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Abstract

This paper puts forward an intertemporal model of a small open economy which allows for the simultaneous analysis of the determination of endogenous growth and external balance. The model assumes infinitely lived, overlapping generations that maximize lifetime utility, and competitive firms that maximize their net present value in the presence of adjustment costs for investment. Domestic securities are assumed perfect substitutes for foreign securities and the economy is small in the sense of being a price taker in international goods and assets markets. The endogenous growth rate is determined solely as a function of the determinants of domestic investment, such as the world real interest rate, the technology of domestic production and adjustment costs for investment. The endogenous growth rate is independent of domestic savings and the preferences of consumers. Given the domestic growth and investment rate, the preferences of consumers determine the current account and external balance. The model can also be used to analyze the implications of budgetary policy. The world real interest rate affects growth negatively but has a positive impact on external balance. The productivity of domestic capital affects growth positively but causes a deterioration in external balance. Government consumption and government debt affect the current account and external balance negatively, but do not affect the endogenous growth rate. This model addresses and resolves the indeterminancy problems that arise in comparable representative household endogenous growth models of small open economies.

Keywords: endogenous growth, external balance, interest rates, productivity, fiscal policy
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1. Introduction

Endogenous growth models have gone a long way towards addressing the main weaknesses of the traditional growth literature, mainly the implication that the long run growth rate of productivity is exogenous.[1]

In the same vein, the intertemporal approach to open economy macroeconomics has gone a long way towards addressing the main weaknesses of previous approaches that did not rely on intertemporal optimization. A number of models that rely on the intertemporal approach are now being used to address the issue of external balance in a consistent way.[2]

This paper puts forward an intertemporal model of a small open economy which allows for the simultaneous analysis of the determination of endogenous growth and external balance.

The model assumes infinitely lived overlapping generations of households, that maximize lifetime utility from consumption, and competitive firms that maximize their net present value in the presence of adjustment costs for investment. Domestic securities are assumed perfect substitutes for foreign securities and the economy is assumed small, in the sense of being a price taker in international goods and assets markets. The endogenous growth rate is determined solely as a function of the determinants of domestic investment, such as the world real interest rate, the technology of domestic production and adjustment costs for investment. The endogenous growth rate is independent of domestic savings and the preferences of consumers. Given the domestic growth and investment rate, the preferences of consumers and budgetary policy determine the domestic savings rate. The current account and external balance are determined by the difference between domestic savings and investment.

The model proposed in this paper can be viewed as the simplest possible intertemporal model for a small open economy, that allows for the simultaneous determination of endogenous growth, external balance and current account dynamics. Because there is an investment schedule which is independent of the savings schedule, the model determines both the growth rate, as a function of the investment rate, and the current account, as the difference between domestic savings and investment. We demonstrate that both adjustment costs for investment and overlapping generations are necessary and sufficient conditions for a balanced growth path with external balance to exist in the absence of

[1] See Romer (1986), Lucas (1988), and a number of comprehensive recent surveys such as Barro and Sala-i-Martin (2004), Aghion and Hewitt (2009), Acemoglou (2009).

[2] See Obstfeld and Rogoff (1995, 1996) for early surveys of models based on the intertemporal approach.

borrowing constraints. We use the model to study the effects of shocks to the world real interest rate, domestic supply shocks, as well as shocks to domestic budgetary policy.[3]

It is shown that a rise in the world real interest rate results in a fall of the endogenous growth rate and an improvement in the net external position of the economy. The current account impoves, because of a fall of domestic investment relative to savings, and this improvement is sustained as the economy gradually converges to a new long run equilibrium with higher net external assets relative to output.

On the other hand, a rise in the aggregate productivity of domestic capital, i.e a positive supply shock, causes a rise in the endogenous growth rate but also a deterioration in the equilibrium net external position. This is because the higher investment leads to a worsening of the current account, which leads to convergence to a new long run equilibrium with lower net external assets relative to output.

Finally we analyse budgetary policy. A tax financed rise in the government consumption to output ratio does not affect the growth rate, but it causes a deterioration in the current account, as the fall in private consumption does not fully compensate for the rise in government consumption. This is because Ricardian equivalence does not hold in this overlapping generations model. The deterioration in the current account leads to convergence to a new long run equilibrium, with lower net external assets relative to output. On the other hand, a one off increase in the government debt to output ratio has similar effects. It does not affect the growth rate, but it causes a deterioration in the current account, as current generations increase consumption relative to output, because Ricardian equivalence does not hold. Again, the deterioration in the current account leads to convergence to a new long run equilibrium, with lower net external assets relative to output.

The rest of the paper is organized as follows: In section 2 we present the basic model of the investment decisions of firms, with “learning by doing” and adjustment costs for investment. We derive the first order conditions for the maximization of the present value

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[3] Alogoskoufis and van der Ploeg (1994), Barro, Mankiw and Sala-i-Martin (1995) and Turnovsky (1996) have presented models that address similar issues for open economies. The Alogoskoufis and van der Ploeg (1994) model is a symmetric model of a two country world, in which the world real interest and investment rates adjust to equate growth rates and the user cost of capital across countries. The Barro, Mankiw and Sala-i-Martin (1995) model relies on borrowing constraints and not adjustment costs for investment. The Turnovsky (1996) model is a small open economy model with adjustment costs for investment and a representative household. Adjustment costs for investment or borrowing constraints solve the problem of the potential discrepancy between the world real interest rate and the domestic marginal productivity of capital in AK endogenous growth models. However, unless the growth rate of domestic output determined by the investment decisions of domestic firms is equal to the growth rate of domestic consumption, determined by the decisions of a representative household, a balanced growth path may not exist. In general, if the real interest is exogenously given and there are no borrowing constraints, there is no mechanism to ensure that these two equilibrium growth rates will coincide in a small open economy with a representative household. Thus, the need to rely on a model with overlapping generations as is the case in this paper.

of domestic firms. In section 3 we discuss the simultaneous determination of the endogenous growth rate and the shadow price of capital. The endogenous growth rate depends solely on the determinants of the investment rate, namely the world real interest rate, the marginal product of capital, the adjustment cost parameter and the depreciation rate. In section 4 we show how the consumption decisions of households and budgetary policy, in an overlapping generations context, determine external balance and current account dynamics. We also introduce government consumption and government debt. In section 5 we use the full model to examine the effects of shocks to the world real interest rate, the domestic productivity of capital, as well as government expenditure and government debt. The final section sums up the conclusions.

2. The Investment Decisions of Firms

We assume a small open economy, consisting of competitive firms that produce an internationally traded good.

2.1 Production

The production function of firm *i* at time *t* is given by,

$$Y_{it} = AK_{it}^{\alpha}(h_t L_{it})^{1-\alpha}, \quad 0<\alpha<1 \tag{1}$$

where *Y* is output, *K* physical capital, *L* the number of employees and *h* the efficiency of labour. The efficiency of labour is the same for all firms.

Following Arrow (1962) we assume *learning by doing*. In particular we assume that the efficiency of labour is a linear function of the aggregate capital labour ratio. Thus,

$$h_t = B\left(\frac{K}{L}\right)_t, \quad 0<\beta<1 \tag{2}$$

where *B* is a constant, and *K/L* is the aggregate capital labour ratio.

Substituting (2) in (1) and aggregating, we get aggregate output as a linear function of aggregate physical capital.

$$Y_t = \bar{A}K_t \tag{3}$$

where,

$$\bar{A} = AB^{1-\alpha} \qquad (4)$$

In what follows we shall refer to $\bar{A}$ as the *aggregate productivity of domestic capital*. Average and marginal aggregate productivity are constant and obviously equal, due to the linearity of the aggregate production function (3).[4]

Due to the linearity of the aggregate production function, the (endogenous) rate of economic growth *g* will be equal to the rate of net capital accumulation, which is in turn determined by the rate of investment. Therefore, we shall have,

$$g = \dot{Y}_t / Y_t = \dot{K}_t / K_t = (I_t / K_t) - \delta = \bar{A}(I_t / Y_t) - \delta \qquad (5)$$

where *I* is gross investment and δ the rate of depreciation.

In this endogenous growth model, the long run rate of growth is determined by the ratio of gross investment to GDP as well as the productivity of capital. Investment in physical capital is the driving force of the long run growth process, so we now turn to the determination of investment.

2.2. Adjustment Costs and the Rate of Investment

Investment is determined by the profit maximization decisions of private firms. We assume that new investment is subject to a marginal adjustment cost which is a function of the ratio of new investment goods to total installed capital (see Lucas 1967, Gould 1968, Abel 1982 and Hayashi 1982).

Thus, the instantaneous profits of firms are given by,

$$Y_{it} - w_t L_{it} - \left[1 + \frac{\phi}{2}\left(\frac{I_{it}}{K_{it}}\right)\right] I_{it} \qquad (6)$$

where *w* is the real wage and ϕ is a positive constant measuring the intensity of the marginal adjustment cost of new investment.

[4] The linearity of the aggregate production function follows from the assumed linearity in the production of human capital (efficiency of labor) in (2). The qualitative implications of this model would be similar to the implications for the transition path in an exogenous growth model.

$\phi\left(\frac{I_{it}}{K_{it}}\right)$ is the marginal adjustment cost.

Each firm selects employment and investment in order to maximize the present value of its profits.

$$V_{it} = \int_{s=t}^{\infty} e^{-rs}\left(Y_{is} - w_s L_{is} - \left[1 + \frac{\phi}{2}\left(\frac{I_{is}}{K_{is}}\right)\right] I_{is}\right) ds \tag{7}$$

under the constraint,

$$\dot{K}_{is} = I_{is} - \delta K_{is} \tag{8}$$

r is the real domestic interest rate.

From the first order condition for a maximum of (7) subject to (8),

$$w_t = (1-\alpha) A \left(\frac{K_{it}}{L_{it}}\right)^{\alpha} h_t^{1-\alpha} \tag{9}$$

$$q_{it} = 1 + \phi\left(\frac{I_{it}}{K_{it}}\right) = 1 + \phi\left(\frac{\dot{K}_{it}}{K_{it}} + \delta\right) \tag{10}$$

$$\left(r + \delta - \frac{\dot{q}_{it}}{q_{it}}\right) q_{it} = \alpha A \left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} h_t^{1-\alpha} + \frac{\phi}{2}\left(\frac{\dot{K}_{it}}{K_{it}} + \delta\right)^2 \tag{11}$$

where q_i is the shadow price of installed physical capital.

The details of deriving the first order conditions (9) to (11) are well known (see Lucas 1967 Gould 1967) and are therefore omitted.

From (9), employment is determined so that the marginal product of labour for the firm equals the real wage. Given that the real wage is the same for all firms, all firms will choose the same capital-labour ratio.

From (10), the shadow price of installed capital is equal to the marginal cost of new investment. This is equal to the cost of purchase of new capital goods, plus the marginal adjustment cost of investment.

From (11), the *user cost of capital* (on the left hand side) is equal to the augmented marginal product of capital (on the right hand side). The marginal product of capital has two components: The marginal product in terms of additional output (the first term on the right hand side) and the reduction of the adjustment cost of future investment (the second term on the right hand side). A higher capital stock today means a smaller marginal adjustment cost for future investment.

It is worth noting that if there are no adjustment costs for investment, then q is equal to one (from (10)). (11) then becomes the well known condition that the real interest rate r is equal to the net marginal product of capital.

Thus, if $\phi = 0$, which implies $q=1$, then, $r = \alpha A \left(\frac{K_{it}}{L_{it}} \right)^{\alpha-1} h_t^{1-\alpha} - \delta$.

We shall return to this special case below, arguing in favor of adjustment costs for investment on both theoretical and empirical grounds.

3. The Endogenous Growth Rate and the Shadow Price of Capital

Aggregating (9) to (11), taking into account (2) to (5), we have the following *aggregate first order conditions*.

$$w_t = (1-\alpha)\bar{A}\left(\frac{K_t}{L_t}\right) \tag{12}$$

$$q_t = 1 + \phi\left(\frac{\dot{K}_t}{K_t} + \delta\right) = 1 + \phi(g+\delta) \tag{13}$$

$$\left(r + \delta - \frac{\dot{q}_t}{q_t}\right) q_t = \alpha \bar{A} + \frac{\phi}{2}\left(\frac{\dot{K}_t}{K_t} + \delta\right)^2 = \alpha \bar{A} + \frac{\phi}{2}(g+\delta)^2 \tag{14}$$

From (13), the growth rate is a linear function of q, the shadow price of capital.

$$g = \frac{q_t - 1}{\phi} - \delta \tag{15}$$

We shall further assume that *domestic bonds are perfect substitutes for foreign bonds* and that *the economy is small*, in the sense that it cannot affect the world real interest rate. Therefore,

$$r = r^* \tag{16}$$

where r^* is the world real interest rate. (16) allows us to treat the real interest rate as exogenous for the small open economy.[5]

Substituting (16) in (14), and solving for q we get,

$$q_t = \frac{1}{r^* + \delta}\left(\alpha \bar{A} + \frac{\phi}{2}(g+\delta)^2\right) \tag{17}$$

We can easily prove the following proposition about the determinants of the long run growth rate g.

Proposition 1: In a small open economy with investment adjustment costs and perfect asset substitutability, endogenous long run growth is determined solely by the world real interest rate, the productivity of capital, the depreciation rate and adjustment costs for investment. Aggregate domestic demand and domestic savings have no impact on the endogenous long run growth rate.

Proof : The proof of proposition 1 is straightforward. From (13) and (17) it follows that,

$$(r^* + \delta)(1 + \phi(g+\delta)) = \alpha \bar{A} + \frac{\phi}{2}(g+\delta)^2 \tag{18}$$

Since (18) depends only the exogenous world real interest rate, the exogenous productivity of domestic capital, the depreciation rate and the adjustment cost parameter, its solution for g will also depend on the same parameters, and will be independent of other factors such as domestic savings.

Proposition 1 is a property which follows directly from the assumption of a small open economy and perfect asset substitutability. As such, it also holds in exogenous growth

[5] In Alogoskoufis (2012) we allow for imperfect substitutability between domestic and foreign bonds. In this case, the domestic real interest rate becomes endogenous and depends on the domestic government debt to output ratio.

open economy models with perfect asset substitutability, during the transition path to long run equilibrium(see Obstfeld and Rogoff 1996). However, this property does not hold in traditional growth models without capital mobility, such as the Solow or Ramsey models, in which the determinants of capital accumulation are domestic savings.

3.1 The Equilibrium Endogenous Growth Rate

Equation (18) is a quadratic equation in g and has two solutions which lie on either side of r^*, the world real interest rate. Only the solution with $g < r^*$ is stable in the sense of satisfying the transversality condition for the maximization of the present value of profits for firms. This solution implies that the equilibrium growth rate g_E is determined by,

$$g_E = r^* - \sqrt{(r^*)^2 - \frac{2}{\phi}\left(\alpha \bar{A} - (r^* + \delta)(1 + \phi\delta)\right) - \delta^2} \qquad (19)$$

Proposition 1 can be confirmed from (19), which shows that the equilibrium endogenous growth rate g_E depends only on the world real interest rate, the productivity of domestic capital, the depreciation rate and the adjustment cost parameter.[6]

Equilibrium q, say q_E, will be determined by substituting (19) in (13).

In what follows we shall assume that the equilibrium growth rate is real, which requires that,

$$(r^*)^2 \geq \frac{2}{\phi}\left(\alpha \bar{A} - (r^* + \delta)(1 + \phi\delta)\right) - \delta^2$$

Under this assumption, one can prove the following two properties.

First, *the equilibrium growth rate depends negatively on the world real interest rate*.

Proof : From (19), the first derivative of the equilibrium growth rate with respect to the world real interest rate is given by,

$$\frac{\partial g_E}{\partial r^*} = -\frac{g_E + \frac{1}{\phi}(1 + \phi\delta)}{\sqrt{(r^*)^2 - \frac{2}{\phi}\left(\alpha \bar{A} - (r^* + \delta)(1 + \phi\delta)\right) - \delta^2}} < 0$$

[6] For example, assuming δ=1%, φ=90, a=0.33 and $\bar{A}$=0.33 (a capital output ratio of 3), a world real interest rate r^* of 3% implies through (19) an equilibrium endogenous growth rate g_E of 2.2%.

Second, *the equilibrium growth rate depends positively on the aggregate productivity of capital*.

Proof : From (19), the first derivative of the the equilibrium growth rate with respect to the aggregate productivity of capital is given by,

$$\frac{\partial g_E}{\partial \bar{A}} = \frac{\alpha / \phi}{\sqrt{(r^*)^2 - \frac{2}{\phi}\left(\alpha \bar{A} - (r^* + \delta)(1 + \phi\delta)\right) - \delta^2}} > 0$$

The difference between the world real interest rate and the endogenous growth rate is given by,

$$r^* - g_E = \sqrt{(r^*)^2 - \frac{2}{\phi}\left(a \bar{A} - (r^* + \delta)(1 + \phi\delta)\right) - \delta^2} \qquad (20)$$

It is straightforward to show that *the world real interest rate endogenous growth differential* is a positive function of the world real interest rate and a negative function of the productivity of domestic capital.

The determination of equilibrium is depicted graphically in Figure 1.

The positively sloped straight line depicts (13). (17) is the curved line, as (17) is a non-linear relation. The only stable equilibrium is at *E*, given that the second equilibrium (not shown in the graph) does not satisfy the transversality condition.

The position of (17) depends (among other factors such as φ and δ) on the world real interest rate and the aggregate productivity of domestic capital.

A rise in world real interest rate causes (17) to shift to the right (downwards). In the new equilibrium, (see Figure 2), both the growth rate and the shadow price of capital fall. Thus, a rise in the world real interest rate causes the domestic investment rate and the long run growth rate to fall. As a result, the differential between the real interest rate and the domestic growth rate also increases (widens).

A rise in the productivity of domestic capital causes (17) to shift to the left (upwards). In the new equilibrium, (see Figure 3), both the growth rate and the shadow price of capital rise. Thus, a rise in the productivity of domestic capital causes the investment rate and the long run growth rate to rise. As a result, the differential between the real interest rate and the domestic growth rate becomes smaller (narrows).

We shall return to the implications of these effects when we also examine external balance. As we have already noted, (see Proposition 1), the rate of growth does not depend on the demand side of the domestic economy. Only factors that affect the investment decisions of firms affect the growth rate. However, as we shall see, the growth rate, and, in addition, the differential between the world real interest rate and the growth rate, have an impact on external balance. The model is recursive. The growth rate in determined by supply side factors, but the current account and external balance are affected by both the demand side and the supply side.

4. Private Consumption, External Balance and the Current Account

To analyse the demand side, we assume a continuous time overlapping generations model (Blanchard (1985), Weil (1989)). In this model, private consumption is determined by forward looking optimizing households. Since the investment rate and the growth rate do not depend on domestic demand, the demand side determines aggregate savings, the net foreign position of the country and the dynamics of the current account.

We assume that the economy consists of infinitely lived households, that have been born at different times in the past. nL_t households are born at each instant, where L_t is total population at instant t, and n, is the rate of growth of the number of households. We shall further assume that each households supplies on unit of labor. Therefore n is also the rate of growth of the labor force.

The household born at instant j chooses chooses consumption to maximize,

$$U_j = \int_{s=j}^{\infty} e^{-\rho s} \ln c_{js} ds \tag{21}$$

where ρ is the pure rate of time preference and c_{js} is the consumption of household j at instant s. Instantaneous utility is assumed logarithmic.[7]

Each household supplies one unit of labor, and its flow budget constraint is given by,

$$\dot{a}_{js} = r^* a_{js} + w_s - c_{js} \tag{22}$$

where a_{js} denotes the total assets (non human wealth) of household j at instant s.

[7] We are using the Weil (1987) version of the Blanchard Weil model, where, without loss of generality, the probability of death is assumed equal to zero.

The total assets of households consist of shares in domestic firms, government bonds and foreign assets.

From the maximization of (21) subject to (22), after aggregating, we can get aggregate private consumption. The details of the maximization and aggregation for these types of models are generally well known (see Blanchard 1985, Weil 1987) and are therefore omitted.

4.1. Private Consumption and Net Foreign Assets

The aggregate consumption function derived from this model is given by,

$$\dot{c}_t = \left(r^* - \rho + n - g_E\right)c_t - n\rho\left(q_E k_t + b_t + f_t\right) \qquad (23)$$

where c is the ratio of private consumption to domestic output, ρ the pure rate of time preference of households, n the population growth rate (which equals the rate of increase of the number of households), k the capital output ratio, b the public debt to output ratio and f the ratio of net foreign assets to output. The sum $qk+b+f$ measures net household wealth as a ratio to total domestic output. Obviously, if net foreign assets are negative (external debt is positive), then household wealth is smaller than the total value of domestic capital and public debt.

Note from (23) that if $n=0$, i.e if we have a representative household economy, a balanced growth path would only exist if,

$$r^* - g_E = \rho$$

This requires from (20) that,

$$\rho = \sqrt{(r^*)^2 - \frac{2}{\phi}\left(a\bar{A} - (r^* + \delta)(1 + \phi\delta)\right) - \delta^2}$$

Given that all the parameters above are exogenously and independently given, it is unlikely that this knife edge condition would hold. Thus, in a representative household model of a small open economy a balanced growth path would not necessarily exist.

The current account is defined as the difference between national savings and domestic investment. As a percentage of domestic output it is defined as,

$$\dot{f}_t = 1 + \left(r^* - g_E\right)f_t - c_t - c_{gt} - \left(g + \delta\right)q_E k_t \qquad (24)$$

where c_g is the ratio of public consumption to output.

The first two terms on the right hand side of (24) define national income relative to domestic output. The last three terms (private consumption, public consumption and investment) define total domestic demand (absorption). In the cost of investment we have allowed for adjustment costs, by multiplying by the equilibrium shadow price of capital. If absorption is smaller than national income, the current account is positive and the country accumulates net foreign assets. In the opposite case, the current account is negative (in deficit) and the country sees its net foreign position worsen.

We next turn to the determination of government consumption and government debt.

4.2 Public Debt and the Government Budget Constraint

The evolution of the public debt to output ratio is determined by the government budget constraint,

$$\dot{b}_t = (r^* - g_E) b_t + c_{gt} - \tau_t \qquad (25)$$

where τ is the ratio of total current (tax) revenue to output. We assume that taxes are lump sum.

Since the real interest rate exceeds the output growth rate, the debt to output ratio increases without limit, unless there is a primary suplus that exactly offsets interest payments on the existing debt.

In what follows we shall assume that the government consumption to output ratio is constant and that the government adjusts total current revenue to achieve a primary surplus that stabilizes the public debt to output ratio.

From (25), this means that,

$$\bar{b} = \frac{\tau - \bar{c}_g}{r^* - g_E} \qquad (26)$$

The bar over b and c_g denotes the government targets for public debt and public consumption respectively.

From (26), one can determine the primary surplus that is required in order to keep a constant public debt to output ratio, as a function of the world real interest rate and the growth rate of output.

It is obvious from (26) that a increase in world real interest rate, which, as we have already determined, causes a reduction in the growth rate as well, requires a corresponding increase in the primary budget surplus. If this does not happen, the public debt to output ratio diverges from the government's target, and becomes unsustainable.

In what follows, we then assume a tax revenue rule that ensures sustainability. This takes the form,

$$\tau = \bar{c}_g + (r^* - g)\bar{b} \tag{27}$$

Under this tax rule, the government debt to output ratio is kept stable, as changes in the other variables (world interest rates, the debt default premium and the growth rate) are not allowed to affect the debt to output ratio, through offsetting changes in the primary surplus.

4.3 Equilibrium Consumption and External Balance

Using the aggregate production function (4),

$$k_t = \bar{A}^{-1} \tag{28}$$

Substituting (28) in (23) and (24) and assuming a constant public debt to output ratio, we get,

$$\dot{c}_t = \left(r^* - \rho + n - g_E\right)c_t - n\rho\left(q_E \bar{A}^{-1} + \bar{b} + f_t\right) \tag{29}$$

$$\dot{f}_t = 1 + \left(r^* - g_E\right)f_t - c_t - \bar{c}_g - \left(g_E + \delta\right)q_E \bar{A}^{-1} \tag{30}$$

We can use (29) and (30) to analyse the determination of the private consumption to output ratio and external balance (the net foreign asset position of the country), as functions of the fiscal policy position and the other parameters of the model. The equilibrium is analysed in Figure 2.

To analyse the equilibrium we have assumed, without loss of generality, that,

$r^*-\rho+n-g>0$

Note that in this overlapping generations model equilibrium does not require that in equilibrium the coefficient of c_t in (29) is zero, as it would in a representative household model.

The equilibrium is determined at the intersection of the $\dot{c}=0$ and $\dot{f}=0$ equilibrium conditions.

From (29), setting $\dot{c}=0$, we get,

$$c_t=\frac{n\rho}{\left(r^*-\rho+n-g_E\right)}\left(q_E\bar{A}^{-1}+\bar{b}+f_t\right) \tag{31}$$

Equation (31) is the equilibrium condition for private consumption and we shall call it the *equilibrium private consumption locus*.

From (30), setting $\dot{f}=0$, we get,

$$c_t=1+\left(r^*-g_E\right)f_t-\bar{c}_g-\left(g_E+\delta\right)q_E\bar{A}^{-1} \tag{32}$$

Equation (32) is the long run equilibrium condition for the balance of payments. We shall call it the *external balance locus*.

When both conditions are satisfied, the economy is in long run equilibrium, with a constant private consumption to output ratio, and constant net foreign assets relative to output. The constant net foreign assets to output ratio determines external balance. The equilibrium is unique.

In the short run, the economy adjusts towards equilibrium consumption with external balance along a unique saddle path. This saddle path is depicted in Figure 4.

To the right of the long run equilibrium position private consumption is higher than what would be required for external balance and the current account is in deficit. Because of the current account deficit, the country is reducing its net foreign assets relative to output. As net foreign assets are reduced, so is the private consumption to output ratio, causing an increase in the national savings rate. As the national savings rate increases, the current

account deficit is gradually reduced, until the economy achieves long run equilibrium with external balance.

To the left of the long run equilibrium position, private consumption is lower than what would be required for external balance, the current account is in surplus, and net foreign assets gradually increase. The gradual increase in net foreign assets causes the private consumption to output ratio to rise, and the national savings rate to fall. As the national savings rate falls, the current account surplus is gradually reduced, until the economy achieves long run equilibrium with external balance.

5. The Effects of Real Interest Rate, Supply and Fiscal Policy Shocks

We can now use the full model to examine the short and long run effects of shocks to the world real interest rate, the aggregate productivity of capital and fiscal policy.

5.1 A Permanent Rise in the World Real Interest Rate

We first consider a permanent rise in the world real interest rate. As we have seen in section 3 (see also Figure 2), this causes a reduction in the investment rate and the long run growth rate. Thus, an increase in the world real interest rate also increases the differential between the real interest rate and the growth rate. The effects on external balance are analysed in Figure 5.

The equilibrium private consumption locus shifts to the right (downwards) and its slope falls. The current account equilibrium locus shifts to the left (upwards) and its slope rises. In the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to GDP are higher. This equilibrium position is approached through an adjustment process along the new saddlepath. Following the rise in the real interest rate private consumption and investment fall relative to output. The rise in savings and the fall in investment generate a current account surplus, and the country starts accumulating net foreign assets. This causes a gradual rise in the private consumption to output ratio, as the wealth of households gradually increases. The economy converges to point E´, which is the new long run equilibrium.

Note also from (27) that a rise in the world real interest rate will require an increase in domestic taxes to stop the public debt to GDP ratio from rising.

5.2 A Permanent Rise in the Productivity of Capital

We next turn to the effects of a supply shock, in the form of a permanent rise in the productivity of domestic capital. As we have seen in section 3 (see also Figure 3), this causes an increase in q, the investment rate and the long run growth rate. An increase in the productivity of domestic capital reduces the differential between the real interest rate

and the growth rate but it also reduces the capital output ratio. The effects on external balance are analysed in Figure 6.

The equilibrium consumption locus shifts to the left (upwards) and its slope rises. The current account equilibrium locus shifts to the right (downwards) and its slope falls. In the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to GDP are lower. This equilibrium position is approached through an adjustment process along the new saddle path. Following the rise in the aggregate productivity of domestic capital private investment rises relative to output. The rise in investment generates a current account deficit, and the country starts decumulating net foreign assets. This causes a gradual fall in the private consumption to output ratio, as the wealth of households gradually decreases. The process stops at point E´ which is the new long run equilibrium.

Long run growth is higher but because the increased investment has been financed through borrowing from abroad, net foreign assets fall relative to domestic output. Private consumption falls as a share of (the increased) domestic output, because the investment rate is higher in the new equilibrium, and a larger share of the increased domestic output is devoted to payments for the external borrowing that has financed the higher investment.

It is also worth noting that, from (27), an increase in the aggregate productivity of domestic capital will require a reduction in domestic taxes relative to output, if the government debt to output ratio were to remain constant. Otherwise, the government debt to GDP ratio will start falling continuously.

5.3 The Effects of Fiscal Policy Shocks

We next turn to fiscal policy shocks. We shall examine two types of shocks. First a permanent rise the government debt to output ratio, and, second a permanent rise in the government consumption to output ratio.

5.3.1. A Permanent Rise in the Government Debt to Output Ratio

Consider first a permanent one off rise in the government debt to output ratio. The analysis is in Figure 7. The equilibrium consumption locus shifts to the left (upwards) In the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to output are lower. This equilibrium position is approached through an adjustment process along the new saddle path. Following the rise in the government debt to output ratio, private consumption rises from c_E to c_0, as Ricardian equivalence does not hold in this model. Future generations are expected to share part of the burden of servicing the higher government debt. The rise in private consumption generates a current account deficit, and the country starts decumulating net foreign assets. This causes a

gradual fall in the private consumption to output ratio, as the wealth of households gradually decreases. The economy gradually converges to E´, which is the new long run equilibrium.

5.3.2. A Permanent Rise in the Government Consumption to Output Ratio

Consider next a permanent one off rise in the government consumption to output ratio. The analysis is in Figure 8. The external balance locus shifts to the right (downwards) In the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to output are lower. This equilibrium position is approached through an adjustment process along the new saddle path. Following the rise in the government consumption to output ratio, private consumption falls from c_E to c_0. However, as Ricardian equivalence does not hold in this model, the fall in private consumption is smaller than the rise in government consumption. Future generations are expected to share part of the burden of paying for the higher government consumption. Thus, domestic savings fall relative to output. This generates a current account deficit, and the country starts decumulating net foreign assets. This causes a further gradual fall in the private consumption to output ratio, as the wealth of households gradually decreases. The economy gradually converges to E´, which is the new long run equilibrium.

6. Conclusions

This paper puts forward an endogenous growth intertemporal model of a small open economy, with adjustment costs for investment and overlapping generations. The model assumes infinitely lived overlapping generations that maximize lifetime utility, and firms that maximize their net present value in the presence of adjustment costs for investment. It is also assumed, along with most of the literature, that domestic securities are perfect substitutes for foreign securities and that the economy is small, taking the world real interest rate as given.

It is shown that the equilibrium endogenous growth rate is determined solely as a function of the determinants of domestic investment, such as the world real interest rate, technology and adjustment costs, and is independent of domestic savings, the preferences of consumers and budgetary policy. Given the domestic investment and growth rate, the preferences of consumers and budgetary policy determine the current account and external balance.

This model addresses and resolves the indeterminancy problems that usually arise in endogenous growth models of small open economies with intertemporally optimizing consumers.

We use the model to analyze the effects of world real interest rate, and domestic supply and budgetary policy shocks. We demonstrate that interest rate shocks have adverse

effects on domestic growth but cause an improvement in external balance. Shocks to the aggregate productivity of domestic capital have positive effects on the growth rate but cause a deterioration in external balance. Finally, shocks in government consumption and government debt have no effects on the growth rate, but they cause a deterioration in external balance.

Figure 1
The Equilibrium Rate of Growth and the Shadow Price of Capital

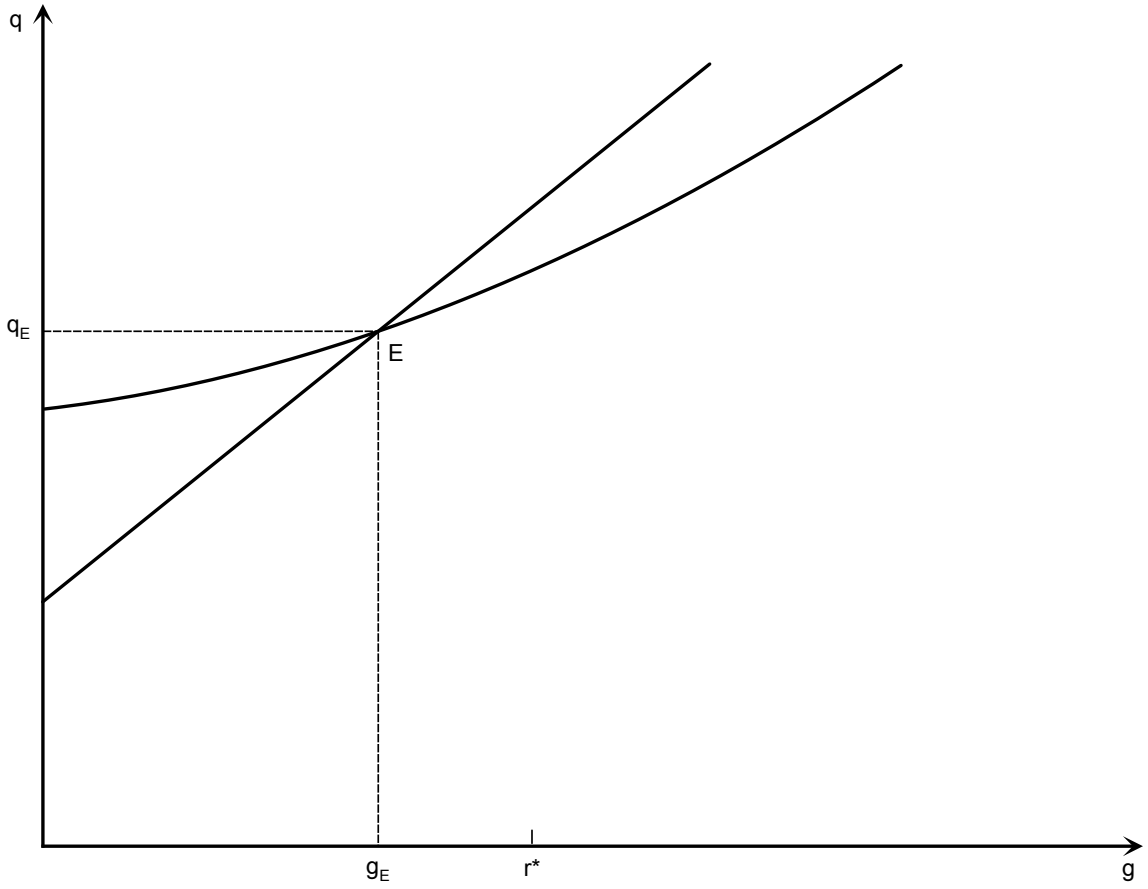


Figure 2
A Rise in the World Real Interest Rate

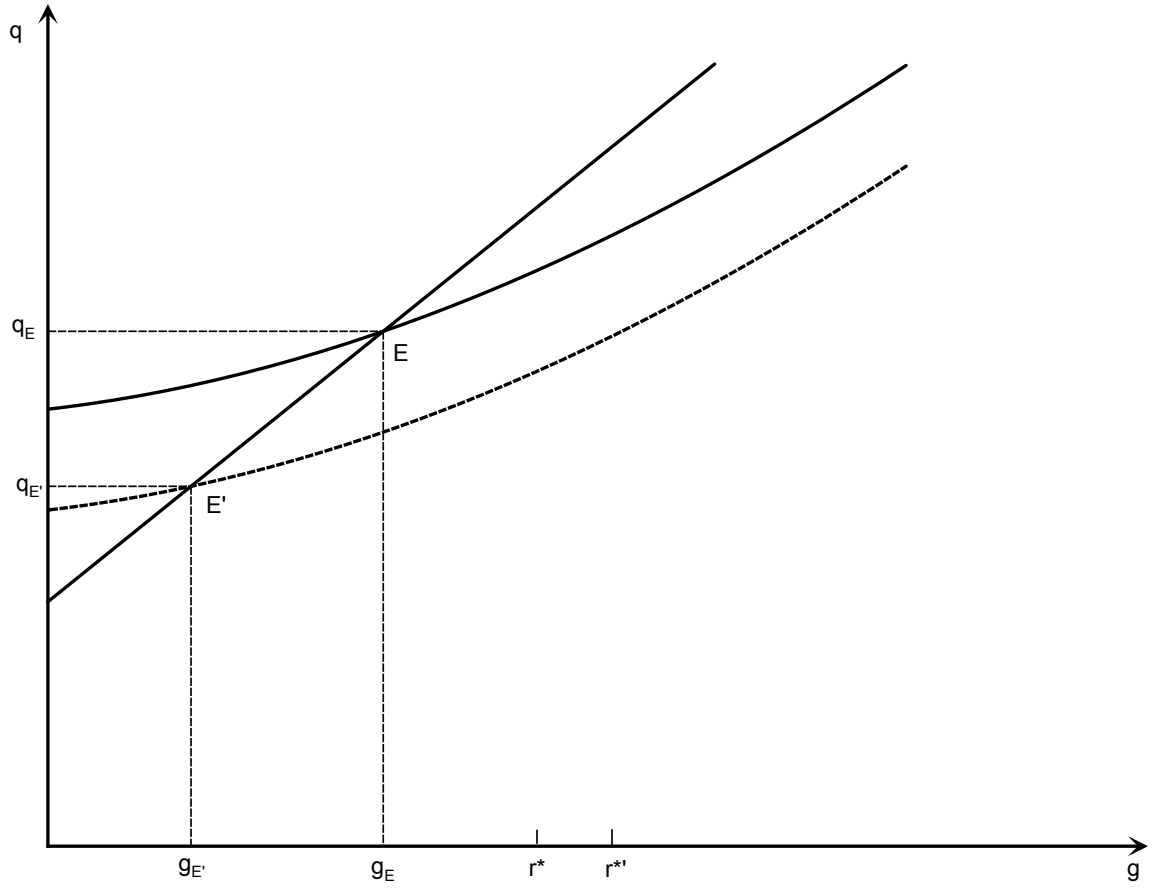


Figure 3
A Rise in the Productivity of Domestic Capital

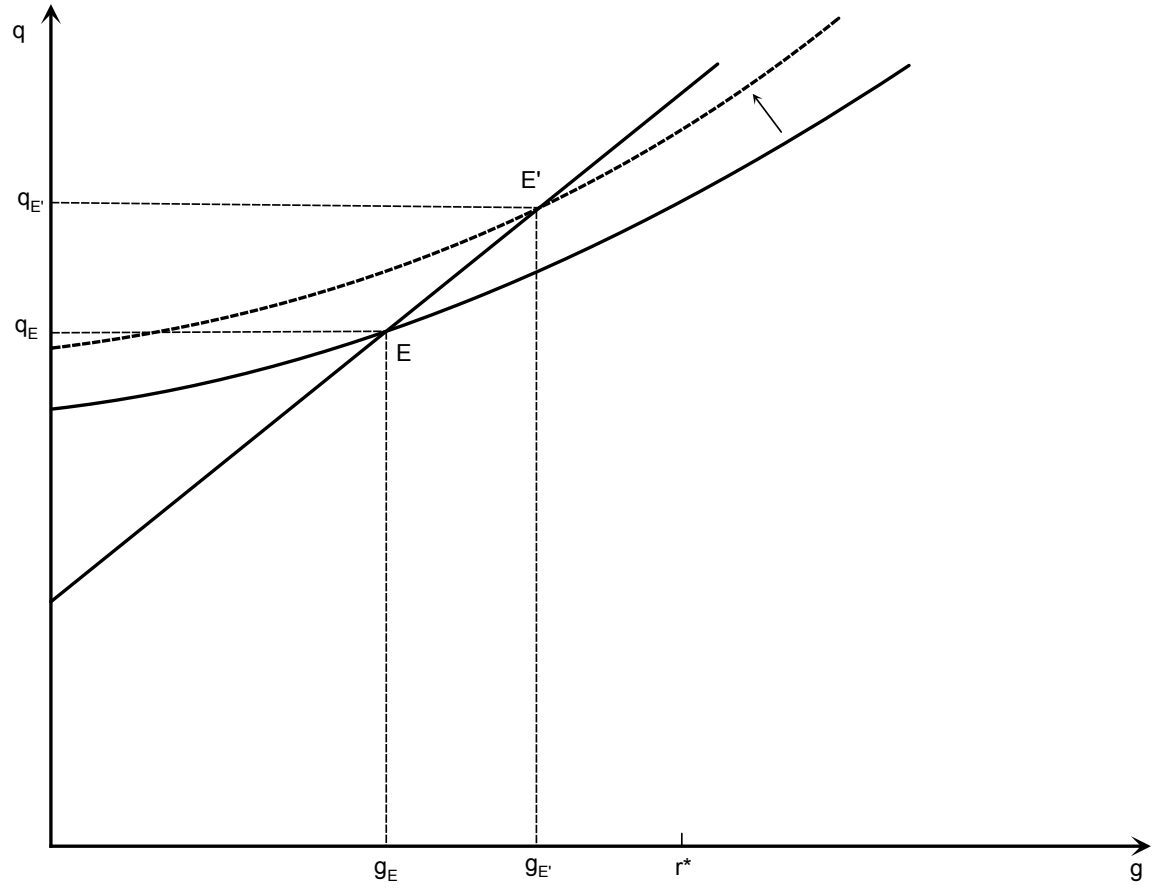


Figure 4
Private Consumption and External Balance

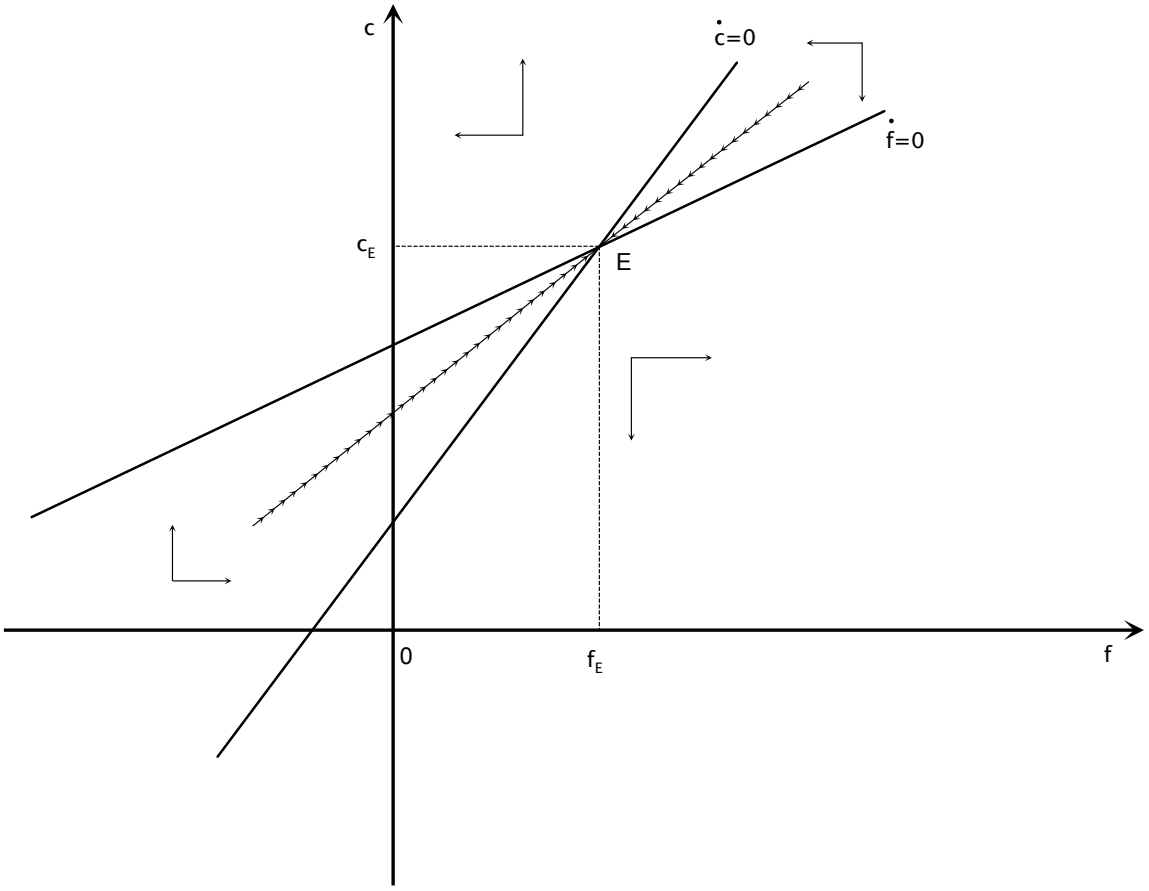


Figure 5
A Permanent Rise in the World Real Interest Rate

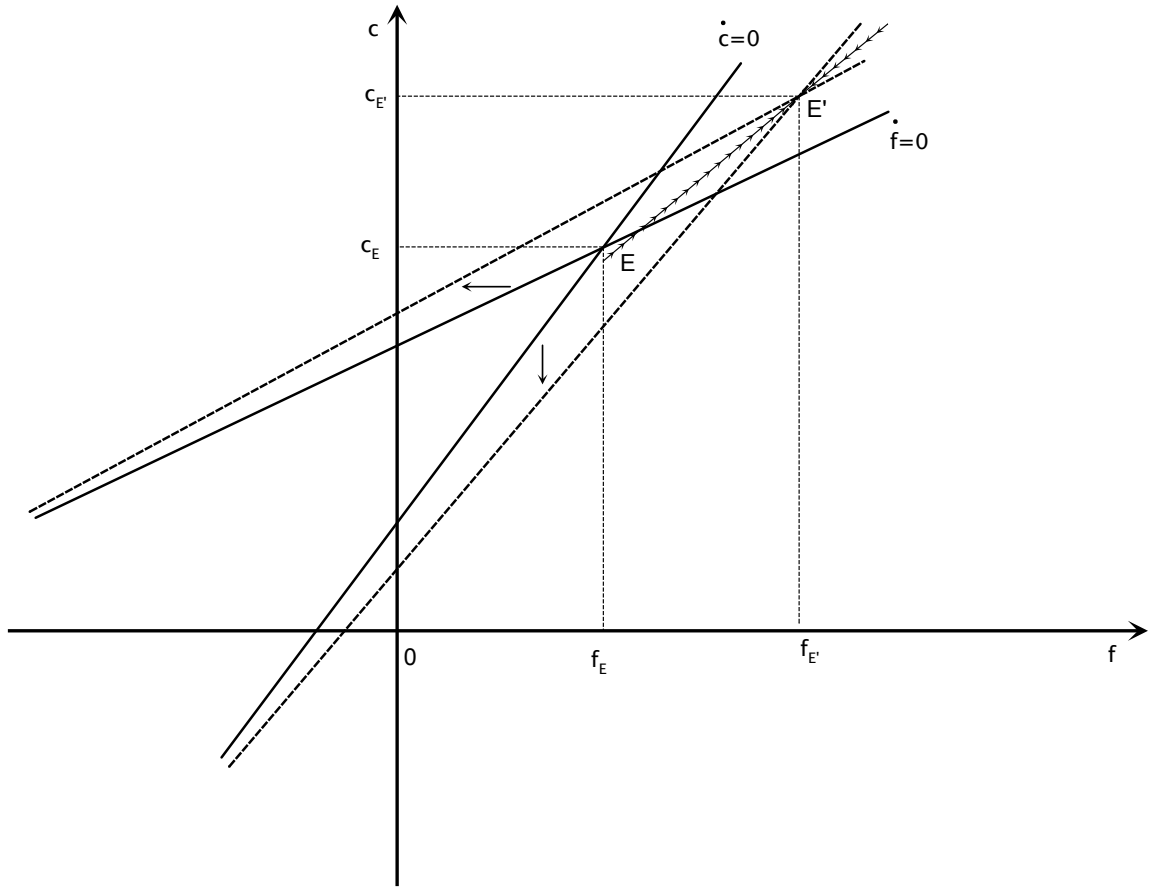


Figure 6
A Permanent Rise in the Domestic Aggregate Productivity of Capital

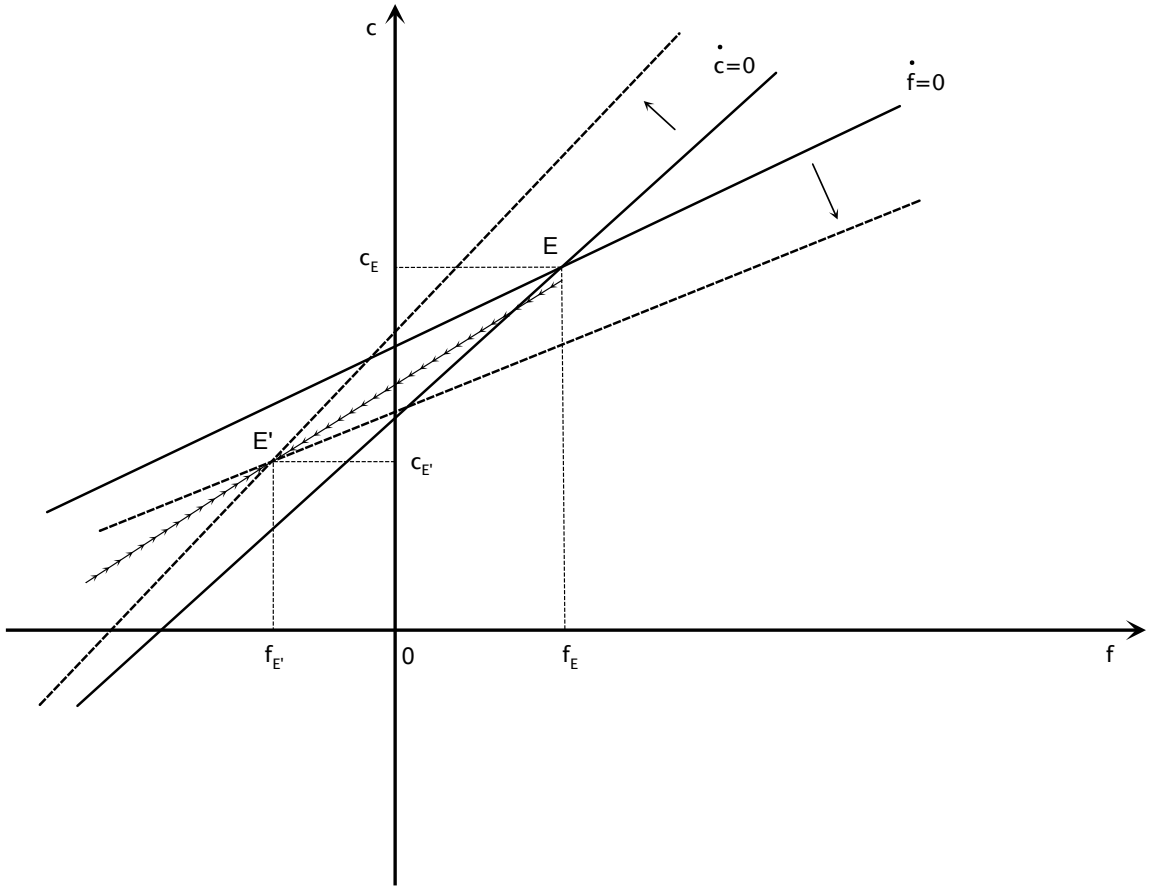


Figure 7
A Permanent Rise in the Government Debt to Output Ratio

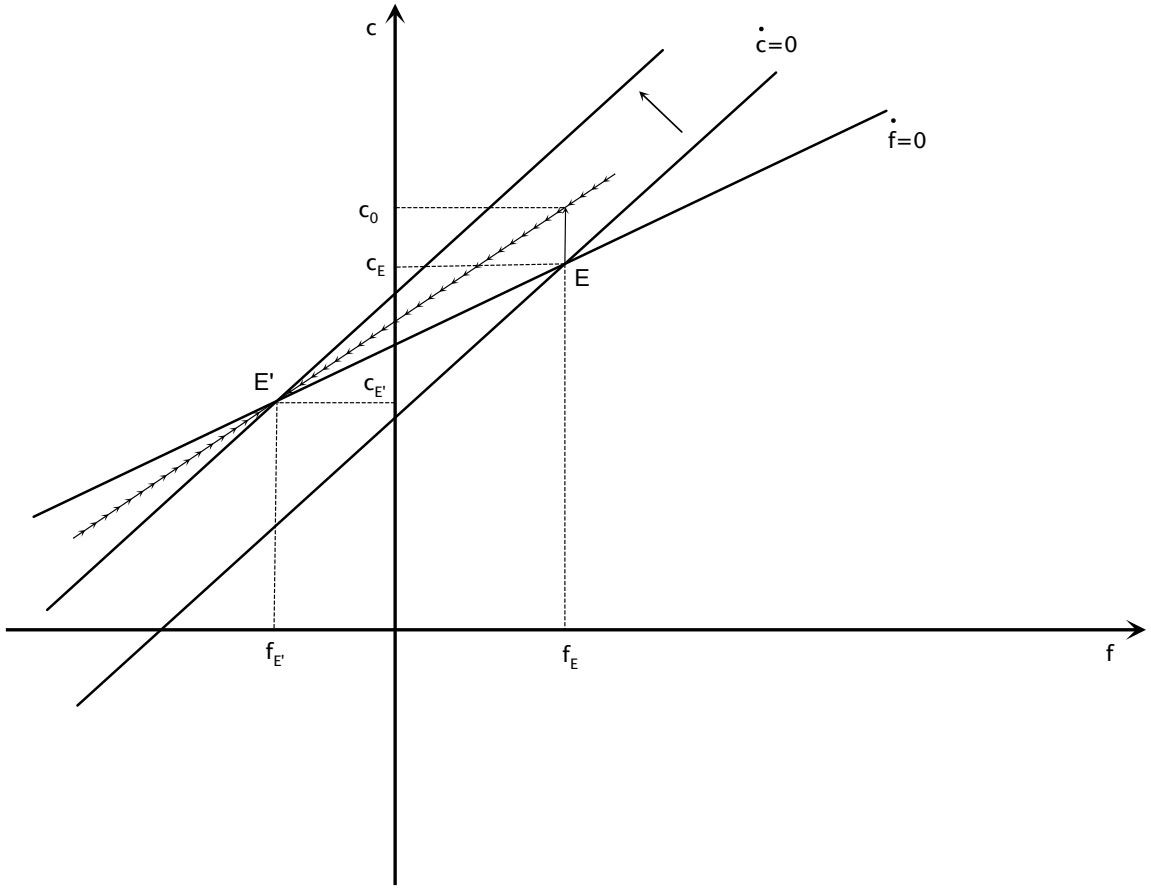
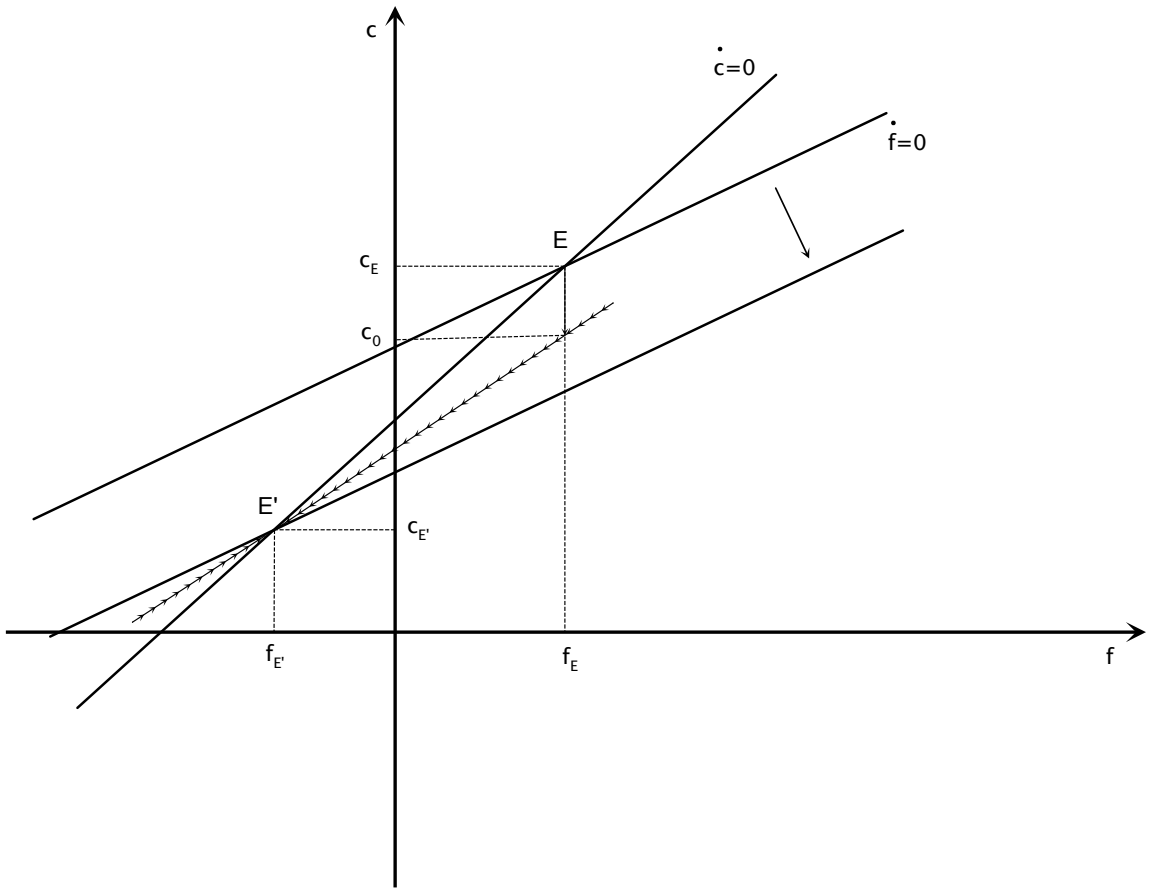


Figure 8
A Permanent Rise in the Government Consumption to Output Ratio



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