THE WELFARE EFFECTS OF LEGAL
UNCERTAINTY AND ITS IMPLICATIONS
FOR ENFORCEMENT PROCEDURES

by

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The Welfare Effects of Legal Uncertainty and its Implications for Enforcement Procedures

by

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Abstract

While the issue of legal uncertainty is widely discussed and considered of fundamental importance among those involved in the design and implementation of public policy and among legal experts and economists, the concept has never up to now been formalised and the welfare “costs of legal uncertainty” has never been analysed using economic theory. Our main objective in this paper will be to extend the framework that we have developed for analysing optimal legal standards (Katsoulacos and Ulph, 2009), in order to examine the welfare implications of legal uncertainty and to compare alternative decision and enforcement procedures of Competition Authorities under varying degrees of Legal Uncertainty. The degree of Legal Uncertainty is made to depend on the information available to the Authority concerning the characteristics of firms and their environment, that determine the harm/benefit generated by their actions, and on the information available to the firms about these characteristics and about the assessment criteria/models used by the Authority. Our analysis shows that:

- There is no automatic equivalence between Effects-Based rules and Legal Uncertainty.
- There is no monotonic link between Legal Uncertainty and welfare. While very great degrees of Legal Uncertainty are welfare reducing, welfare can be higher when there is some degree of Legal Uncertainty than when there is no Legal Uncertainty. This is true also when firms can invest into reducing Legal Uncertainty and it is more likely to be true when procedural uncertainty is also present.
- If the tests on which Effects-Based procedures are based are good enough to enable the Authority to effectively discriminate then Effects-Based procedures will often be welfare superior to Per Se rules even though they involve Legal Uncertainty.

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1. Introduction

An important issue when considering what type of enforcement procedure to use in situations in which regulatory intervention in markets is deemed necessary is that of Legal Certainty⁵. Certain enforcement or decision procedures are thought of, ceteris paribus, as being superior because their “cost” in terms, or as a result, of the legal uncertainty⁶ generated when these procedures are adopted is relatively low. This issue is important for a broad range of regulatory interventions⁷ which are induced by the following set of circumstances: (a) agents are taking actions that are privately beneficial but from a wider social viewpoint may be harmful or beneficial (b) the degree of social harm/benefit varies with the circumstances under which the action is taken (c) the authority/regulator cannot observe the precise circumstances under which any given action is taken.

While the issue of legal uncertainty has been widely alluded to and considered of fundamental importance among those involved in the design and implementation of public policy and, especially among legal experts, the concept has never up to now been formalised and the “cost of legal uncertainty” has never been analysed using economic theory⁸. The present paper attempts to do exactly that.

The context of many of the discussions on legal uncertainty usually involves comparisons of Effects-Based (or what alternatively can be termed discriminating) and Per Se decision rules or enforcement procedures⁹. The idea is that under Per Se rules

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⁵ Legal scholars and social scientists have, of course, discussed the issue in a much wider context. Among early prominent authors, Max Weber, thought of legal certainty as necessary for capitalist progress – see discussion in Amato (1983) with extensive references to legal scholars including Posner’s (1977). In the more specific context with which we are concerned here, Forrester (2000) notes that “Legal certainty....... is very frequently invoked as a prime concern for those responsible for enforcing the competition rules. Advocates General, article writers and the Commission itself have each stated on various occasions how important it was to ensure legal certainty”. In the context of network industry regulation, see de Hauteclouque (2008).

⁶ Lack of ability to predict the outcome of a legal dispute. Amato (1983) defines “legal uncertainty” as a “situation that obtains when the (legal) rule that is relevant to a given act or transaction is said by informed attorneys to have an expected official outcome at or near the 0.5 level of predictability”.

⁷ These include interventions associated with the application of Competition Policy, Sectoral Regulation, Environmental Policy, Tax Compliance mechanisms etc.

⁸ Indeed there are no formal or comprehensive discussions concerning exactly how “legal uncertainty” generates social welfare costs though issues relating to risk-adversity, potentially adverse deterrence effects, costs to firms for obtaining and analyzing information to reduce uncertainty and the fact that penalties are less easy to justify if an action is condemned, are recognized as important considerations. As will become clear below, the emphasis in this paper is on deterrence effects and in an extension we allow firms to reduce uncertainty by incurring some costs.

⁹ Under a Per Se procedure an Authority allows or disallows an entire class of actions without trying to identify more carefully sub-classes of actions that might generally be harmful or generally benign. A discriminating legal standard or
either all actions are allowed or all are disallowed, depending on whether their average harm is negative or positive, whereas under Effects-Based procedures, the Authority will allow some actions and disallow others. When agents make the decision as to whether or not to undertake the action they have to consider the possibility that it might come under scrutiny by the Authority. Consequently, it is argued, under a Per Se rule they are certain how the action will be treated, whereas, under an Effects-Based approach, they do not know for sure what decision would be taken by the Authority. This Legal Uncertainty induced by effects–based procedures is harmful and should lead the Authority to favour Per Se procedures. The issue has gained in importance recently as Competition Authorities (CAs) at both national and EU level have adopted many significant reforms in decision and enforcement procedures, increasing reliance on economics-based methodologies and consequent use of an Effects-Based rather than a Per Se approach to deciding cases.

However arguments about the implications of LU for the choice of legal standards have typically been asserted rather than demonstrated and have not been subjected to rigorous scrutiny. In this paper we adapt the framework of K&U (2009) and use it to subject these ideas to rigorous analysis. Our analysis shows that whether legal uncertainty emerges under an Effects-Based procedure, its nature and extent, the implications for firm behaviour and the consequent choice of legal standard, depends crucially on:

- the information available to the Authority concerning the characteristics of firms and their environment that determine the harm/benefit generated by their actions,
- the information available to the firms about these characteristics and about the assessment criteria/tests and models used by the Authority,
- The costs that the firms will have to incur in order to “reduce” Legal Uncertainty
- The procedural uncertainty that the firms might also face – in terms of incomplete coverage by the CA of the actions that should be investigated and delays in reaching decision under some decision rules.

A number of interesting points and results emerge from our analysis:

*Effects-Based* approach requires the Authority to establish explicit criteria for deeming some actions to be harmful and others benign and to then investigate each case to see which of these criteria it meets. An extreme form of the Effects-Based approach is what in US is termed Rule – of – Reason under which competition authorities have the discretion to apply different economic methodologies and criteria on a case-by-case basis.
It is often presumed that just because *Effects-Based* procedures will make different decisions in different cases, they necessarily create Legal Uncertainty. But this is wrong. Suppose that, even after an investigation, the Authority cannot determine all the relevant characteristics that determine the harm/benefit caused by an action taken by any firm, but nevertheless can accurately measure a subset of these, and that the firms taking the actions also know the value of these characteristics. Suppose the Authority uses a decision rule that it will disallow all actions where the vector of characteristics lies in a particular set but allow all other actions. For example it might disallow all actions where market share exceeds a particular value – having made clear the methodology and all the factors that will take into account in reaching an estimate of market share. And suppose that this decision rule can be communicated effectively to firms. This is an *Effects-Based* rule because different decisions will be made in different cases. Moreover it will typically be subject to both Type I Errors (False Acquittals) and Type II Errors (False Convictions) because there are harm-relevant characteristics that are excluded from the test. But there will be absolutely no Legal Uncertainty. Every firm knows the value of the characteristics that will lead the Authority to make its decision and so will know in advance exactly what decision will be made in its case were it ever to be investigated. Put somewhat differently, it is very important to distinguish between *variability* of treatment and *uncertainty* of treatment. *Effects-Based* procedures generate *variability* of treatment but not necessarily *uncertainty* of treatment. The crucial result we establish is that provided the *Effects-Based* rule can *effectively discriminate* – in the sense defined in K&U (2009) – then welfare under an *Effects-Based* rule with no Legal Uncertainty is higher than under a *Per Se* rule. So not only is *Per Se* not the only way of achieving Legal Certainty, it is not necessarily the best way of achieving it. This result is important because it suggests that Legal Certainty might best be achieved by *Effects-Based* procedures provided the tests are based on factor that are known to firms and that the rule is made transparent.
(ii) In many cases of course these circumstances may not arise, and Authorities may use tests based on factors or characteristics not readily observable by firms and, in addition, may not want or may not be able to fully reveal and explain the nature of the tests to firms. For example firms may know that decisions are based at least in part on a market share test, but not know all the factors that will potentially affect what definition of market will be used. Consequently, the things that firms know about themselves and their environment may only be loosely connected to the characteristics on which the test is based, and so firms do not know for sure whether their action will be disallowed but can only calculate/perceive the probability that it’s being disallowed were it ever to come before the Authority. Moreover, since firms differ in their characteristics and/or environment, in general different firms will attach different probabilities to their action’s being disallowed. So now there is what we call Partial Legal Uncertainty. In particular, suppose that, as in K&U (2009), under Partial Legal Uncertainty, firms:

- know whether their conduct is harmful or benign;
- do not know the values of the underlying characteristics which make their actions harmful or benign;
- but know enough about how the Authority will assess their conduct, that they calculate/perceive the probability of the Authority’s making Type I and Type II errors.

For this case we establish two important results.

1. If the authority uses an Effects-Based procedure then welfare in this situation where there is Legal Uncertainty could still be higher than in the situation described in (i) above where there was no Legal Uncertainty. So Legal Uncertainty can be welfare-enhancing. This is because Legal Uncertainty can generate deterrence effects that are on balance welfare-improving. A fortiori if the Effects-Based rule can effectively discriminate then, given the result we reported in (i) welfare will be higher than under Per Se.
2. Even if under an Effects-Based procedure welfare is lower with Legal Uncertainty than in the situation in (i) where there is no Legal Uncertainty, welfare may still be higher than under Per Se. This is because:
   - If the Effects-Based procedure can effectively discriminate it has lower costs of decision errors
   - Firms whose actions are harmful face a higher probability of having their actions disallowed than firms whose actions are benign and so, given that firms know their type, there is greater deterrence of harmful actions than benign actions.

(iii) In the most extreme case of Complete Legal Uncertainty firms may know so little about the characteristics that the Authority might use in the tests that it will conduct in any specific case and about the nature of the tests that it will employ, that not only do they not know for sure whether their action will be allowed or disallowed and can only calculate/perceive a probability of its being disallowed, but, this probability is unrelated to anything that the firm knows about itself and is common to all firms. Effectively the only thing that firms know is how often on average actions of the type they are contemplating taking are disallowed when they come before the Authority. For this reason this case is referred to as the "average deterrence" case while that in (ii) above is referred to as "marginal deterrence". The key result we establish is that if the CA uses an Effects-Based procedure then welfare under Complete Legal Uncertainty (and average deterrence) will be lower than welfare under Partial Legal Uncertainty (and marginal deterrence) and the likelihood that Effects-Based procedures welfare dominate Per Se is correspondingly smaller. But even in this case of Complete Legal

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10 Vickers (2007) also distinguishes between “discretionary decision making” by an Authority “based on whatever is thought to be desirable in economic terms case by case” and the Effects-Based approach proposed recently in EU. The case-by-case approach can be thought of as Rule of Reason. The Effects-Based approach need not necessarily produce legal uncertainty in the second sense above, as when the Authority uses clearly specified models and criteria that allow firms to anticipate correctly how their conduct will be assessed—in the sense of correctly anticipating when the conduct will be allowed or disallowed depending on whether it is harmful or benign.

11 See Katsoulacos and Ulph (2008a, b); also Immordino and Polo (2008).
Uncertainty welfare may still be higher under an *effects–based* procedure than under *Per Se*.

(iv) Allowing firms to reduce legal uncertainty by incurring some costs does not alter the qualitative nature of the welfare comparisons between Partial and No Legal Uncertainty as stated above.

(v) Procedural uncertainty due to incomplete coverage and decision-making delays has some rather subtle implications for deterrence and may actually *increase* the attractiveness of Partial vs. No Legal Uncertainty.

To summarise:

- There is no automatic equivalence between *Effects-Based* rules and Legal Uncertainty.
- There is no monotonic link between Legal Uncertainty and welfare. While very great degrees of Legal Uncertainty are welfare reducing, welfare can be higher when there is some degree of Legal Uncertainty than when there is no Legal Uncertainty. This is true also when firms can invest into reducing Legal Uncertainty and it is more likely to be true when procedural uncertainty is also present.
- If the tests on which *Effects-Based* procedures are based are good enough to enable the Authority to *effectively discriminate* then *Effects-Based procedures* will often be welfare superior to *Per Se* rules even though they involve Legal Uncertainty.
2. The Model

We begin by setting out a base model which is identical to that employed by Katsoulacos & Ulph (2009) except that, to start with, we ignore all procedural uncertainty by assuming that all cases are investigated (the coverage rate is 1) and that there are no delays in reaching a decision. Later we will consider some extensions.

a. Basic Set Up

There is a population of firms of size 1 that could take a particular type of action. A fraction \( \gamma \), \( 0 < \gamma < 1 \) come from a Harmful environment so, if they take the action and it is not stopped, this generates harm \( h_H > 0 \) The remaining fraction come from a Benign environment, so, if they take the action, this generates harm \( h_B < 0 \). Thus, \( \bar{h} = \gamma h_H + (1-\gamma)h_B \) is the average harm. The type of action is said to be *Presumptively Legal* (resp. *Illegal*) if \( \bar{h} < 0 \) (resp. \( \bar{h} > 0 \)).

In the absence of any intervention by the Competition Authority taking an action will confer a private benefit \( b > 0 \) for the firm. The distribution of \( b \) is independent of the environment from which the firm comes and is given by \( F(b), \ 0 < F(b) < 1; \ F'(b) > 0 \).

b. Competition Authority Decision Procedures

There is a Competition Authority (hereafter CA) which investigates all actions that are taken. It uses one of two decision procedures.

*Per Se* Here it allows all actions if they are *Presumptively Legal* and disallows all actions if they are *Presumptively Illegal*. Note that the fundamental characteristics of Per Se rules are that (i) the CAs decisions are based on the average harm of the type of action considered, so (ii) each action is treated in the same way – they are either all allowed or
disallowed and (iii) only one type of error is made – Type I for presumptively illegal and Type II for Presumptively Legal actions.

**Effects-Based** Under this procedure the CA undertakes an investigation of each action as a result of which it gets a signal of the likely harm caused by the action. This signal, which is only imperfectly correlated with the true harm will be either a “Positive Harm” - indicating that the action is likely to reduce welfare - or a “Negative Harm” - indicating that the action is likely to increase welfare – signal. The probability that a Benign action generates a Negative Harm signal is \( p_B, \; 0 < p_B < 1 \); the probability that a Harmful action generates a Positive Harm signal is \( p_H, \; 0 < p_H < 1 \). We assume that the underlying “model” used by the CA in its investigation to generate the signal has some discriminatory power so that \( p_B + p_H > 1 \) so firms from the Harmful environment are more likely to generate a Positive Harm signal than are firms from the Benign environment, and, correspondingly, firms from the Benign environment are more likely to generate a Negative Harm signal than are firms from the Harmful environment.

The decision rule used by the CA is to allow all actions that give a Negative signal and disallow those that give a Positive signal.

We want to assume that the *Effects-Based* decision rule is not a “straw man” and that this is in fact a sensible decision rule to use given the signals generated by the CA’s investigation. To do this we need to strengthen the assumption we make about the discriminatory power of the CA’s model.

**ASSUMPTION** Assume that the *Effects-Based Rule* can *Effectively Discriminate* – so for *Presumptively Legal* type of action this requires that

\[
q_H = \frac{p_H}{1 - p_B} > \frac{(1 - \gamma)(-h_B)}{\gamma h_H} = s_L > 1 \tag{1}
\]
while for *Presumptively Illegal* type of action this requires that

\[ q_B = \frac{p_B}{1 - p_B} > \frac{\gamma h_h}{(1 - \gamma)(-h_B)} \equiv s_I > 1. \]  \hspace{1cm} (2)

Here \( q_H \) (resp. \( q_B \)) is the measure of the *discriminating quality* of the model the CA uses to generate its signals of potential harm for *Presumptively Legal* (resp. *Illegal*) types of action\(^{12} \) and \( s_L \) (resp. \( s_I \)) is the *strength of the presumption of Legality* (resp. Illegality).

Then, as shown in Katsoulacos & Ulph (2009) whether the act is *Presumptively Legal* or *Illegal* the Cost of Decision Errors (CDE) will be lower under the *Effects-Based* rule than under the *Per Se* decision rule.

Finally assume that under both a *Per Se* and an *Effects-Based* decision procedure:

a. there is no delay in reaching a decision;

b. a firm that has its action disallowed has to pay a penalty \( \Phi > 0 \).

c. **Behaviour of Firms**

Firms have to decide whether or not to take the action, and obviously a factor that will be relevant to them in reaching this decision is the anticipated likely decision by the CA when it investigates the action. So behaviour varies depending on what decision procedure the CA uses.

*Per Se* Here it is obvious that if the action is *Presumptively Legal* then all firms will make benefit \( b > 0 \) by taking it and so all will take the action. If the action is *Presumptively Illegal* then all firms make a profit \(-\Phi\) if they take the action – so none does. So, in an obvious notation, **irrespective of which environment a firm comes**

\(^{12}\) So, for *Presumptively Legal* (resp. *Illegal*) types of action what matters is how good the model is at detecting Harmful (resp. Benign) actions.
from, the fraction of firms deterred from taking the action in each of these two cases is:

\[ F^{PSL} = 0; \quad F^{PSL} = 1 \]

*Effects Based*  Here we distinguish between 3 different information scenarios.

**Scenario 1: No Legal Uncertainty**
Here we assume that even though, after an investigation, the CA cannot determine all the relevant characteristics that determine the harm/benefit caused by an action, nevertheless it can accurately measure a subset of these, and the firms taking the actions also know the value of these characteristics as well as their type (Benign or Harmful). The CA uses a decision rule that will disallow all actions where the vector of characteristics lies in a particular set but allow all other actions. Also, this decision rule can be communicated effectively to firms.

This decision rule is *Effects-Based* as, in contrast to *Per Se*, the CA:
- Does not decide on the basis of average harm
- Individual actions are treated differently
- There are both Type I & II decision errors

In particular, the CA’s “model” disallows a fraction \( 1 - p_B \) of benign actions and allows a fraction \( 1 - p_H \) of harmful actions.

But there is absolutely no Legal Uncertainty. Every firm knows the value of the characteristics that will lead the CA to make its decision and knows in advance exactly what decision will be made in each case.

So we have:

**Proposition 1**  *Per Se* is not the only way of generating Legal Certainty. There can be Legal Certainty under an *Effects Based* decision procedure provided the decision criteria are based on factors that are observable by firms themselves and the decision rule is made known to firms.

Note that under this *Effects Based Rule* with *No Legal Uncertainty* a fraction of
\[ F_{EB}^{E0} = 1 - p_B \]  \hspace{1cm} (3)

of benign actions and a fraction
\[ F_{E0}^{E0} = p_H \]  \hspace{1cm} (3')

of harmful actions will not be undertaken, as the firms know for certain that these actions will be disallowed, where:
\[ 0 < F_{EB}^{E0} < F_{H}^{E0} < 1 \]  \hspace{1cm} (3'')

Notice however that here, as under *Per Se*, the actions not undertaken are not “deterred” in the sense of “deterrence due to uncertainty arising from imperfect ability to predict how a specific action will be treated”.

**Scenario 2: Partial Legal Uncertainty**

Here we assume that firms know what it is that is of concern to the authority and whether or not their action is likely to be harmful or benign (that is they know their type/environment - Harmful or Benign) but they do not know the signal that will be generated as a result of the CA applying its model to their specific case. This may arise either because they know the type of factors that the CA will use in making its decision but they do not know the values of these in their own context\(^{13}\), or because the CA does not reveal what factors (or, all the factors) that it takes into account. So they have *Legal Uncertainty* about what decision will be taken in their case. However, by knowing their type they at least know something that is correlated with the criteria on which the CA will be making its decision.

**Note 1:** There is a rather striking symmetry in the asymmetrical informational position of the CA and firms: the CA would like to know firms’ types but can only observe the signal generated by the application of its “model”; firms would like to know what signal would be generated in their own specific case, but only know their own type.

\(^{13}\) And moreover are not prepared to incur the cost of discovering this.
We assume also that each firm knows enough about the CA’s decision process to know the probability \( p_e, \ e = B, H \) that applies to a firm from their environment\(^{14} \). Let \( b_e, \ e = B, H \) be the minimum value of private benefit for which the expected profit of a firm from environment \( e \) will be zero. This is given by:

\[
\begin{align*}
 b_B &= \frac{(1-p_B)\Phi}{p_B}; \quad b_H = \frac{p_H\Phi}{(1-p_H)}; \quad 0 < b_B < b_H
\end{align*}
\]  

(4)

So under an Effects Based Rule with Partial Legal Uncertainty the fraction of firms from each of the two environments that are deterred from taking the action is:

\[
F_{EBP}^B = F\left( b_B \right); \quad F_{EBP}^H = F\left( b_H \right); \quad 0 < F_{EBP}^B < F_{EBP}^H < 1.
\]  

(5)

Scenario 3: Complete Legal Uncertainty

In this case we assume that firms know neither whether their specific action will be allowed or disallowed if investigated (as in Scenario 2 above), nor do they know what effect their action will have on welfare – they do not know their type. All they know is the average probability of having an action disallowed

\[
\bar{p} = \gamma p_H + (1-\gamma)(1-p_B).
\]

So let:

\[
\tilde{b} = \frac{p_H\Phi}{1-p}; \quad b_B < \tilde{b} < b_H
\]  

(6)

be the critical value of private benefit at which expected profits are zero when firms know only the average probability of having their action disallowed. Then irrespective of which environment a firm comes from under an Effects Based Rule with Complete Legal Uncertainty the fraction of firms that are deterred from taking the action will be:

\[
\]

\(^{14} \) Following Immordino and Polo (2007) this situation can also be referred to as one of marginal deterrence.
\[ F^{EBC} = F\left(\tilde{h}\right); \quad F^{EBP}_B < F^{EBC} < F^{EBP}_H. \]  

Notice the difference in “deterrence” between the case where there is Partial or Complete Uncertainty and “deterrence” when there is no Legal Uncertainty (under Effects-Based or Per Se): it is only in the former two cases that “deterrence” is related to “imperfect ability to predict how a specific action will be treated”.

3. Welfare Comparisons

3.1 Welfare

Since welfare arises solely from those firms that take the action it is clear that welfare in each of these set-ups is as follows:

\[ W^{PSL} = -\tilde{h} > 0; \quad W^{PSI} = 0 \]  

Effects-Based

No Legal Uncertainty

\[ W^{EB0} = \gamma \left( -h_H \right) \left( 1 - F^{EB0}_H \right) + (1 - \gamma) \left( -h_B \right) \left( 1 - F^{EB0}_B \right) \]

\[ = \gamma \left( -h_H \right) \left( 1 - P_H \right) + (1 - \gamma) \left( -h_B \right) p_B \]  

\[ W^{EBP} = \gamma \left( -h_H \right) \left( 1 - P_H \right) \left( 1 - F^{EBP}_H \right) + (1 - \gamma) \left( -h_B \right) p_B \left( 1 - F^{EBP}_B \right) \]

Complete Legal Uncertainty

\[ W^{EBC} = \left[ 1 - F^{EBC} \right] \left[ \gamma \left( -h_H \right) \left( 1 - P_H \right) + (1 - \gamma) \left( -h_B \right) p_B \right]. \]

3.2 Welfare Comparisons
We now carry out a systematic comparison between the levels of welfare under the different decision procedures and between different levels of uncertainty.

### 3.2.1 Per Se vs Effects Based with No Legal Uncertainty

**Proposition 2** If the decision criterion used under an Effects-Based procedure can Effectively Discriminate then when there is No Legal Uncertainty welfare is higher than under Per Se. Formally if the action is Presumptively Legal (resp. Illegal) then

\[
W^{EB0} > W^{PSL} \quad \text{(resp. } W^{EB0} > W^{PSI})\,.
\]

Proof: From (3), (8) and (9) we have

\[
\begin{align*}
W^{EB0} - W^{PSL} &= p_H (\gamma h_H) - (1 - p_B) \left[(1-\gamma)(-h_B)\right] \\
W^{EB0} - W^{PSI} &= p_B \left[(1-\gamma)(-h_B)\right] - (1 - p_B) (\gamma h_H)
\end{align*}
\]

(12)

and the result then follows from (1) and (2).

The implication is that provided:
- the factors on which the decision is made are known by firms themselves;
- the rule is made known to firms;
- and, moreover, the decision rule is sufficiently powerful that the CA can Effectively Discriminate

then not only is Per Se not the only way of generating Legal Certainty (Proposition 1) it is not the best way of doing so.

Proposition 2 has the following Corollary which will be useful later:
**Corollary 2** Welfare under an *Effectively Discriminating Effects-Based* procedure with *No Legal Uncertainty* is strictly positive, i.e.

\[ W_{EB0}^{EB0} > 0 \quad (13) \]

**Proof:** Welfare is greater than under *Per Se* which is either 0 for *Presumptively Illegal* acts or \(-\bar{h} > 0\) for *Presumptively Legal* actions.

**Note 2:** The welfare difference between a *Per Se* decision rule and an *Effects-Based* decision procedure with *No Legal Uncertainty* depends purely on the difference in their costs of decision errors which in turn depends on the discriminatory power of the *Effects –Based* procedure.

### 3.2.2 *No Legal Uncertainty* vs *Partial Legal Uncertainty*

From (9) and (10) we have:

\[ W_{EB0}^{EB0} - W_{EBP}^{EBP} = \left( F_{B}^{EBP} W_{EB0}^{EB0} \right) + \left[ -\left( F_{H}^{EBP} - F_{B}^{EBP} \right) \gamma \left( 1 - p_{H} \right) \bar{h} \right] \quad (14) \]

The first term on the RHS of (14) is what we can call the **absolute deterrence effect** and, from (13) is strictly positive. What this tells us is that if, under *Partial Legal Uncertainty* deterrence was exactly the same for firms from the Harmful environment than for firms from the Benign environment, then this is a factor that would lead to welfare being higher with *No Legal Uncertainty* than with *Partial Legal Uncertainty*. This can be explained given that, comparing (9) to (10), if \( F_{H}^{EBP} = F_{B}^{EBP} = \tilde{F}^{EBP} \) then \( W_{EBP}^{EBP} = (1 - \tilde{F}^{EBP}) W_{EB0}^{EB0} \Rightarrow W_{EBP}^{EBP} < W_{EB0}^{EB0} \); it is due to partial legal uncertainty generating, relative to no-uncertainty, additional deterrence that lowers welfare by \( \tilde{F}^{EBP} \) \( W_{EB0}^{EB0} \) where \( W_{EB0}^{EB0} \) is the average welfare of actions allowed. Deterring an equal number of benign and harmful actions reduces welfare because, as is clear from (10), the (negative) impact on
welfare of deterring a benign action is larger than the (positive) impact on welfare of deterring a harmful action by an effectively discriminating Effects-Based rule. Thus, if under Effects-Based with Partial Legal Uncertainty there were no differential deterrence, its net effect on welfare would be negative compared to Effects-Based with No Legal Uncertainty. The first term on the RHS of (14) can be thought of as the Cost of Partial Legal Uncertainty.

But the second term on the RHS of (14) is negative and represents what we can call the differential deterrence effect. This just reflects the fact that one of the advantages of Partial Legal Uncertainty is that it creates greater deterrence for firms from the harmful environment than for firms from the benign environment.

Given the very general level of our analysis there is no guarantee that one of these terms is larger than the other and hence no restriction that we can place on the sign of the welfare difference between No Legal Uncertainty and Partial Legal Uncertainty. So, for example, depending on precisely where \( b_a \) and \( b_H \) fall in the distribution of private benefits, \( F(b_a) \) can be small and \( F(b_H) - F(b_a) \) large, or \( F(b_a) \) can be large and \( F(b_H) - F(b_a) \) small.

So we have established:

**Proposition 3** Welfare can be higher under Partial Legal Uncertainty than under No Legal Uncertainty.

A very powerful implication of this proposition is that even if the factors that the CA used to make its decision were observable by firms, and if the way they were used to form a signal about potential harm and the critical value at which it was decided to disallow rather than allow were all easily communicable to firms, it may be desirable for the CA not to reveal this information.
An interesting corollary is:

**Corollary 3.1** Other things being equal – same discriminatory power, same deterrence effects – *No Legal Uncertainty* is more to be preferred to *Partial Legal Uncertainty* for *Presumptively Legal* actions than for *Presumptively Illegal* actions.

Proof: For a given level of discriminatory power $W_{EB0}^P$ - and hence the first term on RHIS of (14) will be higher for *Presumptively Legal* acts than for *Presumptively Illegal* types of action.

**Note 3** Notice that because we assume that the discriminating quality of the *Effects-Based* decision procedure is exactly the same under all three information scenarios, so too are the costs of decision errors, consequently the welfare difference in (14) depends solely on deterrence effects.

From **Proposition 2** and **Proposition 3** we can establish the following:

**Corollary 3.2**

(a) If *Partial Legal Uncertainty* welfare dominates *No Legal Uncertainty* then *a fortiori* it welfare dominates *Per Se*.

(b) Even if welfare is lower under *Partial Legal Uncertainty* than under *No Legal Uncertainty* it will still be higher than under *Per Se* provided the gain in lower costs of decision errors from having effective discrimination outweighs the welfare loss from deterrence effects.

### 3.2.3 Partial Legal Uncertainty vs Complete Legal Uncertainty

From (10) and (11) we have:

$$W_{EBP}^P - W_{EBC}^P = \gamma(1 - p_H)h_H(\bar{F}_H^{EBP} - \bar{F}_H^{EBC}) + (1 - \gamma)p_Bh_B(\bar{F}_B^{EBP} - \bar{F}_B^{EBC})$$

(15)
From (7) we see that both terms on the RHS of (15) are positive: the first because having *Partial Legal Uncertainty* deters more harmful actions and the second because it deters fewer benign actions compared to the situation of *Complete Legal Uncertainty*.

So we have:

**Proposition 4** *Partial Legal Uncertainty* welfare dominates *Complete Legal Uncertainty*

The final welfare comparison we wish to make is:

**3.2.4 No Legal Uncertainty vs Complete Legal Uncertainty**

It is straightforward to see from (9) and (11) that

\[ W^{EB0} - W^{EBC} = F^{EBC} W^{EB0} > 0 \]  

(16)

so, all we have in this case is a (larger) **absolute deterrence effect** than was present in the comparison between *No Legal Uncertainty* and *Partial Legal Uncertainty* with no offsetting **differential deterrence effect**. So we have

**Proposition 5** *No Legal Uncertainty* welfare dominates *Complete Legal Uncertainty*.

4. **Extensions**

In this section we consider two extensions of the analysis above. First we allow firms to reduce legal uncertainty by incurring some cost in obtaining and analysing information. Secondly we allow for the presence of procedural uncertainty – incomplete coverage and delays in decision making.
4.1 Firms invest to reduce uncertainty

One of the costs sometimes associated with legal uncertainty is that firms have to incur costs to reduce the uncertainty. Above we have been implicitly assuming that these costs are too high so firms will never incur them.

To take into account this consideration suppose now that in the case of Partial Legal Uncertainty firms can, before they take the action, make an investment that will completely reveal the nature of the test that will be carried out by the CA and the signal the CA will discover and hence the decision the CA will make. That is firms can buy Legal Certainty. Suppose that the cost of the investment is \( C > 0 \).

Take a firm from the Harmful environment with private benefit \( b > 0 \). If it makes the investment then it learns with certainty whether in its case, when investigated, the CA’s “model” will generate a Negative Harm signal. The probability that it belongs to those firms that will give a Negative Harm signal when investigated is \( (1 - p_H) \). So its expected net profits from making the investment will be \( (1 - p_H)b - C \). However if it chooses not to reduce the uncertainty then it will take the action only if the expected net profits from doing so are positive, so its expected profits if it remains uninformed are

\[
MAX \left[ (1 - p_H)b - p_H\Phi, 0 \right].
\]

From this we can see that if \( C \geq p_H\Phi \) the firm will choose not to be informed. However if \( C < p_H\Phi \) then a fraction \( F_{EBP}^{H}(C) = F \left[ b_H^i(C) \right] \)

where

\[
b_H^i(C) = \frac{C}{1 - p_H} < b_H = \frac{p_H\Phi}{(1 - p_H)}
\]

will choose not to become informed and will not take the action while the remaining fraction \( \left[ 1 - F_{EBP}^{H}(C) \right] \) will choose to become informed and will take the action depending on whether they find that they belong to the fraction that will be allowed or to the fraction that will be disallowed. Notice that since
$F^*_{EBPI}(C) < F^*_{EBP}$ more firms will take the action than in the case where all firms remained uninformed.

An analogous discussion holds for firms from the Benign environment. In this case the corresponding value of $b$ is $b^I_b(C) = \frac{C}{p_B} < b_B = \frac{(1-p_B)\Phi}{p_B}$ and a fraction $F^*_{EBPI}(C) = F[b^I_b(C)]$ will choose not to become informed.

So we get the following regimes:

**Regime 1** \( C \geq p_H \Phi \)

Here all firms remain uninformed and welfare under Partial Legal Uncertainty is, as before:

$$W^*_{EBP}(C) = -\left[\gamma \left(1 - F^*_{EBP}\right) (1-p_H) h_H + (1-\gamma) \left(1 - F^*_{EBP}\right) p_B h_B \right]. \quad (17)$$

**Regime 2** \( p_H \Phi > C \geq (1-p_B) \Phi \)

Here all firms from the benign environment remain uninformed while a fraction $\left[1 - F^*_{EBPI}(C)\right]$ of firms from the harmful environment will choose to become informed and will take the action depending on whether they find that they belong to the fraction that will be allowed or to the fraction that will be disallowed. Welfare is therefore:

$$W^*_{EBP}(C) = -\left\{\gamma \left[1 - F^*_{EBPI}(C)\right][ (1-p_H) h_H + C ] + (1-\gamma) \left(1 - F^*_{EBP}\right) p_B h_B \right\}$$

$$= -\left\{\gamma \left[1 - F\left(\frac{C}{1-p_H}\right)\right][ (1-p_H) h_H + C ] + (1-\gamma) \left(1 - F^*_{EBP}\right) p_B h_B \right\} \quad (18)$$
Notice that if \( C \approx p_H \Phi \) then welfare is definitely lower than in Regime 1 where \( C \geq p_H \Phi \). This is because the fraction of firms deterred is effectively the same - i.e. 

\[
F \left( \frac{C}{1 - p_H} \right) \approx F_H^{EBP} - \text{but now there is a group of firms incurring costs to become informed, which, pace discussion above, has no welfare advantage compared to just taking the action and having it allowed/disallowed.}
\]

More generally it is interesting to consider how welfare varies with costs within this regime. Differentiating (18) we get:

\[
\frac{dW^{EBP}}{dC} = \gamma \left[ 1 - F \left( b'_H(C) \right) \right] \left\{ \frac{f \left( b'_H(C) \right)}{1 - F \left( b'_H(C) \right)} \right\} \left( h_H + b'_H(C) \right) - 1 \right\} (19)
\]

There are two separate effects of a unit increase in \( C \). First it imposes higher costs on firms choosing to become informed – which is welfare-reducing. But second it deters more firms from incurring the costs of becoming informed and taking the action – at least with some probability. This is a good thing because it avoids some harm and the incurring of costs which, from the point of view of society are wasteful.

**Regime 3** \( 0 \leq C < (1 - p_b) \Phi \)

Here a fraction of firms from both environments will choose to become informed, and consequently welfare is:

\[
W^{EBP}(C) = -\left\{ \gamma \left[ 1 - F_{H}^{EBPI}(C) \right] \left[ (1 - p_H)h_H + C \right] + (1 - \gamma) \left[ 1 - F_B^{EBPI}(C) \right] (p_Bh_B + C) \right\}
\]

\[
= -\left\{ \gamma \left[ 1 - F \left( \frac{C}{1 - p_H} \right) \right] \left[ (1 - p_H)h_H + C \right] + (1 - \gamma) \left[ 1 - F \left( \frac{C}{p_B} \right) \right] (p_Bh_B + C) \right\}
\]

(20)
There are a number of points to note.

First it follows from (20) and (9) that as $C \to 0$, $W^{EBP}(C) \to W^{EB0}$ - as is intuitively obvious.

Second, once again for values of $C$ close the boundary of Regime 2 above, that is $C \approx (1 - p_h)\Phi$, welfare is discontinuously lower – because deterrence effects are very close to those in the higher regime, but now there is a whole new group of firms incurring learning costs.

Finally to consider more generally how welfare varies with costs on the interior of the regime we can differentiate (20) to get:

$$\frac{dW^{EBP}}{dC} = \gamma \left[ 1 - F\left(b^I_h(C)\right) \right] \left[ \frac{f\left(b^I_h(C)\right)}{1 - F\left(h^I_h(C)\right)} \left(h^I_h + b^I_h(C)\right) - 1 \right] + (1 - \gamma) \left[ 1 - F\left(b^I_h(C)\right) \right] \left[ \frac{f\left(b^I_h(C)\right)}{1 - F\left(h^I_h(C)\right)} \left(h^I_h + b^I_h(C)\right) - 1 \right]. \quad (21)$$

The first term on RHS is as before while the sign of the additional second term arising for firms from the Benign environment is now ambiguous - because the harmful effect of deterring a few more firms from taking a beneficial action could be offset by the gain from ensuring that they do not incur costs of becoming informed.

So, in summary, the recognition that firms might incur costs to become informed and reduce uncertainty has the following implications.

The first is that welfare now explicitly takes account of the costs of becoming informed and, other things (particularly deterrence effects) being equal welfare will be lower under Partial Legal Uncertainty than it otherwise would have been. But other things are
not equal and deterrence effects are also affected by the costs of becoming informed. So the picture is more complex.

The second is that welfare under *Partial Legal Uncertainty* varies from being equal to welfare under *No Legal Uncertainty* – in the case where $C = 0$ - to being equal to welfare under *Partial Legal Uncertainty* as considered in the core part of the paper\(^\text{15}\) - in the case where $C \geq p_\text{H}\Phi$ (Regime 1).

So given the discussion above we can re-state Proposition 3 as follows:

**Proposition 6**  It is possible that $W^{\text{EBP}}(C = p_\text{H}\Phi) > W^{\text{EBP}}(C = 0) = W^{\text{EBO}}$.

Thirdly welfare varies neither continuously nor monotonically with the costs of becoming informed, but rather takes two discontinuous upward jumps as costs increase across the thresholds between the 3 regimes. This has some complex implications as shown in the two Corollaries that follow:

**Corollary 6.1** Even though firms incur costs of becoming informed there may be whole ranges of costs where welfare will still be higher under *Partial Legal Uncertainty* than under *No Legal Uncertainty*, and, from Proposition 2, *a fortiori* under *Pe Se*.

Proof: Certainly in the case where $W^{\text{EBP}}(p_\text{H}\Phi) > W^{\text{EBP}}(0)$ there will be ranges of costs where $W^{\text{EBP}}(C) > W^{\text{EBP}}(0) = W^{\text{EBO}} > W^{\text{PSL}}$. On the other hand even if $W^{\text{EBP}}(p_\text{H}\Phi) < W^{\text{EBP}}(0)$ there still could be ranges of costs where $W^{\text{EBP}}(C) > W^{\text{EBP}}(0) = W^{\text{EBO}} > W^{\text{PSL}}$. Figure 1 in the Annex illustrates.

**Corollary 6.2** Despite the fact that $W^{\text{EBP}}(p_\text{H}\Phi) > W^{\text{EBP}}(0)$ there could be ranges of costs for which $W^{\text{EBP}}(C) < W^{\text{EBP}}(0)$ so the costs of becoming informed impose a real

\(^{15}\) In which the implicit assumption was that C is too high for firms to become informed.
welfare loss which drives welfare below that with No Legal Uncertainty. Figure 2 in Annex illustrates.

4.2 Procedural Uncertainty

So far we have assumed that the only source of uncertainty is that arising from the decision process of the CA, and we have done this by assuming that every action was investigated – the coverage rate was 1 – and that there were no delays in reaching a decision. In this section we want to recognise that there is also an element of procedural uncertainty generated by the process of investigation by a CA – and the aim is to understand not just how this affects welfare but also how this interacts with legal uncertainty – and the comparison between Effects-Based and Per Se procedures.

So let us now suppose the following.

- Whatever decision rule the CA uses, only a fraction $\chi$, $0 < \chi < 1$ of actions that are taken by firms will be investigated by the CA. $\chi$ is what we call the coverage rate.
- If the CA uses a Per Se decision rule then there is no delay in reaching a decision, but if it uses an Effects-Based decision rule then there is a delay $\delta$, $0 \leq \delta \leq 1$ in reaching a decision. This means that a firm will get this fraction of the private benefit but will also generate this fraction of the social harm/benefit.

To understand the effect of these procedural features on firm behaviour and hence welfare consider in turn the two decision procedures.

4.2.1 Per Se Decisions

Notice that under Per Se if an action is Presumptively Legal all firms know that their action will certainly be allowed, so the fact that only a fraction of firms will have their actions investigated and that there might be a delay in the decision to allow them is irrelevant. There are still no deterrence effects and once again welfare is:
Turning to a Presumptively Illegal action, firms know that, from whatever environment they come, if they are investigated their action will be disallowed and they will have to pay a penalty, but there is now a chance that they may not be investigated. So now only a fraction
\[ F^{PSI} = F\left(\frac{\chi\Phi}{1-\chi}\right) \] of firms from each environment will be deterred from taking the action, while of those that do take it a fraction \((1-\chi)\) will not have the action disallowed. Accordingly welfare under Per Se Illegality is now:
\[ W^{PSI} = -\bar{h} \Theta^{PSI} \leq 0, \] where \( \Theta^{PSI} = (1 - F^{PSI})(1 - \chi), \ 0 \leq \Theta^{PSI} \leq 1, \ \Theta^{PSI} = 0 \Leftrightarrow \chi = 1 \) is the fraction of harmful actions that are now taken (that are not deterred and not detected) under Per Se Illegality due to incomplete coverage and is a strictly decreasing function of \( \chi \). So, comparing (24) with (8) we see that the effect of having incomplete coverage is to lower welfare, since fewer actions that are on average harmful are disallowed.

4.2.2 Effects-Based Decisions

It is useful to consider in turn the three information scenarios under Effects-Based procedures.

Scenario 1: No Legal Uncertainty
Here just as with Per Se Legality firms that know they will give a negative signal if investigated will know that their actions will be allowed and so incomplete coverage and a possible delay in reaching a decision are irrelevant – there will be no deterrence effect. On the other hand, just as under Per Se Illegality firms that know they will give a positive signal if investigated also realise that there is a chance they will not be investigated and, even if they are, there could be a delay in reaching a decision enabling them to make a fraction of their private benefit, so the fraction of firms from each environment who will be deterred will be

$$F^E_{EBO} = F\left(\frac{\chi \Phi}{1 - \chi (1 - \delta)}\right).$$

(25)

Notice that this is strictly decreasing in $\delta$ and so

$$F^E_{EBO} \leq F^{PSI}$$

(26)

with strict inequality if the delay is positive.

Welfare is therefore:

$$W^E_{EBO} = \gamma (-h_H) \left[1 - p_H + p_H \Theta^E_{EBO}\right] + (1 - \gamma) (-h_B) \left[p_B + (1 - p_B) \Theta^E_{EBO}\right]$$

(27)

or

$$W^E_{EBO} = \gamma (-h_H) \left[1 - p_H \left(1 - \Theta^E_{EBO}\right)\right] + (1 - \gamma) (-h_B) \left[1 - (1 - p_B) \left(1 - \Theta^E_{EBO}\right)\right],$$

(27')

where $\Theta^E_{EBO} = (1 - F^E_{EBO}) \left[1 - \chi (1 - \delta)\right]$, $0 \leq \Theta^E_{EBO} \leq 1$, is the fraction of actions giving a positive signal that are taken (they are not deterred and not detected) when there is No Legal Uncertainty - so relative to the situation with no procedural uncertainty (compare also to equation (9)) there is an increase in the fraction of harmful actions taken of $p_H \Theta^E_{EBO}$ and an increase in the fraction of benign actions taken of $(1 - p_B) \Theta^E_{EBO}$. $\Theta^E_{EBO}$ is a strictly increasing function of $\delta$, with $\Theta^E_{EBO} = \Theta^{PSI} \leftrightarrow \delta = 0$, and $\Theta^E_{EBO} > \Theta^{PSI}, \delta > 0$, because the longer the delay the fewer firms will be deterred and the larger will be the fraction of harm/benefit generated by those firms that do take the action.
**Scenario 2: Partial Legal Uncertainty**

Firms now recognise that there is only a chance of being investigated, that if they are investigated the probability of having their action disallowed will depend on their type, but that any such decision may incur a delay during which time they can continue to obtain private benefits. So, analogous to (25), the introduction of incomplete coverage and a potential delay in making decisions means that, instead of (4), the crucial values of private benefit below which firms from the two environments will be deterred from taking the action are now defined by:

\[
\begin{align*}
    b_B &= \frac{\chi(1-p_B) \Phi}{1-\chi(1-p_B)(1-\delta)}; \\
    b_H &= \frac{\chi p_H \Phi}{1-\chi p_H(1-\delta)}; \\
    0 < b_B < b_H.
\end{align*}
\]  

(28)

With these definitions, the fraction of firms from each of the two environments that are deterred is still given by (5). Consequently welfare is given by:

\[
W_{EBP}^{\text{EBP}} = \gamma(-h_B)[1-\chi p_H(1-\delta)](1-F_B^{EBP}) + (1-\gamma)(-h_B)[1-\chi(1-p_B)(1-\delta)](1-F_B^{EBP})
\]

(29)

Or

\[
W_{EBP}^{\text{EBP}} = \gamma(-h_B)(1-p_H)(1-F_H^{EBP}) + \gamma(-h_B)p_H\Theta_{H+}^{EBP} + (1-\gamma)(-h_B)p_B(1-F_B^{EBP}) + (1-\gamma)(-h_B)(1-p_B)\Theta_{B+}^{EBP}
\]

(29')

where \( \Theta_{H+}^{EBP} = [1-\chi(1-\delta)](1-F_H^{EBP}) \)

and \( \Theta_{B+}^{EBP} = [1-\chi(1-\delta)](1-F_B^{EBP}) \)

\( \Theta_{H+}^{EBP} \) being the fraction of harmful actions giving a Positive signal that are taken due to procedural uncertainty and \( \Theta_{B+}^{EBP} \) been the fraction of benign actions giving a Positive signal that are taken due to procedural uncertainty (compare also (29') to (10)).

**Scenario 3: Complete Legal Uncertainty**
As in the case where there is no procedural uncertainty, exactly the same proportion of firms from each environment are deterred from taking the action, and is given by

\[
F^{EBC} = F\left(\frac{\chi \Phi}{1 - \chi p(1 - \delta)}\right),
\]

(30)

and so, as above,

\[
F^{EBP}_B < F^{EBC} < F^{EBP}_H.
\]

(31)

Welfare is then

\[
W^{EBC} = (1 - F^{EBC}) \left[ \gamma(\neg h_H)\left[1 - \chi p_H(1 - \delta)\right] + (1 - \gamma)(\neg h_B)\left[1 - \chi(1 - p_B)(1 - \delta)\right] \right].
\]

(32)

How do the welfare comparisons we obtained above go through in this new situation?

4.2.3 Welfare Comparisons

As above we carry out a number of welfare comparisons.

(i) Per Se vs Effects Based with No Legal Uncertainty

From (22), (24) & (27) it is straightforward to show that,

\[
W^{EB0} - W^{PSL} = \left(1 - \Theta^{EB0}_E\right) \left[ \gamma h_H p_H - (1 - \gamma)(\neg h_B)(1 - p_B) \right]
\]

(33)

which is strictly positive if the Effects-Based decision rule can effectively discriminate while
where the first term is positive if the Effects-Based decision rule can effectively discriminate but the second non-positive – and strictly negative if there is positive delay in reaching decisions under an Effects-Based procedure. Comparing (33), (34) to equations (12) we see that procedural uncertainty reduces the difference in welfare under Effects-Based and Per Se procedures in favour of Per Se procedures. Further now, for Presumptively Illegal actions the comparison between Per Se and Effects-Based Rules no longer depends solely on decision cost errors but also on differential deterrence effects arising from procedural differences between the two decision-making processes. Specifically we have:

**Proposition 7**
(i) If an action is Presumptively Legal then an Effect-Based procedure with No Legal Uncertainty, still under procedural uncertainty, welfare-dominates Per Se Legality (though the difference in welfare is now smaller due to the additional deterrence effect of procedural uncertainty under Effects-Based).

(ii) If an action is Presumptively Illegal then an Effect-Based procedure with No Legal Uncertainty welfare-dominates Per Se Illegality (PSI) provided the delay in decision-making under the Effects-Based decision process is sufficiently small. That is, large procedural uncertainty can tilt the balance in favour of PSI even though under Effects-Based there is no Legal Uncertainty.

(ii) **No Legal Uncertainty vs Partial Legal Uncertainty**

From (27) and (29) we can show that:
\[ W^{EB0} - W^{EBP} = \left( F_B^{EBP} W^{EB0} \right) - \left[ \left( F_B^{EBP} - F_B^{EB0} \right)[1 - \chi(1 - \delta)]\left[ \gamma h_H p_H - (1 - \gamma)(-h_B)(1 - p_B) \right] \right] \\
- \left( F_H^{EBP} - F_B^{EBP} \right) \gamma h_H \left( 1 - \chi(1 - \delta) p_H \right) \]

Or

\[ W^{EB0} - W^{EBP} = \left( F_B^{EBP} W^{EB0} \right) - \left[ \left( \Theta_+^{EB0} - \Theta_+^{EBP} \right)[\gamma h_H p_H - (1 - \gamma)(-h_B)(1 - p_B)] \right] \\
- \left( F_H^{EBP} - F_B^{EBP} \right) \gamma h_H \left( 1 - \chi(1 - \delta) p_H \right) \]  

(35)

As before - equation (14) - there is an absolute deterrence effect (the first term on the RHS) which is strictly positive and favours Effects-Based with No Legal Uncertainty. However, this effect is now counteracted by the fact that now an additional fraction of actions giving a positive signal is taken due to procedural uncertainty under Effects-Based with and without Legal Uncertainty. The fraction of actions giving a positive signal that are taken due to procedural uncertainty under Effects-Based and No Legal Uncertainty is greater than the fraction of benign actions giving a positive signal that are taken due to procedural uncertainty under Effects-Based with Partial Legal Uncertainty (\( \Theta_+^{EB0} > \Theta_+^{EBP} \)) – so the second term on the RHS of (35) is positive and now favours Effects-Based with Partial Legal Uncertainty. As before there is again a differential deterrence effect that also favours Effects-Based with Partial Legal Uncertainty – the third term on the RHS of (35).

Thus we have:

**Proposition 8**

(i) Effects-Based with Partial Legal Uncertainty is more likely to dominate Effects-Based with No Legal Uncertainty when there is procedural uncertainty.

(ii) The greater the procedural uncertainty – the smaller, that is, \( \chi(1 - \delta) \), the more likely that Effects-Based with Partial Legal Uncertainty dominates Effects-Based with No Legal Uncertainty.
(iii) **Partial Legal Uncertainty vs. Complete Legal Uncertainty**

From (29) and (32) it is straightforward to show that:

\[
W_{EBP}^E - W_{EBC}^E = \gamma(-h_H)\left[1 - \chi_p(1 - \delta)\right](F_{EBC}^E - F_{EBP}^E) \\
\quad + (1 - \gamma)(-h_B)\left[1 - \chi(1 - P_B)(1 - \delta)\right](F_{EBC}^E - F_{EBP}^E) > 0
\]

So we have:

**Proposition 9**

(i) With procedural uncertainty too, *Effects-Based* with Partial Legal Uncertainty welfare dominates *Effects-Based* with Complete Legal Uncertainty.

(ii) The greater the procedural uncertainty – the smaller, that is, $\chi(1 - \delta)$, the greater the difference in welfare under *Effects-Based* with Partial Legal Uncertainty and welfare under *Effects-Based* with Complete Legal Uncertainty.

5. **Conclusions**

In this paper we have adapted the framework of K&U (2009) and used it to subject to rigorous analysis the implications of legal uncertainty for welfare and for the optimal choice of legal standards. We have shown how these implications depend crucially on:

- the information available to the Authority concerning the characteristics of firms and their environment that determine the harm/benefit generated by their actions,
- the information available to the firms about these characteristics and about the assessment criteria/tests and models used by the Authority,
- The costs that the firms will have to incur in order to “reduce” Legal Uncertainty.
- The procedural uncertainty that the firms might also face – in terms of incomplete coverage by the CA of the actions that should be investigated and delays in reaching decision under some decision rules.

A number of interesting results emerge from our analysis, in particular:
• There is no automatic equivalence between *Effects-Based* rules and Legal Uncertainty.

• There is no monotonic link between Legal Uncertainty and welfare. While very great degrees of Legal Uncertainty are welfare reducing, welfare can be higher when there is some degree of Legal Uncertainty than when there is no Legal Uncertainty. This is true also when firms can invest into reducing Legal Uncertainty and it is *more likely* to be true when procedural uncertainty is also present.

• If the tests on which *Effects-Based* procedures are based are good enough to enable the Authority to *effectively discriminate* then *Effects-Based procedures* will often be welfare superior to *Per Se* rules even though they involve Legal Uncertainty.
Figure 1

Figure 1 shows a graph depicting the relationship between two variables, $W$ and $C$, with $W$ on the y-axis and $C$ on the x-axis. The graph illustrates the function $W^{EBP}(0)$ and $W^{EBP}(p_H\Phi)$, where $p_B\Phi$ and $p_H\Phi$ are indicated on the x-axis. The function $W^{EBP}(0)$ decreases from $W^{EBP}(0)$ to $W^{EBP}(p_H\Phi)$ as $C$ increases from $(1-p_B)\Phi$ to $p_H\Phi$. The graph provides a visual representation of the change in $W$ as $C$ varies within the specified range.
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