ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

DEPARTMENT OF ECONOMICS

WORKING PAPER SERIES 10-2011

Endogenous Spatial Differentiation with Vertical Contracting

Frago Kourandi and Nikolaos Vettas

76 Patission Str., Athens 104 34, Greece
Tel. (++30) 210-8203911 - Fax: (++30) 210-8203301
www.econ.aueb.gr

Endogenous Spatial Differentiation with Vertical Contracting

Frago Kourandi and Nikolaos Vettas*

November 24, 2011

Abstract

We set-up a linear city model with duopoly upstream and downstream. There is a transportation cost when consumers buy from a retailer and when retailers buy from a wholesaler. All location and pricing decisions are endogenous. When the wholesalers choose their locations first, they locate closer to the center relative to the retailers, and relative to when they move simultaneously. Each wholesaler does this to strengthen the strategic position of his retailer by pulling him towards the market center resulting, in equilibrium, to a greater intensity of competition. All firms locate farther away from the market center and industry profits are higher under linear pricing relative to two-part tariffs and in turn relative to vertical integration.

JEL Classification: L13; R32

Keywords: Spatial differentiation; Vertical contracting; Linear city; Strategic commitment

*Kourandi: Department of Economics & Social Sciences, Telecom ParisTech, 46 Rue Barrault, Paris 75013, France; e-mail: kourandi@aueb.gr. Vettas: Department of Economics, Athens University of Economics and Business, 76 Patision Str., Athens 10434, Greece and CEPR, UK; e-mail: nvettas@aueb.gr. We thank, for their suggestions and discussions L. Linnemer, N. Matsushima, M. Peitz, P. Rey and participants in seminars and conferences where previous versions of this work have been presented.

1 Introduction

Motivation and main idea. Thanks to an extensive and important literature we understand many aspects of how oligopolistic firms choose their locations or horizontal differentiation before they compete in prices. Maximal, minimal or intermediate differentiation emerges depending on the balance between a direct demand effect that pulls firms towards the center of the market and a strategic price effect that pushes them away from one another. The literature, however, focuses on the direct market relation between the oligopolists and the final consumers. In contrast, the nature of vertical market relations is very important in the great majority of oligopolies. In other words, how upstream firms (wholesalers) trade with downstream firms (retailers) is interrelated with how downstream firms trade with final consumers. In this paper, we study the horizontal product differentiation and pricing decisions in vertically linked duopolies. Specifically, we endogenize the locations and pricing decisions of wholesalers and retailers on a Hotelling line.

Upstream-downstream trade and product differentiation are two key elements in many oligopolistic industries. Our analysis is particularly relevant for industries where both upstream and downstream oligopolists choose their locations. As usual, a location can be understood literally, in geographical space, and metaphorically, in the space of "horizontal" product characteristics. Consider, for example some agriculture or food industry where there may be oligopoly both upstream and downstream. Geographical location is a central consideration for all firms, since the transportation cost can be a very significant part of the total cost when the products are heavy to carry and perishable. But also in many other cases including concrete, steel, and many other commodities, upstream and downstream location choices are crucial. The "product variety" interpretation of locations is also interesting. Products can be differentiated at the wholesale level and retailers may have to pay a transformation cost so that they modify the features of the product and bring it closer to how they wish to position it in the retail market. For example, milk sold by wholesalers may differ with respect to its fat content and retailers have a cost when using that input to produce daily products, also differentiated with respect to their fat content. Thus, our analysis is applicable to any oligopolistic industry where transportation costs exist both upstream and downstream.

The model. We study a two-tier horizontally differentiated duopoly model. Each downstream firm is able to turn one unit it gets from its upstream firm into one unit in the final market. Upstream and downstream firms are differentiated with respect to their horizontal locations. In our basic model, firms choose any location on the real line while consumers are uniformly distributed within the unit interval. Choices are made sequentially. Upstream firms and then downstream firms choose their locations. Subsequently, upstream and then downstream firms choose their prices. Finally, consumers make their selection

The questions that can be answered in a framework that combines vertical contracting and product differentiation elements refer both to the implications of upstream to downstream pricing for the location choices and to the role that upstream and/or downstream differentiation plays for vertical trade and final prices. The strategic incentives are rich, as each firm's choice affects the subsequent choices of both firms and of the final consumers. For instance, upstream locations affect downstream locations, then wholesale prices and then the final prices. The nature of competition downstream depends on the marginal cost of each retailer and, importantly, in our model this cost becomes endogenous since it depends on the location choices of the wholesalers and the retailers.

Main results. In one-level models of horizontal differentiation (that is, without upstream firms) there are two opposite forces, the "demand" effect, pushing firms close to each other, and the "price competition" effect, pushing them to the opposite direction. In our model there is a third force working through the wholesale prices and the transportation cost that the downstream firms pay when moving to the center of the line. The interaction of all these forces shapes the equilibrium. Somewhat surprisingly, given the complexity of the model, there is a unique subgame perfect equilibrium which can be expressed in closed form. In this, all firms locate outside the unit interval and when the wholesalers choose their locations first, they always locate closer to the city center than the retailers. One set of results refers to the comparison between various types of vertical trade: we first assume linear pricing and then proceed to the case of vertical integration and to two-part tariffs. A second set of results studies how the timing of the location decisions matters.

Our main results can be summarized as follows. First, regardless of the timing of location choices, all firms locate farther away from the center under linear pricing, than under two-part tariffs and in turn than under vertical integration. Second, the social cost, including both the intermediate and the final transportation cost can be also ranked due to the modified location incentives. We find that prices are lower but total social cost is higher under linear pricing, then under two-part tariffs and then under vertical integration. We note that either with linear or two-part tariffs double marginalization emerges and final prices are higher than under vertical integration. Total equilibrium profits are also higher than under vertical integration.

Third, crucial insights are also obtained when we study alternative timing assumptions about the location choices. Regardless of the mode of vertical trade, when wholesalers choose their locations first, they choose to locate closer to the center relative to when location choices are simultaneous (or when retailers move first). They do this to credibly pull their retailers towards the center and offer them a stronger strategic position in the final market. Thus, locations are used as strategic commitment devices. In equilibrium, this strategic behavior leads to more intense competition and lower profits when wholesalers choose their locations first. The industry would, thus, not like the wholesaler to be able to choose their locations first. This result is reminiscent of the strategic delegation and contracting literature (as e.g. in Fershtman and Judd, 1987 or Bonanno and Vickers, 1988) but it operates through the location choices. Upstream players seek to gain unilaterally by offering a way for their downstream partners to credibly commit to more aggressive behavior: in our model more aggressive behavior means a location choice closer to the center. When upstream locations are chosen first, each upstream firm strategically moves closer to the center so that it can offer (through the upstream-downstream transportation cost) a credible way for its downstream firm to also move towards the center. When both upstream firms do this, both downstream are drawn towards the center. In equilibrium, competition gets intensified, with industry profit being lower, than when this strategic effect is absent (e.g. when upstream locations do not proceed the downstream).

Related literature. Our paper brings together two distinct literatures, on product differentiation and on vertical contracting. Each contains a number of very influential papers: for some key results on product differentiation see e.g. Anderson *et al.* (1992), Gabszewicz and Thisse (1992) and for vertical relations see e.g. Rey and Tirole (2007) and Rey and Verge$'$ (2008). As these literatures are too large to survey here, we only discuss work that is more closely related to the specific setting of our model. The strand of the product differentiation literature we are building on, starts with the classic linear-city model of Hotelling (1929), modified by d' Aspremont *et al.* (1979) in a way that the strategic incentives can also be more easily characterized.[1] Nevertheless, only very few papers have examined vertical chain interactions in a horizontal differentiation framework. This creates a significant gap in the literature since in reality many markets have an important vertical element. A key observation here is that in our set-up the marginal costs of the downstream firms are endogenously determined and that all location choices are affected by the terms of vertical trade.[2]

Some papers have studied upstream monopoly with horizontally differentiated firms downstream. Gupta *et al.* (1994) assume that an upstream monopolist sets its wholesale price based on its observation of the locations chosen by the downstream firms and the downstream firms can price discriminate. Beladi *et al.* (2008) also study upstream monopoly and downstream duopoly, with two-part tariffs and the downstream firms not able to produce all varieties. Aiura and Sato (2008) examine an upstream monopolist at the center of the city and supplying two retailers. Retailers do not pay wholesale prices but only a transportation cost and choose their locations and final prices. In our model, there is oligopoly competition both upstream and downstream.

A location-price equilibrium is also analyzed by Brekke and Straume (2004) where each downstream firm has its own supplier, upstream firms bargain about the linear input prices with the downstream firms, but in contrast to our model, upstream firms are not product differentiated. In Allain (2002) and Laussel (2006) two upstream firms are exogenously brand differentiated and two downstream firms are exogenously spatially differentiated, thus, consumers face four different

[1] Many variants of the linear city model have been studied. Among other, Anderson and Neven (1991) solve the location-pricing game when oligopolists compete in quantities. Ziss (1993) examines the d' Aspremont *et al.* model with heterogeneous production costs: if the marginal cost difference is sufficient small, a price and location equilibrium exists in which both firms enter and maximum differentiation emerges. Anderson and Engers (1994) study a price-taking equilibrium in the spatial setting. In Vettas (2003) and Vettas and Christou (2005), firms are horizontally as well as vertically differentiated. Tabuchi and Thisse (1995) and Lambertini (1997) allow firms to locate on the entire real line.

[2] Dobson and Waterson (1996) study exclusivity in an exogenously non-horizontal differentiated successive duopoly at the upstream and downstream level, with consumers facing four varieties. Inderst and Shaffer (2007) analyze the impact of retail mergers on product variety in a non-Hotelling type differentiation model.

products. In our model consumers care directly only about the final product locations and prices - we endogenize the location choices in both levels allowing input prices to be set by the wholesalers. Reisinger and Schnitzer (2010) study successive oligopolies with endogenous entry in the upstream and downstream level where firms are exogenously equidistantly located on a circle. Due to uncertainty in the realizations of the downstream firms' locations, the downstream and upstream locations are not correlated with each other. In contrast, the main feature of our paper is that location choices are interrelated. The closest work to ours probably is by Matsushima (2004) who has also studied the two upstream and two downstream firm structure on the line. However, he studies simultaneous symmetric location choices of upstream and downstream firms within the unit interval, when the upstream firms price discriminate between the retailers and the transportation cost is paid by the wholesalers.[3]

In our paper, by deriving in closed form the subgame perfect equilibrium of the general game with upstream and downstream duopoly when choices are either sequential or simultaneous and all firms can locate anywhere on the real line, we obtain a rich set of results and insights about the interrelation between vertical trade and horizontal product differentiation.

Roadmap. The remainder of the paper is as follows. Section 2 sets up the basic model. Section 3 derives the equilibrium and studies its properties. The modified order of location choices is examined in Section 4. Section 5 examines the vertical integration case and Section 6 studies two-part tariffs. Section 7 concludes.

2 The basic model

We set up a model where upstream and downstream duopolies locate on a line as follows. Consumers are uniformly distributed on a [0,1] interval and have unit demands. There are two upstream firms, A and B, and two downstream firms, X and Y, each choosing a unique location on the entire real line. There is exclusive dealing and the downstream firms have a simple fixed-proportions technology: firm X turns each unit it purchases from firm A into one unit that it can sell to the final consumers; likewise, for firms Y and B.

The locations of firms A and B are denoted by a and $1-b$, respectively while the locations of firms X and Y are denoted by x and $1-y$, respectively (see Figure 1). Thus, for the A and X chain, a and x measure how much to the right of endpoint 0 each firm is located, while for the B and Y chain, b and y measure how much to the left of endpoint 1 each firm is located. Without loss of generality, we consider $1-a-b>0$, so that A is located to the left of B. In the main body of the paper we will also focus on the case where $1-x-y>0$ so that X is also to the left of Y; in Appendix A2 we also discuss the possibility that the downstream locations switch to the "wrong" side of the line relative to their upstream suppliers.

[3] Matsushima (2009) studies the incentives for vertical mergers in a locations model.

A transportation cost has to be paid both in the wholesale and retail market. So that our results are easily comparable to the literature, we follow the often used assumption that this cost is quadratic in distance. A consumer located at point z pays transportation cost $t(x-z)^2$ when purchasing a product from firm X and $t(1-y-z)^2$ when purchasing a product from firm Y. In turn, firm X pays transportation cost $\tau(x-a)^2$ when purchasing a unit from firm A and firm Y pays transportation cost $\tau(y-b)^2$ when purchasing a unit from firm B. These costs, $t>0$ and $\tau>0$, may be understood as real transportation costs (for example depending on the weight or the volume of the product) or as product characteristic transformation costs necessary to convert one unit of the upstream firm's input to one unit of final good. We consider linear pricing at the wholesale and retail level.[4] A final consumer that purchases a unit from downstream firm X or Y pays the price set by this firm (p_X or p_Y) plus the transportation cost between the chosen firm's location and his own location. We also assume that the basic reservation value of each consumer is high enough so that each consumer purchases one unit of the product. A downstream firm who purchases a unit from upstream firm A or B pays the price set by this firm (w_A or w_B) plus the transportation cost between the two trading firms. Apart from location difference, the products are homogeneous. Production costs are assumed zero for simplicity. Note that the final consumers care about the upstream locations and prices indirectly, that is, only to the extent that they affect the downstream locations and prices.

Each of the four firms seeks to maximize its own profit and each consumer his own net surplus. We assume no information asymmetries and proceed to analyze a sequential game where all locations and prices are endogenous. We view locations as longer-run (and more difficult to change) variables than prices. Therefore, the main model we analyze is a five stage game as follows:

(1) Upstream firms A and B simultaneously choose their locations, a, b.

(2) Downstream firms X and Y simultaneously choose their locations, x, y.

(3) Upstream firms simultaneously set linear wholesale prices, w_A, w_B: firm A charges w_A to firm X and firm B charges w_B to firm Y.

(4) Having observed both wholesale prices, downstream firms simultaneously set their (final) product prices, p_X, p_Y.

(5) Having observed the firms' locations and final prices, each consumer purchases one unit of the product from one of the downstream firms X and Y.

We proceed backwards, solving for the subgame perfect Nash equilibria of the game.

[4] Linear pricing is a natural starting point for our problem as it is widely used in practice, makes our results easily comparable to the standard one-level location models and allows us to study the equilibrium profits for upstream and downstream firms. We turn to two-part tariffs in Section 6 and compare the two cases.

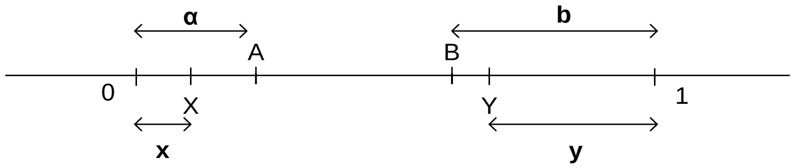


Figure 1: Upstream and downstream locations (example)

In subsequent sections of the paper, we also consider the case when all locations (upstream and downstream) are chosen simultaneously. We also examine (in Appendix A) other extensions of the analysis: when firm locations cannot be on the entire real line but only within the unit interval; and when downstream locations can be at the opposite side on the line relative to their upstream suppliers. But first we proceed to the analysis of our main case.

3 Equilibrium analysis

To obtain a subgame perfect equilibrium, we work backwards. First, we consider trade in the downstream market to find the equilibrium retail prices and consumer choices given the wholesale prices and the locations of all firms. Second, we derive the equilibrium wholesale prices given all firms' locations and anticipating equilibrium behavior in the retail market. Third, we derive the downstream firms' equilibrium locations given the upstream locations and anticipating pricing, wholesale and then retail. Finally, we derive upstream equilibrium locations anticipating equilibrium in all the subsequent stages. Once we derive the equilibrium, we study its properties. As should be expected when dealing with such a five-stage game, the analysis is tedious at times and some of the formal details are omitted, or are presented in brevity to facilitate the continuity of the arguments and the intuition.

3.1 Consumers' choices and retail prices

This part of the analysis corresponds to duopoly competition in the final (retail) market with fixed locations and potentially different unit costs.[5] For some locations and costs, both retailers sell in the market, while for locations and costs that significantly favor one of the retailers, we obtain a corner solution where the rival makes no sales. The costs faced by the retailers are equal to the wholesale price plus the transportation cost they have to pay per unit.

Given the firms' locations and the wholesale prices, we calculate the demand functions for the downstream firms X and Y. Let z be the demand of firm X and $1 - z$ the demand of firm Y. The

[5] Ziss (1993) studies the linear city model with different unit costs. Our analysis in this part parallels his.

firms' profit functions are

$$\Pi_X = (p_X - f_X)z, \ \Pi_Y = (p_Y - f_Y)(1-z),$$

where f_X (respectively f_Y) is the *aggregate* marginal cost, that is, the wholesale price *plus* the transportation cost of firm X (respectively Y) with $f_X \equiv w_A + \tau(x-a)^2$ and $f_Y \equiv w_B + \tau(y-b)^2$. When both firms sell positive quantities, demand for each firm is characterized by the presence of an indifferent consumer located at z, as follows:

$$p_X + t(x-z)^2 = p_Y + t(1-y-z)^2. \tag{1}$$

Of course, the location of the indifferent consumer depends on the downstream firms' locations, product prices and the transportation cost parameter t, that is, $z = z(p_X, p_Y, x, y; t)$; to simplify the exposition we suppress the arguments of this function and solve (1) to obtain

$$z = \frac{1+x-y}{2} + \frac{p_Y - p_X}{2t(1-x-y)}. \tag{2}$$

Further, taking into account the possibility that *all* consumers may choose to purchase from one firm, demand for firm X will be equal to

$$z = \begin{cases} 1 & if \quad \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} \geq 1 \\ \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} & if \quad 0 < \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} < 1 \\ 0 & if \quad \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} \leq 0. \end{cases} \tag{3}$$

We note that the demand and profit functions for the downstream firms are continuous in both firms' prices.

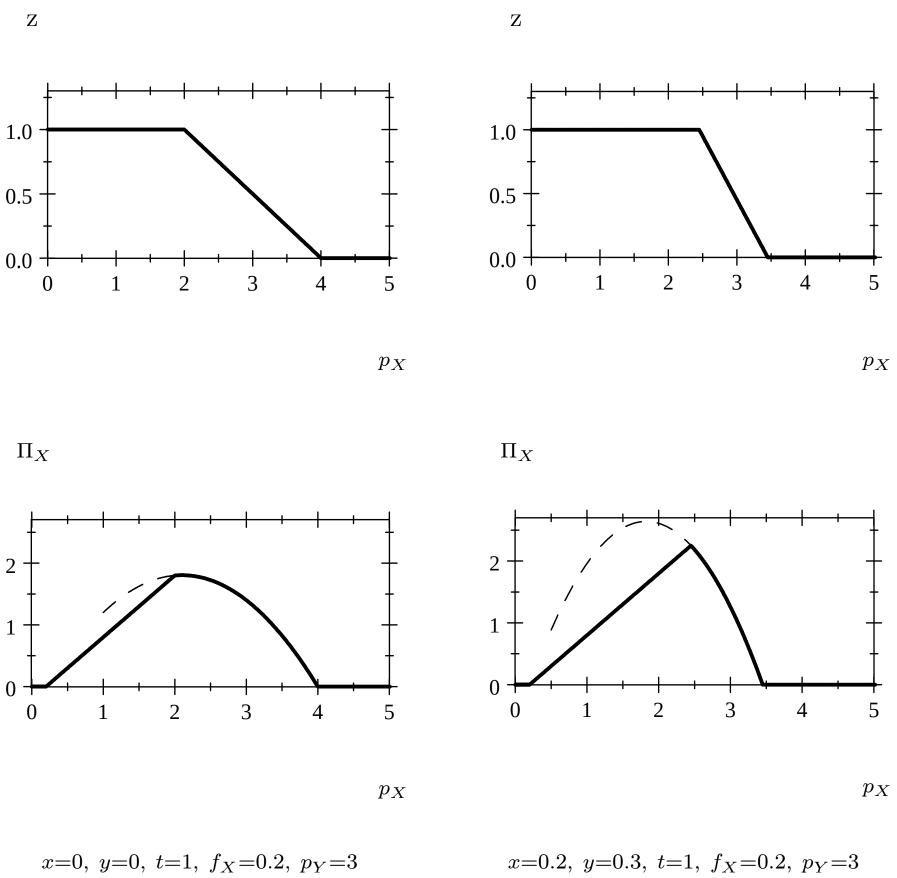


Figure 2: Downstream demand and profits

For intermediate values of the prices, the market is shared, $z \in (0,1)$, and as firm X reduces p_X, the indifferent consumer moves to the right on the line (thus, z increases). When p_X decreases below the threshold implicit in expression (3), firm X captures all the demand ($z = 1$) and its profit is simply equal to its profit margin $p_X - f_X$. Likewise, for high values of p_X, firm X makes no sales and its profit is driven to zero. Figure 2 presents two examples of the demand and profit functions for different values of the parameters.

Given the profit functions for the downstream firms, we now proceed to the characterization of the equilibrium retail prices. Special care should be taken for the characterization of the corner cases:

Lemma 1 *The equilibrium price and profit for firm X are:*

$$p_X = \begin{cases} f_Y - t(1-x-y)(1+y-x) & if \quad f_X \leq \overline{f_X} \\ \frac{1}{3}\left(t(1-x-y)(3+x-y) + f_Y + 2f_X\right) & if \quad \overline{f_X} < f_X < \widehat{f_X} \\ f_X & if \quad f_X \geq \widehat{f_X} \end{cases} \tag{4}$$

and

$$\Pi_X = \begin{cases} f_Y - t(1-x-y)(1+y-x) - f_X & if \quad f_X \leq \overline{f_X} \\ \frac{(t(1-x-y)(3+x-y)+f_Y-f_X)^2}{18t(1-x-y)} & if \quad \overline{f_X} < f_X < \widehat{f_X} \\ 0 & if \quad f_X \geq \widehat{f_X}, \end{cases} \tag{5}$$

where

$$\begin{aligned} \widehat{f_X} &\equiv f_Y + t(1-x-y)(3-y+x), \\ \overline{f_X} &\equiv f_Y - t(1-x-y)(3-x+y). \end{aligned}$$

The expressions for firm Y can be expressed in a symmetric way.[6]

When firm X has low enough aggregate marginal cost (lower than the critical value $\overline{f_X}$) it is the only one that sells in the market, while Y has zero demand. The opposite is true when X has a high aggregate marginal cost (higher than $\widehat{f_X}$). When the difference in the marginal costs of the two firms is not too large, $\overline{f_X} < f_X < \widehat{f_X}$, both downstream firms make sales in the final goods' market.

We note two properties of the downstream market equilibrium. First, the first derivative of the final prices and the profits with respect to the transportation cost parameter t of the consumers is positive: the more differentiated the final goods are, the higher are the prices set by the retailers and the profits they obtain. Second, we observe that $\frac{dp_i}{dfi} = \frac{2}{3} < 1$. Thus, following an increase in his aggregate marginal cost, a retailer passes part of this to the final consumers but absorbs the rest.

3.2 Wholesale prices

In this stage, the two upstream firms compete by setting their wholesale prices. Each upstream firm exclusively supplies its retailer: firm A supplies firm X and firm B supplies firm Y. Even though each upstream firm cannot supply the rival retailer, competition takes place: the wholesale prices shape competition in the subsequent stage where retail prices and final demand is determined. If the wholesale price charged to retailer X by the upstream firm A is high, retailer X has a relative cost disadvantage compared with the rival retailer and, thus, the demand obtained by firm X (and, in turn, by firm A) is low.

Since a downstream firm turns one unit of the good it purchases in the wholesale market into one unit it sells in the retail market, the demand function of firms A and B are $D_A = z$ and $D_B = 1 - z$ and their profit functions are $\Pi_A = w_A z$ and $\Pi_B = w_B(1-z)$. Assuming equilibrium will follow when setting retail prices, as characterized in the previous subsection, the profit function for firm

[6] For brevity we only provide throughout the paper the expressions for firm X and A, while the expressions for firms Y and B are suppressed since they can be written symmetrically.

A can be expressed as:

$$
\begin{aligned}
\Pi_A &= w_A \\
& \quad \text{if } w_A \leq w_B + \tau \left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\
&= w_A \left(\tfrac{2t(1-x-y)(y-x)+w_B-w_A+\tau\left((y-b)^2-(x-a)^2\right)}{6t(1-x-y)} + \tfrac{1}{2}(1+x-y) \right) \\
& \quad \text{if } \left\{ \begin{array}{l} w_A > w_B + \tau \left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\ w_A < w_B + \tau \left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1) \end{array} \right\} \qquad (6) \\
&= 0 \\
& \quad \text{if } w_A \geq w_B + \tau \left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1)\,.
\end{aligned}
$$

Note that the profits are functions of the wholesale prices and all four locations. When the aggregate marginal cost $w_A + \tau(a-x)^2$ faced by firm X is low enough compared to its rival, firm A captures the whole demand via its retailer X ($z = 1$). When the aggregate marginal cost faced by firm X is high enough, its demand (and by implication also the demand of firm A) becomes zero ($z = 0$). For intermediate values of $w_A + \tau(a-x)^2$ we obtain $z \in (0,1)$, both retailers make sales in the final market, and therefore, both wholesalers make sales in the upstream market. We should stress the role that upstream and downstream locations play in our model since they crucially affect the marginal costs of the retailers. For some locations, even if an upstream firm charges a zero wholesale price to its retailer, it cannot obtain positive demand and the aggregate marginal cost of one retailer is always lower than the rival's.[7] This happens when the distance of one retailer from its supplier is very large compared to the rival's distance from its own supplier. Clearly, these cases will tend to emerge when the transportation cost parameter τ is relatively high.

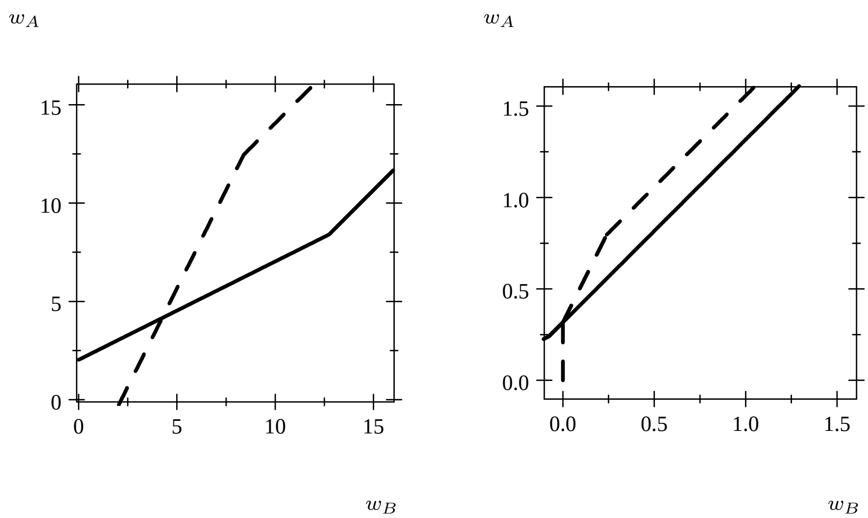
Maximization of the upstream profit functions (see (6) and analogously for B) with respect to

[7] To illustrate, take for example $t = \tau = 1, a = b = 0.2, x = 0.06, y = 0.9$. Then even if $w_B = 0$ firm A takes the whole demand.

the wholesale prices implies the reaction functions of the upstream firms.[8] We obtain, for firm A,

$$
\begin{aligned}
w_A = R_A(w_B) \quad &= \quad w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\
& \quad\quad \text{if } w_B \geq \tau\left((x-a)^2 - (y-b)^2\right) + t\,(y-x+9)\,(1-x-y) \\
\\
&= \quad \tfrac{1}{2} w_B + \frac{t(x-y+3)(1-x-y) - \tau\left((x-a)^2-(y-b)^2\right)}{2} \\
& \quad\quad \text{if } \left\{ \begin{array}{l} w_B > \tau\left((x-a)^2 - (y-b)^2\right) - t\,(x-y+3)\,(1-x-y) \\ w_B < \tau\left((x-a)^2 - (y-b)^2\right) + t\,(y-x+9)\,(1-x-y) \end{array} \right\} \\
\\
&= \quad 0 \\
& \quad\quad \text{if } w_B \leq \tau\left((x-a)^2 - (y-b)^2\right) - t\,(x-y+3)\,(1-x-y)\,.
\end{aligned}
$$

In Figure 3 (I) we present the reaction functions for some parameters values t, τ and locations a, b, x, y for which both firms sell. In Figure 3 (II) we show a case where firm B faces a strong cost disadvantage as it is located far away from its supplier, and this implies a high aggregate marginal cost $w_B + \tau(y-b)^2$ for its retailer. Even if the wholesale price it sets is reduced to zero, firm B cannot obtain positive demand. In equilibrium, firm A serves the whole market via its retailer X. We also note that the wholesale prices are strategic complements, an increase in w_B leads to an increase to w_A and vice versa.



x=0.1, y=0.2, a=0.3, b=0.4, t=2, τ=1 x=00.6, y=0.9, a=b=0.2, t=τ=1

(I)   (II)

Figure 3: $R_A(w_B)$: solid line, $R_B(w_A)$: dash line

From the reaction functions we obtain:

[8] The profit functions are quasi-concave. When both firms have positive demand, from the first order conditions we obtain a unique critical point. At this point, the second order conditions are satisfied, thus, the local critical point is the total max.

Lemma 2 *The equilibrium wholesale price for firm A (as functions of the upstream and downstream locations) is:*

$$\begin{aligned} w_A &= \tau\left((y-b)^2-(x-a)^2\right)+t\,(y-x+3)\,(x+y-1) \\ &\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0 \\ \\ &= \tfrac{\tau\left((y-b)^2-(x-a)^2\right)+t(x-y+9)(1-x-y)}{3} \\ &\quad \text{if } \left\{ \begin{array}{l} \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)>0 \end{array} \right\} \\ \\ &= 0 \\ &\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(x-y+9)\,(1-x-y)\geq 0. \end{aligned} \tag{7}$$

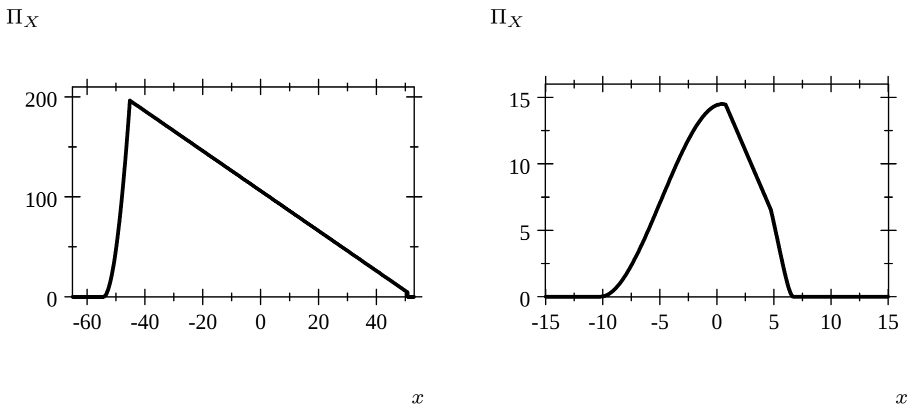
In equilibrium, when the upstream locations are asymmetric enough relative to the downstream locations, the disadvantaged upstream firm sets a zero price and there are no sales for itself (and its retailer) while the rival upstream firm sets the highest price that allows it to capture the whole demand. Otherwise, the market is shared (upstream and downstream). In either case, the equilibrium upstream profit for firm A can be obtained by substituting the equilibrium wholesale price (7) into expression (6).

3.3 Downstream locations

Taking as given the locations of the upstream firms and assuming that the retail and wholesale prices will be subsequently chosen in equilibrium, the downstream firms simultaneously choose their locations to maximize their profits. The profit function for firm X is:

$$\begin{aligned} \Pi_X &= 2t(1-x-y) \\ &\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0 \\ \\ &= \tfrac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t(x-y+9)(1-x-y)\right)^2}{162t(1-x-y)} \\ &\quad \text{if } \left\{ \begin{array}{l} \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)>0 \end{array} \right\} \\ \\ &= 0 \\ &\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(x-y+9)\,(1-x-y)\geq 0. \end{aligned} \tag{8}$$

In our main model firms are allowed to locate anywhere on the real line, that is, also outside the unit interval where consumers are located. Like in the previous steps of our analysis, special care should be taken about the corner cases. Is it possible that for certain wholesale locations, a retailer cannot avoid obtaining zero demand no matter what location it would choose? We find that, if the upstream locations are asymmetric enough, we may obtain a corner solution in the continuation of the game, that is, one retailer finds it impossible to avoid making zero sales. Take, for example, firm B (that supplies firm Y) to be located far away from the unit interval and firm A (that supplies X) to be located at the center of the unit interval. Then, the transportation cost that retailer Y pays when supplied by firm B is high compared to the rival's transportation cost. Based on its cost advantage, firm X maximizes its profits by choosing a location x that allows it to capture the whole market. The alternative, choosing a location far away from the unit interval would lead to zero demand or, on any event, to demand lower than unity and firm X's profits would not be maximized. Figure 4 (I) presents the profit function of firm X when it captures the whole demand. In contrast, in Figure 4 (II) firm X maximizes its profit when it shares the market with firm Y, as its cost advantage is not high enough to allow it to capture all the demand.



a=0.5, b=−100, y=−52, t=τ=1 (I)

a=0.5, b=−10, y=−7, t=τ=1 (II)

Figure 4: Downstream profits

We have to determine when a corner solution will emerge. From the profit function above, we calculate that for $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0$ firm X serves the whole demand. When firm B is located far away from the unit interval, its retailer Y has to decide where to locate in order not to pay a high transportation cost but also get some positive demand. If Y locates at its supplier's location, it minimizes its own transportation cost but it is then located far away from the consumers. In contrast, when it locates in or near the unit interval it is close to the final consumers but faces a high transportation cost. In equilibrium, Y chooses a location y that maximizes the expression $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)$, that is, it minimizes the area where its demand is zero. By direct calculations it follows that $y=\frac{b\tau-4t}{t+\tau}$. Given this location y, firm X maximizes its own profits. This argument holds symmetrically when firm A is located far away from the unit interval and firm X has a cost disadvantage. Of course, when neither firm has a high cost disadvantage in equilibrium the firms share the demand. From this analysis we obtain:[9]

Lemma 3 *The equilibrium downstream locations (as functions of the upstream locations) are:*

$$
\begin{aligned}
(x,y) &= \left(\frac{\left(5t+a\tau-\sqrt{t\tau(a-b-9)(1-a-b)}\right)}{t+\tau}, \frac{b\tau-4t}{t+\tau}\right) \\
&\quad \text{if } 81t+4\tau\left(a-b-9\right)\left(a+b-1\right)\leq 0 \\
&= \left(\frac{\left(16a\tau^2(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)}, \frac{\left(16b\tau^2(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)}\right) \\
&\quad \text{if } \left\{\begin{array}{c} 81t-4\tau\left(a-b+9\right)\left(a+b-1\right)>0 \\ 81t+4\tau\left(a-b-9\right)\left(a+b-1\right)>0 \end{array}\right\} \\
&= \left(\frac{a\tau-4t}{t+\tau}, \frac{\left(5t+b\tau-\sqrt{t\tau(a-b+9)(a+b-1)}\right)}{t+\tau}\right) \\
&\quad \text{if } 81t-4\tau\left(a-b+9\right)\left(a+b-1\right)\leq 0.
\end{aligned} \tag{9}
$$

The equilibrium downstream profits for firm X in this stage can be obtained by substituting the equilibrium downstream locations (9) into the profits (8).

3.4 Upstream locations

In this stage, upstream firms choose their locations to maximize their profits. An upstream firm will never choose a location that will make its own retailer face so high aggregate marginal cost that would make it obtain zero demand in the subsequent stage. Obviously, this is because such an outcome would also lead to zero demand and profits for this upstream firm. Therefore in equilibrium, both upstream firms avoid receiving zero demand and they share the market. The relevant profit functions reduce to:

$$
\begin{aligned}
\Pi_A &= \frac{t\left(9t+2\tau\left(1-a-b\right)\right)\left(81t+4\tau\left(a-b+9\right)\left(1-a-b\right)\right)^2}{108\left(t+\tau\right)\left(9t+4\tau\left(1-a-b\right)\right)^2}, \\
\Pi_B &= \frac{t\left(9t+2\tau\left(1-a-b\right)\right)\left(81t+4\tau\left(b-a+9\right)\left(1-a-b\right)\right)^2}{108\left(t+\tau\right)\left(9t+4\tau\left(1-a-b\right)\right)^2}.
\end{aligned}
$$

From the first order conditions we obtain[10] the equilibrium upstream locations, $a = b = \frac{9t-5\tau-9\sqrt{\tau^2+t^2}}{8\tau}$. Notice that $a < 0$ and $b < 0$, thus, upstream firms are located outside the unit

[9] The second order conditions of the downstream firms are satisfied at the equilibrium locations. Profit margins are positive at the equilibrium point x,y if $a-b+45 > \frac{9(-9t+16\tau(1-a-b))}{4\tau(1-a-b)}$ and $a-b-45 < \frac{9(9t-16\tau(1-a-b))}{4\tau(1-a-b)}$. The second order conditions for firms X and Y are satisfied for $a-b-45 < \frac{243t^2+32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$ and $a-b+45 > \frac{-243t^2-32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$ respectively. This holds as $\frac{9(9t+16\tau(a+b-1))}{4\tau(1-a-b)} < \frac{243t^2+32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$ and $\frac{9(-9t+16\tau(1-a-b))}{4\tau(1-a-b)} > \frac{-243t^2-32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$.

[10] The second order conditions are satisfied in equilibrium.

interval for all values of the parameters t and τ. Combining this result with Lemmas 1-3 we obtain:

Proposition 1 *The unique subgame perfect equilibrium outcome in the five stage game with linear wholesale pricing is:*

$$
\begin{aligned}
a^L &= b^L = \frac{9t - 5\tau - 9\sqrt{\tau^2 + t^2}}{8\tau}, \\
x^L &= y^L = \frac{-\left(5(t+\tau) + 9\sqrt{\tau^2 + t^2}\right)}{8(t+\tau)}, \\
w_A^L &= w_B^L = \frac{27t\left(t + \tau + \sqrt{\tau^2 + t^2}\right)}{4(t+\tau)}, \\
p_X^L &= p_Y^L = \frac{9t\left(9t^3 + 41t^2\tau + 73t\tau^2 + 32\tau^3 + \sqrt{\tau^2 + t^2}\,(-9t + 32\tau)(t+\tau)\right)}{32\tau(t+\tau)^2}, \\
z^L &= 0.5, \\
\Pi_A^L &= \Pi_B^L = \frac{27t\left(t + \tau + \sqrt{\tau^2 + t^2}\right)}{8(t+\tau)}, \\
\Pi_X^L &= \Pi_Y^L = \frac{9t\left(t + \tau + \sqrt{\tau^2 + t^2}\right)}{8(t+\tau)}.
\end{aligned}
$$

We note that, in equilibrium, the two downstream firms and the corresponding two upstream firms share the market equally, all four firms obtain positive profits and all locations are outside the unit interval. Further, upstream firms locate closer to each other (and to the market center) relative to the downstream firms ($a > x$). We discuss the equilibrium properties in detail in the next subsection.

3.5 Equilibrium properties

We discuss now the properties of the equilibrium we have derived. First, we compare our findings to the d' Aspremont *et al.* (1979) model. Second, we study comparative statics. Third, we compare the equilibrium to the social optimum. Finally, we discuss the different forces that shape the equilibrium in our model. The vertical integration case is examined separately in Section 5 below.

No vertical effects. We contrast our results to the standard d' Aspremont *et al.* (1979) model, that is, the case where there is no vertical structure, or alternatively when there are no wholesalers or they play a passive role.[11] With only one stage of competition, in equilibrium, firms would locate at distance -0.25 from each endpoint and equilibrium prices would be equal to $\frac{3t}{2}$.[12] By direct

[11] Note that the d' Aspremont *et al.* (1979) results are also obtained if we assume that firms are vertically integrated *and* each pair of vertically integrated firms (firm X with its supplier A, firm Y with its supplier B) has to choose a *unique* location and final price. In Section 5, we will study the vertical integration case keeping the timing of the game as in our basic model: wholesale prices are zero but upstream and downstream divisions can be at different locations.

[12] The above result directly follows from extending the d' Aspremont *et al.* (1979) model when firms are allowed to locate anywhere on the real line, see e.g. Tabuchi and Thisse (1995) and Lambertini (1997).

comparison to Proposition 1, we find that:

Corollary 1 *In our model, final prices and chain profits are higher than in the d' Aspremont et al. (1979) model. Downstream locations are farther away from the unit interval compared to the upstream locations and the latter are farther away compared to the d' Aspremont et al. (1979) model :*

$$x^L = y^L < a^L = b^L < -0.25.$$

The introduction of the wholesalers in the model increases the final prices since each retailer's marginal cost is increased by the wholesale price. The chain profits are also higher in our vertical structure compared to the d' Aspremont et al. (1979) model. This result is related to other results on "vertical separation" such as Bonanno and Vickers (1988). Upstream firms charge positive wholesale prices and overall profit increases.[13] Here however we also have endogenous locations. As each vertical chain competes with the rival chain at both the upstream and downstream levels, competition is intensified and firms move farther away from the center compared to the case where there is one level of competition.

Comparative statics. Now we turn to a discussion of how equilibrium location and prices respond to changes of the two key parameters, t and τ.

Corollary 2 *As the downstream transportation cost parameter t increases, all four prices and profit levels increase and the upstream firms move closer to the center. However, as t increases, the downstream firms move closer to center when $\tau > t$ and farther away from the center when $\tau < t$.*

We obtain these results by directly differentiating the relevant expressions in Proposition 1: wholesale and retail prices increase ($\frac{dw_A^L}{dt} > 0, \frac{dp_X^L}{dt} > 0$) and this also implies higher profit levels ($\frac{d\Pi_A^L}{dt} > 0, \frac{d\Pi_X^L}{dt} > 0$), with symmetric relations holding for firms B and Y. As should be expected, when final consumers view the products as more differentiated, competition becomes less intense. In the extreme case where t is zero, consumers buy from the cheapest retailer since there is no product differentiation at the downstream level, and this leads to zero wholesale and retail prices and profits for all firms. Also, when t increases upstream location increases ($\frac{da^L}{dt} > 0$) for all values of τ, but downstream location increases ($\frac{dx^L}{dt} > 0$) when $\tau > t$. When final consumers view the products as more differentiated, upstream firms move closer to the center and retailers follow them when retail transportation costs are high. In the extreme case when $t = 0$, all four firms locate at $a^L = b^L = x^L = y^L = -1.75$.

Corollary 3 *As the upstream transportation cost parameter τ increases, the wholesale prices and all four profit levels increase when $\tau > t$, and decrease otherwise. No general conclusion can be reached regarding how the retail prices respond to an increase in τ (they can either increase or*

[13] A more complete discussion is deferred for Section 5 where vertically integrated firms choose different locations for each of their divisions.

decrease). Also, as τ increases, upstream firms move farther away from the center for all values of t. As τ increases, the downstream firms move farther away from the center when $\tau > t$ and move closer to the center when $\tau < t$.

By directly differentiating the relevant expressions in Proposition 1, we obtain:

$$
\begin{aligned}
\frac{dw_A^L}{d\tau} &= \frac{27t^2(\tau-t)}{4\sqrt{\tau^2+t^2}\,(t+\tau)^2}, \\
\frac{dp_X^L}{d\tau} &= \frac{9t^2\left((t+\tau)\left(41\tau^3-32t\tau^2+18t^2\tau+9t^3\right)-9\sqrt{\tau^2+t^2}\left(\tau\left(t\tau+\tau^2+3t^2\right)+t^3\right)\right)}{32\tau^2\sqrt{\tau^2+t^2}\,(t+\tau)^3}, \\
\frac{d\Pi_A^L}{d\tau} &= \frac{27t^2(\tau-t)}{8\sqrt{\tau^2+t^2}\,(t+\tau)^2},\ \frac{d\Pi_X^L}{d\tau} = \frac{9t^2(\tau-t)}{8\sqrt{\tau^2+t^2}\,(t+\tau)^2}.
\end{aligned}
$$

To illustrate, take for example $\tau = 2t$. Then $\frac{dw_A^L}{d\tau} = 0.335\,4$, $\frac{dp_X^L}{d\tau} = 0.410\,7$, $\frac{d\Pi_A^L}{d\tau} = 0.167\,7$, $\frac{d\Pi_X^L}{d\tau} =$

$0.05\,59$. When the upstream transportation cost parameter τ is equal to twice the downstream transportation cost parameter t, an increase in the cost parameter τ, can be partly passed to the final consumers and the upstream firms absorb the rest. In the limit case where τ approaches zero, we obtain: $\lim_{\tau\to 0^+} w_A^L = 13.\,5t$, $\lim_{\tau\to 0^+} p_X^L = 18t$, $\lim_{\tau\to 0^+} \Pi_A^L = 6.\,75t$, $\lim_{\tau\to 0^+} \Pi_X^L = 2.\,25t$

We also obtain $\frac{da^L}{d\tau} < 0$ for all t and $\frac{dx^L}{d\tau} < 0$ for $\tau > t$. Otherwise, for $\tau < t$, we obtain $\frac{dx^L}{d\tau} > 0$. In the limit case where τ goes to zero, we obtain: $\lim_{\tau\to 0^+} a^L = -0.625$, $\lim_{\tau\to 0^+} x^L = -1.\,75$.

Welfare and social optimum. Before we discuss in more detail the forces that shape the equilibrium locations, it is useful to briefly turn to the welfare properties of the equilibrium. Final consumers face unit demand, thus, the social cost (SC) simply equals the transportation cost paid by the retailers plus the transportation cost paid by the final consumers:

$$SC = \int_0^z t(k-x)^2 dk + \int_z^1 t(1-y-k)^2 dk + \tau(x-a)^2 z + \tau(y-b)^2(1-z). \tag{10}$$

By substituting the locations from Proposition 1, we calculate that in equilibrium the total social cost is:

$$SC^L = \frac{18t\left(\sqrt{\tau^2+t^2}\,(-9t+7\tau)+7\tau(t+\tau)+9t^2\right)}{64\tau(t+\tau)} + \frac{t}{12} > 0.$$

A social planner, in contrast, would minimize the total transportation cost (10). Standard calculations imply that the socially optimal locations are $a = b = x = y = 0.25$. Firm X should be located together with its supplier A, and firm Y together with its supplier B, so as to eliminate the transportation costs of the retailers. Also, they are both located inside the unit interval (and splitting the distances equally) to minimize the transportation cost paid by the consumers. The optimum (minimum) social cost is then $SC = \frac{t}{48}$. Therefore:

Proposition 2 *We have $x^L = y^L < a^L = b^L < -0.25 < 0.25$.*

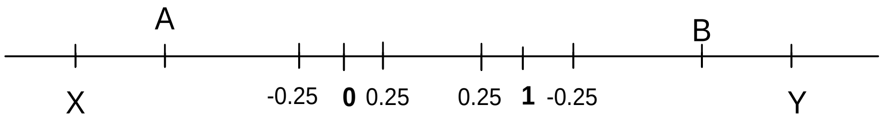


Figure 5: Equilibrium locations

In the equilibrium of our model, firms locate outside the unit interval in contrast to the socially optimum locations which are within the unit interval (see Figure 5). Thus, the corresponding total social cost is strictly higher than the optimal cost and also strictly higher than the cost at the d' Aspremont *et al.* (1979) model.

Equilibrium forces. Now let us discuss the forces behind the determination of the equilibrium locations in more detail. In models of horizontal differentiation with duopolists selling directly to final consumers, there are two opposite forces, one pushing firms close to each other to directly obtain higher demand and one in the opposite direction to reduce the intensity of price competition. In our model, these forces are modified and a third force also emerges. Given the locations of the other three firms, if the downstream firm moves closer to the center of the unit interval this affects its aggregate marginal cost, the wholesale price plus the transportation cost. This effect may be working to bring firms either closer to each other or apart, depending on the locations of the upstream firms. The marginal production cost of the retailers is location dependent.

In a standard linear city duopoly model as in d'Aspremont *et al.* (1979) the marginal production cost is exogenous. Denoting this cost by c, profits are $\Pi_X = (p_X - c)\, z$ for firm X. By the envelope theorem, we obtain:

$$\frac{d\Pi_X}{dx} = (p_X - c)\left(\frac{dz}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{dx}\right),$$

evaluated at the equilibrium prices.[14] The direct demand effect that pushes firms close to each other corresponds to $\frac{dz}{dx}$ while the indirect price competition effect that pushes firms to the opposite direction corresponds to $\frac{dz}{dp_Y}\frac{dp_Y}{dx}$. In our model the marginal production cost for the retailers is endogenous and equal to the wholesale price plus the transportation cost, $f_X = w_A + \tau(a-x)^2$. The profit function of firm X is $\Pi_X = (p_X - f_X)\, z$ and by the envelope theorem we now obtain:

$$\frac{d\Pi_X}{dx} = (p_X - f_X)\left(\frac{dz}{dx} + \left(\frac{dz}{dp_Y}\frac{dp_Y}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_X}\frac{df_X}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_Y}\frac{df_Y}{dx}\right)\right) - \frac{df_X}{dx}z.$$

A change in the location x affects the aggregate marginal costs of both retailers and the final prices directly and indirectly via the change in the marginal costs. The direct demand effect again corresponds to $\frac{dz}{dx}$ but the indirect price competition effect now corresponds to the expression $\frac{dz}{dp_Y}\frac{dp_Y}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_X}\frac{df_X}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_Y}\frac{df_Y}{dx}$ as final prices are affected by the endogenous marginal production

[14]See Tirole (1989), p. 281.

costs of the retailers. A change in x, leads to a change in the wholesale prices and the retailers' transportation cost, therefore, final prices change too. Finally, profits are also affected directly by the change in the marginal cost itself, $\frac{df_X}{dx}$.

Corollary 4 *In addition to the demand effect and the price competition effect, the aggregate marginal cost effect plays a key role in shaping the retailer location incentives. These three forces co-determine the equilibrium locations.*

As the profit margin $(p_X - f_X)$ and the demand z are positive, the following derivatives determine the effect of x on the profit Π_X :

$$\frac{dz}{dx} = \frac{\tau\left((y-b)^2-(x-a)^2\right)+t\left(9-y+x\right)\left(1-x-y\right)}{18t(1-x-y)^2} - \frac{x}{1-x-y},$$

$$\frac{dz}{dp_Y}\frac{dp_Y}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_X}\frac{df_X}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_Y}\frac{df_Y}{dx} = \frac{4\left(t\left(x-5\right)-\tau\left(a-x\right)\right)}{9t(1-x-y)},$$

$$\text{and } \frac{df_X}{dx} = -\frac{2}{3}\left(t\left(x+4\right)+2\tau\left(a-x\right)\right).$$

The price competition effect $\frac{4(t(x-5)-\tau(a-x))}{9t(1-x-y)}$ is negative if firm A is located to the right of firm X ($a > x$) and $x < 5$. Given a, b, y, if retailer X moves to right (that is, if x increases) this may either increase or decrease the aggregate marginal cost that it pays depending on the location of its supplier A: $\frac{df_X}{dx} = \frac{d\left(w_A+\tau(a-x)^2\right)}{dx} = -\frac{2}{3}\left(t\left(x+4\right)+2\tau\left(a-x\right)\right)$ where w_A is given by (7). If firm A is located to the right of firm X ($a > x$) with $x+4 > 0$ then $\frac{df_X}{dx} < 0$, as x increases the marginal cost of firm X decreases, which is a positive effect. So, by increasing x, the marginal cost is reduced, demand is increased (demand effect) and prices fall (price competition effect). But if firm A is located to the left of firm X ($a < x$), the sign of $\frac{df_X}{dx}$ depends on the values of the cost parameters. If $\frac{df_X}{dx} > 0$, this means that, as x increases, the aggregate marginal cost increases too, which may lead to an increase in the retail price and a reduction in demand. Therefore, as firm X moves to the center, the sign of the demand effect and the price competition effect is ambiguous. In equilibrium, the upstream firms are located closer to the unit interval compared to the downstream firms and all four firms are outside the unit interval ($x < a < 0, y < b < 0$). The forces pushing firms (X relative to Y and A relative to B) close to each other are dominated by the forces pushing firms to the opposite direction and this total effect is stronger for the downstream firms.

3.6 Some extensions

In Appendix A we present two extensions of our basic model. First, we study the model where firms are not allowed to locate outside the unit interval and we find that firms are located at the two opposite endpoints (as firms locate always outside the unit interval under unrestricted locations). Second, we allow retailers to possibly locate to the opposite side of their wholesalers on the line.

We find that for some upstream locations and parameter values it could also be an equilibrium in the second stage of the game that the retailers locate at the opposite side of the line relative to the corresponding wholesaler. We also show that the second stage equilibrium profit is lower in this arrangement. Still, we do not find an equilibrium of the entire game when all locations (upstream and downstream) are endogenous and are chosen so that wholesalers' and retailers' locations are chosen at the opposite side of one another.

4 The order of location choices matters

We present now a crucial extension of the basic model which allow us to obtain additional insights into the problem, especially about the location incentives. In this section, we consider what happens when the wholesalers cannot choose their locations first. As we show, the equilibrium locations are affected in a systematic way.

4.1 Simultaneous location choices

In contrast to our analysis thus far, suppose now that all four firms make their location choices simultaneously in the first stage, while the pricing stages of the game remain as before. In the second stage of this modified game the wholesale prices are determined, followed by the final prices, whereas final consumers decide which retailer to buy from. By inserting the wholesale and final prices that we have obtained in our proceeding analysis into the profits of the upstream and downstream firms, we obtain the profit functions depending on the four location choices. We find that $\Pi_A = 3\Pi_X$ and $\Pi_B = 3\Pi_Y$ for all location values and parameters t, τ:

$$\begin{aligned}
\Pi_X &= \frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)\right)^2}{162t(1-x-y)},\\
\Pi_Y &= \frac{\left(\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\right)^2}{162t(1-x-y)},\\
\Pi_A &= \frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)\right)^2}{54t(1-x-y)},\\
\Pi_B &= \frac{\left(\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\right)^2}{54t(1-x-y)}.
\end{aligned}$$

The four firms simultaneously seek to each maximize its profits with respect to its location. From the first order conditions of the upstream firms (and the fact that the corresponding profit margins are positive), we find that each wholesaler chooses to have the same location as the corresponding retailer:

$$a = x \text{ and } b = y.$$

Further, from the first order conditions of the retailers we obtain:[15] $a = x = b = y = -1.75$.

[15] The second order conditions are satisfied.

Thus, we have:[16]

Proposition 3 *In the simultaneous locations choice model, the equilibrium outcome is:*

$$\begin{aligned}
\widehat{a}^L &= \widehat{b}^L = \widehat{x}^L = \widehat{y}^L = -1.75, \\
\widehat{w}_A^L &= \widehat{w}_B^L = 13.5t, \ \widehat{p}_X^L = \widehat{p}_Y^L = 18t, \\
\widehat{z}^L &= 0.5, \\
\widehat{\Pi}_A^L &= \widehat{\Pi}_B^L = 6.75t, \ \widehat{\Pi}_X^L = \widehat{\Pi}_Y^L = 2.25t.
\end{aligned}$$

As in our basic model, all firms are located outside the unit interval, now at -1.75. We observe that the equilibrium locations are independent of the parameter τ, as downstream firms are located at the same point of their suppliers, which means that they pay zero transportation costs in equilibrium. Equilibrium locations are also independent of the parameter t; this is a result similar to the standard linear city model where firms are located in equilibrium at -0.25 for all (quadratic) transportation cost parameters. Compared to that model, firms in our model are located farther away ($-1.75 < -0.25$). The introduction of the wholesalers in the problem, which makes the production costs for the retailers endogenous, pushes the locations farther away. The forces that push the firms closer to each other are dominated by the forces that push them to the opposite direction and the latter are stronger at a model with endogenous production costs.[17,18]

4.2 Commitment incentives

It is important to compare the equilibrium locations in our simultaneous locations game (Proposition 3) with the sequential locations game (Proposition 1). In the previous section, we found that $x < a < 0$ and $y < b < 0$, which means that all firms are located outside the unit interval with the upstream firms to be closer to the center. We can further calculate that for all parameter values $-1.75 < x < a < 0$ and $-1.75 < y < b < 0$. Thus, we find that in the simultaneous locations game firms move farther away from the city center than when upstream locations are chosen first (see

[16] We obtain the same equilibrium outcome when downstream firms choose their locations first and upstream location choices follow. For details see Kourandi and Vettas (2010).

[17] Brekke and Straume (2004) allow wholesale prices to be determined through bargaining between the upstream and downstream firms but they do not study the location choices of the upstream firms. In contrast, we assume that wholesale prices are set by the upstream firms and that their locations are endogenous. When the bargaining power of the upstream firms equals one, and there is no product differentiation at the upstream level ($\tau = 0$) the two models deliver the same results, firms locate at -1.75.

[18] Matsushima (2004) solves the simultaneous restricted locations choice model so that upstream firms price discriminate among downstream firms. He finds that for some parameters values downstream firms are located closer to the center of the unit interval compared to the upstream firms in an effort to reduce the wholesale prices they pay. In our model, there is no such incentive since each retailer has its own supplier and no price discrimination takes place.

Figure 6).

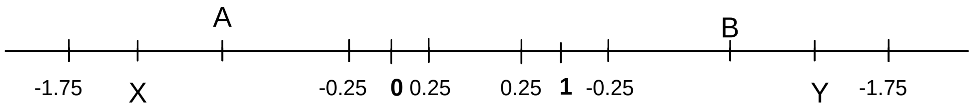


Figure 6: Locations under different timing of the game

In addition, profits for both upstream and downstream firms and wholesale and final prices are *higher* compared to the sequential location choice game. Why then upstream firms do not locate at -1.75 in our basic model, to obtain higher profits? Each upstream firm has a unilateral incentive to deviate from -1.75 and move closer to the center given that its rival is located at -1.75 and that downstream locations will follow equation (9). Each wholesaler does this in an effort to offer a strategic commitment incentive to its own retailer, to help it move credibly closer to the market center and, in this way, strengthen its position in the final market. However, both retailers become more competitive in this way and all firms obtain lower equilibrium profits, that is we could say we have a Prisoners' Dilemma situation in locations when wholesalers choose locations first. Both upstream firms jointly prefer being located at -1.75 (which also leads to $x = y = -1.75$) however there is a unilateral incentive for each to move closer to the unit interval to (indirectly) increase its own demand and enjoy higher profits. For example, if firm B were located at -1.75 then firm A would like to deviate from -1.75 and move closer to the unit interval (a increases) so that it pulls firm X also closer to the unit interval (x increases too). Demand would then unilaterally increase for firms X and A, as well as profits.

Remark 1 *When wholesalers choose their locations first, they locate closer to the center in an effort to also pull their retailers closer to the center, but they end up with lower equilibrium profits than in the simultaneous locations game.*

Thus, we find that firms would prefer to be in a game where they set their locations simultaneously compared to the sequential game where upstream firms move first. When upstream firms set their locations first, they can unilaterally affect the downstream locations and this leads to a stronger competition between them. Likewise, if the location choices were reversed compared to our basic model and downstream firms chose their locations first, the equilibrium locations would coincide with the simultaneous equilibrium locations.[19] If retailers are already located at -1.75, the suppliers have no other choice but to locate on their retailers' locations to maximize their profits. In summary, all four firms prefer the downstream firms to locate first (or that locations are simultaneous) so that upstream firms locate at -1.75 and not move to a location closer to the center.

[19] See Kourandi and Vettas, 2010.

Higher profits can be obtained either with simultaneous location choices or when the downstream firms locate first, compared to our basic model.

This result is reminiscent of other results in the sequential choices and strategic commitment literature. In particular, Fershtman and Judd (1987), Bonanno and Vickers (1988), Brander and Spencer (1985), and others have compared contracting and pricing incentives in structures with a vertical dimension. Owners reward managers for revenue or market shares, upstream firms choose low wholesale prices, governments choose export subsidies, all in an effort to unilaterally strengthen the strategic position of their downstream agents (technically, to shift their reaction functions to a more aggressive position). Here, we identify how this incentive for strategic commitment to aggressive behavior affects upstream and downstream equilibrium locations (and prices) along the real line.

5 Vertical integration

Thus far we have examined the case of vertical separation: all firms, upstream and downstream are independent and seek to maximize their own profit. Here we examine the problem under vertical integration. This study offers a useful benchmark, offering also additional insights into the vertical separation case. Under vertical integration (firm X with its supplier A and firm Y with its supplier B), the wholesalers charge zero wholesale prices to their own retailers in the vertical chain. However, as in the vertical separation model, upstream and downstream divisions of vertically integrated firms are allowed to be at different locations. As before, we start by considering the case where upstream firms choose locations first. Then we study the simultaneous location choices, and finally we compare the equilibrium between the vertical integration and the vertical separation cases. Of course, we are interested both in the locations chosen and in the final prices.

5.1 Upstream locations first

In the first stage of this vertical integration game, each of the two vertically integrated chains chooses the location of its upstream division. In the second stage, each chain chooses the location of its downstream division.[20] In the third stage, the two vertically integrated (VI) chains compete for the consumers by setting the final prices. We proceed backwards to solve for the subgame perfect equilibrium. The third stage of the game remains the same as in our basic model and the final prices for the chain A-X are given by equation (4). Each VI pair of firms maximizes the joint profits of its chain given all the locations chosen. Now the marginal production costs, f_X and f_Y, become simply the transportation costs of the retailers, since the wholesale prices are zero. The equilibrium profits of the vertically integrated chain A-X, Π_X^{VI}, are as follows:

[20] Thus, one may think of each vertically integrated firm as having a factory and a point of sales possibly at different geographical locations.

$$\Pi_X^{VI} = \begin{cases} f_Y - t(1-x-y)(1+y-x) - f_X & if \quad f_X \leq \overline{f_X} \\ \frac{(t(1-x-y)(3+x-y)+f_Y-f_X)^2}{18t(1-x-y)} & if \quad \overline{f_X} < f_X < \widehat{f_X} \\ 0 & if \quad f_X \geq \widehat{f_X}, \end{cases} \tag{11}$$

where $\widehat{f_X} = f_Y + t(1-x-y)(3-y+x)$, $\overline{f_X} = f_Y - t(1-x-y)(3-x+y)$ and $f_X = \tau(a-x)^2$, $f_Y = \tau(y-b)^2$. Similarly, we can calculate the joint profits for the rival chain, B-Y.

In the second stage, taking as given the locations of the upstream divisions and assuming that the retail prices will be subsequently determined in equilibrium, the VI firms simultaneously choose the locations of their downstream divisions to maximize their joint profits. Firms are again allowed to locate anywhere on the real line. Special care should be taken about the possibility that one firm may end up with zero sales, a corner solution. If upstream firms are located asymmetrically enough, this may favor a downstream division and this chain may take the whole demand. Following the same logic as in Subsection 3.3, the cost disadvantaged retailer minimizes the area where its demand is zero. Of course, when neither retailer has such a high cost disadvantage, firms share the demand. From this analysis we obtain the equilibrium downstream locations (as functions of the upstream locations):

$$\begin{aligned} (x,y) &= \left(\tfrac{t+a\tau}{t+\tau}, \tfrac{b\tau-t}{t+\tau}\right) \\ &\quad \text{if } 9t - 4\tau\,(a-b-3)\,(1-a-b) \leq 0 \\ &= \left(\tfrac{3t^2+4t\tau\left(-5a+a^2-b^2+1\right)+16\tau^2 a(a+b-1)}{4(t+\tau)(-3t+4\tau(a+b-1))}, \tfrac{3t^2+4t\tau\left(-5b-a^2+b^2+1\right)+16\tau^2 b(a+b-1)}{4(t+\tau)(-3t+4\tau(a+b-1))}\right) \\ &\quad \text{if } \{9t - 4\tau\,(a-b-3)\,(1-a-b) > 0 \text{ and } 9t + 4\tau\,(a-b+3)\,(1-a-b) > 0\} \\ &= \left(\tfrac{a\tau-t}{t+\tau}, \tfrac{t+b\tau}{t+\tau}\right) \\ &\quad \text{if } 9t + 4\tau\,(a-b+3)\,(1-a-b) \leq 0. \end{aligned} \tag{12}$$

Now, in the first stage of the game, upstream locations will be chosen to maximize the joint profits of each chain. An upstream location will not be chosen so that the corresponding retailer obtains zero demand. Therefore, in equilibrium both vertical chains avoid receiving zero demand and the relevant joint profit functions reduce to:

$$\begin{aligned} \Pi_X^{VI} &= \frac{t\,(3t+2\tau\,(1-a-b))\,(9t+4\tau\,(a-b+3)\,(1-a-b))^2}{36\,(t+\tau)\,(3t+4\tau\,(1-a-b))^2} \\ \Pi_Y^{VI} &= \frac{t\,(3t+2\tau\,(1-a-b))\,(9t+4\tau\,(b-a+3)\,(1-a-b))^2}{36\,(t+\tau)\,(3t+4\tau\,(1-a-b))^2}. \end{aligned}$$

Maximization of the expressions and solving for the equilibrium yields: $a = b = \frac{3t+\tau-3\sqrt{\tau^2+t^2}}{8\tau}$.

Proposition 4 *Under vertical integration and with upstream locations chosen first, the unique* *subgame perfect equilibrium outcome is:*

$$\begin{aligned}
a^{VI} &= b^{VI} = \frac{3t+\tau-3\sqrt{\tau^2+t^2}}{8\tau}, \\
x^{VI} &= y^{VI} = \frac{t+\tau-3\sqrt{t^2+\tau^2}}{8\left(t+\tau\right)}, \\
p_X^{VI} &= p_Y^{VI} = \frac{3t\left(8\tau^3+19t\tau^2+11t^2\tau+3t^3+\sqrt{\tau^2+t^2}\left(-3t+8\tau\right)\left(t+\tau\right)\right)}{32\tau\left(t+\tau\right)^2}, \\
z^{VI} &= 0.5, \\
\Pi_X^{VI} &= \Pi_Y^{VI} = \frac{3t\left(t+\tau+\sqrt{t^2+\tau^2}\right)}{8\left(t+\tau\right)}.
\end{aligned}$$

We note that, in equilibrium, firms obtain positive profits for all parameter values and the retailers are always located outside the unit interval ($x^{VI} = y^{VI} < 0$). Further, upstream firms are located closer to the center relative to the downstream firms ($a^{VI} > x^{VI}$) and upstream firms are located within the unit interval ($a^{VI} > 0$) when $t > \frac{4\tau}{3}$ and outside otherwise.

By substituting the equilibrium locations under VI in the social cost function (10), we calculate that, in equilibrium, the total social cost is:

$$SC^{VI} = \frac{t\left(17t\tau+17\tau^2+27t^2-9\sqrt{\tau^2+t^2}\left(3t-\tau\right)\right)}{96\tau\left(t+\tau\right)}.$$

We postpone the comparison between the vertical integration case and the equilibrium outcome of our basic model until Subsection 5.3.

5.2 Simultaneous location choices

Suppose now that the VI firms choose their upstream and downstream division locations simultaneously in the first stage, while the pricing stage of the game remains the same. The two VI firms choose their upstream and downstream locations simultaneously to maximize their joint profits:

$$\begin{aligned}
\Pi_X^{VI} &= \frac{\left(t(1-x-y)(3+x-y)+\tau(y-b)^2-\tau(x-a)^2\right)^2}{18t(1-x-y)}, \\
\Pi_Y^{VI} &= \frac{\left(t(1-x-y)(3+y-x)+\tau(x-a)^2-\tau(y-b)^2\right)^2}{18t(1-x-y)}
\end{aligned}$$

Profits Π_X^{VI} are maximized with respect to a and x and profits Π_Y^{VI} are maximized with respect to b and y. The second order conditions hold and the first order conditions imply a unique maximum. We find that each wholesaler will be chosen to be at the same location as the corresponding retailer:

$$a = x \text{ and } b = y.$$

Further, from the first order conditions, we obtain: $a = x = b = y = -0.25$. Thus, we have:

Proposition 5 *With simultaneous locations choices, the equilibrium outcome under vertical integration is:*

$$\begin{aligned}\widehat{a}^{VI} &= \widehat{b}^{VI} = \widehat{x}^{VI} = \widehat{y}^{VI} = -0.25,\\ \widehat{p}_X^{VI} &= \widehat{p}_Y^{VI} = \frac{3t}{2},\\ \widehat{z}^{VI} &= 0.5,\\ \widehat{\Pi}_X^{VI} &= \widehat{\Pi}_Y^{VI} = \frac{3t}{4}.\end{aligned}$$

All four firms locate outside the unit interval, now at -0.25. The equilibrium locations are the same as when upstream and downstream locations have to coincide. In other words, the locations here are the ones we would get in the d' Aspremont *et al.* (1979) variant of the Hotelling's model where are no vertical relations and when firms could locate anywhere on the line.

5.3 Comparison

First, we compare the VI cases under the alternative timing assumptions about the order of the location choices and then we compare the VI case to the vertical separation case. We can directly calculate that, for all parameter values, $-0.25 < x^{VI} < a^{VI}$ and $-0.25 < y^{VI} < b^{VI}$. In the simultaneous locations game the VI firms move farther away from the city center relative to the case where upstream locations are chosen first. In addition, joint profits are *higher* compared to the sequential locations choice game. Why then the VI firms do not also choose their upstream locations at -0.25 in the sequential locations choice game, to obtain higher profits? The argument is related to that in Subsection 4.2 but not identical since now we have VI. Each VI firm has a unilateral incentive to deviate from -0.25 and each upstream division moves closer to the center given that its rival upstream division is at -0.25 and that downstream locations will follow equation (12). Each VI does this in an effort to obtain a strategic advantage: by moving its upstream division closer to the center, it forces the downstream division to go closer to the center as well, thus creating a force that pushes the rival downstream division further away from the center than otherwise. In such a way, a given chain unilaterally seeks to strengthen its position in the market. However, both chains become more competitive in this way and both obtain lower equilibrium profits. We conclude that:

Remark 2 *Vertically integrated firms obtain higher equilibrium profit when they set their locations simultaneously (or the downstream locations are chosen first) compared to the sequential game where upstream locations are chosen first.*

It is important now to compare the equilibrium outcome under vertical integration (Proposition 4) to the vertical separation (Proposition 1) case. By direct calculations, we compare the equilibrium locations and obtain:

Proposition 6 *We have* $-1.75 < x^L = y^L < a^L = b^L < -0.25 < x^{VI} = y^{VI} < a^{VI} = b^{VI} < 0.25$.

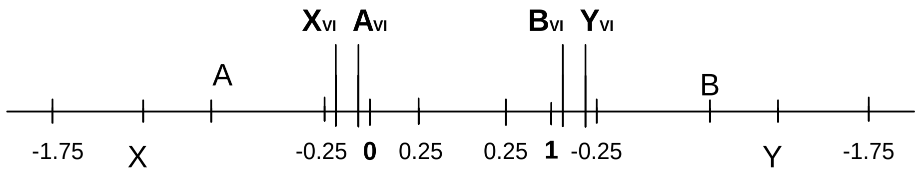


Figure 7: Locations under vertical separation and vertical integration

Under vertical separation, downstream firms locate farther away relative to the upstream firms ($x^L = y^L < a^L = b^L$) and the latter are located farther away relative to the simultaneous locations choices under vertical integration ($a^L = b^L < -0.25$). Moreover, under vertical integration when upstream locations are chosen first, the downstream and upstream firms move closer to the center relative to the vertical separation ($x^L = y^L < a^L = b^L < -0.25 < x^{VI} = y^{VI} < a^{VI} = b^{VI}$) with the upstream firms to be located even closer to the center. However, product differentiation under vertical integration is greater than the social optimum ($x^{VI} = y^{VI} < a^{VI} = b^{VI} < 0.25$). Under vertical integration, the aggregate marginal cost effect is modified, since the wholesale prices are zero. A change in the location of firm X, still affects the marginal cost (f_X) that it pays since it affects its transportation cost but, in contrast to our basic model under vertical separation, there is no effect through the wholesale prices. The equilibrium locations under vertical integration are closer to the center, thus, price competition is less intense when there is no competition in wholesale prices.

Now, let us turn to the final prices. We find that, in equilibrium, the final prices under vertical separation are higher than under vertical integration ($p_X^L = p_Y^L > p_X^{VI} = p_Y^{VI}$). This result did not have to necessarily hold since in our model we have duopoly competition in each stage. We find that each upstream firm raises its price to a level that makes its retailer charge a higher final price than the one we would see in equilibrium under vertical integration. Only part of the negative effect to its retailer of an increased wholesale price is taken into account by the wholesaler, while the remainder of this effect emerges as an externality.

We can obtain some additional insights into the problem by comparing for each symmetric pair of upstream and downstream locations ($a = b,\ x = y$) the final prices under vertical separation and under vertical integration. We find that:

Proposition 7 *For any pair of symmetric locations, final prices under vertical separation are always higher that under vertical integration,* $p_X^L\,(a = b,\ x = y) = p_Y^L\,(a = b,\ x = y) > p_X^{VI}\,(a = b,\ x = y) = p_Y^{VI}\,(a = b,\ x = y)$.

In the result above, we have fixed the upstream and downstream locations, even when these are not equilibrium locations, and compare the vertical separation and vertical integration cases. We find that double marginalization emerges under vertical separation, with positive profit margins at both upstream and downstream levels and higher chain profits compared to the vertical integration case. The idea here is similar to the other models of vertical relations with price competition in the final market (see e.g. Bonanno and Vickers, 1988, and Rey and Stiglitz, 1995). When the retailers' choices are strategic complements (as final prices are in our model) vertical separation tends to imply that wholesale prices will be set at levels that, in equilibrium, will imply higher final prices than under vertical integration. Of course, the important difference in our model is that product differentiation is endogenous and that location and pricing incentives are *interrelated*.

We also compare the social cost under vertical separation to the social cost under vertical integration and have:

Remark 3 *The social cost under vertical separation is higher than under vertical integration.*

Under vertical integration, the equilibrium locations are closer to the social optimum relative to the equilibrium locations under vertical separation, thus, social cost is higher under vertical separation in equilibrium.

6 Two-part tariffs

It is also interesting to examine the problem under two-part tariffs. In addition to a per unit wholesale price that retailers pay when supplied by their suppliers, there can also be a fixed fee in the trade within each vertical chain (F_A and F_B). We will show that this case is in between the linear pricing and the VI case, not only in terms of final prices and industry profits but also in terms of locations. However to analyze this case in a meaningful way we have to modify our model and relax the assumption that the upstream firms have all the pricing bargaining power. This is because then the upstream firms would always capture the entire chain profit leaving zero profits to the retailers and making them indifferent as to which location to choose earlier in the game. Therefore, we now assume that the upstream firm in each vertical chain bargains with the downstream firm over the two-part tariff terms in the third stage of the game. Specifically, we denote by θ the bargaining power of each upstream firm, with $\theta \in (0, 1)$. As before, we start by considering the case where upstream firms choose locations first. Then we study simultaneous location choices, and finally we compare the equilibrium of the game to the linear pricing and to the vertical integration cases.

6.1 Upstream locations first

The timing of the game is the same as in our basic model with the only difference that firms in the third stage of the game bargain over the two-part tariff contract terms. We proceed backwards to find the subgame perfect equilibrium. The retail prices are given again by expression (4) but now the profits of the retailers (by expression (5)) are reduced by the fixed fees, $\Pi_X - F_A$ and $\Pi_Y - F_B$ respectively (note that these fees may be negative).

In the third stage of the game, each vertical chain maximizes the Nash product with respect to the wholesale prices and the fixed fees (w_i, F_i), $i = A, B$. The vertical chain A-X solves:

$$\underset{w_A, F_A}{max} \, (\Pi_A + F_A)^{\theta} (\Pi_X - F_A)^{1-\theta} = (w_A z + F_A)^{\theta} \left((p_X - w_A - \tau(x-a)^2)z - F_A\right)^{1-\theta}.$$

Note that the two chains bargain simultaneously and we are looking for a Nash equilibrium in Nash bargains. In equilibrium, the wholesale price chosen by each chain maximizes its joint profit given the wholesale price chosen by the rival chain. The fixed fee is then used to split the equilibrium joint profit according to the bargaining power of the parties. From the first order condition with respect to the fixed fee, we obtain $F_A = \theta (\Pi_A + \Pi_X) - \Pi_A$ and that firms in the vertical chain maximize their joint profits $\Pi_A + \Pi_X = (p_X - \tau(x-a)^2)z$ with respect to the wholesale price w_A and share these profits according to their bargaining power θ and $1 - \theta$.[21] From this maximization we obtain the reaction functions of the wholesale prices.[22] For w_A we have

$$\begin{aligned}
w_A = R_{A-X}(w_B) \quad &= \quad w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\
&\quad\quad \text{if } w_B \geq \tau\left((a-x)^2 - (b-y)^2\right) + t\,(x-y-5)\,(x+y-1) \\
&= \quad \frac{w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t(x-y+3)(1-x-y)}{4} \\
&\quad\quad \text{if } \left\{ \begin{array}{l} w_B < \tau\left((a-x)^2 - (b-y)^2\right) + t\,(x-y-5)\,(x+y-1) \\ w_B > \tau\left((a-x)^2 - (b-y)^2\right) - t\,(y-x-3)\,(x+y-1) \end{array} \right\} \\
&= \quad \frac{\tau\left((x-a)^2 - (y-b)^2\right) + t(x-y+3)(x+y-1) - w_B}{2} \\
&\quad\quad \text{if } w_B \leq \tau\left((a-x)^2 - (b-y)^2\right) - t\,(y-x-3)\,(x+y-1)
\end{aligned}$$

and analogously for w_B. Solving the system of the two reaction functions we obtain the equilibrium wholesale prices

[21] For a proof see Milliou *et al.* (2003).

[22] For the corners (unit or zero demand) we can argue in a similar way as in the basic model.

$$
\begin{aligned}
w_A &= \tau\left((b-y)^2-(a-x)^2\right)+t\,(y-x+3)\,(x+y-1) \\
&\quad \text{if } t\,(y-x+5)\,(x+y-1)-\tau\left((a-x)^2-(b-y)^2\right)\geq 0 \\
&= \tfrac{\tau\left((b-y)^2-(a-x)^2\right)+t(x-y+5)(1-x-y)}{5} \\
&\quad \text{if } \left\{\begin{array}{l} t\,(y-x+5)\,(x+y-1)-\tau\left((a-x)^2-(b-y)^2\right)<0 \\ t\,(x-y+5)\,(x+y-1)+\tau\left((a-x)^2-(b-y)^2\right)<0 \end{array}\right\} \\
&= 0 \\
&\quad \text{if } t\,(x-y+5)\,(x+y-1)+\tau\left((a-x)^2-(b-y)^2\right)\geq 0
\end{aligned}
$$

and joint profits

$$
\begin{aligned}
\Pi_A+\Pi_X &= \tau\left((y-b)^2-(x-a)^2\right)-t(1-x-y)(1+y-x) \\
&\quad \text{if } t\,(y-x+5)\,(x+y-1)-\tau\left((a-x)^2-(b-y)^2\right)\geq 0 \\
&= \tfrac{\left(t(x-y+5)(x+y-1)+\tau\left((a-x)^2-(b-y)^2\right)\right)^2}{25t(1-x-y)} \\
&\quad \text{if } \left\{\begin{array}{l} t\,(y-x+5)\,(x+y-1)-\tau\left((a-x)^2-(b-y)^2\right)<0 \\ t\,(x-y+5)\,(x+y-1)+\tau\left((a-x)^2-(b-y)^2\right)<0 \end{array}\right\} \\
&= 0 \\
&\quad \text{if } t\,(x-y+5)\,(x+y-1)+\tau\left((a-x)^2-(b-y)^2\right)\geq 0.
\end{aligned}
$$

Then these profits are split according to the rule: $\Pi_A+F_A=\theta\,(\Pi_A+\Pi_X)\,,\ \Pi_X-F_A=(1-\theta)\,(\Pi_A+\Pi_X)\,.$

In the second stage, taking as given the locations of the upstream firms and assuming that the contract terms and retail prices will be subsequently determined in equilibrium, the downstream firms simultaneously choose their locations to maximize their profits, Π_X-F_A and Π_Y-F_B. Firms are again allowed to locate anywhere on the real line and care should be taken about the possibility that one firm may end up with zero sales (if upstream firms are located asymmetrically enough, this may favor a downstream firm and this chain may take the whole demand). Following the same logic as in Subsection 3.3, the cost disadvantaged retailer minimizes the area where its demand is zero. Of course, when neither retailer has such a high cost disadvantage, firms share the demand. From this analysis we obtain the equilibrium downstream locations (as functions of the upstream locations):

$$
\begin{aligned}
(x,y) &= \left(\tfrac{3t+a\tau-\sqrt{t\tau(a-b-5)(1-a-b)}}{t+\tau},\ \tfrac{b\tau-2t}{t+\tau}\right) \\
&\quad \text{if} 4\tau\left(a-b-5\right)\left(1-a-b\right)-25t\geq 0 \\
&= \left(\tfrac{16\tau^2a(a+b-1)+4t\tau\left(-9a-2b+a^2-b^2+3\right)+15t^2}{4(t+\tau)(-5t+4\tau(a+b-1))},\ \tfrac{16\tau^2b(a+b-1)+4t\tau\left(-9b-2a+b^2-a^2+3\right)+15t^2}{4(t+\tau)(-5t+4\tau(a+b-1))}\right) \\
&\quad \text{if } \left\{4\tau\left(a-b-5\right)\left(1-a-b\right)-25t<0 \text{ and } 4\tau\left(b-a-5\right)\left(1-a-b\right)-25t<0\right\} \\
&= \left(\tfrac{a\tau-2t}{t+\tau},\ \tfrac{3t+b\tau-\sqrt{t\tau(b-a-5)(1-a-b)}}{t+\tau}\right) \\
&\quad \text{if } 4\tau\left(b-a-5\right)\left(1-a-b\right)-25t\geq 0.
\end{aligned}
$$

Now, in the first stage of the game, upstream locations are chosen so as to maximize upstream profits. of course, an upstream location will not be chosen so that the corresponding retailer obtains zero demand. Therefore, in equilibrium both vertical chains avoid receiving zero demand. Maximization of the upstream profits and solving for the equilibrium yields: $a=b=\frac{5t-\tau-5\sqrt{\tau^2+t^2}}{8\tau}$. In summary, we obtain:

Proposition 8 *Under two-part tariffs and with upstream locations chosen first, the unique subgame perfect equilibrium outcome is:*

$$
\begin{aligned}
a^T &= b^T = \frac{5t-\tau-5\sqrt{\tau^2+t^2}}{8\tau}, \\
x^T &= y^T = -\frac{t+\tau+5\sqrt{\tau^2+t^2}}{8\left(t+\tau\right)}, \\
w_A^T &= w_B^T = \frac{5t\left(t+\tau+\sqrt{\tau^2+t^2}\right)}{4\left(t+\tau\right)}, \\
F_A^T &= F_B^T = \frac{\left(2\theta-1\right)5t\left(t+\tau+\sqrt{\tau^2+t^2}\right)}{8\left(t+\tau\right)}, \\
p_X^T &= p_Y^T = \frac{5t\left(\sqrt{\tau^2+t^2}\left(-5t+16\tau\right)\left(t+\tau\right)+5t^3+21t^2\tau+37t\tau^2+16\tau^3\right)}{32\tau\left(t+\tau\right)^2}, \\
z^T &= 0.5, \\
\Pi_A^T+\Pi_X^T &= \Pi_B^T+\Pi_Y^T = \frac{5t\left(t+\tau+\sqrt{\tau^2+t^2}\right)}{4\left(t+\tau\right)}, \\
\Pi_A^T+F_A^T &= \Pi_B^T+F_B^T = \frac{5t\theta\left(t+\tau+\sqrt{\tau^2+t^2}\right)}{4\left(t+\tau\right)}, \\
\Pi_X^T-F_A^T &= \Pi_Y^T-F_B^T = \frac{5t\left(1-\theta\right)\left(t+\tau+\sqrt{\tau^2+t^2}\right)}{4\left(t+\tau\right)}.
\end{aligned}
$$

We note that, in equilibrium, the two downstream firms and the corresponding two upstream firms share the market equally and all four firms obtain positive profits. The fixed fees are positive when $\theta > \frac{1}{2}$, otherwise for $\theta < \frac{1}{2}$ the fixed fees become negative and the upstream firms should pay these fees to the retailers to carry their products. Further, all locations are outside the unit interval and upstream firms locate closer to each other (and to the market center) relative to the downstream firms ($0 > a^T > x^T$). Note also that the wholesale prices are still positive, and in this sense, "double marginization" again exists.

By substituting the equilibrium locations under two-part tariffs in the social cost function (10), we calculate that, in equilibrium, the total social cost is:

$$SC^T = \frac{t\left(75t^2 + 53t\tau + 53\tau^2 - 15\,(5t - 3\tau)\sqrt{\tau^2 + t^2}\right)}{96\tau\,(t + \tau)}.$$

We postpone the comparison between the two-part tariffs case and the equilibrium outcome of our basic model with linear prices until Subsection 6.3.

6.2 Simultaneous location choices

Suppose now that the firms choose their upstream and downstream locations simultaneously in the first stage, while the pricing stages of the game remain the same. The four firms simultaneously seek to each maximize its profits with respect to its location. Since upstream and downstream firms share their joint profits according to their bargaining power, each chain maximizes its joint profits simultaneously with respect to the upstream and downstream locations. We find that each wholesaler chooses to have the same location as the corresponding retailer: $a = x$ and $b = y$ and further, from the first order conditions of the retailers we obtain: $a = x = b = y = -0.75$. Thus, we have:

Proposition 9 *In the simultaneous locations choice model, the equilibrium outcome under two-part tariffs is:*

$$\begin{aligned}
\widehat{a}^T &= \widehat{b}^T = \widehat{x}^T = \widehat{y}^T = -0.75, \\
\widehat{w}_A^T &= \widehat{w}_B^T = \frac{5t}{2}, \\
\widehat{F}_A^T &= \widehat{F}_B^T = \frac{5t\,(2\theta - 1)}{4}, \\
\widehat{p}_X^T &= \widehat{p}_Y^T = 5t, \\
\widehat{z}^T &= 0.5, \\
\widehat{\Pi}_A^T + \widehat{\Pi}_X^T &= \widehat{\Pi}_B^T + \widehat{\Pi}_Y^T = \frac{5t}{2}, \\
\widehat{\Pi}_A^T + \widehat{F}_A^T &= \widehat{\Pi}_B^T + \widehat{F}_B^T = \frac{5t\theta}{2}, \\
\widehat{\Pi}_X^T - \widehat{F}_A^T &= \widehat{\Pi}_Y^T - \widehat{F}_B^T = \frac{5t\,(1 - \theta)}{2}.
\end{aligned}$$

All four firms locate outside the unit interval, now at -0.75. As in the sequential location model, the fixed fees are positive when $\theta > \frac{1}{2}$ and otherwise negative.

6.3 Comparison

First, we compare the two-part tariff cases under our alternative timing assumptions about the order of the location choices. Then we will compare the two-part tariff pricing case to the linear pricing case and to the vertical integration case. We calculate that, for all parameter values, $-0.75 < x^T < a^T$ and $-0.75 < y^T < b^T$. In the simultaneous locations game the firms move farther away from the city center relative to the case where upstream locations are chosen first. In addition, profits are *higher* compared to the sequential locations choice game, as well as the wholesale and retail prices. Again, the argument is similar to that in Subsection 4.2 concerning the incentives of the upstream firms to offer a stronger strategic commitment to their downstream firms. We conclude that:

Remark 4 *Firms under two-part tariff pricing obtain higher equilibrium profit when they set their locations simultaneously compared to the sequential game where upstream locations are chosen first.*

It is important now to compare the equilibrium outcome under two-part tariffs (Proposition 8) to the equilibrium outcome under linear pricing (Proposition 1) and to the vertical integration (Proposition 4) case. By direct calculations, we compare the equilibrium locations and obtain:

Proposition 10 *We have* $a^L = b^L < a^T = b^T < a^{VI} = b^{VI}, x^L = y^L < x^T = y^T < x^{VI} = y^{VI}$ *with* $a^i = b^i > x^i = y^i$ *for* $i = L, T, VI$.

Under two-part tariffs upstream and downstream firms locate between the locations chosen under linear pricing and vertical integration. Competition under two-part tariffs is more intense compared to the linear pricing but less intense compared to the vertical integration case. This is also illustrated on the wholesale and retail prices. By direct comparison we obtain:

Proposition 11 *We have* $w_A^L = w_B^L > w_A^T = w_B^T > 0$ *and* $p_X^L = p_Y^L > p_X^T = p_Y^T > p_X^{VI} = p_Y^{VI}$.

Under two-part tariffs, firms can also use fixed fees to capture profits. The presence of this second instrument allows the wholesale price to play a stronger strategic role. As a result with two-part tariffs price competition becomes more intense with lower wholesale and retail prices compared to the linear pricing game. At the same time, these values are higher compared to the vertical integration case. Considering also how equilibrium locations change, total chain profits under linear pricing exceed the chain profits under two-part tariffs and the latter exceed the profits under vertical integration: $\Pi_A^L + \Pi_X^L = \Pi_B^L + \Pi_Y^L > \Pi_A^T + \Pi_X^T = \Pi_B^T + \Pi_Y^T > \Pi_X^{VI} = \Pi_Y^{VI}$.

Finally, by comparing the social cost under the three different cases we obtain:

Remark 5 *The social cost under linear pricing exceeds the social cost under two-part tariffs and that under vertical integration,* $SC^L > SC^T > SC^{VI}$.

Under vertical integration, the equilibrium locations are closer to the social optimum relative to the equilibrium locations under vertical separation both under two-part tariffs and linear pricing, thus, social cost is lower under vertical integration in equilibrium.

7 Conclusion

Our paper contributes to two literatures, on horizontal differentiation and on vertical contracting. We have studied a linear city model with duopoly upstream and downstream. Wholesalers and then retailers choose their locations and then their prices, before consumers make their choices. We can derive in closed form a unique subgame perfect equilibrium of this five stage game and examine its properties. We find that wholesalers choose to become less differentiated (that is, locate closer to the unit interval) than the retailers and that differentiation (in upstream and downstream locations) is greater compared to the vertical integration benchmark, which in turn is greater than in the social optimum. We also find positive profit margins both upstream and downstream and that chain profits and final prices are higher than under vertical integration ("double marginalization"). Thus, vertical separation in our duopoly implies both a higher social cost (of transportation) and higher final prices for the consumers. Our study of two-part tariffs (with upstream-downstream bargaining) shows that in this case industry profits, prices, and locations are in between the cases of linear pricing and vertical integration.

Considering a number of extensions offers additional insights into the problem. We modify the order of location choices and find that when locations are chosen simultaneously by all four firms, the upstream and the downstream firms in each pair choose the same location. This location is also farther away from the city center relative to the case when upstream locations are chosen first: in that case, the wholesalers locate closer to the city center with the goal to also pull their retailers closer to the center, so that they can each strategically strengthen their retailer's position in the downstream market. Still, since this happens for both vertical chains, the end result is a more competitive market (than when all locations are chosen simultaneously) and profits are lower.

While the results presented here take a relatively simple and clear form, the equilibrium calculations are quite involved, as one should expect from a five stage game, especially when care has to be taken for possible corner solutions. Still, a number of extensions and modifications appear promising. In companion work we consider the role of price discrimination at the wholesale level (upstream competition is intensified as firms compete for the demand of each retailer separately) and non exclusive vertical relations (due to the discontinuity of the profit functions under some parameter values the upstream firms always undercut their rival and no equilibrium in pure strategies exist in the wholesale prices). Additional modifications of the product differentiation structure, pricing and contracting and the timing of the game may offer additional important insights, and so would models where consumers may directly care also about the upstream choices (and not only indirectly, as in our model). Of course, empirical work that studies the interplay of product differentiation and vertical contracting will also be of great interest and hopefully our theoretical study of provides some foundations in this direction.

References

[1] Aiura H. and Sato Y. (2008), Welfare properties of spatial competition with location-dependent costs, *Regional Science and Urban Economics*, 38 (1), 32-48.

[2] Allain M.L. (2002), The balance of power between manufacturers and retailers: a differentiation model, *Louvain Economic Review*, 68, 359-370.

[3] Anderson S., de Palma A. and Thisse J.-F. (1992), *Discrete choice theory of product differentiation*, MIT Press.

[4] Anderson S. and Engers M., (1994), Spatial competition with price-taking firms, *Economica*, 61, 125-136.

[5] Anderson S. and Neven D., (1991), Cournot competition yields spatial agglomeration, *International Economic Review*, 32, 4, 793-808.

[6] Beladi H., Chakrabarti A. and Marjit S. (2008), Vertical mergers and downstream spatial competition with different product varieties, *Economics Letters*.

[7] Bonanno G. and Vickers J. (1988), Vertical Separation, *The Journal of Industrial Economics* 36, 3, 257-265.

[8] Brander J. and Spencer B. (1985), Export subsidies and international market share rivalry, *Journal of International Economics* 18, 83-100.

[9] Brekke K. and Straume O. R. (2004), Bilateral monopolies and location choice, *Regional Science & Urban Economics* 34, 275-288.

[10] Christou C. and Vettas N. (2005), Location choices under quality uncertainty, *Mathematical Social Sciences* 50, 268-278.

[11] d'Aspremont C., Gabszewicz J.J. and Thisse J.-F. (1979), On Hotelling's "Stability in competition", *Econometrica* 47, 1145-1150.

[12] Dobson P. and Waterson M. (1996), Exclusive tradinng contracts in successive differentiated duopoly, *Southern Economic Journal*, 63 (2), 361-377.

[13] Fershtman C. and Judd K. (1987), Equilibrium Incentives in Oligopoly, *The American Economic Review* 77, 5, 927-940.

[14] Gabszewicz J.J. and Thisse J.-F. (1992), Location. In: Aumann R., Hart S. (eds.), Handbook of Game Theory, vol. I. North-Holland, Amsterdam, 281-304 (chapter 9).

[15] Gupta B.H., Kats A. and Pal D. (1994), Upstream monopoly, downstream competition and spatial price discrimination, *Regional Science and Urban Economics* 24, 529–542.

[16] Hotelling H. (1929), Stability in competition, *Economic Journal* 39, 41– 57.

[17] Inderst R. and Shaffer G. (2007), Retail mergers, buyer power and product variety, *The Economic Journal* 117, 45-67.

[18] Kourandi F. and Vettas N. (2010), Endogenous spatial differentiation with vertical contracting, CEPR discussion paper No. 7948.

[19] Lambertini L. (1997), Unicity of the equilibrium in the unconstrained Hotelling model, *Regional Science and Urban Economics* 27, 785-798.

[20] Laussel D. (2006), Are manufacturers competing with or through supermarkets? A theoretical investigation, *The B.E. Journal of Theoretical Economics*, 6, 1, 10.

[21] Matsushima N. (2004), Technology of upstream firms and equilibrium product differentiation, *International Journal of Industrial Organization*, 22, 1091– 1114.

[22] Matsushima N. (2009), Vertical mergers and product differentiation, *The Journal of Industrial Economics*, LVII, 4, 812-834.

[23] Milliou C., Petrakis E. and Vettas N. (2003), Endogenous contracts under bargaining in competing vertical chains, CEPR discussion paper No. 3976.

[24] Reisinger M. and Schnitzer M. (2010), Successive oligopolies with differentiated firms and endogenous entry, *Working Paper, University of Munich*, 1-42.

[25] Rey P. and Stiglitz J. (1995), The role of exclusive territories in producers' competition, *The Rand Journal of Economics*, 26, 3, 431-451.

[26] Rey P. and Tirole J. (2007), A Primer on Foreclosure, in: Armstrong M. and Rob Porter R. (eds.), Handbook of Industrial Organization III.

[27] Rey P. and Verge T. (2008), Economics of Vertical Restraints, in: Buccirossi P. (ed.), Handbook of Antitrust Economics, MIT Press (chapter 9).

[28] Tabuchi T. and Thisse J.-F. (1995), Asymmetric equilibria in spatial competition, *International Journal of Industrial Organization* 13, 213-227.

[29] Tirole J. (1989), *The Theory of Industrial Organization*, MIT Press.

[30] Vettas N. (1999), Location as a signal of quality, CEPR discussion paper No. 2165.

[31] Ziss S. (1993), Entry deterrence, cost advantage and horizontal product differentiation, *Regional Science and Urban Economics* 23, 523– 543.

Appendix A

We explore here extensions of our basic model. First, we study the model where firms are not allowed to locate outside the unit interval. Second, we allow retailers to possibly locate to the opposite side of their wholesalers on the line.

Appendix A1: Locations in the unit interval

We modify our basic model so that firms are restricted to locate within the [0,1], an assumption often made in the literature. Our analysis of the game is as in the basic model, however now a, b, x and y cannot take negative values. Thus, we have to solve for the restricted location choices in the first and second stage of the game with the other stages remaining the same.

In the second stage of the game, the downstream firms set their locations and their profit functions are given in Subsection (3.3). Can one of the two retailers in equilibrium serve the whole market? Firm X takes the whole demand if $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0$. Given that all firms are located in the unit interval, this expression can be negative only when $\tau(y-b)^2$ is higher than $\tau(x-a)^2+t\,(y-x+9)\,(1-x-y)$. Thus, firm Y can avoid receiving zero demand by setting $y=b$, that is, locate at the same point as its supplier. Firm Y has some local monopoly power and always serves some consumers located close to that firm. Likewise, firm X can avoid receiving zero demand by setting $x=a$. Therefore, in equilibrium no retailer can serve the whole demand, each retailer can assure at least some sales. The profit functions, thus, reduce to:

$$\Pi_X=\frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)\right)^2}{162t(1-x-y)},$$

and

$$\Pi_Y=\frac{\left(\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\right)^2}{162t(1-x-y)}.$$

From the first order conditions we obtain (note that the profit margin is positive[23]):

$$x=\frac{\left(16a\tau^2\,(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2\right)}{4\,(t+\tau)\,(4\tau\,(1-a-b)+9t)},$$

and

$$y=\frac{\left(16b\tau^2\,(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2\right)}{4\,(t+\tau)\,(4\tau\,(1-a-b)+9t)}.$$

However, the pair of locations (x, y) is positive only if the numerators of x and y are positive (also note that $x,\ y<1$). Otherwise, due to the concavity of the profit functions, the equilibrium locations become zero. We get four cases, depending on the values of $a,\ b$ and the parameter values $t,\ \tau$. In the second stage of the game, both retailers may be located within [0,1], both retailers may be located at the two opposite endpoints or one retailer within [0,1] and the rival at the endpoint.

Nevertheless, in the first stage of the game, the wholesalers choose symmetric locations $(a=b)$,

[23] The profit margin for firm X is positive when $\tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)>0$ and for firm Y when $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)>0$. The second order conditions are satisfied in equilibrium.

thus, we cannot obtain, as an equilibrium outcome, asymmetric locations downstream. Further, from the analysis of the unrestricted locations model, we find that all four firms are located outside the unit interval. Thus, in the restricted locations model, we cannot obtain an equilibrium with all four firms located in [0,1], since there does not exist such an equilibrium in the unrestricted locations model.

In the unrestricted locations model, the forces that push retailers away from the center of the unit interval are stronger relative to the wholesalers. Thus, there are two possible equilibrium outcomes in the restricted locations model. Retailers locate at the two opposite endpoints and the wholesalers either locate within the unit interval or at the endpoints as their corresponding retailers. We prove that when retailers are located at the two opposite endpoints ($x = y = 0$), the wholesalers maximize their profits, in the restricted locations model, by locating at the two opposite endpoints as their retailers ($a = b = 0$).

The following results summarize the equilibrium outcome and the social cost in equilibrium.[24]

Proposition A1 *The equilibrium outcome with locations restricted in the unit interval is:*

$$\begin{aligned} a^* &= b^* = x^* = y^* = 0, \\ w_A^* &= w_B^* = 3t,\ p_X^* = p_Y^* = 4t, \\ z^* &= 0.5, \\ \Pi_A^* &= \Pi_B^* = \frac{3t}{2},\ \Pi_X^* = \Pi_Y^* = \frac{t}{2}. \end{aligned}$$

The social cost is simply equal to the transportation cost of the consumers since this is a unit final demand model (prices do not affect welfare) and the downstream firms are located at the same point as their suppliers:

$$SC = \int_0^{\frac{1}{2}} t(z-x)^2 dz + \int_{\frac{1}{2}}^1 t(1-y-z)^2 dz = \int_0^{\frac{1}{2}} tz^2 dz + \int_{\frac{1}{2}}^1 t(1-z)^2 dz = \frac{t}{12}.$$

As the transportation cost parameter t increases, the social cost increases too. As t increases, the products become more differentiated and the profits of the upstream and downstream firms increase. We observe that the cost parameter τ that reflects the transportation cost paid by the downstream firms when supplied by the upstream firms does not affect the equilibrium prices and profits. This is because in equilibrium the retailers are located at the same point as their suppliers. The wholesale and product prices in the restricted locations model are lower than in our basic model where firms can locate anywhere on the real line. Again, there are three forces that affect the location choices. The demand effect, the price competition effect and the aggregate marginal cost effect. The forces that push firms farther away from each other dominate the forces that push them towards each other and this lead to maximum differentiation. This result was expected since

[24] In equilibrium, the inequality $\overline{f_X} < f_X < \widehat{f_X}$ holds true and is equivalent to $-3t < 0 < 3t$.

in the unrestricted locations model we have obtained that all four firms locate outside the unit interval (and since the profit functions are quasi-concave).

We should also note that if we modify the restricted locations model to have all four firms simultaneously choose their locations, we obtain that again upstream firms locate at the same point as their retailers and that maximum differentiation occurs ($a = x = 0$ and $b = y = 0$) as opposed to the unrestricted locations model where all firms locate at -1.75. So, in the simultaneous locations choice model with locations within [0,1] we obtain the same equilibrium outcome compared to the sequential location choices with restricted locations.

Appendix A2: Retailers locating at the opposite side from the wholesalers

In our analysis we have assumed that upstream firm A locates to the left of B: this is simply a matter of labeling and without loss of generality. We have also proceeded to the equilibrium derivation assuming that each downstream firm locates at the same side of the line as the corresponding upstream (X with A and Y with B). Doing so has allowed us to simplify the exposition in the first two stages of the game and focus on the core arguments. Here, for completeness, we investigate the possibility that, while A is to the left of B, X locates to the right of Y. We find that for some upstream locations and parameter values it could also be an equilibrium in the second stage of the game that the retailers locate at the opposite side of the line relative to the corresponding wholesaler. We also show that second stage equilibrium profit is lower in this arrangement. Still, we do not find an equilibrium of the entire game when all locations (upstream and downstream) are endogenous and are chosen so that wholesalers' and retailers' locations are chosen at the opposite side of one another.

We proceed with the analysis. Let us fix the upstream locations (with $1 - a - b > 0$). Thus far we have assumed that $1 - x - y > 0$. We calculate the profit functions of the downstream firms when firm X is located at the same point of Y (with $1 - x - y = 0$) or to the right of firm Y (with $1 - x - y < 0$). The overall profit function of firm X is then:

$$\Pi_X = \begin{cases} \Pi_X^L & if \quad 1 - x - y > 0 \\ 0 & if \quad 1 - x - y = 0 \\ \Pi_X^R & if \quad 1 - x - y < 0, \end{cases}$$

where Π_X^L is the profit for X when X is to the left (L) of Y (expression (8) in Subsection (3.3)) and

Π_X^R is the profit for X when X is to the right (R) of Y with:

$$
\begin{aligned}
\Pi_X^R &= 2t\left(x+y-1\right) \\
& \quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\left(x-y+9\right)\left(x+y-1\right)\leq 0 \\
&= \tfrac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t(y-x+9)(x+y-1)\right)^2}{162t(x+y-1)} \\
& \quad \text{if } \left\{\begin{array}{c} \tau\left((y-b)^2-(x-a)^2\right)+t\left(y-x+9\right)\left(x+y-1\right)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\left(x-y+9\right)\left(x+y-1\right)>0 \end{array}\right\} \\
&= 0 \\
& \quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\left(y-x+9\right)\left(x+y-1\right)\geq 0.
\end{aligned}
$$

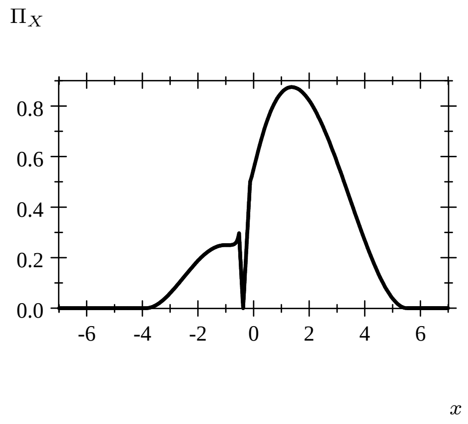
We can also write the profit function of firm Y in an analogous way.

Our main result in this appendix is that for symmetric upstream locations ($a=b$) and sufficiently high transportation cost parameter t there is an additional equilibrium when the downstream locations are set. In Subsection (3.3) we have characterized the equilibrium where X locates to the left of Y. Now, we show that, for some parameter values, there is another equilibrium where X locates to the right of Y.

If there are equilibrium locations, say $(\widehat{x},\widehat{y})$, where X is located to the right of Y these would be given by the maximization of the downstream firms' profits Π_X^R and Π_Y^R respectively. From the first order conditions we have:

$$
\begin{aligned}
\widehat{x} &= \frac{16a\tau^2\left(a+b-1\right)+4t\tau\left(19a+12b+a^2-b^2-11\right)+99t^2}{4\left(t+\tau\right)\left(4\tau\left(a+b-1\right)+9t\right)}, \\
\widehat{y} &= \frac{16b\tau^2\left(a+b-1\right)+4t\tau\left(19b+12a+b^2-a^2-11\right)+99t^2}{4\left(t+\tau\right)\left(4\tau\left(a+b-1\right)+9t\right)}
\end{aligned}
$$

if $9t+2\tau\left(a+b-1\right)>0$, $\frac{(81t+4\tau(a-b-9)(1-a-b))}{9t+4\tau(a+b-1)}>0$ and $\frac{(81t+4\tau(a-b+9)(a+b-1))}{(9t+4\tau(a+b-1))}>0$. The first constraint is necessary to have positive equilibrium profits and the other two to have positive profit margins (to assure that the calculated quantities are not negative). Thus, if such an equilibrium $(\widehat{x},\widehat{y})$ exists, it will satisfy these constraints.



a=b=0, y=1.375, t=τ=1

Figure A2: Downstream profits with X located either to the left or right of Y

Figure A2 illustrates the nature of the problem. When the upstream firms are located at the two opposite endpoints and the transportation cost parameters are equal to one, we obtain $\widehat{y} = 1.375$. In Figure A2, we plot the profit function of firm X allowing X to be located either to the left or to the right of firm Y. When firm X is located to the left of firm Y and far enough from the unit interval and its own supplier, it gets zero demand and profits. There is an interval where firm X is located to the left of firm Y and both firms sell to the final consumers. However, when X is located close to Y, either to the left or to the right, X captures the whole demand. Firm X has a cost advantage compared to firm Y and reduces its retail price to capture the whole demand. This cost advantage is greater when firm X is located to the right of firm Y, since firm X is closer to its supplier, firm A. Thus, the profits of firm X when it captures the whole demand are higher when it chooses the side that is closer to its supplier. There is again an interval where the two retailers share the market, but now firm X is to the right of firm Y. Finally, if firm X is located to the right of firm Y and away from the unit interval, it obtains zero demand since it has a significant cost disadvantage and is located away from [0,1] where consumers live.

We prove that for symmetric upstream locations and some parameter values t, τ, firm X (given $y = \widehat{y} = \frac{11t+4a\tau}{4(t+\tau)}$) obtains higher profits when it locates to the right of firm Y and shares the market with Y compared to the profits obtained when it locates to the left of Y and shares the market or it locates to the left of Y and captures the whole demand or it locates to the right of Y and captures the whole demand. The total maximum of firm X's profits is to the right of firm Y when both firms sell to the final consumers. Figure A2 presents an example. Employing symmetry, we prove that, for certain parameter values and given $\widehat{x}$, the best response of firm Y is $\widehat{y}$.

Proposition A2 *For fixed and symmetric upstream locations and* $t \geq \frac{4\tau(1-2a)}{9}$, *the pair of locations* $\widehat{x} = \widehat{y} = \frac{11t+4a\tau}{4(t+\tau)}$ *constitutes an equilibrium in the second stage of the game.*

We find that: if the transportation cost parameter of the consumers, t, is high enough (compared to τ, or equivalently τ is low enough), it can also be an equilibrium in the second stage of the game that retailers locate at the opposite side of the corresponding wholesaler. In the extreme case where t is positive and τ is zero, retailers pay zero transportation costs when supplied by their wholesalers. This is equivalent to locating at the optimum points on the real line either to the same or the opposite side of their wholesalers. In Figure A2, we assume $a = b = 0$ and $\tau = 1$, therefore we need $t \geq \frac{4}{9}$, which is satisfied since $t = 1$.

Therefore, there can be two equilibria in the second stage of the game for fixed upstream locations, $a = b$, and for $t \geq \frac{4\tau(1-2a)}{9}$. Retailers either locate to the same side (see (x^*, y^*), from equation (9) for $a = b$) or to the opposite side of their wholesalers $(\widehat{x}, \widehat{y})$:

$$x^* = y^* = \frac{-7t + 4a\tau}{4(t+\tau)},\ \Pi_X^L(x^*, y^*) = \Pi_Y^L(x^*, y^*) = \frac{t(9t + 2\tau(1-2a))}{4(t+\tau)}$$

and

$$\widehat{x} = \widehat{y} = \frac{11t + 4a\tau}{4(t+\tau)},\ \Pi_X^R(\widehat{x}, \widehat{y}) = \Pi_Y^R(\widehat{x}, \widehat{y}) = \frac{t(9t - 2\tau(1-2a))}{4(t+\tau)}.$$

We now compare the profits in the two equilibria and find that equilibrium (x^*, y^*) Pareto dominates $(\widehat{x}, \widehat{y})$ since $\Pi_X^L(x^*, y^*) > \Pi_X^R(\widehat{x}, \widehat{y})$.[25] Downstream equilibrium profits are higher when retailers are located closer to their suppliers.

Can this behavior be part of a subgame perfect equilibrium in the whole game? If we make the upstream firms locate at the points indicated in Proposition 1 ($a^* = b^* = \frac{9t-5\tau-9\sqrt{\tau^2+t^2}}{8\tau}$) we calculate that we can have this additional equilibrium $\widehat{x} = \widehat{y} = \frac{31t-5\tau-9\sqrt{\tau^2+t^2}}{8(t+\tau)}$ in the second stage of the game when $t \geq \frac{4}{3}\tau$. Retailers locate either at (x^*, y^*) or at $(\widehat{x}, \widehat{y})$ in the second stage of the game for these fixed upstream locations. He have proved numerically that the pair (a^*, b^*) is not an equilibrium pair of locations in the first stage of the game, when it is anticipated that the retailers, in the equilibrium of the subsequent stage, locate at $(\widehat{x}, \widehat{y})$. Each wholesaler then has an incentive to deviate from (a^*, b^*).

Appendix B (NOT for publication)

Appendix B1: Profit function of firm X for locations x at the left or the right of y. In Subsection (3.3) we provide the profit functions of the downstream firms when $1 - x - y > 0$, that is, firm X is located to the left of firm Y. Now, we study the case where firm X is located at the same point ($1 - x - y = 0$) or to the right of firm Y ($1 - x - y < 0$). When $1 - x - y < 0$ we cannot simply replace in the profit functions (8) and the respective expression for firm Y, x with $1 - y$ and y with $1 - x$ since there is exclusive dealing between firm X and its supplier A and between firm Y and its supplier B. If firm X is located to the right of firm Y, it is still supplied by firm A which is located to the left of firm B. Thus, when firm X is located to the right of firm Y is not completely symmetric to when firm Y is located to the right of firm X, since they have different suppliers.

[25] For symmetric upstream locations, we have $a = b < \frac{1}{2}$.

When $1-x-y=0$, there is no product differentiation downstream, thus, consumers buy one unit of the product from the cheapest retailer. The demand of firm X, becomes:

$$z = D_X = D_A = \begin{cases} 1 & if \quad p_X < p_Y \\ \frac{1}{2} & if \quad p_X = p_Y \\ 0 & if \quad p_X > p_Y. \end{cases}$$

Downstream competition is very intense (Bertrand competition) and pushes final prices to the marginal cost. Firm X faces aggregate marginal cost $f_X = w_A + \tau(x-a)^2$ and firm Y $f_Y = w_B + \tau(y-b)^2$. Thus, final price is set equal to $\max\{f_X, f_Y\}$ and the firm with the lowest aggregate marginal cost captures the whole demand and enjoys positive profits. The profit function of firm X becomes:

$$\Pi_X = \begin{cases} w_B - w_A + \tau\left((y-b)^2 - (x-a)^2\right) & if \quad w_A + \tau(x-a)^2 < w_B + \tau(y-b)^2 \\ 0 & if \quad w_A + \tau(x-a)^2 \geq w_B + \tau(y-b)^2. \end{cases}$$

Since firm X is supplied by firm A, the profit function of firm A is:

$$\Pi_A = \begin{cases} w_A & if \quad w_A < w_B + \tau\left((y-b)^2 - (x-a)^2\right) \\ 0 & if \quad w_A \geq w_B + \tau\left((y-b)^2 - (x-a)^2\right). \end{cases}$$

Downstream competition is transferred to the upstream level and the wholesale prices are reduced to the difference in the transportation costs of the two retailers (minus $\varepsilon \simeq 0$). If firm B charges a wholesale price higher than that level, firm A can undercut w_B and capture the whole demand via its retailer X. Thus:

$$w_A = \begin{cases} \tau\left((1-x-b)^2 - (x-a)^2\right) & if \quad (1-x-b)^2 - (x-a)^2 > 0 \\ 0 & otherwise. \end{cases}$$

The profits of X reduce to zero, $\Pi_X = 0$, as $\tau\left((1-x-b)^2 - (x-a)^2\right) - \left(\tau\left((1-x-b)^2 - (x-a)^2\right)\right) = 0$. In an analogous manner $\Pi_Y = 0$.

When $1-x-y<0$, firm X is located to the right of firm Y. The indifferent consumer is given by:

$$p_Y + t(z - (1-y))^2 = p_X + t(x-z)^2$$

and the demand of firm Y, now z, is:

$$z = D_Y = D_B = \begin{cases} 1 & if \quad \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} \geq 1 \\ \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} & if \quad 0 < \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} < 1 \\ 0 & if \quad \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} \leq 0. \end{cases}$$

The profit functions of firm X and Y are: $\Pi_X^R = (p_X - f_X)(1-z)$ and $\Pi_Y^R = (p_Y - f_Y)z$. The superscript R refers to the case where firm X is to the right of firm Y. From the first order conditions, we obtain the equilibrium final price for firm X (likewise, for firm Y):

$$p_X^R = \begin{cases} f_Y - t(x+y-1)(1+x-y) & if \quad f_X \leq f_Y - t(3-y+x)(x+y-1) \\ \frac{1}{3}\left(t(x+y-1)(3+y-x) + f_Y + 2f_X\right) & if \quad \left\{ \begin{array}{l} f_Y - t(3-y+x)(x+y-1) < f_X \\ f_X < f_Y + t(3-x+y)(x+y-1) \end{array} \right\} \\ f_X & if \quad f_X \geq f_Y + t(3-x+y)(x+y-1) \end{cases}$$

with the respective profits

$$\Pi_X^R = \begin{cases} f_Y - t(x-y+1)(x+y-1) - f_X & if \quad f_X \leq f_Y - t(3-y+x)(x+y-1) \\ \frac{(t(x+y-1)(3+y-x)+f_Y-f_X)^2}{18t(x+y-1)} & if \quad \left\{ \begin{array}{l} f_Y - t(3-y+x)(x+y-1) < f_X \\ f_X < f_Y + t(3-x+y)(x+y-1) \end{array} \right\} \\ 0 & if \quad f_X \geq f_Y + t(3-x+y)(x+y-1). \end{cases}$$

In the third stage, upstream firms seek to maximize their profits $\Pi_A^R = w_A(1-z)$, $\Pi_B^R = w_B z$ with respect to their wholesale prices. Assuming equilibrium in the subsequent stage, the profit function of firm A becomes:

$$\begin{aligned} \Pi_A^R &= 0 \\ &\quad \text{if } w_A \geq w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t(y-x+3)(x+y-1) \\ &= w_A\left(1 - \left(\frac{2t(y-x)(x+y-1) + w_A - w_B + \tau\left((x-a)^2 - (y-b)^2\right)}{6t(x+y-1)} + \frac{1}{2}(1+x-y)\right)\right) \\ &\quad \text{if } \left\{ \begin{array}{l} w_A < w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t(y-x+3)(x+y-1) \\ w_A > w_B + \tau\left((y-b)^2 - (x-a)^2\right) - t(x-y+3)(x+y-1) \end{array} \right\} \\ &= w_A \\ &\quad \text{if } w_A \leq w_B + \tau\left((y-b)^2 - (x-a)^2\right) - t(x-y+3)(x+y-1). \end{aligned}$$

When the aggregate marginal cost faced by firm X, $w_A + \tau(x-a)^2$, is high enough, the demand of firm X reduces to zero, thus, firm A gets zero demand too. For intermediate prices, w_A, the market is shared with firm B and for very low prices firm A captures the whole demand. As in Subsection

(3.2), we calculate the equilibrium wholesale prices for firm A (analogously for firm B):

$$
\begin{aligned}
w_A^R &= \tau\left((y-b)^2-(x-a)^2\right)-t\,(x-y+3)\,(x+y-1) \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)\leq 0 \\
&= \tfrac{\tau\left((y-b)^2-(x-a)^2\right)+t(y-x+9)(x+y-1)}{3} \\
&\quad \text{if } \left\{\begin{array}{l} \tau\left((y-b)^2-(x-a)^2\right)+t\,(y-x+9)\,(x+y-1)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)>0 \end{array}\right\} \\
&= 0 \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(y-x+9)\,(x+y-1)\geq 0.
\end{aligned}
$$

The corresponding profits for firm X are:

$$
\begin{aligned}
\Pi_X^R &= 2t\,(x+y-1) \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)\leq 0 \\
&= \tfrac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t(y-x+9)(x+y-1)\right)^2}{162t(x+y-1)} \\
&\quad \text{if } \left\{\begin{array}{l} \tau\left((y-b)^2-(x-a)^2\right)+t\,(y-x+9)\,(x+y-1)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)>0 \end{array}\right\} \\
&= 0 \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(y-x+9)\,(x+y-1)\geq 0.
\end{aligned}
$$

Subsection (3.3) provides the profits of firm X when located to the left of firm Y (Π_X^L) and this appendix presents the profits of firm X when X is located at the same point or to the right of firm Y (Π_X^R). Thus,

$$
\Pi_X = \left\{\begin{array}{ccc} \Pi_X^L & if & 1-x-y>0 \\ 0 & if & 1-x-y=0 \\ \Pi_X^R & if & 1-x-y<0. \end{array}\right.
$$

■

Appendix B2: Derivation of the downstream equilibrium locations.

In Subsection (3.3) we prove that there exist an equilibrium $x^*=\frac{\left(16a\tau^2(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)}$, $y^*=\frac{\left(16b\tau^2(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)}$ in the second stage of the game, when firm X is located to the left of firm Y. Here, we prove that given $y=y^*$, firm X prefers to locate at the left of firm Y at point x^*, compared to the right of firm Y. For simplicity, we provide the proof for symmetric upstream locations ($a=b$) since this will be the equilibrium outcome.

Initially, we prove that given $y=y^*=\frac{-7t+4a\tau}{4(t+\tau)}$, firm X cannot capture the whole demand either located to the left or to the right of firm Y. Firm X would serve the whole market located to the left of firm Y if: $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0$ or if $x\in(x_1,x_2)$ with $x_1\equiv\frac{20t+4a\tau-3\sqrt{t(9t-16\tau+32a\tau)}}{4(t+\tau)}$, $x_2\equiv\frac{20t+4a\tau+3\sqrt{t(9t-16\tau+32a\tau)}}{4(t+\tau)}$. Since $1-y^*<x_1<x_2$, the inequality $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0$ is not satisfied. Firm X would serve the whole market located to the right of firm Y if: $\tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)\leq 0$ or if $x\in(x_4,x_3)$ with $x_3\equiv\frac{-16t+4a\tau+3\sqrt{t(81t+16\tau-32a\tau)}}{4(t+\tau)}$, $x_4\equiv\frac{-16t+4a\tau-3\sqrt{t(81t+16\tau-32a\tau)}}{4(t+\tau)}$. Since $1-y^*>x_3>x_4$, the inequality $\tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)\leq 0$ is not satisfied.

Thus, we have to prove that firm X prefers to share the market with firm Y and locate to the left of Y than to the right. Firm X maximizes its profits located to the left of Y at $x^*=\frac{-7t+4a\tau}{4(t+\tau)}$ with $\Pi_X^L(x^*,y^*)=\frac{t(9t-2\tau(2a-1))}{4(t+\tau)}$ and to the right of Y at $x^+=\frac{51t+16\tau-20a\tau}{12(t+\tau)}$ with $\Pi_X^R(x^+,y^*)=\frac{(9t-2\tau(2a-1))(9t-16\tau+32a\tau)^2}{8748t(t+\tau)}$ for $9t+16\tau\,(2a-1)>0$. The constraint $9t+16\tau\,(2a-1)>0$ is necessary for both firms to sell when X is located to the right of Y. If a is low enough, that is, upstream firms are located away from the unit interval, firm X can never obtain positive demand when located to the right of firm Y since its supplier A is far away. By direct comparison we obtain: $\Pi_X^L(x^*,y^*)>\Pi_X^R(x^+,y^*)$. ■

Appendix B3: Derivation of the second equilibrium in the second stage of the game for fixed and symmetric upstream locations.

For symmetric upstream locations ($a=b$) and $y=\widehat{y}=\frac{11t+4a\tau}{4(t+\tau)}$, the profits of firm X when it locates to the right of firm Y and firms share the market are maximized at $\widehat{x}=\frac{11t+4a\tau}{4(t+\tau)}$ with $\Pi_X^R(\widehat{x},\widehat{y})=\frac{t(9t+2\tau(2a-1))}{4(t+\tau)}$. On the contrast, when firm X locates to the left of firm Y and firms share the market profits are maximized at $x^-=-\frac{\left(15t+2\tau(a-2)+\frac{1}{2}\sqrt{(9t-8\tau+16a\tau)^2}\right)}{6(t+\tau)}$ with

$$\Pi_X^L(x^-,\widehat{y})=\frac{\left(4\,(-9t+4\tau\,(2a-1))\sqrt{(9t-8\tau+16a\tau)^2}+4\left(-81t^2+288t\tau\,(2a-1)+32\tau^2\,(2a-1)^2\right)\right)^2}{279\,936t\,(t+\tau)\left(9t+4\tau\,(1-2a)+\sqrt{(9t-8\tau+16a\tau)^2}\right)}.$$

When firm X locates to the left of firm Y and captures the whole demand, profits are maximized at $x_m^L=\frac{20t+4a\tau-3\sqrt{t(81t+16\tau(2a-1))}}{4(t+\tau)}$ with $\Pi_X^L(x_m^L,\widehat{y})=\frac{t\left(4\tau(1-2a)-27t+3\sqrt{t(81t+16\tau(2a-1))}\right)}{2(t+\tau)}$.Finally, firm X captures the whole demand located to the right of firm Y by maximizing its profits at $x_m^R=\frac{-16t+4a\tau+3\sqrt{t(9t+16\tau(1-2a))}}{4(t+\tau)}$ with $\Pi_X^R(x_m^R,\widehat{y})=\frac{t\left(-9t-4\tau+8a\tau+3\sqrt{t(9t-16\tau(2a-1))}\right)}{2(t+\tau)}$.

The $(\widehat{x},\widehat{y})$ pair of locations constitutes an equilibrium in the second stage of the game when: $\Pi_X^R(\widehat{x},\widehat{y})\geq\Pi_X^L(x^-,\widehat{y})$, $\Pi_X^R(\widehat{x},\widehat{y})\geq\Pi_X^L(x_m^L,\widehat{y})$ and $\Pi_X^R(\widehat{x},\widehat{y})\geq\Pi_X^R(x_m^R,\widehat{y})$. By direct calculations we find that when $\Pi_X^R(\widehat{x},\widehat{y})\geq\Pi_X^L(x^-,\widehat{y})$, then $\Pi_X^R(\widehat{x},\widehat{y})\geq\Pi_X^L(x_m^L,\widehat{y})$ and $\Pi_X^R(\widehat{x},\widehat{y})\geq\Pi_X^R(x_m^R,\widehat{y})$ hold true. Inequality $\Pi_X^R(\widehat{x},\widehat{y})\geq\Pi_X^L(x^-,\widehat{y})$ is satisfied when $t\geq\frac{4\tau(1-2a)}{9}$. ■