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Abstract

This paper examines the effects of government debt policies with *imperfect* substitutability between securities issued by different countries. It puts forward an intertemporal model of a small open economy to analyze the effects of government debt on the real interest rate, economic growth, private consumption and the balance of payments. The model is an endogenous growth, overlapping generations model with convex adjustment costs for investment, and imperfect substitutability between domestic and foreign bonds. It is shown than an increase in the government debt to output ratio causes the spread between the domestic and the foreign real interest rate to rise and the endogenous growth rate to fall. In addition, when domestic government bonds are relatively close substitutes for foreign bonds, the rise in government debt causes a temporary rise in domestic consumption, as current generations view government debt as wealth. The current account moves into deficit, the economy decumulates net foreign assets, and in the new long run equilibrium both the consumption to output ratio and net foreign assets as a proportion of output fall. When domestic government bonds are not close substitutes for foreign bonds, the rise in government debt causes a temporary fall in domestic consumption, as the negative real interest rate effect of the rise in government debt dominates the wealth effect. In this case the current account improves and in the new long run equilibrium both the consumption to output ratio and net foreign assets as a proportion of output rise.

Keywords: government debt, interest rates, endogenous growth, external balance, fiscal policy
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1. Introduction

The assumption of perfect capital mobility, or perfect substitutability between domestic and foreign securities, has been a mainstay of open economy macroeconomics since the early 1960s, following the pathbreaking contribution of Mundell (1963). It has survived as a core assumption in the more recent intertemporal approach to international macroeconomics, because of its simplicity and ease of use. Differences in interest rates among open economies are usually attributed to expectations about future changes in the exchange rate in the context of international asset markets that trade in otherwise perfectly substitutable securities expressed in different currencies.

Yet the post-2008 experience of a number of small eurozone economies, that have seen their interest rates diverge despite sharing a common currency, points us to other characteristics, besides exchange rate expectations, that differentiate government securities and their rates of return. It would be hard to explain macroeconomic developments in these economies while continuing to use the assumption of perfect substitutability between bonds issued by different governments.

This paper sets out to examine the effects of government debt policies with *imperfect* substitutability between securities issued by different countries. It puts forward an endogenous growth model of a small open economy to study the effects of government debt on the real interest rate, economic growth, private consumption and the balance of payments.

The model is an endogenous growth, overlapping generations model with convex adjustment costs for investment, and imperfect substitutability between domestic and foreign bonds.

It is shown than an increase in the government debt to output ratio causes the spread between the domestic and the foreign real interest rate to rise and the endogenous growth rate to fall. In addition, the effects of government debt on private consumption, the current account and external balance depend on the degree of substitutability between domestic and foreign bonds.

If domestic bonds are fairly close substitutes for foreign bonds, an increase in government debt causes a temporary rise in domestic private consumption, as current generations view government debt as part of their wealth. The current account moves into deficit, the economy decumulates net foreign assets, and in the new long run equilibrium both the consumption to output ratio and net foreign assets as a proportion of output fall.

The opposite effects take place when domestic bonds are relatively imperfect substitutes for foreign bonds. In this case, an increase in government debt causes a temporary fall in domestic consumption, as the real interest rate rises significantly, because of the low

substitutability between domestic and foreign bonds. The rise in the real interest rate causes larger reductions in investment and growth, but more significantly it causes private consumption to fall, as the large real interest rate effect dominates the wealth effect of the rise in government debt. The current account moves into surplus, the economy accumulates net foreign assets, and in the new long run equilibrium both private consumption and net foreign assets rise as a proportion of output.

The rest of the paper is organized as follows: Section 2 presents an endogenous growth model with learning by doing and adjustment costs for investment. In section 3 we derive the equilibrium growth rate and examine its dependence on the domestic real interest rate and the aggregate productivity of domestic capital. In section 4 we model the demand for domestic government securities in the context of an international bond market in which investors view domestic securities as imperfect substitutes for foreign securities. We demostrate that the equilibrium domestic real interest rate depends of the real interest in the rest of the world, but also on the relative supplies of government debt domestically and in the rest of the world. In section 5 we model the demand side and external balance. We use a continuous time overlapping generations models for the determination of private consumption, and derive the full equilibrium. The full model determines the domestic real interest rate, the domestic real wage, the growth rate, as well as equilibrium private consumption and net foreign assets relative to domestic output. In section 6 we examine the effects on government debt on the main endogenous variables, namely the growth rate, the consumption rate and net foreign assets. We also analyse the dynamics of the current account following a change in government debt. The final section sums up the conclusions.

2. Adjustment Costs for Investment and Endogenous Growth

We assume a small open economy, consisting of competitive firms that produce an internationally traded good.

2.1 Production

The production function of firm i at time t is given by,

$$Y_{it} = AK_{it}^{\alpha}(h_t L_{it})^{1-\alpha}, \quad 0<\alpha<1 \tag{1}$$

where Y is output, K physical capital, L the number of employees and h the efficiency of labour. The efficiency of labour is the same for all firms.

Following Arrow (1962) we assume learning by doing. In particular we assume that the efficiency of labour is a linear function of the aggregate capital labour ratio. Thus,

$$h_t = B\left(\frac{K}{L}\right)_t, \quad 0<\beta<1 \tag{2}$$

where B is a constant, and K/L is the aggregate capital labour ratio.[1]

Substituting (2) in (1) and aggregating, we get aggregate output as a linear function of aggregate physical capital.

$$Y_t = \bar{A} K_t \tag{3}$$

where,

$$\bar{A} = AB^{1-\alpha} \tag{4}$$

Due to the linearity of the aggregate production function, the rate of economic growth g is equal to the rate of net capital accumulation, which in turn is determined by the rate of investment. Therefore, we have,

$$g = \dot{Y}_t / Y_t = \dot{K}_t / K_t = (I_t / K_t) - \delta = \bar{A}(I_t / Y_t) - \delta \tag{5}$$

where I is gross investment and δ the rate of depreciation.

In this endogenous growth model, the growth rate is determined by the ratio of gross investment to GDP.

2.2. Adjustment Costs and the Rate of Investment

Investment is determined by the profit maximization decisions of private firms. We assume that new investment is subject to a marginal adjustment cost which is a function of the ratio of new investment goods to total installed capital.[2]

Thus, the instantaneous profits of firms are given by,

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[1] The linearity of (2) is what makes our model an endogenous growth model. The results would hold for the transition path of exogenous growth models as well. For endogenous growth models see Romer (1986), Lucas (1988), and a number of comprehensive recent surveys such as Barro and Sala-i-Martin (2004), Aghion and Hewitt (2009), Acemoglou (2009).

[2] See Lucas (1967), Gould (1968), Abel (1982) and Hayashi (1982) for models of investment with similar adjustment costs.

$$Y_{it} - w_t L_{it} - \left[1 + \frac{\phi}{2}\left(\frac{I_{it}}{K_{it}}\right)\right] I_{it} \tag{6}$$

where w is the real wage and ϕ is a positive constant measuring the intensity of the marginal adjustment cost of new investment.

$\phi\left(\frac{I_{it}}{K_{it}}\right)$ is the marginal adjustment cost.

Each firm selects employment and investment in order to maximize the present value of its profits.

$$\int_{s=t}^{\infty} e^{-rs}\left(Y_{is} - w_s L_{is} - \left[1 + \frac{\phi}{2}\left(\frac{I_{is}}{K_{is}}\right)\right] I_{is}\right) ds \tag{7}$$

under the constraint,

$$\dot{K}_{is} = I_{is} - \delta K_{is} \tag{8}$$

r is the real domestic interest rate.

From the first order condition for a maximum of (7) subject to (8),

$$w_t = (1-\alpha) A \left(\frac{K_{it}}{L_{it}}\right)^{\alpha} h_t^{1-\alpha} \tag{9}$$

$$q_{it} = 1 + \phi\left(\frac{I_{it}}{K_{it}}\right) = 1 + \phi\left(\frac{\dot{K}_{it}}{K_{it}} + \delta\right) \tag{10}$$

$$\left(r + \delta - \frac{\dot{q}_{it}}{q_{it}}\right) q_{it} = \alpha A \left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} h_t^{1-\alpha} + \frac{\phi}{2}\left(\frac{\dot{K}_{it}}{K_{it}} + \delta\right)^2 \tag{11}$$

where q is the shadow price of installed physical capital.

From (9), employment is determined so that the marginal product of labour for the firm equals the real wage. Given that the real wage is the same for all firms, all firms will choose the same capital-labour ratio.

From (10), the shadow price of installed capital is equal to the marginal cost of new investment. This is equal to the cost of purchase of new capital goods, plus the marginal adjustment cost of investment.

From (11), the user cost of capital (on the left hand side) is equal to the marginal product of capital (on the right hand side). The marginal product of capital has two components: The marginal product in production of new output (the first term on the right hand side) and the reduction of the adjustment cost of future investment (the second term on the right hand side). A higher capital stock today means a smaller marginal adjustment cost for future investment.

It is worth noting that if ϕ is equal to zero (no adjustment cost for investment), then q is equal to one (from (10)). (11) then becomes the well known condition that the real interest rate r is equal to the net marginal product of capital.

Thus, if $q=1$, then, $r = \alpha A\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} h_t^{1-\alpha} - \delta$.

3. The Growth Rate and the Shadow Price of Capital

Aggregating (9) to (11), taking into account (2) to (5), we have the following aggregate first order conditions.

$$w_t = (1-\alpha)\bar{A}\left(\frac{K_t}{L_t}\right) = (1-\alpha)\left(\frac{Y_t}{L_t}\right) \tag{12}$$

$$q_t = 1 + \phi\left(\frac{\dot{K}_t}{K_t} + \delta\right) = 1 + \phi(g+\delta) \tag{13}$$

$$\left(r + \delta - \frac{\dot{q}_t}{q_t}\right) q_t = \alpha\bar{A} + \frac{\phi}{2}\left(\frac{\dot{K}_t}{K_t} + \delta\right)^2 = \alpha\bar{A} + \frac{\phi}{2}(g+\delta)^2 \tag{14}$$

Equation (12) determines the real wage. In the steady state, with a rising capital labour ratio and rising productivity of labour, the real wage is rising along with productivity. In fact, the real wage is a constant share of (the rising) output per worker.

Equations (13) and (14) can be used to determine the equilibrium shadow price of capital q and the equilibrium growth rate g as functions of the domestic real interest rate and the exogenous technological and adjustment costs parameters.

Using (13) to substitute for q in (14) we get,

$$(r+\delta)(1+\phi(g+\delta)) = \alpha \bar{A} + \frac{\phi}{2}(g+\delta)^2 \tag{15}$$

In (15) we have imposed the condition of a constant q (and g), since both q and the investment and growth rates are non predetermined variables.

Equation (15) is a quadratic equation in g and has two solutions which lie on either side of r, the domestic real interest rate. Only the solution with $g < r$ is stable in the sense of satisfying the transversality condition for the maximization of the present value of profits for firms. This solution implies that the equilibrium growth rate g_E is determined by,

$$g_E = r - \sqrt{r^2 - \frac{2}{\phi}\left(\alpha \bar{A} - (r+\delta)(1+\phi\delta)\right) - \delta^2} \tag{16}$$

Equilibrium q, say q_E, will be determined by substituting (16) in (13).

In what follows we shall assume that the equilibrium growth rate is a real number, which requires that,

$$(r)^2 \geq \frac{2}{\phi}\left(\alpha \bar{A} - (r+\delta)(1+\phi\delta)\right) - \delta^2$$

Under this assumption, one can prove the following two propositions:

First, *the equilibrium growth rate depends negatively on the domestic real interest rate*.

Proof : From (16), the first derivative of the equilibrium growth rate with respect to the domestic real interest rate is given by,

$$\frac{\partial g_E}{\partial r} = -\frac{g_E + \frac{1}{\phi}(1+\phi\delta)}{\sqrt{r^2 - \frac{2}{\phi}\left(\alpha\bar{A} - (r+\delta)(1+\phi\delta)\right) - \delta^2}} < 0$$

Second, *the equilibrium growth rate depends positively on the aggregate productivity of capital*.

Proof : From (16), the first derivative of the the equilibrium growth rate with respect to the aggregate productivity of capital is given by,

$$\frac{\partial g_E}{\partial \bar{A}} = \frac{\alpha/\phi}{\sqrt{r^2 - \frac{2}{\phi}\left(\alpha\bar{A} - (r+\delta)(1+\phi\delta)\right) - \delta^2}} > 0$$

The difference between the domestic real interest rate and the endogenous growth rate is given by,

$$r - g_E = \sqrt{r^2 - \frac{2}{\phi}\left(\alpha\bar{A} - (r+\delta)(1+\phi\delta)\right) - \delta^2} \qquad (17)$$

It is straightforward to show that *the real interest rate growth differential* is a positive function of the domestic real interest rate and a negative function of the productivity of domestic capital.

The determination of equilibrium is depicted graphically in Figure 1.

The positively sloped straight line depicts (13). (14) is the curved line, as (14) is a non-linear relation. The only stable equilibrium is at *E*, given that the second equilibrium (not shown in the diagram) does not satisfy the transversality condition.

The position of (14) depends (among other factors such as φ and δ) on the domestic real interest rate and the aggregate productivity of domestic capital.

A rise in the real interest rate causes (14) to shift to the right (downwards). In the new equilibrium, (see Figure 2), both the growth rate and the shadow price of capital fall. Thus, a rise in the domestic real interest rate causes the domestic investment rate and the long run growth rate to fall. As a result, the differential between the real interest rate and the domestic growth rate also increases (widens).

A rise in the productivity of domestic capital causes (14) to shift to the left (upwards). In the new equilibrium, (see Figure 3), both the growth rate and the shadow price of capital rise. Thus, a rise in the productivity of domestic capital causes the investment rate and the long run growth rate to rise. As a result, the differential between the real interest rate and the domestic growth rate becomes smaller (narrows).

We shall return to the implications of these effects when we also examine external balance. First however we shall turn to the determination of the domestic real interest rate.

4. Government Debt and the Domestic Real Interest Rate

Unlike most of the literature that assumes perfect substitutability between domestic and foreign assets, we shall assume that domestic bonds are imperfect substitutes for foreign bonds. This will allow us to look at the determinants of the spread between domestic and international interest rates.

We assume that bond returns are determined in the international capital market. International investors are assumed to have the following preferences over domestic government bonds B and government bonds from the rest of the world (foreign bonds) B^*.

$$V(B,B^*) = \left[\left(\frac{B}{Y} \right)^{\theta} + \left(\frac{B^*}{Y^*} \right)^{\theta} \right]^{\frac{1}{\theta}} \tag{18}$$

$1/(1-\theta)$ is the elasticity of substitution between domestic and foreign bonds, and Y and Y^* are domestic output and output in the rest of the world respectively. In (18) it is assumed that the utility of government bonds for foreign investors depends on the size (output) of the issuing country.[3]

Equation (18) is maximized subject to the wealth constraint,

$$\frac{B}{1+r} + \frac{B^*}{1+r^*} = W^* \tag{19}$$

where r is the domestic real interest rate, r^* the foreign real interest rate (exogenous because of the small open economy assumption) and W^* total global wealth allocated to

[3] We follow this approach to modelling differences in the characteristics of different government securities because this is a deterministic model, and the risk characteristics of the various securities are not explicitly modelled. Note that in a stochastic model $1-\theta$ would be the *coefficient of relative risk aversion*.

investment in government bonds (also assumed exogenous). The price of domestic and foreign bonds are *1/(1+r)* and *1/(1+r*)* respectively.

From the maximization of (18) subject to (19), we get,

$$\left(\frac{B}{B^*}\frac{Y^*}{Y}\right)^{\theta-1}=\frac{1+r^*}{1+r} \tag{20}$$

Taking logs on both sides of (20), it follows that the equilibrium domestic real interest rate, r_E, is given by,

$$r_E \simeq r^*+(1-\theta)\left(\frac{b}{b^*}-1\right) \tag{21}$$

where $b=B/Y$ and $b^*=B^*/Y^*$ are the government debt to output ratios in the domestic economy and the rest of the world respectively.

Assuming that the elasticity of substitution is positive ($\theta<1$), the domestic real interest rate spread is a positive function of the domestic government debt to GDP ratio. Because of imperfect substitutability between domestic and foreign bonds, a rise in the government debt to GDP ratio causes a rise in the domestic real interest rate. The rise depends on the inverse of the elasticity of substitution. As the elasticity of substitution tends to infinity, $1-\theta$ tends to zero and domestic bonds become perfect substitutes for foreign bonds. The perfect substitutes case, which is the one mainly used in the open economies literature, implies equality between domestic and foreign real interest rates.

Since we have assumed that the domestic economy is small, in the sense that it cannot affect the world real interest rate, (21) determines the domestic real interest rate solely in terms of the world real interest rate and a spread that depends on the relative government debt to output ratios in the domestic economy and the rest of the world.

Thus, the domestic real interest rate differs from the real interest rate in the rest of the world, only to the extent that the domestic government debt to output ratio differs from the government debt to output ratio in the rest of the world.

From (21),

$$\frac{\partial r_E}{\partial b}=\frac{1-\theta}{b^*}>0$$

A rise in the government debt to output ratio causes the domestic real interest rate to rise, and, through (16), the domestic equilibrium growth rate to fall. As a result, the shadow price of capital in the domestic economy also falls (see Figure 2).

We can now turn to the demand side and the determination of external balance.

5. The Demand Side and the Determination of External Balance

In this section we look at the demand side, analyse the behavior of domestic consumers and we take a closer look at the government budget constraint.

To analyse the demand side, we assume a continuous time overlapping generations model (Blanchard (1995), Weil (1997)). In this model, private consumption is endogenous. Assuming an exogenous fiscal policy, the demand side also determines the net foreign position of the country and the evolution of the current account.

5.1 Private Consumption and Net Foreign Assets

Assuming the Weil version of the model, in which the probability of death is zero, the (optimal) consumption function is given by,

$$\dot{c}_t = \left(r_E - \rho + n - g_E\right)c_t - n\rho\left(q_t k_t + b_t + f_t\right) \tag{22}$$

where c is the ratio of private consumption to income, ρ the pure rate of time preference of households, n the population growth rate (which equals the rate of increase of the number of households), k the capital output ratio, b the government debt to output ratio and f the ratio of net foreign assets to output. The sum $qk+b+f$ measures net household wealth as a ratio to total output. Obviously, if net foreign assets are negative (external debt is positive), then household wealth is smaller than the total value of domestic capital and public debt.

The current account is defined as the difference between national savings and investment. As a percentage of output it is defined as,

$$\dot{f}_t = 1 + \left(r^* - g_E\right)f_t - c_t - c_{gt} - \left(g_E + \delta\right)q_E k_t \quad \text{for } f_t > 0 \tag{23a}$$

$$\dot{f}_t = 1 + \left(r_E - g_E\right)f_t - c_t - c_{gt} - \left(g_E + \delta\right)q_E k_t \quad \text{for } f_t < 0 \tag{23b}$$

where c_g is the ratio of public consumption to output.

In (23a) we have assumed that if net foreign assets are positive their real return is equal to the world real interest rate. If net foreign assets are negative, as in (23b), their real return is equal to the domestic real interest rate.

The first two terms on the right hand side of (23) define national income relative to domestic income (product). The last three terms (private consumption, public consumption and investment) define total domestic demand (absorption). If absorption is smaller than national income, the current account is positive and the country accumulates net foreign assets. In the opposite case, the current account is negative (in deficit) and the country sees its net foreign position worsen.[4]

5.2 Government Debt and the Government Budget Constraint

The evolution of the public debt to output ratio is determined by the government budget constraint,

$$\dot{b}_t = (r_E - g_E) b_t + c_{gt} - \tau_t \tag{24}$$

where τ is the ratio of total current (tax) revenue to output. We shall assume that taxes are lump sum.

To the extent that the domestic real interest rate exceeds output growth, as is the case in this model, the debt to output ratio increases without limit, unless there is a primary suplus that exactly offsets interest payments on the existing debt.

In what follows we shall assume that the government consumption to output ratio is constant, determined by the government, and that the government adjusts total current revenue to achieve a primary surplus that stabilises the public debt to output ratio.

From (24), this means that,

$$\bar{b} = \frac{\tau - \bar{c}_g}{r_E - g_E} \tag{25}$$

The bar over b and c_g denotes the governmental targets for public debt and public consumption respectively.

[4] Our model belongs to the class of models based on the intertemporal approach to external balance. See Obstfeld and Rogoff (1995, 1996) for early surveys of models based on the intertemporal approach.

From (25), one can determine the primary surplus that is required in order to keep a constant public debt to output ratio, as a function of the world real interest rate, the domestic real interest rate premium and the growth rate of output.

It is obvious from (25) that an increase in the domestic real interest rate, which, as we have already demonstrated, causes a reduction in the growth rate as well, requires a corresponding increase in the primary budget surplus. If this does not happen, the public debt to output ratio diverges from the government's target, and becomes unsustainable.

In what follows, we then assume a tax revenue rule that ensures sustainability at all times. This takes the form,

$$\tau = \bar{c}_g + (r_E - g_E)\bar{b} \tag{26}$$

Under this tax rule, the government debt to output ratio is kept continuously stable, as changes in the other variables (world interest rates, the domestic spread and the growth rate) are not allowed to affect the debt to output ratio, since taxes adjust to bring about offsetting changes in the primary surplus.

5.3 Equilibrium

Using the aggregate production function (3),

$$k_t = \bar{A}^{-1} \tag{27}$$

Substituting (27) in (22) and (23) and assuming a constant public debt to output ratio, we get,

$$\dot{c}_t = \left(r_E - \rho + n - g_E\right)c_t - n\rho\left(q_E \bar{A}^{-1} + \bar{b} + f_t\right) \tag{28}$$

$$\dot{f}_t = 1 + \left(r^* - g_E\right)f_t - c_t - \bar{c}_g - \left(g_E + \delta\right)\bar{A}^{-1} \quad \text{for } f_t > 0 \tag{29a}$$

$$\dot{f}_t = 1 + \left(r_E - g_E\right)f_t - c_t - \bar{c}_g - \left(g_E + \delta\right)\bar{A}^{-1} \quad \text{for } f_t < 0 \tag{29b}$$

We can use (28) and (29) to analyse the determination of the private consumption to output ratio and the net foreign asset position of the country, as functions of the fiscal

policy position and the other parameters of the model. The equilibrium is depicted in Figure 4.

To analyse the equilibrium we have assumed, without loss of generality, that,

$r_E-\rho+n-g_E>0$, and that $r_E-g_E > r^*-g_E > 0$.

This assumption is necessary for the satisfaction of the transversality conditions of the firms' and consumers' intertemporal optimization problems.

The equilibrium is determined at the intersection of the $\dot{c}=0$ and $\dot{f}=0$ equilibrium conditions.

From (28), setting $\dot{c}=0$, we get,

$$c_t = \frac{n\rho}{\left(r_E-\rho+n-g_E\right)}\left(q_E\bar{A}^{-1}+\bar{b}+f_t\right) \tag{30}$$

Equation (30) is the equilibrium condition for private consumption and we shall call it the *equilibrium private consumption locus*.

From (29a), setting $\dot{f}=0$, we get,

$$c_t = 1+\left(r^*-g_E\right)f_t-\bar{c}_g-\left(g_E+\delta\right)q_E\bar{A}^{-1} \tag{31a}$$

Equation (31a) is the long run equilibrium condition for the balance of payments, for an economy with positive net foreign assets. We shall call it the *external balance locus*.

When both conditions are satisfied, the economy is in long run equilibrium, with a constant private consumption to output ratio, and constant net foreign assets relative to output. The constant net foreign assets to output ratio determines external balance. The equilibrium is unique.

In the short run, the economy adjusts towards external balance along a unique saddle path which is depicted in Figure 4. To the right of the long run equilibrium position private consumption is higher than what would be required for external balance and the current account is in deficit. The country is reducing its net foreign assets relative to output. To the left, private consumption is lower, the current account is in surplus, and net foreign assets gradually increase.

The corresponding equilibrium for an economy with negative net foreign assets is depicted in Figure 5.

The external balance locus for such an economy is given by,

$$c_t = 1 + (r_E - g_E) f_t - \bar{c}_g - (g_E + \delta) q_E \bar{A}^{-1} \tag{31b}$$

The slope of the external balance locus depends on the domestic real interest rate when net foreign assets are negative, while it depends on the world real interest rate when net foreign assets are positive. In addition, with positive net foreign assets, national income (GNP) is greater than domestic output (GDP), while the opposite holds with negative net foreign assets. Apart from these two differences, the nature of the equilibrium is quite similar in the two cases.

The constant net foreign assets to output ratio defines external balance and the equilibrium is unique. In the short run, the economy adjusts towards external balance along a unique saddle path which is depicted in Figure 5. To the right of the long run equilibrium position private consumption is higher than what would be required for external balance and the current account is in deficit. The country is reducing its net foreign assets relative to output. To the left, private consumption is lower, the current account is in surplus, and net foreign assets gradually increase.

We can now turn to a full analysis of the macroeconomic effects of government debt.

6. The Macroeconomic Effects of Government Debt

In this section we shall consider the macroeconomic effects of a permanent one off rise in the government debt to output ratio.

Because of imperfect substitutability between domestic and foreign bonds, this will cause a rise in the domestic real interest rate (see equation 21). From equation (16), the rise in the domestic real interest rate will cause a reduction in the investment rate, and thus the endogenous growth rate.

6.1 Effects on the Domestic Real Interest Rate

The effect of the rise in government debt on the domestic real interest rate is given by,

$$\frac{\partial r_E}{\partial b} = \frac{1-\theta}{b^*} > 0 \tag{32}$$

The effect will be stronger the lower the elasticity of substitution between domestic and foreign bonds. If domestic bonds are imperfect substitutes for foreign bonds, a change in domestic government debt produces larger changes in the domestic real interest rate.

In addition, the effect will be stronger, the lower the debt to output ratio in the rest of the world. A high debt to output ratio in the rest of the world means than a given rise in the domestic debt to output ratio causes a smaller change in the relative debt to output ratio of the domestic economy versus the rest of the world.[5]

6.2. Effects on the Endogenous Growth Rate

The effect on the endogenous growth rate comes about through the effect of government debt on the domestic real interest rate. It is given by,

$$\frac{\partial g_E}{\partial r_E}\frac{\partial r_E}{\partial b} = -\frac{g_E + \frac{1}{\phi}(1+\phi\delta)}{r_E - g_E}\frac{1-\theta}{b^*} < 0 \tag{33}$$

The effect is negative because of the negative impact of the domestic real interest rate on the investment rate of firms and thus on the growth rate. Again, the size of the effect depends inversely on the elasticity of substitution between domestic and foreign bonds. For perfect substitutes there will be no effect, as the domestic real interest rate is always equal to the real interest rate in the rest of the world. For imperfect substitutes, the effect is negative. The lowest the elasticity of substitution between domestic and foreign bonds, the highest the negative impact of a change in the government debt to output ratio on the domestic growth rate.

Clearly, the effect of a rise in the debt to output ratio on the differential between the real interest rate and the growth rate is therefore positive, and is given by,

$$\frac{\partial(r_E - g_E)}{\partial b} = \frac{1-\theta}{b^*}\left(1 + \frac{g_E + \frac{1}{\phi}(1+\phi\delta)}{r_E - g_E}\right) = \frac{1-\theta}{b^*}\left(\frac{r_E + \frac{1}{\phi}(1+\phi\delta)}{r_E - g_E}\right) > 0 \tag{34}$$

[5] In Alogoskoufis (2012) we analyze this model for the case where domestic bonds are perfect substitutes for foreign bonds. In this case, government debt has no effect on the domestic real interest rate of a small open economy, which is always equal to the world real interest rate. As a result, government debt has no impact on the domestic growth rate either. Turnovsky (1996) also analyzes an intertemporal model of a small open economy with a representative household, assuming that domestic bonds are perfect substitutes for foreign bonds.

Note again that if the elasticity of substitution between domestic and foreign bonds is equal to infinity, i.e $\theta=1$, then government debt has no effect on either the domestic real interest rate or the domestic growth rate. In this case, the domestic real interest rate is always equal to the real interest rate in the rest of the world, which, by the small open economy assumption, does not depend on domestic factors.

6.3 Effects on External Balance

The effects of a rise in government debt on external balance are more complicated. To examine these effects we must see how a change in the government debt to output ratio affects the *equilibrium private consumption locus* (30) and the *external balance locus* (31a), (31b).

There are two countervailing influences of a rise in government debt on domestic private consumption. On the one hand, the rise in the government debt to output ratio tends to increase the current consumption to output ratio, as current generations feel wealthier in the knowledge that part of the increase in taxes to service the higher government debt will be paid by future generations. This reduces private savings. On the other hand, the increase in the real interest rate relative to the growth rate, tends to reduce private consumption relative to domestic output and thus increase private savings.

The net effect is given by taking the first derivative of the equilibrium private consumption locus (30) with respect to the debt to output ratio,

$$\frac{\partial c}{\partial b}=\frac{n\rho}{r_E-\rho-g_E+n}\left(1-\frac{1-\theta}{b^*(r_E-g_E)}\frac{\left(r_E+\frac{1}{\phi}(1+\phi\delta)\right)\left(q_E\bar{A}^{-1}+b+f_t\right)}{(r_E-\rho-g_E+n)}\right) \tag{35}$$

This will be positive, in the sense that an increase in the debt to output ratio causes the equilibrium consumption locus to rise, only if $1-\theta$ is sufficiently close to zero, that is if the elasticity of substitution between domestic and foreign bonds $1/(1-\theta)$ is sufficiently large. We shall term this *the close substitutes case*. If the elasticity of substitution between domestic and foreign bonds is low, that is if domestic bonds are not sufficiently close substitutes for foreign bonds, a rise in domestic government debt will cause a large rise in the real interest rate-growth differential, which will in turn cause domestic private consumption to fall. We shall term this case *the distant substitutes case*.

On the other hand, there is no ambiguity with regard to the shift of the external balance locus following an increase in the government debt to output ratio. Government bonds affect this locus indirectly, through their effects on the growth rate and the real interest rate. One can easily see that an increase in the government debt to output ratio shifts this

locus upwards (and increases its slope) because of the rise in the domestic real interest rate and the fall in the domestic growth rate.

6.3.1 The Close Substitutes Case

In what follows, we first concentrate in the case of a large elasticity of substitution between domestic and foreign bonds. In this case, an increase in government debt tends to increase private consumption demand and reduce savings.

The dynamic effects of a rise in the government debt to output ratio for the case of the close substitutes case are examined in Figure 6.

The initial equilibrium is at point E, with a private consumption to output ratio c_E and a positive net foreign assets to output ratio f_E. At time t_0 there is a sudden unanticipated permanent rise in the government debt to output ratio, which is then stabilized at the new higher level. Both the external balance locus and the equilibrium private consumption locus shift to the left. Their slope also changes, on account of the change in the real interest rate and the growth rate. The new long run equilibrium is at E´ which is associated with a lower equilibrium consumption to output ratio and lower net foreign assets relative to domestic output.

Following the unanticipated rise in public debt private consumption initially rises to c_0 on the saddlepath associated with the new long run equilibrium. Along the saddlepath private consumption is higher than what would be required for external balance, the current account is in deficit and the economy is reducing its net foreign asset position to pay for the current account deficits. External balance is being restored gradually as the economy approaches the new long run equilibrium.

Thus, in the close substitutes case, a rise in the government debt to output ratio causes private consumption to rise temporarily, the current account to deteriorate and in the new equilibrium both the private consumption to output ratio and net foreign assets to output to settle at a lower level. Note that the current account deteriorates in the short run despite the fall in the investment rate which is smaller than the reduction in private savings, due to the assumption of close substitutability between domestic and foreign bonds.

A numerical simulation of a calibrated version of the model (see Table 1) suggests that in the close substitutes case (θ=0.99), the effects of government debt on the real interest rate and the growth rate are relatively small, while the effects on private consumption and the external balance position are relatively large.

6.3.1 The Distant Substitutes Case

The opposite happens in the distant substitutes case. If the elasticity of substitution between domestic and foreign bonds is relatively low, i.e if (35) is negative, a rise in the government debt to output ratio produces large increases in the domestic real interest rate and large reductions in the domestic investment and growth rate. As a result, private consumption also falls and savings increase as the substitution effect dominates the wealth effect.

The effects are depicted in Figure 7.

The initial equilibrium is again at point E, with a private consumption to output ratio c_E and a positive net foreign assets to output ratio f_E. At time t_0 there is a sudden unanticipated permanent rise in the government debt to output ratio, which is then stabilized at the new higher level. The external balance locus shifts to the left, but the equilibrium private consumption locus now shifts to the right, on account of the large rise in the domestic real interest rate which reduces private consumption. Their slope also changes, on account of the change in the real interest rate and the growth rate. The new long run equilibrium is at E´ which is associated with a higher equilibrium consumption and higher net foreign assets relative to domestic output.

Following the unanticipated rise in public debt private consumption initially falls to c_0 on the saddlepath associated with the new long run equilibrium. Along the saddlepath private consumption is lower than what would be required for external balance, the current account is in surplus and the economy is increasing its net foreign asset position due to the current account surpluses. External balance is being restored gradually as the economy approaches the new long run equilibrium.

Thus, if domestic government bonds are not sufficiently close substitutes for foreign bonds, a rise in the government debt to output ratio causes private consumption to fall temporarily and the current account to improve. In the new long run equilibrium both private consumption and net foreign assets settle at a higher level relative to domestic output.

7. Conclusions

This paper puts forward a intertemporal model of a small open economy and analyzes the effects of government debt on the domestic real interest rate, economic growth, private consumption and the balance of payments.

The model is an endogenous growth, overlapping generations model with convex adjustment costs for investment, and imperfect substitutability between domestic and foreign bonds.

It is shown than an increase in the government debt to output ratio causes the spread between the domestic and the foreign real interest rate to rise and the endogenous growth rate to fall.

In addition, when domestic government bonds are sufficiently close substitutes for foreign bonds, the rise in government debt causes a temporary *rise* in domestic consumption, as current generations view government debt as wealth. The current account moves into deficit, the economy decumulates net foreign assets, and in the new long run equilibrium both the consumption to output ratio and net foreign assets as a proportion of output fall.

When domestic government bonds are not sufficiently close substitutes for foreign bonds, the rise in government debt causes a temporary *fall* in domestic consumption, as the negative real interest rate effect of the rise in government debt dominates the wealth effect. In this case the current account improves and in the new long run equilibrium both the consumption to output ratio and net foreign assets as a proportion of output rise.

A numerical simulation of a calibrated version of the model in the case of government securities that are close substitutes, suggests that the effects of government debt on the domestic real interest rate and the endogenous growth rate are relatively small, while the effects on private consumption and net foreign assets are relatively large. The opposite would happen in the case when government securities are not close substitutes.

Figure 1
The Equilibrium Rate of Growth and the Shadow Price of Capital

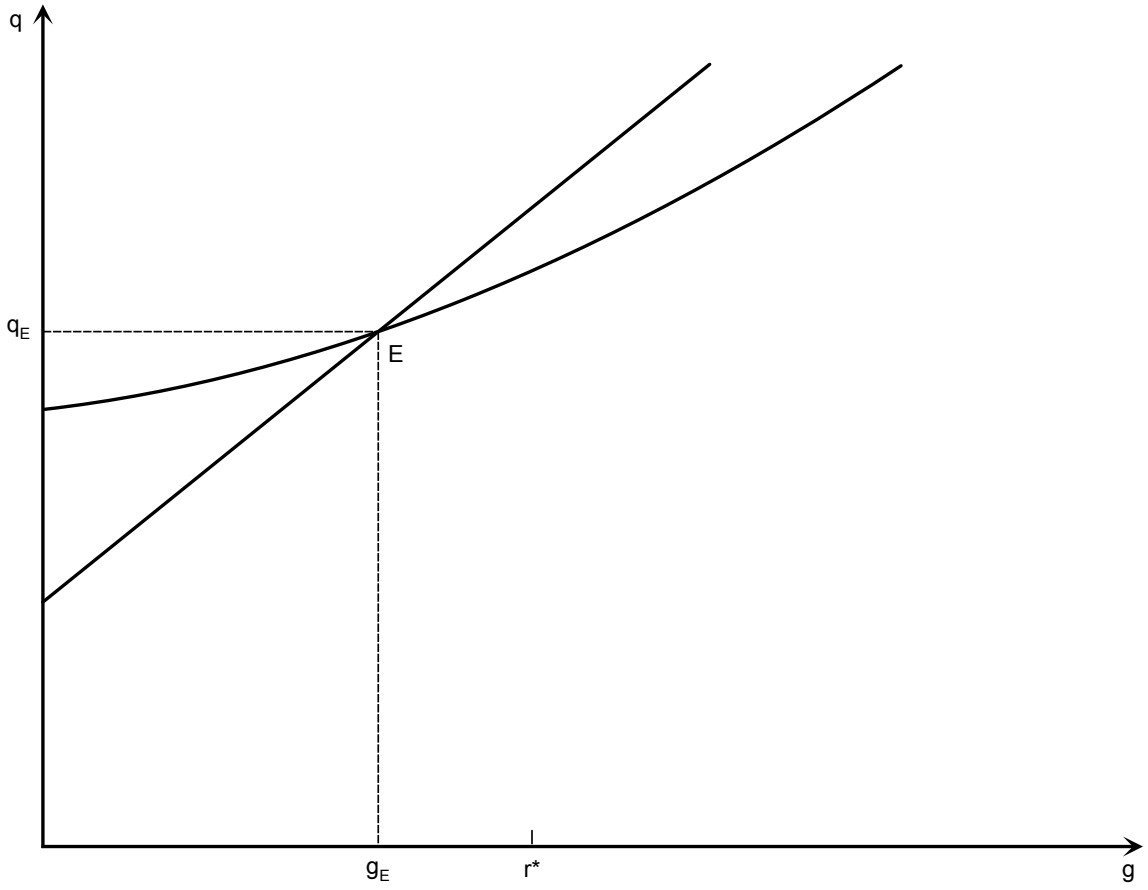

Figure 2
A Rise in the Domestic Real Interest Rate

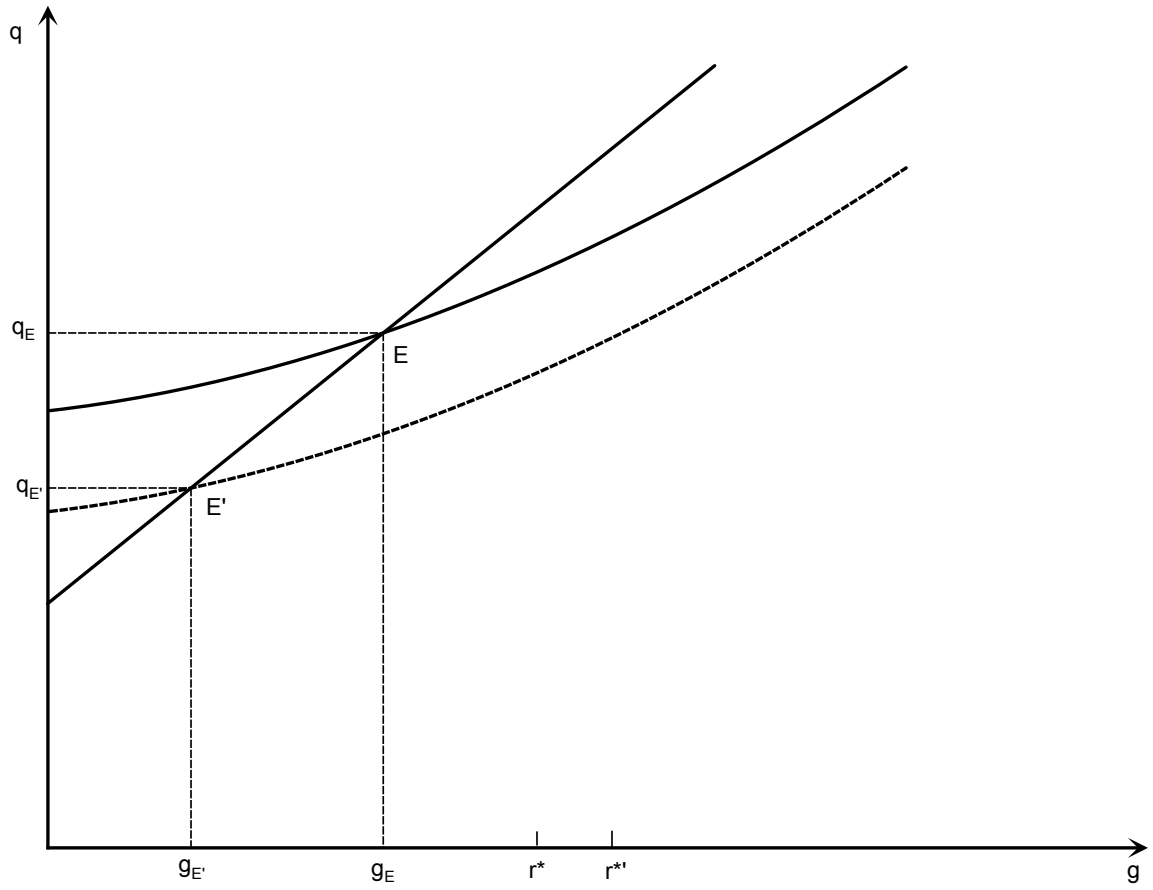

Figure 3
A Rise in the Productivity of Domestic Capital

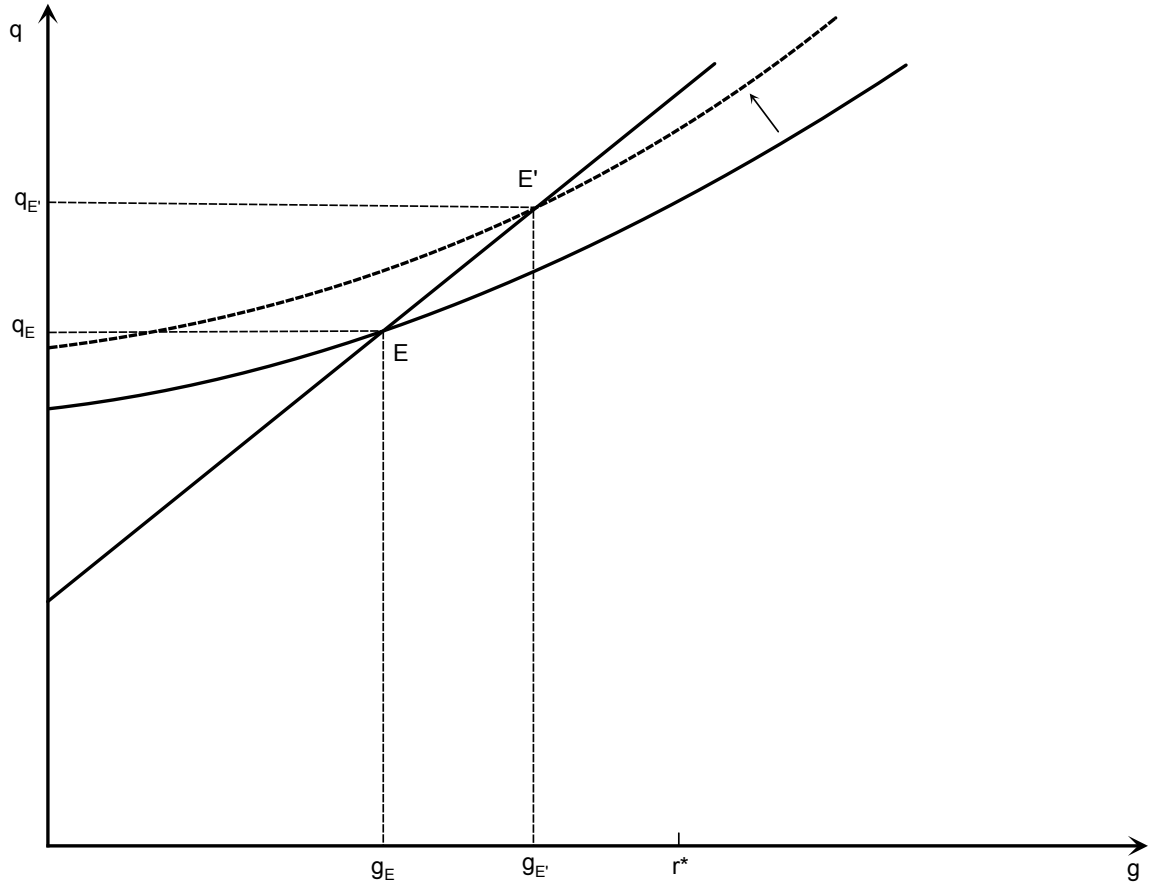

Figure 4
Private Consumption and External Balance with Positive Net Foreign Assets

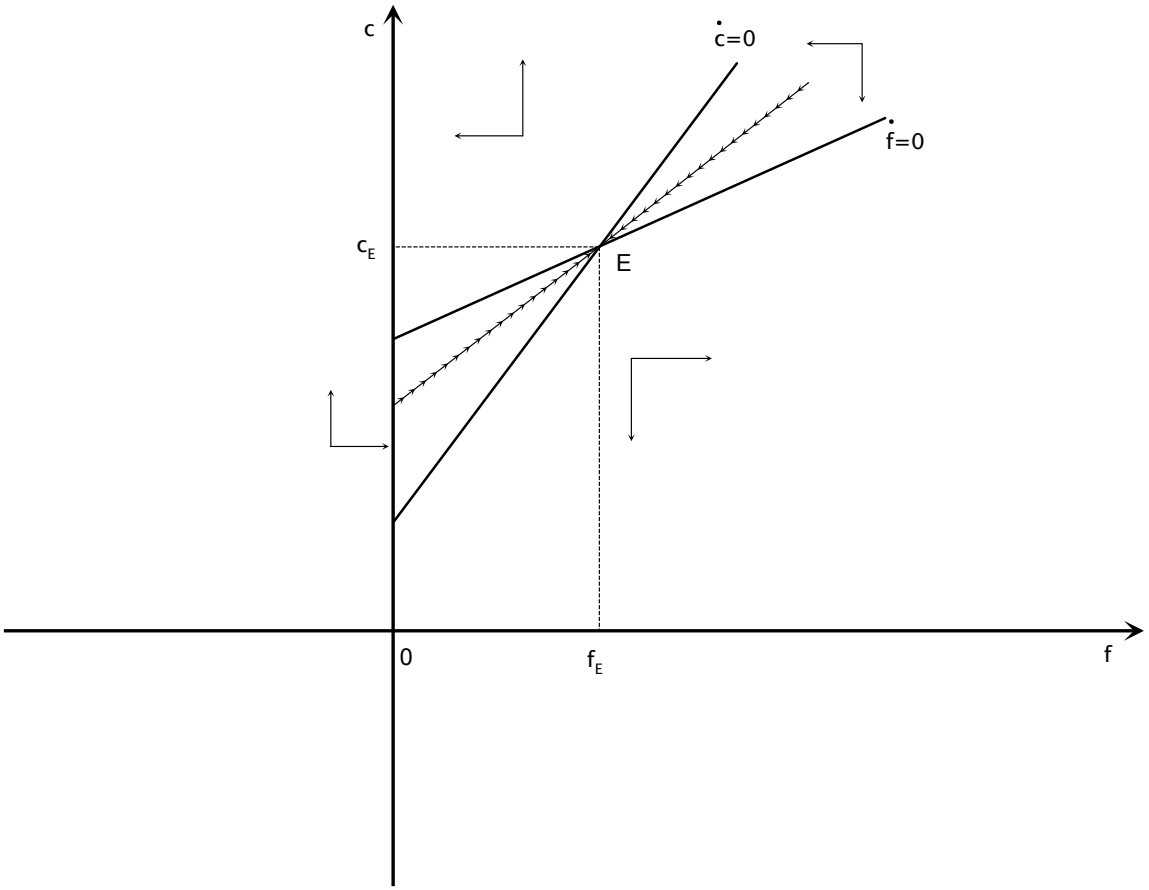

Figure 5
Private Consumption and External Balance
with Negative Net Foreign Assets

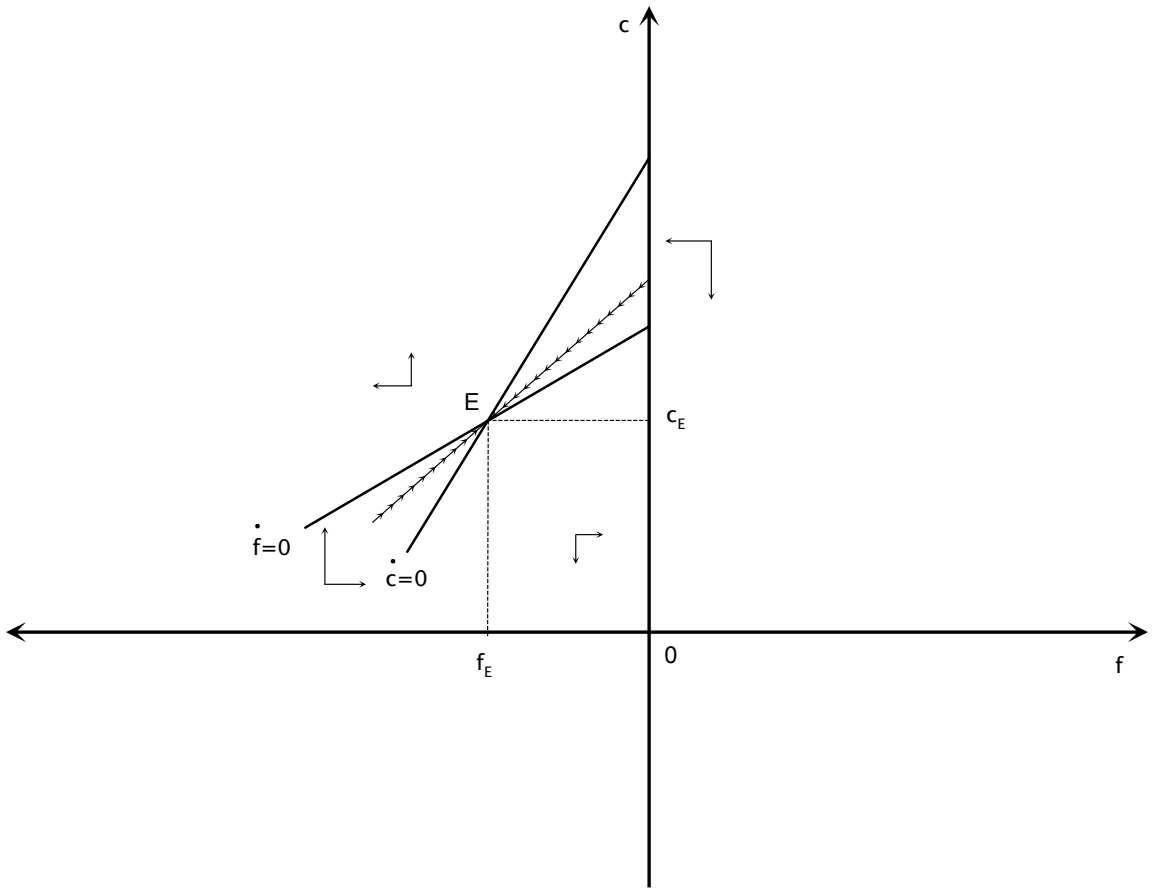

Figure 6
Effects of Government Debt on Private Consumption and External Balance:
The Close Substitutes Case

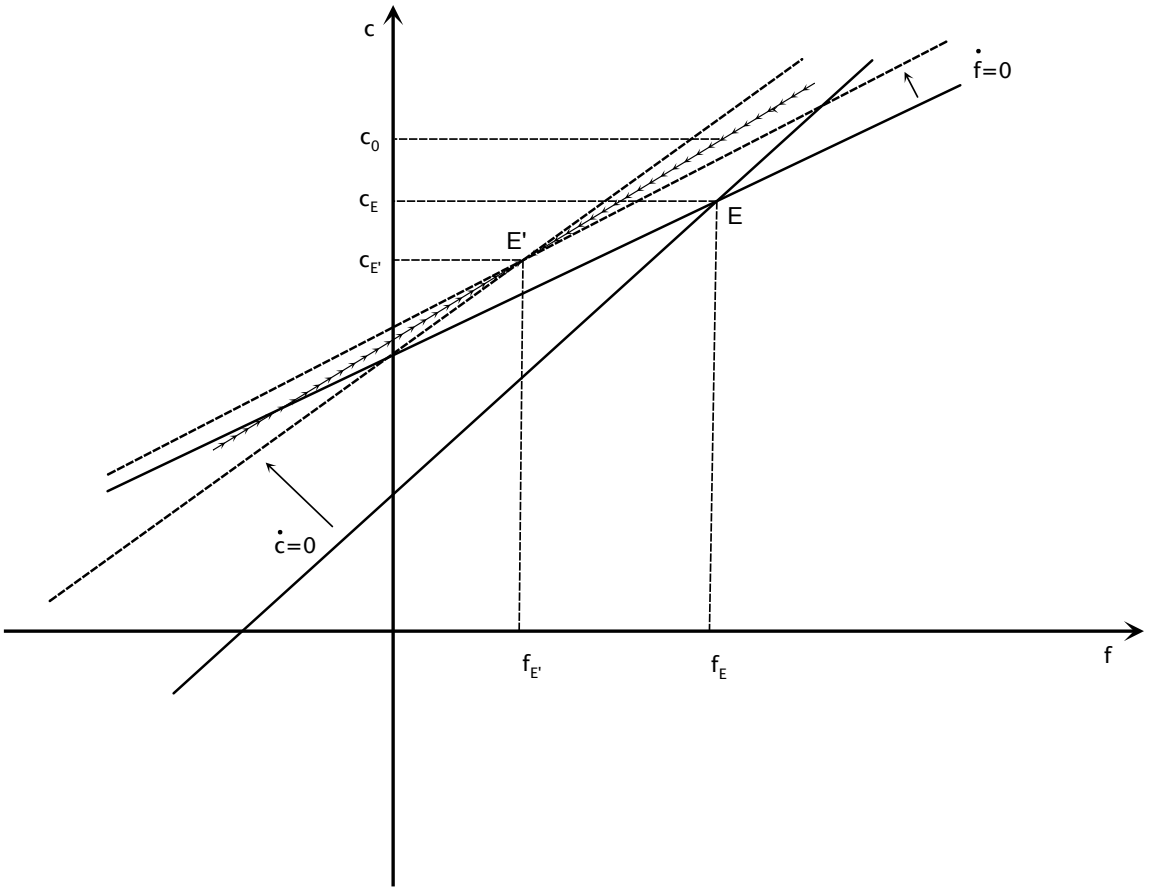

Figure 7
Government Debt and External Balance
The Distant Substitutes Case

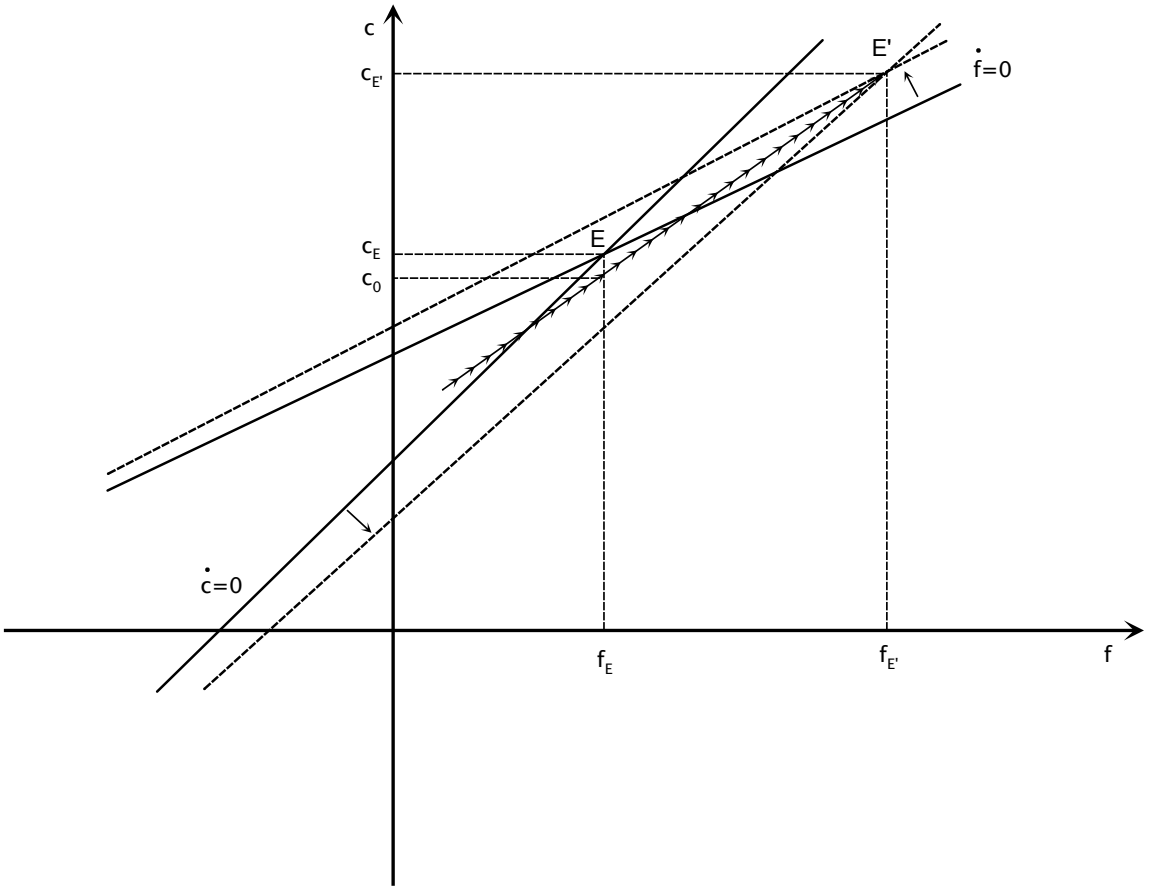

Table 1
Equilibrium Values for Different Government Debt to Output Ratios

b	***r***	***g***	***q***	***c***	***f***
120%	3.60%	1.92%	2.401	115.9%	52.715
121%	3.61%	1.88%	2.382	89.5%	26.858
122%	3.63%	1.84%	2.364	73.1%	11.415
123%	3.64%	1.81%	2.347	61.9%	1.416
124%	3.65%	1.77%	2.330	53.8%	-3.637
125%	3.67%	1.74%	2.314	47.7%	-6.880

Exogenous Parameter Values: $r^*=3\%$, $\delta=1\%$, $\rho=2.5\%$, $a=0.30$, $\bar{A}=0.33$, $\theta=0.99$, $c_g=20\%$, $b^*=75\%$, $n=1\%$, $\phi=48$.

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