Currency Areas, Economic Asymmetries, and the Dynamics of Economic Integration*

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Abstract

We study the relationship between integration and synchronization of business cycles of countries belonging to a group of integrated economies. Although there is a rich empirical research showing the positive relationship between the two, there is no systematic formal discussion addressed to conditions under which the business cycles of a group of integrated economies tend to be synchronized. We are modeling the behaviour of the interacting economics using the theory of dynamic systems. Our methodology reveals that the commovement of business cycles of a group of integrated economies may result from a "mode-locking" phenomenon, i.e., a non-linear process by which weak coupling (integration) economies tend to synchronize their cycles. This is interpreted to mean that integration in a group of economies may not lead to an OCA unless their cycles are synchronized.

Keywords: Currency areas, dynamics of economic integration, stability equilibrium conditions, business cycles.

JEL Classification C 62, E 32, F 15

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1 Introduction

Empirical research seems to indicate that business cycles tend to become more symmetric as countries reduce the impediments of commodity and factor trade (i.e., form common markets), and adopt a common currency (Hess and Shin, 1995; Frankel and Rose, 1996). Starting from this observation one may be tempted to argue that the synchronization of business cycles, following economic integration, is a good OCA criterion, in the sense that it indicates that is optimal for the integrated economy to have its own currency. This is so because synchronization of business cycles presupposes a high degree of correlation of economic activities, implying that shocks will turn out to be symmetric rather than asymmetric, and therefore efforts of national authorities to improve their national interests by adopting independent monetary policies may yield inferior results for all.

On this, the European Commission (1990) views that trade between the industrial economies in Europe is intra-industry and hence higher integration will increase the proportion of the intra-industry trade to total trade leading to a possibility of disappearance of asymmetric shocks. On this reasoning, economic integration of a group of integrated economies is a necessary prerequisite for an OCA, implying simultaneously that the business cycles will be synchronised.

Krugman (1991) has challenged this view and insisted that higher integration leads to regional specialization, and this may work against a common currency. None of these views has a universal validity. As Demopoulos and Yannacopoulos (2000) have demonstrated , they follow logically from different (implicit) assumptions concerning economies of scale: The European Commissions view is the logical outcome of the assumptions that countries are similar in tastes , factor endowments, and technologies, exhibiting increasing returns to scale, with scale economies that are external to the firm and international. The Krugman's view follows from the same assumptions put forward by the European Commission with the difference that scale economies are external to the firm and national. Therefore, a group of integrated economies may not necessarily be an OCA, and in this case, the business cycles of the member countries may not be synchronized due to occurrence of asymmetric shocks.

But even if shocks are asymmetric one may expect that their effects are spread to all economies of the group through trade and financial markets leading to a possible synchronization of business cycles (Frankel and Rose, 1996 ; Alexander and Loef, 2003). This paves the way for our investigation of the relationship between integration and synchronization of business cycles. Although there is a rich empirical research showing the positive relationship between the two (Frankel and Rose, 1996; Frankel and Rose, 1998; Rose and Engel, 2001; Artis and Zhang, 1995; etc.), there is no systematic formal discussion addressed to reasons that the business cycles of a group of integrated economies tend to be synchronized.

In this paper, we try to fill this gap. We suggest that the commovement of business cycles within an integrated economy may result from a *mode-locking* phenomenon, i.e., a non-linear process by which weak coupling (integration) between oscillatory systems (economies) tends to synchronize their cycles. This approach was adopted by Selover and Jensen (1999) in their study for the international business cycles transmission mechanism, and by Yannacopoulos and Yannacopoulos (2001) in a study of a more general macroeconomic model.

The methodology used in this paper is that of non-linear dynamical systems. It first studies the equilibrium conditions of isolated economies. For this purpose, a modified version of the IS-LM model is used to describe the macroeconomic behaviour of each one of the economies involved. It is shown that the equilibrium of the system is a limit cycle. The economic oscillations corresponding to the limit cycle are stable and periodic. The paper proceeds to study the coupling (integration) process of the integrated group of economies via their international trade, as in Gandolfo (1997) and Puu (1997).

The question then to this issue is under what conditions coupling would lead to the synchronization of the business cycles of the integrated economies? It is shown that a necessary (but not sufficient) condition is that cycles must be resonant (i.e. cycles must have similar periods). A possible economic interpretation of this condition is that countries whose cycles satisfy this condition are similar in their economic structure (Selover and Jensen, 1999). If cycles are synchronized, i.e., if the countries exhibit similar behaviour over time then the area may be characterized as an OCA. Another possible case that may arise when economies are resonant is the case of anti-synchronization. In this case, phases of business cycles exhibit a difference equal to 180 degrees, implying that the economic behaviour of the countries of the group is completely asymmetric. Then, the region is not an OCA.

The remainder of this paper is structured as follows: In Section 2 we discuss issues related to our objective, such as, the nature of equilibrium of the economies under isolation, the coupling (integration) of the economies involved, and the dynamics of their integrated process. In Section 3 we present a formal model for the coupled (integrated) economies which illustrates the issues discussed in the paper (Section 4). Section 5 summarizes the conclusions.

2 A non formal exposition of the model

The problem under consideration is how the economic oscillations of these economies are affected by integration, and in particular under what conditions these oscillations are synchronized. The following issues are important for our discussion: (i) the nature of equilibrium of economies under isolation, (ii) the integration of these economies, and (iii) the dynamics of integration, i.e., how the oscillation of one economy affects the oscillation of another. These issues are discussed in turn.

2.1 The nature of equilibrium of the economies in isolation

For the purpose of our analysis every economy in isolation must exhibit business cycles, and these cycles have to be stable and periodic, i.e., the system must return after a time period to the point from which it has started. This is important: the cycles of a system cannot be synchronized unless the oscillations are periodic.

These periodic oscillations are derived from a modified version of a dynamic IS-LM model (as in Torre, 1977; Gabisch and Lorenz, 1989), which is used in this paper for the description of the macroeconomic behaviour of the isolated economies. In this model, savings are assumed to depend on national income and the rate of interest. Investment is assumed to depend on income, interest rate and the capital stock, which plays a key role in this model. The standard assumptions concerning the dependence of investment on the rate of interest and the capital stock are made. However, for the purpose of our analysis we assume that the investment function is S-shaped, which implies that the IS function is S-shaped as well. The rate of interest is determined in the money market and it is assumed that the demand for money satisfies the usual Keynesian assumptions.

Thus, the IS-LM model has three dynamic equations: (i) the IS equation which is S-shaped, (ii) the LM equation, and (iii) an equation for the capital stock. A solution to this model leads to a stable limit cycle (figure 1) which implies that the economic oscillations of every isolated

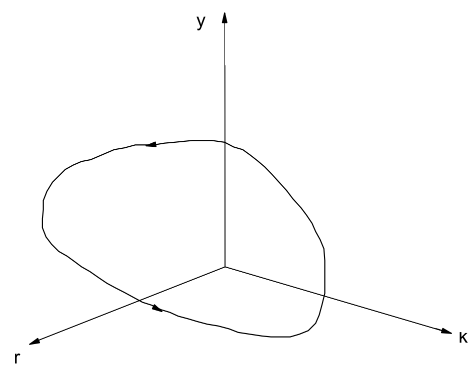

Figure 1: The limit cycle in phase space

economy are stable and periodic as it is required for the purpose of our analysis.

The limit cycle (figure 1) reveals the important role of changes in the capital stock on the model. In fact, the position of the IS curve depends not only on the level of investment, but also - negatively- on the capital stock. When the economy is at boom, capital accumulates. This gradually reduces investment, at any given level of output, shifting down the IS curve. At low equilibrium level, the capital stock of the isolated economy is depreciating faster than it is being replaced. In this last case, the marginal productivity of capital increases causing an increase in investment, shifting up the IS curve. Thus the accumulation of capital during the boom period eventually triggers a depression, while the depreciation of capital during the depression period eventually triggers a boom.

Since our *universe* consists of N economies, there will be N IS-LM dynamic models, and therefore N limit cycles, each of which describes the behaviour of the isolated economies in question. The next question therefore is to discuss the integration of these isolated economies. By integration these economies are *promoted* from closed economies into open economies. This issue is discussed in the next section.

2.2 Coupling

In order to discuss integration, we have to extend our original IS- LM models so as to include an external sector. In this way our N economies (i.e. our N IS-LM models) are connected. Within the context of the present model, integration takes place through coupling. Coupling takes place through trade as in Gandolfo (1977) and Puu (1977) and describes the intensity of trade flows between countries (and by implication the openness of the economies under consideration). It is assumed that the intensity of these trade flows depends on the income of the countries involved, and this dependence may have either a linear or a non-linear form. These export (import) functions define the *topology* of the connections of the economy i with its neighbours. A set of neighbours may be defined in a number of ways ranging from next neighbours on the lattice (local coupling) to global coupling.

An example of local coupling is illustrated in Figure 2. In this figure, $N = 4$ economies are located on a straight line (one dimensional lattice). The trading partner of country 1 is country

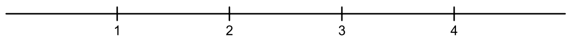

Figure 2: The one dimensional lattice

2, the trading partners of country 2 are countries 1 and 3, and so on. This presentation is an extreme form of the gravity model of international trade which says that the volume of trade between two countries is proportional to their national incomes and inversely proportional to its distance. In this case, the distance between non-neighbours is assumed to be infinite, so that non-neighbours do not trade at all. Thus, the volume of trade between neighbours depends only on their national income. The structure of the coupling can be presented by means of a $\{0.1\}$ matrix in which the zeros denote the uncoupled countries (countries that do not trade) and the ones the coupled countries (trading partners). Thus, we have N economies on a lattice (say a one dimensional lattice) linked (coupled) by trade.

We have shown that the behaviour of every economy is described (under isolation) by a limit cycle. The question then is how (and under what conditions) these limit cycles are modified if these isolated economies are linked by trade (coupled). This paves the way to the next issue which refers to the dynamics of the system.

2.3 Dynamics

This section studies to the dynamics of the system, i.e., how coupling affects the business cycles of the countries under consideration. Obviously, the economies belonging to the same lattice (the same geographical area) interact, and eventually an equilibrium is reached. We suggest that the synchronization of business cycles is a mode-locking phenomenon, i.e., a phenomenon in which two or more coupled oscillators (economies) of similar but not necessarily the same frequencies can lock on to another and move into synchronization with one another.

A necessary condition for synchronization is that the periods of economic cycles of every economy are in resonance. This means that the ratio of the period of economic cycles is a rational number. This will happen even if coupling is weak.

A possible economic interpretation for resonance is that the countries under examination have similar economic structure and this similarity is reflected in the similar periods of their cycles (Selover and Jensen, 1999). In the case of synchronization, the movement of the business cycles is correlated, meaning that the behaviour of all countries of the group is similar. Shocks are symmetric since they affect all member countries in the same way. In this case the group of countries is an OCA.

Notice that synchronization is not the only phenomenon that can emerge in this case of resonance. Other phenomena that are possible are anti-synchonization and localization of economic activity. We restrict our discussion to anti- synchronization. In this case, the behaviour of the countries belonging to the same group is completely anti-symmetric. The shocks do not affect the members of the group in the same way and the integrated group is not an OCA.

The three issues referred to above (equilibrium properties of the isolated economies, trade linkages (coupling), and the dynamics of the system are discussed more formally in the next section.

3 The model

In this section we present a simple model for coupled economies which may be used to illustrate the issues mentioned in the main body of the paper. We will assume that our 'universe' consists of N economies which may interact through trade relations.

3.1 A normal form for the uncoupled economies

We will assume that each isolated economy may exhibit business cycles. These are oscillatory solutions which may appear as the result of a bifurcation of the dynamical system modeling the isolated economy. A common type of bifurcation leading to oscillatory solutions in general classes of business cycle models is the Hopf bifurcation (see e.g. Gabisch and Lorentz 1989, or Gandolfo 1997). In order to be able to provide the most general results we will try to model the isolated economies with the generic form of dynamical system that may produce a Hopf bifurcation, that is the normal form for this typw of bifurcation.

One of the major advances in dynamical systems theory was the theory of normal forms. According to that, **any** dynamical system near a bifurcation point of a specific type (e.g. a Hopf bifurcation) may assume a particular **universal** form under a proper change of variables (which depends on the particular system). That means that **any** model undergoing a Hopf bifurcation leading to limit cycle behaviour may be described by a generic dynamical system which will reproduce all the qualitative results. To get to this generic system one may have to perform local nonlinear coordinate changes (see Appendix A). The great advantage of using normal forms is that in principle one may limit the study of the dynamics to particular types of systems. This is very important when using dynamical systems in modelling say in economics, where the majority of the models are majors simplifications of complex interdependencies between different economic quantities, and thus model dependent results may often be useless.

For the above model we may use the Hopf normal form. The fact that the original business cycle model undergoes a Hopf bifurcation guarantees, through the application of the normal form theorem that close to the equilibrium point there is a set of variables such that the dynamics of the original system may be described by the following set of equations

$$\begin{aligned} \dot{x}_n &= d\mu_{Rn}x_n - \mu_{In}y_n - a_n(x_n^2 + y_n^2)x_n \\ \dot{y}_n &= \mu_{In}x_n + d\mu_{Rn}y_n + a_n(x_n^2 + y_n^2)y_n \end{aligned}$$

or in terms of the complex variable $z_n = x_n + iy_n$,

$$\dot{z}_n = \mu_n z_n - a_n \mid z_n \mid^2 z_n$$

where $\mu = d\mu_{Rn} + i\mu_{In}$ is a complex parameter whose exact value depends on the original system and $a_n \in \mathbb{R}$ is a real parameter whose value again depends on the original system.

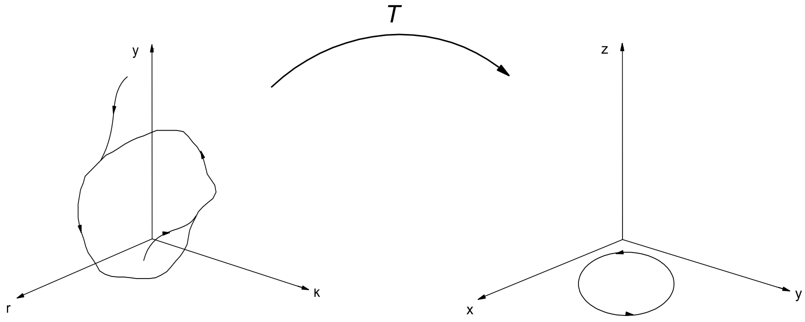

Figure 3: The transformation taking the IS-LM system into normal form

These parameters can be explicitly evaluated using the normal form change of variables. These parameters determine characteristics of the limit cycle, such as amplitude and frequency. The transformation leading the original system into normal form close to the bifurcation point is shown schematically in Figure 3.

The normal form is a two dimensional dynamical system. However, it is important to note that the original model leading to a Hopf bifurcation may be of any dimension, higher than two (see example in Section 4.1). What the normal form theorem states is that there is a local nonlinear change of variables (near a fixed point that undergoes a Hopf bifurcation) such that in the new variables and in a neighbourghood of the fixed point any dynamical system no matter what its dimensionality is will be described by the two dimensional normal form. Notice that this system is two dimensional even though the original system is three dimensional. As a matter of fact, it makes no difference what the dimensionality of the original system is, close enough to the Hopf bifurcation the dynamics for any system will be described by a two dimensional system of the above form. The only system dependent quantities are the definition of x_n, y_n in terms of the original variables of the system. The exact relations giving x_n, y_n in terms of the original variables may be obtained using an algorithmic procedure but the derivation and the expressions are quite cumbersome and they are ommited.

One can see that the above system has a periodic solution of the form

$$\begin{aligned} x_n &= R_n \cos(\mu_{In} t + \phi_n) \\ y_n &= R_n \sin(\mu_{In} t + \phi_n) \end{aligned}$$

or in terms of complex variables

$$z_n = R_n e^{i\mu_{In} t + \phi_n}$$

where $R_n^2 = \frac{d\mu_{Rn}}{a_n}$ is the amplitude of the cycle, and ϕ_n is the phase of oscillations for each economy which will play an important rôle for the arguments of this paper. We will assume in what follows that $sign(\mu_{Rn}) = sign(a_n)$ and that $0 < d << 1$ so that we study limit cycles of very small amplitude. Whether the limit cycle is attracting or not depends on the sign of μ_{Rn}. If $\mu_{Rn} < 0$ the limit cycle is attracting otherwise it is repulsive.

Finally, we may treat the above system in 'action-angle' variables (ρ_n, θ_n). In this new variables the system undergoes a major simplification. Define (ρ_n, θ_n) by the relation

$$z_n = \rho_n e^{i\theta_n}$$

It is straightforward to see that in the new variables the system becomes

$$\begin{aligned} \dot{\rho}_n &= \delta\mu_{Rn}\rho_n - a_n\rho_n^3 \\ \dot{\theta}_n &= \mu_{In} \end{aligned}$$

In this new variables we see that the system is decoupled, a fact that makes it very easy to study the dynamics. The limit cycle solution in the new variables corresponds to the system

$$\begin{aligned} \rho_n &= \sqrt{\frac{d\mu_{Rn}}{a_n}} \\ \dot{\theta}_n &= \mu_{In} \end{aligned}$$

Intuitively ρ_n corresponds to the radius of the limit cycle n and θ_n corresponds to the phase of economy n. Notice that the solution of the differential equation above is trivial and equal to $\theta_n = \mu_{In}t + \phi_n$. This differential equation is called the **phase equation**.

3.2 The model for the coupled economies in normal form

We will now assume that the isolated economies become coupled through international trade. As will be shown explicitly for a specific model that may serve as an example (see Section 4.2) we may consider linear coupling through one of the variables describing the isolated economies. We will choose to present the model for the coupled economies in normal form in order to exploit the simple form of the system in these variables to derive general results on the dynamics.

Performing the coordinate changes taking the individual economies into normal form (see Appendix A) we get the following general form for the coupled economies system in the new variables

$$\begin{aligned} \dot{z}_n &= (d\mu_{Rn} + i\mu_{In})z_n - a_n \mid z_n \mid^2 z_n \\ &+ \epsilon\sum_{k=1}^{N'}(c_{nk}z_k - c_{kn}z_n) + \epsilon\sum_{k=1}^{N'}(d_{nk}\bar{z}_k - d_{kn}\bar{z}_n) \end{aligned} \tag{1}$$

In the above by $\bar{z}$ we denote the complex conjugate of z. The quantities $c_{nk}, d_{nk} \in \mathbb{R}$ are functions of the trade strengths between the different economies which in principle can be evaluated exactly but this evaluation is a cumbersome task. The parameter ϵ in front of the coupling term denotes the fact that there is weak interaction due to trade between the economies. This condition is a technical condition needed in the analysis that follows, but it is conjectured that most of the results presented here will hold qualitatively for higher values of the interaction.

The system described in equation (1) will be the starting point for our study. As argued above, it is a generic model that may describe the effects of interaction between a number of economies. Furthermore, we showed that this model may be derived from a specific economic model, the coupled augmented IS-LM model through normal form theory. The generality of the arguments of normal form theory, guarantees that the qualitative results holding for the above model will hold for more general and more detailed economic models, close to their Hopf bifurcation points.

4 An example: The augmented IS-LM model

In this section we present a particular example, which is commonly used in the theory of business cycle modelling the augmented IS-LM model. We will present also a coupled version of the IS-LM model where coupling occurs through international trade and we will show that this system may be reduced through proper coordinate changes to the generic coupled normal form model presented in the previous section. We choose our isolated model problem to be three dimensional so as to demonstrate the important fact that the normal form will be two dimensional independently of the initial dimensionality of the original system giving rise to the Hopf bifurcation.

4.1 The isolated economies

To model the economies in the absence of interaction (isolation) we will use an augmented IS-LM business cycle model, which can be thought of as the simplest complete business cycle models in the Keynesian tradition (see e.g. Gabisch and Lorentz, 1989). The model is as follows

$$\begin{aligned}\dot{Y}_n &= \alpha(I(Y_n, K_n, r_n) - S(Y_n, r_n))\\ \dot{r}_n &= \beta(L(r_n, Y_n) - \bar{M}_n)\\ \dot{K}_n &= I(Y_n, K_n, r_n) - \delta K_n\end{aligned}$$

where $n = 1, ..., N$ and Y_n is the real income, K_n is the capital stock, r_n is the interest rate, I_n denotes the investment, S_n denotes the saving function, and L_n denotes the money demand function of economy n. Notice that in this model there is no interaction between the economic quantities of economy n with other economies. The functions I_n, S_n and L_n are assumed to satisfy the usual conditions (see e.g. Gabisch and Lorentz, 1989). The function I_n is assumed to be S-shaped.

The dynamics of the isolated model have been studied in detail (Gabisch and Lorentz, 1989) and it has been shown that it undergoes a Hopf type bifurcation. This means that for certain values of the parameters of the model the equilibrium point (Y_n^*, r_n^*, K_n^*) of the economy n may become unstable, giving rise to a limit cycle that is to oscillatory behaviour in time. Depending on the parameters of the model the limit cycle may be stable or unstable. The parameter values for which the Hopf bifurcation occurs is given by the celebrated Hopf Bifurcation Theorem (see e.g. Guckenheimer and Holmes, 1986). For the particular model the Hopf bifurcation occurs at a value $a_n = a_0$ where

$$A_n > 0 \quad B_n > 0 \quad C_n > 0 \quad A_n B_n - C_n = 0$$

where

$$\begin{aligned}A_n &= -(\alpha(I_Y - S_Y) + \beta L_r + (I_K - \delta))\\ B_n &= \beta L_r(I_K - \delta) + \alpha(I_Y - S_Y)(I_K - \delta) - \alpha I_Y I_K\\ &+ \alpha\beta(I_Y - S_Y)L_r - \alpha\beta L_Y(I_r - S_r)\\ C_n &= -det J\end{aligned}$$

where J is the Jacobian of the system at the equilibrium point and all the above quantities depend on the economic parameters of economy n.

The construction of the new normal form variables z_n in which the isolated economy n may be described by the two dimensional Hopf normal form presented in Section 3.1 is in general an algebraically complex procedure. We sketch it in Appendix A.

4.2 A model for coupled economies

Our main assumptions in this section will be that exchange rates are fixed and that there are no capital movements. Our model is in the spirit of a similar model described in Gandolfo (1997).

Let us denote by $IMP_{mn}(Y_n)$ the imports of country n from country m and by $IMP_n(Y_n)$ the total imports of country n. Clearly,

$$IMP_n(Y_n) = \sum_{m=1}^{N'} IMP_{mn}(Y_n)$$

where the dash on the sum means that the term $m = n$ is excluded. By X_n we denote the exports of economy n to all the other economies. Clearly,

$$X_n = \sum_{m=1}^{N'} IMP_{nm}(Y_m)$$

For simplicity we will assume that $IMP_{mn}(Y_n)$ are linear functions that is $IMP_{mn}(Y_n) = C_{mn}Y_n$ where C_{mn} is a measure of the trade strength interaction between economies m and n.

Since all interaction is assumed to occur by trade the coupled augmented IS-LM model becomes

$$\begin{aligned} \dot{Y}_n &= \alpha(I(Y_n, K_n, r_n) - S(Y_n, r_n) + X_n - IMP_n(Y_n)) \\ \dot{r}_n &= \beta(L(r_n, Y_n) - \bar{M}_n) \\ \dot{K}_n &= I(Y_n, K_n, r_n) - \delta K_n \end{aligned}$$

The interaction is through the X_n term which depends on Y_m, $m \neq n$. The above is a system of $3N$ coupled differential equations which as we shall see may have very rich dynamics. Note that Gandolfo (1997) has not made any comments upon the dynamical behaviour of this model apart from the fact that this model may display chaotic behaviour. Since to the first order the coordinate transformation leading to the normal form variables is a linear transformation (see Appendix A) by assuming linear trade relations we see that the couple normal form model will consist of linear coupling between the normal form coordinates of the various economies, that is will lead to a general model of the form presented in section 3.2.

5 Dynamic behaviour of the model

In this section we present some results which may provide some insight on the rich dynamical behaviour of our model.

5.1 The effect of resonances

The first result is a general result showing the importance of resonance between the uncoupled economies in the effect the coupling will have on the N-country economy as a whole. By resonance we mean that the frequencies Ω_i, $i = 1, ..., N$ of the business cycles of the individual economies will satisfy a linear relation of the form

$$m_1\Omega_1 + \cdots m_N\Omega_N = 0$$

for $m_i \in \mathbb{Z}$. The simplest type of resonance one may consider is $\Omega_1 = \Omega_2 = \cdots = \Omega_N$.

One can prove that the coupling will have negligible effects on the economies if these economies are not in resonance. In this case the coupling is only affecting by a small correction the frequency of the cycle of each economy but not its phase. On the contrary, when economies are in resonance, then even weak coupling may affect considerably the dynamics of the system as a whole, in the sense that not only the frequency of the business cycles will be affected but non trivial dynamics will arise for the phases as well. Such dynamics may be synchronization between different business cycles, or anti-synchronization, cycle stopping etc.

This qualitative discussion is summarized in the following proposition (Yannacopoulos and Yannacopoulos, 2001 and references therein)

Proposition 1 *Assume that we take the phase equation for system (1). Define* $\Omega = (\Omega_1, \cdots, \Omega_N)$, $\theta = (\theta_1, \cdots, \theta_N)$. *The dynamics of the phase equation on time intervals of order* $O(1/\epsilon)$ *can be approximated by*

1. $\dot{\theta} = \Omega + \epsilon\omega + O(\epsilon)$ *if* Ω *is non-resonant*
2. $\dot{\theta} = \Omega + \epsilon H(K\theta)$, *if* Ω *is resonant.*

In the above $\omega \in \mathbb{R}^n$ *is a constant vector, whereas* H *is a function of the phases, and* K *is a matrix satisfying* $K\Omega = 0$. *This matrix defines which economies interact with which.*

The above proposition allows us to conclude that only economies which are relatively close, i.e. that have business cycles of similar frequency may face strong effects because of interaction with other economies. Economies which are dissimilar, i.e. that have business cycles with frequencies whose ratios are irrational numbers, will be more or less unaffected by the interaction with other economies.

5.2 Synchronization or anti-synchronization

Let us now assume that we are in the resonant case and we will see how interaction may affect the phases of the business cycles of the economies.

In order to study this problem we will use the so called phase equations for the model. The phase equations describe the evolution of the phase difference of the phases of the economies in the presence of coupling from the phases of the economies in the isolated case. The derivation of the phase equations is described in Appendix B.

Assume for instance that $\mu_{In} = \mu_{Ik}$ for $k \in \mathcal{R}$ where $\mathcal{R}$ is an index set containing the economies which are in the lowest order resonance with economy n. For instance if economy n has the business cycles of the same frequency with say economies 1 and 5 then $\mathcal{R} = \{1, 5\}$.

Following the procedure described in Appendix B we find that the evolution of the phases is given by the following system of ODEs

$$\dot{\Theta}_n = \sum_{k=1,k\in\mathcal{R}}^{N'} c'_{nk}\sin(\Theta_k - \Theta_n) \tag{2}$$

where Θ_n is the deviation of the n economy from the phase in the isolated case, $\Theta_n := \theta_n \mu_{In} t$. The above means that the phase of economy n will only be affected by the phases of the economies in the index set $\mathcal{R}$ which will be the only ones interacting with this economy.

The averaged phase equation (2) despite its simplicity may reveal a richness of behaviours. Notice that $\Theta_k = a_k = const$, $k = 1, ..., N$ with $\Theta_k - \Theta_n = 0$ and/or $\Theta_k - \Theta_n = \pi$ are always solutions of the above phase equation. These solutions do not change in time and are called steady states of the phase equation. The solution $\Theta_k = \Theta_n = 0$ corresponds to synchronisation of the business cycles of economies n and k (i.e. they have the same phases). Whether this behaviour persists or not depends on the stability of this solution. By the term stability we mean that if we change our initial conditions by a small quantity near the steady state solution in the long run the new solution will return to the original solution $\Theta_k = a_k$. Mathematically this means that if we consider solutions of the form $\Theta_k(t) = a_k + \delta\Theta_k(t)$, with $\delta\Theta_k$ small, $\delta\Theta_k(t) \to 0$ as $t \to \infty$. By standard dynamical systems arguments we see that the behaviour in time of $\delta\Theta_k$ is given by the solution of a linearized system

$$\dot{\delta\Theta} = J(a)\delta\Theta$$

where $a = (a_k)$ and by $J(a)$ we denote the Jacobian of the phase equation evaluated at the steady state solution $\Theta_k = a_k$ and $\delta\Theta = (\delta\Theta_1, \cdots, \delta\Theta_N)^T$. The stability depends on the eigenvalues of the matrix $J = J(a)$ which in turn depends on the matrix $[c_{nk}]$ that is on the characteristics of the coupling through trade. On the other hand the solution $\Theta_k = \Theta_n = \pi$ corresponds to anti-synchronization of the business cycles of economies n and k (i.e. the business cycles are out of phase with a phase difference of π). Again the persistence of this behaviour depends on the stability of this solution which in turn depends on the eigenvalues of the matrix $[c_{nk}]$ that is on the characteristics of the coupling through trade. Thus in the resonant case, economies which may start with different phases, as an effect of coupling may either end up in a synchronized state or in an unsynchronized state depending on the characteristics of coupling through trade.

The exact form of the stability conditions depends on the type of resonance and the type of coupling. In the case of first order resonance $\Omega_k = \Omega_n$ for $k \in \mathcal{R}_n$ the Jacobian matrix of the phase equations for the synchronized state becomes $J_s = [J_{nk}]$ where $J_{nk} = c'_{nk}$, $k \in \mathcal{R}_n$ and $J_{nn} = -\sum_{k\in\mathcal{R}_n} c'_{nk}$. For the unsynchronized state $J_a = -J_s$. The synchronized state is stable (and thus attracts all initial conditions) if the matrix J_s has eigenvalues with negative real parts. The antisynchronized state is stable (and thus attracts all nearby initial conditions) if the matrix J_a has eigenvalues with negative real parts.

The general form of the Jacobian will be

$$J(c) = \begin{pmatrix} -\sum_{k,k\neq 1}^{K} c'_{1k}\cos(a_k - a_1) & c'_{12}\cos(a_2 - a_1) & \cdots & c'_{1K}\cos(a_K - a_1) \\ c'_{21}\cos(a_1 - a_2) & -\sum_{k,k\neq 2}^{K} c'_{2k}\cos(a_k - a_2) & \cdots & c'_{2K}\cos(a_K - a_2) \\ \vdots & \cdots & \vdots & \\ c'_{K1}\cos(a_K - a_1) & c'_{K2}\cos(a_K - a_2) & \cdots & -\sum_{k,k\neq K}^{K} c'_{Kk}\cos(a_K - a_k) \end{pmatrix}$$

where we have relabelled the index set $\mathcal{R}$ as $\mathcal{R} = \{1, 2, \cdots, K\}$. In order to characterize the stability of a state $\Theta_k = a_k = const$) we need to see if the eigenvalues of this matrix have positive or negative real parts. If the real parts are positive this will correspond to an unstable steady state, whereas if the real parts are negative this will corespond to a stable steady state.

The above results can be summarized in the following proposition.

Proposition 2 *In the case of first order resonance between the economies*
(i) The synchronized state is stable if the matrix $J_s = [J_{nk}]$ is a stable matrix (has eigenvalues with real part less or equal to 0).
(ii) An antisynchronized state is stable if the matrix $J_a = -J_s$ is a stable matrix.

Concerning the criteria for stability we have the following

Corollary 1 *In the case of first order resonance*
(i) If $c'_{nk} > 0$ for all n and $k \in \mathcal{R}_n$ then the synchronized state is stable.
(ii) If $c'_{nk} < 0$ or all n and $k \in \mathcal{R}_n$ then the anti-synchronized state is stable.

The proof is provided in the Appendix C.

Similar results may be shown to hold in the case of higher order resonances. We do not present here due to lack of space.

We end this section with an example.

Example 1 *Assume that we have 4 economies, the 3 of which are in first order resonance e.g. $\Omega_1 = \Omega_2 = \Omega_3$ and the fourth is non-resonant with the rest e.g. $\Omega_4 = \sqrt{2}\Omega_1$. Then the general form of the phase equation will be*

$$\begin{aligned}
\dot{\Theta}_1 &= c'_{12}\sin(\Theta_2 - \Theta_1) + c'_{13}\sin(\Theta_3 - \Theta_1) \\
\dot{\Theta}_2 &= c'_{21}\sin(\Theta_1 - \Theta_2) + c'_{23}\sin(\Theta_3 - \Theta_2) \\
\dot{\Theta}_3 &= c'_{31}\sin(\Theta_1 - \Theta_3) + c'_{32}\sin(\Theta_2 - \Theta_3) \\
\dot{\Theta}_4 &= 0
\end{aligned}$$

For the stability of the synchronized state we have to study the properties of the linear system

$$\delta\dot{\Theta} = \begin{pmatrix} -c'_{12} - c'_{13} & c'_{12} & c'_{13} \\ c'_{21} & -c'_{21} - c'_{23} & c'_{23} \\ c'_{31} & c'_{32} & c'_{31} - c'_{32} \end{pmatrix} \delta\Theta$$

where $\delta\Theta = (\delta\Theta_1, \delta\Theta_2, \delta\Theta_3)^T$. According to Corollary 1 the synchronized state is stable if $c'_{nk} > 0$. In Figure 4 we show a solution of the phase system for the case where $c'_{nk} = 1$ for all n, k. In this case according to Corollary 1 the synchronized state is stable. This is shown clearly in Figure 4 where we see that eventhough the business cycles for economies 1, 2 and 3 start with a phase difference they end up in the synchonized state. Again according to Corollary 1 the anti-synchronized state is stavle if $c'_{nk} < 0$. In Figure 5 we show a solution of the phase system in the case where $c'_{nk} = -1$ so that according to Corollary 1 the anti-synchronized state is stable. It is seen clearly in Figure 5 that the long time behaviour of the system is locked in the anti-synchronized state. Note that in this case the economies end up in the anti-synchronized state even though they start with more or less equal phases (synchronized).

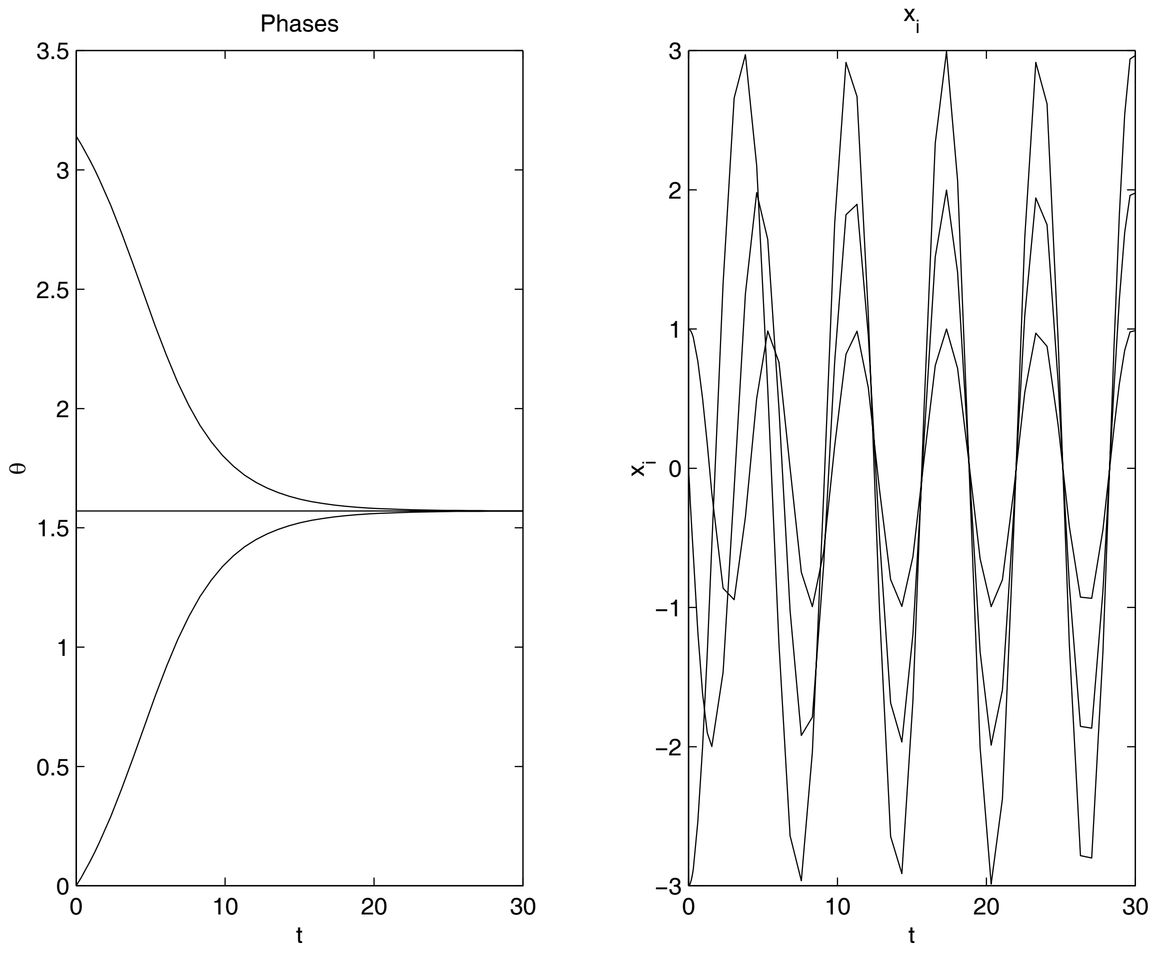

Figure 4: Synchronized state of the economies

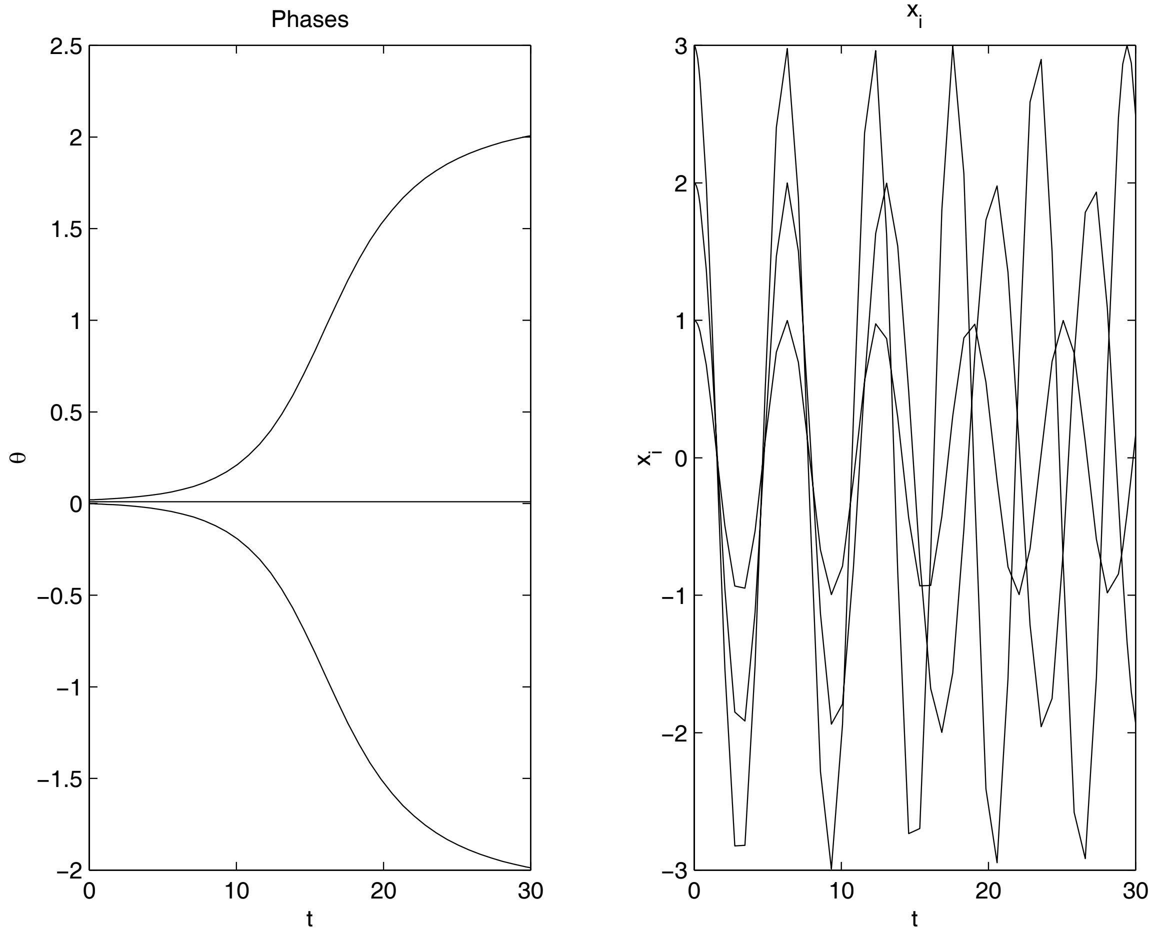

Figure 5: Anti-synchronized state of the economies

6 Concluding Remarks

The issue addressed in this paper is whether economic integration leads to the synchronization of the business cycles in a group of integrated economies. In the positive case, this may be considered as an indicator that the integrated economies are an OCA. Formal analysis has shown that, provided that the economies are in resonance, weak trade interaction between countries may lead either to synchronization or to anti-synchronization of the business cycles of a group of economies depending on the values given to the coupling parameters. In the first case (synchronization), the cycles of countries in isolation which exhibit asymmetric behaviour, are synchronized by integration (coupling) implying that their economies converge. We may say that these countries form an OCA since the synchronization of the business cycles reveals similar economic behaviour implying that shocks affect all countries in a similar way (shocks are symmetric).

In the second case (anti-synchronization) economic integration (coupling) does not lead to economic convergence. We may conjecture that, in this case, shocks do not affect all economies in the same way, and therefore this group of economies is not an OCA. Thus economic integration does not necessarily lead to conditions that favour the adoption of a common currency.

In the light of this discussion, the opposite scenarios proposed by the European Commission and Krugman in their well known dispute appear to be special solutions to the same dynamic model.

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A Reduction to normal form

Let us briefly sketch the reduction of the three dimensional IS-LM model to normal form. In this appendix we drop the subscripts n (referring to the fact that we are studying the n-th economy). Let us assume without loss of generality that the steady state of the system is located at the origin of the system of coordinates $(0,0,0)$. Let us further assume that the Jacobian of the model has two eigenvalues with zero real part and one eigenvalue with negative real part. Then we may find a linear transformation $\mathcal{T}_1$ leading to the new variables (x_1, x_2, Z) in which the IS-LM system locally takes the form

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x_1, x_2, Z) \\ \dot{x}_2 &= -x_1 + f_2(x_1, x_2, Z) \\ \dot{Z} &= -aZ + f_3(x_1, x_2, Z) \end{aligned} \tag{3}$$

where $a \in \mathbb{R}^+$ and f_i, $i = 1, 2, 3$ are nonlinear functions that can easily be calculated using the linear coordinate transformation $\mathcal{T}_1$. According to the center manifold theorem (see eg. Guckenheimer and Holmes, 1986) there exist a local center manifold such that $Z = h(x_1, x_2)$ for some function h to be determined. Since this manifold is invariant under the flow h will be determined by the solution of the first order nonlinear partial differential equation

$$\begin{aligned} \frac{\partial h}{\partial x_1}(x_2 + f_1(x_1, x_2, h(x_1, x_2)) + \frac{\partial h}{\partial x_2}(-x_1 + f_2(x_1, x_2, h(x_1, x_2)) \\ -f_3(x_1, x_2, h(x_1, x_2, h(x_1, x_2)) + ah(x_1, x_2) = 0 \end{aligned}$$

This equation is very hard to be solved but h may be approximated using power series methods. Once h has been obtained the system (3) can be reduced to a two dimensional system on the center manifold,

$$\begin{aligned} \dot{x}_1 &= x_2 + \bar{f}_1(x_1, x_2) \\ \dot{x}_2 &= -x_1 + \bar{f}_2(x_1, x_2) \end{aligned} \tag{4}$$

where $\bar{f}_i(x_1, x_2) = f_i(x_1, x_2, h(x_1, x_2))$, $i = 1, 2$. We may now resort to normal form theory (see eg. Guckenheimer and Holmes, 1986) according to which there exists a nonlinear transformation $\mathcal{T}_2$ which takes $(x_1, x_2) \to (x, y)$ and (x, y) are chosen so that in these variables the system (4) assumes the form

$$\dot{z} = \mu z - a \mid z \mid^2 z$$

in terms of the complex variable $z_n = x_n + iy_n$, where $\mu = d\mu_{Rn} + i\mu_{In}$. The complex number μ is related to the characteristics of the limit cycle and its exact value can be found in terms of the exact form of the original system. The transformation $T = \mathcal{T}_2 \circ \mathcal{T}_1$ is the non-linear transformation taking the IS-LM model into the normal form (see Figure 3).

B Reduction of the full system to the phase equation

In this Appendix we describe the derivation of the phase equations for the coupled system (1). Our first move is to go to the variables (ρ_n, θ_n), Differentiating and separating real and imaginary parts we find (after some algebra) that

$$
\begin{aligned}
\dot{\rho}_n &= d\mu_{Rn}\rho_n - a_n\rho_n^3 + \\
&+ \epsilon \sum_{k=1}^{N'} [c_{nk}\rho_n \cos(\theta_k - \theta_n) - c_{kn}\rho_n \\
&+ d_{nk}\rho_k \cos(\theta_k + \theta_n) - d_{kn}\rho_n \cos(\theta_k + \theta_n)] \\
\dot{\theta}_n &= \mu_{In} + \epsilon \sum_{k=1}^{N'} \left[c_{nk}\frac{\rho_k}{\rho_n} \sin(\theta_k - \theta_n) - \right. \\
&- \left. d_{nk}\frac{r_k}{r_n} \sin(\theta_k + \theta_n) + d_{kn} \sin(\theta_k + \theta_n) \right]
\end{aligned}
$$

We will now project the above system of equations onto the limit cycles $\mathcal{M}$ of the uncoupled system. This is equivalent in a sense to setting $\rho_n^2 = \frac{d\mu_{Rn}}{a_n}$ which in turn leads to the phase equation

$$
\dot{\theta}_n = \mu_{In} + \epsilon \sum_{k=1}^{N'} (c'_{nk} \sin(\theta_k - \theta_n) - d'_{nk} \sin(\theta_k + \theta_n) + d_{kn} \sin(\theta_k + \theta_n) \tag{5}
$$

where

$$
\begin{aligned}
c'_{nk} &= c_{nk} \left(\frac{a_n \mu_{Rk}}{a_k \mu_{Rn}} \right) \\
d'_{nk} &= d_{nk} \left(\frac{a_n \mu_{Rk}}{a_k \mu_{Rn}} \right)
\end{aligned}
$$

To study the long run effects of coupling we may perform averaging to the phase equation above. If we write the phase equation system as $\dot{\theta}_n = h_n(\theta_1, ..., \theta_N)$ then the long run dynamics of the phase equation is provided by the differential equation

$$
\dot{\Theta}_n = Q(\Theta_1, ..., \Theta_N)
$$

where Θ_n is the deviation of the n economy from the phase in the isolated case, $\Theta_n := \theta_n \mu_{In} t$ and

$$
Q_n(\Theta) = \lim_{T\to\infty} \int_0^T h_n(\Omega t + \Theta) dt
$$

In order to get more insight for the term Q_n let us rewrite it as follows

$$
\begin{aligned}
Q_n(\Theta) &= \sum_{k=1}^{N'} c'_{nk} \frac{1}{T} \int_0^T \sin((\mu_{Ik} - \mu_{In})t + (\Theta_k - \Theta_n))dt \\
&- d'_{nk} \frac{1}{T} \int_0^T \sin((\mu_{Ik} + \mu_{In})t + (\Theta_k + \Theta_n))dt \\
&+ d'_{kn} \frac{1}{T} \int_0^T \sin((\mu_{Ik} + \mu_{In})t + (\Theta_k + \Theta_n))dt
\end{aligned}
$$

The value of the trigonometric integral depend on the ratios between μ_{Ik}, μ_{In} and thus the impact of coupling depends on the ratio of frequencies between the business cycles of the individual economies.

Assume for instance that $\mu_{In} = \mu_{Ik}$ for $k \in \mathcal{R}$ where $\mathcal{R}$ is an index set containing the economies which are in the lowest order resonance with economy n. For instance if economy n has the business cycles of the same frequency with say economies 1 and 5 then $\mathcal{R} = \{1, 5\}$. Performing the integrations we see that the surviving terms will give us

$$\dot{\Theta}_n = \sum_{k=1, k\in\mathcal{R}}^{N'} c^{'}_{nk} \sin(\Theta_k - \Theta_n)$$

The above means that the phase of economy n will only be affected by the phases of the economies in the index set $\mathcal{R}$ which will be the only ones interacting with this economy.

C Proof of Corollary 1

The proof of Corollary 1 is based on a theorem on stability matrices (see Marcus and Minc, 1992, 3.3.5. p. 159) according to which a matrix $A = [a_{ij}] \in \mathbb{C}^{n\times n}$ is semistable if

$$Re(a_{ii}) \leq -\sum_{i=j, j\neq i}^{n} \mid a_{ij} \mid$$

If the inequalities are strict then the matrix is stable. In the case where A is real the matrix must also be non-singular. Recall that a matrix is called stable (semi-stable) if the real parts of all its eigenvalues are negative (nonpositive). By directly applying this result to the Jacobian of the phase equation we obtain Corollary 1.

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