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Money, Growth and External Balance in a Small Open Economy

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Abstract

This paper puts forward an intertemporal model of a small open economy to analyze the effects of money, government debt and real shocks on growth, inflation and external balance. The model is an endogenous growth, overlapping generations model, with money in the utility function, convex adjustment costs for investment, and perfect substitutability between domestic and foreign bonds. It is shown that the growth rate depends only on the world real interest rate, the productivity of domestic capital, the adjustment cost parameter for investment and the depreciation rate. It does not depend on money, budgetary policies or the preferences of domestic consumers. Consumption of goods and services and external balance, in addition to the world real interest rate and the domestic productivity of capital, depend on money, budgetary policies and the preferences of domestic consumers. The model is used to analyze the full effects of real and monetary shocks. Monetary growth is not superneutral in this model, as it affects domestic consumption and the net foreign position of the economy.

Keywords: money, endogenous growth, external balance, current account, interest rates, government debt, monetary policy, fiscal policy
JEL Classification: F4 E4 O42 D9 E6

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1. Introduction

This paper puts forward an intertemporal model of a small open economy to study the determination of the long run growth rate and external balance and the effects of money, budgetary policies and real shocks.

The model is an endogenous growth, overlapping generations model with money in the utility function, convex adjustment costs for investment, and perfect substitutability between domestic and foreign bonds.[1]

It is shown that the model possesses a recursive equilibrium, in which the endogenous growth rate is determined solely as a function of the determinants of domestic physical capital investment, such as the world real interest rate, technology and adjustment costs. Because of the small open economy and perfect asset substitutability assumptions, the growth rate is independent of domestic savings and its determinants, such as the preferences of consumers, budgetary policies or monetary growth. Given the domestic investment and growth rate, domestic savings determine the current account and external balance.

We use the model to analyze the implications of real shocks, such as permanent changes in the world real interest rate and the domestic productivity of capital, budgetary policies, such as a permanent changes in the government consumption and the government debt to output ratios and monetary shocks, such as permanent changes in the rate of growth of the money supply.

A permanent rise in the world real interest rate results in a fall of the endogenous growth rate and an improvement in the net external position of the economy. The current account impoves, because domestic investment and private consumption fall relative to domestic output. This improvement in the current account is sustained as the economy gradually converges to a new long run equilibrium with rising private consumption and net external assets relative to output. With a constant rate of growth of the money supply, the inflation rate increases, because of the fall in the endogenous growth rate.

A permanent rise in the aggregate productivity of domestic capital, i.e a positive supply shock, causes a rise in the endogenous growth rate but also a deterioration in the equilibrium net external position. This is because the higher investment leads to a deterioration of the current account, which leads to convergence to a new long run equilibrium with lower net external assets relative to output. With a constant rate of growth of the money supply, the inflation rate falls, because of the increase in the endogenous growth rate.

[1] In Alogoskoufis (2012) we present a model without money, in order to concentrate on the real determinants of endogenous growth and external balance in this class of small open economy models.

A tax financed permanent rise in the government consumption to output ratio does not affect the growth rate, but it causes a deterioration in the current account, as the fall in private consumption does not fully compensate for the rise in government consumption. This is because Ricardian equivalence does not hold in this overlapping generations model. The deterioration in the current account leads to convergence to a new long run equilibrium, with lower net external assets and private consumption relative to output. Steady state inflation is not affected, if the rate of growth of the money supply remains constant.

A one off permanent increase in the government debt to output ratio has similar effects. It does not affect the growth rate, but it causes a deterioration in the current account, as current generations increase consumption relative to output, because Ricardian equivalence does not hold. Again, the deterioration in the current account leads to convergence to a new long run equilibrium, with lower net external assets and private consumption relative to output. Steady state inflation is not affected, if the rate of growth of the money supply remains constant.

We finally examine the effects of a permanent increase in the rate of growth of the money supply. An increase in the rate of growth of the money supply results in an equiproportional increase in steady state inflation. However, money is not superneutral in this model. Although it does not affect the growth rate, it affects private consumption, because of the substitutability between consumption and real money balances in the preferences of domestic consumers. Under the assumption that the elasticity of substitution between consumption and real money balances is not extremely high, a rise in the rate of growth of the money supply causes an initial fall in the private consumption to output ratio. This results in an improvement in the current account and the gradual accumulation of net foreign assets. In the new steady state both private consumption and net foreign assets are higher as a fraction of domestic output.[2]

The rest of the paper is organized as follows: Section 2 presents an endogenous growth model with learning by doing and adjustment costs for investment. In section 3 we derive the equilibrium growth rate and examine its dependence on the world real interest rate and the aggregate productivity of domestic capital. In section 4 we model consumption

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[2] The literature on money and growth started with Tobin (1965). Sidrauski (1967) argued in favor of the super neutrality of money, using a representative household model. The literature has since grown exponentially. The superneutrality of money highlighted by Sidrauski does not hold in overlapping generations models. van der Ploeg and Alogoskoufis (1994) examined money and endogenous growth using a closed economy model of overlapping generations, similar in many respects to the one used here. In their paper, the growth rate is determined by the savings rate, and, as monetary growth increases private savings, growth is affected positively by monetary growth. In the small open economy model of this paper, private savings do not affect the growth rate, but the net external position of the economy. Hence, money growth does not affect the growth rate. Money is not super neutral though as it affects the ratio of private consumption to income and the accumulation of net foreign assets.

and money demand. We use a continuous time overlapping generations model, with money in the utility function. In section 5 we derive the conditions for external balance, using the equilibrium condition in the goods market. External balance is defined as a constant ratio of net external assets to domestic output. In section 6 we analyze the dynamics of government debt and budgetary policies, while in section 7 we derive the relationship between monetary growth and inflation. In section 8 we analyze equilibrium in the full model, which determines the domestic real wage, the growth rate, as well as equilibrium private consumption and net foreign assets relative to domestic output. Current account dynamics arise in the adjustment towards long run equilibrium. In sections 9 and 10 we examine the effects of permanent shocks to the world real interest rate, the productivity of domestic capital and the government consumption and debt ratios on the main endogenous variables, namely the growth rate, the consumption rate, the inflation rate and net foreign assets relative to domestic output. We also analyse the dynamics of the current account. Section 11 focuses on the effects of monetary growth. The final section sums up the conclusions.

2. Adjustment Costs for Investment and Endogenous Growth

We assume a small open economy, consisting of competitive firms that produce an internationally traded good.

2.1 Production

The production function of firm i at time t is given by,

$$Y_{it} = AK_{it}^{\alpha}(h_t L_{it})^{1-\alpha}, \quad 0<\alpha<1 \tag{1}$$

where Y is output, K physical capital, L the number of employees and h the efficiency of labour. The efficiency of labour is the same for all firms.

Following Arrow (1962) we assume learning by doing. In particular we assume that the efficiency of labour is a linear function of the aggregate capital labour ratio. Thus,

$$h_t = B\left(\frac{K}{L}\right)_t, \quad 0<\beta<1 \tag{2}$$

where B is a constant, and K/L is the aggregate capital labour ratio.[3]

[3] The linearity of (2) is what makes our model an endogenous growth model. The results would hold for the transition path of exogenous growth models as well. For endogenous growth models see Romer (1986), Lucas (1988), and a number of comprehensive recent surveys such as Barro and Sala-i-Martin (2004), Aghion and Hewitt (2009), Acemoglou (2009).

Substituting (2) in (1) and aggregating, we get aggregate output as a linear function of aggregate physical capital.

$$Y_t = \bar{A} K_t \tag{3}$$

where,

$$\bar{A} = AB^{1-\alpha} \tag{4}$$

Due to the linearity of the aggregate production function, the rate of economic growth *g* is equal to the rate of net capital accumulation, which in turn is determined by the rate of investment. Therefore, we have,

$$g = \dot{Y}_t / Y_t = \dot{K}_t / K_t = (I_t / K_t) - \delta = \bar{A}(I_t / Y_t) - \delta \tag{5}$$

where I is gross investment and δ the rate of depreciation.

In this endogenous growth model, the growth rate is determined by the ratio of gross investment to domestic output.

2.2. Adjustment Costs and the Rate of Investment

Investment is determined by the profit maximizing decisions of private firms. We assume that new investment is subject to a marginal adjustment cost, which is a function of the ratio of new investment goods to total installed capital.[4]

Thus, the instantaneous profits of firms are given by,

$$Y_{it} - w_t L_{it} - \left[1 + \frac{\phi}{2}\left(\frac{I_{it}}{K_{it}}\right)\right] I_{it} \tag{6}$$

where w is the real wage and ϕ is a positive constant measuring the intensity of the marginal adjustment cost of new investment.

[4] See Lucas (1967), Gould (1968), Abel (1982) and Hayashi (1982) for models of investment with similar adjustment costs.

$\phi\left(\frac{I_{it}}{K_{it}}\right)$ is the marginal adjustment cost.

Each firm selects employment and investment in order to maximize the present value of its profits.

$$\int_{s=t}^{\infty} e^{-rs}\left(Y_{is} - w_s L_{is} - \left[1+\frac{\phi}{2}\left(\frac{I_{is}}{K_{is}}\right)\right] I_{is}\right) ds \tag{7}$$

under the constraint,

$$\dot{K}_{is} = I_{is} - \delta K_{is} \tag{8}$$

r is the real domestic interest rate.

From the first order condition for a maximum of (7) subject to (8),

$$w_t = (1-\alpha)A\left(\frac{K_{it}}{L_{it}}\right)^{\alpha} h_t^{1-\alpha} \tag{9}$$

$$q_{it} = 1+\phi\left(\frac{I_{it}}{K_{it}}\right) = 1+\phi\left(\frac{\dot{K}_{it}}{K_{it}}+\delta\right) \tag{10}$$

$$\left(r+\delta-\frac{\dot{q}_{it}}{q_{it}}\right) q_{it} = \alpha A\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} h_t^{1-\alpha} + \frac{\phi}{2}\left(\frac{\dot{K}_{it}}{K_{it}}+\delta\right)^2 \tag{11}$$

where q is the shadow price of installed physical capital.

From (9), employment is determined so that the marginal product of labour for the firm equals the real wage. Given that the real wage is the same for all firms, all firms will choose the same capital-labour ratio.

From (10), the shadow price of installed capital is equal to the marginal cost of new investment. This is equal to the cost of purchase of new capital goods, plus the marginal adjustment cost of investment.

From (11), the user cost of capital (on the left hand side) is equal to the marginal product of capital (on the right hand side). The marginal product of capital has two components: The marginal product in production of new output (the first term on the right hand side) and the reduction of the adjustment cost of future investment (the second term on the right hand side). A higher capital stock today means a smaller marginal adjustment cost for future investment.

It is worth noting that if ϕ is equal to zero (no adjustment cost for investment), then q is equal to one (from (10)). (11) then becomes the well known condition that the real interest rate r is equal to the net marginal product of capital.

Thus, if $q=1$, then, $r = \alpha A\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} h_t^{1-\alpha} - \delta$.

3. The Growth Rate and the Shadow Price of Capital

Aggregating (9) to (11), taking into account (2) to (5), we have the following aggregate first order conditions.

$$w_t = (1-\alpha)\bar{A}\left(\frac{K_t}{L_t}\right) = (1-\alpha)\left(\frac{Y_t}{L_t}\right) \tag{12}$$

$$q_t = 1+\phi\left(\frac{\dot{K}_t}{K_t}+\delta\right) = 1+\phi(g+\delta) \tag{13}$$

$$\left(r+\delta-\frac{\dot{q}_t}{q_t}\right)q_t = \alpha\bar{A}+\frac{\phi}{2}\left(\frac{\dot{K}_t}{K_t}+\delta\right)^2 = \alpha\bar{A}+\frac{\phi}{2}(g+\delta)^2 \tag{14}$$

Equation (12) determines the real wage. In the steady state, with a rising capital labour ratio and rising productivity of labour, the real wage is rising along with productivity. In fact, the real wage is a constant share of (the rising) output per worker.

Equations (13) and (14) can be used to determine the equilibrium shadow price of capital q and the equilibrium growth rate g as functions of the domestic real interest rate and the exogenous technological and adjustment costs parameters.

Using (13) to substitute for q in (14) we get,

$$\left(r+\delta\right)\left(1+\phi(g+\delta)\right)=\alpha\bar{A}+\frac{\phi}{2}\left(g+\delta\right)^{2} \tag{15}$$

In (15) we have imposed the condition of a constant q (and g), since both q and the investment and growth rates are non predetermined variables.

Equation (15) is a quadratic equation in g and has two solutions which lie on either side of r, the domestic real interest rate. Only the solution with $g < r$ is stable in the sense of satisfying the transversality condition for the maximization of the present value of profits for firms. This solution implies that the equilibrium growth rate g_E is determined by,

$$g_E = r-\sqrt{r^2-\frac{2}{\phi}\left(\alpha\bar{A}-(r+\delta)(1+\phi\delta)\right)-\delta^2} \tag{16}$$

Equilibrium q, say q_E, will be determined by substituting (16) in (13).

We have already assumed that our economy is small and open, taking world prices as given. We shall further assume that domestic assets are perfect substitutes for international assets. These two assumption imply that the domestic real interest rate is equal to the world real interest rate, which is exogenous for a small open economy. Thus,

$$r = r^* \tag{17}$$

where r^* is the world real interest rate.

Substituting (17) in (16) we end up with,

$$g_E = r^*-\sqrt{(r^*)^2-\frac{2}{\phi}\left(\alpha\bar{A}-(r^*+\delta)(1+\phi\delta)\right)-\delta^2} \tag{18}$$

In what follows we shall assume that the equilibrium growth rate is a real number, which requires that,

$$(r^*)^2 \geq \frac{2}{\phi}\left(\alpha\bar{A}-(r^*+\delta)(1+\phi\delta)\right)-\delta^2$$

Under this assumption, one can prove the following three propositions:

First, *the equilibrium growth rate does not depend on aggregate national savings.*

Proof : From (18), the equilibrium growth rate depends on the world real interest rate, the aggregate productivity of domestic capital, the depreciation rate and the parameter that determines the intensity of investment adjustment costs. These are the only factors that affect the investment rate and the growth rate. Because of the assumption of perfect asset substitutability (perfect capital mobility), aggregate national savings are not a determinant of domestic investment, unlike models without capital mobility (such as the Solow Swan (1956) model) where aggregate savings determine the investment rate.

Second, *the equilibrium growth rate depends negatively on the world real interest rate*.

Proof : From (18), the first derivative of the equilibrium growth rate with respect to the world real interest rate is given by,

$$\frac{\partial g_E}{\partial r^*} = -\frac{g_E + \frac{1}{\phi}(1+\phi\delta)}{\sqrt{(r^*)^2 - \frac{2}{\phi}\left(\alpha \bar{A} - (r^*+\delta)(1+\phi\delta)\right) - \delta^2}} < 0$$

Third, *the equilibrium growth rate depends positively on the aggregate productivity of capital*.

Proof : From (18), the first derivative of the the equilibrium growth rate with respect to the aggregate productivity of capital is given by,

$$\frac{\partial g_E}{\partial \bar{A}} = \frac{\alpha / \phi}{\sqrt{(r^*)^2 - \frac{2}{\phi}\left(\alpha \bar{A} - (r^*+\delta)(1+\phi\delta)\right) - \delta^2}} > 0$$

The difference between the world real interest rate and the endogenous growth rate is given by,

$$r^* - g_E = \sqrt{(r^*)^2 - \frac{2}{\phi}\left(\alpha \bar{A} - (r^*+\delta)(1+\phi\delta)\right) - \delta^2} \tag{19}$$

It is straightforward to show that *the real interest rate growth differential* is a positive function of the world real interest rate and a negative function of the productivity of domestic capital.

The determination of equilibrium is also depicted graphically in Figure 1.

The positively sloped straight line depicts (13). (14) is the curved line, as (14) is a non-linear relation. The only stable equilibrium is at *E*, given that the second equilibrium (not shown in the diagram) does not satisfy the transversality condition.

The position of (14) depends (among other factors such as ϕ and δ) on the world real interest rate and the aggregate productivity of domestic capital.

A rise in the real interest rate causes (14) to shift to the right (downwards). In the new equilibrium, (see Figure 2), both the growth rate and the shadow price of capital fall. Thus, a rise in the domestic real interest rate causes the domestic investment rate and the long run growth rate to fall. As a result, the differential between the real interest rate and the domestic growth rate also increases (widens).

A rise in the productivity of domestic capital causes (14) to shift to the left (upwards). In the new equilibrium, (see Figure 3), both the growth rate and the shadow price of capital rise. Thus, a rise in the productivity of domestic capital causes the investment rate and the long run growth rate to rise. As a result, the differential between the real interest rate and the domestic growth rate becomes smaller (narrows).

We shall return to the full implications of these effects when we also examine external balance.

4. Private Consumption and Money Demand

On the demand side we assume a continuous time overlapping generations model (Blanchard (1985), Weil (1987)) with money in the utility function. In this model, private consumption and money demand are determined by forward looking optimizing households (see also van der Ploeg and Alogoskoufis 1994).

We assume that the economy consists of infinitely lived households, that have been born at different times in the past. nL_t households are born at each instant, where L_t is total population at instant *t*, and *n* is the rate of growth of the number of households (and population). We shall further assume that each household supplies one unit of labor. Therefore *n* is also the rate of growth of the labor force.

4.1 Consumption and Money Demand of Households

The household born at instant *j* chooses chooses consumption and real money balances to maximize,

$$U_j = \int_{s=j}^{\infty} e^{-\rho s} \ln\left(\gamma c_{js}^{\varepsilon} + (1-\gamma) m_{js}^{\varepsilon}\right)^{\frac{1}{\varepsilon}} ds \tag{20}$$

subject to the instantaneous budget constraint,

$$\dot{a}_{js} = r_s a_{js} + w_{js} - \tau_{js} - c_{js} - \left(r_s + \pi_s\right) m_{js} \tag{21}$$

and the household's solvency (no-Ponzi game) condition,

$$\lim_{t\to\infty} e^{-\int_{s=j}^{t} r_s\, ds} a_{jt} = 0 \tag{22}$$

where ρ is the pure rate of time preference, $1/(1-\varepsilon)$ is the elasticity of substitution between consumption and real money balances, c_{js} is the consumption of household j at instant s, m_{js} are real money holdings of household j at instant s, a_{js} is non-human wealth of household j at instant s, w_{js} is non asset (labor) income of household j at instant s and τ_{js} are lump sum taxes (net of transfers) of household j at instant s. r_s is the real interest rate and π_s is the (expected) inflation rate. Instantaneous utility is assumed logarithmic in its combined arguments, implying that the elasticity of intertemporal substitution is equal to unity.[5]

Comprehensive consumption x consists of consumption of goods plus interest foregone on money holdings,

$$x_{js} = c_{js} + (r_s + \pi_s) m_{js} \tag{23}$$

Maximization of (20) subject to (21) and (22) yields an Euler equation for comprehensive consumption and the static first order condition that the marginal rate of substitution between goods and real money balances equals the opportunity cost of holding real money balances. These take the form,

$$\dot{x}_{js} = (r_s - \rho) x_{js} \tag{24}$$

$$c_{js} = \left(\frac{\gamma}{1-\gamma} (r_s + \pi_s) \right)^{\frac{1}{1-\varepsilon}} m_{js} \tag{25}$$

We can use the first order conditions (24) and (25) to derive aggregate consumption and aggregate money demand.

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[5] We are using the Weil (1987) version of the Blanchard Weil model, where, without loss of generality, the probability of death is assumed equal to zero.

4.2 Aggregate Consumption and Aggregate Money Demand

From (24) and the household's present value budget constraint it follows that,

$$x_{js} = \rho\left(a_{js} + \int_{v=s}^{\infty} \left(w_{jv} - \tau_{jv}\right) e^{-\int_{\psi=s}^{v} r_{\psi}\, d\psi}\, dv \right) \tag{26}$$

Comprehensive consumption is linear in total wealth because of the assumption of the constant elasticity of intertemporal substitution. Furthermore, as we have assumed that the elasticity of intertemporal substitution is equal to 1, the propensity to consume out of wealth is independent of the real interest rate and only depends on the pure rate of time preference.

The size of the cohort born at time *j* equals nL_j, where $L_j=exp(nj)$ is the population size at time *j*. Population aggregates are defined as,

$$X_t = n \int_{j=-\infty}^{t} x_{jt} e^{nj}\, dj \tag{27}$$

Aggregating over cohorts, assuming that newly born households do not inherit any wealth, and replacing the domestic real interest rate by the world real interest rate, yields,

$$\dot{X}_t = (r^* - \rho + n)X_t - n\rho A_t \tag{28}$$

$$\dot{A}_t = r^* A_t + W_t - T_t - X_t \tag{29}$$

$$M_t = \lambda C_t \quad , \quad \lambda = \left(\frac{\gamma}{1-\gamma}(r^* + \pi) \right)^{-\frac{1}{1-\varepsilon}} \tag{30}$$

Equations (28) and(29) determine the evolution of aggregate comprehensive consumption and aggregate household wealth.

Equation (30) is the *aggregate money demand function* in this model. Aggregate money demand is proportional to aggregate comsumption. λ*,* the ratio of real money balances to consumption, depends negatively on the nominal interest rate. The elasticity of money demand with respect to the nominal interest rate is equal to minus the elasticity of substitution between consumption and real money balances in the preferences of consumers.

Equation (30) can be used to decompose comprehensive consumption into goods demand and money demand.

From (30) and the definition of comprehensive consumption,

$$X_t = C_t + (r^* + \pi)M_t = \left(1 + \lambda(r^* + \pi)\right)C_t \tag{31}$$

From (31),

$$\dot{X}_t = \left(1 + \lambda(r^* + \pi)\right)\dot{C}_t \tag{32}$$

Combining (28), (31) and (32), the evolution of *aggregate consumption of goods and services* is determined by,

$$\dot{C}_t = (r^* - \rho + n)C_t - n\rho\left(1 + \lambda(r^* + \pi)\right)^{-1} A_t \tag{33}$$

(33) is the aggregate consumption function in this model.

We shall assume that households hold four classes of assets. Shares in domestic firms, government bonds, money and net foreign assets (foreign assets held by domestic residents net of domestic assets held by foreigners). Net foreign assets determine the external position of the economy. If net foreign assets are positive, then gross national income (GNP) is higher than gross domestic product (GDP), while if net foreign assets are negative, GNP is lower than GDP.

From (33), under these assumptions, the evolution of aggregate consumption is then determined by,

$$\dot{C}_t = (r_t - \rho + n)C_t - n\rho\left(1 + \lambda(r^* + \pi)\right)^{-1}\left(q_E K_t + B_t + M_t + F_t\right) \tag{34}$$

which, using the aggregate money demand function (30), can be rewritten as,

$$\dot{C}_t = \frac{1}{\left(1 + \lambda(r^* + \pi)\right)}\Big([(r^* - \rho + n)(1 + \lambda(r^* + \pi) - n\rho\lambda]C_t - n\rho\left(q_E K_t + B_t + F_t\right)\Big) \tag{35}$$

$q_E K$ is the real value of shares in domestic firms, B the real value of government bonds, M real money balances and F real net foreign assets.

We can express (35) in terms of shares in domestic output Y, using lowercase letters to denote such shares.

$$\dot{c}_t = \frac{1}{\left(1+\lambda(r^*+\pi)\right)}\left([(r^*-\rho+n-g_E)(1+\lambda(r^*+\pi))-n\rho\lambda]c_t - n\rho\left(q_E \bar{A}^{-1} + b_t + f_t \right)\right) \quad (36)$$

where from (3),

$$k_t = \bar{A}^{-1} \quad (37)$$

We next move on to discuss the determination of external balance.

5. The Determination of External Balance

Net foreign assets F evolve as a result of current account imbalances, i.e imbalances between national savings and investment, according to,

$$\dot{F}_t = Y_t + r^* F_t - C_t - C_{gt} - (g_E + \delta) q_E K_t \quad (38)$$

where C_g is real government consumption.[6]

Expressing (38) in terms of shares in domestic output,

$$\dot{f}_t = 1 + \left(r^* - g_E\right) f_t - c_t - c_{gt} - \left(g_E + \delta\right) q_E \bar{A}^{-1} \quad (39)$$

Equation (39) determines the current account as a share of domestic output. We shall define *external balance* as a situation in which net foreign assets are a constant fraction of domestic output. The condition for external balance to hold is,

$$\dot{f}_t = 0 \quad (40)$$

which from (39) requires,

[6] (38) is derived from the national income identity in an open economy.

$$1+\left(r^{*}-g_{E}\right)f_{t}-c_{t}-c_{gt}=\left(g_{E}+\delta\right)q_{E}\bar{A}^{-1} \tag{41}$$

Thus, for external balance to hold, national savings (on the left hand side of (41)) must be equal to domestic investment (on the right hand side of (41)).[7]

6. Government Debt and the Government Budget Constraint

The evolution of the public debt to output ratio is determined by the government budget constraint,

$$\dot{b}_{t}=\left(r^{*}-g_{E}\right)b_{t}+c_{gt}-\tau_{t}-\mu m_{t} \tag{42}$$

where τ is the ratio of total current (tax) revenue to output, and μ is the rate of growth of the money supply. We shall assume taxes are lump sum.

To the extent that the domestic real interest rate exceeds output growth, as is the case in this model, the debt to output ratio increases without limit, unless there is a primary surplus which together with seigniorage exactly offsets interest payments on the existing debt.

In what follows we shall assume that the government consumption to output ratio is constant, determined by the government, and that the government adjusts total current revenue to achieve a primary surplus that stabilises the public debt to output ratio.

From (42), this means that,

$$\bar{b}=\frac{\tau_{t}+\mu m_{t}-\bar{c}_{g}}{r^{*}-g_{E}} \tag{43}$$

The bar over b and c_g denotes the governmental targets for the public debt and public consumption to output respectively.

From (43), one can determine the primary surplus that is required in order to keep a constant public debt to output ratio, as a function of the world real interest rate, the growth rate of output and seigniorage.

[7] Our model belongs to the class of models based on the intertemporal approach to external balance. See Obstfeld and Rogoff (1995, 1996) for early surveys of models based on the intertemporal approach.

It is obvious from (43) that an increase in the world real interest rate, which, as we have already demonstrated, causes a reduction in the growth rate as well, requires a corresponding increase in the primary budget surplus. If this does not happen, the public debt to output ratio diverges from the government's target, and becomes unsustainable.

In what follows, we then assume a tax revenue rule that ensures sustainability at all times. This takes the form,

$$\tau_t = \bar{c}_g + (r^* - g_E)\bar{b} - \mu m_t \tag{44}$$

Under this tax rule, the government debt to output ratio is kept continuously stable, as changes in the other variables (the world interest rate, the growth rate, the rate of monetary growth and real money balances) are not allowed to affect the debt to output ratio. Their effects on the debt to output ratio are neutralized through offsetting changes in lump sum taxes.

7. Inflation and Monetary Growth

So far, we have not addressed the question of how the domestic inflation rate is determined. To address this question, we must examine equilibrium in the money market.

Expressing the money demand function (30) as a share of domestic output we get,

$$m_t = \lambda c_t \tag{45}$$

The change in the money supply as a share of domestic output is determined by,

$$\dot{m}_t = (\mu - \pi - g_E)m_t \tag{46}$$

Combining (45) and (46) we get,

$$\dot{m}_t = (\mu - \pi - g_E)\lambda c_t \tag{47}$$

In long run equilibrium with constant money demand and consumption as a share of output we shall have,

$$\dot{m}_t = 0 \Rightarrow \pi = \mu - g_E \tag{48}$$

We have already determined that because of the assumption of perfect substitutability between domestic and international assets, the growth rate does not depend on domestic demand factors. Hence, in the steady state, money growth only affects inflation. An increase in the rate of growth of the money supply results in an equiproportional increase in inflation.

However, as we shall show below, monetary growth is not superneutral. It has real effects in this model, since inflation affects consumption and net foreign assets through the nominal interest rate.

8. Steady State Equilibrium and Current Account Dynamics

We can now use the full model to examine general equilibrium and the adjustment of the economy towards internal and external balance.

The full model consists of the investment (and growth) function (18), the consumption function (36), the goods market equilibrium condition (current account equation) (39), the money demand equation (45), and the money market equilibrium condition (47). Assuming constant government targets for the government consumption to output ratio and the government debt to output ratio, as well as a constant rate of growth of the money supply, the model can be collected as,

$$g_E = r^* - \sqrt{(r^*)^2 - \frac{2}{\phi}\left(\alpha \bar{A} - (r^*+\delta)(1+\phi\delta)\right) - \delta^2} \tag{18}$$

$$\dot{c}_t = \frac{1}{\left(1+\lambda(r^*+\pi)\right)}\left([(r^*-\rho+n-g_E)(1+\lambda(r^*+\pi)) - n\rho\lambda]c_t - n\rho\left(q_E \bar{A}^{-1} + b_t + f_t\right)\right) \tag{36′}$$

$$\dot{f}_t = 1 + \left(r^* - g_E\right)f_t - c_t - \bar{c}_g - \left(g_E+\delta\right)q_E \bar{A}^{-1} \tag{39′}$$

$$m_t = \left(\frac{\gamma}{1-\gamma}(r^*+\pi)\right)^{-\frac{1}{1-\varepsilon}} c_t \tag{45}$$

$$\dot{m}_t = (\mu - \pi - g_E)\left(\frac{\gamma}{1-\gamma}(r^*+\pi)\right)^{-\frac{1}{1-\varepsilon}} c_t \tag{47}$$

The determination of the growth rate has already been analyzed and shown to depend only on the determinants of domestic investment, such as the world real interest rate, the aggregate productivity of capital and other supply side parameters, such as the adjustment cost parameter and the depreciation rate of capital.

Equations (36′) and (39′) can be used to determine the equilibrium consumption to output ratio and equilibrium net foreign assets relative to domestic output. The adjustment path to this equilibrium will determine the dynamics of the current account.

The equilibrium is depicted in Figure 4. To analyse the equilibrium we assume throughout that,

$r^*-\rho+n-g_E>0$, and that $r^*-g_E>0$.

These assumptions are necessary for the transversality conditions of producers and consumers to hold.

The steady state is determined at the intersection of the $\dot{c}=0$ and $\dot{f}=0$ conditions.

From (36′), setting $\dot{c}=0$, we get,

$$c_t=\frac{n\rho}{(r^*-\rho+n-g_E)(1+\lambda(r^*+\pi))-n\rho\lambda}\left(q_E\bar{A}^{-1}+\bar{b}+f_t\right) \tag{49}$$

Equation (49) is the equilibrium condition for private consumption and we shall call it the *equilibrium private consumption locus*.

From (39′), setting $\dot{f}=0$, we get,

$$c_t=1+\left(r^*-g_E\right)f_t-\bar{c}_g-\left(g_E+\delta\right)q_E\bar{A}^{-1} \tag{50}$$

Equation (50) is the long run equilibrium condition for the balance of payments. We shall call it the *external balance locus*.

When both conditions are satisfied, the economy is in steady state equilibrium, with a constant private consumption (and real money balances) to output ratio, and constant net foreign assets relative to output. The constant net foreign assets to output ratio determines external balance. The equilibrium is unique.

Equilibrium inflation is determined from (48), as the difference of the rate of growth of the money supply from the growth rate of output.

In the short run, the economy adjusts towards the steady state along a unique saddle path which is depicted in Figure 4. To the right of the steady state equilibrium position private consumption is higher than what would be required for external balance and the current account is in deficit. The country is reducing its net foreign assets relative to output. To the left, private consumption is lower, the current account is in surplus, and net foreign assets gradually increase.

The corresponding steady state equilibrium for an economy with negative net foreign assets is depicted in Figure 5.

With positive net foreign assets, national income (GNP) is greater than domestic output (GDP), while the opposite holds with negative net foreign assets. Apart from these two differences, the nature of the equilibrium is quite similar in the two cases.

We can now turn to an analysis of the macroeconomic effects of real shocks, budgetary policies and monetary growth. We shall analyze permanent shocks and how they affect both the steady state and the short run adjustment process. We start with the analysis of shocks to the world real interest rate and the productivity of domestic capital.

9. The Macroeconomic Effects of Real Shocks

We first consider a *permanent rise in the world real interest rate*. As we have seen in section 3 (see also Figure 2), this causes a reduction in q, the investment rate and the long run growth rate. Thus, an increase in the world real interest rate also increases the differential between the real interest rate and the growth rate.

The effects on external balance are analysed in Figure 6. The equilibrium private consumption locus shifts to the right (downwards) and its slope falls. The current account equilibrium locus shifts to the left (upwards) and its slope rises. In the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to GDP are higher. This equilibrium position is approached through an adjustment process along the new saddlepath. Following the rise in the real interest rate private consumption and investment fall relative to output. The rise in savings and the fall in investment generate a current account surplus, and the country starts accumulating net foreign assets. This causes a gradual rise in the private consumption to output ratio, as the wealth of households gradually increases. The economy converges to point E′, which is the new long run equilibrium.

Note also from (44) that a rise in the world real interest rate will require an increase in domestic taxes to stop the public debt to GDP ratio from rising.

Finally, with a constant rate of growth of the money supply, from equation (48), the decrease in the long run growth rate will result in a rise in steady state inflation.

We next turn to the effects of a supply shock, in the form of a *permanent rise in the productivity of domestic capital*. As we have seen in section 3 (see also Figure 3), this causes an increase in q, the investment rate and the long run growth rate. An increase in the productivity of domestic capital reduces the differential between the real interest rate and the growth rate but also reduces the capital output ratio. The effects on private consumption and the external balance are analysed in Figure 7.

The equilibrium consumption locus shifts to the left (upwards) and its slope rises. The current account equilibrium locus shifts to the right (downwards) and its slope falls. In the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to GDP are lower. This equilibrium position is approached through an adjustment process along the new saddlepath. Following the rise in the aggregate productivity of domestic capital private investment rises relative to output. The rise in investment generates a current account deficit, and the country starts decumulating net foreign assets. This causes a gradual fall in the private consumption to output ratio, as the wealth of households gradually decreases. The process stops at point E´ which is the new long run equilibrium.

Long run growth is higher but, because the increased investment has been financed through borrowing from abroad, net foreign assets fall relative to domestic output. Private consumption falls as a share of (the increased) domestic output, because the investment rate is higher in the new equilibrium, and a larger share of the increased domestic output is devoted to payments for the external borrowing that has financed the higher investment.

It is also worth noting that, from (44), an increase in the aggregate productivity of domestic capital will require a reduction in domestic taxes relative to output, if the government debt to output ratio were to remain constant. Otherwise, the government debt to GDP ratio will start falling continuously.

Finally, with a constant rate of growth of the money supply, from equation (48), the increase in the long run growth rate will result in a fall in equilibrium inflation.

10. The Macroeconomic Effects of Budgetary Policies

In this section we turn to the macroeconomic effects of permanent one off rises in the government debt to output ratio and the government consumption to output ratio.

Because of perfect substitutability between domestic and foreign bonds, such policies do not affect either the domestic real interest rate (which is equal to the world real interest rate) or the domestic growth rate (and q), which is determined by equation (18).

Consider first a permanent one off rise in the government debt to output ratio. The analysis is in Figure 8. The equilibrium consumption locus shifts to the left (upwards). In the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to output are lower. This equilibrium position is approached through an adjustment process along the new saddlepath. Following the rise in the government debt to output ratio, private consumption rises from c_E to c_0, as consumers treat the increase in government debt as an increase in wealth. Ricardian equivalence does not hold in this model, as future generations are expected to share part of the burden of servicing the higher government debt. The rise in private consumption generates a current account deficit, and the country starts decumulating net foreign assets. This causes a gradual fall in the private consumption to output ratio, as the wealth of households gradually decreases. The economy gradually converges to E′, which is the new long run equilibrium.

The growth rate is not affected by the increase in the government debt to output ratio. Since the rate of growth of the money supply remains constant, the inflation rate is not affected either.

Consider next a permanent one off rise in the government consumption to output ratio. The analysis is in Figure 9. The external balance locus shifts to the right (downwards). In the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to output are lower. This equilibrium position is approached through an adjustment process along the new saddlepath. Following the rise in the government consumption to output ratio, private consumption falls from c_E to c_0. However, as Ricardian equivalence does not hold in this model, the fall in private consumption is smaller than the rise in government consumption. Future generations are expected to share part of the burden of paying for the higher government consumption. Thus, domestic savings fall relative to output. This generates a current account deficit, and the country starts decumulating net foreign assets. This causes a further gradual fall in the private consumption to output ratio, as the wealth of households gradually decreases. The economy gradually converges to E′, which is the new long run equilibrium.

Again, neither the growth rate, nor the inflation rate are affected with a constant rate of growth of the money supply.

11. The Macroeconomic Effects of Monetary Growth

We next turn to the macroeconomic effects of a permanent rise in the rate of monetary growth μ. In the steady state, this causes an increase in the rate of inflation by the same proportion, and an increase in nominal interest rates.

The growth rate and q are not affected by the increase in monetary growth and inflation.

An increase in monetary growth and inflation will, from (30), result in a fall in λ, the ratio of real money balances to private consumption.

The change in λ following a change in the rate of inflation is given by,

$$\frac{\partial\lambda}{\partial\pi} = -\frac{1}{1-\varepsilon}\frac{\lambda}{r^*+\pi} < 0 \tag{51}$$

This is a substitution effect, between consumption and real money balances. Holding real money balances becomes more expensive with a higher inflation rate.Real money balances fall relative to consumption of goods and services.

Apart from this substitution effect, the fall in real money balances will cause a wealth effect, which will tend to depress aggregate comprehensive consumption and therefore aggregate consumption of goods and services.

The total impact of an increase in steady state monetary growth and inflation can be calculated from taking the first derivative of the steady state consumption function (49) with respect to inflation. This results in,

$$\frac{\partial c}{\partial\pi} = \frac{n\rho\left\{\frac{\lambda}{1-\varepsilon}\left(\varepsilon(r^*-\rho+n-g_E)-\frac{n\rho}{r^*+\pi}\right)\right\}}{\left((r^*-\rho+n-g_E)(1+\lambda(r^*+\pi))-n\rho\lambda\right)^2}\left(q_E\bar{A}^{-1}+\bar{b}+f\right) \tag{52}$$

From (52), the impact of an increase in inflation on the steady state consumption function will be unambiguously negative, if the elasticity of substitution between consumption and real money balances is not too large. In fact, a rise in inflation will reduce the private consumption to income ratio if,

$$\varepsilon < \frac{n\rho}{(r^*+\pi)(r^*-\rho+n-g_E)} \tag{53}$$

Condition (53) is satisfied in the case of Cobb Douglas utility, as a unitary elasticity of substitution implies $\varepsilon=0$. It is also satisfied in all cases where the elasticity of substitution between consumption and real money balances (or equivalently the interest rate elasticity of money demand) is smaller than one, as this implies $\varepsilon<0$ which satisfies (53).

In the analysis that follows we shall assume that (53) holds.[8]

In Figure 10 we analyze the effects of an increase in the rate of growth of the money supply, holding all other parameters constant. Lump sum taxes are also assumed to adjust to compensate for the change in revenue from seigniorage (see equation (44)).

A permanent increase in the rate of growth of the money supply causes an increase in steady state inflation by the same proportion. The equilibrium consumption locus shifts to the right (downwards). Consumption falls in the short term, as real money balances fall. However, in the new long run equilibrium both the consumption to domestic output ratio, and the ratio of net foreign assets to output are higher. This equilibrium position is approached through an adjustment process along the new saddlepath. Following the rise in money growth and the inflation rate, private consumption falls from c_E to c_0, as real money balances fall, causing a reduction in consumer wealth. The fall in private consumption generates a current account surplus, and the country starts accumulating net foreign assets. This causes a gradual rise in the private consumption to output ratio, as the wealth of households gradually increases. The economy converges to E′, which is the new long run equilibrium, with a higher consumption to output ratio and higher net foreign assets.

12. Conclusions

This paper has put forward an intertemporal model of a small open economy to analyze the effects of money, government debt and real shocks on growth, inflation and external balance.

The model is an endogenous growth, overlapping generations model with money in the utility function, convex adjustment costs for investment, and perfect substitutability between domestic and foreign bonds.

It is shown that the growth rate depends only on the world real interest rate, the productivity of domestic capital, the adjustment costs parameter for investment and the depreciation rate. It does not depend on money, budgetary policies or the preferences of domestic consumers.

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[8] This is also in accordance with the evidence on the interest rate elasticity of money demand for countries with moderate inflation rates.

Consumption of goods and services and external balance, in addition to the world real interest rate and the domestic productivity of capital, depend on money, budgetary policies and the preferences of domestic consumers.

The model is used to analyze the full effects of real, budgetary and monetary shocks. Monetary growth is not superneutral in this model, as it affects domestic consumption and the net foreign position of the economy.

Figure 1
The Equilibrium Rate of Growth and the Shadow Price of Capital

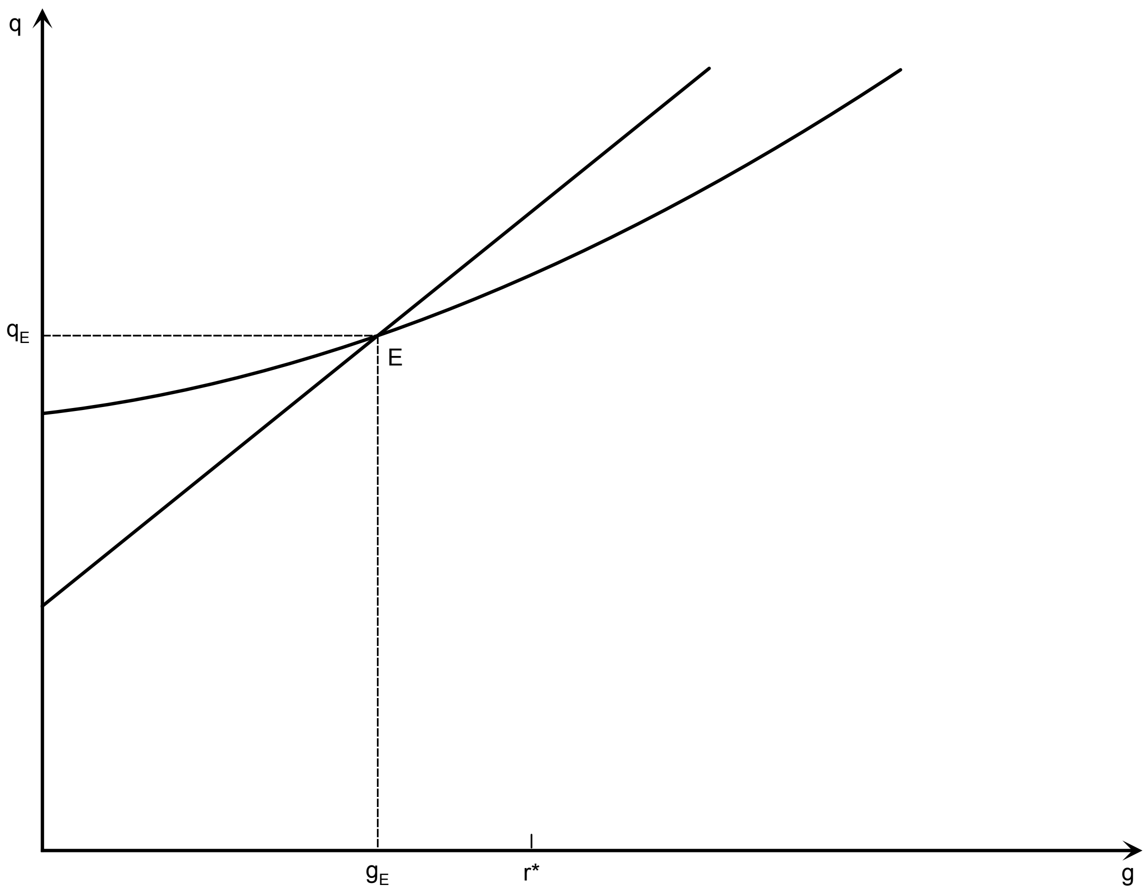


Figure 2
A Rise in the World Real Interest Rate

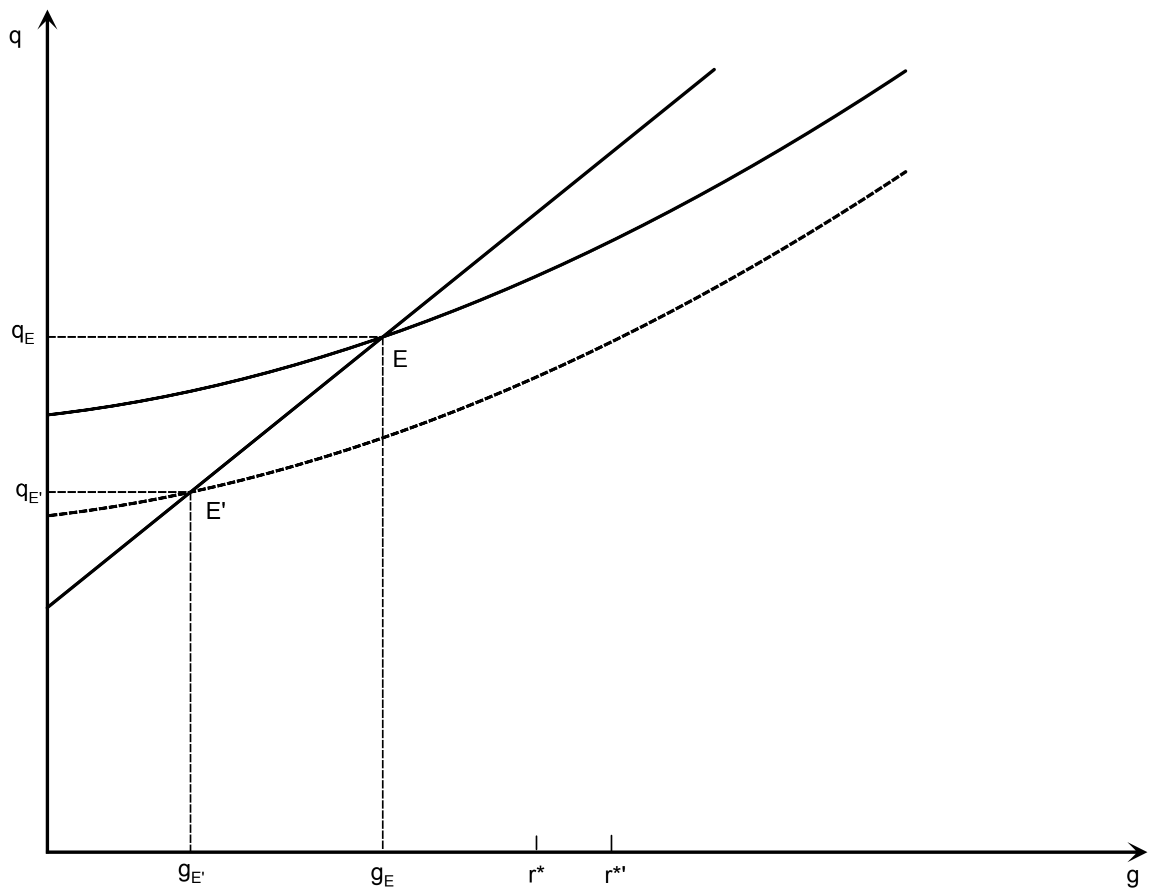


Figure 3
A Rise in the Productivity of Domestic Capital

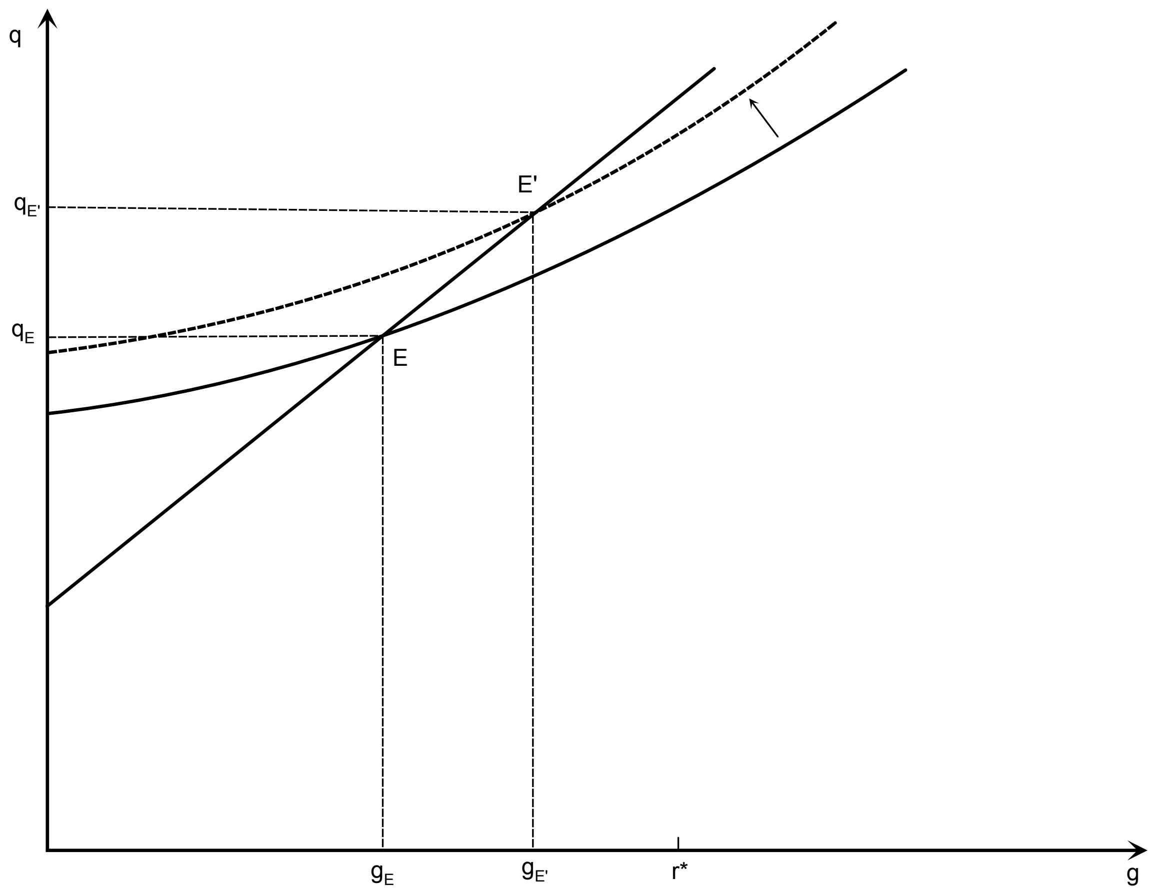


Figure 4
Private Consumption and External Balance with Positive Net Foreign Assets

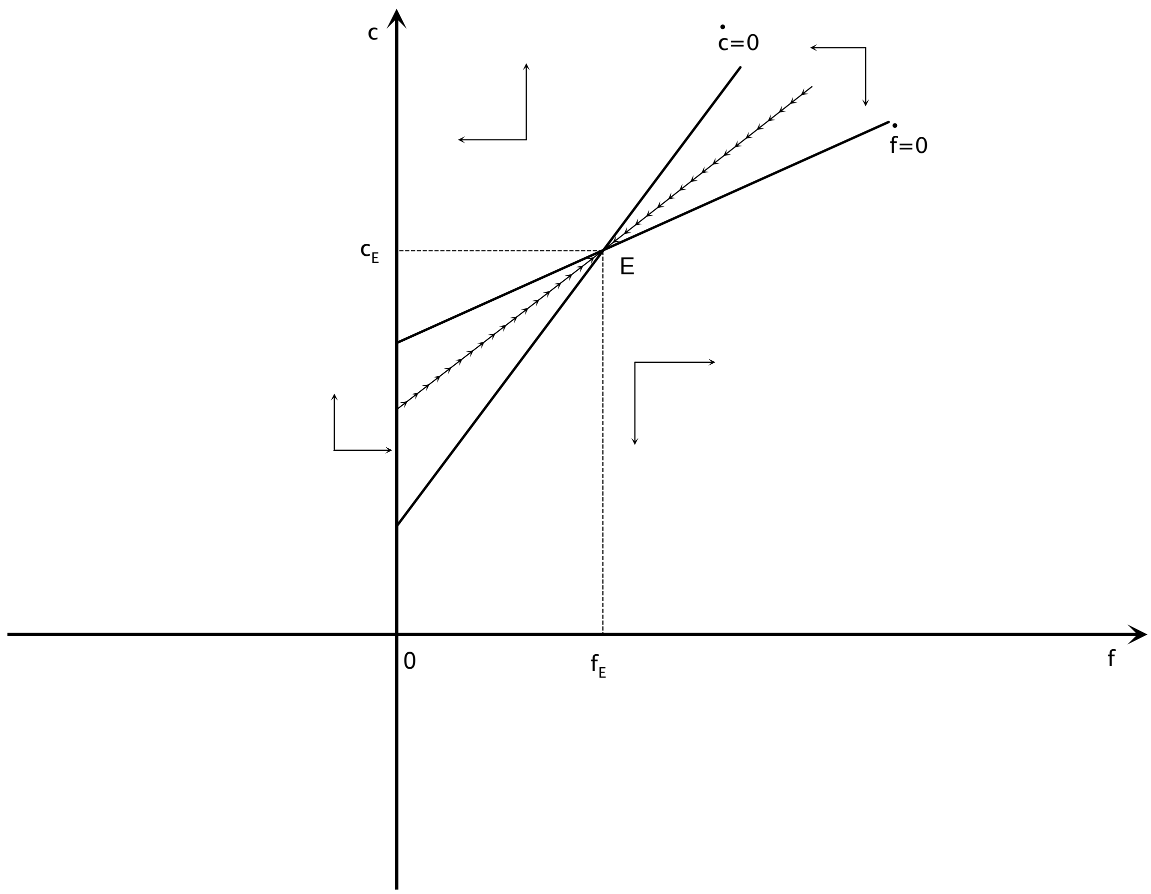


Figure 5
Private Consumption and External Balance
with Negative Net Foreign Assets

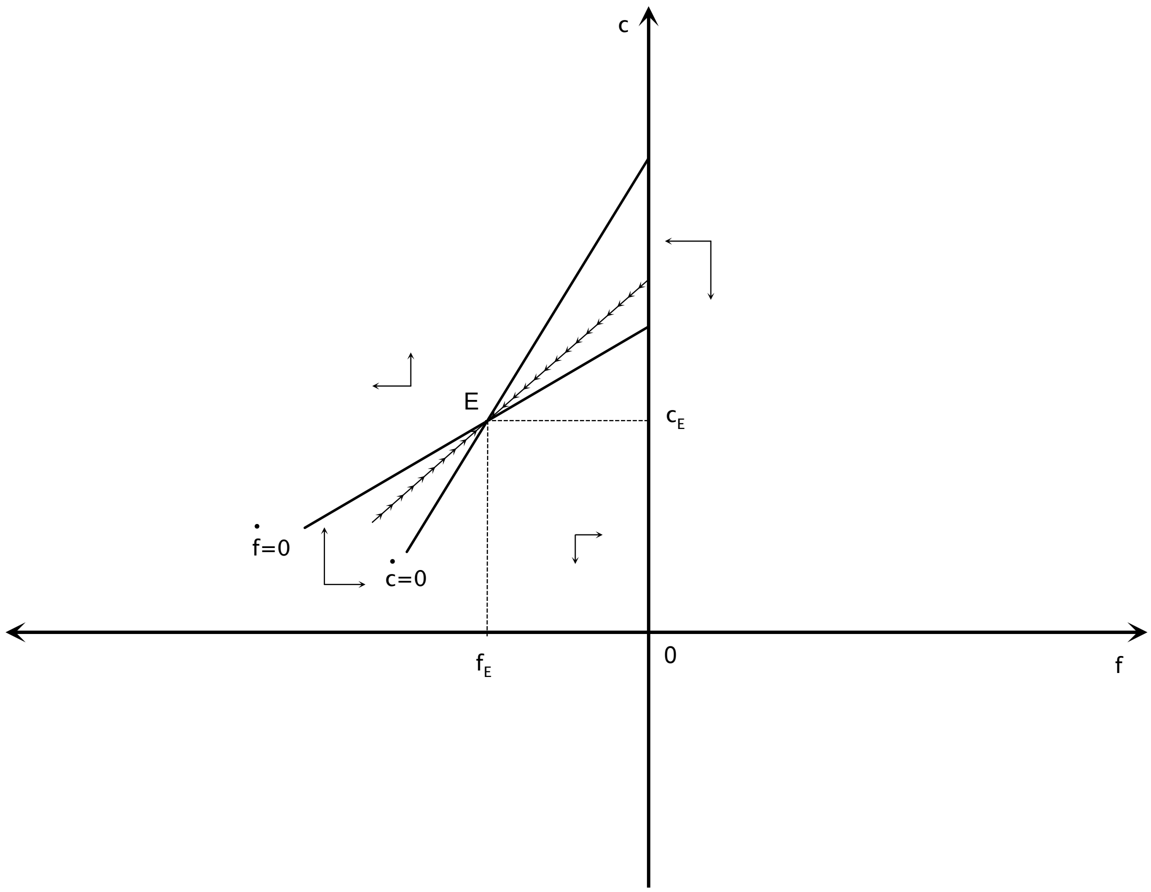


Figure 6
A Permanent Rise in the World Real Interest Rate

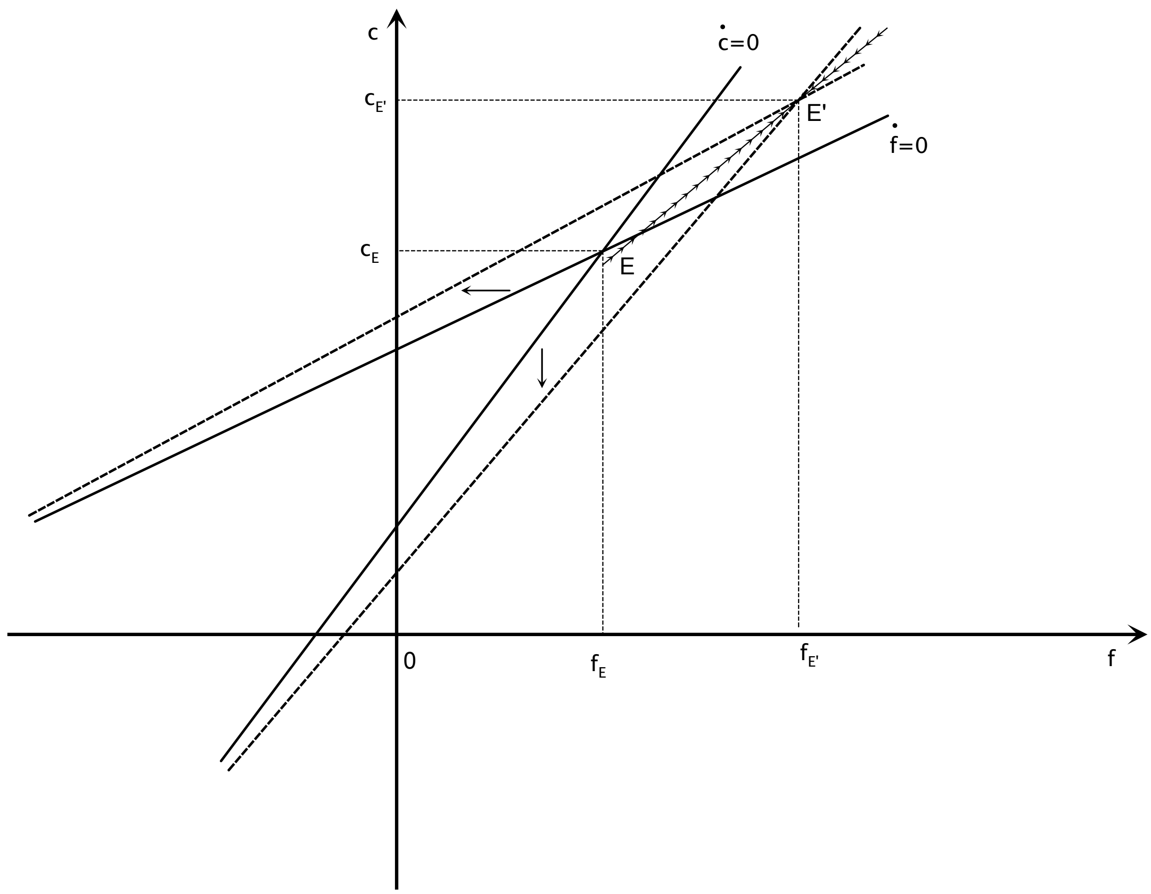


Figure 7
A Permanent Rise in the Domestic Aggregate Productivity of Capital

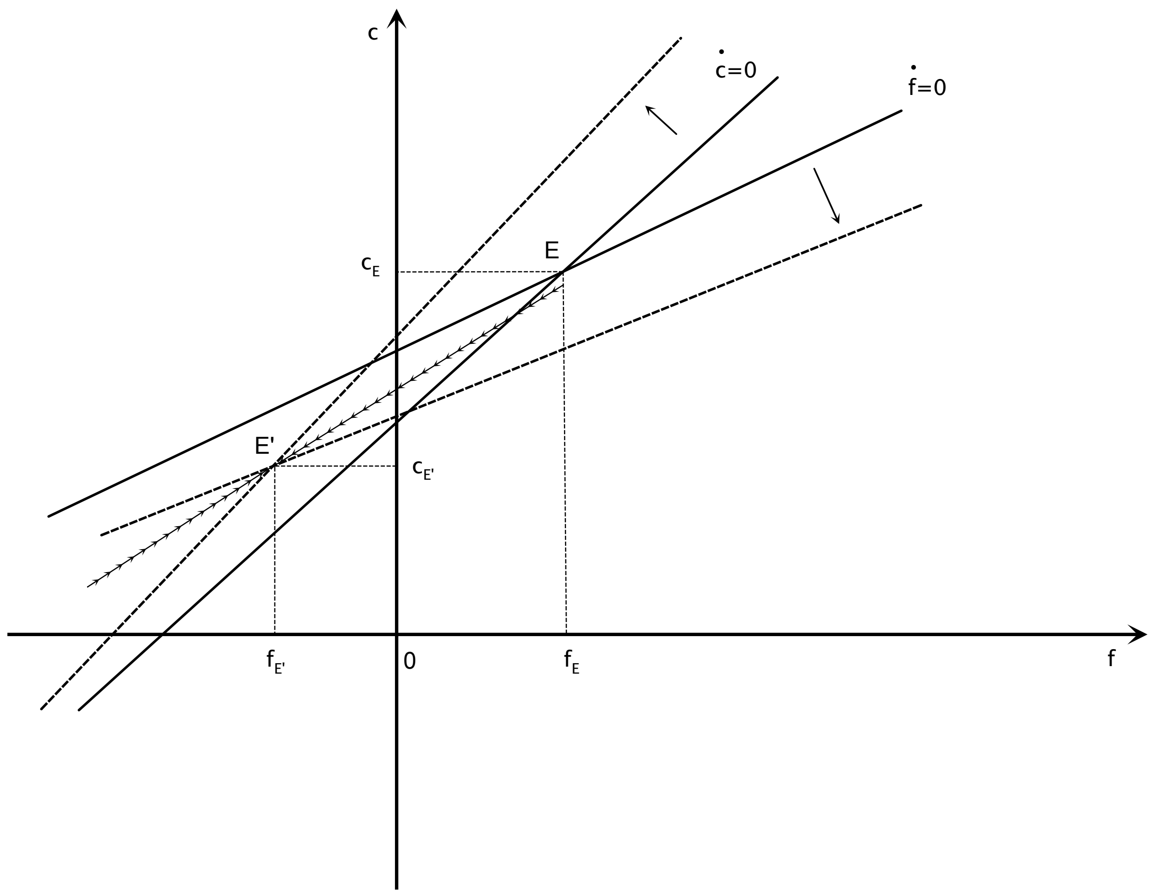


Figure 8
A Permanent Rise in the Government Debt to Output Ratio

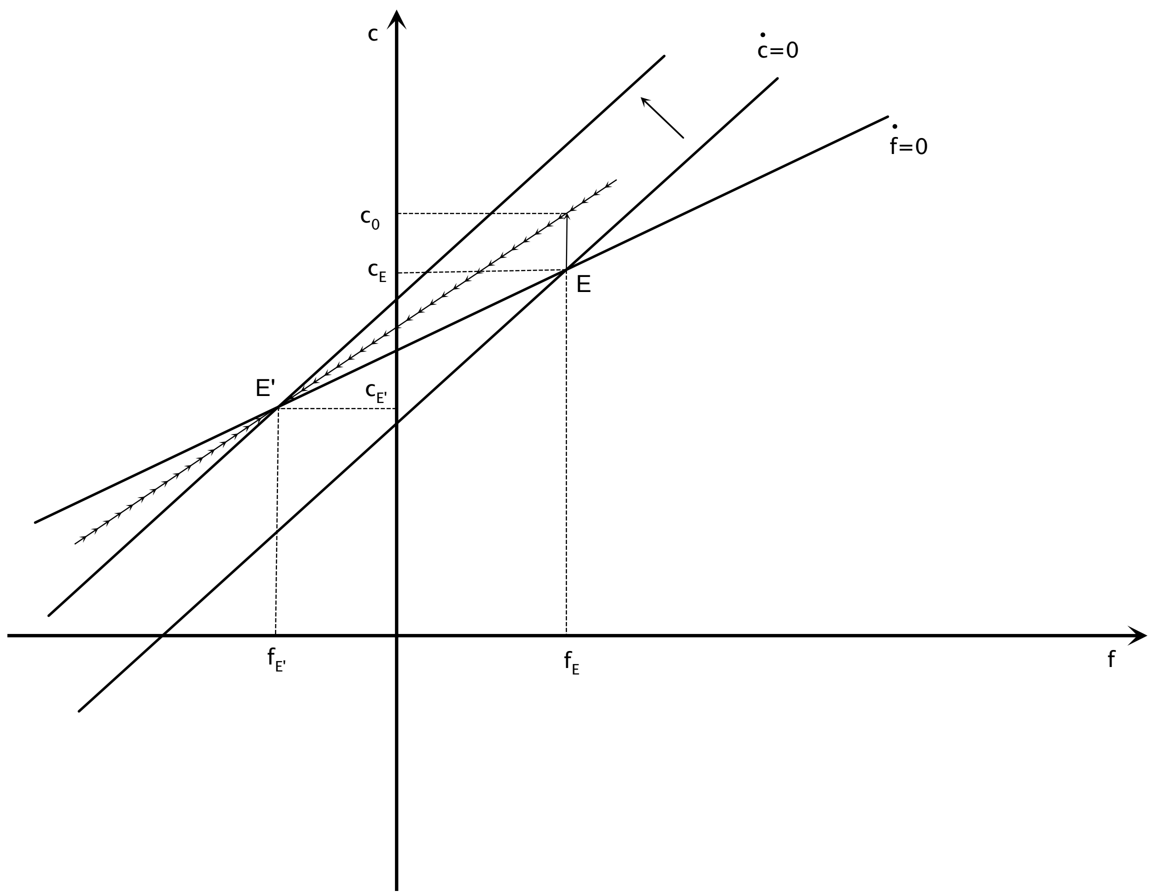


Figure 9
A Permanent Rise in the Government Consumption to Output Ratio

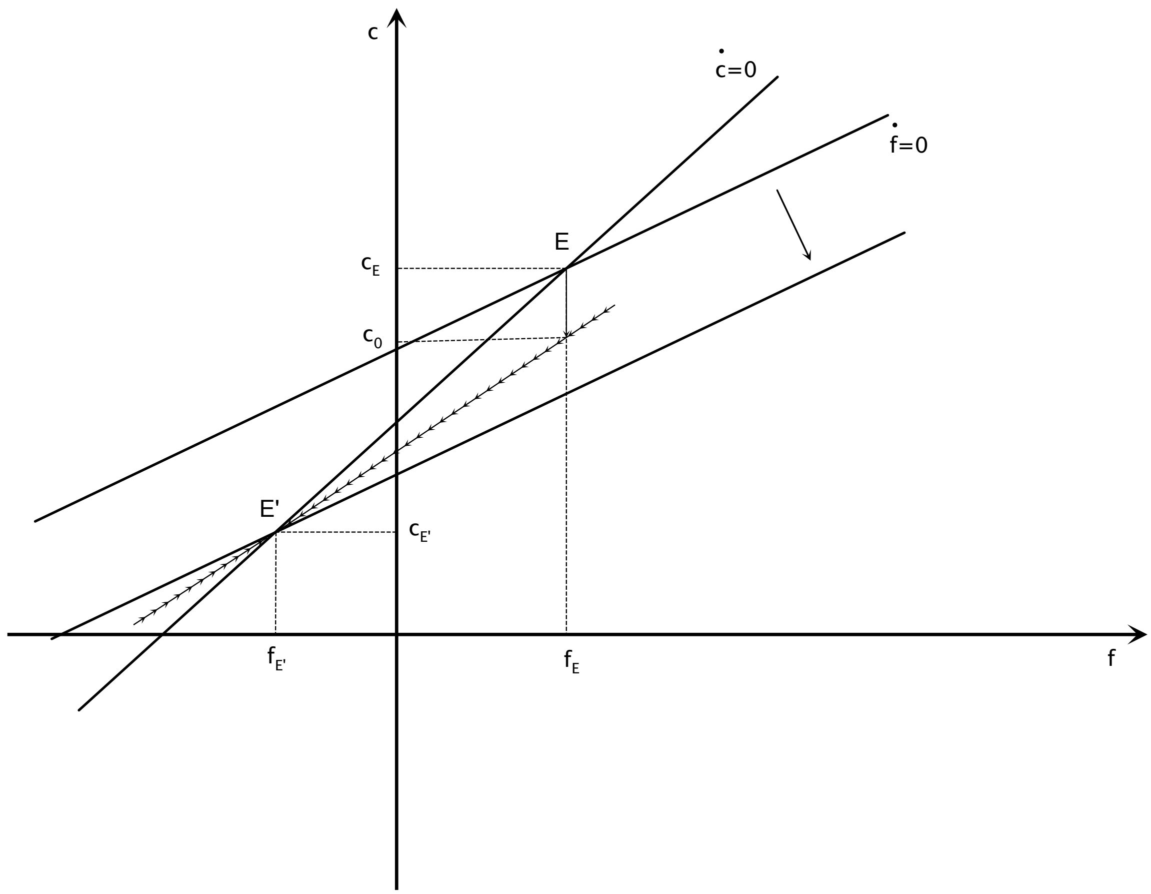
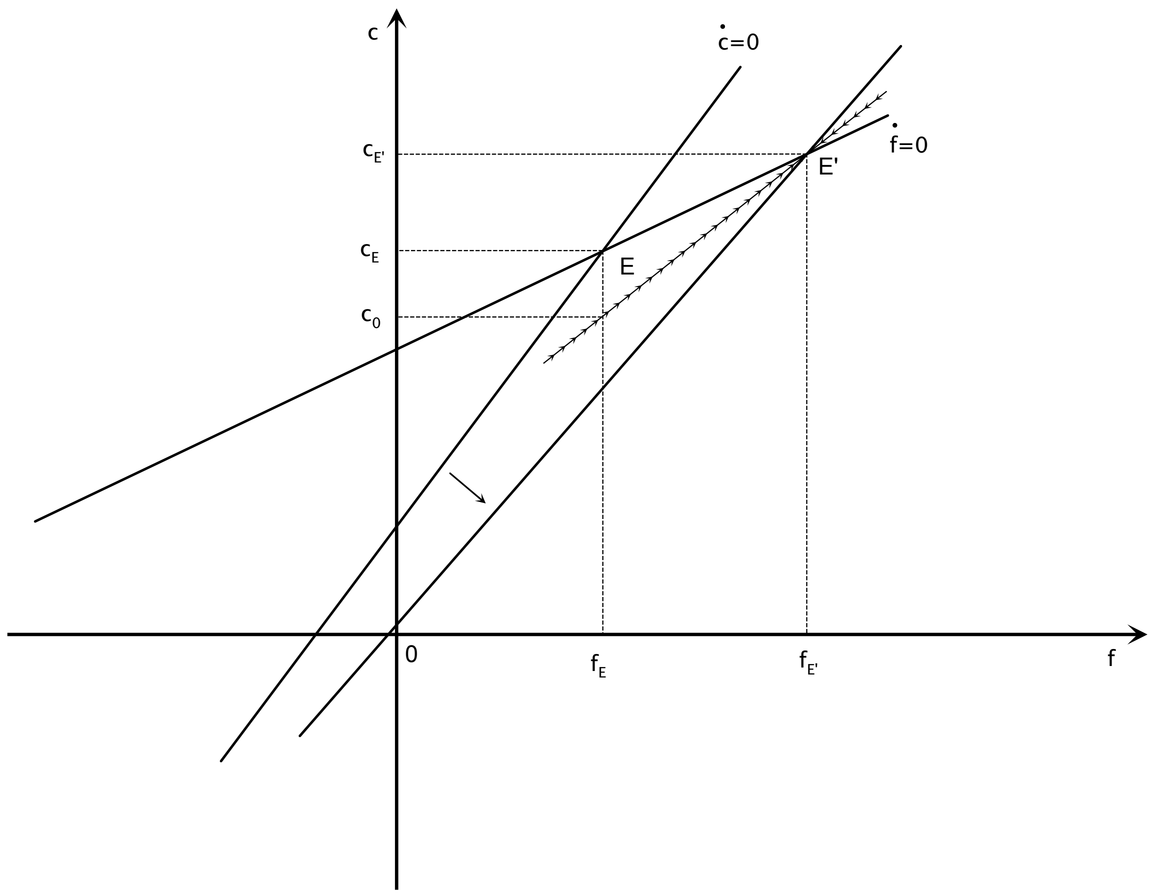


Figure 10
A Permanent Rise in the Rate of Growth of the Money Supply



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