon)\tau\Gamma^2 k_t^2 \left[(1-\eta)E + \eta m_t - \dfrac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right]}{1 + (1+\lambda)(1-\upsilon)\tau\Gamma k_t \left[(1-\eta)E + \eta m_t - \dfrac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right]}, \tag{A9.11}$$

and

$$m_{t+1} = (1-\eta)E + \eta m_t - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}, \tag{A9.12}$$

which, in steady-state, becomes

$$m = \frac{1}{\eta}\left\{\frac{1}{\left[(1-\tau)(1-\gamma)\Gamma\lambda-(1+\lambda)\right]\Gamma\tau(1-\upsilon)k} - \left((1-\eta)E - \frac{p\Gamma k}{1+\upsilon\tau\Gamma k}\right)\right\}, \tag{A9.13}$$

and

$$m = E - \frac{1}{1-\eta}\frac{p\Gamma k}{1+\upsilon\tau\Gamma k}. \tag{A9.14}$$

In what follows, we use the same parameter values as above except for the value of η. Now the algebra is relative simple even with a value of η different than 0.5; hence, we use $\eta = 0.8$, which diminishes the role of *E*. Nevertheless, as mentioned above, our results hold for a wide range of parameter values.

If there is no pollution abatement, i.e., $\upsilon = 0$, equations (A9.11) and (A9.12) simplify to

$$k_{t+1} = \frac{(1-\tau)(1-\gamma)\lambda\tau\Gamma^2 k_t^2\left[(1-\eta)E+\eta m_t - p\Gamma k_t\right]}{1+(1+\lambda)\tau\Gamma k_t\left[(1-\eta)E+\eta m_t - p\Gamma k_t\right]}. \tag{A9.15}$$

and

$$m_{t+1} = (1-\eta)E + \eta m_t - p\Gamma k_t, \tag{A9.16}$$

and, in steady-state,

$$m = \frac{1}{\eta}\left\{\frac{1}{\left[(1-\tau)(1-\gamma)\Gamma\lambda-(1+\lambda)\right]\Gamma\tau k} - \left((1-\eta)E - p\Gamma k\right)\right\}, \tag{A9.17}$$

and

$$m = E - \frac{1}{1-\eta}p\Gamma k. \tag{A9.18}$$

These two loci are drawn in Figure A8. There are two steady state equilibria. For example, if $\lambda = 0.4$ then the two equilibria are $(k_1, m_1) = (0.005, 9.827)$ and $(k_2, m_2) = (0.262, 0.173)$. Moreover, when $\lambda = 0.4$, at both equilibria the eigenvalues are real; one of them has modulus greater and the other less one. Thus, both equilibria are saddle-path stable.

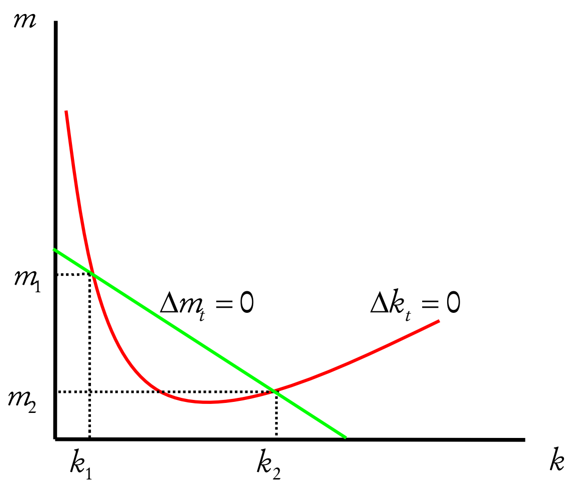

Figure A8. Multiple Equilibria with Environmental Quality as in Eq. $(8'')$

Nevertheless, as was the case with the previous specification regarding the evolution equation of environmental quality, the stability properties of the equilibria change with λ. In particular, at $\lambda = 0.174$ there are also two equilibria: (0.029, 9.829) and (0.238, 1.071). Evaluated at the second equilibrium, the system has two complex eigenvalues, $0.817 \pm 0.576i$, which have modulus 1, i.e., the equilibrium is non-hyperbolic. Neither of these eigenvalues is a second, third, nor fourth root of unity.[14] Moreover, the derivative of the modulus of the eigenvalues, evaluated at $\lambda = 0.174$, is equal to $1.468 > 0$. It follows then from the Hopf Bifurcation Theorem, also known as Andronov-Hopf and Naimark-Sacker Theorem, that there exists a limit cycle in the neighbourhood of the equilibrium for either $\lambda > 0.174$ or $\lambda < 0.174$ (see Azariadis 1993, Theorem 8.5, p. 100). To find out whether the cycle emerges for values of λ higher or lower than 0.174 we need to apply some additional rather technical tests (see Devaney 2003, Theorem 8.8, p. 249). Nevertheless, using numerical methods, we can find that the cycle emerges for values of $\lambda >$ 0.174. For example if $\lambda = 0.4$, then there is a two-period cycle: (0.202, 0.075) and (0.315, 0.547). Moreover, since the steady-state equilibrium $(k_2, m_2) = (0.262, 0.173)$ that the cycle surrounds is unstable in the saddle-path sense, it follows that the cycle is attracting. Finally, numerical investigations have shown that cycles of higher periodicity may also emerge.

[14] An nth root of unity, $n = 1, 2, \ldots$, is a complex number z satisfying the equation $z^n = 1$.

If there is pollution abatement, that is $\upsilon > 0$, then the system is described by equations (A9.2) and (A9.3). Once again, as with the previous specification regarding the evolution of the environmental quality, if, as in Lemma 2, the percentage of tax revenue that the government allocates to abatement (υ) is high enough $(\upsilon > p/(\tau(1-\eta)E))$, then there will be a unique steady-state equilibrium (see Figure A9). For the values specified above and $\lambda = 0.4$, there is a unique steady-state equilibrium $(k_1, m_1) = (0.009, 9.663)$, which is saddle-path stable. Hence, no cycle emerges. A country that starts with an initial capital stock that is greater than k_1 and not on the saddle path will be able to grow unboundedly.

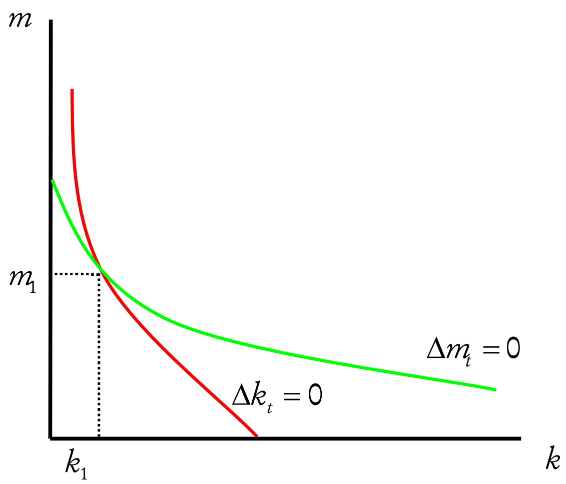

Figure A9. Pollution Abatement with Environmental Quality as in Eq. $(8'')$

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