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Retrieving inflation expectations and risk premia effects from the term structure of interest rates

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Abstract

This paper suggests an empirically attractive Gaussian dynamic term structure model to retrieve estimates of real interest rates and inflation expectations from the nominal term structure of interest rates which are net of inflation risk premium effects. The paper shows that this model is consistent with the data and that time-variation of inflation risk premium and real interest rates can explain the puzzling behavior of the spread between long and short-term nominal interest rates to forecast changes in inflation rates, especially over short-term horizons. The estimates of inflation risk premium effects retrieved by the model tend to be negative and significant, which implies that investors in the bond market require less compensation for holding nominal bonds compared to inflation-indexed bonds. This is more evident during the recent financial crisis.

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1 Introduction

There is recently growing interest in the literature in retrieving market expectations about inflation and inflation risk premium based on the term structure of nominal and real interest rates (see, e.g., Christensen, Lopez and Rudebusch (2010), D'Amico, Kim and Wei (2010), Grishchenko and Huang (2012)) or inflation swap rates (see e.g., Haubrich, Pennacchi and Ritchken (2012)). The real interest rates are obtained from inflation-indexed bonds, such as the treasury inflation protected securities (TIPS) and/or inflation swap rates. Since inflation-indexed bonds are available for long-term maturities (i.e., five years, or longer) and data on inflation swap rates start from 2003, the above studies are focused on retrieving inflation expectations and inflation risk premia from term structure data over long-term horizons. Thus, little is known about market's inflation expectations and risk premium effects over short-term horizons (i.e., up to one-year ahead), which is of great interest for monetary policy authorities on forecasting inflation, accurately. Furthermore, estimates of inflation expectations obtained from inflation-indexed bond markets are not net of the inflation risk premia effects.

To provide estimates of expected future inflation rates and inflation risk premium effects, especially over short-term horizons, in this paper we estimate an arbitrage-free, affine Gaussian dynamic term structure model (GDTSM) based on nominal interest rates, real consumption growth and inflation rate. Our model enables us to retrieve estimates for the real term structure of interest rates by fitting the GDTSM into nominal term structure and real consumption data, simultaneously. Exploring information from real consumption data can help in better capturing the dynamics of the real term structure of interest rates (see, e.g., Berardi and Torous (2005)). As in the empirical literature (see, e.g., Litterman and Scheinkman (1991)) and, more recently, Argyropoulos and Tzavalis (2012), our affine GDTSM assumes that nominal interest rates are spanned by three common factors. Two of them are unobserved and are assumed that also span the real term structure of interest rates and real consumption growth. The third factor, which spans the nominal term structure, is taken to be the current inflation rate, which is an observed variable. These assumptions are often made in the empirical literature of the term structure (see Ang and Piazzesi (2003), Dewachter and Lyrio (2006), Ang, Bekaert and Wei (2008)). Although there is little macroeconomic structure in our model[1], we specify our factor dynamics in a general way which allows for feedback and/or contemporaneous

[1] Models with more structural macroeconomic specification in the literature are found in Hordahl, Tristani and Vestin (2006) , Rudebusch and Wu (2008), among others. Also, standard new keynesian macro-finance models which encompass financial and macro variables can be found in Hordahl and Tristani (2012).

correlation effects between inflation and real interest rates. These specifications are in line with those assumed by Diebold, Rudebusch and Aruoba (2006) and Christensen et al. (2010).

To retrieve estimates of the two unobserved factors spanning the nominal and real term structure of interest rates, we rely on the approach of Pearson and Sun (1994). According to this, a number of zero-coupon (discount) interest rates are used as instruments to obtain the unobserved factors. This can be done by inverting the pricing relationship of zero-coupon bonds implied by the GDTSM. However, this approach relies on the assumption that these zero-coupon bond instruments are priced without measurement errors, which may not be true in practice. To overcome this problem, instead of observed values of interest rates, we suggest employing their projected values on principal component factors spanning the term structure of interest rates, across a very broad set of maturity intervals (see Argyropoulos and Tzavalis (2013)). Since it is based on a very large set of different maturity interest rates, principal component (PC) analysis can provide term structure factors which constitute well diversified portfolios of interest rates (see also Joslin, Singleton and Zhu (2011)). These can diminish the effects of interest rate measurement errors on the estimates of the unobserved factors of the nominal or real term structure of interest rates, considered by our model.

The results of the paper lead to a number of interesting conclusions. First, they show that our model can provide estimates of real interest rates and expected inflation rates which are very close to those provided in the literature based on survey data and/or inflation indexed bonds. Second, they indicate that inflation risk premia tend to be negative and more volatile over short-term horizons, compared to long-term ones. This is more evident during the recent financial crisis, where the magnitude of inflation risk premium is found to considerably increase. These results challenge empirical approaches based on the difference between nominal and real yields (implied by inflation-indexed bonds) to retrieve market expectations about future inflation rates. These expectations are not net of inflation risk premia effects. The negative sign of the inflation risk premium implies that investors would prefer to hold nominal bonds rather than inflation-indexed bonds. This can be attributed to the fact that the latter can be thought of as a more liquid category of assets than the former one, especially during financial crisis.

Another interesting conclusion that can be drawn from the results of the paper is that, as the maturity horizon increases, the volatility of inflation risk premium to decline, considerably. A similar conclusion can be drawn for the volatility of real interest rates, too. These results can explain the failure of the term spread between nominal interest rates to forecast future changes in inflation rates over short-term horizons, noticed by many studies in the literature (see, e.g., Mishkin (1990), and Tzavalis and Wickens (1996)). By adjusting this term spread for time-varying real interest rates and inflation risk premium effects, the paper provides clear cut evidence that we can successfully forecast future inflation rates from the nominal term structure of interest rates, as is predicted by the expectation hypothesis.

The paper is organized as follows. Section 2 presents the GDTSM and provides the closed form solution of inflation risk premia, implied by this model. In Section 3, we fit the model into the data and present estimates of its parameters, as well as real interest rates, inflation expectations and inflation risk premia effects from the data. This section also examines if the nominal term structure can successfully forecast future inflation rates, after being adjusted for real interest rates and inflation risk premium effects. Finally, Section 4 concludes the paper.

2 Model setup

2.1 Assumptions and basic relationships

In this section, we present the main assumptions and formulas of the Gaussian dynamic term structure model (GDTSM) used in our analysis.

Consider that bond prices and interest rates in the economy are driven by K-state (unobserved) variables at time t, denoted as x_{it}, stacked into a $K-$dimension column vector X_t. These variables obey the following uncorrelated Gaussian vector processes:[2]

$$dX_t = k(\theta - X_t)dt + \Sigma dW_t, \tag{1}$$

where W_t denotes a $K-$dimensional Wiener process, k and Σ are $(K \times K)$- dimension matrices of the mean reversion speed and volatility and θ is a $K-$dimensional vector of the long-run means of state variables x_{it}.

In this economy, real consumption and price levels, denoted as C_t and P_t, respectively, obey the following Gaussian processes:[3]

$$\frac{dC_t}{C_t} = \vartheta_t dt + \Sigma_c dW_t \tag{2}$$

and

$$\frac{dP_t}{P_t} = \pi_t^e dt + \Sigma_P dW_t, \tag{3}$$

where ϑ_t is the drift of the growth rate of real consumption and π_t is the instantaneous expected inflation rate. In equilibrium, ϑ_t equals the instantaneous real interest rate, denoted as r_t^*. This is the return of a

[2] See also Vasicek (1977), Dai and Singleton (2002), Ang and Piazessi (2003), Ahn (2004), Berardi and Torous (2005).

[3] See, e.g., Boudoukh (1993), Veronesi (2000), Bansal and Yaron (2004), Berardi and Torous (2005), Berardi (2009).

real bond paying one unit of consumption.[4] The instantaneous real interest rate r_t^* and inflation rate π_t^e are both affine in the state variables, i.e.,

$$r_t^* = \delta_0^* + \delta_1^{*\prime} X_t \tag{4}$$

and

$$\pi_t^e = \delta_0^\pi + \delta_1^{\pi\prime} X_t, \tag{5}$$

where δ_0^* and δ_0^π are scalars, and δ_1^* and δ_1^π are $K-$dimension column vectors of loading coefficients of state variables x_{it} on r_t^* and π_t^e, respectively. The expected inflation and real consumption growth rates from current period t to future period $t+\tau$, are also affine in state variables x_{it}. These can be written as follows:[5]

$$E_t\left[\Delta_\tau P_{t+\tau}\right] = g_0(\tau) + g_1(\tau)' X_{it} \tag{6}$$

and

$$E_t\left[\Delta_\tau C_{t+\tau}\right] = \psi_0(\tau) + \psi_1(\tau)' X_{it} \tag{7}$$

where $E_t\left[\Delta_\tau P_{t+\tau}\right] = E_t[\ln(P_{t+\tau}/P_t)]$, $E_t\left[\Delta_\tau C_{t+\tau}\right] = E_t[\ln(C_{t+\tau}/C_t)]$, $g_0(\tau)$ and $\psi_0(\tau)$ are scalars, and $g_1(\tau)$ and $\psi(\tau)$ are $K-$dimension column vectors defined as follows:

$$g_1(\tau) = (I - e^{-k'\tau})(k')^{-1}\delta_1^\pi \quad \text{and} \quad \psi_1(\tau) = (I - e^{-k'\tau})(k')^{-1}\delta_1^*.$$

In the above economy, the current, t-time price of a real bond, denoted as $B_t^*(\tau)$, paying one unit of consumption in future period $t+\tau$ can be derived by the conditional expectation of the marginal rate of substitution between periods t and $t+\tau$, i.e.,

$$B_t^*(\tau) = E_t\left(\frac{m_{t+\tau}}{m_t}\right), \tag{8}$$

where m_t denotes the instantaneous stochastic discount factor (or pricing kernel) of one unit of real consumption. m_t, is assumed that obeys the following stochastic process:

$$\frac{dm_t}{m_t} = -r_t^* dt - \Lambda_t^{*\prime} dW_t,$$

where Λ_t^* is a $(K \times 1)$ column vector of risk pricing functions associated with the innovations of each factor x_{it}, for all i. Under the assumptions of the vector of stochastic process X_t (see (1)), the conditional expectation of $\frac{dm_t}{m_t}$ at time t, is given as $E_t\left(\frac{dm_t}{m_t}\right) = -r_t^* dt$.

[4] See, e.g., Lucas (1978) and Veronesi (2000).

[5] See, e.g., Risa (2001), Berardi and Torous (2005).

The current price of a nominal bond with maturity interval τ, denoted $B_t(\tau)$, paying one dollar in period $t+\tau$ is given as

$$B_t(\tau) = E_t\left(\frac{m_{t+\tau}}{m_t}\frac{P_t}{P_{t+\tau}}\right) = E_t\left(\frac{M_{t+\tau}}{M_t}\right), \tag{9}$$

where M_t is the continuous time stochastic discount factor in nominal terms. Since real bonds can be thought as nominal bonds which pay realized inflation upon their maturity date, the real and nominal discount factors m_t and M_t, respectively, are linked through the following relationship:

$$M_t = m_t/P_t.$$

Stochastic discount factor M_t is assumed that obeys the following Gaussian process:

$$\frac{dM_t}{M_t} = -r_t dt - \Lambda_t' dW_t, \tag{10}$$

where r_t is the instantaneous nominal interest rate and Λ_t is a K-dimension column vector consisting of the risk pricing functions associated with state variables x_{it}, for all i. As r_t^*, nominal rate r_t is affine to state variables x_{it}, i.e.,

$$r_t = \delta_0 + \delta_1' X_t \tag{11}$$

where δ_0 is a scalar and δ_1 is a K-dimension column vector of loading coefficients. Since the risk pricing functions, collected in Λ_t, evaluate $K-$ independent sources of risk associated with state variables x_{it}, following Duffee (2002) we assume that Λ_t is also affine in X_t, i.e.,

$$\Lambda_t = \Sigma^{-1}\left(\lambda_0 + \lambda_1 X_t'\right), \tag{12}$$

where λ_0 is a K-dimension column vector of scalars and λ_1 is a $(K \times K)$-dimension diagonal matrix of loading coefficients, with elements $\lambda_{1,ii}$. The above assumptions of risk pricing functions imply that, under risk neutral measure Q, the risk neutral dynamics of state vector X_t can be written as follows:

$$dX_t = k^Q(\theta^Q - X_t)dt + \Sigma dW_t^Q, \tag{13}$$

where $k^Q = k + \lambda_1$ and $\theta^Q = k^{Q^{-1}}(k\theta - \lambda_0)$.

Substituting (10), (11) and (12) into (9), and assuming that nominal bonds prices $B_t(\tau)$ are exponentially affine to vector of state variables x_{it}, we can derive the following closed form solution of $B_t(\tau)$:[6]

$$B_t(\tau) = e^{-A(\tau) - D(\tau)' X_t}, \tag{14}$$

[6] See, e.g., Dai and Singleton (2002) and Fisher (2004).

where $A(\tau)$ is a scalar function and $D(\tau)$ is a K-dimension column vector, defined as $D(\tau) = (D_1(\tau), D_2(\tau),..., D_K(\tau))'$. This collects the loading coefficients of factors x_{it} on bond pricing formula (14). From the last formula, we can obtain the corresponding pricing formula of nominal discount (zero-coupon) interest rates $R_t(\tau)$, with maturity interval τ, as follows:

$$R_t(\tau) = (1/\tau)\left[A(\tau) + D(\tau)'X_t\right], \tag{15}$$

defined as the nominal term structure of interest rates. Following similar steps to the above, we can derive a pricing formula of real discount interest rates $R_t^*(\tau)$, with maturity interval τ, i.e.,

$$R_t^*(\tau) = (1/\tau)\left[a(\tau) + d(\tau)'X_t\right], \tag{16}$$

referred to as real term structure of interest rates. Note that, in practice, the dimension of the vector of state variables x_{it} spanning real interest rates can be reduced by one, or a higher number, of variables, if one assumes that real interest rates are spanned by a smaller number of factors than nominal interest rates. The same is true for instantaneous real rate r_t^*. This is an empirical matter (see, e.g., Dewachter and Lyrio (2006), Argyropoulos and Tzavalis (2012), or our empirical analysis in Section 3).

Closed form solutions of value functions $A(\tau)$, $D(\tau)$ and $a(\tau)$, $d(\tau)$ can be obtained by solving a set of ordinary differential equations under no arbitrage profitable conditions (see Duffie and Kan (1996)). For our Gaussian dynamic term structure model, described above, these solutions for the K-dimension vector $D(\tau)$ are given as follows:[7]

$$D(\tau) = \left(I - e^{-k^{Q\prime}\tau}\right)\left(k^{Q\prime}\right)^{-1}\delta_1. \tag{17}$$

These impose a set of cross-section restrictions on the loading coefficients of x_{it} on interest rates $R_t(\tau)$, given by relationship (15). Analogous to the above are the functional forms of the vector of loading coefficients $d(\tau)$, for the real interest rates relationship (16).

The GDTSM, described above, enables us to derive an analytic solution for the expected excess holding period return of a τ-period to maturity discount bond over the short-term interest rate (here, instantaneous rate r_t). This return is referred to as term premium (see, e.g., Tzavalis and Wickens (1997), Bolder (2001) and Duffee (2002)) and is given as follows:

$$E_t\left[h_{t+1}(\tau) - r_t\right] = -D(\tau)'\Sigma\Lambda_t \tag{18}$$

$$= -D(\tau)'\lambda_0 + \Gamma(\tau)'X_t, \text{ using (12)}, \tag{19}$$

[7] See Risa (2001), Dai and Singleton (2002), Kim and Orphanides (2012).

where $\Gamma(\tau)' = -D(\tau)'\lambda_1$ and $h_{t+1}(\tau)$ constitutes the one-period return of buying a nominal discount bond at time t and selling it one period after. To calculate return $h_{t+1}(\tau)$ in discrete-time, we can assume continuously compounded interest rates, implying $R_t(\tau) = -(1/\tau)\log B_t(\tau)$. Then, $h_{t+1}(\tau) - r_t$ can be written as follows:

$$hpr_{t+1}(\tau) \equiv h_{t+1}(\tau) - r_t = \log\left(\frac{B_{t+1}(\tau-1)}{B_t(\tau)}\right) - r_t = -(\tau-1)\left[R_{t+1}(\tau-1)\right] + \tau R_t(\tau) - r_t. \tag{20}$$

As noted by Argyropoulos and Tzavalis (2013), joint estimation of relationship (18) and interest rates formula (15) helps to better identify from the data the mean reversion and price of risk parameters of the model, collected in matrices k and λ_1, respectively. This happens because expected excess holding period returns $E_t\left[h_{t+1}(\tau) - r_t\right]$ are linear in λ_1, as shown by (18).

2.2 The τ-period Fisher equation and inflation risk premia

Based on the relationships presented in the previous section, in this section we will derive the relationship between nominal interest rates $R_t(\tau)$, real interest rates $R_t^*(\tau)$ and expected inflation τ-periods ahead, for all maturity intervals τ. This relationship is referred in the literature as τ-period Fisher equation. In our framework, it will be used to obtain an analytic relationship of inflation risk premium in terms of state variables x_{it} underlying nominal and real term structures of interest rates. This can be proved very useful in practice, as it can be employed to distinguish inflation expectations from inflation risk premium effects. This can not be done based on nominal interest rates and real interest rates implied by inflation-indexed bonds. The latter imply crude estimates of inflation expectations, which are not net of inflation risk premium effects.

The τ-Fisher equation can be derived by using equations (8) and (9). This implies the following relationship between nominal and real bond prices:[8]

$$\begin{aligned}
B_t(\tau) &= E_t\left(\frac{m_{t+\tau}}{m_t}\frac{P_t}{P_{t+\tau}}\right) \\
&= E_t\left(\frac{m_{t+\tau}}{m_t}\right) \times E_t\left(\frac{P_t}{P_{t+\tau}}\right) + cov\left(\frac{m_{t+\tau}}{m_t};\frac{P_t}{P_{t+\tau}}\right) \\
&= B_t^*(\tau) \times E_t\left(\frac{P_t}{P_{t+\tau}}\right) \times \left(1 + \frac{cov\left(\frac{m_{t+\tau}}{m_t};\frac{P_t}{P_{t+\tau}}\right)}{E_t\left(\frac{m_{t+\tau}}{m_t}\right) \times E_t\left(\frac{P_t}{P_{t+\tau}}\right)}\right)
\end{aligned}$$

The last relationship can be written in a more compact form as

$$B_t(\tau) = B_t^*(\tau) E_t(P_t/P_{t+\tau}) IP_t(\tau), \tag{21}$$

[8] See, e.g., Cochrane (2001), Kim and Wright (2005), Berardi (2009), Christensen et al (2010), D' Amico et al (2010).

where

$$IP_t(\tau) \equiv 1 + \frac{cov\left(m_{t+\tau}/m_t; P_t/P_{t+\tau}\right)}{E_t\left(m_{t+\tau}/m_t\right) E_t(P_t/P_{t+\tau})} \tag{22}$$

gives the definition of inflation risk premium $IP_t(\tau)$, over maturity interval τ. Taking logarithms of the last relationship and multiplying by $-(1/\tau)$ gives the τ-period Fisher equation:[9]

$$R_t(\tau) = R_t^*(\tau) + \pi_t^e(\tau) + \wp_t(\tau), \tag{23}$$

where $\pi_t^e(\tau) \equiv (1/\tau)E_t\left[\ln(\Delta_\tau P_{t+\tau})\right] = (1/\tau)E_t \ln\left(P_{t+\tau}/P_t\right)$ is the expected inflation rate, at time t, for τ-periods ahead and $\wp_t(\tau) = -(1/\tau)\ln\left(IP_t\right)$ reflects inflation risk premium effects.

Using relationships (15), (16) and (6), equation (23) implies the following closed form solution of inflation risk premium effects:

$$\wp_t(\tau) = (1/\tau)\left[A(\tau) + D(\tau)'X_t\right] - (1/\tau)\left[a(\tau) + d(\tau)'X_t\right] - (1/\tau)[g_0(\tau) + g_1(\tau)'X_t]. \tag{24}$$

This is affine to vector of state variables X_t, where $a(\tau)$ and $d(\tau)$ take analogous functional forms to $A(\tau)$ and $D(\tau)$, respectively (see 17). Finally, from equations (23) and (24) it can be clearly seen that the break-even-inflation (BEI) rate, defined in the literature as the difference between nominal and real rates, i.e., $BEI(\tau) \equiv R_t(\tau) - R_t^*(\tau) = \pi_t^e(\tau) + \wp_t(\tau)$, provides estimates of inflation expectations of the bond market which are not net of inflation risk premium effects.

3 Empirical analysis

In this section, we estimate the GDTSM presented in the previous section and retrieve inflation expectations from the nominal term structure of interest rates, $R_t(\tau)$, adjusted for inflation risk premium effects. Our analysis is organized as follows. First, we describe our data and carry out principal component (PC) analysis to estimate the unknown common factors, denoted as pc_{it}, spanning $R_t(\tau)$, for all τ. This analysis will also determine the maximum number of state variables x_{it} underlying $R_t(\tau)$, for all τ. This happens because principal component factors pc_{it} constitute portfolios of yields, driven by variables x_{it}. Next, we present efficient unit root tests for $R_t(\tau)$ to examine if these series contain a unit root in their autoregressive component. These tests are crucial in setting up the appropriate econometric framework of estimating our GDTSM from the data. Third, we estimate and test the model based on a rich set of data, which

[9] Note that in our analysis, we assume that Jensen's inequality term $-(1/\tau)[\ln(E_t(P_t/P_{t+\tau}) - E_t(\ln(P_t/P_{t+\tau}))]$ is negligible (see Buraschi and Jiltsov (2005), D' Amico et al (2010), inter alia).

consists of nominal interest rates, real consumption growth rate, inflation rate and excess holding period returns. To retrieve estimates of unobserved state variables x_{it}, underlying $R_t(\tau)$, we modify Pearson's and Sun (1994) approach, denoted as P-S. According to this approach, estimates of x_{it} are retrieved from observed values of $R_t(\tau)$, or transformations of them like term spread $R_t(\tau) - r_t$, by inverting the discount (zero-bond) interest rates relationship (15), implied by the GDTSM. Our modification of this approach is focused on minimizing the effects of possible measurement errors in nominal interest rates $R_t(\tau)$ on the retrieved estimates of x_{it}. This is done by inverting relationship (15) based on projected values of $R_t(\tau)$, or $R_t(\tau) - r_t$, on principal component factors pc_{it}. The latter constitute well diversified portfolios of interest rates (yields), as mentioned above, which diversify away measurement errors in $R_t(\tau)$ on the estimates of x_{it} (see Argyropoulos and Tzavalis (2013)). Finally, our analysis compares the estimates of the real interest rates and inflation expectations obtained by our model to those implied by inflation-indexed bonds and survey data. This part of empirical work is focused on examining how important are inflation risk premium effects in forecasting future inflation rates over short and long-term horizons.

3.1 Data

Our data consists of discount (zero-coupon) interest rates of the US economy, calculated by zero coupon or coupon-bearing bonds.[10] These series are of monthly frequency and cover the period from 1997:7 to 2009:10. They span a very large cross-section set of different maturity intervals τ, from one month to five years (60 months). Inflation rate is calculated as the seasonally adjusted 12-month percentage change of Consumer Price Index for All Urban Consumers (CPI-U), as is often assumed when pricing inflation-indexed bonds, known as TIPS (Treasury inflation protected securities). The real consumption series C_t, used in our analysis, is calculated based on the seasonally adjusted annual real personal consumption expenditures. This series is taken from the federal reserve economic data archive (FRED, see code PCE96).

3.1.1 Principal component (PC) analysis

Our PC analysis is based on a large set of different maturity nominal interest rates $R_t(\tau)$, ranging from 1 to 60 months maturity intervals. The results of our PC analysis are presented in Table 1. Table 2 presents some descriptive statistics of the estimates of principal component factors pc_{it}, obtained by our analysis. These include correlation coefficients of them with the long-term 5-years interest rate, defined as $z_{1t} \equiv R_t(60)$, and

[10] They are obtained from the data archive of J. Huston McCulloch, http://www.econ.ohio-state.edu/jhm/ts/ts.html

the term spread between this rate and the short-term one, defined as $z_{2t} \equiv R_t(60) - r_t$. The latter is found to be closely correlated with the second principal component factor spanning the nominal term structure, referred to as slope factor (see, e.g., Ang and Piazzesi (2003), or bellow).

Table 1: Number of factors pc_{it}	1	2	3
% variation explained in $\Delta R_t(\tau)$	93.20	99.17	99.99
% variation explained in $R_t(\tau)$	98.48	99.95	99.99

Notes: The table presents the percentage (%) of the total variation of nominal rates $R_t(\tau)$ explained by the number of principal component factors pc_{it}, for i ={1,2,3}. These factors are retrieved by PC analysis based on a set of N=60 nominal rates, ranging from 1 to 60 months maturity intervals.

The results of Table 1 clearly indicate that three principal components pc_{it}, for $i = \{1, 2, 3\}$, explain 99.99% of the total variation in the levels (or first differences) of the nominal term structure of interest rates $R_t(\tau)$, for all τ. The results of our PC analysis are consistent with those reported by Litterman and Scheinkman (1991) and Bliss (1997). The first principal component factor, denoted as pc_{1t}, explains the largest part of the total variation in nominal rates $R_t(\tau)$, i.e., 98.48%. This can be also confirmed by the variance and minimum (min) and maximum (max) values of this factor, reported in Table 2, which are the biggest ones, in term of magnitude, among the three principal component factors. This factor is often interpreted as level factor, as it can explain parallel shifts in $R_t(\tau)$, across all maturity intervals τ. Together with the second principal component factor, pc_{2t}, they explain the 99.95% of this variation. The remaining percentage, which is actually, very small is explained by the third principal component factor pc_{3t}. The second and third principal component factors are referred in the literature as slope and curvature factors, as they determine the slope (or term spread $R_t(\tau) - r_t$) of the nominal term structure curve and its changes, respectively. It is interesting to note at this point that principal component factors pc_{it} do not correspond one-to-one to state variables x_{it}, underlying our GDTSM, for all i. This can be justified from interest rates relationship (15), which imply that $R_t(\tau)$ and, hence, pc_{it} constitute linear transformations of x_{it}, for all i. It can be also confirmed later on by the estimates of x_{it}, obtained by fitting our GDTSM into our data.

The results of Table 2 indicate that the first two principal component factors pc_{1t} and pc_{2t} are very highly correlated with observed variables z_{1t} and z_{2t}, namely $R_t(60)$ and $R_t(60) - r_t$, respectively. Thus, they can capture most of the time-variation of pc_{1t} and pc_{2t}. These results indicate that z_{1t} and z_{2t} should be employed as the right choice of interest rates variables (instruments) in obtaining estimates of unobserved state variables x_{it} from our data, exploiting interest rates pricing relationship (15) and applying our extension of P-S methodology, described before.

Table 2: Summary statistics of principal component factors pc_{it}

	pc_{1t}	pc_{2t}	pc_{3t}
Mean	0.00	0.00	0.00
Variance	152.14	2.28	0.25
Min	-22.87	-3.07	-0.52
Max	21.92	5.38	1.70
		Correlation Coefficients	
z_{1t}	0.97	0.23	0.07
z_{2t}	-0.80	0.70	0.04

Notes: The table presents summary statistics of principal component factors pc_{it}. Max stands for the maximum value of pc_{it}, while Min. for the minimum. Variables z_{1t} and z_{2t} are defined as follows: $z_{1t} \equiv R_t(60)$ and $z_{2t} \equiv R_t(60) - r_t$, where r_t is the one-month interest rate.

3.1.2 Unit root tests

To test for a unit root in the level of nominal interest rates $R_t(\tau)$, we carry out a second generation ADF unit root test, known as efficient ADF (E-ADF) test (see, e.g., Elliot, Rothenberg and Stock (1996) and Ng and Perron (2001)). This test is designed to have maximum power against stationary alternatives to unit root hypothesis which are local to unity. Thus, it can improve the power performance of the standard ADF statistic, often used in practice to test for a unit root in $R_t(\tau)$.

The values of E-ADF unit root test statistic are reported in Table 3. This is done for interest rates $R_t(\tau)$, with maturity intervals $\tau = \{1, 3, 6, 12, 24, 36, 48, 60\}$ months. Note that, in addition to E-ADF, the table also presents values of P_T unit root test statistic, suggested by Elliott et al. (1996) as alternative to E-ADF. To capture a possible linear deterministic trend in the levels of $R_t(\tau)$, occurred during our sample, both E-ADF and P_T statistics assume that the vector of deterministic components D_t employed to detrended series $R_t(\tau)$ contains also a deterministic trend.

Table 3: Efficient unit root tests for interest rates $R_t(\tau)$

τ :	1	3	6	12	24	36	48	60
ϕ	0.99	0.99	0.99	0.98	0.98	0.98	0.97	0.97
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)
E-ADF	-2.13	-2.12	-2.10	-2.26	-2.21	-2.11	-2.09	-2.08
P_T	4.95*	5.04*	5.19*	3.53**	3.84**	4.69*	5.13*	5.11*

Notes: The table presents unit root tests for interest rates $R_t(\tau)$, across different maturity intervals τ. ϕ denotes the autoregressive coefficient of the auxiliary regression, employed to carry out the tests. Standard errors are in parentheses. E-ADF and P_T are the efficient unit root test statistics suggested by Elliott et al. (1996). Critical values of these test statistics are provided by Elliott et al. (1996). (*) and (**) mean significance at 5% and 1% levels, respectively.

The results of Table 3 clearly indicate that, despite the fact that the values of the autoregressive coefficients ϕ of the auxiliary regressions employed to carry out the tests are found to be very close to unity, the unit root hypothesis can not be rejected against its stationary alternative, for all $R_t(\tau)$ considered. This is true at 5%, or 1% significance levels. The estimates of the autoregressive coefficient ϕ reported in the table indicate that interest rates $R_t(\tau)$ exhibit a very fast mean reversion towards their long-run mean, especially those of shorter maturity intervals (i.e., $1, 3$ and 6 months). These results indicate that $R_t(\tau)$ constitute stationary series. Thus, standard asymptotic theory can be applied to conduct inference on the parameters of our GDTSM, presented in Section 2.

3.2 Econometric specification and estimation of the GDTSM

To estimate the GDTSM presented in Section2, we make the following assumptions, also made in the literature (see introduction). First, given the results of our PC analysis, we assume that the number of state variables x_{it} underlying our model is $K = 3$. Second, we assume that the first two of these two state variables, i.e., x_{1t} and x_{2t}, jointly span nominal interest rates $R_t(\tau)$, for all τ, and real consumption growth rate, defined as $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$. These two factors are collected in the 2-dimension column vector $X_t^* = (x_{1t}, x_{2t})'$. As in Ang and Piazzesi (2003) and Diebold et al. (2006), the third state variable x_{3t} will be taken to be inflation rate π_t, which is an observed variable. Thus, the vector of state variables X_t is specified as follows: $X_t \equiv (X_t^*, \pi_t)'$. This specification of X_t allows us to capture any feedback and/or contemporaneous effects between the vector of unobserved variables $X_t^* = (x_{1t}, x_{2t})'$, determining real consumption growth, and inflation rate π_t. It can thus provide short-run forecasts of future inflation rate π_t, without assuming othogonality between real and nominal variables. Finally, we assume that the loading coefficients of x_{1t} and x_{2t} on real and nominal short-term interest rates r_t^* and r_t are the same, for the first two state variables. That is, we have $\delta_{11}^* = \delta_{11}$ and $\delta_{12}^* = \delta_{12}$, while δ_{13} is the loading coefficient of the inflation factor π_t.

The system of equations employed to estimate the GDTSM is based on the following relationships of Section 2: (1), (6), (7), (15) and (18). Below, we write these relationships in regression form as follows:

$$\Delta X_{t+1} = const + (\Phi - I)X_t + \omega_{t+1} \tag{25}$$

$$\Delta R_{t+1}(\tau) = const + D(\tau)' E_t(\Delta X_{t+1}) + e_{t+1}(\tau) \tag{26}$$

$$\Delta c_{t+1} = const + \psi_1(\tau)' X_t^* + \xi_{t+1}, \text{ and} \tag{27}$$

$$hpr_{t+1}(\tau) = const + \Gamma(\tau)' X_t + \varsigma_{t+1}(\tau), \tag{28}$$

where I is the identity matrix of dimension (3×3),

$$\Delta X_{t+1} \equiv \begin{bmatrix} \Delta x_{1t+1} \\ \Delta x_{2t+1} \\ \Delta \pi_{t+1} \end{bmatrix} \text{ and } \Phi \equiv \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix},$$

with diagonal elements defined in terms of continuous-time mean-reversion parameters as $\phi_{ii} = e^{-k_i \Delta t}$, for all i, and ω_{t+1}, $e_{t+1}(\tau)$, ξ_{t+1} and $\varsigma_{t+1}(\tau)$ constitute scalars (or a vector in case of ω_{t+1}) of error terms.

The above system, given by equations (25)-(28), consists of four different sets of simultaneous regressions. The first set, which captures the dynamics of vector of state variables ΔX_{t+1} (see (25)), assumes that the matrix of autoregressive coefficients Φ of X_t is not diagonal. This allows for possible feedback effects between all variables of vector X_t. In the estimation, the elements of the vector of error terms ω_{t+1} are also allowed to be correlated to each other. The above specification of vector ΔX_t also preserves the structure of inflation rate relationship (5), assumed by our GDTSM.

The second set of regressions of the above system (see 26) corresponds to nominal interest rates relationship (15), augmented with error terms $e_{t+1}(\tau)$. These errors can be taken to reflect possible measurement errors of interest rates $R_t(\tau)$ in relationship (15). These errors may be quite substantial for long-term discount interest rates (i.e., for $\tau > 12$ months), as these rates are approximated by fitting spline functions (or by applying dynamic programming methods) to non zero-coupon bond prices with very long maturity intervals, which are less liquid assets. Note that regression (26) is given in first-differences of its variables, $\Delta R_t(\tau)$. This is done in order to directly accommodate estimates of the expected values of their independent variables (i.e., $E_t(\Delta X_{t+1})$). The latter are obtained by simultaneously estimating all sets of regressions of the system. Note that, for $\tau = 1$ month, (26) gives relationship (11), for the short-term nominal interest rate r_t.

Finally, the third and fourth set of regressions of the system (i.e., equations (27) and (28)), correspond to real consumption and expected excess returns relationships of the GDTSM, given by equations (7) and (18), respectively. The specification of consumption growth rate regression (27) assumes that real consumption C_t is determined by the two unobserved factors x_{1t} and x_{2t}. This reflects upon evidence that real consumption and/or output growth depends on two term structure of interest rates factors (i.e., short-term rate r_t and spread $R_t(\tau) - r_t$).[11] As argued in Section 2, the inclusion of the set of excess holding period returns regressions (28) into the system will help to better identify from our data price of risk parameters $\lambda_{1,ii}$ of

[11] See, for instance, Harvey (1988), Berardi and Torous (2005), and Argyropoulos and Tzavalis (2012).

risk pricing functions Λ_{it}, for all i.

To estimate system of equations (25)-(28), we will employ the Generalized Method of Moments (GMM) (see Hansen (1982)). This method can provide asymptotically efficient estimates of the parameters of the system which are robust to possible heteroscedasticity and/or serial correlation of errors ω_{t+1}, $e_{t+1}(\tau)$, ξ_{t+1} and $\varsigma_{t+1}(\tau)$. In this estimation procedure, we will impose the no-arbitrage conditions implied by equation (17) on the slope coefficients of the sets of regressions (26), (27) and (28), i.e., on elements of matrix $D(\tau)$, and vectors $\psi_1(\tau)$ and $\Gamma(\tau)$. These constitute a set of cross-section restrictions on the parameters of the system which can be tested by our data based on Sargan's overidentifying restrictions test statistic.

As noted before, to obtain estimates of the vector of unobserved state variables x_{1t} and x_{2t} from our data, by inverting pricing relationship (15), we will rely on estimates of interest rate variables $z_{1t} \equiv R_t(60)$ and $z_{2t} \equiv R_t(60) - r_t$. These will be obtained by regressing them on principal component factors pc_{it}. These regressions will be estimated, simultaneously, with our system of equations (25)-(28). By construction, the above estimates of variables z_{1t} and z_{2t} will be orthogonal to any measurement errors inherent in them, as the latter are diversified away in principal component factors pc_{it}.

3.2.1 Estimation results

GMM estimates of the key parameters of the system of equations (25)-(28) of our GDTSM, namely loading coefficients of state variables x_{it} on short-term interest rate r_t, δ_{1i}, mean reversion and price of risk parameters k_{ii} and $\lambda_{1,ii}$, for all i, the elements of matrix Φ and the correlation matrix of the residuals of stochastic processes of x_{it} (see 25), denoted as $\hat{\omega}_{it+1}$, are given in Table 4. Note that, in brackets, next to the diagonal estimates of matrix Φ, the table reports values of the mean reversion parameters k_{ii} of x_{it}, for all i, based on relationship $\phi_{ii} = e^{-k_i \Delta t}$. The above all estimates are obtained using a set of interest rates $R_t(\tau)$ and excess holding period returns $hpr_t(\tau)$, with maturity intervals τ ={3,6,9,24,36} months. As instruments, we have used lagged values of the ten year (120 months) nominal interest rate, the spread between the two year (24 months) and one-month nominal interest rates, and inflation rate π_t (see Table 4). In addition to the above estimates, the table also presents estimates of Sargan's overidentifying restrictions test statistics, denoted as J.

Table 4: GMM estimates of system (25)-(28)

$\Delta X_{t+1} = const + \Phi X_t + \omega_{t+1}$,
$\Delta R_{t+1}(\tau) = const + D(\tau)' E_t(\Delta X_{t+1}) + e_{t+1}(\tau)$,
$\Delta c_{t+1} = const + \psi_1(\tau)' X_t^* + \xi_{t+1}$
$hpr_{t+1}(\tau) = const + \Gamma(\tau)' X_t + \varsigma_{t+1}(\tau)$, where $\psi_1(\tau) = (I - e^{-k'\tau})(k')^{-1}\delta_1^*$,
$k^{\mathcal{Q}} = k + \lambda_1$, $D(\tau) = (I - e^{-k^{\mathcal{Q}\prime}\tau})(k^{\mathcal{Q}\prime})^{-1}\delta_1$ and $\Gamma(\tau)' = -D(\tau)'\lambda_1$.

	x_{1t}	x_{2t}	π_t
δ_{1i}	1.46	-1.05	0.002
	(0.003)	(0.002)	(0.0004)
ϕ_{1i}	-0.01 [k_{11} =0.13]	0.001	-0.0003
	(0.003)	(0.0001)	(0.0008)
ϕ_{2i}	0.001	-0.03 [k_{22} =0.35]	-0.0005
	(0.0001)	(0.006)	(0.001)
ϕ_{3i}	0.12	-0.33	-0.18 [k_{33} =2.50]
	(0.05)	(0.03)	(0.06)
$\lambda_{1,ii}$	-0.009	-0.05	-0.86
	(0.001)	(0.003)	(0.40)

Variance-covariance matrix of residuals $\hat{\omega}_{it+1}$

	$\widehat{\omega}_{1t+1}$	$\widehat{\omega}_{2t+1}$	$\widehat{\omega}_{3t+1}$
$\widehat{\omega}_{1t+1}$	0.38	0.93	-0.10
$\widehat{\omega}_{2t+1}$		0.72	-0.08
$\widehat{\omega}_{3t+1}$			0.61

$J = 118.84$ (p-value = 0.11)
Instruments: constant, $R_{t-h}(120)$ for $h = \{1, 2\}$; $R_t(24) - r_t, \pi_{t-h}$ for $h = \{1, 2, 3, 4\}$

Notes: The table presents GMM estimates of parameters of the system of equations (25)-(28). Heteroscedasticity and autocorrelation consistent (Newey-West) standard errors are shown in parentheses. J is Sargan's overidentifying restriction test. In the estimation, we impose the following restrictions on the slope coefficients of the loading coefficients of state variables x_{it} on $\Delta R_{t+1}(\tau)$, Δc_{t+1} and $hpr_{t+1}(\tau)$: $\psi_1(\tau) = (I - e^{-k'\tau})(k')^{-1}\delta_1^*$, $k^Q = k + \lambda_1$, $D(\tau) = (I - e^{-k^{\mathcal{Q}\prime}\tau})(k^{Q\prime})^{-1}\delta_1$ and $\Gamma(\tau)' = -D(\tau)'\lambda_1$, implied by the following structural equations (15), (7) and (18) of our GDTSM, respectively. δ_{1i}^* are assumed equal to δ_{1i}, for i = {1,2}.

The first conclusion that can be drawn from the results of the table is that our GDTSM specification is consistent with the data. This can be justified by the value of J statistic reported in the table, indicating that the cross-section restrictions imposed on the matrix and vectors of coefficients of the system $D(\tau)$, $\Gamma(\tau)$ and $\psi_1(\tau)$, respectively, can not be rejected at 1%, or 5%, probability levels. The results of the table indicate that estimates of mean-reversion and price of risk parameters k_{ii} and $\lambda_{1,ii}$ are significant at 5% level, for all i. The significance of the estimates of $\lambda_{1,ii}$ means that the risks associated with variation in all state variables x_{it} (including inflation rate) are priced in the market. The negative sign of $\lambda_{1,ii}$, for all i, is consistent with the risk averse behavior of bond market investors. The latter decrease the values of mean reversion parameters k_{ii} under the risk neutral measure Q, collected in vector k^Q. The reported estimates of

k_{ii} indicate that, among the three state variables x_{it}, the first two (i.e. x_{1t} and x_{2t}), spanning both the real and nominal term structure of interest rates, as well as real consumption growth are very persistent, given that k_{ii} have values very close to zero. This does not happen with the estimates of k_{ii} for inflation rate π_t. These results imply that shocks in state variables x_{1t} and x_{2t} will have more persistent effects on nominal term structure of interest rates than inflation shocks.

Another interesting conclusion that can be drawn from the results of the table is that there significant feedback effects from state variables x_{1t} and x_{2t} on future inflation rate π_{t+1}, but not inversely. These results can be justified by the estimates of the elements of matrix Φ and their standard errors, reported in the table. These show that the estimates of autoregressive coefficients ϕ_{31} and ϕ_{32}, capturing feedback effects of state variables x_{1t} and x_{2t} on π_{t+1}, are different than zero at 5% level. On the other hand, the estimates of ϕ_{13} and ϕ_{23}, capturing feedback effects of π_t on x_{1t+1} and x_{2t+1}, are not different than zero. Taking these results together with those of the estimates of the correlation coefficients among residual terms $\hat{\omega}_{it+1}$, for all i, which show very little (almost zero) contemporaneous correlation between inflation rate π_{t+1} and state variables x_{1t+1} and x_{2t+1} shocks, one can conclude that the direction of causality between these three variables is from x_{1t} and x_{2t} on π_t, and not inversely. This result enables us to safely assume that residuals $\hat{\omega}_{it+1}$, for $i = 3$, constitute inflation rate shocks. The effects of these shocks on the inflation risk premium effects will be investigated later on, in Subsection 3.2.3.

The very slow mean reversion of state variables x_{1t} and x_{2t}, noted above, can be also confirmed by the inspection of the estimates of them obtained through the estimation of our GDTSM. These are graphically presented in Figure 1. These estimates are presented vis-a-vis those of the first two principal component factors pc_{1t} and pc_{2t}, obtained by the PC analysis of Subsection 3.1.1. As was expected, x_{it} are closely correlated with pc_{it}, for $i = \{1, 2\}$, but they do not have one-to-one correspondence. These results imply that employing principal component factors to proxy state variables x_{1t} and x_{2t} may not correctly capture the latter. The correlation coefficients between pc_{it} and x_{it}, for all i, including inflation rate π_t, are reported in Table 5. As said before, the close correlation between x_{it} and pc_{it} can be attributed to the fact that pc_{it} constitute linear transformations of x_{1t} and x_{2t}. They imply that employing principal component factors to proxy state variables x_{1t} and x_{2t} may not correctly capture the latter. The results of Table 5 also indicate that there is little correlation between inflation rate and state variables x_{it}, or principal components factors pc_{it}, which is consistent with the results of Table 4.

Table 5: Correlation coefficients among pc_{it} and x_{it}

	pc_{1t}	pc_{2t}	pc_{3t}	x_{1t}	x_{2t}	π_t
pc_{1t}	1.00	0.00	0.00	0.88	-0.46	0.10
pc_{2t}		1.00	0.00	0.38	0.92	-0.21
pc_{3t}			1.00	0.00	0.00	0.02
x_{1t}				1.00	0.63	-0.15
x_{2t}					1.00	-0.23
π_t						1.00

Notes: The table presents correlation coefficients between pc_{it} and x_{it}, for all i. Note that state variable x_{3t} is also defined as $x_{3t} \equiv \pi_t$.

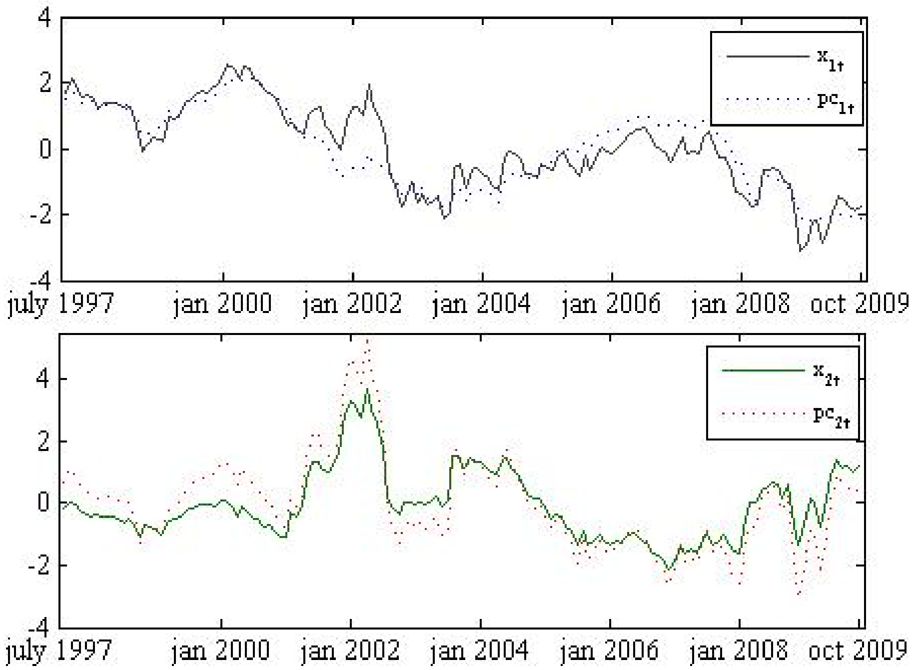

Figure 1. Estimates of state variables x_{it} and principal component factors pc_{it}, for $i = \{1, 2\}$.

3.2.2 Comparison to market estimates of inflation expectations and real interest rates

To see how closely real interest rates $R_t^*(\tau)$ and inflation expectations (denoted as $\pi_t^e(\tau)$) implied by our GDTSM model are to those reported in the market, in Figures 2 and 3 we report estimates of them, over our sample. Figures 2A and 2B compare the estimates of $R_t^*(\tau)$ obtained by our model to those based on survey data and inflation indexed bonds, respectively. In particular, Figure 2A also presents values of $R_t^*(\tau)$ taken from the Cleveland fed survey (see also Haubrich et al. (2012)), which are available for $\tau = 12$ months.[12]

[12] http://www.clevelandfed.org/research/data/inflation_expectations

Figure 2B presents values of $R_t^*(\tau)$ implied by the 5-year zero coupon TIPS rate. These are taken from Gürkaynak, Sack and Wright (2010).[13]. The following table presents values of the correlation coefficients between the estimates of our model for $R_t^*(\tau)$ and those of the market, described above, denoted as $R_t^{*,M}(\tau)$. Note that $R_t^{*M}(\tau)$.are not measured net of risk premium effects, as $R_t^*(\tau)$ in our model.

Table 6A: Values of correlation coefficients $Corr(R_t^{*,M}(\tau); R_t^*(\tau))$

τ (in months)	1	12	60
$Corr(R_t^{*,M}(\tau); R_t^*(\tau))$	0.77	0.76	0.70

Notes: The table presents values of the correlation coefficient between our model estimates of real rates $R_t^*(\tau)$ and those of the market, denoted as $R_t^{*,M}(\tau)$, for different maturity intervals τ.

The results of Figures 2A-2B and Table 6A clearly indicate that our estimates of real interest rates $R_t^*(\tau)$ are very close to those implied by the survey and TIPS' market term structure data. The correlation coefficients between our model estimates of $R_t^*(\tau)$ and the market ones, $R_t^{*M}(\tau)$, are found to be 0.76 and 0.70, respectively. The biggest deviations between series $R_t^*(\tau)$ and $R_t^{*M}(\tau)$ are observed during the period of recent financial crisis, i.e., 2008-2009. This can be obviously attributed to the effects of the recent financial crisis on $R_t^{*M}(\tau)$. Fears of credit and liquidity risks, triggered by this financial crisis, may have driven the yields of TIPS up, given that these are less liquid assets than nominal bonds.

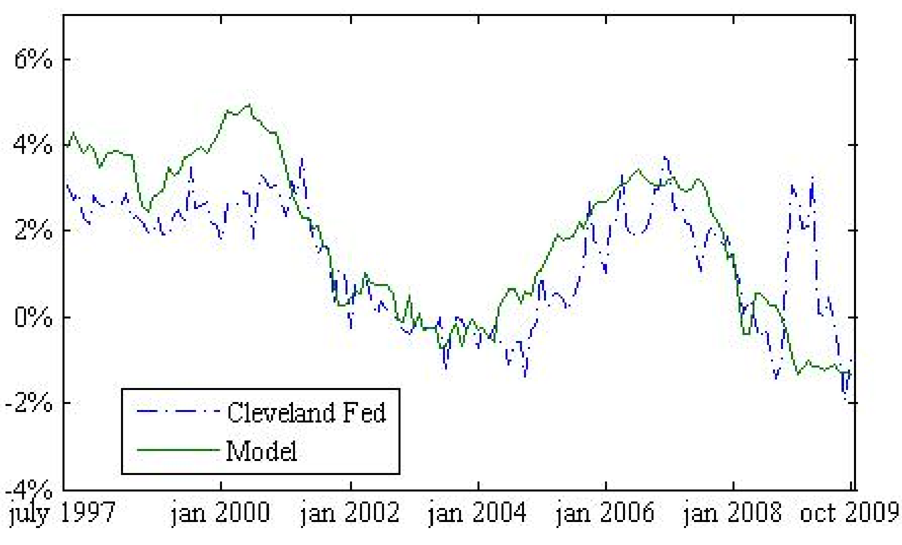

Figure 2A. Survey based values real interest rate $R_t^*(\tau)$, for 12 months, against estimates of it obtained by the estimates of our GDTSM.

[13] http://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html

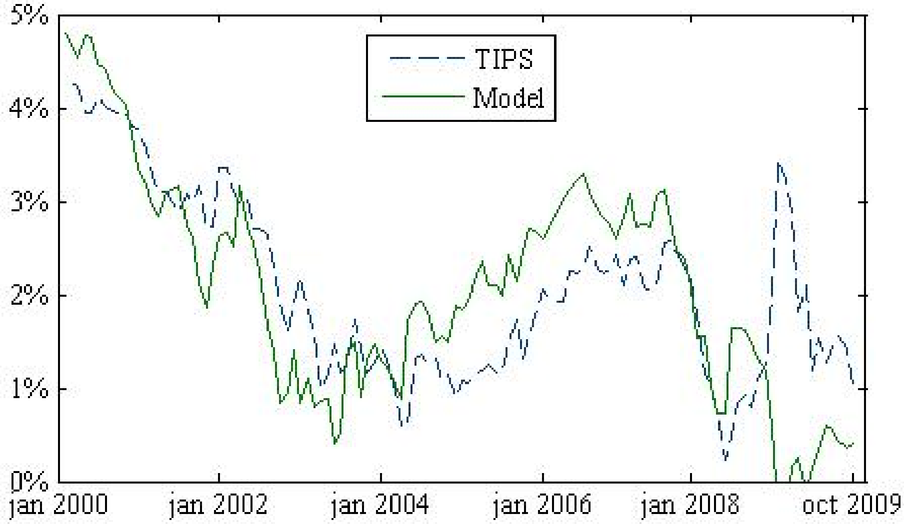

Figure 2B. TIPS implied values of real interest rate $R_t^*(\tau)$, for 60 months, against estimates of it based on the estimates of our GDTSM.

Similar conclusions to the above can be drawn for the inflation expectations obtained by our model over τ-periods ahead, $\pi_t^e(\tau)$, based on relationship (6). As Figure 3 shows, these are very close to those based on the Cleveland fed survey data denoted as $\pi_t^{e,M}(\tau)$, for $\tau = 36$ months.[14] Values of the correlation coefficients between $\pi_t^e(\tau)$ and $\pi_t^{e,M}(\tau)$, for different maturity intervals τ, are given in Table 6B, below. These values are very close to unity.

Table 6B: Values of correlation coefficients $Corr\left(\pi_t^{e,M}(\tau); \pi_t^e(\tau)\right)$

τ (in months)	12	36	60
$Corr\left(\pi_t^{e,M}(\tau); \pi_t^e(\tau)\right)$	0.92	0.95	0.96

Notes: The table presents values of the correlation coefficient between inflation expectations obtained by our model (denoted as $\pi_t^e(\tau)$) and those based on the Cleveland fed survey data (denoted as $\pi_t^{e,M}(\tau)$), for different maturity intervals τ.

[14] As real interest rates, note that the values of expected inflation implied by the TIPS data are not net of inflation risk premia effects. These are calculated as $BEI(\tau) \equiv R_t(\tau) - R_t^*(\tau)$, which equals to $\pi_t^e(\tau) + \wp_t(\tau)$. See Subsection 2.2.

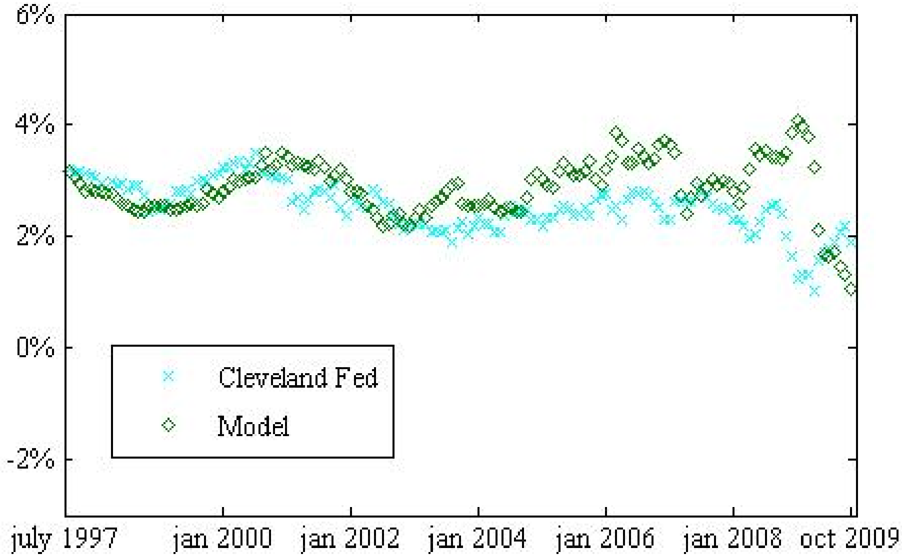

Figure 3: Inflation expectations based on survey data for 36 months ahead against those obtained by the estimates of the GDTSM.

3.2.3 Estimates of inflation risk premium effects

In this section, we estimate the inflation risk premia effects $\wp_t(\tau)$, based on our GDTSM estimates, and investigate some of their key features. Recall that $\wp_t(\tau)$ can be calculated from our GDTSM as follows:

$$\wp_t(\tau) = (1/\tau)\left[A(\tau) + D(\tau)'X_t\right] - (1/\tau)\left[a(\tau) + d(\tau)'X_t\right] - (1/\tau)[g_0(\tau) + g_1(\tau)'X_t],$$

see equation (24). Figure 4 presents estimates of $\wp_t(\tau)$ based on our model versus those implied by survey based Cleveland fed real yields data, denoted as $\wp_t^M(\tau)$. As real yields (or inflation expectations) implied by TIPS or Cleveland fed data are not net of risk premium effects, to obtain estimates of $\wp_t^M(\tau)$ based on market data we have relied on estimates of inflation expectations also based on Cleveland fed survey data (see fn 14). Values of the correlation coefficients between $\wp_t^M(\tau)$ and $\wp_t(\tau)$, together with some descriptive statistics of them are reported in Table 7.[15]

The results of Table 7 and Figure 4 indicate that our model estimates of $\wp_t(\tau)$ are closely related to those implied by the TIPS' yields. The correlation coefficients between these two alternative measures of $\wp_t(\tau)$ vary between 0.65 and 0.67 values. Both of the above sets of estimates of $\wp_t(\tau)$ vary between negative and

[15] Note that the table does not present values of correlation coefficients $Corr(\wp_t^M(\tau); \wp_t(\tau))$ for the set of short-term maturities $\tau = \{3{,}6\}$, since TIPS' are less liquid for such maturity intervals.

positive values. They tend to take negative values for most periods of the sample and, especially, during the recent financial crisis. This can be also confirmed by the mean values of $\wp_t(\tau)$, reported in the table. From relationship (22), it can be seen that a negative value of $\wp_t(\tau)$ means a positive value of the covariance between marginal utility ratio $m_{t+\tau}/m_t$ and inverted price level change $P_t/P_{t+\tau}$, which is consistent with the consumption smoothing attitude of investors. It also implies that

nominal interest rates $R_t(\tau)$ are less than the sum of real rates $R_t^*(\tau)$ and expected future inflation rates $\pi_t^e(\tau)$, predicted by the Fisher equation. The latter means that investors would prefer to hold nominal bonds rather than inflation-indexed bonds. This may be also attributed to the fact that the latter are less liquid assets.

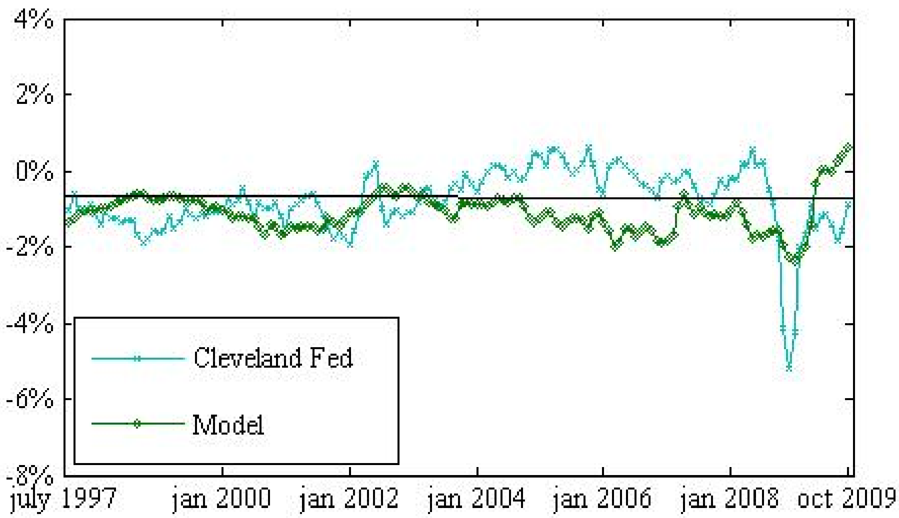

Figure 4: Inflation risk premia effects, for $\tau = 36$ months, implied by the estimates of our GTSDM and Cleveland Fed survey on yields and inflation expectation.

Table 7: Descriptive statistics of risk premium effects $\wp_t(\tau)$

τ (in months)	3	6	12	36	60
Mean	-1.53	-1.51	-1.47	-1.31	-1.17
St.Dev.	1.32	1.14	0.94	0.71	0.60
$Corr(\wp_t^M(\tau);\wp_t(\tau))$	-	-	0.65	0.68	0.67

Notes: The table presents descriptive statistics, i.e., the mean and standard deviation (St.Dev), of risk premium effects $\wp_t(\tau)$, as well as values of correlation coefficients between estimates of the risk premium effects implied by our GDTSM and those based on market data (TIPS or Cleveland fed survey based yields denoted as $\wp_t^M(\tau)$), for different maturity intervals τ.

Another interesting conclusion that can be drawn from the results of Table 7 is that both the mean and volatility (standard deviation) of inflation risk premium effects $\wp_t(\tau)$ decline with maturity interval τ. This is also consistent with evidence provided by Grishchenko and Huang (2012), based on market and survey data. As can be seen from the closed-form solution of $\wp_t(\tau)$, given by equation (24), the decrease of the mean and volatility values of $\wp_t(\tau)$ with τ can be attributed to the fact that state variables x_{it} affecting $\wp_t(\tau)$ are offset to each other and they are scaled by maturity interval τ. It can be also attributed to the fact that inflation shocks, which affect directly $\wp_t(\tau)$, have a lower degree of persistency on the level of inflation rate π_t (or the other two state variables), as is implied by the estimates of the mean reversion parameters reported in Table 4. The latter can be more clearly seen by the graphs of impulse response functions ($IRFs$) of the effects of a 1% positive inflation shock on $\wp_t(\tau)$, presented in Figure 5. These $IRFs$ are calculated based on the following relationship:

$$\wp_t(\tau) = \mathcal{G}(\tau)' X_t, \quad \text{where } X_t = \Phi X_{t-1} + \omega_t,$$

$\mathcal{G}(\tau)$ is a (3×1)-dimension vector defined as $\mathcal{G}(\tau) = (1/\tau)(g_{11}(\tau),\, g_{12}(\tau),\, (g_{13}(\tau) - D_\pi(\tau))$' (see equation (**24**)). In particular, Figure 5 presents $IRFs$ of a 1% positive inflation shock on $\wp_t(\tau)$, for maturity intervals of $\tau = \{12, 36, 60\}$ months.

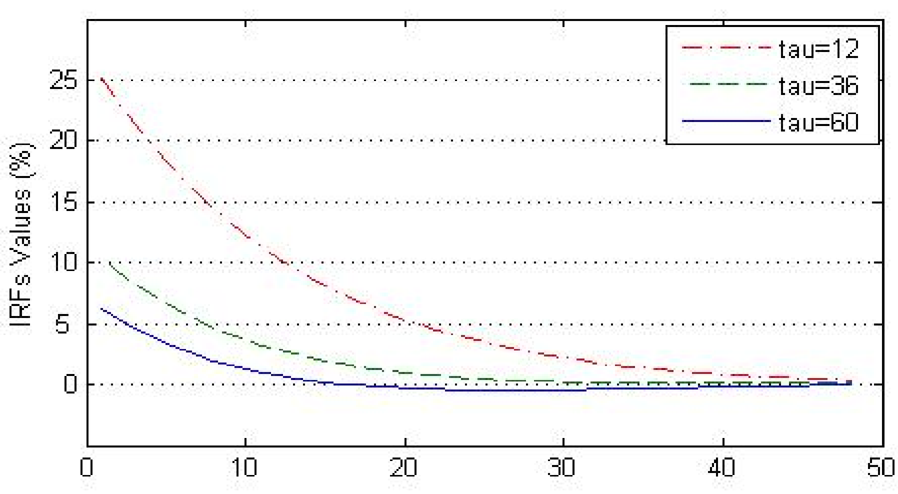

Figure 5. Impulse response functions ($IRFs$) of inflation risk premium effects $\wp_t(\tau)$ to 1% positive inflation shocks.

To calculate these $IRFs$, we assume that inflation rate π_t is uniquely determined by its own (inflation) shocks. This can be empirically justified by the estimates of the elements of the variance-covariance matrix of residuals $\hat{\omega}_{it+1}$, for $i = \{1, 2, 3\}$, reported in Table 4. These imply that the degree of correlation between the shocks of state variables x_{1t} and x_{2t}, and that of inflation is very close to zero. Inspection of the $IRFs$, presented in Figure 5, confirm our arguments about the effects of inflation shocks on $\wp_t(\tau)$ across different maturity intervals τ, made above. They clearly indicate that these effects are positive, for all τ. They are stronger at shorter maturity intervals τ (e.g., $\tau = 12$), whereas they decay faster for longer τ (i,.e., $\tau = 60$).

3.3 Forecasting inflation from the term structure

Having obtained estimates of risk premium effects $\wp_t(\tau)$ and real interest rates $R_t^*(\tau)$ based on our GDTSM, in this section we examine if time variation of these two variables can explain the failures of the nominal spread $R_t(\tau) - r_t$ (or $R_t(\tau) - R_t(s)$, for $\tau > s$), to provide forecasts of future inflation which are consistent with the predictions of the rational expectations hypothesis of the term structure (REHTS). To this end, the following regression model has been employed in the literature:

$$\pi_t(\tau) - \pi_t(s) = a_{\tau,s} + b_{\tau,s}(R_t(\tau) - R_t(s)) + \varepsilon_t(\tau, s), \tag{29}$$

(see, e.g., Mishkin (1990)), where $\pi_t(\tau) - \pi_t(s)$ is the change of inflation rates between future periods $t + \tau$ and $t + s$. If real interest rates $R_t^*(\tau)$ and risk premium effects $\wp_t(\tau)$ are constant, then the REHTS predicts that $\beta_{\tau,s} = 1$, for all $\tau \neq s$.

Table 8 presents GMM estimates of the slope coefficients of regression model (29), for different τ and s. In the estimation procedure, as instrument we employ lagged values of nominal and real spreads, as well as a proxy of inflation risk premium effects based on the Cleveland fed survey (see the notes of the table). By employing instrumental variables, GMM estimation procedure may mitigate the effects of simultaneity bias between $R_t(\tau) - R_t(s)$ and $\varepsilon_t(\tau, s)$, due to the omission of time-varying real interest rates and risk premium effects from the RHS of (29), on the estimates of coefficients $a_{\tau,s}$ and $b_{\tau,s}$.

The results of the table are consistent with those of Mishkin (1990). They show that the nominal term spread $R_t(\tau) - R_t(s)$ contains information about future inflation rate changes only at the long-end of the term structure of nominal interest rates, i.e., for pairs of maturity intervals like $(\tau, s) = \{(36, 12), (60, 12)\}$. For the pairs of maturity intervals $(\tau, s) = \{(12, 3), (36, 3)\}$, which considers short-term forecasting horizons, regression model (29) fails to predict the future changes of inflation rates $\pi_t(\tau) - \pi_t(s)$. In this case, the estimates of slope coefficient $b_{\tau,s}$ are far away from unity. Note that, for $(\tau, s) = \{(12, 3)\}$, they take negative values.

Table 8: GMM estimates of inflation forecasting equation (29) adjusted for time-varying real interest rates effects

Model: $\pi_t(\tau) - \pi_t(s) = a_{\tau,s} + b_{\tau,s}(R_t(\tau) - R_t(s)) + \varepsilon_t(\tau, s)$

Maturity intervals (τ, s)	$a_{\tau,s}$	$b_{\tau,s}$	J-(p-value)
(12,3)	0.24	-1.86	0.97
	(0.40)	(1.80)	
(36,3)	0.01	0.17	0.50
	(0.40)	(0.31)	
(36,12)	-0.20	0.88	0.11
	(0.25)	(0.55)	
(60,12)	-0.36	0.67	0.30
	(0.23)	(0.30)	

Notes: The table presents GMM estimates of inflation forecasting regression model (29). Heteroscedasticity and autocorrelation consistent (Newey-West) standard errors are shown in parentheses. J-(p-value) gives the p-value of Sargan's overidentifying restriction test with 11 degrees of freedom. In the GMM estimation procedure, we employ as instruments the following variables: $\boldsymbol{\iota}_{1t-i} = R_t(36) - R_t(3)$, $\iota_{2t-i} = R_t^{*,M}(36) - R_t^{*,M}(1)$, $\iota_{2t-i} = \wp_t^M(36) - \wp_t^M(3)$, for $i = 1, 2, 3, 4$.

To examine if the above puzzling behavior of term spread $R_t(\tau) - R_t(s)$ can be attributed to the time variation of inflation risk premium effects $\wp_t(\tau)$ and/or real term interest rates $R_t^*(\tau)$, next we have estimated the following version of model (29), adjusting for these two components of nominal rates:

$$\pi_t(\tau) - \pi_t(s) = c_{\tau,s} + b_{\tau,s}(R_t(\tau) - R_t(s) - \theta_t(\tau, s)) + u_t(\tau, s), \tag{30}$$

where

$$\theta_t(\tau, s) = (R_t^*(\tau) - R_t^*(s)) - (\wp_t(\tau) - \wp_t(s))$$

captures the inflation risk premia and real term structure effects, jointly. This regression model is based on relationship (23)). Under the REHTS, it implies that $b_{\tau,s} = 1$. GMM estimates of the above regression model is given in Tables 9A and 9B. Table 9A presents estimates of (30), where nominal term spread $R_t(\tau) - R_t(s)$ is adjusted only for time-varying real interest rates effects, i.e., $\theta_t(\tau, s) \equiv R_t^*(\tau) - R_t^*(s)$. Table 9B adjusts $R_t(\tau) - R_t(s)$ for both real interest rates and inflation risk premia effects, i.e., $\theta_t(\tau, s) \equiv (R_t^*(\tau) - R_t^*(s)) - (\wp_t(\tau) - \wp_t(s))$.

Table 9A: GMM estimates of inflation forecasting regression (30) adjusted for time-varying real interest rates and risk premium effects

Model: $\pi_t(\tau) - \pi_t(s) = c_{\tau,s} + b_{\tau,s}(R_t(\tau) - R_t(s) - \theta_t(\tau,s)) + u_t(\tau,s)$ where $\theta_t(\tau,s) \equiv (R_t^*(\tau) - R_t^*(s))$

Maturity intervals (τ, s)	$a_{\tau,s}$	$b_{\tau,s}$	J-(p-value)
(12,3)	-0.50	1.40	0.92
	(0.08)	(0.52)	
(36,3)	0.15	0.45	0.60
	(0.29)	(0.22)	
(36,12)	0.23	0.22	0.99
	(0.34)	(0.80)	
(60,12)	0.43	0.85	0.68
	(0.26)	(0.09)	

Notes: The table presents GMM estimates of inflation forecasting regression (30), adjusted for real interest rates effects $R_t^*(\tau) - R_t^*(s)$. Heteroscedasticity and autocorrelation consistent (Newey-West) standard errors are shown in parentheses. J-(p-value) gives the p-value of Sargan's overidentifying restriction test with 11 degrees of freedom. In the GMM estimation procedure, we employ as instruments the following variables: $\iota_{1t-i} = R_t(36) - R_t(3)$, $\iota_{2t-i} = R_t^{*,M}(36) - R_t^{*,M}(1)$, $\iota_{2t-i} = \wp_t^M(36) - \wp_t^M(3)$, for $i = 1, 2, 3, 4$.

The results of Tables 9A and 9B clearly indicate that adjusting term spread $R_t(\tau) - R_t(s)$ both for time-varying real term structure and inflation risk premium effects can explain its failure to forecast future changes of inflation rates $\pi_t(\tau) - \pi_t(s)$, over short-term horizons. The estimates of slope coefficients $b_{\tau,s}$ of regression model (30), allowing for time-varying real term structure and inflation risk premium effects, become close to unity, which is consistent with the predictions of the REHTS. As the analysis of our previous sections has shown, the variation of these two effects cease with maturity interval τ, which can explain the success of term spread $R_t(\tau) - R_t(s)$ to forecast future inflation rate changes $\pi_t(\tau) - \pi_t(s)$ at the long-end of the term structure (see Table 8). Finally, note that support for regression model (30) can be obtained by the p-values of Sargan's overidentifying restrictions tests, denoted as J, reported in the tables. These can not reject the orthogonality conditions (overidentifying restrictions) of the model by the data.

Table 9B: GMM estimates of inflation forecasting equations (30)

Model: $\pi_t(\tau) - \pi_t(s) = c_{\tau,s} + b_{\tau,s}(R_t(\tau) - R_t(s) - \theta_t(\tau,s)) + u_t(\tau,s)$,
where $\theta_t(\tau,s) \equiv (R^*_t(m) - R^*_t(n)) - (\wp_t(m) - \wp_t(n))$

Maturity intervals (τ, s)	$a_{\tau,s}$	$b_{\tau,s}$	J-(p-value)
(12,3)	-0.22	0.72	0.98
	(0.15)	(0.35)	
(36,3)	0.35	0.81	0.68
	(0.19)	(0.18)	
(36,12)	-0.42	0.83	0.93
	(0.13)	(0.14)	
(60,12)	0.66	0.80	0.85
	(0.28)	(0.16)	

Notes: The table presents GMM estimates of inflation forecasting regression (30), adjusted for real interest rates effects $R^*_t(\tau) - R^*_t(s)$. Heteroscedasticity and autocorrelation consistent (Newey-West) standard errors are shown in parentheses. J-(p-value) gives the p-value of Sargan's overidentifying restriction test with 11 degrees of freedom, we employ as instruments the following variables: $\iota_{1t-i} = R_t(36) - R_t(3)$, $\iota_{2t-i} = R^{*,M}_t(36) - R^{*,M}_t(1)$, $\iota_{2t-i} = \wp^M_t(36) - \wp^M_t(3)$, for $i = \{1,2,3,4\}$.

4 Conclusions

This paper has suggested a Gaussian dynamic term structure model (GDTSM) with the aim of examining how important the inflation risk premium and/or real interest rates effects are on predicting future changes in inflation from the nominal term structure of interest rates. The model enables us to retrieve from nominal interest rates, real consumption growth and inflation rates, estimates of real interest rates and inflation expectations which are net of inflation risk premium effects. Inflation-indexed bonds, employed for this purpose, provide biased estimates of real interest rates and inflation expectations, which depend on inflation risk premium effects. Furthermore, these bonds are illiquid assets over short-term horizons.

The paper provides a number of interesting findings, which can be proved very useful in forecasting future inflation rates and/or retreiving real interest rates from the term structure of interest rates, in practice. First, it shows that the model is consistent with the data and, thus, can efficiently describe the dynamics of nominal and real interest rates, as well as of inflation rates observed in reality. The real interest rates and inflation expectations retrieved by the model are close to those implied by survey data and inflation-indexed bonds. The latter are often provided over longer horizons. Second, the inflation risk premium estimated by our model is found to be negative for some intervals of our sample and very volatile, especially over term-term horizons. This means that investors require less compensation for holding nominal bonds, compared to real (inflation-indexed) ones. This attitude of investors may be due to the fact that inflation-indexed bonds are less liquid assets. Third, real interest rates are also volatility over short-term maturity intervals. These together with inflation risk premium effects can explain the failures of the nominal term spread to forecast future inflation rates over short-term horizons.

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