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Abstract

We analyze a simple dynamic framework where sellers are capacity constrained over the length of the game. Buyers act strategically in the market, knowing that their purchases may affect future prices. The model is examined when there are single and multiple buyers, with both linear and non-linear pricing. We find that, in general, there are only mixed strategy equilibria and that sellers get a rent above the amount needed to satisfy the market demand that the other seller cannot meet. Buyers would like to commit not to buy in the future or hire agents with instructions to always buy from the lowest priced seller. Furthermore, sellers’ market shares tend to be maximally asymmetric with high probability, even though they are \textit{ex ante} identical.

\textit{JEL numbers:} D4, L1

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1 Introduction

In many durable goods markets, sellers who have market power and intertemporal capacity constraints face strategic buyers who make purchases over time. There may be a single buyer, as in the case of a government that purchases military equipment or awards construction projects, such as for bridges, roads, or airports, and chooses among the offers of a few large available suppliers. Or, there may be a small number of large buyers, such as in the case of airline companies that order aircraft or that of shipping companies that order cruise ships, where the supply could come only from a small number of large, specialized companies. The capacity constraint may be due to the production technology: a construction company that undertakes to build a highway today may not have enough engineers or machinery available to compete for an additional large project tomorrow, given that the projects take a long time to complete; a similar constraint is faced by an aircraft builder that accepts an order for a large number of aircraft. Or, the capacity constraint may simply correspond to the flow of a resource that cannot exceed some level: thus, if a supplier receives a large order today, he will be constrained on what he can offer in the future. This effect may be indirect, if the resource is a necessary ingredient for a final product (as often in the case of pharmaceuticals). More generally, cases like the ones mentioned above suggest a need to study dynamic oligopolistic price competition under capacity constraints, when buyers are also strategic. Although this topic is both important and interesting, it has not been treated yet in the literature.

To obtain some first insights into the problem, consider the following simple setting. Take two sellers of some homogeneous product, say aircraft, to fix ideas. Each seller cannot supply more than a given number of aircraft over two periods. Suppose that there is only one large buyer in this market, this may be the defense department, with a demand that exceeds the capacity of each seller but not that of both sellers combined. Let the period one prices be lower for one seller than the other. Then, if the buyer’s purchases exhaust the capacity of the low priced seller, only the other

1 Anton and Yao (1990) provide a critical survey of the empirical literature on competition in defense procurement - see also Burnett and Kovacic (1989) for an evaluation of relevant policies. In an empirical study of the defense market, Greer and Liao (1986, p.1259) find that “the aerospace industry’s capacity utilization rate, which measures propensity to compete, has a significant impact on the variation of defense business profitability and on the cost of acquiring major weapon systems under dual-source competition”. Ghemawat and McGahan (1998) show that order backlogs, that is, the inability of manufacturers to supply products at the time the buyers want them, is important in the U.S. large turbine generator industry and affects firms’ strategic pricing decisions. Likewise, production may take significant time intervals in several industries: e.g., for large cruise ships, it can take three years to build a single ship and an additional two years or more to produce another one of the same type. Jofre-Bonet and Pesendorfer (2003) estimate a dynamic procurement auction game for highway construction in California - they find that, due to contractors’ capacity constraints, previously won uncompleted contracts reduce the probability of winning further contracts.
seller will remain active in the second period and, unconstrained from any competition, he will charge the monopoly price. A number of questions arise. Anticipating such behavior, how should the buyer behave? Should he split his orders in the first period, in order to preserve competition in the future, or should he get the best deal today? Given the buyer’s possible incentives to split orders, how will the sellers behave in equilibrium? Should sellers price in a way that would induce the buyer to split or not to split his purchases between the sellers? How do sellers’ equilibrium profits compare with the case of only a single pricing stage? Does the buyer have an incentive to commit to not making purchases in the future? Are there incentives for the buyer to vertically integrate with a seller?

An additional set of questions emerges when there is more than one buyer. Would the buyers like to coordinate their purchases? Is buyer coordination possible in equilibrium? Are the seller equilibrium market shares identical, since the sellers are identical?

We consider a set of simple dynamic models with the following key features. There are two incumbent sellers who choose their capacities and a large number of potential sellers who can enter and choose their capacities after the incumbents. Capacity choices are thus endogenized and determine how much a firm can produce over the entire game. Next, sellers set first-period prices and then buyers decide how many units they wish to purchase from each seller. The situation is repeated in the second period, given the remaining capacity of the firms; sellers set prices and buyers decide which firm to purchase from. We examine separately the cases of a single buyer (monopsony) and that of two or more buyers (oligopsony).

Our main results are as follows. First, entry is always blockaded - the capacity level for each incumbent seller is such that there is no profitable entry by other sellers. Given these capacity levels, a pure strategy subgame perfect equilibrium fails to exist. This is due to a combination of two phenomena. First, buyers have an incentive to split his orders in the first period if the prices are close, in order to keep strong competition in the second period. This in turn, gives the sellers incentives to raise their prices. Second, if prices get “high,” each seller has a unilateral incentive to lower his price, and sell all his capacity. We characterize the mixed strategy equilibrium and show that it has two important properties. Buyer may have a strict incentive to split their orders with positive probability: for certain realizations of the equilibrium prices (that do not differ too much) a buyer chooses to buy in the first period from both a high price seller and a low price seller. Further, we also show that the sellers make a positive economic rent above the profits of serving the buyer’s residual demand, if the other seller sold all of his units. There are three main implications
that follow from this result. First, buyers would like to commit to not make purchases in the second period, so as to induce strong price competition in the first period (that is, a buyer is hurt when competition takes place in two periods rather than in one). This is consistent with the practice in the airline industry, where airliners have options to buy airplanes in the future. Specifically, the common practice of airline companies when they are purchasing aircraft is to order a specific number (to be delivered over 2-4 years) and at the same time to agree on a significant number of “option aircrafts”, with these options possible to be exercised over a specified time interval of say 5-7 years. So airline companies choose not to negotiate frequently with the sellers and place new orders as their needs may increase over time, but instead they negotiate at one time in a way that covers their possible needs over the foreseeable future. Second, a buyer has the incentive to instruct its purchasing agents to always buy from the lowest priced firm. This is consistent with many government procurement rules that do not allow discretion to its purchasing officers. In other words, in equilibrium, a buyer is hurt by his ability to behave strategically over the two periods and would like to commit to myopic behavior, if possible. Finally, the firm has a strict incentive to vertically integrate with one of the suppliers.

We get additional results when there are multiple buyers. First, buyers would like to coordinate their purchasing strategies in the first period to maintain strong competition in the second period. Second the inability of buyers to coordinate in equilibrium, makes it highly likely that sellers' markets shares in both the first period and for the entire game can be quite asymmetric, even though sellers are ex ante identical. Finally, the most important general conclusion is that regardless of the number of buyers, in the equilibrium of the model, sellers make higher profit than what would be justified by residual demand, which is also the profit what they would be making if competition was taking place in a single period and/or buyers were not strategic.

This paper studies competition with strategic buyers and sellers under dynamic (that is, intertemporal) capacity constraints. As such, it is broadly related to two literatures. First, the literature on capacity-constrained competition starts with the classic work of Edgeworth (1897) who shows that competition may lead to the “nonexistence” of a price equilibrium or, as is sometimes described, price “cycles”. Subsequent work that has studied pricing under capacity constraints in-

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2This appears to be common practice in the industry. For example, in 2002 EasyJet signed a contract with Airbus for 120 new A319 aircraft and agreed to have the option to buy in addition up to the same number of aircraft for about the same price. While the agreed aircraft were being gradually delivered, EasyJet exercised in 2006 the option and placed and order for an additional 20 units to account for projected growth with delivery set between then and 2008. Similarly, an order was placed in 2006 by GE Commercial Aviation to buy 30 Next generation 737s from Boeing and also to agree for an option for an additional 30.
cludes Beckman (1965), Levitan and Shubik (1972), Osborne and Pitchik (1986) and Dasgupta and Maskin (1986b).³ Other papers have studied the choice of capacities in anticipation of oligopoly competition ⁴ and the effect of capacity constraints on collusion.⁵ All this work refers to capacity constraints that operate period-by-period, that is, there is a limit on how much can be produced or sold in each period that does not depend on past decisions. Dynamic capacity constraints, the focus of our paper, have received much less attention in the literature. Griesmer and Shubik (1963), with prices in a discrete set, and Dudey (1992), with real prices (?), study games where capacity-constrained duopolists face a finite sequence of buyers with unit demands and a common value. Under certain conditions, the sellers maximize in equilibrium their joint profits. Ghemawat (1997, ch.2) and Ghemawat and McGaham (1998) characterize mixed strategy equilibria in a two-period duopoly (with one seller having initially half of the capacity of the other). Dudey (2006) presents conditions so that a Bertrand outcome is consistent with capacities chosen by the sellers before the buyers arrive. In the above mentioned papers, capacity constraints are not dynamic and demand is modeled as invariant and independent across periods. The key distinguishing feature of our work is that the buyers (and not just the sellers) are strategic and the evolution of capacities across periods depends on the actions of both sides of the market.⁶

Second, the present paper is related to the body of research where both buyers and sellers are large, strategic, buyers and bilateral oligopoly considerations arise.⁷ In particular, it is related to


⁵See e.g. Brock and Scheinkman (1985), Lambson (1987), Rotemberg and Saloner (1989), and Compte, Jenny and Rey (2002).

⁶In recent work, Bhaskar (2001) shows that by acting strategically a buyer can increase his net surplus when buyers are capacity constrained. In that model, however, there is a single buyer who has unit demand in each period for a perishable good, and so “order splitting” cannot be studied: the buyer can chose not to buy in a given period, he receives zero value in that period, but gets future units at lower prices. Our model allows both for a single and for multiple buyers. Buyers view the good as durable (receive value over both periods) and may wish to split their orders, possibly buying from the more expensive supplier in the first period. Our focus and set of results is, thus, quite different. Equilibrium behavior is such that the buyers get hurt as a result of their strategic behavior because it alters the pricing incentives for the sellers. In our model, in equilibrium a buyer never chooses not to buy at all in the first period.

⁷Aspects of bilateral oligopoly have been studied, among other papers, in Horn and Wolinsky (1988), Dobson and Waterson (1997), Hendricks and McAfee (2000) and Inderst and Wey (2003).
other work that has examined when a buyer influences the degree of competition among (potential) suppliers, as in the context of “split awards” and “dual-sourcing”. Rob (1986) studies procurement contracts that allow selection of an efficient supplier, while providing incentives for product development. Anton and Yao (1987, 1992) consider models where a buyer can buy either from one seller or split his order and buy from two sellers. They find conditions under which a buyer will split his order and characterize seemingly collusive equilibria. Related studies on dual-sourcing are offered by Riordan and Sappington (1987) and Demski, Sappington and Spiller (1987). Our work differs in two important ways. The intertemporal links are at the heart of our analysis: the key issue is how purchasing decisions today affect the sellers' remaining capacities tomorrow. In contrast, the work mentioned above focuses on static issues and relies on cost asymmetries. Strategic purchases from competing sellers and a single buyer in a dynamic setting are also studied under “learning curve” effects; see e.g. Cabral and Riordan (1994) and Lewis and Yildirim (2002, and 2005 for switching costs). In our case, by buying from one seller you make that seller less competitive in the following period (and in fact inactive, when the buyer is left with no capacity) - in the learning curve case, the more you buy from a seller, the more competitive you make that seller, as his unit cost decreases.  

The remainder of the paper is organized as follows. The model is set up in Section 2. Section 3 characterizes the equilibrium with one buyer and discusses a number of implications of the equilibrium properties. The duopsony case is presented in Section 4 - subsequently, the analysis is also generalized to the case of an arbitrary number of buyers. We conclude in Section 5. Proofs not required for the continuity of the presentation are relegated to an Appendix.

2 The model

Buyers and sellers interact over two periods. We examine two market structures on the buyers' side, one where there is a single buyer (monopsony) and the other where there are two buyers (duopsony); we also demonstrate how our results generalize when there are more than two buyers. There are two identical incumbent sellers and many identical potential entrants on the seller side of the market. The product is perfectly homogeneous and perfectly durable over the lifetime of the

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8In Bergemann and Välimäki (1996), sellers set prices and a buyer chooses which seller to purchase from, affecting how competitive each seller could be in subsequent periods. The action there comes from experimentation, not from capacities. Strategic competition with capacity constraints is also part of Yanelle’s (1997) model of financial intermediation.
model. All sellers and buyers, have a common discount factor $\delta$.

Each buyer values each of the first two units $V$ in each period that he has the unit and a third unit at $V_3$ in period 2. Thus, for the first two units that a buyer buys in period 1, he gets consumption value $V$ in each period. We assume that $V \geq V_3 > 0$.

At the start of the game the incumbent sellers simultaneously choose their capacities. The potential entrants observe the incumbents’ capacity choices and then simultaneously choose whether to enter and their capacity choice if they enter. We assume that the cost of capacity for any seller is small, $\varepsilon$, but positive. The marginal cost of production is 0 if total sales across both selling periods one and two do not exceed capacity and infinite otherwise. The capacity choice is the maximum that the seller can produce over the two periods. Thus, each seller has capacity at the beginning of the second period equal to his initial capacity minus the units he sold in the first period.

Throughout the analysis, we will examine subgames where the capacity choice at the start of the game for each incumbent seller is equal to $3N - 1$, where $N$ is the number of buyers (e.g. in monopsony each seller has 2 units and in duopsony each seller has 5 units) and that entry is always blockaded. We demonstrate that these are the equilibrium capacity choices when we work via backward induction in the analysis.

In each period, each of the sellers sets a per unit price for his available units of capacity. Each buyer chooses how many units he wants to purchase from each seller at the price specified, as long as the seller has enough capacity. If the demand by buyers is greater than a seller’s capacity, then they are rationed. The rationing rule that we use is that each buyer is equally likely to get his order filled. The rationed buyers can buy from the other seller as many units as they want.

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9To clarify, the maximum gross value that a buyer could obtain over both periods and evaluated at the beginning of the first period is equal to $2V(1+\delta)+\delta V_3$. Our specification is consistent with growing demand. Note that, in general, the first and second units could have different values (say $V_1 \geq V_2$). Also, we could allow the demand of the third unit to be random. It is straightforward to introduce either of these cases in the model, with no qualitative change in the results, only at the cost of some additional notation.

10It will be clear that if the marginal cost of producing an additional unit is sufficiently high, then the results still hold. Also, fixed costs would not change the results as long as fixed costs are below a seller’s equilibrium profit. If the fixed costs were above a seller’s equilibrium profit, then the firm would not enter.

11We focus on the core case where each seller sets a simple unit price, that is competition when there is no price discrimination among buyers or among units. The flavor of our results would be the same if discriminatory pricing was allowed with multiple buyers. In the Appendix we discuss implications of non-linear pricing.

12Our results would not change qualitatively if the sellers could choose which buyer to ration, as long as each buyer has a positive probability of being rationed.
assume that sellers commit to their prices one period at a time and that all information is common knowledge and symmetric. We derive the set of symmetric subgame perfect equilibria of the game.

Let us now discuss why we have adopted this modelling strategy. We analyze a dynamic bilateral oligopoly game, where all players are “large” and therefore expected to have market power. In such cases, one wants the model to reflect the possibility that each player can exercise some market power. By allowing the sellers to make price offers and the buyers to choose how many units to accept from each seller, all players indeed have market power in our model. It follows that quantities and prices evolve from the first period to the second as a joint result of the strategies adopted by the buyers and the sellers. If, instead, we allowed the buyers to make price offers, then the buyers would have all the market power. This would not be realistic, particularly in the case when there are at least as many buyers as sellers. Also it would not be interesting, because trivially the buyers would be able to extract the entire surplus from the sellers. In fact, anticipating such a scenario, sellers would not be willing to pay even an infinitesimal entry cost and thus this market would never open.\textsuperscript{13} There are further advantages of this modeling strategy. First, it makes the results easy to compare between the monopsony and the oligopsony cases. Second, it makes our results more easily comparable with other papers in the literature, in particular the ones mentioned in the Introduction with intertemporal capacity constraints, where the prices are indeed set by the sellers.\textsuperscript{14} Third, there may be agency (moral hazard) considerations that contribute to why we typically see in reality the sellers making offers.\textsuperscript{15}

The interpretation of the timing of the game is immediate in case the sellers’ supply comes from an existing stock (either units that have been produced at an earlier time, or some natural resource

\textsuperscript{13}Of course, there are other structures that would allow buyers and sellers to each keep part of the market surplus, involving some form of multilateral bargaining. However these appear less robust and more complicated (and in particular, more dependent on the modeling details) than the structure we have adopted here. In any game where the sellers would have some control over setting prices and the buyers over choosing where to buy from, it is expected that equilibrium behavior would reflect the same qualitative features we emphasize here.

\textsuperscript{14}This is whether the buyers are strategic (Bhaskar, 2001) or not (e.g. Dudey, 1992 or Ghemawat and McGaham, 1998).

\textsuperscript{15}In general we see the sellers making offers, even with a single buyer, like when the Department of Defense (DOD) is purchasing weapon systems. The DOD may do this to solve possible agency problems between the agent running the procurement auction and the DOD. If an agent can propose offers, it is much easier for sellers to bribe the agent to make high offers than if sellers make offers, which can be observed by the regulator. This is because the sellers can bribe the agent to make high offers to each of them, but competition between the sellers would give each seller an incentive to submit a bid to grab all the sells and it would be quite difficult for the agent to accept one offer that was much higher than another.
Figure 1: Timing

that the firm controls). One simple way to understand the timing, in the case where production takes place in every period is illustrated in Figure 1. The idea here is that actual production takes time. Thus, orders placed in period one are not completed before period two orders arrive. Since each seller has the capacity to only work on a limited number of units at a time, units ordered in period one restrict how many units could be ordered in period two. In such a case, since our interpretation involves delivery after the current period, the buyers’ values specified in the game should be understood as the present values for these future deliveries (and the interpretation of discounting should be also accordingly adjusted).

3 Monopsony

We first examine the single buyer case ($N = 1$), that is, monopsony. We are constructing a subgame perfect equilibrium, and thus we work backwards by starting from period 2.

3.1 Second period

There are several cases to consider, depending on how many units the buyer has bought from each seller in period one. We will use, throughout the paper, the convention of calling a seller with $i$ units of remaining capacity seller $i$.

*Buyer bought two units in period 1.* If the buyer bought a unit from each of the sellers in period 1, then the price in period 2 is 0 due to Bertrand competition. If the buyer bought both units from the same firm, then the other firm would be a monopolist in period 2 and charge $V_3$. Thus, period 2 equilibrium profit of a seller that has one remaining unit of capacity is 0 and that of a seller with two remaining units of capacity is $V_3$. 
Buyer bought one unit in period 1. In this case, the buyer has demand for two units, one of the sellers has a capacity of 1 unit, seller 1, while the other has a capacity of 2 units, seller 2. We demonstrate that there is no pure strategy equilibrium in period 2 by the following Lemma.

**Lemma 1** If the buyer bought one unit in period 1 in the monopsony model, then there is no pure strategy equilibrium in period 2.

**Proof.** First, notice that the equilibrium cannot involve seller 2 charging a zero price: that seller could increase his profit by raising his price (as seller 1 does not have enough capacity to cover the buyer’s entire demand). Thus, seller 1 would also never charge a price of zero. Suppose now that both sellers charged the same positive price. One, if not both, sellers have a positive probability of being rationed. A rationed seller could defect with a slightly lower price and raise his payoff. Suppose that the prices are not equal: \( p_i < p_j \leq V_3 \). Clearly, seller \( i \) could increase his payoff by increasing his price since he still sells the same number of units. Similarly, seller \( i \) can improve his payoff by increasing his price if \( p_i < V_3 \leq p_j \). Finally, if \( V_3 \leq p_i < p_j \), seller \( j \) makes 0 profit and can raise his payoff by undercutting firm \( i \)'s price.

There is a unique mixed strategy equilibrium which we provide in the following Lemma (see also Figure 2 for an illustration).

**Lemma 2** If the buyer bought one unit in period 1 in the monopsony model, then there is a unique mixed strategy equilibrium. Both sellers mix on the interval \([V_3/2, V_3]\). Seller 1’s price distribution is \( F_1(p) = 2 - \frac{V_3}{p} \), with an expected profit of \( V_3/2 \). Seller 2’s price distribution is \( F_2(p) = 1 - \frac{V_3}{2p} \) for \( p < V_3 \), with a mass of 1/2 at price \( V_3 \), and expected profit equal to \( V_3 \). Seller 2’s price distribution first order stochastically dominates seller 1’s distribution.

**Proof.** See Appendix A1.

By Lemma 2, we obtain two key insights that run throughout the paper. The first concerns the calculation of the equilibrium sellers’ profits and the second regards the ranking of the sellers’ price distributions. The seller with two units of capacity can always guarantee himself a payoff of at least \( V_3 \), since he knows that, no matter what the other seller does, he can always charge \( V_3 \) and sell at least one unit. This is the high-capacity seller’s security profit level. The high-capacity seller’s security profit puts a lower bound on the price offered in period 2. In the situation examined at Lemma 2, the lowest price is \( V_3/2 \): the seller will never charge a lower price because he can at
most sell two units and would do better by selling one unit at $V_3$. This puts a lower bound of $V_3/2$ on the period 2 profit of seller 1, the low-capacity seller; this, in turn, is equal to the profit that the low-capacity seller can guarantee to himself, given that the high-capacity seller will not choose a strictly dominated price.\(^{16}\) Competition between the two sellers fixes their profits at their respective security levels.\(^{17}\)

Lemma 2 can be generalized to any case where there is a low-capacity seller that cannot cover the demand and a high-capacity seller that can cover the demand, including the case where there are multiple buyers. The steps in the analysis are the same as the ones presented for Lemma 2.

We can then state:

**Lemma 3** Suppose that in period 2 the buyers have value for $B$ units and the capacity of the low-capacity seller is $C$, with $C < B$. Then there is no pure strategy equilibrium. In the unique mixed strategy equilibrium, the high-capacity seller’s profit is $V_3(B - C)$ and the low capacity seller’s profit is $C \frac{V_3(B - C)}{B}$. The support of the prices is from $V_3(B - C)/B$ to $V_3$.

Note that given the structure of demand and capacity, the situation described here will always be the case whenever we have asymmetric capacities in period 2: the low capacity seller’s capacity will be strictly lower than the demand while the high capacity seller’s capacity will be at least as high as the demand.

To understand Lemma 3 note that the high-capacity seller’s security profit, is $V_3(B - C)$. This is so because the low-capacity seller can supply only up to $C$ of the $B$ units that the buyers demand and buyer are willing to pay up to $V_3$. This high-capacity seller’s security profit puts a lower bound on the price offered in period 2. Given the high-capacity seller can sell at most $B$ units (that is the total demand), he will never charge a price below $V_3(B - C)/B$, since a lower price would lead to profit lower than his security profit. Since the high-capacity seller would never change a price below $V_3(B - C)/B$, this level also puts a lower bound on the price the low-capacity seller would charge and, as that seller has $C$ units he could possibly sell, his profit becomes $C \frac{V_3(B - C)}{B}$. The details of the formal proof are identical to Lemma 2.

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\(^{16}\)Note that, while $V_3/2$ is not the “security” profit of the low-capacity seller, it becomes that after one round of elimination of strictly dominated strategies.

\(^{17}\)Mixed strategy equilibria are also characterized in Ghemawat and McGahan (1998) for the case where one seller has double the capacity of the other.
The second insight deals with the incentives for aggressive pricing. We find that the seller with larger capacity will price less aggressively than the seller with smaller capacity in period 2. The larger capacity seller knows that he will make sales even if he is the highest price seller, while the smaller capacity seller makes no sales if he is the high price seller. So, the low capacity seller always has incentives to price more aggressively. More precisely, the high-capacity seller price distribution first-order stochastically dominates the price distribution of the low capacity seller. This general property has important implications for the quantities sold and the market shares over the entire game.

We now examine the remaining period-two case (subgame).

Buyer bought no units in period 1. Each seller enters period 2 with 2 units of capacity, while the buyer demands 3 units. Using Lemma 3, each player’s expected second-period equilibrium payoff is $V_3$; this is the security profit of each seller. The equilibrium behavior in the second period is now summarized:

**Lemma 4** Second period competition for a monopsonist falls into one of three categories. (i) If only one seller is active (the rival has zero remaining capacity), that seller sets the monopoly price, $V_3$, and extracts the buyer’s entire surplus. (ii) If each seller has enough capacity to cover by himself the buyer’s demand then the price is zero. (iii) If the buyer’s demand exceeds the capacity of one seller but not the aggregate sellers’ capacity, then there is no pure strategy equilibrium. In the mixed strategy equilibrium, a seller with two units of capacity has expected profit equal to $V_3$ and a seller with one unit of capacity has expected profit equal to $V_3/2$. 

![Figure 2: Mixed strategy equilibrium](image)
3.2 First period

Now, we go back to period 1. First, we demonstrate that the buyer will always buy two units in equilibrium and that there is no pure strategy equilibrium. We then characterize equilibrium payoffs and discuss the properties of equilibria.

**Proposition 5** The buyer buys two units in period 1.

We sketch the proof here; the formal proof is in Appendix A2. First, prices must be positive, since the seller knows that even if he does not sell a unit in period 1, he will make positive profits in period 2. We then show that a buyer will never buy three units in period 1. For the buyer to buy three units, he must buy two units from the low priced seller at a positive price and one from the other seller. If he only buys one unit from each seller in period 1, the price for the third unit bought in period 2 is zero due to Bertrand competition; thus, the buyer will never buy three units. We next argue that the price never exceeds a bound such that the buyer prefers buying one unit from each seller as opposed to only one unit from the low priced seller. This is because this price is greater than $\delta V_3$, which by Lemma 4 is greater than a seller’s expected profit in period 2 if he makes no sales in period 1. Thus, two units will always be purchased in any equilibrium.

A feature of the equilibrium is the incentive of the buyer to split his order. This is captured by the following result.

**Lemma 6** The buyer prefers to buy one unit from each seller as opposed to buying two units from the lowest priced seller if the difference in prices is less than $\delta V_3$.

This is an important result. It says that a buyer prefers to split his order if the discounted price differential is lower than the discounted price of a third unit when facing a monopolist. The price of a third unit when splitting an order is zero, while if the buyer does not split an order it is $V_3$. This value is the expected discounted payoff to a seller of not selling a unit in period 1, which makes sense since the third unit will always be bought by the buyer so there is no efficiency loss.

The next proposition demonstrates that there is no pure strategy equilibrium (symmetric or asymmetric) in the entire game.

**Proposition 7** There is no pure strategy equilibrium in the monopsony model.
Proof. See Appendix A3. ■

The result of no pure strategy equilibrium is due to two phenomena. First, as depicted in Lemma 6, the buyer’s incentive to split his orders if the prices are close, within $\delta V_3$. This gives the sellers incentives to raise price. On the other hand, if prices get “high,” then sellers have incentives to drop their prices, and sell two units immediately. This cycling feature is common in games with capacity constraints.

Thus far, we have proved there is no pure-strategy equilibrium and the buyer always buys two units in period 1. Now we further characterize the (mixed-strategy) symmetric equilibria of the game.\(^{18}\) First, we prove in Appendix A4 that the sellers’ price distributions must be sufficiently wide, so that the buyer will accept either 0, 1, or 2 units from a particular buyer. Figure 3 illustrates what can happen with the equilibrium prices distributed on some the interval $[\bar{p}, \bar{p}]$. If a seller sets a price between $p$ and $p + \delta V_3$, he will sell either 1 or 2 units; the other buyer will never undercut his price by more than $\delta V_3$ so the seller will always sell at least one unit and if the other seller’s price is greater than his price by more than $\delta V_3$ he will sell two units. If the seller sets a price between $p + \delta V_3$ and $\bar{p} - \delta V_3$ he will sell either 0, 1 or 2 units. If the prices are within $\delta V_3$ of each other, the buyer will want to split his order between the sellers (Lemma 4). Otherwise, the buyer will buy two units from the low priced seller. Finally, for prices between $\bar{p} - \delta V_3$ and $\bar{p}$, the seller can never sell two units, since the other seller’s price will never be more than $\delta V_3$ above his price. He will sell 1 unit if the prices are within $\delta V_3$, and 0 units otherwise. Since we show that $\bar{p} - p \geq 2\delta V_3$ we establish the following important property.

Remark 1 In the monopsony model, splitting of orders by the buyer between the two sellers occurs in equilibrium with positive probability: if the difference if the two prices is smaller than $\delta V_3$, the buyer buys one unit from each seller.

In Appendix A4, we also prove that the lowest price, $\bar{p}$, offered by the sellers in a mixed strategy equilibrium of the monopsony model is greater than $\delta V_3$. It immediately then follows that:

\(^{18}\)Given that we have well-defined payoffs in each of the period-two subgames, we can guarantee existence of a (symmetric) mixed strategy Nash equilibrium in period-one prices and, consequently, existence of a subgame-perfect equilibrium in the entire game. We can use Theorem 5 in Dasgupta and Maskin (1986a): in the first period, discontinuities in each firm’s profit function occur only at a small number of discrete own prices of each firm (even though these do not have to be only points where both firms set equal prices). Individual profit functions are bounded and weakly lower semicontinuous in own prices and the sum of the profits is upper semicontinuous. Further application of Theorem 6 establishes symmetry. Section 2 of Dasgupta and Maskin (1986b) illustrates how existence arguments can be applied in capacity constrained price competition.
Figure 3: Period 1 acceptances: each area (x,y) indicates the price realizations for which the buyer buys x units from firm A and y units from firm B.

**Proposition 8** In the monopsony model the expected profit of each seller is greater than $\delta V_3$.

Thus, in equilibrium, the sellers receive rents above satisfying the residual demand after the buyer bought the other seller’s capacity (or the static Bertrand competition), $\delta V_3$. Why is this the case? By Lemma 4a seller knows that if he makes no sales in period 1, his expected profit is $\delta V_3$. This gives a seller the incentive to raise his price above $\delta V_3$ to take a chance of no sells in period 1, since by Lemma 6 a seller knows that even if he has the highest price he will make a sell as long as the price difference is less than $\delta V_3$. Since there is no cost of increasing his price above $\delta V_3$ and a potential benefit, the seller can improve his payoff.

### 3.3 Equilibrium properties and analysis

As we saw above (Proposition 8), in the equilibrium of the monopsony model each seller’s profit exceeds $\delta V_3$. Note that our equilibrium is efficient and that the buyer’s payment is equal to the total profit of the two sellers. Therefore, in equilibrium the buyer obtains a gross value equal to $2V(1 + \delta) + \delta V_3$ and pays an amount that exceeds $2\delta V_3$. That the equilibrium expected profit is greater than $\delta V_3$ for each seller is an important property and we further discuss some of its implications in the following subsection. We illustrate three strategies that the buyer can use to reduce his expected payments and still preserve efficiency. First, the buyer benefits if he can
commit to make all his purchases at once, effectively making the game collapse into a one-shot interaction. This is equivalent to the buyer having an option to buy units in period 2 at the period 1 prices. Second, we show that the buyer has an incentive to commit to (myopic) period-by-period minimization of his purchase costs. This can be done by the buyer hiring an agent and requiring the agent to buy from the lowest priced seller. Government procurement often works this way. Third, we demonstrate that the buyer will benefit by merging with one of the sellers. These three observations help to demonstrate the fundamental force that drives the equilibrium: due to strategic considerations, the buyer does not always purchase from the lowest priced seller when he plans to make further purchases, giving sellers the incentive to raise their prices above the static equilibrium level.

Our first observation is:

**Corollary 9** *In the monopsony model, the buyer would like to commit to not buying any units in period 2.*

The idea is that the equilibrium profit level described in Proposition 8 is larger than in the static equilibrium (when the buyer commits to buying all goods in period 1). This is by the following argument. We found in Lemma 4 that the second-period expected equilibrium profit if no units are sold in period one is $V_3$. If all competition took place in one period, the sellers’ expected payoff would be $\delta V_3$, since the strategic situation would be exactly the same as the last period with all sellers having full capacity (and the buyer’s valuation for the third unit, as of period 1, equal to $\delta V_3$). Thus, each seller’s profit in the one-shot situation would be $\delta V_3$. Since the allocation is always efficient, lower seller profit implies higher buyer profit. Thus, the buyer’s surplus is higher if he can commit to only buying once.

The behavior described in Corollary 9 would require, of course, some vehicle of commitment that would make future purchases not possible. This is an interesting result and can be viewed as consistent with the practice of airliners placing a large order that often involves the option to purchase some planes in the future at the same price for firm orders placed now. Such behavior is sometimes attributed to economies of scale – our analysis shows that such behavior may emerge for reasons purely having to do with how sellers compete with one another. In particular, it is easy to see that a game where sellers set prices for both units purchased in periods 1 and 2 in period 1 is exactly the same as if the game was only played in a single period.  

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19 It is also easy to see that the buyer would be better off if he could commit to *reduce* his demand to only
Our second observation is:

**Corollary 10** The buyer would like to commit to myopic behavior and to make his purchases on the basis of static optimization in each period.

Suppose that the buyer could commit to behaving myopically (that is, to not behaving strategically across periods). In other words, while valuations are the same as assumed in the model, now the buyer does not recognize the link between the periods and views his purchases in each period as a separate problem. Thus, the buyer within each period purchases a unit from the seller that charges the lowest price (as long as this price is below his reservation price). There are two possible ways to generate a pure strategy equilibrium for this model. First, in equilibrium each seller charges \( \frac{\delta V_3}{2} \) in the first period and the buyer purchases two units from one or the other seller. Then, the seller that has not sold his two units in the first period, charges a price of \( V_3 \) in the second period and the buyer purchases one unit from that seller. Thus, total payment for the buyer is \( 2\delta V_3 \). To establish that this is an equilibrium, first note that the buyer indeed behaves optimally, on a period by period basis. Second, neither seller has a profitable deviation. In period 1, if a seller lowers his price below \( \frac{\delta V_3}{2} \), he then sells both units and obtains a lower profit. If he raises his price, he sells no units in the first period but obtains a profit equal to \( V_3 \) in the second.

The possibility that the buyer may split his order (he is indifferent, given the myopia assumption, between splitting his order and not splitting) may be viewed as a weakness of the equilibrium described just above. This can be easily addressed, if we introduce a smallest unit of account, \( \Delta \). The equilibrium has one seller charging \( \frac{\delta V_3}{2} - \Delta \) and the other seller charging \( \frac{\delta V_3}{2} \) in the first period and the buyer buying two units from the low priced seller. The seller that made no sales in the first period, charges \( V_3 \) in the second period and the buyer purchases one unit from that seller. Thus, total payment in present value terms for the buyer is \( 2\delta V_3 - 2\Delta \). Clearly, the equilibrium

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\( \Delta \) to establish that this is an equilibrium, first note that the buyer behaves optimally. Second, neither seller has a profitable deviation. Clearly, no seller can gain from lowering his price If the low priced seller raises his price to \( \frac{\delta V_3}{2} \), the equilibrium can have the buyer splitting his order (as the prices would be equal) and lowering this seller’s profit. Thus, there are no profitable deviations.
payoffs are essentially the same under both approaches.

What drives this result is that now a seller knows that if he sets a higher price than his rival he cannot sell a unit in period one (and can only obtain a second period profit of $V_3$). The above comparison may provide a rationale for purchasing policies that large buyers have in place that require purchasing at each situation strictly from the lowest priced seller. In particular a government may often assume the role of such a large buyer. It is often observed that, even when faced with scenarios like the one examined here, governments require that purchasing agents absolutely buy from the low-priced supplier, with no attention paid to the future implications of these purchasing decisions. While there may be other reasons for such a commitment policy (such as preventing corruption and bribes for government agents), our analysis suggests that by “tying its hands” and committing to purchase from the seller that sets the lowest current price, the government manages to obtain a lower purchasing cost across the entire purchasing horizon. We find, in other words, that delegation to such a purchasing agent that maximizes in a myopic way is beneficial, since it ends up intensifying competition among sellers.\footnote{Strategic delegation has been also shown to be (unilaterally) beneficial by providing commitment to some modified market behavior in other settings (see e.g. Fershtman and Judd, 1987, and Vickers, 1985). In our case, the key is the separation from the subsequent period and the commitment to myopia.}

A further implication of Proposition 8 is:

**Corollary 11** In the monopsony model with linear prices, the buyer has a strict incentive to buy one of the sellers, that is, to become vertically integrated.

This result is based on the following calculations. By vertically integrating, and paying the equilibrium profit of a seller when there is no integration, $\pi$, the total price that the buyer will pay is $\pi + \delta V_3$ since the other buyer would change the monopoly price $V_3$ for a third unit (sold in period 2). This total payment is strictly less than the total expected payment $(2\pi)$ that he would otherwise make in equilibrium. Thus, even though the other seller will be a monopolist, the buyer’s payments are lower, since the seller that has not participated in the vertical integration now has lower profits. We note here that the seller who had the lowest prices could not profitably buy the other seller’s capacity to sell a third unit to the buyer. Since the equilibrium price is greater than $\delta V_3$, the buyer will only buy two units. More generally, a seller who is a potential seller of capacity
to the other seller will only sell a unit at the expected profit of the unit. Since this is equal to the buyer’s valuation of the unit, there are no gains of trade between sellers.\footnote{We note here that the seller who had the lowest prices could not profitably buy the other seller’s capacity to sell a third unit to the buyer. Since the equilibrium price is greater than \(\delta V_3\), the buyer will only buy two units. More generally, a seller who is a potential seller of capacity to the other seller will only sell a unit at the expected profit of the unit. Since this is equal to the buyer’s valuation of the unit, there are no gains of trade between sellers.}

### 3.4 Initial capacity choices by sellers

Thus far, we have conducted the analysis in this assuming that each seller has 2 units of initial capacity. We now argue that these capacities are indeed the ones chosen in equilibrium by each and incumbent and there is no entry.

If the incumbents have a total of less than four units of capacity, then an entrant can profitably enter, since if there are four or fewer units of capacity among three sellers, the price in period 1 will always be strictly above \(0\); if each seller had only one unit of capacity, they would never charge a price less than \(\delta V_3\) in period 1, while if one seller had two units of capacity he will never charge a price less than \(\delta V_3/2\). This creates a positive profit opportunity for an entrant. Entry to increase industry capacity will lower the profit of an incumbent who chose a capacity of one. A market configuration of either four sellers each having one unit of capacity or one seller having two units of capacity and two sellers each having one unit of capacity results in lower prices than when two sellers each have two units of capacity, since sellers with lower levels of capacity price more aggressively as demonstrated in Lemma 2.

On the other hand, if each incumbent chooses two units of capacity an entrant cannot profitably enter the market. This is because of the following argument. First, it is straightforward to demonstrate in any subgame in period 2 the price will equal \(0\). Given this fact, a firm knows it can make no profits from sales in period 2. Going back to period 1, the equilibrium has all sellers charging price \(0\), where no seller can profitably deviate by charging a higher price; all the buyer demand for three units can be satisfied by the other two sellers.

Finally, there is no possible way for a seller to improve their profit by increasing their capacity above 2 units. The price for a unit in period 2 is \(0\), unless the buyer bought two units from a seller with capacity 2. But, this means that the seller with a capacity of greater than 2 units will have an equilibrium profit of \(\delta V_3\) less his capacity cost. If the seller with high capacity sold all three units in period 1, then it must be at a price of \(0\), since this would be price in period 2 when both seller
have a capacity of at least 1. Thus, the seller’s profit falls by adding more capacity than 2 and we have the following

**Proposition 12** in the monopsony model, the equilibrium capacity choice by each incumbent is two units and there is no entry.

4 Duopsony

Now, we study strategic issues raised when there are two buyers. Each buyer has the same demand as in the monopsony case, and each seller has now has a capacity of five units. We will show that the same economic phenomena occurs in duopsony as in monopsony and demonstrate that it can be generalized for any number of buyers.

4.1 Second period

We first consider equilibrium behavior in the possible period-two subgames. Mixed strategy equilibria arise unless either both sellers can cover the market or one seller is a monopolist. The construction of equilibria is similar to that for the monopsony case and based on Lemma 3. The key results from the period 2 analysis, required for our subsequent analysis of period 1, are summarized in the following Lemma.

**Lemma 13** In the second period of a duopsony (i) the expected payoff for a seller with full capacity is $V_3$; (ii) If one seller sold 4 units and the other seller sold no units, then the expected price is $EP_2 > V_3/2$; (iii) If each seller has enough capacity to satisfy the market demand, then each seller’s profit is 0.

Part (ii) of the Lemma follows because, if demand is for two units the high capacity seller, and subsequently the low capacity seller, will never charge less than $V_3/2$ in the mixed strategy equilibrium.

4.2 First period

Now, we go to period 1. As in the monopsony case, there will be no pure strategy equilibrium. We will now show that the expected profit for each seller is again strictly greater than $\delta V_3$, the profit in the one shot game, and that the buyers split their orders between sellers with positive probability.
As in the monopsony model, a buyer will always buy at least two units. That is, the sellers will always choose prices such that buyers prefer to buy in period 1 than to wait till period 2 for the first two units. Each seller has five units of capacity at the beginning of period 1.

Suppose that the prices offered by sellers are $P_L$ and $P_H$, where $P_L \leq P_H$. For now, assume that a buyer will buy only two units in period 1. We will later examine the case when a buyer may buy three units in period 1. Let $\alpha$ be the probability in the symmetric equilibrium that a buyer will buy two units from the low priced seller and $(1 - \alpha)$ the probability that he will split his orders between the two sellers. It is never the case that a buyer will buy two units from the high priced seller, since if a buyer splits his order between the two sellers, he will guarantee a price of 0 in period 2 for a third unit, since each seller will have at least two units of capacity and market demand is two. A buyer is indifferent between buying two units from the low priced seller and splitting her order if

$$-2P_L - \alpha \delta EP_2 = -P_L - P_H$$

where by Lemma 13 $EP_2 > V_3/2$ is the expected price that the buyer will pay in period 2 if one seller sells 4 units in period 1 and the other sellers sell 0. Solving for $\alpha$ we have

$$\alpha = \frac{P_H - P_L}{\delta EP_2}$$

Note that if prices are equal, the buyer always splits his order and if the price difference is greater than $\delta EP_2$, then the buyer always buys both units from the low priced seller. This is because the value of splitting an order to obtain the unit in period 2 for a price of 0 instead of $EP_2$ is not worth the added cost of buying a unit from the high priced seller.

The profit of a seller if he charges a price lower than the other seller is

$$\pi = \alpha^2 [4P + \delta V_3/2] + 2\alpha(1 - \alpha) [3P] + (1 - \alpha)^2 [2P]$$

The first term is his profit if both buyers buy 2 units from him plus his expected profit in period 2, $\delta V_3/2$. The second term is his profit if one buyer buys two units from him and the other buyer buys one unit. This occurs with probability $2\alpha(1 - \alpha)$ and by Lemma 13 the seller makes no profit in period 2. The final term is the seller’s profit if both buyers buy one unit from each seller. The seller’s profit is zero in period 2 in this case. Equation (1) simplifies to

$$\pi = \alpha^2 \delta V_3/2 + 2\alpha P + 2P$$

Differentiating (2) with respect to $P$ we have

$$\frac{\partial \pi}{\partial P} = 2 \left[1 - \frac{P}{\delta EP_2}\right] + \alpha \left[2 - \frac{\delta V_3}{\delta EP_2}\right]$$
The important thing to note is that this profit is strictly increasing in $P$ for any $P$ less than $\delta EP_2$, since $\delta EP_2 > \delta V_3/2$.

The profit of a seller if he is the high priced seller is

$$\pi = \alpha^2 \delta V_3 + 2\alpha (1 - \alpha) [P] + (1 - \alpha)^2 [2P]$$

Equation (3) simplifies to

$$\pi = \alpha^2 \delta V_3 - 2\alpha P + 2P$$

Differentiating (4) with respect to $P$ we have

$$\frac{\partial \pi}{\partial P} = 2 \left[ 1 - \alpha - \frac{P}{\delta EP_2} + \alpha \frac{\delta V_3}{\delta EP_2} \right]$$

The important thing to note is that there exists a $P_0$ greater than $\delta V_3$ such that profit is strictly increasing in $P$ if $P < P_0$.

Thus, we have just shown that a seller’s profit is always increasing, whether he is the low or priced seller, for any price less than a price greater than $\delta V_3/2$. Since the low priced seller has expected sales of greater than 2 units, a seller’s expected profit is greater than $\delta V_3$, which is the single shot payoff.

Finally, we need to argue if the low priced seller charges a price less than $\delta V_3/2$ that he would not want to have his fifth unit of capacity purchased in period 1 and would improve his payoff by raising his price. This is the case, since his payoff from not having it accepted, $\delta EP_2$ is greater than prices less than $\delta V_3/2$. Thus, we have the following result

**Proposition 14** In the duopsony model, sellers’ expected payoffs are greater than in the static model, and buyers split their orders across sellers with positive probability.

As in the monopsony model, the following two corollaries follow directly from the proposition.

**Corollary 15** The buyers have a strict incentive to commit not to buy in the future or to have a policy to only buying from the lowest price seller.
This follows from the result that in the static model, each seller’s expected equilibrium profit is $\delta V_3$, see Lemma 3.

**Corollary 16** A buyer has a strict incentive to vertically integrate with a seller.

Suppose that a buyer unilaterally buys a seller. This increases his expected profit because a buyer can buy a seller, by paying the equilibrium profit. This is the buyer’s expected cost. He can then satisfy his demand for 3 units and have two extra units of supply and obtain an additional positive profit by selling to the other buyer.

As with the monopsony case, there will be no entry in the game and each incumbent will choose five units of capacity. To see this, note that if the total incumbent capacity is less than ten units, then an entrant can profitably enter the market. Furthermore, it is straightforward to show that if an incumbent has more than five units that additional units will either go unsold or be sold for a price of 0. Finally, no entrant can profitably enter the market if each incumbent has 5 units, since the period 1 subgame has each firm charging 0.

It is interesting to note that the equilibrium market shares of the firms may be quite asymmetric both in terms of revenue and profits: one seller sells four units in period 1 and possibly one in period 2 for an expected profit greater than $2.5\delta V_3$, while the other sells at most two units in period 2 with an expected profit of $\delta V_3$. This result, is complementary to other studies of asymmetries in the literature - see e.g. Saloner (1987), Gabszewicz and Poddar (1997) and Besanko and Doraszelski (2004). In these studies, capacity asymmetries are typically due to the selling firms’ incentives to invest strategically in anticipation of a market competition stage. In contrast, in our analysis, the asymmetries are due to the fact that the buyers are large players and choose strategically which firms they should purchase from - the fact that they cannot fully coordinate their behavior in equilibrium, but need to mix, gives rise to asymmetric sellers’ market shares.

### 4.3 $N$ buyers

Our analysis can be applied to the case when there are $N$ buyers, each with the same demand as in the preceding analysis. Each seller now has a capacity of $3N – 1$ units. Similar arguments as the ones used in the monopsony and duopsony models can be used to establish that the sellers will each select this capacity level to prevent entry.

It can also be shown that there will be a mixed strategy equilibrium where each seller’s expected
profit is greater than $\delta V_3$; as before, a seller can always guarantee a payoff of $\delta V_3$ if he makes no sales in period 1 and thus this makes the sellers less aggressive in period 1 and leads to higher equilibrium prices. Again, the buyers will have an incentive to commit not buying in the future and to have a policy to always buy from the lowest priced supplier. In the one period pricing game each seller makes an expected profit of $\delta V_3$, their security profit, and the supports of the pricing distribution are $\left[ \frac{\delta V_3}{3N-1}, \delta V_3 \right]$, where the lowest price seller sells $3N - 1$ units and the high price seller sells a single unit.

Note that equilibrium is efficient and that each of the $N$ buyers will have a gross surplus equal to $2V(1 + \delta) + \delta V_3$ independent of $N$ and a net payment of $2\delta V_3/N$. Thus the net surplus for each buyer is strictly increasing with $N$. The reason why buyers benefit by having more buyers in the market is that sellers are more aggressive since if they are the low priced seller they sell more units and obtain higher profits.

5 Conclusion

Capacity constraints play an important role in oligopolistic competition. In this paper, we have examined markets where both sellers and buyers act strategically. Sellers have intertemporal capacity constraints, as well as the power to set prices. Buyers decide which sellers to buy from, taking into consideration that their current purchasing decisions affect the intensity of sellers’ competition in the future. Capacity constraints imply that a pure strategy equilibrium fails to exist. Instead, sellers play a mixed strategy with respect to their pricing, and the buyer may split his orders. Importantly, we find that the sellers enjoy higher profits than what they would have in an one-shot interaction (or, equivalently, the competitive profit from satisfying residual demand). The buyers are hurt, in equilibrium, by their ability to behave strategically over the two periods, since this behavior allows the sellers to increase their prices above their rival’s and still sell their products. Thus, the buyers have a strict incentive to commit not to buy in the future, or to commit to myopic, period-by-period, maximization (perhaps by delegating purchasing decisions to agents), as well as to vertically integrate with one of the sellers. We find that, since there has to be some buyers’ miscoordination, the equilibrium under duopsony implies asymmetric market shares for the sellers.

While we have tried to keep the model as simple as possible, our qualitative results appear robust to modified formulations. Perhaps the most important ones refer to how the capacity constraints function. In the model, if a seller sells one unit today, his available capacity decreases tomorrow by
exactly one unit. In some of the cases for which our analysis is relevant, like the ones mentioned in
the Introduction, it may be that the capacity decreases by less than one unit, in particular, if we
adopt the view that each unit takes time to build and, thus, occupies the firm’s production capacity
for a certain time interval. Similarly, instead of the unit cost jumping to infinity once capacity is
reached, in some cases it may be that the unit cost increases in a smoother way: cost curves that
are convex enough function in a way similar to capacity constraints. We believe the spirit of our
main results is valid under such modifications, as long as the crucial property that by purchasing
a unit from a seller you decrease this seller’s ability to supply in the subsequent periods holds.

This is, to our knowledge, the first paper that considers capacity constraints and buyers’ strate-
gic behavior in a dynamic setting. A number of extensions are open for future work. While non
trivial, these present theoretical interest and, at the same time, may make the analysis more directly
relevant for certain markets. First, one may wish to examine the case where the products offered
by the two sellers are differentiated. Is there a distortion because buyers strategically purchase
products different from their most preferred ones, simply with the purpose of intensifying com-
petition in the future? A second interesting extension is when the sellers have asymmetric initial
capacities. Which seller sells faster? Do buyers have an incentive to favor a seller with a larger or
with a smaller remaining capacity? Is competition more intense when capacities are more or less
symmetric? Finally, in our model, price determination takes the form that sellers set prices in each
period. Alternative formulations are also possible. For instance, sellers may be able to make their
prices dependent on the buyers’ purchasing behavior e.g. by offering a lower price to a buyer that
has not purchased in the past - or does not currently purchase - a unit from the rival seller. Our
setting may allow us to examine such “loyalty discount” discriminatory schemes.

**Nonlinear pricing.** Our analysis under monopsony illustrates how a buyer’s strategic behav-
ior modifies the pricing incentives for the sellers, allowing them to charge in equilibrium higher
prices than otherwise. Before turning to the issue of coordination among buyers, it may be useful
to conclude this analysis with a remark that highlights some aspects of the equilibrium pricing
incentives. In our analysis, each seller sets in each period a single price per unit, that is, pricing
is linear. It should not be too surprising that the application of nonlinear pricing would lead to
different results. This case would be relevant when a seller can price the sale of one unit separately
from the sale of two units. We sketch the analysis for such a case in Appendix A5 and we summarize
here as follows.\(^{23}\)

\(^{23}\)Complete details are available in the discussion paper version of this paper, Biglaiser and Vettas (2004).
Remark 2 With a monopsonist under non-linear pricing, there are unique pure strategy equilibrium payoffs with each seller making profit equal to $\delta V_3$. In period 1, both sellers charge $\delta V_3$ for both a single unit and two units and the buyer buys either two or three units.

In this case, the ability of each seller to price each of his units separately allows us to derive an equilibrium where the sellers make no positive rents: their equilibrium payoffs are equal to the profit from satisfying the residual demand, after the buyer bought the other seller’s capacity. Further, the buyer has no incentive to commit to not making purchases in period 2, and to hire an agent to commit to buy only a single unit, and has no incentive to vertically integrate by buying one of the sellers. The reason for these results in the monopsony case is that with non-linear pricing the sellers can price “as if” all purchases are done in a single period. This makes competition more stiff and results in a better outcome from the buyer’s point of view. The equilibrium involves a two-part tariff, with the fixed fee equal to a seller’s discounted monopoly profit in the next period, and all units are priced at marginal cost, which we have normalized to 0. This equilibrium was not possible with linear prices, because each seller would have an incentive to raise his price to induce the buyer to split his order. Obviously, since under nonlinear pricing sellers obtain lower equilibrium profit than under linear pricing; if they could have a choice they would tend to prefer a linear pricing regime.

Appendix


First, we argue that the players choose prices in the interval $[V_3^2, V_3]$. Suppose that seller 2, asked a price $p$ less than $V_3/2$. If seller 1 charges a price less than $p$, then seller 2 will sell 1 unit, while if seller 1 charges a price higher than $p$, seller 2 sells 2 units. Seller 2 could improve his payoff no matter what prices seller 1 asks by asking for $V_3 - \epsilon$ for $\epsilon$ very small and selling at least one unit for sure, since $V_3 - \epsilon > 2p$. Since seller 2 will charge a price of at least $V_3/2$, then so will seller 1; otherwise, seller 1 could increase his price and still guarantee a sell of 1 unit. Thus, both sellers charge at least $V_3/2$. Now, we argue that price will be no more than $V_3$. Take the highest price $p$ offered in equilibrium greater than $V_3$. First, assume that there is not a mass point by both sellers at this price. This offer will never be accepted by the buyer, since he will always buy the second unit from the lower priced seller and his valuation for a third unit is $V_3 < p$. The seller could always improve his payoff by charging a positive price less than $V_3/2$. Second, if there is a mass point by both sellers, then at least one of them is rationed with positive probability and a seller can slightly
undercut his price and improve his payoff. Thus, all prices will be between $V_3/2$ and $V_3$.

Now, we argue that the expected equilibrium period 2 payoffs are $V_3/2$ for seller 1 and $V_3$ for seller 2. Given that the equilibrium prices are between $V_3/2$ and $V_3$, we know that the profits for seller 1 is at least $V_3/2$ and for seller 2 at least $V_3$. First, we argue that it can never be the case that both sellers will have an atom at the highest price $p_H$; later we further show that seller 2 will have a mass point at $p_H$. If both did, then there is a positive probability of a seller being rationed, and a seller could improve his payoff by slightly lowering his price. Thus, a seller asking $p_H$ knows that he will be the highest priced seller. If he is seller 1 he will not make a sell, while if he is seller 2 he will make a sell of one unit. If seller 2 charges $p_H$ he knows that his payoff will be $p_H$, thus $p_H$ must equal $V_3$. If the lowest price offered in equilibrium, $p_L$, were greater than $V_3/2$, then seller 2 could improve his payoff by offering $p_L - \epsilon > V_3/2$, with the buyer buying two units from the seller and thus improve his payoff above $V_3$. Thus, the lowest price is $V_3/2$. Since both sellers must offer this price, seller 1’s expected payoff must be $V_3/2$.

We now find the equilibrium price distributions. Let $F_i$ be the distribution of seller $i$’s price offers. Seller 1’s price distribution is then determined by indifference for seller 2:

$$p [F_1(p) + 2(1 - F_1(p))] = V_3, \quad (A1.1)$$

since seller 2’s expected payoff is $V_3$ by the earlier argument. Seller 2’s payoff is calculated as follows. When seller 2 charges price $p$, then with probability $F_1(p)$ seller 1’s price is lower and seller 2 sells one unit, while with probability $1 - F_1(p)$ seller 1’s price is higher and seller 2 sells both his units. Solving equation (A1.1), we obtain:

$$F_1(p) = 2 - \frac{V_3}{p}.$$  

Seller 2’s price distribution is a little more complicated. For $p < V_3$, it is determined by

$$p [1 - F_2(p)] = \frac{V_3}{2}. \quad (A1.2)$$

Seller 1 sells one unit if his price is lower than the rival’s and this happens with probability $1 - F_2(p)$; otherwise, he sells no units. This equals seller 1’s expected profit $V_3/2$ by Lemma 2. Condition (A1.2) implies

$$F_2(p) = 1 - \frac{V_3}{2p}.$$  

There is a mass of $1/2$ at price $V_3$. Simple arguments can be used to establish that the equilibrium pricing distributions must be continuous and that the only mass point may be located at $V_3$ for seller 2.

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Appendix A2: Proof of Proposition 1.

First, we define some notation and buyer payoffs. Then we proceed to prove the proposition in a series of Lemmas. Suppose that the prices in period 1 are \(p_H\) and \(p_L\), with \(p_H \geq p_L\). Note that pricing in period one could, in principle, be determined via either pure or mixed strategies. In the former case, \(p_H\) and \(p_L\) are the prices set by the two sellers, whereas in the latter these are realizations of the mixed strategies. We use Lemma 4 in computing the payoffs.

The buyer’s payoff if he buys one unit from each of the firms in period 1 is

\[ W_1 \equiv 2V(1 + \delta) - p_H - p_L + \delta V_3. \]

In this case, the buyer gets one unit for free in the following period (since competition drives the price to zero).

The buyer’s payoff if he buys both units from firm \(L\) is

\[ W_2 \equiv (1 + \delta)2V - 2p_L. \]

In this case, the buyer faces a monopolist and pays \(V_3\) in period 2.

The buyer’s expected payoff if he buys only one unit from firm \(L\) is

\[ W_3 \equiv (1 + \delta)V - p_L + \delta [V + V_3 - E\min[p_1, p_2] - Ep_2]. \]

In the following period, the buyer will buy two additional units. He will pay the lowest price offered in the following period for the second unit and will buy the third unit from the seller who has two units of capacity in period 2, since either he is the low priced seller or the other seller has no more capacity.

The buyer’s expected payoff if he buys two units from the lowest priced seller and one from the highest priced seller is

\[ W_4 \equiv 2V(1 + \delta) - 2p_L - p_H + \delta V_3. \]

Now, the series of Lemmas. These are numbered (A1-A5) independently from the Lemmas in the main body of the paper.

**Lemma A1** The sellers set strictly positive prices in period 1.

**Proof.** If a seller set a price of 0, then either the buyer would buy two units from that seller or one from each of the sellers. In either case, the seller makes 0 profit. If the buyer buys two units from that seller, then the seller can sell no more in period 2. If the seller sells one unit, then
the buyer must have bought one unit from the other seller, since \( W_1 > W_3 \) at a 0 price in period 1. The seller could raise his price so that he gets no sells in period 1, and improve his payoff by Lemma 2.

It follows directly that:

**Lemma A2** The buyer never buys three units in period 1.

**Proof.** By Lemma A1, both prices are positive. Since \( W_1 > W_4 \) if \( p_L > 0 \), then buying two units always dominates buying three units.

We now argue in the following three lemmas that no price will be above \( V + \delta [E \min[p_1, p_2] + Ep_2] \equiv p^C \) from which the Proposition will be proven.

**Lemma A3** The buyer prefers to buy one unit from each of the sellers instead of only one unit from the low price seller if \( p_H < p^C \). Thus the buyer will not buy any units from a seller charging \( p_H > p^C \), when \( p_H > p_L \).

**Proof.** Compare \( W_1 \) and \( W_3 \).

**Lemma A4** In any equilibrium, each price offered by each seller in period 1 is an offer which results in his selling at least one unit with positive probability.

**Proof.** Let \( \tilde{p} \) be the highest price offered in any (possibly mixed strategy) equilibrium by a seller. Suppose that in equilibrium \( \tilde{p} \) is never accepted. By Lemma 4 the seller’s expected payoff of making this offer is \( \delta V_3 \). Let \( \underline{p} \) be the lowest price offered in equilibrium by the other seller. By Lemmas 3 and A1 if the difference between the two prices set is less than \( \delta V_3 \), then the highest price will always be accepted, as long as it is less than \( p^C \). By Lemma 5, \( \underline{p} > 0 \). A player offering \( \underline{p} \) can defect and offer a price \( p \) that is the minimum of \( [\underline{p} + \delta V_3, p^C] \) and know that it will be accepted, and increase his payoff. Thus, no offer is made that is always rejected.

**Lemma A5** In any equilibrium, no seller will offer a price above \( p^C \).

**Proof.** Let \( \overline{p}_i \) be the highest price offered in any equilibrium by seller \( i \). Suppose that \( \overline{p}_i \geq \overline{p}_j \), \( i \neq j \), and \( \overline{p}_i > p^C \). If \( \overline{p}_i > \overline{p}_j \), then seller \( i \)'s offer will never be accepted by Lemma A3. Then seller \( i \)'s payoff is \( \delta V_3 \). Seller \( i \) can clearly improve his payoff by making an offer of \( \delta V_3 + \epsilon \) (note that \( \delta V_3 + \epsilon < p^C \)). Suppose now that \( \overline{p}_i = \overline{p}_j \equiv \overline{p} \). There could not be a mass point at \( \overline{p} \) by each seller, since only one unit will be bought and that seller could increase his payoff by a slight undercut in price. If there is no mass at \( \overline{p} \), then there is no possibility that the offer will be accepted. But, this contradicts Lemma A4.
Thus, we have proved Proposition 1.

**Appendix A3: Proof of Proposition 2.**

Suppose we have a pure strategy equilibrium with prices $p_H$ and $p_L$, where $p_H \geq p_L$. A pure strategy equilibrium could exist only if both sellers offered $p^C$ and the buyer was purchasing a unit from each seller. At any other price, at least one of the sellers could defect and improve their payoff. To see this, we need to look at various cases. First, suppose that the lower offer in equilibrium, $p_L$, is greater than $\delta V_3$. If $p_L > p_H - \delta V_3$, then the buyer will split his order by Lemma 6. Seller $L$ could improve his profit by increasing his offer. If $p_L < p_H - \delta V_3$, then the buyer will buy both units from seller $L$. Seller $H$ will have a payoff of $\delta V_3$. Seller $H$ can improve his payoff by making an offer that is accepted. If $p_L = p_H - \delta V_3$, then either seller $L$ is not selling 2 units or seller $H$ is not selling any units. One of the sellers has an incentive to defect. To see this, suppose that $\alpha \in [0,1]$ is the probability that the buyer splits his order between the sellers. Then the payoff to seller $L$ is $\pi_L = \alpha p_L + (1-\alpha)2p_L$. The payoff to seller $H$ is $\pi_H = \alpha p_H + (1-\alpha)\delta V_3$, which equals $\pi_H = \alpha p_L + \delta V_3$ by assumption that $p_L = p_H + \delta V_3$. But seller $H$'s payoff must be at least as large as $p_L + \delta V_3 - \epsilon$ for all positive $\epsilon$, since he could always guarantee an acceptance by dropping his price $\epsilon$. Thus, $\alpha$ would have to equal 1. But, if $\alpha = 1$, then seller $L$ could improve his payoff by raising his price.

Now, suppose that $p_L \leq \delta V_3$. If $p_L > p_H - \delta V_3$, then the buyer will split his order by Lemma 6. Seller $L$ could improve his profit by increasing his offer. If $p_L < p_H - \delta V_3$, then the buyer will buy both units from seller $L$. If $\frac{\delta V_3}{2} < p_L$, then seller $H$ can improve his payoff by making an offer that is accepted. If $p_L < \frac{\delta V_3}{2}$, then seller $L$ could raise his offer to $p_H$ the buyer will split his order and the low seller’s profit increases. As before, if $p_L = p_H - \delta V_3$, then one of the sellers could do better by defecting.

An equilibrium with both sellers offering $p^C$ could arise only if $p^C \geq 2(p^C - \delta V_3)$ or if $2\delta V_3 \geq p^C$. This is because a defection by a seller that gets the buyer to buy two units from the seller will reduce his profits. Otherwise a seller would defect. This is equivalent to $2\delta V_3 \geq V_2 + \delta [E \min[p_1, p_2] + E p_2]$. But, this condition never can hold, since both $p_1$ and $p_2$ are greater than $V_3/2$.

**Appendix A4: Proof of Proposition 3.**

We know that $p - p \geq \delta V_3$; otherwise a seller could increase his payoff by moving mass from lower parts of the price distribution to higher parts and still get accepted. Suppose that $p^C < 2\delta V_3$ and for now assume that the equilibrium price distribution is continuous. Define three regions...
as follows: region 1 where \( p \in [p, \bar{p} - \delta V_3] \), region 2 where \( p \in [\bar{p} - \delta V_3, \bar{p} + \delta V_3] \) and region 3 where \( p \in [\bar{p} + \delta V_3, \bar{p}] \). A price offered in region 1 will be accepted for either 1 or for 2 units. A price in region 2 will always be accepted for 1 unit. A price in region 3 will be accepted either for a single unit or no units. But, if there is an offer in region 2, then a seller can always improve his payoff by moving all the probability mass in region 2 to a price of \( \bar{p} + \delta V_3 \). Thus, there would be a gap in the price offer distribution.

Suppose that there was a gap in the price offer distribution in region 2. Then prices offered in region 1 would all be moved to the top of region 1 at a price of \( \bar{p} - \delta V_3 \), since whether the offer is accepted either once or twice is independent of the price in region 1. But, if sellers move up all their mass to \( \bar{p} + \delta V_3 \), then the price distribution would only be \( \delta V_3 \), but then any price in the interior distribution is inferior to either a price at the bottom or the top of the distribution. Thus, we would have a two-point distribution. But this cannot be an equilibrium. Suppose one player made an offer of \( \bar{p} \) and the other at \( \bar{p} \). Then either the buyer accepts 2 units at the low price or splits his order. In the former case, the high bidder could increase his payoff by reducing his offer slightly, while in the latter case the low bidder could increase his payoff by a bid reduction.

Let \( \pi \) be the equilibrium payoff. The equilibrium pricing equations are as follows. If \( p < \bar{p} + \delta V_3 \),

\[
p[2 - F(p + \delta V_3)] = \pi. \tag{A4.1}
\]

If \( \bar{p} + \delta V_3 < p < \bar{p} - \delta V_3 \),

\[
dV_3 F(p - \delta V_3) + P[2 - F(p + \delta V_3) - F(p - \delta V_3)] = \pi. \tag{A4.2}
\]

If \( p > \bar{p} - \delta V_3 \),

\[
dV_3 F(p - \delta V_3) + P(1 - F(p - \delta V_3)) = \pi. \tag{A4.3}
\]

Some further important facts about the equilibrium follow.

Substituting \( p = \bar{p} \) and \( p = \bar{p} + \delta V_3 \) into (A4.1), setting the two resulting values equal to each and manipulating the equation, we obtain

\[
dV_3 [2 - F(\bar{p} + 2\delta V_3)] = \bar{p} [F(p + 2\delta V_3) - F(\bar{p} + \delta V_3)].
\]

Since \( 2 - F(p + 2\delta V_3) \geq 1 \) and \( F(p + 2\delta V_3) - F(\bar{p} + \delta V_3) < 1 \), it must be the case that \( p > \delta V_3 \). This completes the proof.

Appendix A5: Monopsony with non-linear pricing.
We sketch here the analysis in the monopsony case under the assumption that each of the sellers can offer a menu of prices in each period; a price if it sells one unit and a price if it sells two units. The details of the analysis are available in the accompanying discussion paper, Biglaiser and Vettas (2004).

First, we have to consider equilibrium in the various period 2 subgames. Second period competition in the case of a monopsonist under non-linear pricing falls into one of three categories. If only one seller is active (the rival has zero remaining capacity), that seller sets the monopoly price, $V_3$, and extracts the entire buyer’s surplus. If both sellers have enough capacity to cover the buyer’s demand, there is (Bertrand) pricing at zero. Finally, if the buyer’s demand exceeds the capacity of one seller but not the aggregate sellers’ capacity then there is a pure strategy equilibrium: a seller with two units charges $V_3$ for two units (and a price at least as high for one unit) and a seller with one unit charges 0. It follows from these three cases that a seller gives up second-period profit $V_3$ when, by selling one (or two) units in period 1, his remaining capacity drops from 2 units to 1 (or 0, respectively). Clearly, he would demand at least the discounted present value of that amount to sell one unit in period one.

Now we turn to period 1. It can be shown that there is a unique pure strategy equilibrium price paid for the goods. The equilibrium prices are for each seller to charge $\delta V_3$ for both a single unit and two units. The period 1 actions are that the buyer either buys one good from each seller, two units from one of the sellers and none from the other, or two units from one seller and one unit from the other. The total amount paid by the buyer and the revenue that each seller receives is the same, no matter which of the three actions the buyer takes in period 1.

From the above it follows that in the monopsonist case under non-linear pricing the equilibrium payoff of each seller over the entire game is equal to $\delta V_3$.

References


9, 60-79.


