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Effective monetary policy rules under recessionary conditions

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Abstract

This paper reveals that the central banks of three world leading economies, the US, UK and Japan are all characterized by an asymmetric monetary policy behavior. This is mainly antinflationary in the expansion regime and anticyclical in the recession. The paper shows that the level of output gap deviations above, or below, which regime-switching in the stance of monetary policy of the above three countries occurs is different than zero, often assumed in practice. For the US and UK, this is found to be negative and far away from zero, which means that the CBs of these two countries tend to switch their policy to an anticyclical one when the economy is under quite severe recessionary conditions. For Japan, this happens before the recessionary conditions become so apparent to the economy. To examine the effectiveness of the above monetary policy rule to dampen economic fluctuations, the paper simulates a New Keynesian model. This exercise clearly indicates that this monetary policy rule is more efficient in preventing deep and prolonged recessions in the economy than a policy which is passive in the recession regime, or it is entirely antinflationary.

JEL Classification: E52, C13, C30

Keywords: Threshold monetary policy rules, regime-switching, recessionary conditions, New-Keynesian models

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1. Introduction

There is recently growing research interest in estimating non-linear monetary policy rule models allowing for regime-switching under different business cycle conditions, i.e. between the recession and expansion regimes of the economy (see, e.g., Bec *et al.* (2002), Dolado *et al* (2002), Davig and Leeper (2007, 2008) and Kazanas *et al.* (2011)). Monetary policy rules allowing for regime-switching may be proved beneficiary for the monetary authorities due to the regime-switching expectation formation effects, which can improve the efficiency of monetary policy (see, e.g., Davig and Leeper (2008), and Liu, Waggoner and Zha (2009). Most of the above studies reveal that the behavior of the central bank (CB) of many developed countries, including the US tends to follow only antinflationary policies, especially in the expansion regime. For the recession regime, they show that central banks (CBs) tend to follow a passive interest rate policy with respect to negative output gap deviations. These results are quite puzzling, given that the recent financial crisis started in year 2008 have initiated substantial drops in the short-term lending interest rates of the most developed countries CBs to sustain economic growth.

In this paper, we re-examine the above evidence including also data from the recent financial crisis period. This is done based on a threshold forward-looking monetary policy rule model, which employs output gap deviations from their target level as a state variable to capture the different economic regimes of the economy (see, e.g., Stock and Watson (2003)).[1] The data used in our analysis comes from three world leading economies, the US, UK and Japan (JP), which have received a strong degree of independence since year 1979 and have officially announced their strong commitment mainly on inflation targeting. In contrast to backward, forward looking monetary policy rule models assume that changes in the CBs interest rate reflect expectations about future levels of inflation and real output deviations from their target levels, and they are considered as optimal policy models in the literature (see Svensson (2003) and Nobay and Peel (2003)). The threshold specification of the model enables us to capture regime shifts in its structural parameters which have

[1] Threshold models of monetary policy rules have been also used in many recent studies (see, e.g., Gredig (2007), Kim *et al.* (2005), Taylor and Davradakis (2006)). These models can allow for a convex relationship between inflation and unemployment, or output, rates (see Dolado *et al*. (2000)).

economic policy interest. These shifts can be directly linked to changes in business cycle conditions.

The econometric method that the paper considers to estimate the suggested monetary policy rule model has been recently suggested by Kourtelos *et al*. (2011). This method has the following two interesting properties. First, it counts for any endogeneity effects of the threshold variable on parameter estimates. Ignoring them can lead to wrong inference about the actual behavior of monetary authorities. Second, it assumes that the threshold parameter of the model is unknown and, thus, it is estimated by the data. A sample estimate of this parameter can highlight how strong recessionary conditions should be in order for CBs to change their stance of policy. Furthermore, estimating the threshold parameter from the data rather than treating it as known (e.g., zero), as often assumed in practice (see, e.g., Bec *et al.* (2002)) and Surico (2003)), will lead to more accurate estimates of the parameters of a threshold monetary policy rule model, if the true value of the threshold parameter is different than zero.

The results of the empirical analysis of the paper lead to a number of interesting conclusions. First, they clearly show that the CBs of all three countries examined are characterized by an asymmetric monetary policy behavior. This is mainly antinflationary in the expansion regime and anticyclical in the recession. For the US and UK, the anticyclical stance of the CB in the recession regime becomes more apparent when including data from the recent financial crisis. For Japan, this policy seems that characterizes the behavior of its CB over the whole sample. This can be obviously attributed to the persistent recessionary conditions which have been holding for this country since the nineties.

A second conclusion that can be drawn from our empirical analysis is that the value of the threshold parameter above, or below, which regime-switching in the stance of monetary policy occurs is clearly different than zero. This is true for all three countries examined. For the US and UK, this is found to be negative and far away from zero, which means that the CBs of these two countries tend to switch their policy to an anticyclical one if the economy is under quite severe recessionary conditions. On the other hand, for Japan, this switching happens before the economy enters into the recession regime. This can be taken as evidence that the monetary authorities of this country are characterized by a more proactive attitude against recessionary conditions.

To assess the policy implications of the above results on the effectiveness of monetary policy to dampen economic fluctuations under different business conditions, the paper has simulated a small-scale New Keynesian macroeconomic model. The results of this exercise clearly indicate that a strong anticyclical monetary policy during recessions is necessary to prevent severe and persistent drops in output, compared to a passive policy abandoning the monetary rule in the recession regime, or focusing only on inflation targeting. This result means that the reaction of the CBs of the three countries examined to reduce substantially lending rate i_t in response to the last financial crisis has prevented the economies of these countries from severe recessionary conditions.

The paper is organized as follows. Section 2 presents the threshold monetary policy rule model suggested by the paper. Section 3 describes the sources of our data and estimates the model. Section 4 conducts the simulation exercise of the New Keynesian model based on the estimates of the model, presented in Section 3. Section 5 concludes the paper.

2. A forward looking threshold model of monetary policy rule

Consider the following threshold forward looking monetary policy rule:

$$i_t = \begin{cases} (1-\rho_1)\left(\alpha_1 + \beta_1\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma_1 E_t(\tilde{y}_{t+k})\right) + \rho_1 i_{t-1} + \varepsilon_t, & \text{if } q_t \le \overline{q} \\ (1-\rho_2)\left(\alpha_2 + \beta_2\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma_2 E_t(\tilde{y}_{t+k})\right) + \rho_2 i_{t-1} + \varepsilon_t, & \text{if } q_t > \overline{q} \end{cases} \tag{1}$$

where i_t is the short-term lending interest rate of the CB, $E_t(\cdot) \equiv E(\cdot \mid \Omega_t)$ is the conditional on the current information set of the economy, at time *t*, denoted as Ω_t, expectations' operator, π_{t+n} is the rate of inflation *n*-periods ahead, π^* is the CB's target level for inflation, $\tilde{y}_{t+k}$ denotes the real output gap deviations *k*-periods ahead, often measured as a percentage change of the real GDP from its potential level (see, also Data Section 3.1), ε_t is an i.i.d. (independent and identically distributed) error term with zero mean, which reflects a monetary policy shock, and, finally, q_t and q denote the threshold variable and parameter of model (1), respectively. The latter is assumed to be an unknown parameter, which will be estimated by our data.

Threshold model (1) allows the target level of short term nominal rate i_t, denoted as i_t^*, which is set by the CB to respond to expected future inflation and output deviations from their target levels, to switch between the recession (denoted "1") and expansion (denoted "2") regimes of the economy, defined by conditions $q_t \le q$ and $q_t > q$, respectively, as follows:

$$i_t^* = \begin{cases} a_1 + \beta_1\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma_1\tilde{y}_{t+k}, & \text{if } q_t \le q \\ a_2 + \beta_2\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma_2\tilde{y}_{t+k}, & \text{if } q_t > q. \end{cases}$$

Model (1) also takes into account the smoothing attitude of the CB in the above two regimes. This implies that the actual level of short term interest rate i_t adjusts partially to its target level i_t^* as follows:

$$i_t = (1-\rho_s)i_t^* + \rho_s i_{t-1} + \varepsilon_t, \qquad s = \{1,2\},$$

where autoregressive parameters ρ_s capture the degree of smoothness of interest rate process i_t. The remaining adjustment of i_t is due to monetary policies adopted in previous periods.[2] If there is no regime switching, model (1) reduces to the standard forward looking monetary policy rule model assumed in the literature (see, e.g., Clarida *et al.* (1999), and Rudebusch and Svensson (1999)), which consider $a_1 = a_2 = a$, $\beta_1 = \beta_2 = \beta$, $\gamma_1 = \gamma_2 = \gamma$ and $\rho_1 = \rho_2 = \rho$, i.e.

[2] See, e.g., Clarida *et al.* (1999). The tendency of central banks to smooth changes in short term interest rates stems from various reasons as the fear of instability of capital markets, the loss of credibility from sudden large policy reversals or the need for consensus building to support a policy change. Moreover, the central banks may regard interest rates smoothing as a learning device due to imperfect market information.

$$i_t = (1-\rho)\left(a + \beta\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma \tilde{y}_{t+k}\right) + \rho i_{t-1} + \varepsilon_t . \tag{2}$$

To identify the two regimes "1" and "2", reflecting different business conditions, in our empirical analysis we will assume that threshold variable q_t is given by the current's period output gap deviations $\tilde{y}_t$. That is, it will be defined as $q_t \equiv \tilde{y}_t$. As mentioned in the introduction, this assumption is more realistic, as CBs are more likely to rely their lending rate policy decisions on the more recent announcements about $\tilde{y}_t$ rather than those of previous periods (see also Taylor and Davradakis (2006)). It can be also justified by New Keynesian macroeconomic models, which link i_t to the contemporaneous levels of inflation rate π_t and output gap deviations $\tilde{y}_t$ (see Section 4). Furthermore, variable $\tilde{y}_t$ can also capture financial instability and/or imbalances conditions, given that the latter are immediately related to output gap deviations (see, e.g., Crockett (1997), Bernanke and Gertler (2001), and Davig and Leeper (2008)).

Compared to other approaches allowing for regime-switching in monetary policy rule models, threshold model (1) has some very attractive features. In particular, compared to the intervention dummy approach, which captures regime shifts by using dummy variables relying on ex post and out of sample information, model (1) considers regime-switching as part of model specification. This can be identified endogenously from the data. It can also allows for regime-switching expectation formation effects noted in the introduction, as it does not take a regime as given. Intervention dummy variable approach assumes that, after a regime shift, agents believe that will lie in the new regime permanently. Compared to Markov chain regime-switching models of monetary policy rules (see, e.g., Rabanal (2004), Sims and Zha (2006), Benhabib (2009), Castelnuovo *et al*. (2008)), threshold model (1) allows state variable q_t to be correlated with monetary policy shock ε_t. This is a less restrictive assumption than the orthogonality condition assumed between ε_t and the state, regime-switching variable q_t, imposed by Markov chain regime-switching models. Thus, threshold model (1) allows for any contemporaneous monetary policy interactions between monetary shocks ε_t and threshold variable q_t (i.e., $\tilde{y}_t$, in our analysis), which reflect state conditions of the economy.

3. Empirical Analysis

In this section, we estimate threshold monetary policy rule model (1) and discuss the estimation results. This is done for three world leading economies: the US, UK and Japan. The central banks of these three countries have received by government a significant degree of independence in conducting monetary policy and have announced their commitment on targeting inflation since the seventies, where the above three economies have started experiencing very high inflation rates.[3] The estimation of model (1) for these countries can indicate if, in addition to inflation, their CBs concern also about output fluctuations. In particular, it can reveal if their behavior critically changes during recession periods and it becomes anticyclical.

To investigate possible consequences of ignoring regime-switching due to changes in business cycle conditions (regimes) on conducting monetary policy, our empirical analysis starts with estimating the standard, linear forward looking monetary policy rule model, given by equation (2), which assumes no regime-switching. This model is often estimated in practice. Comparison between the estimates of models (1) and (2) will also reveal which of these two models constitutes the best specification of the data.

3.1. Data

For the US, our data set consists of quarterly observations covering the period from 1970:I to 2011:IV. As short rate i_t, we consider the average Federal Funds rate in the first month of each quarter expressed at annual rates. Inflation rate π_t is measured as the annual rate of the GDP deflator, defined as $P_t = \frac{\text{nominal } GDP_t}{\text{real } GDP_t} \cdot 100$, i.e. $\pi_t = \frac{P_t - P_{t-4}}{P_{t-4}} \cdot 100$. Output gap deviations $\tilde{y}_t$ are measured as the percentage change

[3] More specifically, the Fed changed the course of its policy in October 1979, when its new chairman Paul Volker signaled his intentions to fight inflation. In the same year, Margaret Thatcher assumed power in UK and inflation fighting became the main issue of the Bank of England. For the Bank of Japan, the price stabilization policies started after the first oil-price shock at the end of year 1973, much earlier than year 1979. Despite its lowest score of central bank independence, the central bank of Japan was always committed to antinflationary monetary policy among OECD countries (see, e.g., Cargill *et al*. (1997)).

(rate) of the real GDP with respect to potential real GDP constructed by the Congressional Budget Office (CBO), i.e. $\tilde{y}_t = \frac{\text{real GDP}_t - \text{real potential GDP}_t}{\text{real potential GDP}_t} \cdot 100$. The real GDP and real potential GDP are expressed in annual rates with the year 2005 being the base year.

For the UK, our data set covers the period from 1979:I to 2011:II. As interest rate i_t, we use the average of the base rate of the Bank of England measured in the first month of each quarter, while inflation rate π_t is measured as the annual percentage change of the consumer price index. Output deviations $\tilde{y}_t$ are defined as the US. Finally, for Japan, our data set covers the period from 1973:I to 2011:IV. We use the "Call-Money" rate in the first month of each quarter as i_t. π_t is taken to be the annual rate of the GDP deflator, while deviations $\tilde{y}_t$ are measured as for the US and UK. The data for the US were downloaded from the Federal Reserve Bank of St. Louis, while for the UK and Japan were obtained from the OECD database.

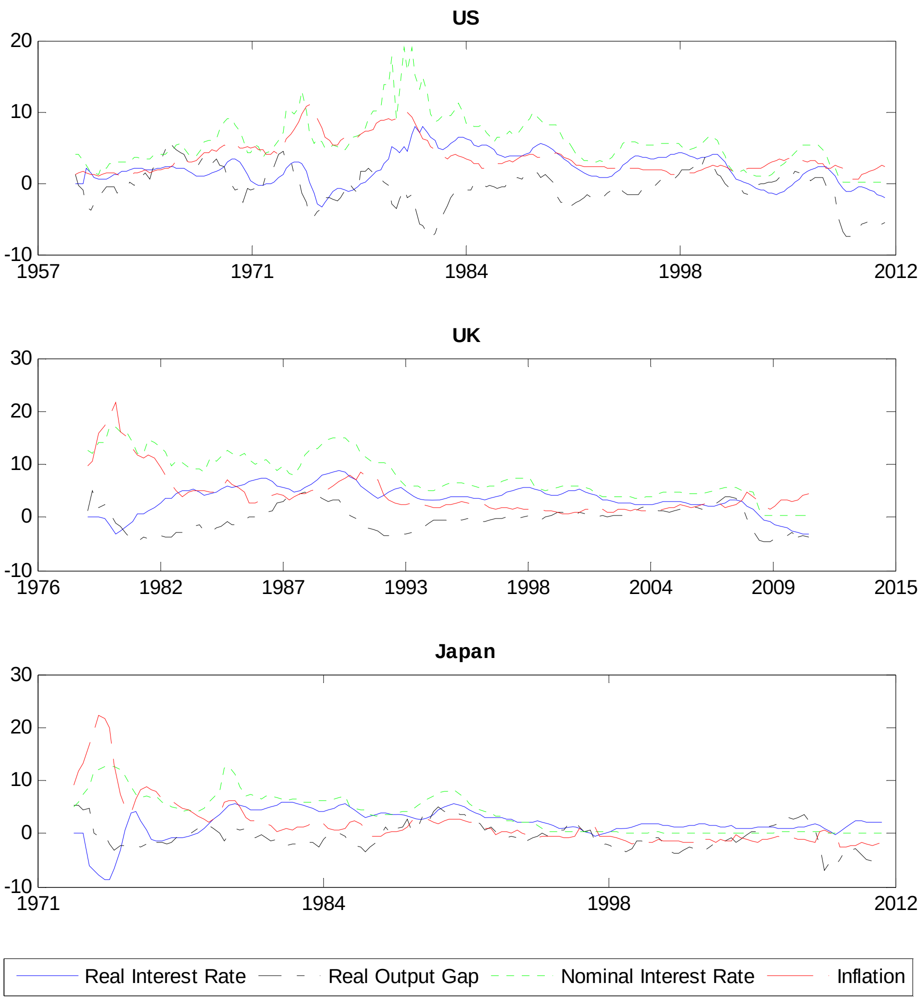
Table 1: Descriptive Statistics

	US (1960:I-2011:IV)		*UK (1979:I-2011:II)*		*JP (1973:I-2011:IV)*	
Variables	*Mean*	*Std. Dev.*	*Mean*	*Std. Dev.*	*Mean*	*Std. Dev.*
i_t	5.66	3.55	7.82	4.13	3.69	3.62
r_t	2.10	2.30	3.57	2.61	1.95	2.66
π_t	3.59	2.35	4.35	3.95	1.81	4.51
$\tilde{y}_t$	-0.59	2.64	-0.25	2.42	-0.54	2.39

Notes: Mean stands for the sample mean, while Std. Dev for standard deviation.

Table 1 presents some basic descriptive statistics of variables i_t, π_t and $\tilde{y}_t$, as well as of the short term real interest rate, denoted as r_t. These statistics are the mean and standard deviation of the sample distribution of the above variables. To calculate real interest rate r_t, we follow Kim's *et al.* (2005) approach, often used in the literature. According to this, r_t is defined as the four quarter moving average of the current and past three-periods interest and inflation rates differences $i_t - \pi_t$. Plots of series i_t, r_t, π_t and $\tilde{y}_t$ are presented in Figure 1, for all three countries.

Figure 1: Plots of series i_t, r_t, π_t and $\tilde{y}_t$



Inspection of the plots of Figure 1 and the results of Table 1 reveal some common features of series i_t, r_t, π_t and $\tilde{y}_t$, across the three countries. This may be attributed to the fact that the CBs of the three countries have followed analogous monetary policies, over our sample. First, the mean values of real interest rate are positive, implying that the values of nominal interest rate i_t are set to higher levels than those of inflation rate π_t. This may be attributed to the antinflationary policy followed by the three CBs, for most of the periods of our sample. Second, the mean values of inflation and nominal interest rates of the US and UK are higher than those of Japan. This can be attributed to the highly coordinated wage settlements in Japan (see, e.g., Sakamoto (2004)). These tend to keep inflation rates at very low levels. A third conclusion that can be drawn from the results of Table 1 is that the mean and standard deviation of the output gap deviations series $\widetilde{y}_t$ are very close to each other, for all three countries. This can be taken as evidence that the CBs of these three countries concern about large output gap deviations from their target levels.

3.2 Estimation of the standard monetary policy rule model

Table 2 presents parameter estimates of the standard monetary policy rule model (2). As shown in Section 2, this model constitutes a restricted version of threshold model (1), which assumes $a_1 = a_2 = a$, $\beta_1 = \beta_2 = \beta$, $\gamma_1 = \gamma_2 = \gamma$ and $\rho_1 = \rho_2 = \rho$. The reported estimates in the table are obtained by replacing the expected values of the variables entering into the RHS of model (2) with their realized values and, then, estimating the resulting model based on the generalized method of moments (GMM) procedure. This method relies on the following moment (orthogonality) conditions:

$$E\left[i_t - (1-\rho)\left(\alpha' + \beta\pi_{t+n} + \gamma\tilde{y}_{t+k}\right) - \rho i_{t-1} \mid \mathbf{z_t}\right] = \mathbf{0},$$

where, $\alpha' = \alpha - \beta\pi^*$ and $\mathbf{z_t}$ is a vector of instrumental variables used in the GMM estimation procedure.[4] This vector includes the constant and one up to four lagged values of the following variables: i_t, π_t, $\widetilde{y}_t$ and the unemployment rate. The last variable is considered as a strong instrument in capturing the future movements in $\widetilde{y}_t$, as it reflects changes in business cycle conditions (see, e.g., Stock and Watson (2003)). For all three countries, the unemployment rate is defined as the seasonally adjusted civilian unemployment rate including persons 16 years of age and older at

[4] In order to obtain an estimate of target inflation rate π^* from the data, note that in the estimation procedure we set a to the sample mean of nominal interest rate i_t, reported in Table 1 (see also Clarida *et al* (1998)).

the last month of each quarter. These series were also obtained from the OECD data base. To compare the fit of standard monetary policy rule model (2) into the data to that of threshold model (1), Table 2 presents values of the following metrics: the MSE (mean squared error) and Theil's inequality coefficient.

The number of lead-periods n and k considered in the GMM estimation procedure of model (2) are set to n=1 and k=0, respectively.[5] This means that the three CBs examined react to one quarter ahead expected values of inflation rate, π_{t+1}, and the currently observed level of output deviations, $\tilde{y}_t$. Due to the forward looking and overlapping nature of future inflation rate π_{t+1}, in the estimation procedure we allow for a moving average variance-covariance matrix of the error term with four lags. The optimal weighting matrix of the GMM procedure is obtained based on the least squares estimation of the model, in the first step (see Hayashi Chapter 3).

The results of Table 2 indicate that the monetary authorities of the three countries examined have mainly adopted an antinflationary monetary policy over our sample. In particular, the bigger than unity estimates of slope coefficient β found for the US and UK mean that the attitude of the CBs of these two countries towards inflation deviations from their target levels is aggressive. These results are consistent with the literature (see, e.g. Rudebusch (2001), Clarida *et al*. (1998)). For Japan, the smaller than unity estimate of β (i.e. β=0.90) indicates that this country has followed a more accommodating inflation policy around its target level. This is a quite surprising result given the strong antinflationary attitude of Japanese governments, since the early seventies. The estimates of the target level of inflation π^* reported in Table 2 are very close to those reported in the literature.[6] From these estimates of π^*, we obtain the following values of real interest rate r_t: 2.42 for the US, 3.20 for the UK and 2.20 for Japan, which are very close to those reported in Table 1.

[5] This specification of n and k has been also used in many studies estimating the forward looking version of the monetary policy rule (see, e.g., Clarida *et al.* (1998, 1999)). We have found that they provide the better fit of the model in terms of the MSE.

[6] See, for instance, Trehan and Wu (2007) who assume an inflation target 3% for the US.

Table 2: Estimates of standard monetary policy rule model (2), $i_t = (1-\rho)\left[\bar{\imath} + \beta E_t(\pi_{t+1} - \pi^*) + \gamma E_t \tilde{y}_t\right] + \rho i_{t-1} + \varepsilon_t$

	US	*UK*	*JAPAN*
β	1.22***	1.30***	0.90***
	(0.22)	(0.23)	(0.11)
	[5.51]	[5.86]	[8.46]
γ	0.92***	1.86***	0.75***
	(0.23)	(0.37)	(0.23)
	[3.97]	[4.99]	[3.22]
π^*	3.24***	4.62***	1.49***
	(0.35)	(0.56)	(0.56)
	[9.35]	[8.18]	[2.68]
ρ	0.88***	0.93***	0.93***
	(0.02)	(0.02)	(0.02)
	[39.72]	[48.27]	[58.33]
MSE	1.303	0.568	0.375
Theil	0.169	0.087	0.119

Notes: The reported estimates in the table are obtained based on the GMM estimation procedure using the Newey-West optimal weighting matrix allowing for serial correlation of order 4. Standard errors are in parentheses and t-statistics are in brackets. MSE stands for mean squared error and Theil for Theil's inequality coefficient. ***, **, * denote 1%, 5%, 10% significance.

Regarding the slope coefficient γ, capturing the response of monetary policy to the output gap deviations, the results Table 2 indicate that the CBs of all three countries have followed an anticyclical policy, over the whole sample. That is, when output gap deviations variable $\tilde{y}_t$ become negative, the CBs will decrease lending rate i_t to boost economic growth. The values of γ reported in the table are very high, compared to those reported in other studies, which provide evidence of a weak response of monetary authorities to cyclical deviations in $\tilde{y}_t$ (see, e.g., Clarida *et al.* (1998)). Note that, among the three countries, the strongest anticyclical policy seems to be followed by the UK monetary authorities, as the estimate of γ is found to be clearly bigger than unity. Finally, note that the estimates of the autoregressive parameter ρ are close to unity, for all three countries. These estimates of ρ indicate a strong attitude of the CB to smooth changes in nominal rates, over time.

The differences between our above estimates of slope coefficient γ and those reported in other studies, mentioned before, may be attributed to the inclusion in our sample of data from the recent financial crisis. As will be clearly shown the next subsection, during this period the attitude of the monetary authorities of the three countries examined became strongly anticyclical, due to the recessionary conditions held in their economies and the recent financial crisis. To examine the robustness of the estimates of the standard monetary policy rule model (2) to possible structural breaks occurred during our sample, we have carried out Bai and Perron's (2003) sequential multiple breaks test.[7] This test has shown that there are structural breaks in the coefficients of model (2). These are found to occur at the following dates: 1969:1 (66:3,69:2), 1980:3 (80:1,80:4), 1989:2 (89:1,91:1) and 2000:3 (98:4,01:2) for the US, 1987:1 (86:4,91:2) and 2001:1 (00:3,01:2) for the UK, and 1980:3 (79:3,81:4) and 1991:2 (91:1,93:1) for Japan, where confidence intervals of the dates of breaks are reported in parentheses. For all three countries, a break is also found to occur in year 2008, where the recent financial crisis began. As will be seen by the estimates of our threshold model (1) in the next section, most of the above dates are associated with business cycles troughs.

3.3 Estimation of the threshold monetary policy rule model (1)

In this section, we present estimates of the parameters of threshold monetary policy rule model (1). To estimate the model, we consider the same values of the lead periods *n* and *k* with those assumed in the estimation of linear model (2), i.e. *n*=1 and *k*=0, and we write the model in a more compact notation as

$$\begin{aligned} i_t &= \left(c_1 + c_2 E_t(\tilde{\pi}_{t+1}) + c_3 E_t(\tilde{y}_t) + c_4 i_{t-1}\right) I(\tilde{y}_t \le \overline{q}) \\ &\quad + \left(c_1' + c_2' E_t(\tilde{\pi}_{t+1}) + c_3' E_t(\tilde{y}_t) + c_4' i_{t-1}\right) I(\tilde{y}_t > \overline{q}) + \varepsilon_t \end{aligned} \tag{3}$$

where

$$c_1 = (1-\rho_1)\alpha_1,\ c_2 = (1-\rho_1)\beta_1,\ c_3 = (1-\rho_1)\gamma_1,\ c_4 = \rho_1$$

and

$$c_1' = (1-\rho_2)\alpha_2,\ c_2' = (1-\rho_2)\beta_2,\ c_3' = (1-\rho_2)\gamma_2,\ c_4' = \rho_2,$$

and $I(.)$ denotes an indicator function of regimes $s=\{1,2\}$. The vector of parameters of this model is defined as $\boldsymbol{\theta} = (\alpha_1, \beta_1, \gamma_1, \rho_1, \alpha_2, \beta_2, \gamma_2, \rho_2)'$.

[7] Note that the implementation of this testing procedure is based on a reduced (backward-looking) form of model (2), which treats the deviations of inflation and output as predetermined variables. This is given as $i_t = \alpha + \varphi i_{t-1} + b\pi_{t-1} + cy_{t-1} + e_t$.

Since threshold variable $\widetilde{y}_t$ can be contemporaneously correlated with monetary shock ε_t (or interest rate i_t), in the estimation procedure of model (3) we will treat q_t as an endogenous variable, otherwise the estimates of threshold parameter $\overline{q}$ and vector $\boldsymbol{\theta}$ will be biased and inconsistent. To this end, we will estimate the model based on a recently suggested method for forward looking threshold models by Kourtelos *et al.* (2011). This relies on consistent estimates of the threshold parameter $\overline{q}$, obtained in a first step.[8] Given them, in a second step, consistent estimates of vector $\boldsymbol{\theta}$ can be derived, based on the GMM estimation procedure. This can be done by splitting the sample into the sub-samples identified by the sample estimate of $\overline{q}$.

More specifically, to derive consistent estimates of $\overline{q}$ the method of Kourtelos *et al.* (ibid) solves out the following problem over all possible values of $\overline{q}$ belonging to set Q:[9]

$$\hat{\overline{q}} = \arg\min_{\overline{q}\in Q} S_n(\overline{q}), \tag{4}$$

where $\hat{\overline{q}}$ denotes the estimator of $\overline{q}$ and

$$\begin{aligned} S_n(\overline{q}) \;=\; & \sum\nolimits_{i=1}^{n}\Big[\Big(i_t - c_1 - c_2\hat{\tilde{\pi}}_{t+1} - c_3\hat{y}_t - c_4 i_{t-1} - \kappa\lambda(\overline{q} - d_2' z_t)\Big) I(\tilde{y}_t \le \overline{q}) \\ & + \Big(i_t - c_1' - c_2'\hat{\tilde{\pi}}_{t+1} - c_3'\hat{y}_t - c_4' i_{t-1} - \kappa\lambda'(\overline{q} - d_2' z_t)\Big) I(\tilde{y}_t > \overline{q})\Big] \end{aligned}$$

is the sum of squared errors of model (1) conditional on LS based estimates of expected values $E_t(\pi_{t+1})$ and $E_t(y_t)$, denoted as $\hat{\pi}_{t+1}$ and $\hat{y}_t$, respectively. These are obtained based on the following reduced form regressions:

$$\tilde{\pi}_t = \mathbf{d}_1'\mathbf{z}_t + e_t \qquad \text{and} \qquad \tilde{\mathrm{y}}_t = \mathbf{d}_2'\mathbf{z}_t + v_t,$$

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[8] The method of Kourtelos *et al.* allows threshold variable $\widetilde{y}_t$ to be correlated with error term ε_t, or i_t. It constitutes an extension of Caner's and Hansen (2004) two-stage least squares (or GMM) method of forward looking threshold models. The latter treats the threshold variable as exogenous.

[9] Following the literature (see, e.g., Kapetanios (2000)), to avoid very extreme values of $\overline{q}$, which may be meaningless and/or leave a very small number of observations in each of the subsamples, we search for estimates of it from the 15th up to 85th percentile of its sample distribution, for all three countries.

respectively, estimated in the first step, where $\mathbf{z}_t$ is a vector of instrumental variables, and e_t and v_t are disturbance terms. Terms $\kappa\lambda(\bar{q}-\mathbf{d}_2'\mathbf{z}_t)$ and $\kappa\lambda'(\bar{q}-\mathbf{d}_2'\mathbf{z}_t)$, which are entered into the sum of squared errors $S_n(\bar{q})$, constitute bias correction terms of conditional expectation $E(i_t \mid \mathbf{z}_t)$, which are due to the endogenous nature of the threshold variable q_t implying $E(v_t\varepsilon_t) \neq 0$, where $\kappa = E(v_t\varepsilon_t)$. The lambda functions of these two terms are defined as follows:

$$\lambda(\bar{q}-\mathbf{d}_2'\mathbf{z}_t) = E(v_t \mid \mathbf{z}_t, \tilde{y}_t \leq \bar{q}) = \frac{\varphi(\bar{q}-\mathrm{d}_2'\mathrm{z}_t)}{\Phi(\bar{q}-\mathrm{d}_2'\mathrm{z}_t)}$$

and

$$\lambda'(\bar{q}-\mathbf{d}_2'\mathbf{z}_t) = E(v_t \mid \mathbf{z}_t, \tilde{y}_t > \bar{q}) = \frac{\varphi(\bar{q}-\mathbf{d}_2'\mathbf{z}_t)}{1-\Phi(\bar{q}-\mathbf{d}_2'\mathbf{z}_t)}.$$

These are the inverse Mills ratio bias correction terms, where $\varphi(\cdot)$ and $\Phi(\cdot)$ denote the normal and cumulative probability density functions, respectively.[10]

Table 3 reports estimates of the vector of parameters of threshold model (3) (or (1)), $\boldsymbol{\theta} = (\alpha_1, \beta_1, \gamma_1, \rho_1, \alpha_2, \beta_2, \gamma_2, \rho_2)'$, and threshold parameter $\bar{q}$ based on the estimation procedure described above. The confidence intervals of the estimates of $\bar{q}$ reported in the table are calculated based on Caner and Hansen's (2004) procedure, as is extended by Kourtellos *et al.* (2011).[11] In addition to the above parameter estimates, Table 3 reports also values of metric MSE and Theil's coefficient, as well as values of test statistic $SupWopt(\bar{q})$, for all $\bar{q} \in Q$. This test statistic is defined as

[10] The adjustment of model (3) (or sum $S_n(\bar{q})$) with the two bias correction terms $\kappa\lambda(\bar{q}-\mathbf{d}_2'\mathbf{z}_t)$ and $\kappa\lambda'(\bar{q}-\mathbf{d}_2'\mathbf{z}_t)$ can be easily seen by noticing that, when $E(v_t\varepsilon_t) \neq 0$, the following relationship holds under the assumption that error terms ε_t and v_t are normally distributed:

$$\begin{aligned} E(i_t \mid \mathbf{z}_t) \;=\; & \left(c_1 + c_2 E_t(\tilde{\pi}_{t+1}) + c_3 E_t(\tilde{y}_t) + c_4 i_{t-1} + \kappa\lambda(\bar{q}-\mathbf{d}_2'\mathbf{z}_t)\right) I(\tilde{y}_t \leq \bar{q}) \\ & + \left(c_1' + c_2' E_t(\tilde{\pi}_{t+1}) + c_3' E_t(\tilde{y}_t) + c_4' i_{t-1} + \kappa\lambda'(\bar{q}-\mathbf{d}_2'\mathbf{z}_t)\right) I(\tilde{y}_t > \bar{q}) \end{aligned}$$

[11] Kourtellos *et al.* (2011) proposed to compute a heteroskedasticity corrected asymptotic confidence intervals of the estimates of $\bar{q}$ using a quadratic polynomial (see, e.g., Hansen (2000)). One difference here is that the nuisance parameters in the conditional variance are estimated via a polynomial regression in $\hat{q}$ and $\hat{q}^2$ instead of q and q^2.

$$SupWopt(\overline{q}) \equiv \sup_{\overline{q} \in Q} Wopt(\overline{q}),$$

where $Wopt(\overline{q})$ is Chow's optimal statistic. The latter tests the following null hypothesis:

$$H_0 : \alpha_1 = a_2, \beta_1 = \beta_2, \ \gamma_1 = \gamma_2 \text{ and } \rho_1 = \rho_2,$$

against its alternative

$$H_a : \quad \alpha_1 \neq a_2, \ \beta_1 \neq \beta_2, \ \gamma_1 \neq \gamma_2 \ or \ \rho_1 \neq \rho_2,$$

for a given (known) value of threshold parameter $\overline{q}$. The definition of statistic $Wopt(\overline{q})$ is given in the appendix.

Test statistic $SupWopt(\overline{q})$ can test if the above hypothesis H_0 is true against its alternative H_a in the case of an unknown value of parameter $\overline{q}$, which can be estimated by the data by solving problem (3), for all $\overline{q} \in Q$. This null hypothesis means that the monetary policy rule is linear, given by equation (2). Rejection of this hypothesis against its alternative H_a constitutes evidence in favor of our threshold monetary policy rule model (1). Thus, testing this hypothesis is crucial in investigating if the monetary policy rule changes according to the different business conditions of the economy. [12]

[12] Note that Statistic $SupWopt(\overline{q})$ has a non conventional distribution, which is the supremum of a chi-square distribution. Since threshold parameter $\overline{q}$ is not identified under the above null hypothesis, to derive critical values of the asymptotic distribution of statistic $SupWopt(\overline{q})$ and calculate the *p*-value of it, we have followed the bootstrap procedure suggested by Hansen (1996). In particular, we define the pseudo dependent variable $i_t^*(\overline{q}) = \hat{\varepsilon}_t(\overline{q})h_t$, where $\hat{\varepsilon}_t(\overline{q})$ are the estimated residuals under the unrestricted model for all $\overline{q} \in Q$ and $h_t \sim N(0,1)$. Then, using this pseudo dependent variable instead of i_t and keeping fixed the values of regressors, we re-estimate threshold model (3) based on the GMM method and calculate test statistic $Wopt(\overline{q})$, for all values $\overline{q}$. From this sequence of statistics $Wopt(\overline{q})$, we obtain the $SupWopt(\overline{q})$. The above procedure is repeated 1000 times and the *p*-value of test statistic $SupWopt(\overline{q})$ is calculated as the average number of times that the simulated values of $SupWopt(\overline{q})$ exceed their sample estimate.

Table 3: Estimates of threshold monetary policy model (1),

$$i_t = \begin{cases} (1-\rho_1)\left[\alpha_1 + \beta_1 \mathrm{E}_t(\tilde{\pi}_{t+1}) + \gamma_1 \mathrm{E}_t(\tilde{y}_t)\right] + \rho_1 i_{t-1} + \varepsilon_t, \text{ if } \tilde{y}_t \le \overline{q} \\ (1-\rho_2)\left[\alpha_2 + \beta_2 \mathrm{E}_t(\tilde{\pi}_{t+1}) + \gamma_2 \mathrm{E}_t(\tilde{y}_t)\right] + \rho_2 i_{t-1} + \varepsilon_t, \text{ if } \tilde{y}_t > \overline{q} \end{cases}$$

Parameters	*US*	*UK*	*JAPAN*
α_1	4.94** (2.24)	10.76*** (1.86)	5.71*** (1.19)
β_1	-0.72 (0.60)	1.05*** (0.12)	0.93*** (0.10)
γ_1	0.99** (0.47)	2.45*** (0.60)	1.39** (0.58)
ρ_1	0.88*** (0.02)	0.89*** (0.01)	0.92*** (0.01)
α_2	6.81*** (0.83)	11.31*** (1.27)	2.26*** (0.44)
β_2	2.58*** (0.70)	2.41*** (0.48)	1.56*** (0.13)
γ_2	1.03* (0.56)	0.70* (0.43)	0.91*** (0.16)
ρ_2	0.92*** (0.02)	0.93*** (0.02)	0.73*** (0.05)
SupWopt (p-value)	0.00	0.00	0.00
Threshold value (95% C.I.)	-2.30 [-2.30,-0.80]	-1.07 [-3.01,-0.76]	0.22 [-0.14,0.54]
Percentile	0.20	0.36	0.68
MSE	1.118	0.535	0.345
Theil	0.157	0.084	0.115

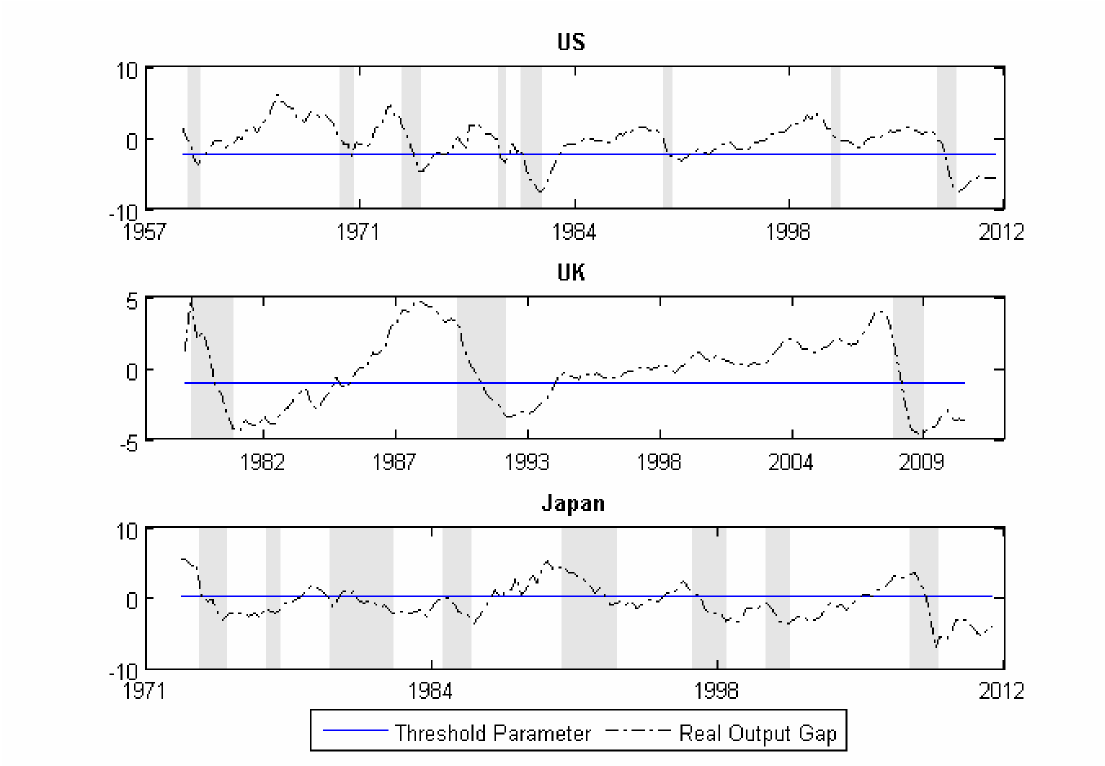
Notes: The table reports estimates of threshold model (1), or (3), based on the GMM estimation procedure, using the Newey-West optimal weighting matrix with 4 lags. Standard errors are in parentheses. MSE stands for mean squared error and Theil for Theil's inequality coefficient. ***, **, * denote 1%, 5%, 10% significance.

The results of Table 3 clearly indicate that the CBs of all three countries examined respond asymmetrically to inflation and output gap rate deviations from their target values depending on the regime of the economy. The *p*-values of statistic *SupWopt*, reported in the table, clearly reject the null hypothesis H_0, for all three countries, and it provides strong evidence in favor of threshold monetary policy rule model (1). Further support of this model compared to the linear monetary policy rule model (2) can be obtained by the values of the MSE metric and Theil's coefficient reported in the table. The values of these metrics are smaller for model (1) than model (2), implying that it fits better into the data.

The results of the table clearly indicate that the CBs of all three countries examined follow mainly a strong antinflationary policy in the expansion regime of the economy, “2”, and an anticyclical policy in the recession regime, “1”. The estimates of beta coefficients, capturing the response of interest rate i_t to the expected inflation deviations $E_t(\pi_{t+1}) - \pi^*$, are much bigger in regime “2” rather than “1”. The opposite happens for the estimates of gamma coefficients, reflecting the response of i_t to output gap deviations $\tilde{y}_t$. The estimates of these coefficients are found to be bigger in regime “1” rather than in “2”. Note that, in regime “1”, the estimate of beta coefficient is not different than zero for the US. These results clearly indicate that the CBs of all three countries examined follow an anticyclical policy during recessions. The close to unity estimates of autoregressive coefficient ρ_s in regime “1”, capturing the smoothing attitude of CBs, indicate the high level of persistency of this anticyclical policy. Finally, note that the higher estimates of gamma coefficient in regime “1”, γ_1, found for model (1), compared to those for model (2) (see Table 2), means that ignoring regime-switching will tend to underestimate the reaction of the CBs of the three countries to recessionary conditions in the economy.

Regarding the estimates of threshold parameter $\overline{q}$, the results of Table 3 indicate that these are negative and different than zero for the US and UK. For Japan, they are positive, but not different than zero. In particular, $\overline{q}$ is found to be: -2.30 for US, -1.07 for UK and 0.22 for Japan. Figure 2 presents graphically the estimates of $\overline{q}$ against the values of output gap deviations $\tilde{y}_t$, for all three countries. As can be seen from this figure, the estimates of $\overline{q}$ are consistent with the business cycle troughs of 61:q1, 70:q4, 75:q1, 80:q3, 82:q4, 91:q1, 01:q4 for US (see shaded areas), officially announced by the National Bureau of Economic Research, the troughs of 81:q1, 92:q2 for the UK (see Birchenhall *et al.* (2001)) and the troughs of 75:q1, 77:q4, 83:q1, 86:q4, 93:q4, 99:q1, 02:q1 for Japan, announced by the Economic Social Research Institute of this country. They also capture the period of the recent financial crisis, associated with the bankruptcy of Lehman Brothers in September of year 2008.

Figure 2: Plots of series $\widetilde{y}_t$ against the estimates of threshold parameter $\overline{q}$



Notes: Shaded areas indicate recessionary periods, according to official announcements.

The above estimates of $\overline{q}$ together with their confidence intervals (CI), as well as their percentiles of the sample distribution of $\widetilde{y}_t$ reported in Table 3, clearly indicate that the monetary authorities of the US and UK tend to reduce interest rate i_t when the economy is under quite severe recessionary conditions. In contrast, the Japanese monetary authorities seem to follow a more proactive policy against recessions. These authorities are more likely to reduce the level of i_t before the economy enters into recession conditions, where $\widetilde{y}_t$ takes negative values. Finally, note that the different estimates of $\overline{q}$ across the three countries found by our empirical analysis reveal that assuming a known (e.g., zero) value for this parameter in order to distinguish the recession regime from the expansion will almost certainly lead to inaccurate estimates of the beta and gamma slope coefficients of threshold monetary policy rule models.

The results of Table 3 are in contrast to results reported in the literature, which do not provide strong evidence supporting the view that the CBs of all three countries examined follow anticyclical monetary policy rules, especially for the US (see, e.g., Clarida *et al*. (1998), Rudebusch (2001)). This is also true for models which allow for regime-switching (see, e.g., Davig and Leeper (2007)). These differences may be attributed to the inclusion of data from the recent financial crisis period in our sample, i.e. after year 2008. This period is characterized by deep recessionary conditions, for all three countries. As can be seen from Figure 1, it is associated with substantial drops in the lending rate i_t of the CBs of the US and UK. To see if the above argument can be supported by the data, in Table 4 we report estimates of threshold model (3) excluding the after year 2007 interval of our sample, when the recent financial crises began.

Table 4: Estimates of model (1) excluding period 2008-2011,

$$i_t = \begin{cases} (1-\rho_1)\left[\alpha_1 + \beta_1 \mathrm{E}_t(\tilde{\pi}_{t+1}) + \gamma_1 \mathrm{E}_t(\tilde{y}_t)\right] + \rho_1 i_{t-1} + \varepsilon_t, \text{ if } \tilde{y}_t \le \overline{q} \\ (1-\rho_2)\left[\alpha_2 + \beta_2 \mathrm{E}_t(\tilde{\pi}_{t+1}) + \gamma_2 \mathrm{E}_t(\tilde{y}_t)\right] + \rho_2 i_{t-1} + \varepsilon_t, \text{ if } \tilde{y}_t > \overline{q} \end{cases}$$

	US	*UK*	*JAPAN*
α_1	2.27 (1.57)	8.47*** (0.72)	7.27*** (2.19)
β_1	-0.48 (0.42)	0.72*** (0.06)	1.02*** (0.17)
γ_1	-0.42 (0.23)	-0.05 (0.32)	2.64* (1.40)
ρ_1	0.81*** (0.03)	0.64*** (0.06)	0.94*** (0.01)
α_2	8.07*** (0.99)	10.23*** (1.05)	2.51*** (0.47)
β_2	2.55*** (0.66)	2.02*** (0.40)	1.45*** (0.14)
γ_2	0.97* (0.53)	1.13** (0.43)	1.00*** (0.18)
ρ_2	0.92*** (0.02)	0.91*** (0.02)	0.73*** (0.06)
SupWopt (*p*-value)	0.00	0.00	0.00
Threshold value (95% C.I.)	-2.35 [-2.35,-0.80]	-1.07 [-1.07,-0.76]	0.21 [-0.10,0.35]
Percentile	0.15	0.32	0.67
MSE	1.166	0.48	0.376
Theil	0.154	0.076	0.113

The results of Table 4 support our argument made above. They clearly indicate that, with the exception of Japan, the US and UK monetary authorities were mainly focused on dampening inflation deviations from their target level before year 2008. That is, they do not seem to concern about output deviations. In particular, during this period they were tending to follow a passive monetary policy with respect to output gap deviations even in the recession regime. The estimates of gamma parameter γ_1, capturing the response of the CB to recessionary conditions, are found to be negative and insignificant for the above two countries, which is consistent with evidence reported in the literature mentioned before. The anticyclical monetary policy followed by Japan for the period before year 2008 can be obviously attributed to the prolonged recessionary conditions held in this country since the early nineties. This can be confirmed from Figure 2 and the dates of business cycle troughs announced by the Economic Social Research Institute of this country, mentioned before.

Summing up, the results of this section clearly indicate that the strong anticyclical monetary policy of the US and UK monetary authorities supported by the estimates of our threshold monetary policy rule model (1) can be attributed to a large extent to the recent financial crisis.

4. Assessing alternative monetary policies under recessionary conditions

The estimates of the threshold monetary policy rule model (1) presented in the previous section raise an important question on the effectiveness of alternative monetary policy rule scenarios to dampen output fluctuations under recessionary conditions. Based on these estimates, in this section we will simulate a small scale New Keynesian (NK) model with the aim of investigating both qualitatively and quantitively if an anticyclical policy in the recession regime can be proved more efficient, compared to a policy abandoning the monetary policy rule. That is, a policy which does not react to output deviations from their target rate, which can thus be characterized as passive. Such a policy is assumed in many theoretical studies (see, e.g., Liu et al (2009)). Answering this question can also show if the reaction of the CBs of the three countries considered to reduce substantially lending rate i_t in response to the last financial crisis has prevented the economies of these countries from very severe recessionary conditions.

To answer the above question, we will obtain impulse response functions (IRFs) of the three macroeconomic variables of threshold monetary policy model (1), i.e. $\tilde{\pi}_{t+k}$, y_{t+k} and i_{t+k} for k=0,1,2,…. quarters ahead, to exogenous demand and supply shocks and we also will calculate the standard deviations of these variables. As mentioned above, this will be done under two different monetary policy rule scenarios. The first will assume an anticyclical policy in the recession regime, while the second will consider a passive monetary policy. The supply shock that we will consider in our analysis is a cost-push structural shock, while the demand shock will be a structural shock affecting the IS curve.

More specifically, the NK model that we consider is given as follows:[13, 14]

$$\tilde{\pi}_t = bE_t(\tilde{\pi}_{t+1}) + \kappa \tilde{y}_t + z_{S,t}, \tag{5.a}$$

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\delta}\left[\tilde{i}_t - E_t(\tilde{\pi}_{t+1})\right] + z_{D,t} \tag{5.b}$$

and

$$\begin{aligned} \tilde{i}_t &= \left[(1-\rho_1)\left(\beta_1 E_t(\tilde{\pi}_{t+1}) + \gamma_1 \tilde{y}_t\right) + \rho_1 \tilde{i}_{t-1}\right] I(\tilde{y}_t \le \overline{q}) \\ &\quad + \left[(1-\rho_2)\left(\beta_2 E_t(\tilde{\pi}_{t+1}) + \gamma_2 \tilde{y}_t\right) + \rho_2 \tilde{i}_{t-1}\right] I(\tilde{y}_t > \overline{q}), \end{aligned} \tag{5.c}$$

where $\tilde{\pi}_{t+1} \equiv \pi_{t+1} - \pi^*$, $\tilde{i}_t \equiv i_t - \overline{i}$ is the deviation of interest rate i_t from its mean $\overline{i}$, b is a discount factor, δ is the relative risk aversion coefficient and $\kappa = \delta \frac{(1-\omega b)(1-\omega)}{\omega}$ is a function of how frequently price adjustments occur (see Calvo (1983)), where ω captures the degree of price stickiness in the economy. In equations (5.a) and (5.b), the two variables $z_{S,t}$ and $z_{D,t}$ represent two exogenous and regime-independent

[13] See, for instance, Davig and Leeper (2007), Farmer *et al* (2008).

[14] This model is linearized around a steady state inflation rate and output of zero to keep the analysis simple.

aggregate supply and demand processes which have the following the autoregressive structure of lag order one:

$$z_{S,t} = \rho_S \; z_{S,t-1} + \varepsilon_{S,t}, \qquad \text{and} \qquad z_{D,t} = \rho_D \; z_{D,t-1} + \varepsilon_{D,t},$$

where $|\rho_S| < 1$ and $|\rho_D| < 1$, while $\varepsilon_{S,t}$ and $\varepsilon_{D,t}$ constitute two i.i.d. zero-mean structural error terms which have $E(\varepsilon_{S,t}\varepsilon_{D,t}) = 0$, for all *t* and *s*. These two error terms respectively represent the two exogenous supply and demand shocks, mentioned before.

In the NK model defined by equations (5.a)-(5.c), the first equation (5.a) describes the change in the aggregate price level (or inflation deviation $\tilde{\pi}_t$) from its target level, as a function of its expected future level and current's period output gap rate $\tilde{y}_t$. This relationship can be derived from the aggregation of optimal price-setting decisions by monopolistically competitive firms in an environment in which each firm adjusts its price with a constant probability at any period (see, e.g., Calvo 1983). Equation (5.b) combines a standard Euler equation for consumption with a market clearing condition equating aggregate consumption and output. This is the IS equation which determines the current level of aggregate output of economy (or the output gap rate $\tilde{y}_t$), as a function of the ex-ante real rate and expected future output. Finally, equation (5.c) is the Central Bank's threshold monetary policy model (1), which is estimated in our previous section.

Model (5.a)-(5.c) can be written into the following structural-equation form:

$$\mathbf{B}(\tilde{y}_t \le \overline{q})\mathbf{x}_{\mathrm{t}} = \mathbf{A}(\tilde{y}_t \le \overline{q})E_t(\mathbf{x}_{t+1}) + \mathbf{D}(\tilde{y}_t \le \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{z}_t \qquad (6.a)$$

and

$$\mathbf{B}(\tilde{y}_t > \overline{q})\mathbf{x}_t = \mathbf{A}(\tilde{y}_t > \overline{q})E_t(\mathbf{x}_{t+1}) + \mathbf{D}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{z}_t, \qquad (6.b)$$

with

$$\mathbf{z}_t = \mathbf{R}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{x}_t = \left[\tilde{\pi}_t, \tilde{y}_t, \tilde{i}_t\right]'$ is the vector of endogenous variables, $\mathbf{z}_t = \left[z_{S,t}, z_{D,t}, 0\right]'$ is a vector which contains exogenous processes $z_{S,t}$ and $z_{D,t}$, $\boldsymbol{\varepsilon}_t = \left[\varepsilon_{S,t}, \varepsilon_{D,t}, 0\right]'$ is a vector which contains exogenous shocks $\varepsilon_{S,t}$ and $\varepsilon_{D,t}$, and

$$\mathbf{B}(\tilde{y}_t \le \overline{q}) = \begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1 & \frac{1}{\delta} \\ 0 & -(1-\rho_1)\gamma_1 & 1 \end{bmatrix}, \qquad \mathbf{A}(\tilde{y}_t \le \overline{q}) = \begin{bmatrix} b & 0 & 0 \\ \frac{1}{\delta} & 1 & 0 \\ (1-\rho_1)\beta_1 & 0 & 0 \end{bmatrix},$$

$$\mathbf{D}(\tilde{y}_t \le \overline{q}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho_1 \end{bmatrix}, \qquad \mathbf{B}(\tilde{y}_t > \overline{q}) = \begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1 & \frac{1}{\delta} \\ 0 & -(1-\rho_2)\gamma_2 & 1 \end{bmatrix},$$

$$\mathbf{A}(\tilde{y}_t > \overline{q}) = \begin{bmatrix} b & 0 & 0 \\ \frac{1}{\delta} & 1 & 0 \\ (1-\rho_2)\beta_2 & 0 & 0 \end{bmatrix}, \qquad \mathbf{D}(\tilde{y}_t > \overline{q}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho_2 \end{bmatrix},$$

and $\mathbf{R} = \begin{bmatrix} \rho_S & 0 & 0 \\ 0 & \rho_D & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

The above model implies the following matrix of transition probabilities between regimes “1” and “2” from time t-1 to t:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} = 1 - p_{11} \\ p_{21} = 1 - p_{22} & p_{22} \end{bmatrix},$$

where

$$p_{11} = \Pr(\beta_t = \beta_1 \wedge \gamma_t = \gamma_1 \wedge \rho_t = \rho_1 \mid \beta_{t-1} = \beta_1 \wedge \gamma_{t-1} = \gamma_1 \wedge \rho_{t-1} = \rho_1)$$

and

$$p_{22} = \Pr(\beta_t = \beta_2 \wedge \gamma_t = \gamma_2 \wedge \rho_t = \rho_2 \mid \beta_{t-1} = \beta_2 \wedge \gamma_{t-1} = \gamma_2 \wedge \rho_{t-1} = \rho_2)\,.$$

Solving out the system of equations (6.a)-(6.b) for vector $\mathbf{x}_t$ gives the following threshold regime-switching rational expectations (TRSRE) model:

$$\mathbf{x}_t = \mathbf{B}(\tilde{y}_t \leq \overline{q})^{-1}\mathbf{A}(\tilde{y}_t \leq \overline{q})E_t(\mathbf{x}_{t+1}) + \mathbf{B}(\tilde{y}_t \leq \overline{q})^{-1}\mathbf{D}(\tilde{y}_t \leq \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{B}(\tilde{y}_t \leq \overline{q})^{-1}\mathbf{z}_t \qquad (7.a)$$

and

$$\mathbf{x}_t = \mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{A}(\tilde{y}_t > \overline{q})E_t(\mathbf{x}_{t+1}) + \mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{D}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{z}_t \qquad (7.b)$$

The rational expectation equilibrium (REE) solution of this model can be written in the following minimum state variable (MSV) form:[15]

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t \leq \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t \leq \overline{q})\mathbf{z}_t \qquad (8.a)$$

and $$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t > \overline{q})\mathbf{z}_t, \qquad (8.b)$$

where matrices $\mathbf{\Omega}(\cdot)$ and $\mathbf{\Gamma}(\cdot)$ are defined analytically in the Appendix. This REE solution implies that the vector of endogenous variables $\mathbf{x}_t$ depends on the monetary policy regime of the economy at time *t*, as well as its lag values $\mathbf{x}_{t-1}$ and the vector of exogenous processes $\mathbf{z}_t$. In the Appendix, we present some conditions which guarantee the forward convergence, mean square stability and determinacy of this solution. The latter means that it is a uniquely bounded REE.

The REE solution (8.a)-(8.b) can be used to derive the IRFs of the three economic variables $\tilde{\pi}_{t+k}$, $\tilde{y}_{t+k}$ and $\tilde{i}_{t+k}$, at time *t+k*, to structural shocks $\varepsilon_{S,t}$ and $\varepsilon_{D,t}$, as well as their standard deviations.[16] To this end, we need to calculate matrices $\mathbf{\Omega}(\cdot)$ and $\mathbf{\Gamma}(\cdot)$. This can be done numerically based on the forward method suggested by Cho(2009). In so doing, we need to assign values of the vector of structural parameters of the NK model (5.a)-(5.b) entered in matrices $\mathbf{B}(\cdot)$, $\mathbf{A}(\cdot)$, $\mathbf{D}(\cdot)$ and **R** defining matrices $\mathbf{\Omega}(\cdot)$ and $\mathbf{\Gamma}(\cdot)$. Actually, two sets of parameters are required. The first is invariant to monetary policy regime. This includes the subjective discount factor *b*, the relative risk aversion parameter *δ*, the degree of stickiness *ω* and the autoregressive

[15] This is analogous to that of Markov regime-switching rational expectation models (see, e.g. Cho and Moreno (2008), and Cho (2009)).

[16] In the Appendix, we show how these IRFs can be obtained from the system of equations (8.a)-(8.b).

coefficients ρ_S and ρ_D. Following Davig and Leeper (2007), Clarida et al (2000), Liu et al (2009), the above parameters are set equal to the following values: b=0.99, δ=1.0, ω=0.67, κ=0.17, $\rho_S = 0.85$ and $\rho_D = 0.85$. These values of the autoregressive coefficients ρ_S and ρ_D guarantee that the forward convergence condition (FCC) of the TRSRE model (7.a)-(7.b) hold for a broad set of values of the remaining parameter of it.

The second set of parameter values required in determining the REE solution (8.a)-(8.b) is monetary policy regime dependent. These are set equal to their values implied by the coefficient estimates reported in Table 3. Finally, the values of the transition probabilities p_{11} and p_{22} are calculated ex post, based on the number of times that the monetary policy rule stayed in regimes "1" and "2", respectively, over our whole sample, according to the estimates of the threshold parameter $\overline{q}$ reported in Table 3. These values are found to be: $p_{11} = 0.80$ and $p_{22} = 0.95$ for US, $p_{11} = 0.93$ and $p_{22} = 0.95$ for UK, and $p_{11} = 0.94$ and $p_{22} = 0.88$ for Japan. As in other empirical studies (see, e.g., Davig and Leeper (2007)), they indicate a very high degree of persistence of both regimes "1" and "2".

Before turning into the analysis of the IRFs implied by (8.a)-(8.b), it must be noted at this point that the estimates of threshold model (1) always imply that the TRSRE model (7.a)-(7.b) is determinate, i.e. its REE solution is uniquely bounded. Moreover, for all three countries the REE solution of this model is found to be mean squared stable and forward convergent.[17] The last condition rules out rational bubbles in the REE solution of the TRSRE model, while mean square stability condition guarantees that all possible REE solutions of the model will be bounded.

[17] Mean square stability of the REE of the TRSRE model (8.a)-(8.b) requires the following condition to hold: $r_\sigma(\mathbf{\Sigma}_\Omega) < 1$, while determinacy requires $r_\sigma(\mathbf{\Sigma}_\mathrm{F}) < 1$, where $r_\sigma(.)$ denotes the maximum eigenvalue of matrices $\mathbf{\Sigma_F}$ or $\mathbf{\Sigma_\Omega}$ defined in the Appendix. Necessary conditions for determinacy are mean square stability and forward convergence. The values of the above maximum eigenvalues that we have found are as follows: $r_\sigma(\mathbf{\Sigma_\Omega}) = 0.33 < 1$ and $r_\sigma(\mathbf{\Sigma}_\mathrm{F}) = 0.96 < 1$ for US, $r_\sigma(\mathbf{\Sigma_\Omega}) = 0.36 < 1$ and $r_\sigma(\mathbf{\Sigma_F}) = 0.88 < 1$ for UK and $r_\sigma(\mathbf{\Sigma_\Omega}) = 0.33 < 1$ $r_\sigma(\mathbf{\Sigma_F}) = 0.99 < 1$ for Japan.

Table 5: Standard deviations of $\tilde{\pi}_t$, $\tilde{y}_t$ and $\tilde{i}_t$ under different monetary policy scenarios in the recession regime

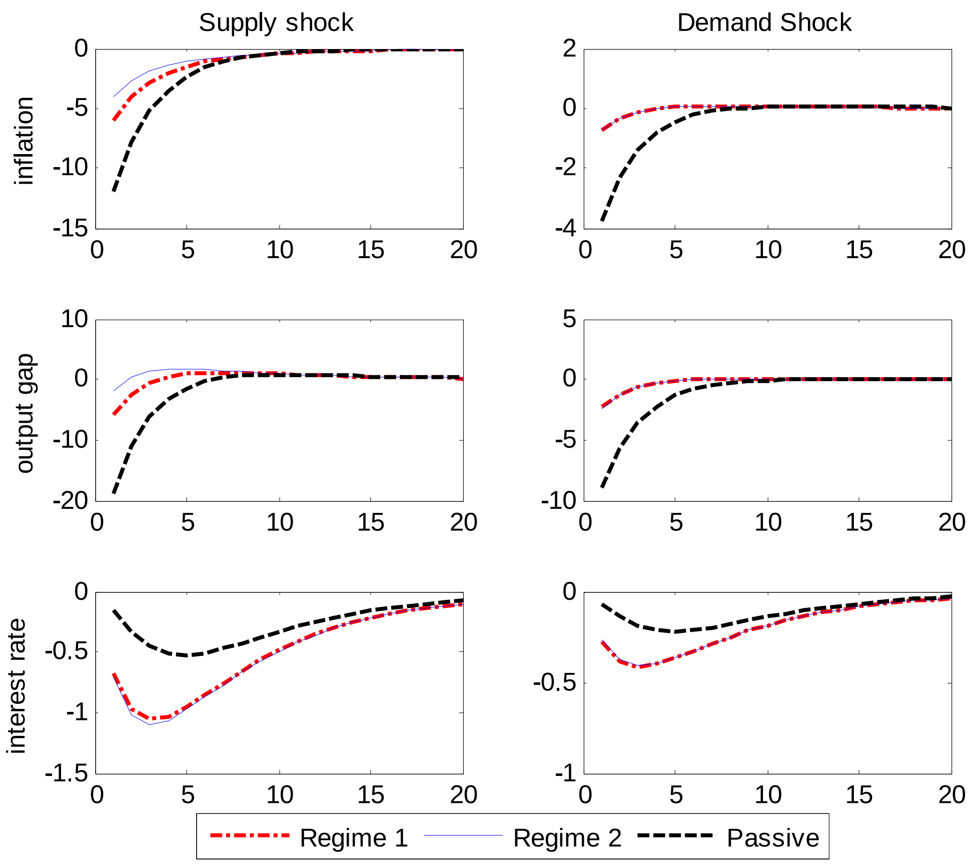
	Policy scenarios	$\sigma_{\tilde{\pi}}$	$\sigma_{\tilde{y}}$	$\sigma_{\tilde{i}}$
US	Active in regime "1"	0.0652	0.0631	0.0296
	Passive in regime "1"	0.0907	0.135	0.0228
UK	Active in regime "1"	0.0645	0.0593	0.0314
	Passive in regime "1"	0.1187	0.1896	0.0133
JAPAN	Active in regime "1"	0.0818	0.0601	0.041
	Passive in regime "1"	0.1078	0.1654	0.0069

Notes: The table presents estimates of the standard deviations of economic variables $\tilde{\pi}_t$, y_t and i_t obtained through the REE solution of the NK model (8a)-(8b), following one-percent (i.e. 0.01) standard deviation negative supply and demand shocks $\varepsilon_{S,t}$ and $\varepsilon_{D,t}$, respectively. The table presents two sets of results. The first is based on the sample estimates of Table 3, which consider an active (anticyclical) policy in the recession regime, while the second considers the case of a passive monetary policy in this regime.

Table 5 presents values of the standard deviations of variables $\tilde{\pi}_t$, $\tilde{y}_t$ and $\tilde{i}_t$ obtained through the REE solution of the NK model (8a)-(8b), following one-percent (i.e. 0.01) standard deviation negative supply and demand shocks $\varepsilon_{S,t}$ and $\varepsilon_{D,t}$, respectively. The table presents two sets of results. The first is based on the sample estimates of Table 3, which consider an active (anticyclical) policy in both regimes of the economy. The second assumes that monetary policy is passive in the recession regime, "1". That is, it sets β_1 and γ_1 to values close to zero, assuming that the CB policy is only driven by its attitude to smooth interest rates. The above estimates are based on 100,000 simulations. Figures 3A-3C graphically present estimates of IRFs of $\tilde{\pi}_{t+k}$, $\tilde{y}_{t+k}$ and $\tilde{i}_{t+k}$, using equations (8.a)-(8.b).[18] These IRFs allow for regime-switching in a future period $t+k$, if the economy lies in any of the two regimes considered, at time t. Thus, it allows for regime-switching expectation formation effects.

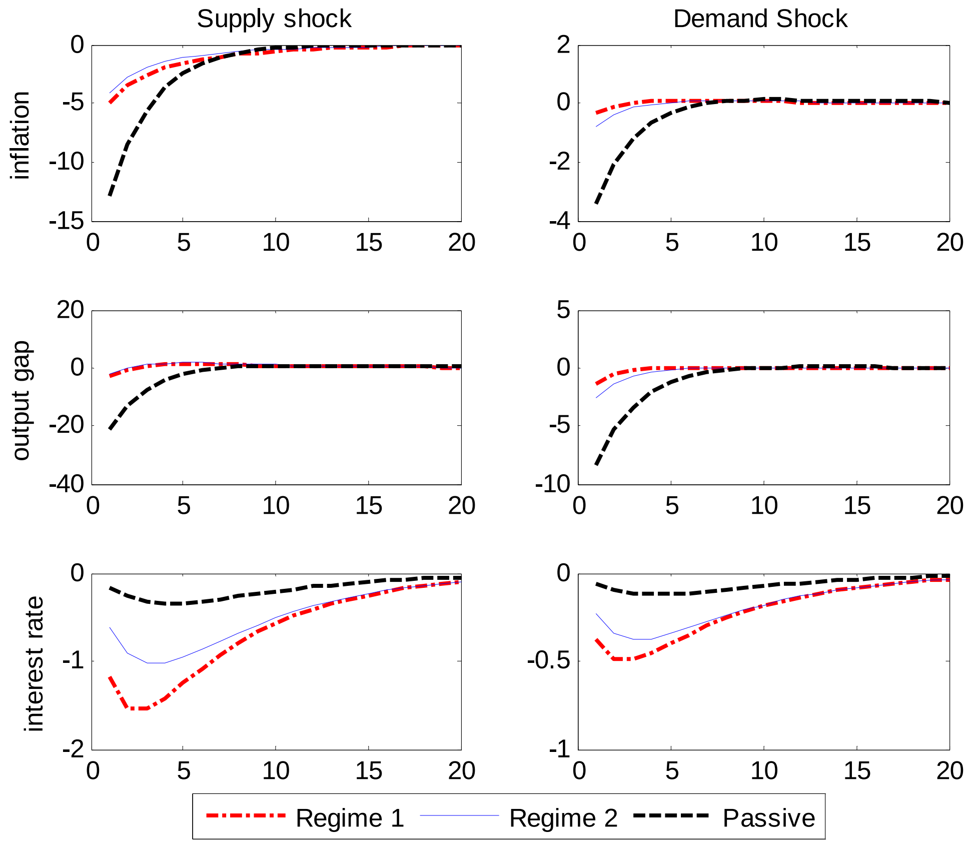
[18] Note that effects of positive shocks were also examined, but these do not changes the main conclusions of our analysis. These produce IRFs which are symmetric to those given in Figures 3A-3C.

Figure 3A: Impulse response functions (IRFs) for the US



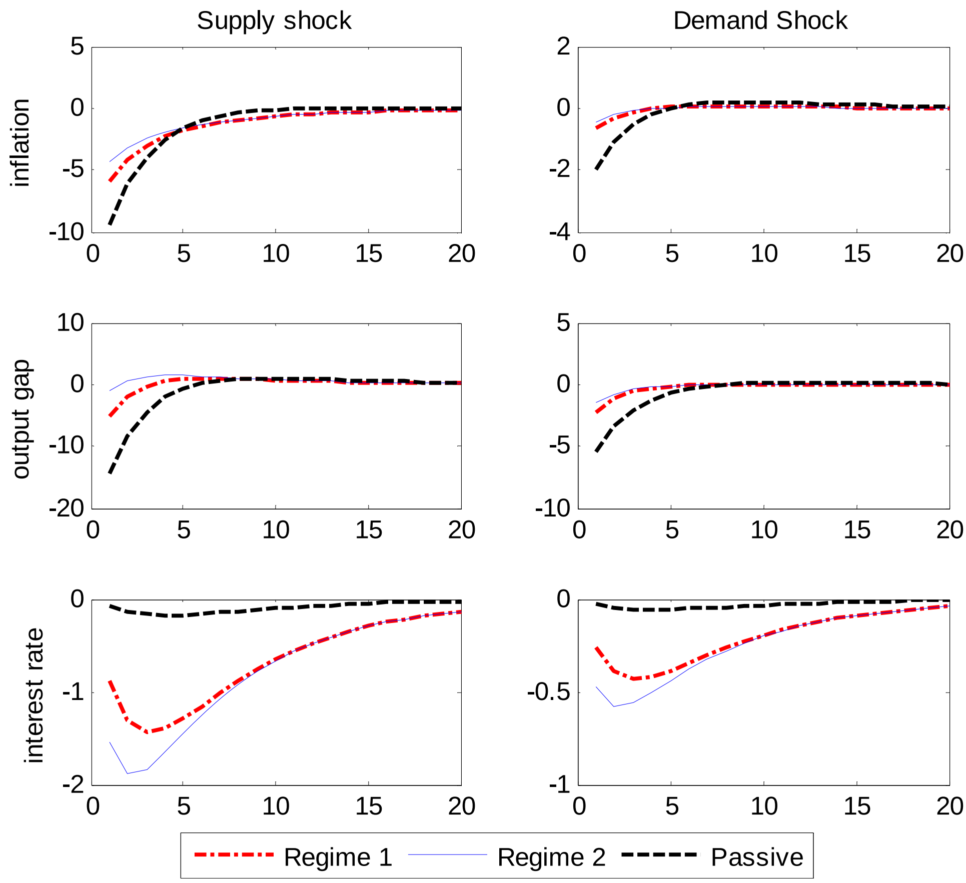
The results of Table 5 and those of the IRFs of Figures 3A-3C lead to a number of interesting conclusions, which have important economic policy implications. First, they clearly show that, by following an active monetary policy responding to output gap deviations $\tilde{y}_t$, the CBs of all three countries examined can prevent substantial falls in the level of $\tilde{y}_t$ and increases in its volatility. In particular, the results of the table indicate that, if the monetary policy rule was passive in the recession regime (regime "1"), then the standard deviations of $\tilde{y}_t$ would be increased by 113.95% for the US, 219.73% for UK and 175.21% for Japan. The IRFs presented in Figures 3A-3C clearly indicate that these excess increases in the volatility of series $\tilde{y}_t$ are due to substantial falls of the level of this series caused by a negative supply or demand shock in the case that monetary policy becomes passive in regime "1". A passive reaction of monetary authorities to supply or demand shocks in this regime of the economy leads to smaller falls in the level of interest rate i_t than an active reaction. Thus, it can cause bigger falls in the levels of output and inflation rate deviations $\tilde{y}_t$ and $\tilde{\pi}_t$, respectively. These deviations are more persistent than those implied by an active reaction of monetary authorities. The bigger inflation rate drops generated in the case of the passive monetary policy considered in the recession regime are necessary in order for the economy to reach its steady state level, after a negative demand or supply shock occurs.

Figure 3B: Impulse response functions (IRFs) for the UK



Finally, another interesting conclusion that can be drawn from the results of Table 5 and Figures 3A-3B is that, for the US and Japan, supply shocks tend to cause bigger and more persistent output gap and inflation deviations compared to the demand shocks. This is not true for the UK. For this economy, either demand or supply shocks cause the smallest inflation or output gap deviations, among the three countries. Given that in our simulation study we assume the same degree of stickiness for all three economies, the above IRF differences can be attributed to the fact that the CB of the UK respond more strongly under recessionary conditions to supply or demand shocks, compared to the two other countries. As can be seen by Table 3, the estimates of slope coefficients β_1 and γ_1 coefficients are bigger for the UK in the recession regime than the two other countries examined.

Figure 3C: Impulse response functions (IRFs) for JAPAN



5. Conclusions

In this paper, we have employed a threshold monetary policy rule model with the aim of investigating if the central banks of three word leading economies, the US, UK and Japan, respond asymmetrically to the inflation and output deviations from their target levels, according to the two phases of the business cycle, i.e. the recession and expansion regimes of the economy. These regimes can be captured by the output gap deviation, which is used as the threshold variable of the model.

The paper relies on a new econometric framework to estimate the suggested model. This allows the threshold variable to be endogenous, as is more likely to happen in practice. In addition to this, it assumes that the threshold parameter is unknown and thus, it is estimated by the data. Ignoring the endogenous nature of the threshold variable or assuming a known value for the threshold parameter may lead to biased estimates of the parameters of the monetary policy rule model and, hence, to wrong inference about the attitude of the central bank against inflation and output gap deviations from their target levels.

The empirical analysis of the paper leads to a number of interesting conclusions. It clearly indicates that the central banks of all three countries examined are characterized by an asymmetric monetary policy behavior. This is found to be mainly antinflationary in the expansion regime of the economy and anticyclical in the recession. For the US and UK, the anticyclical stance of the CB in the recession regime becomes more apparent when including data from the recent financial crisis, began in year 2008. For Japan, it seems that characterizes the behavior of its central bank over the whole sample. The paper also shows that the level of output gap deviations above, or below, which the central banks of the three countries change the stance of their policy is different than zero. For the US and UK, this level is found to be negative and far away from zero, which means that the central banks of these two countries tend to switch their policy to an anticyclical one when the economy is under quite severe recessionary conditions. For Japan, this happens before the economy enters into the recession regime.

To examine the policy implications of the above results on the efficiency of monetary policy in dampening economic fluctuations under recessionary conditions, the paper has simulated a small-scale New Keynesian macroeconomic model. The results of this exercise clearly indicate that a strong anticyclical monetary policy during recessions is necessary to prevent severe and persistent drops in output, compared to a passive policy abandoning the monetary rule in the recession regime of the economy, or a policy focusing only on inflation targeting. These result imply that the reaction of the CBs of the US and UK to reduce substantially their lending short term rate in response to the last financial crisis has prevented the economies of these two countries from severe recessionary conditions.

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A Appendix

A.1 Definition of Chow's optimal test statistic $Wopt(\overline{q})$

Test statistic $Wopt(\overline{q})$ assumes a known value of the threshold parameter $\overline{q}$ and is defined as follows:

$$Wopt(\overline{q}) = T\begin{bmatrix} \hat{\boldsymbol{\theta}}_1(\overline{q}) - \hat{\boldsymbol{\theta}}_2(\overline{q}) \\ \tau\hat{\boldsymbol{\theta}}_1(\overline{q}) + (1-\tau)\hat{\boldsymbol{\theta}}_2(\overline{q}) \end{bmatrix}' \hat{\mathbf{V}}^{-1} \begin{bmatrix} \hat{\boldsymbol{\theta}}_1(\overline{q}) - \hat{\boldsymbol{\theta}}_2(\overline{q}) \\ \tau\hat{\boldsymbol{\theta}}_1(\overline{q}) + (1-\tau)\hat{\boldsymbol{\theta}}_2(\overline{q}) \end{bmatrix},$$

where $\hat{\boldsymbol{\theta}}_1 = \left(\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4\right)'$ and $\hat{\boldsymbol{\theta}}_2 = \left(\hat{c}_1', \hat{c}_2', \hat{c}_3', \hat{c}_4'\right)'$ are the vector of parameters of threshold model (1) under the two different regimes "1" and "2", respectively, $\tau = \frac{T_1}{T}$, where T_1 is the number of observations of the subsample corresponding to regime "1", and $\hat{\mathbf{V}}$ is a consistent estimator of the variance-covariance matrix of the following vector of differences $\left[\hat{\boldsymbol{\theta}}_1(\overline{q}) - \hat{\boldsymbol{\theta}}_2(\hat{\overline{q}}), \tau\hat{\boldsymbol{\theta}}_1(\overline{q}) + (1-\tau)\hat{\boldsymbol{\theta}}_2(\overline{q})\right]$ given as

$$\mathbf{V} = \mathbf{R}\left(\mathbf{M}'\mathbf{G}\mathbf{M}\right)^{-1},$$

where

$$\mathbf{R} = \begin{bmatrix} \mathbf{I}_4 & -\mathbf{I}_4 \\ \tau\mathbf{I}_4 & (1-\tau)\mathbf{I}_4 \end{bmatrix}, \ \mathbf{M} = \begin{bmatrix} \frac{1}{T}\mathbf{z}_1'\mathbf{x}_1 & \mathbf{0}_{k\times 4} \\ \mathbf{0}_{k\times 4} & \frac{1}{T}\mathbf{z}_2'\mathbf{x}_2 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} \tau\hat{\boldsymbol{\Sigma}}_1 & \mathbf{0}_{k\times k} \\ \mathbf{0}_{k\times k} & (1-\tau)\hat{\boldsymbol{\Sigma}}_2 \end{bmatrix},$$

$$\hat{\mathbf{\Sigma}}_1 = \mathbf{z}_1' \begin{bmatrix} \hat{\varepsilon}_{11}^2 & 0 & . & 0 \\ 0 & \hat{\varepsilon}_{12}^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \hat{\varepsilon}_{1T_1}^2 \end{bmatrix} \mathbf{z}_1, \quad \hat{\mathbf{\Sigma}}_2 = \mathbf{z}_2' \begin{bmatrix} \hat{\varepsilon}_{21}^2 & 0 & . & 0 \\ 0 & \hat{\varepsilon}_{22}^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \hat{\varepsilon}_{2(T-T_1)}^2 \end{bmatrix} \mathbf{z}_2,$$

k is the number of instruments, $\mathbf{z}_1, \mathbf{z}_2, \mathbf{x}_{1,}\ \mathbf{x}_2$ are matrices whose columns consist of time series observations of the instrumental variables and repressors employed in the estimation of threshold model (3) which correspond to the two subsamples implied by regimes "1" and "2", respectively, and, finally, $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$ denote the residuals of the threshold model for the two above subsamples.

A.2 Solution of TRSRE Model

In this section of the appendix, we present more analytically the rational expectations equilibrium (REE) solution of the TRSRE model (10.a)-(10.b), given by equations (11.a)-(11.b). In particular, we give the definitions of matrices $\mathbf{\Omega}(\cdot)$ and $\mathbf{\Gamma}(\cdot)$ involved in this solution, as well as those of matrices $\mathbf{\Sigma}_{\mathbf{\Omega}}$ and $\mathbf{\Sigma}_F$ whose maximum values determine the mean square stability and determinacy conditions. The above solution can be obtained following the same steps as Cho (2009), for the Markov chain regime-switching model.

The REE solution (11.a)-(11.b) can be obtained by solving forward the system of equations (10.a)-(10.b) and imposing the forward condition ruling out rational bubbles in equilibrium. This will yield

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t \leq \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t \leq \overline{q})\mathbf{z}_t$$

and

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t > \overline{q})\mathbf{z}_t$$

where

$$\mathbf{\Omega}(\tilde{y}_t \leq \overline{q}) = \lim_{k\to\infty} \mathbf{\Omega}_k(\tilde{y}_t \leq \overline{q}),\ \ \mathbf{\Omega}(\tilde{y}_t > \overline{q}) = \lim_{k\to\infty} \mathbf{\Omega}_k(\tilde{y}_t > \overline{q}),$$

$$\mathbf{\Gamma}(\tilde{y}_t \leq \overline{q}) = \lim_{k\to\infty} \mathbf{\Gamma}_k(\tilde{y}_t \leq \overline{q}) \ \text{ and } \ \mathbf{\Gamma}(\tilde{y}_t > \overline{q}) = \lim_{k\to\infty} \mathbf{\Gamma}_k(\tilde{y}_t > \overline{q})$$

and

$$\mathbf{\Omega}_1\left(\tilde{y}_t \le \overline{q}\right) = \mathbf{B}(\tilde{y}_t \le \overline{q})^{-1}\mathbf{D}(\tilde{y}_t \le \overline{q}),\ \mathbf{\Omega}_1\left(\tilde{y}_t > \overline{q}\right) = \mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{D}(\tilde{y}_t > \overline{q}),$$

$$\mathbf{\Gamma}_1(\tilde{y}_t \le \overline{q}) = \mathbf{B}(\tilde{y}_t \le \overline{q})^{-1},\ \mathbf{\Gamma}_1(\tilde{y}_t > \overline{q}) = \mathbf{B}(\tilde{y}_t > \overline{q})^{-1},$$

$$\mathbf{\Omega}_k\left(\tilde{y}_t \le \overline{q}\right) = \mathbf{\Phi}_{k-1}\left(\tilde{y}_t \le \overline{q}\right)^{-1}\mathbf{B}(\tilde{y}_t \le \overline{q})^{-1}\mathbf{D}(\tilde{y}_t \le \overline{q}),$$

$$\mathbf{\Omega}_k\left(\tilde{y}_t > \overline{q}\right) = \mathbf{\Phi}_{k-1}\left(\tilde{y}_t > \overline{q}\right)^{-1}\mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{D}(\tilde{y}_t > \overline{q}),$$

$$\mathbf{\Gamma}_k(\tilde{y}_t \le \overline{q}) = \mathbf{\Phi}_{k-1}\left(\tilde{y}_t \le \overline{q}\right)^{-1}\mathbf{B}\left(\tilde{y}_t \le \overline{q}\right)^{-1} + E_t\left[\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}\right)\mathbf{\Gamma}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right]\mathbf{R},$$
$$\mathbf{\Gamma}_k(\tilde{y}_t > \overline{q}) = \mathbf{\Phi}_{k-1}\left(\tilde{y}_t > \overline{q}\right)^{-1}\mathbf{B}\left(\tilde{y}_t > \overline{q}\right)^{-1} + E_t\left[\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q}\right)\mathbf{\Gamma}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right]\mathbf{R},$$

with

$$\mathbf{\Phi}_{k-1}\left(\tilde{y}_t \le \overline{q}\right) = \left(\mathbf{I} - E_t\left[\mathbf{B}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}\right)^{-1}\mathbf{A}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}\right)\mathbf{\Omega}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right]\right),$$
$$\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}\right) = \mathbf{\Phi}_{k-1}\left(\tilde{y}_t \le \overline{q}\right)^{-1}\mathbf{B}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q})^{-1}\mathbf{A}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}),$$

$$\mathbf{\Phi}_{k-1}\left(\tilde{y}_t > \overline{q}\right) = \left(\mathbf{I} - E_t\left[\mathbf{B}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q})^{-1}\mathbf{A}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q})\mathbf{\Omega}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right]\right),$$
$$\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q}\right) = \mathbf{\Phi}_{k-1}\left(\tilde{y}_t > \overline{q}\right)^{-1}\mathbf{B}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q})^{-1}\mathbf{A}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q}).$$

Matrices $\mathbf{\Sigma}_{\mathbf{\Omega}}$ and $\mathbf{\Sigma}_F$ are defined as follows

$$\mathbf{\Sigma}_{\mathbf{\Omega}} = \left[p_{ji}\mathbf{\Omega}\left(\tilde{y}_t \le \overline{q}, \tilde{y}_t > \overline{q}\right) \otimes \mathbf{\Omega}\left(\tilde{y}_t \le \overline{q}, \tilde{y}_t > \overline{q}\right)\right]$$
$$\mathbf{\Sigma}_F = \left[p_{ji}\mathbf{F}\left(\tilde{y}_t \le \overline{q}, \tilde{y}_t > \overline{q}\right) \otimes \mathbf{F}\left(\tilde{y}_t \le \overline{q}, \tilde{y}_t > \overline{q}\right)\right]$$

A.3 Impulse Response Functions of TRSRE Model – IRFs

To see how the IRFs of the REE of the TRSRE model are obtained, first note that the forward solution of the TRSRE model is given as

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t \le \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t \le \overline{q})\mathbf{z}_t$$

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t > \overline{q})\mathbf{z}_t,$$

where $\mathbf{z}_t = \mathbf{R}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t,$. The one-step ahead prediction of $\mathbf{x}_{t+1}$ conditional on the t-time information set is given as

$$E_t\mathbf{x}_{t+1} = \mathbf{F_1}(\tilde{y}_t \le \overline{q})\mathbf{x}_t + \mathbf{G_1}(\tilde{y}_t \le \overline{q})\mathbf{z}_t\,,\ E_t\mathbf{x}_{t+1} = \mathbf{F_1}(\tilde{y}_t > \overline{q})\mathbf{x}_t + \mathbf{G_1}(\tilde{y}_t > \overline{q})\mathbf{z}_t$$

where

$$\mathbf{F_1}(\tilde{y}_t \le \overline{q}) = E\left[\boldsymbol{\Omega}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t \le \overline{q}\right],$$

$$\mathbf{F_1}(\tilde{y}_t > \overline{q}) = E\left[\boldsymbol{\Omega}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t > \overline{q}\right],$$

$$\mathbf{G}_1(\tilde{y}_t \le \overline{q}) = E\left[\boldsymbol{\Gamma}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t \le \overline{q}\right]\mathbf{R}\,,$$

$$\mathbf{G}_1(\tilde{y}_t > \overline{q}) = E\left[\boldsymbol{\Gamma}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t > \overline{q}\right]\mathbf{R}\,.$$

The k-step ahead prediction of $\mathbf{x}_{t+k}$ is then given as

$$E_t\mathbf{x}_{t+k} = \mathbf{F}_k(\tilde{y}_t \le \overline{q})\mathbf{x}_t + \mathbf{G}_k(\tilde{y}_t \le \overline{q})\mathbf{z}_t \text{ and } E_t\mathbf{x}_{t+k} = \mathbf{F}_k(\tilde{y}_t > \overline{q})\mathbf{x}_t + \mathbf{G}_k(\tilde{y}_t > \overline{q})\mathbf{z}_t\,,$$

where

$$\mathbf{F}_k(\tilde{y}_t \le \overline{q}) = E\left[\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\boldsymbol{\Omega}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t \le \overline{q}\right],$$

$$\mathbf{F}_k(\tilde{y}_t > \overline{q}) = E\left[\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\boldsymbol{\Omega}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t > \overline{q}\right],$$

$$\mathbf{G}_k(\tilde{y}_t \le \overline{q}) = E\left[\left(\mathbf{G}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) + \mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\boldsymbol{\Gamma}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right) \mid \tilde{y}_t \le \overline{q}\right]\mathbf{R},$$

$$\mathbf{G}_k(\tilde{y}_t > \overline{q}) = E\left[\left(\mathbf{G}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) + \mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\boldsymbol{\Gamma}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right) \mid \tilde{y}_t > \overline{q}\right]\mathbf{R}$$

for $k = 2,3,\ldots$. For $k = 0$ we define $\mathbf{F}_0(\cdot) = \mathbf{I}_n$ and $\mathbf{G}_0(\cdot) = \mathbf{0}_{n\times m}$ where n is the number of endogenous variables and m the number of exogenous.

Given the above definitions, the impulse response functions (IRFs) of $\mathbf{x}_{t+k}$ to the l-th innovation at time t conditional on the state can be calculated by the following expressions:

$$\mathbf{IRF}_k\left(\tilde{y}_t \le \overline{q}\right) = \left(\mathbf{F}_k\left(\tilde{y}_t \le \overline{q}\right)\boldsymbol{\Gamma}\left(\tilde{y}_t \le \overline{q}\right) + \mathbf{G}_k\left(\tilde{y}_t \le \overline{q}\right)\right)\mathbf{e}_l\,,$$

$$\mathbf{IRF}_k\left(\tilde{y}_t > \overline{q}\right) = \left(\mathbf{F}_k\left(\tilde{y}_t > \overline{q}\right)\boldsymbol{\Gamma}\left(\tilde{y}_t > \overline{q}\right) + \mathbf{G}_k\left(\tilde{y}_t > \overline{q}\right)\right)\mathbf{e}_l\,,$$

for $k = 0,1,2,3,...$ where $\mathbf{e}_l$ is an indicator vector of which the l-th element is 1 and 0 elsewhere.