ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

DEPARTMENT OF ECONOMICS

WORKING PAPER SERIES **23-2013**

The power performance of fixed-T panel unit root tests allowing for structural breaks in their deterministic components

Yiannis Karavias and Elias Tzavalis

76 Patission Str., Athens 104 34, Greece
Tel. (++30) 210-8203911 - Fax: (++30) 210-8203301
www.econ.aueb.gr

The power performance of fixed-T panel unit root tests allowing for structural breaks in their deterministic components

Yiannis Karavias[a] and Elias Tzavalis[b]

a: School of Economics, University of Nottingham

Granger Centre for Time Series Econometrics

b: Department of Economics, Athens University of Economics & Business

Abstract

The asymptotic local power of least squares based fixed-T panel unit root tests allowing for a structural break in their individual effects and/or incidental trends of the AR(1) panel data model is studied. These tests correct the least squares estimator of the autoregressive coefficient of this panel data model for its inconsistency due to the individual effects and/or incidental trends of the panel. The limiting distributions of the tests are analytically derived under a sequence of local alternatives, assuming that the cross-sectional dimension of the tests (N) grows large. It is shown that the considered fixed-T tests have local power which tends to unity fast only if the panel data model includes individual effects. For panel data models with incidental trends, the power of the tests becomes trivial. However, this problem does not always appear if the tests allow for serial correlation of the error term.

JEL classification: C22, C23

Keywords: Panel data, unit root tests, structural breaks, local power, serial correlation, incidental trends

*Corresponding author. University of Nottingham, Nottingham NG7 2RD, UK. Tel: +441159515480

Email addresses: ioannis.karavias@nottingham.ac.uk (Y. Karavias), etzavalis@aueb.gr (E. Tzavalis).

1 Introduction

There is recently growing interest in developing panel data unit root tests allowing for a break in their deterministic components, namely in their individual effects and/or individual linear trends (see, Carrion-i-Silvestre et al. (2005), Harris et al (2005), Karavias and Tzavalis (2012, 2013), Chan and Pauwels (2011), Bai and Carrion-i-Silvestre (2012), Hadri et al. (2012) and Pauwels et al. (2012)). As is aptly noted by Perron (1989) in the single time-series literature, not accounting for a break point in the level and/or deterministic trend of economic series can lead to a unit root test which can hardly reject the null hypothesis of unit roots from its alternative of stationary series. Panel unit root tests suffer from this problem too.

This paper investigates the power properties of fixed-T panel unit root tests that allow for structural breaks. These tests are appropriate for panels with few time series observations and many cross-section units, often met in practice (see, e.g., Arellano (2003)). The asymptotic theory employed consider the time dimension (T) as fixed and the cross section dimension (N) as going to infinity. In particular, the paper studies the asymptotic local power of Harris' and Tzavalis (1999) and Karavias' and Tzavalis (2012) panel unit root tests allowing for a structural break in their deterministic components. The first (denoted as HT) was extended by Karavias and Tzavalis (2013) to allow for structural breaks. The second (denoted as KT) allows, in addition to structural breaks, for serial correlation in the error term of the individual series of the panel.[1] Both the above tests are based on the least squares (LS) estimator of the autoregressive coefficient of the AR(1) panel data model. This estimator is corrected for its inconsistency due to the individual effects (both individual in-

[1] Note that a version of the KT test for the case of no structural breaks has been suggested by Kruiniger and Tzavalis (2002), and Moon and Peron (2008) for the case that T is large and the error term is a white noise process.

tercepts and individual intercepts along with incidental trends are considered) of the panel. In the case that the error term is serially correlated, the LS estimator must be also corrected for its inconsistency due to the serial correlation of the error term. The latter can be easily done in the framework considered by Karavias and Tzavalis (2012) (see KT test), which adjusts the LS estimator of the autoregressive coefficient of the AR(1) panel data model only for the inconsistency of its numerator.

The paper makes a number of contributions into the literature of panel data unit root tests, which have important practical implications. First, it shows that, for the standard panel data model with individual intercepts, the HT test has higher asymptotic local power than the KT test. This happens because the HT does not require a consistent estimator of the variance of the error term, compared to the KT test. The HT test is invariant to this nuisance parameter, as it adjusts the LS estimator for its inconsistency of both its numerator and numerator. Second, the paper shows that, as with panel unit root tests that do not allow for a break, the HT and KT tests have trivial asymptotic local power if incidental trends are included in the deterministic components of the AR(1) panel data model. The allowance for a break in the deterministic components of this model does not save the tests from this problem. Third, the tests can increase their power if they allow for serial correlation of the error term. In this case, the KT test can have non-trivial asymptotic local power, even for the panel data model with incidental trends. The increase of the power of this test in this case can be attributed to the serial correlation effects on the inconsistency correction of the LS estimator. The above results are confirmed through a Monte Carlo exercise. This exercise also provides interesting small sample results on the power performance of the tests and shows the usefulness of the asymptotic approximation.

The paper is organized as follows: Section 2 presents the assumptions on the data generating process required by the two tests considered. Section 3 derives the limiting distributions of the tests. For the KT test allowing for serial correlation effects, this is done in Section 4. Section 5 carries out the Monte Carlo exercise. Section 6 concludes the paper. All proofs are given in the appendix.

2 Models and Assumptions

Consider the following AR(1) dynamic panel data models allowing for a common structural break in their deterministic components (individual effects and/or individual linear trends) at time point T_0, for all individual units of the panel i:

$$M1: \ \ y_i = a_i^{(1)} e^{(1)} + a_i^{(2)} e^{(2)} + \zeta_i, \qquad i = 1, 2, .., N,$$

$$M2: \ \ y_i = a_i^{(1)} e^{(1)} + a_i^{(2)} e^{(2)} + \beta_i^{(1)} \tau^{(1)} + \beta_i^{(2)} \tau^{(2)} + \zeta_i, \quad i = 1, ..., N$$

where

$$\zeta_i = \varphi \zeta_{i-1} + u_i,$$

$\varphi \in (-1, 1]$, $y_i = (y_{i1}, ..., y_{iT})'$ and $y_i = (y_{i0}, ..., y_{iT-1})'$ are $(TX1)$ vectors, $u_i = (u_{i1}, ..., u_{iT})$ is the $(TX1)$ vector of error terms u_{it}, a_i and β_i denote the individual effects and slope coefficients of the linear (incidental) trends of the panel. In particular, a_i is defined as $a_i = a_i^{(1)}$ if $t \leq T_0$ and $a_i = a_i^{(2)}$ if $t > T_0$, while $e^{(1)}$ and $e^{(2)}$ are $(TX1)$-column vectors defined as follows: $e_t^{(1)} = 1$ if $t \leq T_0$ and 0 otherwise, and $e_t^{(2)} = 1$ if $t > T_0$ and 0 otherwise. Slope coefficients β_i are defined as $\beta_i = \beta_i^{(1)}$ if $t \leq T_0$ and $\beta_i = \beta_i^{(2)}$ if $t > T_0$, while $\tau^{(1)}$ and $\tau^{(2)}$ are $(TX1)$-column vectors defined as follows: $\tau_t^{(1)} = t$ if $t \leq T_0$, and zero otherwise, and

$\tau_t^{(2)} = t$ if $t > T_0$, and zero otherwise. Throughout the paper, we will denote the fraction of the sample that the break occurs as λ, i.e. $\lambda = \frac{T_0}{T} \in I = \left\{\frac{2}{T}, \frac{3}{T}, ..., \frac{T-1}{T}\right\}$.

The above models nest in the same framework both the null hypothesis of unit roots in φ, i.e., $\varphi = 1$, and its alternative of stationarity, $\varphi < 1$. They can be written in a nonlinear form as follows:

$$
\begin{aligned}
y_i &= \varphi y_{i-1} + (1-\varphi)(a_i^{(1)} e^{(1)} + a_i^{(2)} e^{(2)}) + u_i, \qquad i = 1, 2, ..., N \quad \text{and} \\
y_i &= \varphi y_{i-1} + \varphi \beta_i^{(1)} e^{(1)} + \varphi \beta_i^{(2)} e^{(2)} + (1-\varphi)(a_i^{(1)} e^{(1)} + a_i^{(2)} e^{(2)}) + (1-\varphi)(\beta_i^{(1)} \tau^{(1)} + \beta_i^{(2)} \tau^{(2)}) + u_i,
\end{aligned}
$$

respectively. The "within group" least squares (LS) (known also as least squares dummy variables (LSDV)) estimator of autoregressive coefficient φ of the models can be written as follows:

$$
\hat{\varphi}_m^{(\lambda)} = \left(\sum_{i=1}^{N} y'_{i-1} Q_m^{(\lambda)} y_{i-1} \right)^{-1} \left(\sum_{i=1}^{N} y'_{i-1} Q_m^{(\lambda)} y_i \right), \qquad m = \{1, 2\},
$$

where $Q_m^{(\lambda)}$ is the (TXT) "within" transformation (annihilator) matrix of the individual series of the panel y_{it}. $Q_m^{(\lambda)}$ is defined as $Q_m^{(\lambda)} = I - X_m^{(\lambda)} \left(X_m^{(\lambda)\prime} X_m^{(\lambda)} \right)^{-1} X_m^{(\lambda)\prime}$, for $m = \{1, 2\}$, where $X_1^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$ for model $M1$ and $X_2^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$ for model $M2$. This estimator is inconsistent due to the within transformation of the data, which wipes off the individual effects and/or incidental trends of the panel, as well as its initial conditions y_{i0}. Thus, fixed-T panel unit root tests based on it must rely on a correction of estimator $\hat{\varphi}_m^{(\lambda)}$ for its inconsistency (asymptotic bias) (see, e.g., Harris and Tzavalis (1999, 2004)). To study the asymptotic local power of these tests, define the autoregressive coefficient φ as $\varphi_N = 1 - \frac{c}{\sqrt{N}}$. Then, the hypotheses of interest become

$$
H_0\colon\ c = 0 \ \text{ and } \ H_a\colon\ c > 0,
$$

where c is the local to unity parameter. The limiting distributions of the tests based on LSDV estimator $\hat{\varphi}_m^{(\lambda)}$ will be derived under the sequence of local alternatives φ_N, by making the following quite general assumption:

Assumption 1: (b1) $\{u_i\}$ constitutes a sequence of independent normally distributed random vectors of dimension $(TX1)$ with means $E(u_i) = 0$ and variance-autocovariance matrices $E(u_i u_i') = \Gamma \equiv [\gamma_{ts}]$, $\forall\ i \in \{1, 2, ..., N\}$, where $\gamma_{ts} = E(u_{it}u_{is}) = 0$ for $s = t + p_{\max} + 1, ..., T$ and $t < s$. (b2) $\gamma_{tt} > 0$ for at least one $t = 1, ..., T$. (b3) The $4 + \delta - th$ population moments of Δy_i, $i = 1, ..., N$ are uniformly bounded. That is, for every $l \in R^T$ such that $l'l = 1$, $E(|l'\Delta y_i|^{4+\delta}) < B < +\infty$ for some B, where Δ is the difference operator. (b4) $l'Var(vec(\Delta y_i \Delta y_i')l > 0$ for every $l \in R^{0.5T(T+1)}$ such that $l'l = 1$. (b5) $E(u_{it}y_{io}) = E\left(u_{it}a_i^{(1)}\right) = E\left(u_{it}a_i^{(2)}\right) = 0$ and $\forall\ i \in \{1, 2, ..., N\}$, $t \in \{1, 2, ..., T\}$. (b6) $E\left(u_{it}\beta_i^{(1)}\right) = E\left(u_{it}\beta_i^{(2)}\right) = 0$, $\forall\ i \in \{1, 2, ..., N\}$, $t \in \{1, 2, ..., T\}$, $E(a_{it}^{(\lambda)}\beta_{it}^{(\lambda)}) = 0$, $\forall$ $i \in \{1, 2, ..., N\}$.

Assumption 1 enables us to derive the limiting distribution of the fixed-T panel data unit root tests of Harris and Tzavalis (1999, 2004) (denoted as HT), based on LS estimator $\hat{\varphi}_m^{(\lambda)}$ (denoted as HT), as was extended by Karavias and Tzavalis (2012) to allow for a common break in the deterministic components of models $M1$ and $M2$. It also allows the derivation of this limiting distribution for Karavias' and Tzavalis (2012) fixed-T panel data unit root tests (denoted as KT), based on $\hat{\varphi}_m^{(\lambda)}$, allowing for a structural break under heteroscedasticity and/or serial correlation of error term u_{it}. Condition (b1) of the assumption permits the variance matrix of error terms u_{it}, $\Gamma = E(u_i u_i')$, to have general form heteroscedasticity and serial correlation. The latter is assumed to have maximum order p_{max}, which is less than the time dimension of the panel, T. If $\Gamma = \sigma^2 I$, where I is the (TXT) identity matrix, then

Assumption 1 is consistent with the assumptions of Harris and Tzavalis (1999) panel data unit root tests, considering the simpler case of $u_{it} \sim IID(0, \sigma_u^2)$.

Conditions (b2)-(b4) qualify application of the Markov LLN and the Lindeberg -Levy central limit theorem (CLT) to derive the limiting distribution of the HT and KT tests, as $N \to \infty$, under the assumptions of condition (b1). More specifically, conditions (b2) and (b4) guarantee regularity so that the variance of the errors and its estimator will not be zero. Condition (b3) implies that $Var(y_{i0}) < +\infty$, which is consistent with assumptions like constant, random and mean stationary initial conditions y_{i0}. Covariance stationary of y_{i0}, implying $Var(y_{i0}) = \frac{\sigma^2}{1-\varphi_N^2}$ (see Kruiniger (2008) and Madsen (2010)) is not considered. This is because, as is also aptly noted by Moon et al. (2007), this assumption implies that $Var(y_{i0}) \to \infty$ when $\varphi_N \to 1$, which means that the variance of the initial condition increases with the number of cross-section units. This is not meaningful for cross-section data sets. Finally, (b5)-(b6) constitute weak conditions under which the limiting distribution of the tests can be derived when $c > 0$; (b5) is required for Model $M1$, while (b6) for model $M2$. Under these two conditions, the limiting distribution of the tests under H_a: $c > 0$ becomes invariant to nuisance parameters a_i and β_i, as well as the initial conditions y_{i0} of the panel.

To study the asymptotic local power of the tests, we will rely on the slope parameter, denoted as k, of local power functions of the form

$$\Phi(z_a + ck),$$

where Φ is the standard normal cumulative distribution function and z_a denotes the *α-level* percentile. Since Φ is strictly monotonic, a larger k means greater power, for the same value of c. If k is positive, then the tests will have non-trivial power. If it is zero, they will have trivial power, which is equal to a, and if it is negative they will be biased.

3 The limiting distribution of the tests if $u_{it} \sim NIID(0, \sigma^2)$

This section presents the limiting distribution of the HT and KT test statistics under the sequence of local alternatives $\varphi_N = 1 - \frac{c}{\sqrt{N}}$. The HT test corrects both the numerator and denominator of LS estimator $\hat{\varphi}_m^{(\lambda)}$ for its inconsistency, while the KT corrects only the numerator of $\hat{\varphi}_m^{(\lambda)}$. This enables the KT test to be easily extended to allow for serial correlation in error terms u_{it}. But, in contrast to HT, this test statistic requires a consistent estimator of the variance of u_{it}, σ_u^2, to adjust for the inconsistency of estimator $\hat{\varphi}_m^{(\lambda)}$.

3.1 Model $M1$

For model $M1$, the HT test allowing for a break is based on the following statistic:

$$V_{HT,1}^{(\lambda)-1/2}\sqrt{N}(\hat{\varphi}_1^{(\lambda)} - 1 - B_1(\lambda)),$$

where $B_1(\lambda) = p\lim(\hat{\varphi}_1^{(\lambda)} - 1) = \frac{tr(\Lambda' Q_1^{(\lambda)})}{tr(\Lambda' Q_1^{(\lambda)} \Lambda)}$ is the inconsistency of LS estimator $\hat{\varphi}_1^{(\lambda)}$ under H_0: $c = 0$, $V_{HT,1}^{(\lambda)} = \frac{2tr(A_{HT,1}^{(\lambda)2})}{tr(\Lambda' Q_1^{(\lambda)} \Lambda)^2}$, with $A_{HT,1}^{(\lambda)} = \frac{1}{2}(\Lambda' Q_1^{(\lambda)} + Q_1^{(\lambda)} \Lambda) - B_1(\lambda)(\Lambda' Q_1^{(\lambda)} \Lambda)$, is the variance of the limiting distribution of the corrected for its inconsistency estimator $\hat{\varphi}_1^{(\lambda)}$, i.e. $(\hat{\varphi}_1^{(\lambda)} - 1 - B_1(\lambda))$. The KT test is based on statistic

$$V_{KT,1}^{(\lambda)-1/2}\hat{\delta}_1^{(\lambda)}\sqrt{N}\left(\hat{\varphi}_1^{(\lambda)} - \frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} - 1\right),$$

where $\frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} \equiv \frac{\hat{\sigma}_u^2 tr(\Lambda' Q_1^{(\lambda)})}{\frac{1}{N}\sum_{i=1}^N y'_{i,-1} Q_1^{(\lambda)} y_{i-1}}$ is also consistent estimator of the inconsistency of $\hat{\varphi}_1^{(\lambda)}$, which relies on a consistent estimator of its numerator, $\hat{\delta}_1^{(\lambda)} = \frac{1}{N}\sum_{i=1}^N y'_{i,-1} Q_1^{(\lambda)} y_{i,-1}$ is the denominator of $\hat{\varphi}_1^{(\lambda)}$ scaled by N, $V_{KT,1}^{(\lambda)} = 2\sigma_u^4 tr(A_{KT,1}^{(\lambda)2})$, with $A_{KT,1}^{(\lambda)} = \frac{1}{2}(\Lambda' Q_1^{(\lambda)} + Q_1^{(\lambda)}\Lambda - \Psi_1^{(\lambda)} - \Psi_1^{(\lambda)\prime})$, is the variance of the limiting distribution of $\left(\hat{\varphi}_1^{(\lambda)} - \frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} - 1\right)$, where $\Psi_1^{(\lambda)}$ is a (TXT)-dimension matrix having in its main diagonal the corresponding elements of matrix $\Lambda' Q_1^{(\lambda)}$, and zeros elsewhere, implying $tr(\Psi_1^{(\lambda)}) = tr(\Lambda' Q_1^{(\lambda)})$. This matrix is designed so as, in adjusting the numerator of estimator $\hat{\varphi}_1^{(\lambda)}$ for its inconsistency, to subtract from it sample moments of it which capture its inconsistency effects due to the within transformation of the individual series y_{it} of the panel. This means that the following sum of population moments are left for inference about null hypothesis H_0: $c = 0$:

$$E\left[u'_i(\Lambda' Q_1^{(\lambda)} - \Psi_1^{(\lambda)})u_i\right] = 0, \text{ for all } i.$$

For model $M1$, this sum of moments implies a consistent estimator of variance σ_u^2 under null hypothesis H_0: $c = 0$, which can be taken as $\hat{\sigma}_u^2 = \frac{\sum_{i=1}^N \Delta y'_i \Psi_1^{(\lambda)} \Delta y_i}{N tr(\Psi_1^{(\lambda)})}$, where Δ is the difference operator.[2]

In the next theorem, we give the limiting distribution of the HT and KT statistics, defined above for model $M1$, under the sequence of local alternatives $\varphi_N = 1 - \frac{c}{\sqrt{N}}$.

Theorem 1 *Let conditions (b1)-(b5) of Assumption 1 hold and* $u_{it} \sim NIID(0, \sigma^2)$. *Then,*

[2]It can be easily seen that, under H_0: $c = 0$, we have $p\lim \hat{\sigma}_u^2 = p\lim \frac{1}{tr(\Psi_1^{(\lambda)})N}\sum_{i=1}^N tr(\Psi_1^{(\lambda)}\Delta y_i \Delta y'_i) = \sigma_u^2 \frac{tr(\Lambda' Q_1^{(\lambda)})}{tr(\Psi_1^{(\lambda)})} = \sigma_u^2$, since $tr(\Psi_1^{(\lambda)}) = tr(\Lambda' Q_1^{(\lambda)})$.

under $\varphi_N = 1 - \frac{c}{\sqrt{N}}$*, we have*

$$V_{HT,1}^{(\lambda)-1/2}\sqrt{N}(\hat{\varphi}_1^{(\lambda)} - 1 - B_1(\lambda)) \xrightarrow{d} N\left(-ck_{HT,1}, 1\right)$$

$$\text{and} \qquad V_{KT,1}^{(\lambda)-1/2}\hat{\delta}^{(\lambda)}\sqrt{N}\left(\hat{\varphi}_1^{(\lambda)} - \frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} - 1\right) \xrightarrow{d} N\left(-ck_{KT,1}, 1\right),$$

as $N \longrightarrow \infty$*, where*

$$k_{HT,1} = \frac{T(T-2)\left[T^2(3\lambda^2 - 3\lambda + 1) - 1\right]}{4T^2(2\lambda^2 - 2\lambda + 1) - 8}\sqrt{\frac{T^4\Phi_1 + T^2\Phi_2 + 240}{T^6R_1 + T^5R_2 + T^4R_3 + T^2R_4 + 216T - 136}}$$

$$\text{and} \;\; k_{KT,1} = \frac{\sqrt{3}(T-2)}{\sqrt{T^2(2\lambda^2 - 2\lambda + 1) + 6T + 10 - \frac{4(-\frac{1}{T} + 2(\lambda-1)\lambda T)}{(\lambda-1)\lambda}}},$$

where R_1, R_2, R_3, R_4 *and* Φ_1, Φ_2 *are polynomials of* λ *defined in the appendix (see proof of the theorem).*

The limiting distributions given by Theorem 1 imply that the asymptotic local power function of test statistics HT and KT depend on the values of slope parameters k_{HT} and k_{KT}, respectively. In Table 1, we present values of these parameters, for different values of T and λ. The results of these tables indicate that the asymptotic local power behavior of the two tests is different. The HT test has much higher power than the KT. The power of the test is much bigger when the break is in the beginning or towards the end of the sample, i.e. $\lambda = \{0.25, 0.75\}$.[3] On the other hand, the power of the KT test reaches its maximum point when the break is in the middle of the sample, $\lambda = \{0.50\}$. The power of the HT test increases with T, i.e. $k_{HT,1} = O(T)$. The power of the KT test increases with T for

[3] Analogous evidence is provided for single time series unit root tests allowing for breaks, based on a model selection Bayesian approach (see Meligkotsidou et al. (2011)).

relatively small T. As T grows large, the test has no power gains. This can be seen from $\lim_T k_{KT,1} = \frac{\sqrt{3}}{\sqrt{2\lambda^2-2\lambda+1}}$, which is independent of T. These results can be more clearly seen by the three-dimension Figures 1 and 2, presenting values of $k_{HT,1}$ and $k_{KT,1}$, for different values of λ and T.

The above differences between the HT and KT tests can be attributed to the way that each test corrects for the inconsistency of the LS estimator $\hat{\varphi}_1^{(\lambda)}$. As mentioned before, the HT test is based on a correction of LS estimator $\hat{\varphi}_1^{(\lambda)}$ for the inconsistency of both its numerator and denominator. On the other hand, the KT test is based on an adjustment of estimator $\hat{\varphi}_1^{(\lambda)}$ only for the inconsistency of its numerator, which additionally requires a consistent estimator of the variance of error term u_{it}, σ_u^2. The later reduces the local power of the test. Finally, another interesting result of Theorem 1 is that, under the sequence of local alternatives considered, the break function parameters do not enter the asymptotic distribution of both tests. Thus, the magnitude of the break does not affect local power of the tests. Furthermore, local power is also robust to initial condition y_{i0} asymptotically, which means that the magnitude of y_{i0} also does not affect the power of the test (see also Harvey and Leybourne (2005) and Harris et al. (2010)).

Scaling appropriately the HT and KT test statistics by T and assuming that T,$N \rightarrow \infty$, with $\frac{\sqrt{N}}{T} \rightarrow 0$, it can be shown (see appendix) that, under $\varphi_{N,T} = 1 - \frac{c}{T\sqrt{N}}$, the limiting distributions of the large-T versions of the tests are given as follows:

Corollary 1 *Let conditions (b1)-(b5) of Assumption 1 hold and* $u_{it} \sim NIID(0, \sigma^2)$. *Then,* *under* $\varphi_{N,T} = 1 - \frac{c}{T\sqrt{N}}$, *we have*

$$V_{HT,1}^{*(\lambda)-1/2} T\sqrt{N}(\hat{\varphi}_1^{(\lambda)} - 1 - B_1(\lambda)) \xrightarrow{L} N\left(-ck_{HT,1}^{*}, 1\right),$$

$$\text{and} \qquad V_{KT,1}^{*(\lambda)-1/2} \hat{\delta}^{(\lambda)} T\sqrt{N}\left(\hat{\varphi}_1^{(\lambda)} - \frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} - 1\right) \xrightarrow{d} N\left(-ck_{KT,1}^{*}, 1\right),$$

as T, $N \to \infty$, *with* $\frac{\sqrt{N}}{T} \to 0$, *where*

$$k_{HT,1}^{*} = \frac{3\lambda^2 - 3\lambda + 1}{4(2\lambda^2 - 2\lambda + 1)}\sqrt{\frac{\Phi_1}{R_1}} \quad \text{and } k_{KT,1}^{*} = 0, \tag{1}$$

and

$$V_{HT,1}^{*(\lambda)} = \frac{36R_1}{\Phi_1(2\lambda^2 - 2\lambda - 1)^2} \text{ and } V_{KT,1}^{*(\lambda)} = \frac{36(2\lambda^4 - 4\lambda^3 + 3\lambda^2 - \lambda)}{12(\lambda - 1)\lambda(2\lambda^2 - 2\lambda + 1)^2}$$

respectively denote the local power slope coefficients and the variances of the limiting distributions of the large-T *versions of the* HT *and* KT *test statistics.*

Values of power slope coefficients $k_{HT,1}^{*}$ and $k_{KT,1}^{*}$, for different values of λ, are reported in Table 2. These indicate that, in contrast to the HT test, the large-T extension of the KT test does not have asymptotic local power.[4] This test can be thus thought of as more appropriate for short panels. The results of the table also indicate that the large-T extension of the HT test has less power than its fixed-T version. It is also found that power takes its highest values in the beginning and towards the end of the sample, i.e., for $\lambda = \{0.10, 0.90\}$, as for its fixed-T version. The smaller power of the large-T versions of the tests, compared to their fixed-T ones can be attributed to the faster rate of convergence of the alternative hypotheses to the null, i.e. $\varphi_{N,T} = 1 - \frac{c}{T\sqrt{N}}$ compared to $\varphi_N = 1 - \frac{c}{N}$ (see also Harris et al. (2010)).

[4] An analogous result has been derived by Moon and Perron (2008) for this test in the case of no break.

3.2 Model $M2$

For model $M2$, which additionally considers incidental trends in the deterministic components of individual panel series y_{it}, the HT and KT test statistics are defined analogously to those for model $M1$. The HT test admits the same formulas, but with $Q_2^{(\lambda)}$ instead of $Q_1^{(\lambda)}$, $B_2(\lambda) = p\lim(\hat{\varphi}_2^{(\lambda)} - 1) = \frac{tr(\Lambda' Q_2^{(\lambda)})}{tr(\Lambda' Q_2^{(\lambda)}\Lambda)}$ denotes the inconsistency of LS estimator $\hat{\varphi}_2^{(\lambda)}$, $V_{HT,2}^{(\lambda)} = \frac{2tr(A_{HT,2}^{(\lambda)2})}{tr(\Lambda' Q_2^{(\lambda)}\Lambda)^2}$, with $A_{HT,2}^{(\lambda)} = \frac{1}{2}(\Lambda' Q_2^{(\lambda)} + Q_2^{(\lambda)}\Lambda) - B_2(\lambda)(\Lambda' Q_2^{(\lambda)}\Lambda)$, is the variance of the limiting distribution of $(\hat{\varphi}_2^{(\lambda)} - 1 - B_2(\lambda))$. However, for the KT test, $\hat{\sigma}_u^2 = \frac{\sum_{i=1}^N \Delta y_i' \Psi_1^{(\lambda)} \Delta y_i}{Ntr(\Psi_1^{(\lambda)})}$ is no longer a consistent estimator of σ_u^2 in the case of model $M2$, due to the presence of individual coefficients (effects) β_i under null hypothesis H_0: $c = 0$ implying

$$\frac{1}{N}\sum_{i=1}^{N} E(\Delta y_i \Delta y_i') = \beta_N^{(1)} e^{(1)} e^{(1)\prime} + \beta_N^{(2)} e^{(2)} e^{(2)\prime} + \sigma_u^2 I, \tag{2}$$

where $\beta_N^{(1)} = \frac{1}{N}\sum_{i=1}^N E((\beta_i^{(1)})^2)$ and $\beta_N^{(2)} = \frac{1}{N}\sum_{i=1}^N E((\beta_i^{(2)})^2)$. To render the KT test statistic invariant to these effects, Karavias and Tzavalis (2012) suggested the following estimator of σ_u^2:

$$\hat{\sigma}_u^2 = \frac{\sum_{i=1}^N \Delta y_i' \Theta_2^{(\lambda)} \Delta y_i}{Ntr(\Theta_2^{(\lambda)})},$$

with

$$\Theta_2^{(\lambda)} = \Psi_2^{(\lambda)} + \frac{tr(\Lambda' Q_2^{(\lambda)} M^{(1)})}{trace(M^{(1)} J_1)} M^{(1)} + \frac{tr(\Lambda' Q_2^{(\lambda)} M^{(2)})}{trace(M^{(2)} J_2)} M^{(2)}, \tag{3}$$

where $\Psi_2^{(\lambda)}$ is a diagonal matrix of (TXT)-dimension having in its main diagonal the elements of the main diagonal of the matrix $\Lambda' Q_2^{(\lambda)}$, $J_1 = e^{(1)} e^{(1)\prime}$ and $J_2 = e^{(2)} e^{(2)\prime}$ and $M^{(1)} = J_1 - diag\{e^{(1)}\}$ and $M^{(2)} = J_2 - diag\{e^{(2)}\}$, where $diag\{e^{(r)}\}$, $r = \{1, 2\}$, are matrices that have zeros everywhere except from their main diagonal which have the elements of vectors

$e^{(r)}$. Matrix $\Theta_2^{(\lambda)}$ plays the same role as $\Psi_1^{(\lambda)}$, for the KT test statistic in the case of model $M1$. It provides an estimator of σ_u^2 which enables us to correct the numerator of LS estimator $\hat{\varphi}_2^{(\lambda)}$ for its inconsistency, due to the within transformation of the individual series of the panel, while in parallel providing a number of sample moments upon which inference about unit roots can be drawn. This implies that the variance of the limiting distribution of the adjusted for its inconsistency estimator $\left(\hat{\varphi}_2^{(\lambda)} - 1 - \frac{\hat{b}_2^{(\lambda)}}{\hat{\delta}_2^{(\lambda)}}\right)$ will be given as $V_{KT,2}^{(\lambda)} = 2\sigma_u^4 tr(A_{KT,2}^{(\lambda)2})$, with $A_{KT,2}^{(\lambda)} = \frac{1}{2}(\Lambda' Q_2^{(\lambda)} + Q_2^{(\lambda)}\Lambda - \Theta_2^{(\lambda)} - \Theta_2^{(\lambda)\prime})$.

The next theorem derives the limiting distribution of the HT and KT statistics under the sequence of local alternatives local alternatives $\varphi_N = 1 - \frac{c}{\sqrt{N}}$.

Theorem 2 *Let conditions (b1)-(b6) of Assumption 1 hold and $u_{it} \sim NIID(0, \sigma^2)$. Then, under $\varphi_N = 1 - \frac{c}{\sqrt{N}}$, we have*

$$V_{HT,2}^{(\lambda)-1/2}\sqrt{N}(\hat{\varphi}_2^{(\lambda)} - 1 - B_2(\lambda)) \xrightarrow{L} N\left(-ck_{HT,2}, 1\right) \quad \textit{and}$$

$$V_{KT,2}^{(\lambda)-1/2}\hat{\delta}_2^{(\lambda)}\sqrt{N}\left(\hat{\varphi}_2^{(\lambda)} - 1 - \frac{\hat{b}_2^{(\lambda)}}{\hat{\delta}_2^{(\lambda)}}\right) \xrightarrow{d} N\left(-ck_{KT,2}, 1\right),$$

as $N \to \infty$, where

$$k_{HT,2} = 0 \textit{ and } k_{KT,2} = 0.$$

The results of the theorem indicate that the well known incidental trends problem of panel data unit root tests (see e.g. Moon et al. (2007)) also exists even if the tests allow for break and T is fixed. Both the HT and KT test statistics have trivial power. This result holds for the large-T version of the tests, too.

4 Power of the KT tests if error terms u_{it} are serially correlated

In this section, we consider the case that the variance-covariance matrix of error terms u_{it} has a more general form than $\Gamma = \sigma_u^2 I$, assumed in the previous section. That is, we assume that $\Gamma = [\gamma_{ts}]$, where $\gamma_{ts} = E(u_{it}u_{is}) = 0$ for $s = t + p_{\max} + 1, ..., T$ and $t < s$. This means that u_{it} allow for heteroscedasticity and serial correlation of maximum lag order $p_{\max}$. Our analysis enables us to investigate the combined effects of a structural break and serial correlation in u_{it} on the asymptotic local power of panel unit roots. As only the KT test is extended to allow for serially correlated errors u_{it} (see, e.g., Karavias and Tzavalis (2012)), our analysis will be focused on this test.

For both models $M1$ and $M2$, the KT test statistic under the above assumptions about u_{it} has analogous forms to those presented in the previous section. What changes is that, in order to take into account for an p-order serial correlation in u_{it} which will be appeared in the p-upper and p-lower secondary diagonals of matrix Γ, selection matrices $\Psi_1^{(\lambda)}$ and $\Psi_2^{(\lambda)}$ now are defined differently. They constitute (TXT)-dimension matrices having in their main diagonals and their p-lower and p-upper diagonals the corresponding elements of matrices of $\Lambda' Q_1^{(\lambda)}$ and $\Lambda' Q_2^{(\lambda)}$, respectively. Thus, they will be henceforth denoted by the subscript "p", as $\Psi_{p,1}^{(\lambda)}$ and $\Psi_{p,2}^{(\lambda)}$. Furthermore, in the same reasoning, matrix $M_p^{(1)}$ has elements $m_{ts}^{(1)} = 0$ if $\gamma_{ts} \neq 0$, and $m_{ts}^{(1)} = 1$ if $\gamma_{ts} = 0$, matrix $M_p^{(2)}$ has elements $m_{ts}^{(2)} = 0$ if $\gamma_{ts} \neq 0$, and $m_{2ts} = 1$ if $\gamma_{ts} = 0$. For model $M2$, the corresponding matrix to $\Theta_2^{(\lambda)}$ now will be denoted with the subscript "p" as

$$\Theta_{p,2}^{(\lambda)} = \Psi_{p,2}^{(\lambda)} + \frac{tr(\Lambda' Q_2^{(\lambda)} M_p^{(1)})}{trace(M_p^{(1)} J_1)} M_p^{(1)} + \frac{tr(\Lambda' Q_2^{(\lambda)} M_p^{(2)})}{trace(M^{(2)} J_2)} M_p^{(2)}, \tag{4}$$

where matrix $M_p^{(1)}$ selects the elements of matrix $\beta_N^{(1)} e^{(1)} e^{(1)\prime} + \beta_N^{(2)} e^{(2)} e^{(2)\prime} + \Gamma$ consisting only of individual slope coefficient effects $\beta_N^{(1)}$, for $t, s \leq T_0$. For t or $s > T_0$, all elements of $M_p^{(1)}$ are set to $m_{ts}^{(1)} = 0$. On the other hand, matrix $M_p^{(2)}$ selects the elements of matrix $\beta_N^{(1)} e^{(1)} e^{(1)\prime} + \beta_N^{(2)} e^{(2)} e^{(2)\prime} + \Gamma$ consisting only of effects $\beta_N^{(2)}$, for $t, s > T_0$.

For both models $M1$ and $M2$, the consistent estimator of the inconsistency of the LS estimator $\hat{\varphi}_m^{(\lambda)}$ for the KT test is defined as

$$\frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} = \frac{tr(\Psi_{p,1}^{(\lambda)} \hat{\Gamma})}{\frac{1}{N} \sum_{i=1}^{N} y_{i,-1}' Q_1^{(\lambda)} y_{i,-1}} \text{ and } \frac{\hat{b}_2^{(\lambda)}}{\hat{\delta}_2^{(\lambda)}} = \frac{tr(\Theta_{p,2}^{(\lambda)} \hat{\Gamma})}{\frac{1}{N} \sum_{i=1}^{N} y_{i,-1}' Q_1^{(\lambda)} y_{i,-1}}$$

respectively, where $\hat{\Gamma} = \frac{1}{N} \sum_{i=1}^{N} \Delta y_i \Delta y_i'$ constitutes an estimator of variance-covariance matrix Γ under null hypothesis H_0: $c = 0$. This is consistent for model $M1$. For model $M2$, it is premultiplied by matrix $\Theta_{p,2}^{(\lambda)}$ to become net of the individual effects $\beta_N^{(1)}$ and $\beta_N^{(2)}$. The variance of the limiting distribution of the adjusted for its inconsistency estimator $\hat{\varphi}_m^{(\lambda)}$, $\left(\hat{\varphi}_m^{(\lambda)} - \frac{\hat{b}_m^{(\lambda)}}{\hat{\delta}_m^{(\lambda)}} - 1\right)$, is given as $V_{KT,1}^{(\lambda)} = 2tr\left((A_{KT,1}^{(\lambda)} \Gamma)^2\right)$, with $A_{KT,1}^{(\lambda)} = \frac{1}{2}(\Lambda' Q_1^{(\lambda)} + Q_1^{(\lambda)} \Lambda - \Psi_{p,1}^{(\lambda)} - \Psi_{p,1}^{(\lambda)\prime})$, for model $M1$, and $V_{KT,2}^{(\lambda)} = 2tr\left((A_{KT,2}^{(\lambda)} \Gamma)^2\right)$, with $A_{KT,2}^{(\lambda)} = \frac{1}{2}(\Lambda' Q_2^{(\lambda)} + Q_2^{(\lambda)} \Lambda - \Theta_{p,2}^{(\lambda)} - \Theta_{p,2}^{(\lambda)\prime})$, for model $M2$. [5]

In the next theorem, we provide the limiting distribution of the KT test under the sequence of local alternatives $\varphi_N = 1 - \frac{c}{\sqrt{N}}$, for model $M1$ allowing for serial correlation in u_{it}. As shown in Karavias and Tzavalis (2012), the limiting distribution of the test for this model can be derived assuming that the maximum order of serial correlation of u_{it}, $p_{\max}$, is given as

$$p_{\max} = [T/2 - 2]^*,$$

[5] Note that, for notation simplicity, subscript "p" is suppressed from the notation of $\hat{b}_m^{(\lambda)}$, $V_{KT,1}^{(\lambda)}$ and $V_{KT,2}^{(\lambda)}$.

where $[.]^*$ denotes the greatest integer function.

Theorem 3 *Let conditions (b1)-(b5) of Assumption 1 hold. Then, under* $\varphi_N = 1 - \frac{c}{\sqrt{N}}$, *we have*

$$V_{KT,1}^{(\lambda)-1/2}\hat{\delta}_1^{(\lambda)}\sqrt{N}\left(\hat{\varphi}_1^{(\lambda)} - \frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} - 1\right) \xrightarrow{d} N(-ck_{KT,1}, 1), \;\; \textit{for model } M1,$$

as $N \to \infty$, *where*

$$k_{KT,1} = \frac{tr(F'Q_1^{(\lambda)}\Gamma) + tr(\Lambda'Q_1^{(\lambda)}\Lambda\Gamma) - tr(\Psi_{p,1}^{(\lambda)}\Lambda\Gamma) - tr(\Lambda'\Psi_{p,1}^{(\lambda)}\Gamma)}{\sqrt{2tr((A_{KT,1}^{(\lambda)}\Gamma)^2)}}.$$

The results of the theorem indicate that the asymptotic local power of the KT test now depends also on the values of the variance-covariance parameters γ_{ts}, affecting the power slope parameter $k_{KT,1}$. This can increase, or reduce, the local power of the test depending on the sign of γ_{ts}. To see this more clearly, in Table 3, we present estimates of the power slope parameter $k_{KT,1}$ assuming that error terms u_{it} follow a MA(1) process:

$$u_{it} = \varepsilon_{it} + \theta\varepsilon_{it-1},$$

where $\varepsilon_{it} \sim NIID(0, \sigma_\varepsilon^2)$. Note that the table also considers the case that $\theta = 0$ (i.e., there is no serial correlation), but the KT test allows for serial correlation of order $p = 1$. This case can show if the KT test loses significant power if a higher order of serial correlation p is assumed than the correct one. The results of the table also show that the KT test has always power if $\theta \geq 0$ or the break point T_0 is at the middle of the sample (i.e., $\lambda = 0.5$), as in the case of no serial correlation (see Table 1). The finding that the test has power even if $\theta = 0$, for all cases of T_0 considered, indicates that it can be safely applied to test for unit roots even if higher than the correct order of serial correlation is assumed.[6] As was expected, the power of the test in this case is always less compared to that when the correct lag order $p = 0$ is considered. This can be attributed to the fact that the test exploits less moment conditions in drawing inference about unit roots, by assuming $p = 1$ when $\theta = 0$.

Another interesting conclusion that can be drawn from the results of the table is that, when $\theta > 0$, the power of the KT test becomes bigger than that of its version which does not allow for serial correlation u_{it}, presented in the previous section (see Table 2). We have found that this result can be mainly attributed to the presence of terms $tr(\Psi_{p,1}^{(\lambda)}\Lambda\Gamma)$ and $tr(\Lambda'\Psi_{p,1}^{(\lambda)}\Gamma)$ in the function of slope coefficient $k_{KT,1}$, given by Theorem 3. These have a positive effect on $k_{KT,1}$ (i.e., $tr(\Psi_{p,1}^{(\lambda)}\Lambda\Gamma) + tr(\Lambda'\Psi_{p,1}^{(\lambda)}\Gamma) < 0$) when $\theta > 0$ and a negative effect when $\theta < 0$ (i.e., $tr(\Psi_{p,1}^{(\lambda)}\Lambda\Gamma) + tr(\Lambda'\Psi_{p,1}^{(\lambda)}\Gamma) > 0$).[7] As T increases, the above sign effects of the sign of θ on the KT test are amplified. These power gains of the KT test for model $M1$, when $\theta > 0$, may be also attributed to the fact that a positive value of θ adds to the variability of individual panel series y_{it}, driving further away the limiting distributions of the test under the null and alternative hypotheses.

For model $M2$, the limiting distributions of the KT test under $\varphi_N = 1 - \frac{c}{\sqrt{N}}$ and serially correlated error terms u_{it} are given in the next theorem. Note that, for this model, the maximum order of serial correlation allowed by the KT test is given as

$$p_{\max} = \begin{cases} \frac{T}{2} - 3, \text{if } T \text{ is even and } T_0 = \frac{T}{2} \\ \leq \min\{T_0 - 2, T - T_0 - 2\} \text{ otherwise,} \end{cases}$$

[6] We have found that this is true even for $p > 1$.

[7] The sum of traces $tr(F'Q_1^{(\lambda)}\Gamma) + tr(\Lambda'Q_1^{(\lambda)}\Lambda\Gamma)$ affects the power of the KT test, too. However, because this constitutes a parabola function which opens upwards, its effect on $k_{KT,1}$ is almost symmetrical with respect to the sign of θ. Thus, the relationship between $k_{KT,1}$ and θ is mainly determined by $tr(\Psi_{p,1}^{(\lambda)}\Lambda\Gamma) + tr(\Lambda'\Psi_{p,1}^{(\lambda)}\Gamma)$.

see Karavias and Tzavalis (2012).

Theorem 4 *Let conditions (b1)-(b6) of Assumption 1 hold. Then, under* $\varphi_N = 1 - \frac{c}{\sqrt{N}}$, *we have*

$$V_{KT,2}^{(\lambda)-1/2}\hat{\delta}_2^{(\lambda)}\sqrt{N}\left(\hat{\varphi}_2^{(\lambda)} - 1 - \frac{\hat{b}_2^{(\lambda)}}{\hat{\delta}_2^{(\lambda)}}\right) \xrightarrow{d} N\left(-ck_{KT,2}, 1\right), \text{ for model } M2,$$

as $N \to \infty$, *where*

$$k_{KT,2} = \frac{tr(F'Q_2^{(\lambda)}\Gamma) + tr(\Lambda' Q_2^{(\lambda)}\Lambda\Gamma) - tr(\Theta_{p,2}^{(\lambda)}\Lambda\Gamma) - tr(\Lambda\Theta_{p,2}'^{(\lambda)}\Gamma)}{\sqrt{2tr((A_{KT,2}^{(\lambda)}\Gamma)^2)}}.$$

The results of the theorem indicate that, if it allows for serial correlation in u_{it}, the KT test can have non-trivial power even in the case of incidental trends. Table 4 presents values of $k_{KT,2}$ for the case that u_{it} follows $MA(1)$ process: $u_{it} = \varepsilon_{it} + \theta\varepsilon_{it-1}$. This is done for different values of θ and T. As in Table 3, we also consider the case that $\theta = 0$.

The results of Table 4 indicate that, for model $M2$, the KT test has non-trivial power only if $\theta < 0$. If $\theta = 0$, the test has trivial power while for $\theta > 0$, the test is biased. For $\theta < 0$, the power of the test increases with T. For a given T, it becomes bigger if the break point T_0 is located towards the end of the sample, i.e. $\lambda = 0.75$. These results are in contrast to those for model $M1$, presented in Table 3, where the KT test is found to have more power if $\theta > 0$. This can be attributed to the interaction between matrix Γ and annihilator matrix $Q_2^{(\lambda)}$, entering the trace terms $tr(.)$, on the power slope parameter $k_{KT,2}$ and, in particular, on terms $tr(\Theta_{p,2}^{(\lambda)}\Lambda\Gamma)$ and $tr(\Lambda\Theta_{p,2}'^{(\lambda)}\Gamma)$. Calculations of these terms show that negative values of θ reverse the power reduction effects coming from detrending of the individual panel series through matrix $Q_2^{(\lambda)}$. In contrast to model $M1$, this now happens only when $\theta < 0$. As for model $M1$, the above gains in power of the KT test, when $\theta < 0$, may be also attributed to the reduction in the variability of series y_{it}, which a negative value of θ implies. The series behave more like being generated from a model with a common trend. As shown by Moon et al. (2007), in this case the incidental parameter problem disappears.

5 Monte Carlo results

In this section, we conduct a Monte Carlo study to examine if the asymptotic local power functions of the HT and KT tests, implied by the results of the previous section, provide good approximations of their small sample ones. This is done based on 5000 iterations. For each iteration, we calculate the size of the tests at 5% level (i.e., for $c = 0$) and their power (i.e., for $c = 1$). This is done separately for the cases that $u_{it} \sim NIID(0,1)$ and $u_{it} = \varepsilon_{it} + \theta\varepsilon_{it-1}$, with $\theta \in \{-0.8, -0.5, 0, 0.5, 0.8\}$. The N and T-dimensions of the panel data models are assumed as follows: $N \in \{100, 300, 1000\}$ and $T \in \{8, 10, 15, 20\}$, while the break fraction is taken to be $\lambda \in \{[0.25T], [0.5T], [0.75T]\}$, where $[\cdot]$ denotes integer part. The nuisance parameters of the models are set to the following values: $y_{i0} = 0$, $a_i^{(\lambda)} = 0$ and $\beta_i^{(\lambda)} = 0$, for all i, as they do not affect the limiting distribution of the tests.

Tables 5 and 6 present the results of our simulation study for the case that $u_{it} \sim NIID(0,1)$. The last column of the tables gives the theoretical values (TV) of the power function and the nominal size of the tests, at $a = 5\%$. For model $M1$, the results of Table 6 indicate that both the HT and KT tests have size and power values which are very close to their theoretical ones. Furthermore, the results confirm that the HT test has more power towards the beginning and the end of the sample while the KT test has more power in the middle. As was also predicted by the theory, the HT test has higher power than the KT test. The small sample power of this test is very close to that predicted by its asymptotic local power function (see column TV) even for small N (e.g., N=100). However, this is not always true for the KT test, which needs very high N in order its power to converge to its theoretical value. For model $M2$, the results of Table 6 indicate that, for large N, both HT and KT tests have trivial power, as it was expected. However, in small samples (e.g., $N = 100$), both tests have some non-trivial power. This can be obviously attributed to second, or higher, order effects of the true power function, which cannot be approximated by the first-order approximation considered in our analysis. Note that, for model $M2$, the KT test has slightly higher small sample power than the HT.

Tables 7, 8, 9 and 10 present the results of our simulation study for the KT test allowing for serial correlation in error terms u_{it}, assuming $u_{it} = \varepsilon_{it} + \theta\varepsilon_{it-1}$. This is done for models $M1$ and $M2$, and $T \in \{8, 10\}$. The maximum order of serial correlation allowed by the KT test is set to $p = 1$, which matches that of the MA process of u_{it}. The results of these tables are also consistent with theory. For model $M1$, the KT test has significant power when $\theta > 0$. This converges to its theoretical value, reported in the last column of the table, quite fast as N increases. For negative values of θ, the test has also significant power. This happens for λ=$\{0.75\}$, as was predicted by the theory. Note that both the theoretical and small sample values of the power function of the KT are higher than their corresponding values in the absence of serial correlation (see Table 5). This is also consistent with the theory and can be attributed to the serial correlation effects of u_{it} on the power function of the test, discussed in the previous section.

For model $M2$, the results of Tables 9 and 10 indicate that the KT test has smaller power than for model $M1$. As was expected by the theory, the power of the test is non-trivial if $\theta < 0$. The KT test has also some small sample power if $\theta > 0$, which qualifies its use in practice. As was argued before, this power can be attributed to second, or higher, order effects of the true power function, which are not approximated efficiently by our asymptotic approximations. Finally, another interesting conclusion which can be drawn from the results of our simulation study reported in Tables (7)-(10) is that, when $\theta<0$, a break towards the end of the sample increases the power of the KT test. When $\theta > 0$, the power of the test is maximized at the middle of the sample. These results apply to both models $M1$ and $M2$. They are also consistent with the theoretical results reported in Table 4.

6 Conclusions

This paper analyzes the asymptotic local power properties of least-squares based fixed-T panel unit root tests allowing for a structural break in the deterministic components of the AR(1) panel data model, namely its individual effects and/or slope coefficients of its individual linear (incidental) trends. This is done by assuming that the cross-section dimension of the panel data models (N) grows large. Thus, the results of our analysis concern mainly applications of the above tests to short panels, often used in empirical microeconomic studies.

The paper derives the limiting distributions under the sequence of local alternatives of extensions of Harris and Tzavalis (1999) panel unit root tests (denoted as HT) allowing for a structural break (see Karavias and Tzavalis (2013)) and Karavias' and Tzavalis (2012) recently developed panel data unit root tests (denoted as KT). In addition to a structural break, the last test also allows for serial correlation in the error terms of the AR(1) panel data model. Both of these tests are based on the least squares dummy variables estimator of the autoregressive coefficient of the AR(1) panel data model which is corrected for its inconsistency due to the deterministic components of the panel and/or serial correlation effects of the error term.

The results of the paper lead to a number of interesting conclusions. First, they show that, for the standard AR(1) panel data model with white noise error terms and individual effects, both the HT and KT tests have significant asymptotic local power. The HT test has much higher power than the KT. The power of this test increases with T, in contrast to the KT test. The latter is found to be more appropriate for small T. This happens because, to adjust for the inconsistency of the least squares estimator, the KT test requires consistent estimation of the variance of the error term, which leads to a reduction of its power. The HT test does not depend on this nuisance parameter, as it adjusts the least squares estimator for both the inconsistency of its numerator and denominator, and thus the variance of the error term is cancelled out. The HT test is found to have more power when the break is towards the beginning or the end of the sample, while the KT test has more power when the break is towards the middle of the sample.

Second, both the HT and KT tests have asymptotically trivial power in the case that the AR(1) allows also for incidental trends. The allowance for a common break in the slope coefficients of the incidental trends does not change the behavior of the tests. This problem does not always exist for the KT test extended for serial correlation of the error term. In this case, the paper presents circumstances that the KT test has non-trivial power. In particular, this happens when the error term follows a MA(1) procedure with negative serial correlation. The power of the KT in this case can be attributed to the effects of the serial correlation of error term on the adjustment of the least squares estimator of the autoregressive coefficient for its inconsistency, upon which the KT test is based on. In contrast to large-T panel data unit root tests, the power function of fixed-T tests depend on the values of nuisance parameters capturing serial correlation effects which can affect the asymptotic (over N) power of the tests. The above results are confirmed through a Monte Carlo simulation exercise.

This exercise has shown that the empirical probabilities of rejection are very close to their theoretical values, which means that the asymptotic theory provides a good approximation of small sample results of fixed-T panel data unit roots.

References

[1] Arellano, M., 2003, Panel data econometrics, Oxford University Press, UK

[2] Bai J., Carrion-I-Silvestre, J.L., 2009. Structural Changes, Common Stochastic Trends, and Unit Roots in Panel Data. Review of Economic Studies, vol. 76(2), 471-501.

[3] Carrion-i-Silvestre, J.L., Del Barrio-Castro, T., Lopez-Bazo, E., 2005. Breaking the panels: An application to real per capita GDP. Econometrics Journal, 8, 159-175.

[4] Chan, F., Pauwels, L.L., 2011. Model specification in panel data unit root tests with an unknown break. Mathematics and Computers in Simulation. 81, 1299–1309.

[5] Hahn, J., Kuersteiner, G., 2002. Asymptotically unbiased inference for a dynamic panel model with fixed effects when both n and T are large. Econometrica. 70, 1639-1657.

[6] Hadri K., Larsson, R., Rao, Y., 2012. Testing for stationarity with a break in panels where the time dimension is finite. Bulletin of Economic Research, 64, s123-s148.

[7] Harris D., Harvey D., Leybourne S., and Sakkas N., 2010. Local asymptotic power of the Im-Pesaran-Shin panel unit root test and the impact of initial observations. Econometric Theory 26, 311-324.

[8] Harris, D., Leybourne, S., and McCabe, B., 2005. Panel Stationarity Tests for Purchasing Power Parity with Cross-Sectional Dependence. Journal of Business & Economic Statistics, vol. 23, 395-409.

[9] Harris, R., Tzavalis, E., 1999. Inference for unit roots in dynamic panels where the time dimension is fixed. Journal of Econometrics, 91, 201-226.

[10] Harris, R., Tzavalis, E., 2004. Inference for unit roots for dynamic panels in the presence of deterministic trends: Do stock prices and dividends follow a random walk ? Econometric Reviews 23, 149-166.

[11] Harvey, D.I., Leybourne S.J., 2005. On testing for unit roots and the initial observation. Econometrics Journal 8, 97–111.

[12] Karavias, Y., and Tzavalis, E., 2012. Generalized fixed-T Panel Unit Root Tests Allowing for Structural Breaks. Granger Centre Discussion Paper Series, No 12/02.

[13] Karavias, Y., and Tzavalis, E., 2013. Testing for unit roots in short panels allowing for structural breaks. Computational Statistics and Data Analysis. (In press). http://dx.doi.org/10.1016/j.csda.2012.10.014

[14] Kruiniger, H., 2008. Maximum Likelihood Estimation and Inference Methods for the Covariance Stationary Panel AR(1)/Unit Root Model. Journal of Econometrics 144, 447-464.

[15] Kruiniger, H., and E., Tzavalis, 2002. Testing for unit roots in short dynamic panels with serially correlated and heteroscedastic disturbance terms. Working Papers 459, Department of Economics, Queen Mary, University of London, London.

[16] Madsen E., 2010. Unit root inference in panel data models where the time-series dimension is fixed: a comparison of different tests. Econometrics Journal 13, 63-94.

[17] Meligkotsidou, L., Tzavalis, E., Vrontos I.D., 2011. A Bayesian analysis of unit roots and structural breaks in the level, the trend and the error variance of autoregressive models of economic series. Econometric Reviews, 30 (2), 208-249.

[18] Moon, H.R., Perron, B., 2008. Asymptotic local power of pooled t-ratio tests for unit roots in panels with fixed effects. Econometrics Journal 11, 80-104.

[19] Moon, H.R., Perron, B., Phillips, P.C.B., 2007. Incidental trends and the power of panel unit root tests. Journal of Econometrics, 141(2), 416-459.

[20] Pauwels, L.L., Chan, F., Mancini, G.T., 2012. Testing for Structural Change in Heterogeneous Panels with an Application to the Euro's Trade Effect. Journal of Time Series Econometrics, 4(2), Article 3.

[21] Perron, P., 1989. The great crash, the oil price shock, and the unit root hypothesis. Econometrica, 57, 1361-1401.

[22] Schott, J.R., 1996. Matrix Analysis for Statistics, Wiley-Interscience.

6.1 Appendix

In this appendix, we provide proofs of the theorems and the corollary presented in the main text of the paper.

Proof of Theorem 1: First, we derive the limiting distribution of the HT test statistic, under the sequence of local alternatives $\varphi_N = 1-\frac{c}{\sqrt{N}}$. Define vector $w = (1, \varphi_N, \varphi_N^2, ..., \varphi_N^{T-1})'$

and matrix

$$\Omega = \begin{pmatrix} 0 & . & . & . & . & . & 0 \\ 1 & 0 & & & & & . \\ \varphi_N & 1 & . & & & & . \\ \varphi_N^2 & \varphi_N & . & . & & & . \\ . & & . & . & . & & . \\ . & & & . & 1 & 0 & . \\ \varphi_N^{T-2} & \varphi_N^{T-3} & . & . & \varphi_N & 1 & 0 \end{pmatrix}$$

Under null hypothesis H_0: $c = 0$, we have $\Omega = \Lambda$. The first order Taylor expansions of Ω and w yields

$$\begin{aligned} \Omega &= \Lambda + F(\varphi_N - 1) + o_p(1) \text{ and} \qquad (5) \\ w &= e + f(\varphi_N - 1) + o_P(1), \qquad (6) \end{aligned}$$

respectively, where $F = \frac{d\Omega}{d\varphi_N} \mid_{c=0}$ and $f = \frac{dw}{d\varphi_N} \mid_{c=0}$. Based on the above definitions of w and Ω, vector y_{i-1} can be written as

$$y_{i-1} = wy_{i0} + \Omega X_1^{(\lambda)} \gamma_i^{(\lambda)} + \Omega u_i, \qquad (7)$$

where $\gamma_i^{(\lambda)} = (a_i^{(1)}(1 - \varphi_N), a_i^{(2)}(1 - \varphi_N))' = (1 - \varphi_N)(a_i^{(1)}, a_i^{(2)})'$. Using last relationship of

y_{i-1}, the HT test statistic for model $M1$ can be written under $\varphi_N = 1 - \frac{c}{\sqrt{N}}$ as follows:

$$
\begin{aligned}
& \sqrt{N}(\hat{\varphi}_1^{(\lambda)} - \varphi_N - B_1(\lambda)) \qquad (8) \\
= & \sqrt{N}\left(\frac{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} (\varphi_N y_{i-1} + X_1^{(\lambda)} \gamma_i^{(\lambda)} + u_i)}{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} y_{i-1}} - \varphi_N - B_1(\lambda) \right), \\
= & \sqrt{N}\left(\frac{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} u_i}{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} y_{i-1}} - B_1(\lambda) \frac{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} y_{i-1}}{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} y_{i-1}} \right), \\
= & \frac{\frac{1}{\sqrt{N}}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} u_i - \frac{1}{\sqrt{N}} B_1(\lambda) \sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} y_{i-1}}{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} y_{i-1}} = \frac{(A) - (B)}{(C)}. \qquad (9)
\end{aligned}
$$

Next, we derive asymptotic results of each of quantities (A), (B) and (C), defined by (9). Substituting (7) in (A), we have

$$
\begin{aligned}
(A) \equiv & \frac{1}{\sqrt{N}}\sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} u_i = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} \left(y_{i0} w' + \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime} \Omega' + u'_i \Omega' \right) Q_1^{(\lambda)} u_i \\
= & \frac{1}{\sqrt{N}}\sum_{i=1}^{N} \left(y_{i0} w' Q_1^{(\lambda)} u_i + \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime} \Omega' Q_1^{(\lambda)} u_i + u'_i \Omega' Q_1^{(\lambda)} u_i \right)
\end{aligned}
$$

Using relationships (5)-(6), we can find the following limits of the summands entering into the last relationship of (A). First, it can be shown that

$$
\begin{aligned}
\frac{1}{\sqrt{N}}\sum_{i=1}^{N} y_{i0} w' Q_1^{(\lambda)} u_i = & \frac{1}{\sqrt{N}}\sum_{i=1}^{N} y_{i0}(e' + f'(\varphi_N - 1)) Q_1^{(\lambda)} u_i + o_P(1), \\
= & \frac{1}{\sqrt{N}}\sum_{i=1}^{N} y_{i0} e' Q_1^{(\lambda)} u_i + \frac{c}{N}\sum_{i=1}^{N} y_{i0} f' Q_1^{(\lambda)} u_i + o_P(1), \\
= & o_P(1), \qquad (10)
\end{aligned}
$$

since $e'Q_1^{(\lambda)} = 0$ and $E(y_{i0}u_i) = 0$ by assumption (b5), and

$$\begin{aligned}
& \frac{1}{\sqrt{N}}\sum_{i=1}^{N} \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime}\Omega' Q_1^{(\lambda)} u_i \\
= & \frac{1}{\sqrt{N}}\sum_{i=1}^{N} \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime}(\Lambda' + F'(\varphi_N - 1) + o_p(1)) Q_1^{(\lambda)} u_i, \\
= & \frac{1}{\sqrt{N}}\sum_{i=1}^{N} \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime}\Lambda' Q_1^{(\lambda)} u_i + \frac{c}{N}\sum_{i=1}^{N} \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime} F' Q_1^{(\lambda)} u_i + o_p(1), \\
= & \frac{c}{N}\sum_{i=1}^{N} (a_i^{(1)}, a_i^{(2)})' X_1^{(\lambda)\prime}\Lambda' Q_1^{(\lambda)} u_i + \frac{c^2}{N^{3/2}}\sum_{i=1}^{N} (a_i^{(1)}, a_i^{(2)})' X_1^{(\lambda)\prime} F' Q_1^{(\lambda)} u_i + o_p(1), \\
= & o_p(1),
\end{aligned} \tag{11}$$

since $E(a_i^{(\lambda)} u_i) = 0$ by assumption (b5). Finally, we have

$$\begin{aligned}
\frac{1}{\sqrt{N}}\sum_{i=1}^{N} u_i'\Omega' Q_1^{(\lambda)} u_i &= \frac{1}{\sqrt{N}}\sum_{i=1}^{N} u_i' \left(\Lambda' + F'(\varphi_N - 1) + o_p(1)\right) Q_1^{(\lambda)} u_i, \\
&= \frac{1}{\sqrt{N}}\sum_{i=1}^{N} u_i'\Lambda' Q_1^{(\lambda)} u_i - \frac{c}{N}\sum_{i=1}^{N} u_i' F' Q_1^{(\lambda)} u_i + o_p(1),
\end{aligned}$$

where

$$\frac{c}{N}\sum_{i=1}^{N} u_i' F' Q_1^{(\lambda)} u_i \xrightarrow{p} c\sigma_u^2 tr(F'Q_1^{(\lambda)}) \quad \text{and} \tag{12}$$

$$\sqrt{N}\left(\frac{1}{N}\sum_{i=1}^{N} u_i'\Lambda' Q_1^{(\lambda)} u_i - \sigma_u^2 tr(\Lambda' Q_1^{(\lambda)})\right) \xrightarrow{d} N(0, V_{HT,A}), \tag{13}$$

where $V_{HT,A}$ is the variance of the last limiting distribution. Based on the asymptotic results given by equations (10)-(13), we can show that

$$(A) \equiv \frac{1}{\sqrt{N}}\sum_{i=1}^{N} y_{i-1}' Q_1^{(\lambda)} u_i \xrightarrow{d} N\left(-c\sigma_u^2 tr(F'Q_1^{(\lambda)}), V_{HT,A}\right). \tag{14}$$

To derive asymptotic results for summand (B), write it as follows:

$$\begin{aligned}(B) &\equiv \frac{1}{\sqrt{N}} B_1(\lambda) \sum_{i=1}^{N} y_{i-1}' Q_1^{(\lambda)} y_{i-1} \\ &= \frac{1}{\sqrt{N}} B_1(\lambda) \sum_{i=1}^{N} \left(y_{i0} w' + \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime} \Omega' + u_i' \Omega' \right) Q_1^{(\lambda)} \left(y_{i0} w + \Omega X_1^{(\lambda)} \gamma_i^{(\lambda)} + \Omega u_i \right).\end{aligned}$$

By similar arguments to those applied to derive results (10)-(13), we can prove the following asymptotic results:

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(y_{i0}^2 w' Q_1^{(\lambda)} w + y_{i0} w' Q_1^{(\lambda)} \Omega X_1^{(\lambda)} \gamma_i^{(\lambda)} + y_{i0} w' Q_1^{(\lambda)} \Omega u_i \right) = o_p(1), \quad (15)$$

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime} \Omega' Q_1^{(\lambda)} w y_{i0} + \gamma_i^{(\lambda)\prime} X_1^{(\lambda)\prime} \Omega' Q_1^{(\lambda)} \Omega X_1^{(\lambda)} \gamma_i^{(\lambda)} + \Omega X_1^{(\lambda)} \gamma_i^{(\lambda)} \Omega u_i) = o_p(1), \quad (16)$$

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(u_i' \Omega' Q_1^{(\lambda)} w y_{i0} + u_i' \Omega' Q_1^{(\lambda)} \Omega X_1^{(\lambda)} \gamma_i^{(\lambda)} \right) = o_p(1), \quad (17)$$

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{N} u_i' \Omega' Q_1^{(\lambda)} \Omega u_i = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} u_i' (\Lambda' + F'(\varphi_N - 1)) Q_1^{(\lambda)} (\Lambda + F(\varphi_N - 1)) u_i + o_p(1), \quad (18)$$

where

$$\sqrt{N} \left(\frac{1}{N} \sum_{i=1}^{N} u_i' \Lambda' Q_1^{(\lambda)} \Lambda u_i - \sigma_u^2 tr(\Lambda' Q_1^{(\lambda)} \Lambda) \right) \xrightarrow{d} N(0, V_{HT,B}), \quad (19)$$

$$-\frac{c}{N} \sum_{i=1}^{N} u_i' F' Q_1^{(\lambda)} \Lambda u_i \xrightarrow{p} \sigma_u^2 tr(F' Q_1^{(\lambda)} \Lambda), \quad (20)$$

$$-\frac{c}{N} \sum_{i=1}^{N} u_i' \Lambda' Q_1^{(\lambda)} F u_i \xrightarrow{p} \sigma_u^2 tr(\Lambda' Q_1^{(\lambda)} F) \quad \text{and} \quad (21)$$

$$\frac{c^2}{N^{3/2}} \sum_{i=1}^{N} u_i' F' Q_1^{(\lambda)} F u_i = o_p(1). \quad (22)$$

Based on the above results, given by equations (15)-(22), it can be shown that

$$(B) \equiv \frac{1}{\sqrt{N}} B_1(\lambda) \sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} y_{i-1} \xrightarrow{d} N\left(-c\sigma_u^2 B_1(\lambda)[tr(F' Q_1^{(\lambda)} \Lambda) + tr(\Lambda' Q_1^{(\lambda)} F)], B_1^2(\lambda) V_{HT,B}\right). \tag{23}$$

Finally, following similar arguments to the above, we can easily show that, for quantity (C), the following asymptotic result holds:

$$(C) \equiv \frac{1}{N} \sum_{i=1}^{N} y'_{i-1} Q_1^{(\lambda)} y_{i-1} \xrightarrow{p} \sigma_u^2 tr(\Lambda' Q_1^{(\lambda)} \Lambda). \tag{24}$$

Using asymptotic results (14), (23) and (24), equation (9) implies that

$$\sqrt{N}(\hat{\varphi}_1^{(\lambda)} - \varphi_N - B_1(\lambda)) \xrightarrow{d} N\left(-c\frac{tr(F' Q_1^{(\lambda)}) - 2B_1(\lambda) tr(F' Q_1^{(\lambda)} \Lambda)}{tr(\Lambda' Q_1^{(\lambda)} \Lambda)}, V_{HT,1}^{(\lambda)}\right), \tag{25}$$

$$\text{or} \sqrt{N}(\hat{\varphi}_1^{(\lambda)} - 1 - B_1(\lambda)) \xrightarrow{d} N\left(-c\frac{tr(F' Q_1^{(\lambda)}) + tr(\Lambda' Q_1^{(\lambda)} \Lambda) - 2B_1(\lambda) tr(F' Q_1^{(\lambda)} \Lambda)}{tr(\Lambda' Q_1^{(\lambda)} \Lambda)}, V_{HT,1}^{(\lambda)}\right),$$

since $tr(\Lambda' Q_1^{(\lambda)}) - B_1(\lambda) tr(\Lambda' Q_1^{(\lambda)} \Lambda) = 0$. Note that the analytic formula of variance $V_{HT,1}^{(\lambda)}$ of the last limiting distribution is the same with that of the HT test under null hypothesis H_0: $c = 0$, given by $V_{HT,1}^{(\lambda)} = \frac{2tr(A_{HT,1}^{(\lambda)2})}{tr(\Lambda' Q_1^{(\lambda)} \Lambda)^2}$. This does not depend on local parameter c. It remains the same under the null and sequence of local alternative hypotheses (see, e.g., Madsen (2010) and Karavias and Tzavalis (2013)), given as $V_{HT,1}^{(\lambda)} = \frac{2tr(A_{HT,1}^{(\lambda)2})}{tr(\Lambda' Q_1^{(\lambda)} \Lambda)^2}$. Scaling by $V_{HT,1}^{(\lambda)-1/2}$

the above limiting distribution yields

$$V_{HT,1}^{(\lambda)-1/2}\sqrt{N}(\hat{\varphi}_1^{(\lambda)}-1-B_1(\lambda))\xrightarrow{d}N\left(-ck_{HT,1},1\right),\ \text{ with} \tag{26}$$

$$k_{HT,1} = \frac{tr(F'Q_1^{(\lambda)})+tr(\Lambda'Q_1^{(\lambda)}\Lambda)-2B_1(\lambda)tr(F'Q_1^{(\lambda)}\Lambda)}{\sqrt{2tr(A_{HT,1}^{(\lambda)2})}}.$$

Substituting into the above formula of $k_{HT,1}$ the following identities:

$$tr(F'Q_1^{(\lambda)}\Lambda) = tr(\Lambda'Q_1^{(\lambda)}F) = \tag{27}$$

$$= \frac{6}{144}(3\lambda^2-3\lambda+1)T^3-\frac{1}{12}(2\lambda^2-2\lambda+1)T^2-\frac{1}{24}T+\frac{1}{6}$$

$$tr(\Lambda'Q_1^{(\lambda)}\Lambda)+tr(F'Q_1^{(\lambda)})+tr(\Lambda'Q_1^{(\lambda)}) = 0, \tag{28}$$

$$tr(F'Q_1^{(\lambda)}) = -\frac{T^2}{6}(2\lambda^2-2\lambda+1)+\frac{T}{2}-\frac{4}{6}, \tag{29}$$

$$tr(\Lambda'Q_1^{(\lambda)}) = -\frac{T-2}{2}, \tag{30}$$

$$tr(\Lambda'Q_1^{(\lambda)}\Lambda) = \frac{T^2}{6}(2\lambda^2-2\lambda+1)-\frac{2}{6}, \tag{31}$$

$$tr(A_{HT,1}^{(\lambda)2}) = tr\left[\left(\frac{1}{2}(\Lambda'Q_1^{(\lambda)}+Q_1^{(\lambda)}\Lambda)-B_1(\lambda)(\Lambda'Q_1^{(\lambda)}\Lambda)\right)^2\right], \tag{32}$$

$$tr\left(\left(\Lambda'Q_1^{(\lambda)}+Q_1^{(\lambda)}\Lambda\right)^2\right) = \frac{T^2}{6}(2\lambda^2-2\lambda+1)+T-\frac{7}{3}, \tag{33}$$

$$tr\left(\left(\Lambda'Q_1^{(\lambda)}\Lambda\right)^2\right) = \frac{1}{90}(2\lambda^4-4\lambda^3+6\lambda^2-4\lambda+1)T^4 + \frac{1}{36}(2\lambda^2-2\lambda+1)T^2-\frac{7}{90}, \tag{34}$$

$$tr\left(\left(\Lambda'Q_1^{(\lambda)}+Q_1^{(\lambda)}\Lambda\right)\left(\Lambda'Q_1^{(\lambda)}\Lambda\right)\right) = \frac{T-2}{2}, \tag{35}$$

yields the results of Theorem 1, for the HT test statistic. Note that $2tr(A_{HT,1}^{(\lambda)2})$ can be analytically written as

$$2tr(A_{HT,1}^{(\lambda)^2}) = \frac{D}{S}, \text{ where}$$

$$\begin{aligned}
D &= T^6 R_1 + T^5 R_2 + T^4 R_3 + T^2 R_4 + 216T - 136, \\
S &= T^4 \Phi_1 + T^2 \Phi_2 + 240, \\
R_1 &= 40\lambda^6 - 120\lambda^5 + 204\lambda^4 - 208\lambda^3 + 162\lambda^2 - 78\lambda + 17, \\
R_2 &= -216\lambda^4 + 432\lambda^3 - 528\lambda^2 + 312\lambda - 78, \\
R_3 &= 216\lambda^4 - 432\lambda^3 + 588\lambda^2 - 372\lambda + 108, \\
R_4 &= -120\lambda^2 + 120\lambda - 144, \\
\Phi_1 &= 240\lambda^4 - 480\lambda^3 + 480\lambda^2 - 240\lambda + 60 \text{ and} \\
\Phi_2 &= -480\lambda^2 + 480\lambda - 240.
\end{aligned}$$

To derive the limiting distribution of the KT test under the sequence of local alternatives $\varphi_N = 1 - \frac{c}{\sqrt{N}}$, write

$$\begin{aligned}
\hat{\delta}_1^{(\lambda)}\sqrt{N}\left(\hat{\varphi}_1^{(\lambda)} - \frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} - \varphi_N\right) &= \hat{\delta}_1^{(\lambda)}\sqrt{N}\left(\varphi_N + \frac{\frac{1}{N}\sum_{i=1}^{N} y_{i-1}' Q_1^{(\lambda)} u_i}{\frac{1}{N}\sum_{i=1}^{N} y_{i-1}' Q_1^{(\lambda)} y_{i-1}} - \frac{\hat{b}_1^{(\lambda)}}{\hat{\delta}_1^{(\lambda)}} - \varphi_N\right), \\
&= \sqrt{N}\left(\frac{1}{N}\sum_{i=1}^{N} y_{i-1}' Q_1^{(\lambda)} u_i - \hat{\sigma}_u^2 tr(\Lambda' Q_1^{(\lambda)})\right), \\
&= \sqrt{N}\left(\frac{1}{N}\sum_{i=1}^{N} y_{i-1}' Q_1^{(\lambda)} u_i - \frac{1}{N}\sum_{i=1}^{N} \Delta y_i' \Psi_1^{(\lambda)} \Delta y_i\right),
\end{aligned}$$

$$= \frac{1}{\sqrt{N}}\sum_{i=1}^{N} y_{i-1}' Q_1^{(\lambda)} u_i - \frac{1}{\sqrt{N}}\sum_{i=1}^{N} \Delta y_i' \Psi_1^{(\lambda)} \Delta y_i, \qquad (36)$$

where Δy_i can be written as

$$\Delta y_i = u_i + (\varphi_N - 1)y_{i-1} + X_1^{(\lambda)}\gamma_i^{(\lambda)}. \tag{37}$$

The limiting distribution of the KT test under $\varphi_N = 1 - \frac{c}{\sqrt{N}}$ can be proved by obtaining asymptotic results for the two summands entering into equation (36), i.e., $\frac{1}{\sqrt{N}}\sum_{i=1}^{N} y'_{i-1}Q_1^{(\lambda)}u_i$ and $\frac{1}{\sqrt{N}}\sum_{i=1}^{N} \Delta y'_i \Psi_1^{(\lambda)}\Delta y_i$, following analogous to the proof of (26) steps. The formula of slope power parameter $k_{KT,1}$ is given as

$$k_{KT,1} = \frac{tr(F'Q_1^{(\lambda)}) + tr(\Lambda' Q_1^{(\lambda)}\Lambda)}{\sqrt{2tr(A_{KT,1}^{(\lambda)^2})}}. \tag{38}$$

Substituting the following identities into the above formula of $k_{KT,1}$:

$$tr(A_{KT,1}^{(\lambda)^2}) = tr\left(\left(\frac{1}{2}(\Lambda' Q_1^{(\lambda)} + Q_1^{(\lambda)}\Lambda - \Psi_1^{(\lambda)} - \Psi_1^{(\lambda)\prime})\right)^2\right), \tag{39}$$

$$\begin{aligned}
tr(\Psi_1^{(\lambda)}\Lambda) &= tr(\Lambda'\Psi_1^{(\lambda)}) = 0, && (40)\\
2tr(A_{KT,1}^{(\lambda)^2}) &= 2tr(P^{(\lambda)}) - 2tr(Z^{(\lambda)2}), \text{ with } Z^{(\lambda)} = \frac{1}{2}(\Psi_1^{(\lambda)\prime} + \Psi_1^{(\lambda)}) && (41)\\
\text{and } P^{(\lambda)} &= \frac{1}{2}(\Lambda' Q_1^{(\lambda)})^2 + \frac{1}{2}\Lambda' Q_1^{(\lambda)}\Lambda, && (42)\\
tr\left((\Lambda' Q_1^{(\lambda)})^2\right) &= -\frac{T^2}{12}(2\lambda^2 - 2\lambda - 1) + \frac{T}{2} - \frac{5}{6} \text{ and} && (43)\\
tr(Z^{(\lambda)2}) &= \frac{-\frac{1}{T} + 2(\lambda - 1)\lambda T}{6(\lambda - 1)\lambda} - 1 && (44)
\end{aligned}$$

yields the results of Theorem 1, for the KT statistic.

Proof of Corollary 1: The results of the corollary and, in particular, those of equation

(1) can be derived based on analogous arguments to those applied for the proof of Theorem 1.

To obtain the analytic formula of $k^*_{HT,1}$, given by equation (1), scale (8) by T, replace φ_N with $\varphi_{N,T}$, and apply asymptotic theory for $N \to \infty$, as in Theorem 1. Then, we will have

$$T\sqrt{N}(\hat{\varphi}_1^{(\lambda)} - \varphi_{N,T} - B_1(\lambda)) \xrightarrow{d} N\left(-c\frac{tr(F'Q_1^{(\lambda)}) - 2B_1(\lambda)tr(F'Q_1^{(\lambda)}\Lambda)}{tr(\Lambda' Q_1^{(\lambda)}\Lambda)}, T^2 V_{HT,1}^{(\lambda)}\right).$$

Multiplying with $\left(T^2 V_{HT,1}^{(\lambda)}\right)^{-1/2}$ and using $\varphi_{N,T} = 1 - \frac{c}{T\sqrt{N}}$, the last limiting distribution can be written as

$$T\left(T^2 V_{HT,1}^{(\lambda)}\right)^{-1/2}\sqrt{N}(\hat{\varphi}_1^{(\lambda)} - 1 - B_1(\lambda)) \xrightarrow{d} N\left(-c\frac{1}{T}k_{HT,1}, 1\right) \tag{45}$$

where $k_{HT,1} = \frac{tr(F'Q_1^{(\lambda)}) + tr(\Lambda' Q_1^{(\lambda)}\Lambda) - 2B_1(\lambda)tr(F'Q_1^{(\lambda)}\Lambda)}{\sqrt{2tr(A_{HT,1}^{(\lambda)2})}}$ (see proof of Theorem 1). By taking the limit $T \to \infty$ of $k_{HT,1}$, $V_{HT,1}^{(\lambda)}$ and $\varphi_{N,T}$, (45) can be written as

$$T\left(V_{HT,1}^{*(\lambda)}\right)^{-1/2}\sqrt{N}(\hat{\varphi}_1^{(\lambda)} - 1 - B_1(\lambda)) \xrightarrow{d} N\left(-ck^*_{HT,1}, 1\right), \text{ where}$$

$$\begin{aligned} k^*_{HT,1} &\equiv \lim_T \frac{1}{T}k_{HT,1} = \frac{3\lambda^2 - 3\lambda + 1}{4(2\lambda^2 - 2\lambda + 1)}\sqrt{\frac{\Phi_1}{R_1}} \quad \text{and} \\ V_{HT,1}^{*(\lambda)} &\equiv \lim_T T^2 V_{HT,1}^{(\lambda)} = \frac{36R_1}{\Phi_1(2\lambda^2 - 2\lambda - 1)^2}. \end{aligned}$$

The analytic formulas of the last two limits are derived based on the results of identities (27)-(35). The above results have been derived by taking sequentially limits, first for $N \to \infty$ and, then, for $T \to \infty$. Joint convergence in N, T requires the extra assumption that $\frac{\sqrt{N}}{T} \to 0$, see also Moon and Perron (2008). However, for $c = 0$ there is no need to specify the relative rate of convergence between N and T (see Hahn and Kuersteiner (2002) and Karavias and Tzavalis (2013)).

The formulas of $k^*_{KT,1}$ and $V^{*(\lambda)}_{KT,1}$, given by the corollary for the large-T version of the KT test, can be derived by following similar steps to the above. Then, using the results of identities (39)-(44), we can obtain

$$k^*_{KT,1} \equiv \lim_T \frac{1}{T} k_{KT,1} = 0 \text{ and } V^{*(\lambda)}_{KT,1} \equiv \lim_T T^2 V^{(\lambda)}_{KT,1} \frac{36(2\lambda^4 - 4\lambda^3 + 3\lambda^2 - \lambda)}{12(\lambda-1)\lambda(2\lambda^2 - 2\lambda + 1)^2}.$$

Proof of Theorem 2: To prove the theorem, we will follow analogous steps to those for the proof of Theorem 1. We now will rely on relationships (7) and (37), where now vector $\gamma_i^{(\lambda)}$ is defined as

$$\gamma_i^{*(\lambda)} = \begin{pmatrix} (1-\varphi_N)a_i^{(1)} + \varphi_N \beta_i^{(1)} \\ (1-\varphi_N)a_i^{(2)} + \varphi_N \beta_i^{(2)} \\ (1-\varphi_N)\beta_i^{(1)} \\ (1-\varphi_N)\beta_i^{(2)} \end{pmatrix} = e_* \nu_i + (1-\varphi_N)\mu_i,$$

due to the presence of individual trends under $\varphi_{N,T} = 1 - \frac{c}{T\sqrt{N}}$, where $\mu_i = (\alpha_i^{(1)} - \beta_i^{(1)}, \alpha_i^{(2)} - \beta_i^{(2)}, \beta_i^{(1)}, \beta_i^{(2)})'$, $e_* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ and $\nu_i = (\beta_i^{(1)}, \beta_i^{(2)})'$. The non-standardized HT test statistic for model $M2$ can be written as follows:

$$\begin{aligned}
&\sqrt{N}(\hat{\varphi}_2^{(\lambda)} - \varphi_N - B_2(\lambda)) \\
= \ &\sqrt{N}\left(\frac{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1}Q_2^{(\lambda)}(\varphi_N y_{i-1} + X_2^{(\lambda)}\gamma_i^{*(\lambda)} + u_i)}{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1}Q_2^{(\lambda)}y_{i-1}} - \varphi_N - B_2(\lambda)\right) \\
= \ &\frac{\frac{1}{\sqrt{N}}\sum_{i=1}^{N} y'_{i-1}Q_2^{(\lambda)}u_i - \frac{1}{\sqrt{N}}B_2(\lambda)\sum_{i=1}^{N} y'_{i-1}Q_2^{(\lambda)}y_{i-1}}{\frac{1}{N}\sum_{i=1}^{N} y'_{i-1}Q_2^{(\lambda)}y_{i-1}} = \frac{(A') - (B')}{(C')},
\end{aligned}$$

where $(A') \equiv \frac{1}{\sqrt{N}}\sum_{i=1}^{N} y'_{i-1}Q_2^{(\lambda)}u_i$, $(B') \equiv \frac{1}{\sqrt{N}}B_2(\lambda)\sum_{i=1}^{N} y'_{i-1}Q_2^{(\lambda)}y_{i-1}$ and $(C') \equiv \frac{1}{N}\sum_{i=1}^{N} y'_{i-1}Q_2^{(\lambda)}y_{i-1}$. As in the proof of Theorem 1, next we derive asymptotic results of (A'), (B') and (C'), using $\gamma_i^{*(\lambda)} = e_*\nu_i + (1-\varphi_N)\mu_i$. The most important ones are the following:

$$\begin{aligned}
&\sqrt{N}\left(\frac{1}{N}\sum_{i=1}^{N} \nu'_i e'_* X_2^{(\lambda)\prime}\Lambda' Q_2^{(\lambda)}\Lambda X_2^{(\lambda)} e_*\nu_i - tr(e'_* X_2^{(\lambda)\prime}\Lambda' Q_2^{(\lambda)}\Lambda X_2^{(\lambda)} e_* E(\nu_i\nu'_i))\right) \xrightarrow{d} N(0, V_{HT,4}) \\
&\frac{c}{N}\sum_{i=1}^{N} \nu'_i e'_* X_2^{(\lambda)\prime} F' Q_2^{(\lambda)}\Lambda e_*\nu_i \xrightarrow{p} c\,tr(e'_* X_2^{(\lambda)\prime} F' Q_2^{(\lambda)}\Lambda e_* E(\nu_i\nu'_i)) \\
&\frac{c}{N}\sum_{i=1}^{N} \nu'_i e'_* X_2^{(\lambda)\prime}\Lambda' Q_2^{(\lambda)} F X_2^{(\lambda)} e_*\nu_i \xrightarrow{p} c\,tr(e'_* X_2^{(\lambda)\prime}\Lambda' Q_2^{(\lambda)} F e_* E(\nu_i\nu'_i)) \\
&\frac{c}{N}\sum_{i=1}^{N} \mu'_i X_2^{(\lambda)\prime}\Omega' Q_2^{(\lambda)}\Omega X_2^{(\lambda)} e_*\nu_i \xrightarrow{p} c\,tr(X_2^{(\lambda)\prime}\Lambda' Q_2^{(\lambda)}\Lambda X_2^{(\lambda)} e_* E(\nu_i\mu'_i)) \\
&\frac{c}{N}\sum_{i=1}^{N} \nu'_i e'_* X_2^{(\lambda)\prime}\Lambda' Q_2^{(\lambda)}\Lambda X_2^{(\lambda)}\mu_i \xrightarrow{p} c\,tr(e'_* X_2^{(\lambda)\prime}\Lambda' Q_2^{(\lambda)}\Lambda X_2^{(\lambda)} E(\mu_i\nu'_i))
\end{aligned}$$

Given these results, the proof of Theorem 2 for the test statistic HT follows immediately, after using the following identities:

$$
\begin{aligned}
tr(e_*' X_2^{(\lambda)\prime} \Lambda' Q_2^{(\lambda)} \Lambda X_2^{(\lambda)} e_* E(\nu_i \nu_i')) &= 0 \\
tr(e_*' X_2^{(\lambda)\prime} F' Q_2^{(\lambda)} \Lambda e_* E(\nu_i \nu_i')) - tr(e_*' X_2^{(\lambda)\prime} \Lambda' Q_2^{(\lambda)} F e_* E(\nu_i \nu_i')) &= 0 \\
\text{and } tr(X_2^{(\lambda)\prime} \Lambda' Q_2^{(\lambda)} \Lambda X_2^{(\lambda)} e_* E(\nu_i \mu_i')) - tr(e_*' X_2^{(\lambda)\prime} \Lambda' Q_2^{(\lambda)} \Lambda X_2^{(\lambda)} E(\mu_i \nu_i')) &= 0.
\end{aligned}
$$

The proof of the second result of the theorem, i.e., $k_{KT,2} = 0$, can be proved by following analogous steps to the above and using the following identities:

$$
\begin{aligned}
tr(e_*' X_2^{(\lambda)\prime} \Theta_2^{(\lambda)} \Lambda X_2^{(\lambda)} e_* E(\nu_i \nu_i')) - tr(e_*' X_2^{(\lambda)\prime} \Lambda' \Theta_2^{(\lambda)} X_2^{(\lambda)} e_* E(\nu_i \nu_i')) &= 0 \\
\text{and} \quad tr(X_2^{(\lambda)\prime} \Theta_2^{(\lambda)} X_2^{(\lambda)} e_* E(\nu_i \mu_i')) - tr(e_*' X_2^{(\lambda)\prime} \Theta_2^{(\lambda)} X_2^{(\lambda)} E(\mu_i \nu_i')) &= 0.
\end{aligned}
$$

Proof of Theorem 3: This can be proved by following analogous steps to the proof of Theorem 1, for the KT test statistic, by setting $E(u_i u_i') = \Gamma$ instead of $\sigma_u^2 I_T$.

Proof of Theorem 4: This can be proved by following analogous steps to the proof of Theorems 2 and 3, for the KT test statistic.

7 Tables

Table 1: Values of $k_{HT,1}$ and $k_{KT,1}$ for model $M1$

	$k_{HT,1}$				$k_{KT,1}$			
$\lambda\backslash T$	8	10	15	20	8	10	15	20
0.25	3.18	4.12	6.11	7.75	1.85	1.86	1.96	2.10
0.50	2.93	3.62	5.32	6.99	2.12	2.23	2.34	2.39
0.75	3.18	3.81	5.78	7.75	1.85	2.04	2.09	2.10

Table 2: Values of slope parameters $k^*_{HT,1}$ and $k^*_{KT,1}$

λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
k^*_{HT}	0.433	0.394	0.360	0.338	0.332	0.338	0.360	0.394	0.433
k^*_{KT}	0	0	0	0	0	0	0	0	0

Table 3: Values of $k_{KT,1}$ for model $M1$, with $u_{it} = \varepsilon_{it} + \theta\varepsilon_{it-1}$

T	T_0	$\theta = -0.8$	$\theta = -0.5$	$\theta = 0.0$	$\theta = 0.5$	$\theta = 0.8$
8	2	-1.40	-0.63	1.58	2.71	2.89
	4	0.25	0.61	1.89	2.86	3.04
	6	1.28	1.36	1.58	1.66	1.68
10	2	-1.62	-0.69	1.65	2.60	2.73
	5	0.07	0.56	2.12	3.05	3.21
	7	0.82	1.10	1.82	2.12	2.16
15	3	-1.55	-0.48	1.81	2.50	2.58
	7	-0.41	0.36	2.31	3.10	3.21
	11	0.52	1.01	1.95	2.20	2.23
20	5	-1.52	-0.31	2.00	2.61	2.68
	10	-0.54	0.38	2.38	3.02	3.10
	15	0.33	1.07	2.00	2.22	2.24

Table 4: Values of $k_{KT,2}$ for model $M2$, with $u_{it} = \varepsilon_{it} + \theta\varepsilon_{it-1}$

T	T_0	$\theta = -0.8$	$\theta = -0.5$	$\theta = 0.0$	$\theta = 0.5$	$\theta = 0.8$
8	4	0.08	0.070	0	-0.09	-0.11
10	5	0.20	0.15	0	-0.12	-0.14
	7	0.66	0.46	0	-0.21	-0.24
15	3	0	0	0	0	0
	7	0.47	0.32	0	-0.13	-0.15
	11	0.75	0.53	0	-0.20	-0.23
20	5	0.17	0.11	0	-0.03	-0.04
	10	0.70	0.45	0	-0.15	-0.17
	15	0.80	0.54	0	-0.17	-0.20

Table 5: Simulated values of $k_{HT,1}$ and $k_{KT,1}$ for model $M1$, with $u_{it} \sim NIID(0, \sigma_u^2)$

N				100	300	1000	TV
T=8	$\lambda = 0.25$	c=0	HT	0.048	0.060	0.059	0.050
			KT	0.054	0.050	0.050	0.050
		c=1	HT	0.775	0.853	0.894	0.938
			KT	0.352	0.428	0.474	0.583
	$\lambda = 0.5$	c=0	HT	0.054	0.055	0.053	0.050
			KT	0.048	0.052	0.052	0.050
		c=1	HT	0.768	0.828	0.866	0.901
			KT	0.487	0.546	0.608	0.682
	$\lambda = 0.75$	c=0	HT	0.064	0.055	0.051	0.050
			KT	0.063	0.055	0.051	0.050
		c=1	HT	0.889	0.906	0.926	0.938
			KT	0.375	0.453	0.490	0.583
T=10	$\lambda = 0.25$	c=0	HT	0.059	0.053	0.053	0.050
			KT	0.058	0.049	0.047	0.050
		c=1	HT	0.900	0.960	0.973	0.993
			KT	0.288	0.384	0.458	0.585
	$\lambda = 0.5$	c=0	HT	0.057	0.046	0.047	0.050
			KT	0.063	0.050	0.051	0.050
		c=1	HT	0.878	0.927	0.957	0.976
			KT	0.451	0.527	0.603	0.720
	$\lambda = 0.75$	c=0	HT	0.056	0.060	0.053	0.050
			KT	0.052	0.048	0.044	0.050
		c=1	HT	0.940	0.968	0.976	0.985
			KT	0.339	0.456	0.541	0.653

Table 6: Simulated values of $k_{HT,2}$ and $k_{KT,2}$ for model $M1$, with $u_{it} \sim NIID(0, \sigma_u^2)$

N				100	300	1000	TV
T=8	$\lambda = 0.25$	c=0	HT	0.047	0.040	0.051	0.050
			KT	0.056	0.062	0.057	0.050
		c=1	HT	0.076	0.068	0.065	0.050
			KT	0.087	0.073	0.069	0.050
	$\lambda = 0.5$	c=0	HT	0.054	0.056	0.052	0.050
			KT	0.050	0.060	0.050	0.050
		c=1	HT	0.065	0.060	0.046	0.050
			KT	0.073	0.061	0.060	0.050
	$\lambda = 0.75$	c=0	HT	0.057	0.053	0.047	0.050
			KT	0.060	0.057	0.057	0.050
		c=1	HT	0.061	0.065	0.052	0.050
			KT	0.102	0.080	0.062	0.050
T=10	$\lambda = 0.25$	c=0	HT	0.055	0.050	0.042	0.050
			KT	0.056	0.062	0.058	0.050
		c=1	HT	0.095	0.070	0.068	0.050
			KT	0.108	0.087	0.074	0.050
	$\lambda = 0.5$	c=0	HT	0.052	0.047	0.054	0.050
			KT	0.060	0.058	0.061	0.050
		c=1	HT	0.070	0.063	0.054	0.050
			KT	0.090	0.073	0.055	0.050
	$\lambda = 0.75$	c=0	HT	0.060	0.047	0.045	0.050
			KT	0.069	0.052	0.051	0.050
		c=1	HT	0.083	0.069	0.059	0.050
			KT	0.092	0.078	0.064	0.050

Table 7: Simulated values of power $k_{KT,1}$ for model $M1$, with u_{it}=$\varepsilon_{it} + \theta\varepsilon_{it-1}$ $(T = 8)$						
N			100	300	1000	TV
$\theta = -0.8$	$\lambda = 0.25$	$c = 0$	0.047	0.049	0.053	0.050
		$c = 1$	0.060	0.057	0.063	0.001
	$\lambda = 0.50$	$c = 0$	0.047	0.056	0.053	0.050
		$c = 1$	0.052	0.054	0.054	0.082
	$\lambda = 0.75$	$c = 0$	0.056	0.053	0.059	0.050
		$c = 1$	0.054	0.061	0.049	0.358
$\theta = -0.5$	$\lambda = 0.25$	$c = 0$	0.052	0.053	0.044	0.050
		$c = 1$	0.070	0.0102	0.086	0.011
	$\lambda = 0.50$	$c = 0$	0.050	0.047	0.048	0.050
		$c = 1$	0.093	0.104	0.125	0.151
	$\lambda = 0.75$	$c = 0$	0.045	0.055	0.055	0.050
		$c = 1$	0.073	0.075	0.100	0.391
$\theta = 0.5$	$\lambda = 0.25$	$c = 0$	0.047	0.041	0.057	0.050
		$c = 1$	0.375	0.477	0.580	0.858
	$\lambda = 0.50$	$c = 0$	0.050	0.044	0.044	0.050
		$c = 1$	0.678	0.769	0.825	0.888
	$\lambda = 0.75$	$c = 0$	0.046	0.049	0.042	0.050
		$c = 1$	0.544	0.644	0.652	0.509
$\theta = 0.8$	$\lambda = 0.25$	$c = 0$	0.055	0.052	0.056	0.050
		$c = 1$	0.403	0.512	0.598	0.894
	$\lambda = 0.50$	$c = 0$	0.047	0.046	0.060	0.050
		$c = 1$	0.769	0.830	0.875	0.919
	$\lambda = 0.75$	$c = 0$	0.045	0.052	0.053	0.050
		$c = 1$	0.632	0.696	0.739	0.514

Table 8: Simulated values of $k_{KT,1}$ for model $M1$, with u_{it}=$\varepsilon_{it} + \theta\varepsilon_{it-1}$ ($T = 10$)

N			100	300	1000	TV
$\theta = -0.8$	$\lambda = 0.25$	$c = 0$	0.058	0.056	0.048	0.050
		$c = 1$	0.047	0.051	0.054	0
	$\lambda = 0.50$	$c = 0$	0.050	0.045	0.060	0.050
		$c = 1$	0.054	0.058	0.068	0.058
	$\lambda = 0.75$	$c = 0$	0.049	0.055	0.047	0.050
		$c = 1$	0.049	0.046	0.047	0.205
$\theta = -0.5$	$\lambda = 0.25$	$c = 0$	0.049	0.058	0.044	0.050
		$c = 1$	0.089	0.086	0.108	0.009
	$\lambda = 0.50$	$c = 0$	0.046	0.046	0.053	0.050
		$c = 1$	0.078	0.100	0.118	0.140
	$\lambda = 0.75$	$c = 0$	0.049	0.054	0.055	0.050
		$c = 1$	0.072	0.080	0.097	0.293
$\theta = 0.5$	$\lambda = 0.25$	$c = 0$	0.054	0.054	0.041	0.050
		$c = 1$	0.278	0.391	0.505	0.830
	$\lambda = 0.50$	$c = 0$	0.062	0.051	0.050	0.050
		$c = 1$	0.681	0.789	0.856	0.921
	$\lambda = 0.75$	$c = 0$	0.053	0.050	0.058	0.050
		$c = 1$	0.580	0.693	0.783	0.683
$\theta = 0.8$	$\lambda = 0.25$	$c = 0$	0.056	0.043	0.055	0.050
		$c = 1$	0.273	0.411	0.481	0.861
	$\lambda = 0.50$	$c = 0$	0.049	0.058	0.050	0.050
		$c = 1$	0.752	0.825	0.895	0.941
	$\lambda = 0.75$	$c = 0$	0.051	0.058	0.054	0.050
		$c = 1$	0.654	0.780	0.823	0.698

Table 9: Simulated values of power $k_{KT,2}$ for model $M2$, with u_{it}=$\varepsilon_{it} + \theta\varepsilon_{it-1}$ ($T = 8$)

N			100	300	1000	TV
$\theta = -0.8$	$\lambda = 0.50$	$c = 0$	0.047	0.040	0.045	0.050
		$c = 1$	0.047	0.050	0.056	0.059
$\theta = -0.5$	$\lambda = 0.50$	$c = 0$	0.049	0.050	0.057	0.050
		$c = 1$	0.057	0.054	0.050	0.057
$\theta = 0.5$	$\lambda = 0.50$	$c = 0$	0.048	0.054	0.042	0.050
		$c = 1$	0.054	0.048	0.043	0.041
$\theta = 0.8$	$\lambda = 0.50$	$c = 0$	0.052	0.051	0.052	0.050
		$c = 1$	0.047	0.046	0.036	0.039

Table 10: Simulated values of power $k_{KT,2}$ for model $M2$, with u_{it}=$\varepsilon_{it} + \theta\varepsilon_{it-1}$ ($T = 10$)

N			100	300	1000	TV
$\theta = -0.8$	$\lambda = 0.50$	$c = 0$	0.044	0.047	0.052	0.050
		$c = 1$	0.046	0.048	0.060	0.075
	$\lambda = 0.75$	$c = 0$	0.051	0.048	0.045	0.050
		$c = 1$	0.058	0.066	0.072	0.164
$\theta = -0.5$	$\lambda = 0.50$	$c = 0$	0.057	0.055	0.049	0.050
		$c = 1$	0.061	0.051	0.074	0.068
	$\lambda = 0.75$	$c = 0$	0.054	0.046	0.050	0.050
		$c = 1$	0.093	0.077	0.089	0.119
$\theta = 0.5$	$\lambda = 0.50$	$c = 0$	0.050	0.060	0.053	0.050
		$c = 1$	0.066	0.044	0.047	0.038
	$\lambda = 0.75$	$c = 0$	0.051	0.054	0.052	0.050
		$c = 1$	0.071	0.052	0.038	0.031
$\theta = 0.8$	$\lambda = 0.50$	$c = 0$	0.053	0.060	0.055	0.050
		$c = 1$	0.057	0.043	0.032	0.036
	$\lambda = 0.75$	$c = 0$	0.060	0.059	0.049	0.050
		$c = 1$	0.057	0.041	0.029	0.029

Figure 1: HT slopes in the absence of serial correlation

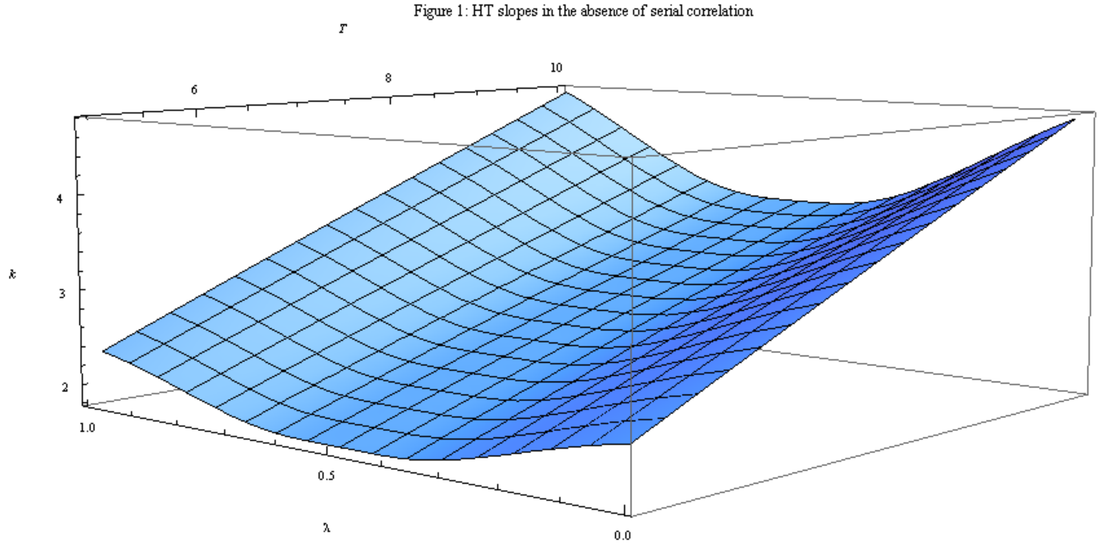

Figure 2: KT slopes in the absence of serial correlation