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Growth**

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Real Term Structure Forecasts of Consumption Growth

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Abstract

This paper suggests an affine term structure model of real interest rates to predict changes in real consumption growth. The model is estimated, jointly, by real interest rates and consumption data, and it is found to be consistent with the consumption smoothing hypothesis. The paper shows that the real term structure is spanned by two common factors, which can be given the interpretation of the level and slope factors, respectively. The risks associated with these factors are priced in the market. Both of these factors can explain the information content of the short-term real interest rate and its term spread with longer term interest rate in forecasting future real consumption growth, over different periods ahead.

JEL classification: G12, E21, E27, E43

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1 Introduction

There are few studies in the literature estimating term structure models of real interest rates, in contrast to the vast amount of studies on the nominal term structure models (see, e.g., Dai and Singleton (2002), Ang et al (2006), for a survey). This may be attributed to the lack of availability of real interest rates, for different maturity intervals.

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Estimating real term structure models is useful for two main reasons. First, it can indicate the number of factors spanning the real term structure and it will estimate their mean reversion and associated prices of risk. The results of this analysis can be compared to those on the nominal term structure of interest rates. Second, it can explain if the real term structure contains information about future real consumption growth.

The information content of the real term structure of interest rates about real consumption growth has been studied in a number of studies in the literature (see, e.g., Harvey (1988), Plosser and Rouwenhorst (1994), Chapman (1997), Rendu de Lint and Stolin (2003), Berardi and Torous (2005), Tsang (2008)).¹ These studies show that the term spread between long and short-term real (or nominal) interest rates appear to contain information about future real consumption growth and economic activity, at short or long horizons. As noted by Harvey (1988), this information of the term spread can be attributed to the desire of investors to smooth their consumption over time. This is consistent with the predictions of the consumption capital asset pricing. In addition to the term spread, evidence suggests that the level of real (or nominal) short-term real interest rate also contains information about the future real consumption growth beyond that implied by the term spread.

This paper contributes into the above literature on many fronts. First, using real consumption and term structure data, instead of nominal, it estimates an empirically tractable Gaussian dynamic term structure model and derives estimates of the underlying unobserved factors spanning the term structure of real interest rates. Then, it examines if this model fits satisfactorily into the data and tests its cross-section restrictions implied by no-profitable arbitrage conditions in the bond market. This is done based on an econometric framework, which apart from real interest rates and excess holding period returns, it also includes consumption data. Second, it employs the model to investigate if the short-term real interest rate and its spread with real long-term interest rates can predict future real consumption growth, over different horizons ahead. To this end, the paper derives closed form formulas of the slope coefficients of these two variables in consumption growth regressions, where they are regressors.

The results of the paper lead to a number of interesting conclusions. First, they show that our term structure model is consistent with the consumption smoothing hypothesis. Second, it finds that there are two common factors which can explain almost all of variation of the term structure of real interest rates. These factors are closely correlated with their estimates retrieved from the data based on principal component analysis. The parameter estimates of the real term structure model indicate that the first of the two factors spanning the real term structure exhibits very slow

¹Note that there is also a close related literature which studies and confirms the leading indicator property of the term structure for real economic activity and consumption growth and, in particular, of the term spread between the long and short term interest rates (see, e.g., Stock and Watson (1989), Estrella (1997) and Jardet (2004)). These papers however rely on the term spread between nominal interest rates, following Donaldson et al (1990).

mean reversion, while its associated price of risk is very small. This factor can explain level shifts in real interest rates. The opposite happens for the second factor, which determines the slope of the term structure. This factor has a much higher mean-reversion and price of risk than the first factor, and it can explain the ability of the real term spread to forecast future real consumption growth over different horizons ahead.

The paper is organized as follows. Section 2 presents the real term structure of interest rates and derives closed form solutions of the slope coefficients of the consumption growth regression model, using the short-term interest rate and its spread with a long-term interest rate as regressors. Section 3 carries out the empirical analysis. This section also includes unit root tests and principal component analysis for real interest rates. The unit root tests can confirm if real interest rates constitute stationary series, as it is assumed by affine term structure models. The principal component analysis can indicate the number of unknown factors spanning the real term structure of interest rates. Our empirical analysis is based on data from the US economy. Section 4 concludes the paper and summarizes some of its more important results.

2 Model setup

Consider an economy with production and stochastic investment opportunity sets (see, e.g., Cox, Ingersoll and Ross (CIR) (1985a, 1985b), or Longstaff and Schwartz (1992)). The investment opportunity set consists of contingent claims (e.g., zero-coupon bonds), a riskless asset and a stochastic production process. We assume that this economy is characterized by K state variables, denoted as x_{it} , at time t , where $i = 1, 2, \dots, K$. These variables obey the following uncorrelated Gaussian processes:²

$$dx_{it} = k_i(\theta_i - x_{it})dt + \sigma_i dW_{it}, \quad i = 1, 2, \dots, K, \quad (1)$$

where W_{it} denotes a Wiener process, k_i and σ_i are the mean-reversion and volatility parameters of processes x_{it} , while θ_i are their long-run means. These state variables constitute common factors which determine real consumption C_t in the economy. If inflation is a constant rate (see, e.g., Harvey (1988)), it can be proved that real consumption growth $\frac{dC_t}{C_t}$ obeys the following process:

$$\frac{dC_t}{C_t} = \left(\sum_{i=1}^K x_{it} - c \right) dt + \sigma_c dW_{ct}, \quad (2)$$

²See also Vasicek (1977), Dai and Singleton (2002), Ahn (2004), Berardi and Torous (2005).

where c is a constant which depends on inflation rate and the proportion between consumption and wealth, and W_{ct} is a Wiener process. By solving forward equation (2), it can be shown that the expected growth rate of real consumption from current period t to a future period $t + \tau$ (i.e., τ -horizons ahead) is given as

$$E_t [\Delta_\tau c_{t+\tau}] = \psi_0(\tau) + \sum_{i=1}^K \psi_{1i}(\tau)x_{it}, \quad (3)$$

where $\Delta_\tau c_{t+\tau} = \ln(C_{t+\tau}/C_t)$ and $\psi_{1i}(\tau) = (1 - e^{-k_i\tau})(k_i)^{-1}$.

In the above economy, the real price of a zero-coupon bond with a τ -period maturity, denoted as $P_t(\tau)$ and, hence, its associated real interest rate, denoted as $R_t(\tau)$, can be derived by solving the following pricing kernel relationship:

$$P_t(\tau) = E_t \left(\frac{M_{t+\tau}}{M_t} \right), \quad (4)$$

where $\frac{M_{t+\tau}}{M_t}$ is the pricing kernel. This is assumed that is given as

$$\frac{dM_t}{M_t} = -r_t dt - \sum_{i=1}^K \Lambda_{it} dW_{it}, \quad (5)$$

where r_t is the instantaneous real interest rate or short - term rate and Λ_{it} are the risk pricing functions, for $i = 1, 2, \dots, K$. For short-term rate r_t , it is assumed that

$$r_t = A(0) + \sum_{i=1}^K B_i(0)x_{it}. \quad (6)$$

The risk pricing functions Λ_{it} evaluate the K -independent sources of risk associated with factors x_{it} . Following Duffee (2002), we will assume that functions Λ_{it} are linear in factors x_{it} , i.e.

$$\Lambda_{it} = \sigma_i^{-1} \left(\lambda_i^{(0)} + \sum_{i=1}^K \lambda_i^{(1)} x_{it} \right). \quad (7)$$

Substituting equations (1), (5), (6) and (7) into pricing kernel equation (4) yields the following zero-coupon real bond pricing formula:

$$P_t(\tau) = e^{-A(\tau) - B(\tau)' X_t}, \quad (8)$$

where X_t is a $(KX1)$ -dimension vector collecting all state variables (factors) x_{it} , i.e. $X_t = (x_{1t}, x_{2t}, \dots, x_{Kt})'$, $A(\tau)$ is a scalar function and $B(\tau)$ is a $(KX1)$ -dimension vector of valued functions, defined as $B(\tau) = (B_1(\tau), B_2(\tau), \dots, B_K(\tau))'$, which collects the loading coefficients of factors x_{it} on bond pricing formula (8). From this, we can obtain a pricing formula for real interest rates of zero-coupon bonds $R_t(\tau)$, with maturity interval τ , as

$$R_t(\tau) = (1/\tau) [A(\tau) + B(\tau)' X_t], \text{ for } \tau = 1, 2, \dots, N \quad (9)$$

Closed form solutions of value functions $A_i(\tau)$ and $B_i(\tau)$ can be obtained by solving a set of ordinary differential equations under no arbitrage profitable conditions (see Duffie and Kan (1996)). For our Gaussian dynamic term structure model (GDTSM), described above, these solutions for $B_i(\tau)$ are analytically given as follows:³

$$B_i(\tau) = B_i(0)(1 - e^{-\tilde{k}_i\tau})(\tilde{k}_i)^{-1}, \text{ where } \tilde{k}_i = k_i + \lambda_i^{(1)}, \quad (10)$$

where \tilde{k}_i constitutes a risk-neutral measure of mean-reversion parameter k_i . These solutions imply a set of cross-section restrictions on the term structure loading coefficients $B_i(\tau)$, for all i , which can be tested, in practice.

The above GDTSM of real interest rates implies that the expected excess holding period real return of a τ -period to maturity zero-coupon bond over short-term interest rate r_t , referred to as term premium (see, e.g., Tzavalis and Wickens (1997), Bolder (2001) and Duffee (2002)), is given as follows:

$$\begin{aligned} E_t [h_{t+1}(\tau) - r_t] &= - \sum_{i=1}^K B_i(\tau) \sigma_i \Lambda_{it} \\ &= - \sum_{i=1}^K B_i(\tau) \lambda_i^{(0)} - \sum_{i=1}^K B_i(\tau) \lambda_i^{(1)} x_{it}. \end{aligned} \quad (11)$$

Joint estimation of the last relationship and interest rates formula (9) (with, or without, cross-section restrictions (10)) will enable us to identify the price of risk slope coefficients $\lambda_i^{(i)}$, which determine the time-varying part of the term premium. To calculate excess return $h_{t+1}(\tau) - r_t$ in discrete-time, we consider the one-period (e.g., one-month) interest rate as short-term interest rate, r_t , and we assume continuously compounded interest rates, implying $R_t(\tau) = -\frac{1}{\tau} \log P_t(\tau)$. Then, $h_{t+1}(\tau) - r_t$ can be written as follows:

$$h_{t+1}(\tau) - r_t = \log \left(\frac{P_{t+1}(\tau-1)}{P_t(\tau)} \right) - r_t = -(\tau-1) [R_{t+1}(\tau-1)] + \tau R_t(\tau) - r_t.$$

2.1 Term structure forecasts of consumption growth

The forecasting implications of the term structure of real interest rates $R_t(\tau)$ about future consumption growth τ -periods ahead, defined as $\Delta_\tau c_{t+\tau}$ where $c_t = \log C_t$, can be investigated by equations (3), (9) and (6). These equations show that both $\Delta_\tau c_{t+\tau}$ and $R_t(\tau)$, for all τ , are driven by K common unobserved factors x_{it} , for $i = 1, 2, \dots, K$.

³See, e.g., Dai and Singleton (2002), Kim and Orphanides (2012).

Substituting out these factors from relationships (3) and (9) implies that $\Delta_\tau c_{t+\tau}$ can be written as a linear function of short-term rate r_t and its spreads with long-term interest rates, defined as $Sp_t(\tau_L) \equiv R_t(\tau_L) - r_t$, where $R_t(\tau_L)$ denotes a long-term interest rate with maturity interval τ_L . To see this more rigorously, assume that the number of common factors x_{it} are $K = 2$, as will be confirmed by our empirical analysis in the next section. Then, equations (9) and (6) imply the following system of equations for short-term interest rate r_t and spread $Sp_t(\tau_L)$:

$$\begin{aligned} Z_t &\equiv \begin{bmatrix} r_t \\ Sp_t(\tau_L) \end{bmatrix} \\ &= \begin{bmatrix} A(0) \\ (1/\tau_L)A(\tau_L) - A(0) \end{bmatrix} + \begin{bmatrix} B_1(0) & B_2(0) \\ (1/\tau_L)B_1(\tau_L) - B_1(0) & (1/\tau_L)B_2(\tau_L) - B_2(0) \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}, \end{aligned}$$

which can be written in a more compact notation as

$$Z_t = A^* + B^* X_t, \quad (12)$$

where $A^* \equiv \begin{bmatrix} A(0) \\ (1/\tau_L)A(\tau_L) - A(0) \end{bmatrix}$, $B^* \equiv \begin{bmatrix} B_1(0) & B_2(0) \\ (1/\tau_L)B_1(\tau_L) - B_1(0) & (1/\tau_L)B_2(\tau_L) - B_2(0) \end{bmatrix}$ and $X_t = (x_{1t}, x_{2t})'$.

Based on equation (12), we can derive the following relationship:

$$E_t[\Delta_\tau c_{t+\tau}] = \psi_0(\tau) - \Psi_1(\tau)' B^{*-1} A^* + \Psi_1(\tau)' B^{*-1} Z_t, \quad (13)$$

where $\Psi_1(\tau)' \equiv (\psi_{11}(\tau), \psi_{12}(\tau))$ (see (3)). This can be done by writing equation (3) in a matrix form and substituting out the vector of common factors X_t from it. This relationship indicates that term spread $Sp_t(\tau_L)$ and short-term rate r_t contains information about future consumption growth $\Delta_\tau c_{t+\tau}$, over different horizons τ . Thus, it can theoretically justify the use of the following linear regression model to forecast consumption growth $\Delta_\tau c_{t+\tau}$:

$$\Delta_\tau c_{t+\tau} = const + \gamma_1(\tau)r_t + \gamma_2(\tau)Sp_t(\tau_L) + u_{t+\tau}, \quad (14)$$

where $u_{t+\tau}$ denotes a disturbance (error) term. According to (13), the slope coefficients of this regression model $\gamma_1(\tau)$ and $\gamma_2(\tau)$ are given in closed form as

$$\gamma_1(\tau) = \frac{\psi_{11}(\tau)[\overline{B}_2(\tau_L) - B_2(0)] - \psi_{12}(\tau)[\overline{B}_1(\tau_L) - B_1(0)]}{B_1(0)[\overline{B}_2(\tau_L) - B_2(0)] - B_2(0)[\overline{B}_1(\tau_L) - B_1(0)]} \quad (15)$$

$$\gamma_2(\tau) = \frac{\psi_{12}(\tau)B_1(0) - \psi_{11}(\tau)B_2(0)}{B_1(0)[\overline{B}_2(\tau_L) - B_2(0)] - B_2(0)[\overline{B}_1(\tau_L) - B_1(0)]} \quad (16)$$

where $\bar{B}_i(\tau_L) = (1/\tau_L)B_i(\tau_L)$ and $B_i(\tau_L) = B_i(0)(1 - e^{-\tilde{k}_i\tau_L})(\tilde{k}_i)^{-1}$, for $\tau = \tau_L$.

The analytical solutions of slope coefficients $\gamma_1(\tau)$ and $\gamma_2(\tau)$, given by formulas (15) and (16), indicate that the degree of information contained in r_t and $Sp_t(\tau_L)$ about $\Delta_\tau c_{t+\tau}$ depends on the values of loading coefficients $B_i(0)$, the mean-reversion and price of risk parameters k_i and $\lambda_i^{(1)}$, respectively, or the risk-neutral mean reversion parameter \tilde{k}_i , as well as maturity interval τ . Next, we analyze the effects of k_i on $\gamma_1(\tau)$ and $\gamma_2(\tau)$, assuming for analytic convenience that $\lambda_i^{(i)} = 0$. The results of this analysis can be used to explain the pattern of the estimates of $\gamma_1(\tau)$ and $\gamma_2(\tau)$ with τ , observed in practice.

First, consider the case of $B_1(0)=B_2(0)=1.00$ (as assumed in many theoretical studies, see Kim and Orphanides (2012), *inter alia*). In this case, formulas (15) and (16) indicate that, if both k_i become very close to zero (i.e., $k_1 \rightarrow 0$ and $k_2 \rightarrow 0$), we have: $\psi_i(\tau) \rightarrow \tau$ and $\bar{B}_i(\tau_L) \rightarrow 1$, for $i = \{1, 2\}$, and, hence, $[\bar{B}_i(\tau_L) - B_i(0)] \rightarrow 0$ and $\gamma_i(\tau) \rightarrow 0$. This means that r_t and $Sp_t(\tau_L)$ do not have information about $\Delta_\tau c_{t+\tau}$, for all τ , which is not consistent with empirical evidence showing that both $\gamma_1(\tau)$ and $\gamma_2(\tau)$ are positive. If one of k_i 's is different than zero, e.g. $k_2 \neq 0$, and the other is very close to it (i.e., $k_1 \rightarrow 0$), then $\gamma_1(\tau) \rightarrow \tau$ and $\gamma_2(\tau) \rightarrow \frac{\psi_{12}(\tau)-\tau}{B_2(\tau_L)-1}$, since $[\bar{B}_1(\tau_L) - B_1(0)] \rightarrow 0$ and $[\bar{B}_2(\tau_L) - B_2(0)]=\bar{B}_2(\tau_L) - 1$. This case predicts that $\gamma_2(\tau)$ is close to zero, which is not also consistent with evidence provided in the literature (see introduction).

Second, consider the case where $B_1(0)=1.00$ and $B_2(0)=-1.00$, which are close to our estimates of $B_i(0)$ reported in Section 3.2. Then, formulas (15) and (16) imply that, for $k_1 \rightarrow 0$ and $k_2 \rightarrow 0$, we have: $\psi_i(\tau) \rightarrow \tau$ and $\bar{B}_i(\tau_L) \rightarrow 1$ (as above) and, hence, $[\bar{B}_1(\tau_L) - B_1(0)] \rightarrow 0$ and $[\bar{B}_2(\tau_L) - B_2(0)] \rightarrow 2$, implying that $\gamma_i(\tau) \rightarrow \tau$, for all i . These results mean that the forecasting ability of r_t and $Sp_t(\tau_L)$ about consumption growth $\Delta_\tau c_{t+\tau}$ increases linearly with τ , when $k_i \rightarrow 0$. This can be attributed to the fact that shocks to factors x_{it} tend to have permanent effects on the level of interest rates and real consumption, due to their high persistency. These effects are not offset each other in the term spread $Sp_t(\tau_L)$. This can be confirmed by Figure 1, which presents values of $\gamma_1(\tau)$ and $\gamma_2(\tau)$ for different values of k_1 and k_2 , when $B_1(0)=1.00$ and $B_2(0)=-1.00$. For this case of $B_i(0)$'s, the graphs of the figure indicate that r_t and $Sp_t(\tau_L)$ have forecasting power on $\Delta_\tau c_{t+\tau}$. This happens for a wide spectrum of values of $\gamma_1(\tau)$ and $\gamma_2(\tau)$. Note that both coefficients $\gamma_1(\tau)$ and $\gamma_2(\tau)$ take their higher values in the case where k_1 and k_2 are mean reverting. In the case of $k_i \rightarrow 0$ and $k_2 > 0$, which is consistent with our estimates of Section 3, we have $\gamma_2(\tau) > \gamma_1(\tau)$. These results imply that mean reversion increases the forecasting power of both of r_t and $Sp_t(\tau_L)$ on $\Delta_\tau c_{t+\tau}$, as is expected by the theory.

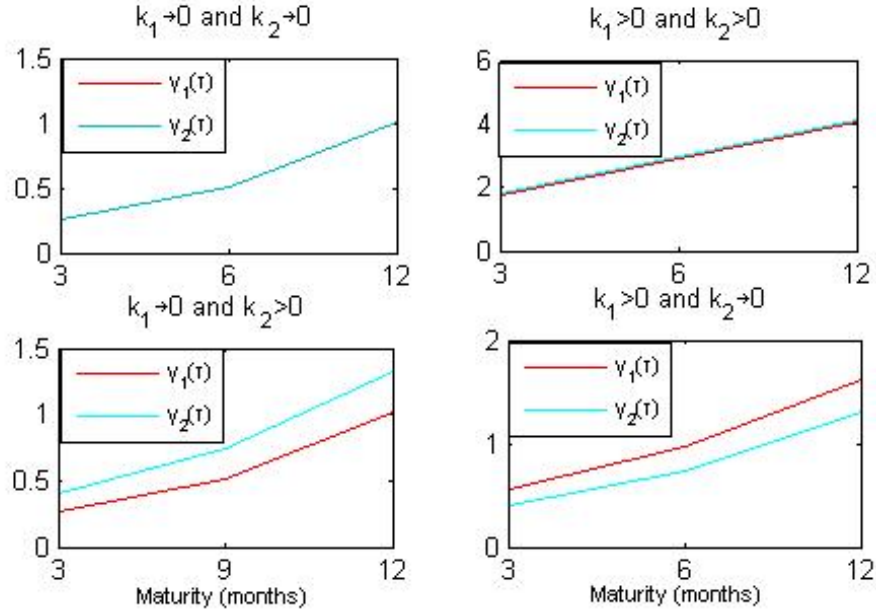


FIGURE 1. The slope coefficients $\gamma_1(\tau)$ and $\gamma_2(\tau)$ as shown in (15) for $B_2(0) < 0$, calculated for different mean reversion coefficient values of the underlying factors.

3 Empirical analysis

Based on data on real interest rates and real consumption, in this section we estimate and test the GDTSM presented in the previous section and we examine if this term structure model can explain the pattern of the slope coefficients of short-term interest rate r_t and spread $Sp_t(\tau_L)$, $\gamma_1(\tau)$ and $\gamma_2(\tau)$, respectively, observed in reality.

Our empirical analysis is based on monthly US real interest rates of zero-coupon bonds covering the period from 1997:07 to 2009:10. These series are taken from the archive of J. Huston McCulloch (Economics Department, Ohio University).⁴ Our consumption data set consists of monthly observations of seasonally adjusted personal real consumption expenditures for the above period, taken from the Federal Reserve Economic Data (FRED) (see code PCEC96). Figure 2 plots all real interest rates series used in our empirical analysis, covering a wide spectrum of maturity intervals from one month to five years (i.e. $\tau = 60$ months). This very broad set of real interest rates will be used in our analysis to examine the number of factors spanning the real term structure and to consider alternative maturity interval long-term spreads $Sp_t(\tau_L)$ as regressors in (14).

⁴<http://www.econ.ohio-state.edu/jhm/ts/ts.html>

As it can be seen from Figure 2, the real term structure of interest rates does not exhibit substantial volatility neither over their cross-section (maturity) dimension nor over the time interval of our data, with the exception of period 2001-2003 and year 2008. Between January 3, 2001 and June 25, 2003, which covers the first period, the Fed lowered its lending, short-term interest rate from 6.5% to 1.0%, which constitutes its lowest level since year 1996. This is done by the Fed to avoid a further slowing down of the US economy and to boost the stock market. Note that, during this period, the US stock market was in the bear regime due to the terrorist attack of September 11, 2001 and the collapses of the Enron and WordCom companies in year 2002 (see, e.g., Ghosh and Constantinides (2010), and Dendramis et al (2012)). In year 2008, which is the second period of a bond market turmoil during our sample, the recent financial crisis, associated with the collapse of Lehman Brothers in September 16, 2008, began. In this year, the US nominal interest rates increased substantially to reflect the higher credit risk levels of the US economy, compared to those in previous years.

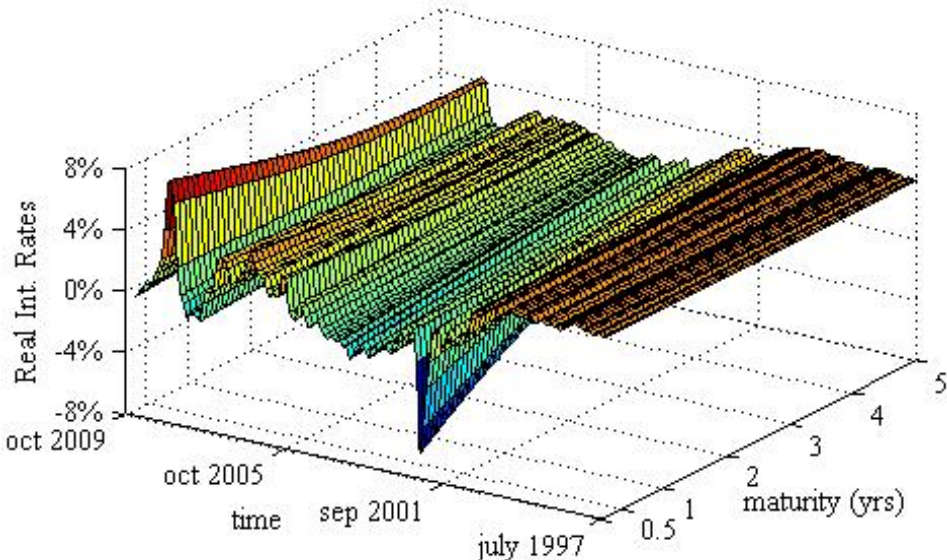


FIGURE 2. The term structure of real interest rates in the US.

Our empirical analysis has the following order. First, we carry out unit root tests for all real interest rates $R_t(\tau)$ employed in our estimation and testing procedures. These tests are critical in choosing the correct econometric framework for estimating and testing our GDTSM, avoiding any spurious regression effects. The latter can appear in estimating (14), if interest rate r_t or spread $Sp_t(\tau_L)$ are integrated series of order one. Second, we conduct principal component (PC) analysis with the aim of determining the maximum number of common factors (state variables) K

which explain the total variation of $R_t(\tau)$ in our sample. Since principal component factors constitute well diversified portfolios of interest rates which are net of measurement or pricing errors effects in $R_t(\tau)$, they can be employed as instruments in the estimation of the GDTSM to minimize the bias effects of the above errors on the parameter estimates of the model (see, e.g., Argyropoulos and Tzavalis (2012)). Third, we estimate and test the GDTSM, with and without consumption data. The estimates of the model are then used to examine if the pattern of slope coefficients $\gamma_1(\tau)$ and $\gamma_2(\tau)$ observed in reality is consistent with that predicted by the theory. In this part of the paper, we also examine if the random walk model of real consumption constitutes a better forecasting model than (14).

3.1 Unit root tests

To test for a unit root in the level of real interest rates $R_t(\tau)$,⁵ we carry out a second generation ADF unit root test, known as efficient ADF (E-ADF) test (see, e.g., Elliott et al. (1996), Elliott (1999), and Ng and Perron (2001)). This test is designed to have maximum power against stationary alternatives to unit root hypothesis which are local to unity. Thus, it can improve the power performance of the standard ADF statistic often used in practice to test for a unit root in $R_t(\tau)$.⁶

Values of E-ADF unit root test statistic are reported in Table 1. This is done for real interest rates $R_t(\tau)$, with maturity intervals $\tau = \{1, 3, 6, 12, 24, 36, 48, 60\}$ months. Note that, in addition to E-ADF, the table also presents values of P_T unit root test statistic, suggested by Elliott et al. (1996) as alternative to E-ADF. To capture a possible linear deterministic trend in the levels of $R_t(\tau)$ during our sample, both statistics E-ADF and P_T assume that the vector of deterministic components D_t employed to detrend $R_t(\tau)$ contains also a deterministic trend, i.e. $D_t = [1, t]$. The results of the table clearly indicate that, despite the fact that the values of the autoregressive coefficients ϕ are close to unity, the unit root hypothesis for $R_t(\tau)$ is rejected against its stationary alternative, for all τ considered. This is true at 5%, or 1% significance levels. The estimates of the autoregressive coefficient ϕ reported in the table indicate that $R_t(\tau)$ exhibit a slow mean reversion towards their long-run mean, especially those of longer maturity intervals of 36 and 60 months.

⁵Evidence of high persistency in real interest rates can be found in Neely and Rapach (2008).

⁶Evidence provided in the literature on unit root tests for interest rates series is mixed. Earlier studies based on single time series unit root tests, such as the standard ADF test, can not reject the null hypothesis of a unit root (see, e.g., Hall et al. (1992)). On the other hand, more recent studies based on panel data tests or Bayesian panel data methods tend to reject this hypothesis (see, e.g. Constantini and Lupi (2007) and Meligotsidou et al (2010)).

TABLE 1: Efficient unit root tests of real interest rates $R_t(\tau)$

τ :	1	3	6	12	24	36	48	60
$\phi - 1$	-0.14 (0.03)	-0.14 (0.03)	-0.14 (0.03)	-0.13 (0.03)	-0.10 (0.03)	-0.09 (0.03)	-0.08 (0.03)	-0.09 (0.03)
ϕ	0.86	0.86	0.86	0.87	0.90	0.91	0.92	0.91
E-ADF	-4.22**	-4.18**	-4.10**	-3.92**	-3.47*	-3.05*	-2.89*	-2.95*
P_T	2.56**	2.61**	2.70**	2.94**	3.71**	4.78*	5.32*	5.14*

Notes: Standard errors are in parentheses. E-ADF and P_T are the efficient unit root test statistics suggested by Elliott et al. (1996). To carry out E-ADF test, we rely on the following auxiliary regression: $\Delta R_t(\tau)^d = (\phi - 1)R_{t-1}(\tau)^d + \sum_{l=1}^p \vartheta_l \Delta R_{t-l}(\tau)^d + \varepsilon_t$, where $R_t(\tau)^d \equiv R_t(\tau) - D_t' \widehat{\delta}_{GLS}(\bar{\phi})$, $D_t = [1, t]$ is the detrended interest rate series $R_t(\tau)$. $\widehat{\delta}_{GLS}(\bar{\phi})$ is the GLS estimator of the quasi differences of interest rates, defined as $d(R_t(\tau)|\bar{\phi}) = R_t(\tau) - \bar{\phi}R_{t-1}(\tau)$ at the value of local parameter $\bar{\phi} = 1 - 13.5/T$, on vector of deterministic components $D_t = [1, t]$. P_T is defined as $P_T = \frac{SSR(\bar{\phi}) - \bar{\phi}SSR(1)}{\widehat{\omega}^2}$, where $SSR(\phi) = \sum \widehat{\nu}_t^2(\phi)$ is the sum of squared residuals $\widehat{\nu}_t(\phi) = d(R_t(\tau)|\bar{\phi}) - D_t' \widehat{\delta}_{GLS}(\bar{\phi})$ and $\widehat{\omega}^2$ is an estimator of the residual spectrum at frequency zero. Critical values of test statistics E-ADF and P_T are provided by Elliott et al. (1996). The lag-order of the auxiliary regressions p used to carry out the tests are chosen based on the SIC criterion. This is found to be $p = 1$, for all maturity intervals τ examined. (*) and (**) mean significance at 5% and 1% levels, respectively.

3.2 Principal component analysis

Principal components (PC) analysis can retrieve a K number of common factors spanning the term structure of real interest rates $R_t(\tau)$ (or their first differences $\Delta R_t(\tau)$), denoted as pc_{it} , for $i = 1, 2, \dots, K$. This can be done by the spectral decomposition of the variance-covariance matrix of $R_t(\tau)$, for $\tau = 1, 2, \dots, N$, denoted as Σ_R , i.e.

$$\Sigma_R = \Omega \Theta \Omega',$$

where $N > K$, Θ is a diagonal matrix of dimension $(N \times N)$.⁷ The elements of Θ are the eigen values of matrix Σ_R . Ω is a $(N \times N)$ -dimension orthogonal matrix whose columns are the eigen vectors corresponding to the eigen values of matrix Σ_R . Given estimates of Ω and Θ , the $(K \times 1)$ -dimension vector of principal component factors pc_{it} , defined as $PC_t = (pc_{1t}, pc_{2t}, \dots, pc_{Kt})'$, can be retrieved from the $(N \times 1)$ -dimension vector of interest rates series $R_t(\tau)$, denoted as R_t , as follows:

$$PC_t = \Omega'(R_t - \bar{R}),$$

where \bar{R} is the sample mean of vector R_t . Note that, due to the rotation problem of PC analysis, pc_{it} may not correspond one-to-one to unobserved factors x_{it} , for all i . However, they will be very highly correlated with x_{it} , as

⁷As shown in Bai and Ng (2002), and Bai (2003) consistent estimates of principal component factors can be obtained by PC analysis of interest rates $R_t(\tau)$, if the following condition holds: $\frac{\sqrt{T}}{N} \rightarrow 0$, where T is the total number of interest rates $R_t(\tau)$ observations and N is their cross-section dimension across different maturity intervals τ .

they constitute portfolios of $R_t(\tau)$. Furthermore, as is noted in Joslin and et al (2011), their estimates will diversify away any measurement or pricing error in $R_t(\tau)$.

Our PC analysis relies on a set of $N = 60$ real interest rates $R_t(\tau)$, spanning a very wide maturity spectrum from one month to five years (sixty months). This is a large cross-section set of $R_t(\tau)$ which guarantees that the retrieved by the PC analysis common factors pc_{it} will efficiently span the real term structure of interest rates and its unobserved factors x_{it} . Figure 3 graphically presents the estimates of pc_{it} , for $i = 1, 2$, retrieved by our PC analysis. These correspond to the first two largest in magnitude eigen values of matrix Σ_R , which are found to explain 99.96% (or 99.87%) of the total variation of the levels of $R_t(\tau)$ (or their first differences $\Delta R_t(\tau)$), over all τ , as shown in the following table:⁸

total number of PCs	1	2
% variation explained in $R_t(\tau)$	94.83	99.96
% variation explained in $\Delta R_t(\tau)$	92.83	99.87

The results of this table clearly indicate that the first two principal component factors pc_{1t} and pc_{2t} , obtained through our PC analysis, explain almost all the variation of the term structure of real interest rates $R_t(\tau)$.⁹ The first factor pc_{1t} explains the largest part of this variation, which is about 95% of the levels of $R_t(\tau)$, or 93% of their differences $\Delta R_t(\tau)$. Its remaining part, which is about 5% for $R_t(\tau)$ (or 7%, for $\Delta R_t(\tau)$), is explained by the second factor pc_{2t} . Although the proportion of the total variation of $R_t(\tau)$ explained by pc_{2t} is very small, this can explain the slope of the real term structure. As can be seen by the graphs of pc_{1t} and pc_{2t} , given by Figure 3, most of the variation of pc_{1t} can be attributed to the turmoils of the US bond market in period 2001-2003 and year 2008. These turmoils have caused shifts in the levels of $R_t(\tau)$ (see also Figure 2). The second factor pc_{2t} has been also affected by

⁸Note that the relative variation of the first two pc_{it} is calculated as

$$\frac{\sum_{i=1}^2 v_i}{tr(V)},$$

where v_i is the eigen value of matrix Σ_R and $tr(V)$ stands for the trace of the matrix of the eigen values of Σ_R , denoted as V .

⁹Note that these two factors explain a total variation in real interest rates $R_t(\tau)$ which is analogous in magnitude to that of nominal interest rates captured by three factors (see, e.g., Litterman and Scheinkman (1991), and, more recently, Argyropoulos and Tzavalis (2012)).

these events, but at a less extent. This factor oscillates less than pc_{1t} over the whole sample.

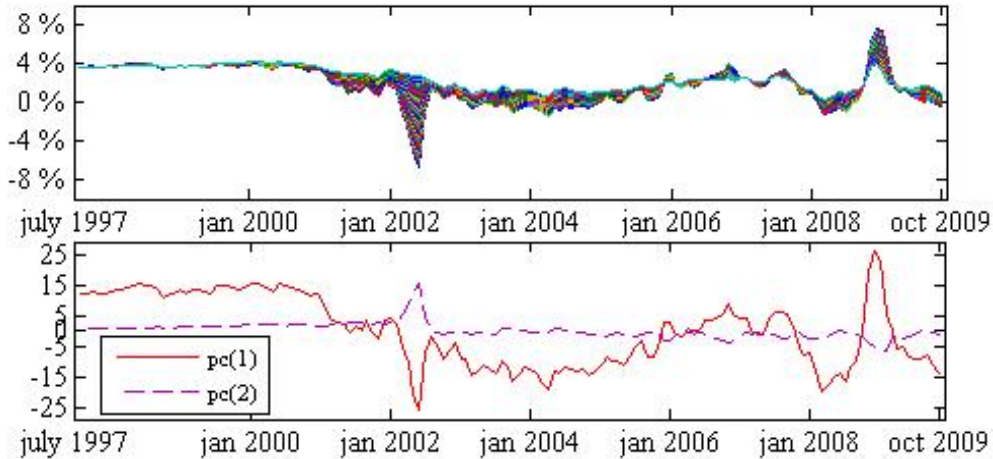


FIGURE 3. The real term structure and the first two principal components.

To gain some economic insight of principal component factors pc_{1t} and pc_{2t} , in Figure 4 we graphically present estimates of their loading coefficients on the first differences of interest rates $\Delta R_t(\tau)$. In Tables 2A and 2B we report some useful descriptive statistics for them, including E-ADF and P_T unit root test statistics.¹⁰ The results of these tables allow us to investigate stochastic features of pc_{it} , which have economic meaning. Table 2A also reports values of the correlation coefficients of pc_{1t} and pc_{2t} with the level of the two year long-term interest rate $R_t(24)$ and the spread between the five-year and one-month interest rates, denoted $Sp_t(60) \equiv R_t(60) - r_t$. These two variables are found to have the maximum degree correlation with pc_{1t} and pc_{2t} , respectively.

¹⁰Similar graphs of the loading coefficients of the first two pc_{it} factors are obtained for the levels of interest rates $R_t(\tau)$.

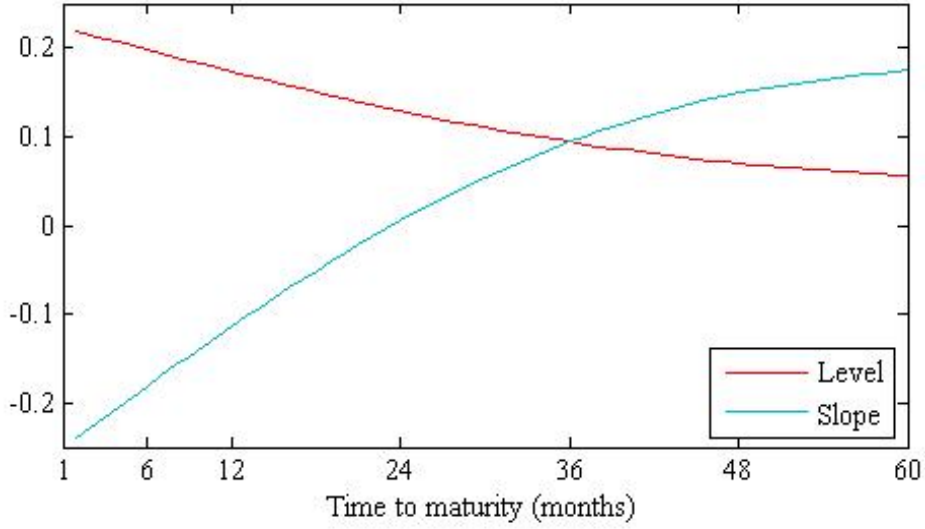


FIGURE 4. Loading coefficients of principal components on the real term structure.

TABLE 2A : Summary statistics of interest rates PCs

Factors	pc_{1t}	pc_{2t}
Mean	0.00	0.00
Max. Value	26.05	15.42
Min. Value	-25.67	-7.39
Variance	128.69	6.96
ρ_1	0.93	0.87
ρ_2	0.82	0.67
ρ_3	0.73	0.51
Correlation coefficients		
$R_t(24)$	0.99	
$Sp_t(60)$		0.75

Notes: Max stands for maximum, while Min. for minimum. ρ_j are the autocorrelations of the principal components pc_{it} , $i = 1, 2$, of lag order $j = 1, 2, 3$

TABLE 2B: Unit root tests for interest rates PCs

	pc_{1t}	pc_{2t}
$\phi_i - 1$	-0.11 (0.03)	-0.22 (0.04)
ϕ_i	0.89	0.78
p	1	1
E-ADF	-3.58**	-5.05**
P_T	3.49**	1.90**

Notes: Standard errors are in parentheses. E-ADF and P_T are the efficient unit root test statistics suggested by Elliott et

al. (1996). See Table 1. The lag order p of the dynamic (first difference) terms of the E-ADF regressions chosen are based on the SIC criterion. (*) and (**) mean significance at 5% and 1% levels.

The results of Table 2B clearly indicate that both principal component factors pc_{1t} and pc_{2t} constitute stationary series. These results are consistent with those on unit root tests for real interest rates $R_t(\tau)$, reported in Table 1. Figure 4 indicates that the loading coefficients of the first factor pc_{1t} on $\Delta R_t(\tau)$ decays with maturity interval τ , but with a very slow rate. On the other hand, the loading coefficients of the second factor pc_{2t} on $\Delta R_t(\tau)$ increases with maturity interval τ , but with a much faster rate than that of first factor pc_{1t} . These patterns of the loading coefficients of pc_{1t} and pc_{2t} on $\Delta R_t(\tau)$ are similar to those found in the empirical literature of the nominal term structure (see, e.g., Litterman and Scheinkman (1991)). Thus, first principal component factor pc_{1t} can be interpreted as a "level" factor. This can explain almost parallel shifts in the whole term structure of real interest rates. This factor is found to be highly correlated with the levels of real interest rates, e.g., the two-year interest rate $R_t(24)$. The second principal component factor pc_{2t} can be given the interpretation of a "slope" factor, since it determines the slope of the real term structure. This factor is found to have maximum correlation with long-term spread $Sp_t(60)$ (see Table 2A). The very high values of correlation coefficients of pc_{1t} and pc_{2t} with $R_t(24)$ and $Sp_t(60)$, respectively, reported in Table 2A, means that, in the estimation of the GDTSM, variables $R_t(24)$ and $Sp_t(60)$ can be employed as appropriate instruments (vehicles) to retrieve estimates of unobserved factors x_{it} from our data, by inverting pricing relationship (9).

The close relationship between x_{it} and pc_{it} , expected by the theory of Section 2, means that pc_{it} can be also employed to provide forecasts of consumption growth rate $\Delta_\tau c_{t+\tau}$. Table 3 presents least squares (LS) estimates of the slope coefficients of the regression of $\Delta_\tau c_{t+\tau}$ on principal component factors pc_{it} , for $\tau = 1, 2, 3, 6, 9$ and 12 months. This can be thought of as an alternative consumption forecasting model to (13). Note that, in addition to the first two principal component factors pc_{1t} and pc_{2t} , in this regression model we also include the third principal component factor pc_{3t} , as a regressor. This is done in order to examine if this factor, whose effect on real term structure variation is almost zero, has any significant information about $\Delta_\tau c_{t+\tau}$.

The results of Table 3 clearly indicate that pc_{1t} and pc_{2t} contain significant information about $\Delta_\tau c_{t+\tau}$, as expected by the theory. This information tends to increase with τ . Consistently with the results of our PC analysis, the third factor pc_{3t} is found to have no information about $\Delta_\tau c_{t+\tau}$, for all τ . Summing up, the results of this section imply that two factors can sufficiently explain almost all the variation of the real term structure. The first two principal component factors of the real term structure obtained by our PC analysis are found to have substantial forecasting power on future consumption growth up to one year ahead.

TABLE 3: Forecasting consumption by principal component factors

Model: $\Delta_\tau c_{t+\tau} = c_0(\tau) + c_1(\tau)pc_{1t} + c_2(\tau)pc_{2t} + c_3(\tau)pc_{3t} + \varepsilon_{t+\tau}$						
τ	1	2	3	6	9	12
$c_1(\tau)$	0.006 (0.003)	0.01 (0.004)	0.02 (0.007)	0.04 (0.02)	0.05 (0.03)	0.08 (0.03)
$c_2(\tau)$	0.02 (0.01)	0.05 (0.01)	0.08 (0.02)	0.13 (0.07)	0.19 (0.11)	0.19 (0.12)
$c_3(\tau)$	-0.18 (0.15)	-0.32 (0.26)	-0.33 (0.34)	-0.41 (0.76)	-0.01 (1.20)	1.27 (1.93)
R^2	0.06	0.13	0.18	0.21	0.23	0.25

Notes: Standard errors are in parentheses. These are corrected for heteroscedasticity and forward-looking moving average serially correlated errors based on Newey–West method.

3.3 Estimation of the real term structure model

Having established good grounds to support that two common factors can explain almost all the variation of the real term structure of interest rates, in this section we estimate the GDTSM presented in Section 2, assuming $K=2$. This model consists of the following structural equations:

$$\Delta R_{t+1}(\tau) = const + B_1(\tau)E_t[\Delta x_{1t+1}] + B_2(\tau)E_t[\Delta x_{2t+1}] + \eta_{t+1}(\tau), \quad \tau = 0, 1, 2, \dots, N \quad (17)$$

$$h_{t+1}(\tau) = const - \lambda_1^{(1)}B_1(\tau)x_{1t} - \lambda_1^{(2)}B_2(\tau)x_{2t} + e_{t+1}(\tau), \quad \tau = 1, 2, \dots, N \quad (18)$$

$$\Delta x_{it+1} = const + (e^{-k_i \Delta t} - 1)x_{it} + \varpi_{it+1}, \quad i = 1, 2 \quad (19)$$

$$\Delta c_{t+1} = const + \psi_1(\tau)x_{1t} + \psi_2(\tau)x_{2t} + v_{t+1} \quad (20)$$

These correspond to the theoretical formulas (9), (11), (1) and (3) of the GDTSM, presented in Section 2. Note that, for the real short-term rate r_t , equation (17) assumes that $\Delta r_{t+1} = const + B_1(0)E_t[\Delta x_{1t}] + B_2(0)E_t[\Delta x_{2t}]$, which corresponds to formula (6). The expectation terms $E_t[\Delta x_{1t}]$ and $E_t[\Delta x_{2t}]$ are estimated through equation (19) of the system. Apart from any possible misspecification errors, the error term of equation (17) $\eta_{t+1}(\tau)$ can reflect measurement or pricing errors (see, e.g., Diebold et al (2006)). These can be attributed to the fact that long-term zero coupon bond prices constitute approximations of coupon-bearing bond prices.

The above system of equations, in addition to equations (17) and (19) often used to estimate affine term structure models of nominal interest rates (see, e.g., Dai and Singleton (2002), and Ang, Piazzesi and Wei (2006)), also includes the set of excess return equations (18). As mentioned in Section 2, this set of equations helps to identify key parameters of the term structure model from the data, like the mean reversion and price of risk parameters k_i and $\lambda_i^{(1)}$, respectively. The latter determines the time-varying component of the term premium, as shown by equation (11).

To estimate the system of equations (17)-(20), we employ the Generalized Method of Moments (*GMM*) (see Hansen (1982)). This method can provide asymptotically efficient estimates of the vector of parameters of the systems which are robust to possible heteroscedasticity and/or serial correlation of error terms $\eta_{t+1}(\tau)$, $e_{t+1}(\tau)$, ϖ_{it+1} and $v_{t+\tau}(\tau)$. In the estimation procedure, we impose the no-arbitrage restrictions given by equation (10) on loading coefficients $B_i(\tau)$. The values of unobserved factors x_{it} involved in the system will be obtained by inverting the following interest rates pricing relationship (12), i.e.,

$$X_t = B^{*-1}(Z_t - A^*),$$

following Pearson's and Sun (1994) approach, where B^* is defined by (12) and vector of series Z_t consists of $z_{1t} = R_t(24)$ and $z_{2t} = Sp_t(60)$. As shown in Table 2A, $R_t(24)$ and $Sp_t(60)$ are found to have the maximum degree of correlation with the principal component factors pc_{1t} and pc_{2t} , respectively, and thus may be less affected by measurement errors. All constants of the system are left unrestricted in the estimation procedure, as they can reflect possible imperfections of the bond market. As $R_t(\tau)$, real consumption growth $\Delta_1 c_{t+1}$ is given in percentage terms, i.e. $\Delta_1 c_{t+1} \equiv 100 \ln[c(t+1)/c(t)]$, and is also annualized.

Tables 4 and 5 present GMM estimates of the mean-reversion and price of risk parameters of system (17)-(20) k_i and $\lambda_i^{(1)}$, with and without including consumption growth equation (20) in it, respectively. Comparison of these two different sets of estimates for k_i and $\lambda_i^{(1)}$ can show if they remain robust to the inclusion of consumption data, when estimating the GDTSM. To estimate both specifications of the above system based on the GMM, we use lagged values of $R_t(24)$ and $Sp_t(60)$ as instrumental variables.

Each of Tables 4 and 5 present two different sets of estimates of k_i and $\lambda_i^{(1)}$. The first relies on values of unobserved factors x_{it} retrieved from the data through relationship $X_t = B^{*-1}(Z_t - A^*)$, often used in practice. See Panels A of the tables. The second set is based on a procedure which slightly modifies the above procedure, suggested by Argyropoulos and Tzavalis (2012). This replaces the observed values of vector Z_t in relationship $X_t = B^{*-1}(Z_t - A^*)$ with their projected values of the elements Z_t on principal component factors pc_{1t} and pc_{2t} . These are obtained based on the following regressions:

$$z_{it} = const_i + d_{i1}pc_{1t} + d_{i2}pc_{2t} + \varsigma_t, \text{ for } i = 1, 2, \quad (21)$$

which are estimated simultaneously with our system of equations (17)-(20). This procedure minimizes the effects of pricing, or measurement, errors of interest rates $R_t(\tau)$ on retrieving estimates of unobserved factors x_{it} through $X_t = B^{*-1}(Z_t - A^*)$. This can be attributed to the fact that principal component factors pc_{it} constitute well diversified

portfolios of interest rates $R_t(\tau)$, if a large set of $R_t(\tau)$ is used to retrieve them. Thus, they can eliminate the effects of measurement or pricing errors in $R_t(\tau)$, or $Sp_t(\tau)$, on the estimates of x_{it} .

TABLE 4: GMM estimates of system (17)-(20)

$\Delta R_{t+1}(\tau) = const + \sum_{i=1}^2 B_i(\tau)E_t[\Delta x_{it+1}] + \eta_{t+1}(\tau), \tau = \{0, 1, 2, \dots, N\}$ $h_{t+1}(\tau) - r_t = const - \sum_{i=1}^2 \lambda_i^{(1)} B_i(\tau)x_{it} + e_{t+1}(\tau)$ $\Delta x_{it+1} = const + (e^{-\tilde{k}_i \Delta t} - 1)x_{it} + \varpi_{it+1}$ $\Delta_1 c_{t+1} = const + \sum_{i=1}^2 \psi_{1i}(\tau)x_{it} + v_{t+1}(\tau)$ where $B_i(\tau) \equiv B_i(0)(1 - e^{-\tilde{k}_i \tau})/\tilde{k}_i \tau$, $\tilde{k}_i = k_i + \lambda_i^1$, $\psi_{1i}(\tau) = (1 - e^{-\tilde{k}_i \tau})/\tilde{k}_i$				
	Panel A		Panel B	
	x_{1t}	x_{2t}	x_{1t}	x_{2t}
$B_i(0)$	0.52 (4×10^{-4})	-0.83 (2×10^{-4})	0.15 (0.001)	-0.75 (0.001)
k_i	0.12 (0.001)	1.38 (0.002)	0.26 (2×10^{-4})	2.08 (0.001)
$\lambda_i^{(1)}$	-0.07 (0.01)	-0.12 (0.01)	-0.06 (1×10^{-4})	-0.19 (5×10^{-4})
d_{1i}			0.20 (5×10^{-5})	-0.28 (1×10^{-4})
d_{2i}			0.05 (6×10^{-5})	0.46 (1×10^{-4})
	$J(102) = 126.50$ (p -value=0.05)		$J(112) = 135.32$ (p -value=0.07)	
	Instruments: 1 (for constant), $R_t(24)$, $Sp_t(60)$, $Sp_{t-i}(60)$, for $i = 1, 2, 3, 4$.			

Notes: The table presents GMM estimates of parameters k_i and λ_i of the system of equations (17)-(20), including consumption growth equation. Panel A presents estimates of k_i and λ_i , for $i=1,2$, based on the observed values of the variables of vector Z_t in inverting relationship (12), while Panel B presents GMM estimates of these parameters based on projected values of vector Z_t on principal component factors pc_{1t} and pc_{2t} (see equation (21)). The estimates of the slope coefficients d_{1i} and d_{2i} of this regression are also given in the table. Heteroscedasticity and autocorrelation consistent (Newey-West) standard errors are shown in parentheses. $J(\cdot)$ is Sargan's overidentifying restriction test, distributed as chi-squared with degrees of freedom given in parentheses. These are equal to the number of orthogonality conditions employed in the GMM estimation procedure minus that of the parameters estimated.

TABLE 5: GMM estimates of system (17)-(19)

$\Delta R_{t+1}(\tau) = const + \sum_{i=1}^2 B_i(\tau) E_t[\Delta x_{it+1}] + \eta_{t+1}(\tau), \tau = \{0, 1, 2, \dots, N\}$ $h_{t+1}(\tau) - r_t = const - \sum_{i=1}^2 \lambda_i^{(1)} B_i(\tau) x_{it} + e_{t+1}(\tau)$ $\Delta x_{it+1} = const + (e^{-\tilde{k}_i \Delta t} - 1) x_{it} + \varpi_{it+1}$ where $B_i(\tau) = B_i(0)(1 - e^{-\tilde{k}_i \tau})/\tilde{k}_i \tau$ and $\tilde{k}_i = k_i + \lambda_i^{(1)}$.				
	Panel A		Panel B	
	x_{1t}	x_{2t}	x_{2t}	x_{2t}
$B_i(0)$	0.88 (5×10^{-4})	-0.63 (3×10^{-4})	0.15 (0.001)	-0.73 (0.0008)
k_i	0.06 (3×10^{-4})	2.18 (0.002)	0.26 (2×10^{-4})	2.10 (0.001)
$\lambda_i^{(1)}$	-0.0005 (1×10^{-4})	-0.38 (8×10^{-4})	-0.06 (1×10^{-4})	-0.18 (5×10^{-4})
d_{1i}			0.20 (5×10^{-5})	-0.27 (1×10^{-4})
d_{2i}			0.05 (7×10^{-5})	0.45 (1×10^{-4})
	$J(96) = 121.37$ (p -value: 0.05)		$J(106) = 132.40$ (p -value: 0.05)	
	Instruments: 1 (for constant), $R_t(24)$, $Sp_t(60)$, $Sp_{t-i}(60)$, for $i = 1, 2, 3, 4$.			

Notes: Panel A presents GMM estimates of parameters k_i and λ_i of the system of equations (17)-(19), without including in it consumption growth equation (20). These are based on observed values of vector Z_t when inverting relationship (12). Panel B presents GMM estimates of k_i and λ_i of the above system based on projected values of vector Z_t on principal component factors pc_{it} (see equation (21)). The estimates of the slope coefficients d_{1i} and d_{2i} of this regression are also given in the table. Heteroscedasticity and autocorrelation consistent (Newey-West) standard errors are reported in parentheses. $J(\cdot)$ is Sargan's overidentifying restriction test, distributed as chi-squared with degrees of freedom given in parentheses. These are equal to the number of orthogonality conditions employed in the GMM estimation procedure minus that of the parameters estimated.

The results of Tables 4 and 5 lead to a number of interesting conclusions. First, they show that the specification of our two factor GDTSM, presented in Section 2, is consistent with the data, which supports the consumption smoothing hypothesis. This is true independently of whether consumption growth equation (20) is included in the estimation of system of equations (17)-(20), or not. This result can be justified by the value of Sargan's overidentifying restrictions test statistic, denoted as $J(\cdot)$, reported in the table. At 5% significance level, J statistic can not reject the orthogonality conditions implied by the system of structural equations (17)-(20) and the instruments used by the GMM estimation procedure of it. This implies that the cross-section restrictions imposed on loading coefficients $B_i(\tau)$ of the GDTSM can not reject the no-arbitrage conditions (10) implied by the theory.

The estimates of parameters k_i and $\lambda_i^{(1)}$ reported in Tables 4 and 5 are close to those found in many studies estimating GDTSM based on nominal interest rates (see, e.g., Ang et al. (2003), and Duffee (2005)). In particular, the estimates of k_i imply a very slow mean reversion for the first unobserved factor x_{1t} , which is very close to zero, and a much faster for the second factor x_{2t} . The reported values of mean-reversion parameter k_2 imply values of the

autoregressive coefficient ϕ of the descretized process (1) for x_{2t} , which are much smaller than those implied by the estimates of unit root auxiliary autoregressive models for $R_t(\tau)$ and pc_{it} . This can be obviously attributed to the fact that $R_t(\tau)$ and pc_{it} constitute linear transformations of unobserved factors x_{1t} and x_{2t} , which exhibit different degree of mean reversion.

Regarding the estimates of price of risk parameters $\lambda_i^{(1)}$, the results of the tables indicate that these are significant for both factors x_{1t} and x_{2t} . This result means that time-varying risk premia effects associated with both factors x_{1t} and x_{2t} are priced in the bond market. According to (11), the negative values of $\lambda_1^{(1)}$ and $\lambda_2^{(1)}$ imply that term premium embodied the real term structure is positive. Note that the estimate of $\lambda_2^{(1)}$, related to the second factor x_{2t} is bigger in absolute value than that of factor x_{1t} . As will be seen latter on, this factor captures the slope of the term structure. Its higher price in absolute terms reduces the mean-reversion parameter k_2 of factor x_{2t} under the risk neutral measure, due to risk aversion effects.

The different sets of values of parameters k_i and $\lambda_i^{(1)}$ reported in Tables 4 and 5 are quite close between the alternative systems of equations estimated, with and without consumption growth equation (20) (i.e., Tables 4 and 5), and across the two methods employed to retrieve estimates of unobserved factors x_{it} (see Panels A and B), i.e. based on observed values of Z_t or projected values of them on principal component factors pc_{it} . The last set of estimates provides more robust estimates of k_i and $\lambda_i^{(1)}$ for both specifications of system (17)-(20), with and without consumption growth equation. This may be attributed to the fact that the estimates of x_{it} based on the projected values of Z_t on pc_{it} are smoother than those based on the actual values of Z_t .

To see if x_{1t} and x_{2t} are closely related to principal component factors pc_{1t} and pc_{2t} , in Figures 5 and 6 we graphically present estimates of them vis-a-vis those of pc_{1t} and pc_{2t} presented in Figure 3. The estimates of x_{1t} and x_{2t} presented in the figures are based on the parameter estimates of system (17)-(20) relying on the projected values of Z_t , reported by Panel B of Table 4. Inspection of the graphs of the above figures clearly indicate that, as was expected, there is a very close relationship between the estimates of x_{1t} and pc_{1t} , and between x_{2t} and pc_{2t} . However, there is no one-to-one correspondence between x_{it} and pc_{it} , for $i=\{1,2\}$. The estimates of x_{it} are smoother than those of pc_{it} , especially for factor x_{1t} . These results imply that, in estimating GDTSMs, replacing unobserved factors x_{it} with estimates of principal component factors may lead to inaccurate estimates of the parameters of these models.

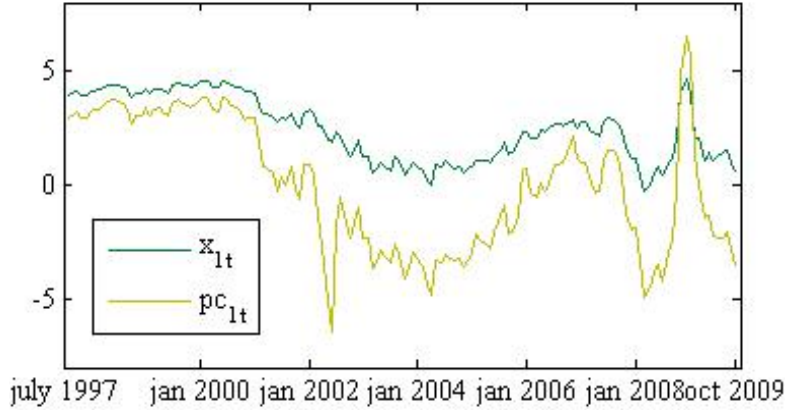


FIGURE 5. Estimates of factor x_{1t} versus principal component factor pc_{1t} .

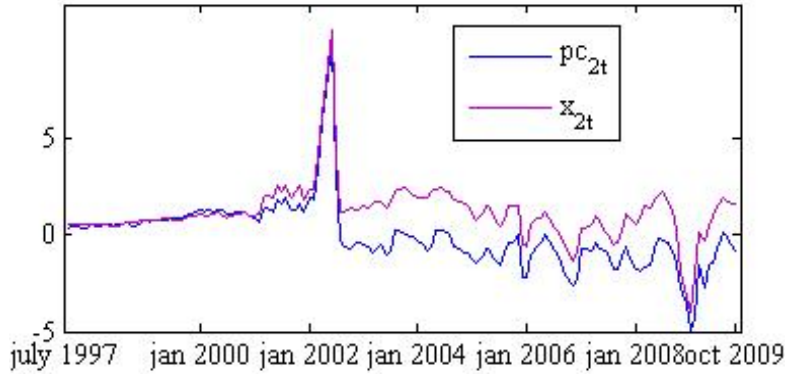


FIGURE 6. Estimates of factor x_{2t} versus principal component factor pc_{2t} .

3.4 Real term structure forecasts of consumption growth

In this section, we examine if the forecasting ability short-term real rate r_t and term spread $Sp_t(\tau_L)$ about future consumption growth $\Delta_{\tau}c_{t+\tau}$, found in the literature (see related studies in the introduction), is in accordance with the theory. Our analysis is mainly interested in examining if the estimates of the key parameters of the GDTSM k_i and λ_{it} can match the pattern of the LS estimates of the slope coefficients of the consumption forecasting regression model (14), $\gamma_1(\tau)$ and $\gamma_2(\tau)$, observed in practice. In addition to this, we also examine the out-of-sample forecasting ability of model (14) relative to that implied by the random walk model of real consumption with drift, suggested by

Hall (1978). As is noted in the literature (see, e.g. Duffee (2005)), the latter is a hard model to beat in forecasting real consumption level, or its growth rate.

Table 6A presents LS estimates of the slope coefficients of regression model (14),

$$\Delta_{\tau}c_{t+\tau} = const + \gamma_1(\tau)r_t + \gamma_2Sp_t(\tau_L) + u_{t+\tau}.$$

This is done for two different spreads of interest rates: $Sp_t(60) = R_t(60) - r_t$ and $Sp_t(36) = R_t(36) - r_t$, and for $\tau = 1, 3, 6, 9, 12$ months ahead. Table 6B presents values of some metrics and test statistics evaluating the in-sample and out-of-sample forecast performance of the above model for $\Delta_{\tau}c_{t+\tau}$ and that implied by the RW model with drift for $c_{t+\tau} = \log C_{t+\tau}$. These metrics include the mean square and absolute errors, denoted as MSE and MAE, respectively. The test statistics employed are those of Diebold and Mariano (1995), denoted DM,¹¹ and Giacomini and Rossi (2005), denoted as GR. The latter is an out-of-sample forecast performance statistic which can test if the forecasts of a model can break down, due to unforeseen breaks-events.¹² To calculate the out-of-sample values of the above metrics and statistics, we rely on recursive estimates of model (14) and the RW model for consumption after period 2004:01, by adding one observation at a time and, then, re-estimating the models by the LS method until the end of sample. The number of the out-of-sample observations used to calculate the above metrics and test statistics are given as $n \equiv T - \tau - m + 1$, where m is our sample window. Note that, for model (14), the tables presents two sets of results. The first employes spread $Sp_t(60) \equiv R_t(60) - r_t$ as regressor, while the second uses $Sp_t(36) \equiv R_t(36) - r_t$.

¹¹DM test statistic is based on the loss difference $d_t = L(u_{t+\tau}^{Model(14)}) - L(u_{t+\tau}^{RW})$. It is defined as $DM = \frac{\bar{d}}{(\widehat{\sigma}_{\bar{d}}^2/T)^{1/2}}$, where $\widehat{\sigma}_{\bar{d}}$ is a consistent estimate of the asymptotic (long-run) variance of $\sqrt{T}\bar{d}$.

¹²The GR statistic is based on the testing principle that, if the forecast performance of a model does not break down, then there should be no difference between its expected out-of-sample and in-sample performance. It is defined as $GR_{m,n,t} = \frac{\overline{SL}_{m,n}}{\widehat{\sigma}_{m,n}^2/\sqrt{n}}$, where $\overline{SL}_{m,n}$ is the average surprise loss given as $\overline{SL}_{m,n} = n^{-1} \sum_{t=m}^{t-\tau} \left\{ (L(\Delta_{\tau}c_{t+\tau}) - \widehat{f}_t(\gamma_1(\tau), \gamma_2(\tau)) - m^{-1} \sum_{j=t-m+1}^t L(\Delta_{\tau}c_{t+\tau}, \widehat{f}_t(\gamma_1(\tau), \gamma_2(\tau)))) \right\}$, for $t = m, \dots, T - \tau$, where $n \equiv T - \tau - m + 1$ is the number of out-of-sample observations and m is the sample window of our initial estimates. $\widehat{\sigma}_{m,n}^2$ is given in Corollary 4 of Giacomini and Rossi (2005). $GR_{m,n,t}$ converges in distribution to a Standard Normal $N(0, 1)$ as $m, n \rightarrow \infty$.

TABLE 6A: Real consumption growth forecasts

Model: $\Delta_t c_{t+\tau} = const + \gamma_1(\tau)r_t + \gamma_2(\tau)Sp_t(\tau L) + u_{t+\tau}$						
τ	1	2	3	6	9	12
	$Sp_t(60) \equiv R_t(60) - r_t$					
$\gamma_1(\tau)$	0.08 (0.03)	0.17 (0.05)	0.26 (0.07)	0.50 (0.19)	0.76 (0.34)	0.97 (0.43)
$\gamma_2(\tau)$	0.10 (0.05)	0.21 (0.07)	0.33 (0.10)	0.60 (0.26)	0.90 (0.44)	1.06 (0.53)
R^2	0.04	0.11	0.16	0.20	0.23	0.24
	$Sp_t(36) \equiv R_t(36) - r_t$					
$\gamma_1(\tau)$	0.08 (0.02)	0.16 (0.04)	0.24 (0.06)	0.46 (0.17)	0.69 (0.31)	0.87 (0.40)
$\gamma_2(\tau)$	0.13 (0.05)	0.27 (0.08)	0.40 (0.12)	0.73 (0.31)	1.06 (0.52)	1.23 (0.63)
R^2	0.05	0.12	0.17	0.21	0.23	0.23

Notes: Newey-West standard errors corrected for heteroscedasticity and moving average errors up to $\tau - 1$ periods ahead are reported in prostheses. R^2 is the coefficient of determination.

The results of Tables 6A and 6B indicate that short-term real interest rate r_t and spread $Sp_t(\tau)$ contains significant information about future consumption growth $\Delta_\tau c_{t+\tau}$, for all τ . This is true for both cases of term spread $Sp_t(\tau)$ considered. The values of R^2 , reported in Table 6A, imply that the forecasting ability of model (14) increases with τ . As was expected, the values of R^2 are similar to those of principal component factors model forecasting future consumption growth $\Delta_\tau c_{t+\tau}$, reported in Table (3). The values of the MSE and MAE reported in Table 6B clearly indicate that the forecast performance of model (14) is better than that of the RW model, for all τ . This is true for both the in-sample and out-of-sample exercises. The better performance of model (14) than the RW model is also supported by the values of DM statistic. The negative values of this statistic indicate that this model provides smaller in magnitude forecast errors than the RW model, especially as τ increases. These values clearly reject the null hypothesis that the two models have the same forecasting ability, at 5% significance level. Further support for model (14) in forecasting $\Delta_\tau c_{t+\tau}$ can be obtained by the GR test statistic. The values of this statistic reported in the table indicate that this model can produce out-of-sample consumption growth forecasts which are stable and consistent with its in-sample forecasts up to three-months ahead.

TABLE 6B: Forecasting performance

τ	1	2	3	6	9	12	1	2	3	6	9	12
	In-sample						Out-of-sample					
	Model: $\Delta_t c_{t+\tau} = const + \gamma_1(\tau)r_t + \gamma_2(\tau)Sp_t(\tau_L) + u_{t+\tau}$											
	$Sp_t(\tau_L) \equiv R_t(60) - r_t$											
<i>MSE</i>	0.17	0.27	0.39	1.12	2.20	3.42	0.14	0.32	0.53	1.75	3.47	5.52
<i>MAE</i>	0.30	0.40	0.49	0.83	1.22	1.54	0.29	0.42	0.56	0.97	1.37	1.68
<i>DM</i>	-1.46	-2.18	-2.71	-2.72	-2.75	-3.02	-1.47	-2.18	-2.71	-2.71	-2.74	-3.02
<i>GR</i>							-0.41	1.34	2.16	5.94	8.89	10.90
	$Sp_t(\tau_L) \equiv R_t(36) - r_t$											
<i>MSE</i>	0.17	0.27	0.40	1.12	2.20	3.49	0.14	0.31	0.53	1.72	3.46	5.60
<i>MAE</i>	0.30	0.40	0.49	0.83	1.22	1.54	0.28	0.42	0.55	0.96	1.37	1.68
<i>DM</i>	-1.57	-2.32	-2.83	-2.85	-2.80	-2.86	-1.58	-2.32	-2.38	-2.85	-2.80	-2.86
<i>GR</i>							-0.44	1.29	2.06	5.88	8.84	11.05
	<i>Random Walk</i>											
<i>MSE</i>	0.18	0.31	0.47	1.42	2.86	4.62	0.16	0.38	0.54	2.20	4.51	7.28
<i>MAE</i>	0.31	0.43	0.53	0.88	1.23	1.55	0.31	0.47	0.62	1.06	1.48	1.87
<i>GR</i>							2.81	2.76	2.80	6.56	9.35	11.76

Notes: The table presents values of the MSE and MAE metrics and DM and GR statistics assessing the forecasting performance of model $\Delta_t c_{t+\tau} = const + \gamma_1(\tau)r_t + \gamma_2(\tau)Sp_t(\tau_L) + u_{t+\tau}$ and that implied by the random walk (RW) model of the level of real consumption with drift. DM and GR denote the Diebold-Mariano and Giacomini-Rossi test statistics, respectively. These statistics follow the standard normal distribution. Note that the GR test statistic is an out-of-sample test statistic, which can test the stability of the out-of-sample forecasts compared to the in-sample ones. To calculate the out-of-sample values of the above metrics and statistics, we rely on recursive estimates of model (14) and the RW model for consumption after period 2004:01, adding one observation at a time and, then, re-estimating the models until the end of sample. The total number of observations used in our out-of-sample forecasting exercise is $n \equiv T - \tau - m + 1 = 69$, where m is our sample window.

Another interesting conclusion that can be drawn from the results Table 6A is that the LS estimates of the slope coefficients $\gamma_1(\tau)$ and $\gamma_2(\tau)$ of model (14) increase with τ . To examine if these estimates of $\gamma_1(\tau)$ and $\gamma_2(\tau)$ can match those implied by the parameter estimates of the GDTSM, over different τ , in Table 7 we present estimates of the latter against the LS estimates. The estimates of $\gamma_1(\tau)$ and $\gamma_2(\tau)$ implied by the GDTSM are derived based on relationship (15) and the estimates of parameters k_i and $\lambda_i^{(1)}$ reported in Panels A and B of Table 4.

The results of Table 7 clearly indicate that the pattern of the LS estimates of coefficients $\gamma_1(\tau)$ and $\gamma_2(\tau)$ with maturity horizon τ , reported in Table 6A, is consistent with that implied by the estimates of our GDTSM. The implied by the GDTSM estimates of $\gamma_1(\tau)$ and $\gamma_2(\tau)$ are close to their LS estimates, even for the forecasting period of $\tau = 12$ months. These lie within the two standard deviations confidence interval of the LS estimates of them. As is predicted by the analysis of subsection 2.1, the estimates of slope coefficient $\gamma_2(\tau)$ are bigger than those of $\gamma_1(\tau)$, since the second factor x_{2t} driving the real term structure is strongly mean reverting. This is true for both sets of implied values of $\gamma_1(\tau)$ and $\gamma_2(\tau)$, reported in the table. These results are also consistent across the two different spreads $Sp_t(\tau,)$, i.e. $Sp_t(60) = R_t(60) - r_t$ and $Sp_t(36) = R_t(36) - r_t$, considered in our analysis.

TABLE 7: GDTSM versus LS estimates of $\gamma_1(\tau)$ and $\gamma_2(\tau)$

$Sp_t(\tau_L) = R_t(60) - r_t$						
τ	Implied estimates by the GDTSM				LS estimates	
	(Table 4, Panel A)		(See Table 4, Panel B)		(see Table 6A)	
	$\gamma_1(\tau)$	$\gamma_1(\tau)$	$\gamma_1(\tau)$	$\gamma_2(\tau)$	$\gamma_1(\tau)$	$\gamma_2(\tau)$
1	0.06	0.13	0.06	0.15	0.08	0.10
2	0.13	0.26	0.14	0.30	0.17	0.21
3	0.20	0.39	0.22	0.43	0.26	0.33
6	0.43	0.73	0.51	0.82	0.50	0.60
9	0.67	1.05	0.82	1.18	0.76	0.90
12	0.92	1.30	1.13	1.52	0.97	1.06
$Sp_t(\tau_L) = R_t(36) - r_t$						
τ	Implied estimates by the GDTSM				LS estimates	
	(Table 4, Panel A)		(Table 4, Panel B)		(see Table 6A)	
	$\gamma_1(\tau)$	$\gamma_2(\tau)$	$\gamma_1(\tau)$	$\gamma_2(\tau)$	$\gamma_1(\tau)$	$\gamma_2(\tau)$
1	0.06	0.12	0.05	0.12	0.08	0.13
2	0.13	0.23	0.11	0.24	0.16	0.27
3	0.20	0.35	0.17	0.36	0.24	0.40
6	0.42	0.66	0.41	0.68	0.46	0.73
9	0.66	0.95	0.68	0.99	0.69	1.06
12	0.91	1.21	0.96	1.26	0.87	1.23

Notes: The table reports theoretical values of slope coefficients $\gamma_1(\tau)$ and $\gamma_2(\tau)$ based on relationship (15) and the estimates of the mean-reversion and risk price function parameters of our GDTSM k_i and $\lambda_i^{(1)}$, reported in Panels A and B of Table 4, respectively. This is done against the LS estimates of these coefficients, reported in Table 6A.

4 Conclusions

This paper suggests a Gaussian dynamic real term structure model to explain the ability of the short-term real interest rate and its term spread with longer term real interest rates to forecast future changes in real consumption growth. The paper fits the model into real term structure and consumption data from the US economy, and it provides a number of interesting results which are consistent with the consumption smoothing hypothesis.

First, it shows that two stationary common factors can explain most of the variation of the real term structure of interest rates. The first of these two factors, which exhibits very slow mean reversion, can explain persistent shifts in the levels of real interest rates. This factor is found to be affected more strongly by the recent financial crisis and the stock market crises of period 2001-2003, which also affected the US bond market. The second factor, which has higher degree of mean reversion, can explain the slope of the real term structure.

Second, the estimates of the price of risk parameters reported by the paper indicate that both of the above factors are priced in the market and, thus, they can explain time variation of excess holding period returns of the market. The estimates of the price of risk and mean-reversion parameters of the two term structure factors retrieved by our

data are also found to be consistent with the cross-section restrictions of the real term structure model suggested by the paper. These restrictions arise by ruling out profitable arbitrage conditions of the market. They are tested based on a structural system of equations consisting of real interest rates, excess holding period real returns, reflecting term premia effects, and real consumption growth.

Finally, the paper rigorously shows that the forecasting ability of the short-term real interest rate and its spread with long-term real interest rates about future real consumption growth, over different periods ahead, can be consistently explained by the common factor representation of the real term structure and consumption growth. This forecasting model of consumption growth is found to perform better than that implied by the random walk model of real consumption level. The ability of the term spread to forecast future consumption growth can be attributed to the high degree of mean reversion of the second common factor driving the real term structure of interest rates.

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