Economics Research Workshop

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How much bargaining power does a worker have?
Targets, uncertainty and informational heterogeneity in a Nash-bargaining wage-determination framework, with a Two-tier Stochastic Frontier model to measure them all.

Chapter of PhD Thesis

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What

- **A Nash-bargaining wage-determination model under uncertainty and informational heterogeneity, based on target-wages rather than on reservation wages.**

- **A new Two-tier Stochastic Frontier model, with statistical dependence inside the error term, and allowing for regressor endogeneity.**

- **An empirical application.**

The two-player Nash Bargaining model

Nash (1950, 1953)

- Two self-interested Expected Utility maximizers: 1,2

- A strictly positive GAIN, if they collaborate.

- A non-negative payoff if they don't collaborate
  - (So No Penalty in case of no agreement).

- **The issue: if agreement is reached, how to split the gain?**

The two-player Nash Bargaining model

- **FORMALITIES**

- Players' "no-agreement" payoffs: $s_0\left(1\right),\ s_0\left(2\right) \geq 0$

  (these are the "credible threats" of the game)

- Players' split of the pie: $s\left(1\right),\ s\left(2\right) > 0$
  - (decision variables somewhere in here)

- Players' "Surplus" functions :

$$S_1 = s\left(1\right) - s_0\left(1\right)$$
$$S_2 = s\left(2\right) - s_0\left(2\right)$$

The two-player Nash Bargaining model

- **The Nash bargaining Solution (see Roth 1979) :**

$$\arg\max\{S_1 \cdot S_2\} = \arg\max\left\{\left(s(1)-s_0(1)\right)\cdot\left(s(2)-s_0(2)\right)\right\}$$

is the unique equilibrium that satisfies the axioms of Expected Utility Theory and is Pareto optimal.

- **The "asymmetric case"**

$$\arg\max\left\{S_1^{\eta} \cdot S_2^{1-\eta}\right\} = \arg\max\left\{\left(s(1)-s_0(1)\right)^{\eta}\cdot\left(s(2)-s_0(2)\right)^{1-\eta}\right\},\quad 0<\eta<1$$

is still the "Nash bargaining solution"

Map to wage determination (deterministic set up)

- **Players: Firm (f) and employee (e)**
- **GAIN: the worker's output,** p **(certain magnitude)**
- **Employee surplus function :**
  - **No-agreement payoff :** $s_0(e) = \underline{\omega}$ **("reservation wage")**
  - **Split of the pie :** $s(e) = \omega$ **(actual wage)**
  - **Surplus function:** $S_e = \omega - \underline{\omega}$
- **Firm surplus function:**
  - **No-agreement payoff :** $s_0(f) = 0$
  - **Split of the pie :** $s(e) = p - \omega$
  - **Surplus function:** $S_e = p - \omega$

Map to wage determination (deterministic set up)

- **Set also** $0<\eta<1$ **to reflect "relative bargaining power of the employee"**

- **Then, objective function:** $S_e^{\eta} \cdot S_f^{1-\eta} = \left(\omega - \underline{\omega}\right)^{\eta} \cdot \left(p - \omega\right)^{1-\eta}$

and Nash Bargaining solution:

$$\omega^{*} = \eta p \; + \; \left(1-\eta\right)\underline{\omega}$$

… a convex combination of "maximum willingness to pay" (p)

and "reservation wage" (minimum acceptable wage) $\underline{\omega}$

Productivity/output uncertainty

- **Assume now that the output of the worker is uncertain.**
- An immediate thought is to substitute an expected value for the previously certain amount. In such a case
- **Objective function:** $S_e^{\eta} \cdot S_f^{1-\eta} = (\omega - \underline{\omega})^{\eta} \cdot \left(E\left(p \middle| I_f\right) - \omega\right)^{1-\eta}$

where I_f is the Information Set of the firm.

Nash Bargaining solution:

$$\omega^* = \eta E\left(p \middle| I_f\right) + (1-\eta)\underline{\omega}$$

It appears trivial… BUT

Productivity/output uncertainty

$$\omega^* = \eta E\left(p \middle| I_f\right) \ + \ \left(1-\eta\right)\underline{\omega}$$

…has issues regarding econometric implementation (and not only)

1) The conditional expectation function relates to many variables that are never available as data (so a lot of "omitted variables" issues).
2) The unobservable η will differ from transaction to transaction, it is a random variable: varying coefficients per observation.
3) Also $\left(1-\eta\right)\underline{\omega}$ is unobservable and will go into the "error term": non-zero mean, correlation with the regressors.
4) It does not take into account **how the surplus functions change during the negotiation process itself.**
5) Last but not least, it does not give us a Two-tier stochastic frontier model, which is what I am doing in my PhD.

Productivity/output uncertainty – a new take

<u>Start with:</u>

"It does not take into account **how the surplus functions change during the negotiation process itself".**

- Think of the negotiation process as a sequence of arguments, offers and counter-offers.
- At some point in the negotiation process, the firm makes an offer, ω_f^T
- Since we will be working econometrically with completed transactions (realized matches), an offer in this sequence eventually will exceed the reservation wage of the worker.
- **Freeze the frame**: at that point the options for the worker are NOT anymore
  - "Walk away and take payoff $\underline{\omega}$ " OR "continue bargaining"

They have now have become

- "Agree and take payoff $\omega_f^T > \underline{\omega}$ OR "continue bargaining"

Which is better. **The new "credible threat" of the worker is to *accept the firm's offer.***

Productivity/output uncertainty – a new take

The surplus function of the worker is now $\boxed{S_e = \omega - \omega_f^T}$

Analogously the firm will face a counter-offer $\omega_e^T < E\left(p \middle| I_f\right)$

and its "credible threat" has now become to accept this counter-offer.
The options now are

"Agree and take payoff $E\left(p \middle| I_f\right) - \omega_e^T$ "

OR "continue bargaining" $E\left(p \middle| I_f\right) - \omega$

The surplus function of the firm is now

$$S_f = \left[E\left(p \middle| I_f\right) - \omega\right] - \left[E\left(p \middle| I_f\right) - \omega_e^T\right] \Rightarrow \boxed{S_f = \omega_e^T - \omega}$$

Productivity/output uncertainty – a new take

Since offers of the one party become credible threats of the other party, they are **credible commitments**, and so must be guided by some self-interested overall strategy/target (that may shift during the negotiation process), some pivotal magnitude.

Condense the sequential procedure into this focal point, treating now the offers / counter-offers as these encompassing targets.

We have transformed the objective function into

$$S_e^{\eta} \cdot S_f^{1-\eta} = \left(\omega - \omega_f^T\right)^{\eta} \cdot \left(\omega_e^T - \omega\right)^{1-\eta}$$

and the Nash Bargaining solution into

$$\omega^* = \eta\omega_e^T \quad + \quad (1-\eta)\,\omega_f^T$$

and the issue now becomes…

Productivity/output uncertainty – a new take

…HOW THESE TARGETS ARE FORMED/CAN BE MODELED?

Enter: The information Set of the worker $I_e, \quad I_e \neq I_f, \quad I_e \cap I_f \neq \varnothing$

- **Information heterogeneity (private information etc).**
- **But also, some common knowledge.**

Consider the **symmetric-information/common-knowledge expected output**

$$E\left(p \middle| I_e \cap I_f\right)$$

What is common (and relevant) knowledge here?

Things like the worker's "official" attributes, say, her resumé.

Also, this is updated/changed during the negotiation process, by some

$$v, \quad E(v) = 0$$

and we have the equilibrium common-knowledge expected output

$$\boxed{\mu(\mathbf{x}) = E\left(p \middle| I_e \cap I_f\right) + v}$$

Productivity/output uncertainty – a new take

Can we relate the worker's and the firm's equilibrium targets to this conditional expectation?

WORKER : "I am not just that. I am MORE than that", because

- Individualism
- $\mu(\mathbf{x})$ does not use all the information the worker possesses

"MORE" = $g \geq 0$ **"self-evaluation premium"**

$$\boxed{\omega_e^T = \mu(\mathbf{x}) + g}$$

FIRM : "It is NEVER AS GOOD as it looks on paper", because

- **"Potential" is not realized production**
- **Credentials cannot guarantee efficiency**
- **"NEVER AS GOOD" =** $d \geq 0$

"prudential discount"

$$\boxed{\omega_f^T = \mu(\mathbf{x}) - d}$$

Productivity/output uncertainty – a new take

$$\omega^* = \eta\omega_e^T + (1-\eta)\omega_f^T$$

$$\omega^* = \eta\left[\mu(\mathbf{x}) + g\right] + (1-\eta)\left[\mu(\mathbf{x}) - d\right]$$

$$\omega^* = \eta\left[E\left(p \middle| I_e \cap I_f\right) + v + g\right] + (1-\eta)\left[E\left(p \middle| I_e \cap I_f\right) + v - d\right]$$

$$\omega^* = E\left(p \middle| I_e \cap I_f\right) + \left[v + \eta g - (1-\eta)d\right]$$

Productivity/output uncertainty – a new take

$$\omega^* = E\left(p \middle| I_e \cap I_f\right) + \left[v + \eta g - (1-\eta)d\right]$$

$$y = f(\mathbf{x}) + \varepsilon$$

$$\varepsilon = v + w - u, \qquad w, u \geq 0$$

We achieved:

- The conditional expected value is more closely related to, and so better approximated by, the data usually available.
- No varying coefficients in sight.
- The composite error term has the Two-Tier Stochastic Frontier **(2TSF)** structure.

Productivity/output uncertainty – a new take

$$\omega_i^* = E\left(p_i \middle| I_e \cap I_f\right) + \left[v_i + \eta_i g_i - (1-\eta_i)d_i\right], \quad i = 1,\dots,n$$

$$y_i = \mathbf{x}_i'\beta + \varepsilon_i$$

$$\varepsilon_i = v_i + w_i - u_i, \qquad w_i, u_i \geq 0$$

REMAINING ISSUE	SOLUTION
w, u are (negatively) correlated by construction	A new 2TSF specification with internal dependence
v may be correlated with $\mathbf{x}$	A Copula to account for regressor endogeneity

The 2TSF Correlated Exponential specification

$$y_i = \mathbf{x}_i'\beta + \varepsilon_i, \quad \varepsilon_i = v_i + w_i - u_i, \quad w_i, u_i \geq 0$$

$$v_i \sim N\left(0, \sigma_v^2\right), \quad w_i, u_i \sim \text{Freund's Bivariate Exponential Extension}$$

(Freund 1961)

Then the density and distribution function of the composite error term are

$$f_\varepsilon(\varepsilon) = \sqrt{2\pi}\phi(\varepsilon/\sigma_v)\cdot\left[mb'\exp\left\{\tfrac{1}{2}\omega_2^2\right\}\Phi(-\omega_2) + (1-m)a'\exp\left\{\tfrac{1}{2}\omega_3^2\right\}\Phi(\omega_3)\right]$$

$$F_\varepsilon(\varepsilon) = \Phi(\varepsilon/\sigma_v) + \sqrt{2\pi}\phi(\varepsilon/\sigma_v)\left[m\exp\left\{\tfrac{1}{2}\omega_2^2\right\}\Phi(-\omega_2) - (1-m)\exp\left\{\tfrac{1}{2}\omega_3^2\right\}\Phi(\omega_3)\right]$$

$$\omega_2 \equiv \frac{\varepsilon}{\sigma_v} + b'\sigma_v, \quad \omega_3 \equiv \frac{\varepsilon}{\sigma_v} - a'\sigma_v, \quad a', b' > 0, \quad 0 < m < 1$$

ϕ, Φ are the Standard Normal density and distribution functions

The 2TSF Correlated Exponential specification

$$y_i = \mathbf{x}_i'\beta \; + \; \varepsilon_i, \quad \varepsilon_i = v_i + w_i - u_i, \quad w_i, u_i \geq 0$$

$$f_\varepsilon(\varepsilon) = \sqrt{2\pi}\phi(\varepsilon/\sigma_v)\cdot\left[mb'\exp\left\{\tfrac{1}{2}\omega_2^2\right\}\Phi(-\omega_2) + (1-m)a'\exp\left\{\tfrac{1}{2}\omega_3^2\right\}\Phi(\omega_3)\right]$$

The specification :

- Nests the independence case -when $mb' = (1-m)a'$
- Has a respectable range for Pearson's correlation coefficient $(-1/3,\, 1)$
- The marginal moments of w, u are not identifiable
- The moments of their difference $z \equiv w - u$ are identifiable

  (And this is what we really care about)
- It is rather easy-going with Maximum Likelihood Estimation.

Regressor endogeneity and Copulas

- A "Copula", denoted by C , is a multivariate joint distribution function whose marginals are all **Uniform (0,1)**

- **Sklar's theorem (essense):**

Let $X_i, i=1,...,m$ be random variables with marginal distribution functions $F_i(x_i),\ i=1,...,m$, marginal density functions $f_i(x_i),\ i=1,...,m$ and let $H(X_1,...,X_m)$ be their joint distribution function. Then there exists a Copula such that

$$H(X_1,...,X_m) = C_{X_1 \cdots X_m}\left(F_1(X_1),...,F_m(X_m)\right)$$

- The **joint density** can then be written

$$h(x_1,...,x_m) = c_{X_1 \cdots X_m}\left(F_1(x_1),...,F_m(x_m)\right) \cdot \prod_{i=1}^{m} f_i(x_i)$$

where $c_{X_1 \cdots X_m}\left(F_1(x_1),...,F_m(x_m)\right)$ is the **copula *density, capturing all of dependence.***

Maximum Likelihood with a copula

The joint density of the regressors and the error term can therefore be written

$$h\left(x_1,\dots,x_m,\varepsilon\right)=c_{X_1\cdots X_m}\left(F_1\left(x_1\right),\dots,F_m\left(x_m\right),F_\varepsilon\left(\varepsilon\right)\right)\cdot\prod_{i=1}^{m} f_i\left(x_i\right)\cdot f_e\left(\varepsilon\right)$$

The marginals of the regressors can be ignored in the log-likelihood, no parameters of interest there. So , the effective likelihood per observation is

$$\ln\ell=\ln c\left(F_1\left(x_1\right),\dots,F_m\left(x_m\right),F_\varepsilon\left(\varepsilon\right)\right)\; + \;\ln f_e\left(\varepsilon\right)$$

The Gaussian copula and the Log-likelihood

We will use the **Gaussian copula density** which is

$$\ln c_i^G = -\frac{1}{2}\ln\det(\mathrm{R}) - \frac{1}{2}\mathbf{q}_i'\left(\mathrm{R}^{-1} - I_{m+1}\right)\mathbf{q}_i$$

Where **R** is a **correlation** matrix, and **q** contains the transformed regressors

$$\Phi^{-1}\left(\hat{F}_i\left(x_i\right)\right)$$

and the transformed CDF of the error term. After one more simplification, the log-likelihood becomes

$$\breve{L} = -\frac{n}{2}\ln\det\left(\breve{\mathrm{R}}\right) - \frac{1}{2}\sum_{i=1}^{n}\mathbf{q}_i'\breve{\mathrm{R}}^{-1}\mathbf{q}_i + \frac{1}{2}\sum_{i=1}^{n}\left[\Phi^{-1}\left(F_\varepsilon\left(y_i - \mathbf{x}_i'\beta;\boldsymbol{\theta}\right)\right)\right]^2$$
$$+\sum_{i=1}^{n}\ln f_\varepsilon\left(y_i - \mathbf{x}_i'\beta;\boldsymbol{\theta}\right)$$

An empirical application

DATA: Koop and Tobias (2004), using the regression specification of Tsionas (2012)

- National Longitudinal Survey of Youth (NLSY), USA.
- 2,178 white males 17,919 observations (panel)
- 1979-1993 (fifteen years),
- Tsionas (2012) specification: data pooling.
- **Regressors :** Education (in years) and its square, "potential experience" and its square, a time-invariant measure of "ability", a constant term, a deterministic time trend and its square.
- **Dependent variable** : log hourly wage in 1993-real terms, (semi-log specification).
- Regressors to be included in the copula density : Education, Potential Experience, Ability.

An empirical application

Regressor	OLS	Ind. Exp. 2TSF	Corr. Exp. 2TSF	Corr. Exp. 2TSF with Copula
constant	0.3356	0.3363	0.1980	-0.0087
	(0.1168)	(0.1139)	(0.0976)	(0.1480)
Residual Skewness	-0.4638	-0.4773	-0.4763	-0.5087
Residual Excess Kurtosis	1.5552	1.5808	1.5798	1.6062
σ_{ε}	0.4722	0.4716	0.4728	0.4748

Estimates are rounded to 4th decimal. Heteroskedasticity robust standard errors (HC2) are provided in parentheses.

An empirical application

- The regression coefficient estimates (not shown) are close in all cases (not identical), and estimated with high accuracy.
- The variance of the composite error term is virtually the same
- So it appears that OLS is consistent except for the constant term.
- **BUT**
- Skewness and Excess Kurtosis point towards the use of a 2TSF specification.
- The benchmark **Independence** 2TSF Exponential Specification **or** the 2TSF **Correlated** Specification**?**

Parameter	Corr. Exp. 2TSF	Corr. Exp. 2TSF with Copula
a'	4.7140	4.6355
b'	2.8857	2.9226
m	0.3104	0.36478

$$mb' = 0.89 \neq \neq 3.25 = (1-m)a'$$

- So **Correlated** specification (consistent with the theoretical model)

Regressor endogeneity

	Corr. Exp. 2TSF with Copula	Std error
$\hat{\text{C}}\text{orr}\left(\text{Education}, \, \varepsilon\right)$	-0.0232	(0.0157)
$\hat{\text{C}}\text{orr}\left(\text{Potential Exper}, \, \varepsilon\right)$	−0.0802***	(0.0298)
$\hat{\text{C}}\text{orr}\left(\text{Ability}, \, \varepsilon\right)$	−0.0047	(0.0250)

- The "Potential Experience" regressor appears endogenous (a little)
- **This makes sense**: it is "Actual Experience" measured with error – and the error resides in the composite error term of the regression, creating correlation.
- So OLS will be inconsistent (a little).

An empirical application

- **So we adopt the 2TSF Correlated Exponential Specification with a Copula**
- **What can we learn about our sample?**

Average net effect of bargaining performance	$E\left(\exp\{w-u\}\right)$	+8,17%	On initial common-information expected output $E\left(p \middle| I_f \cap I_e\right)$
Average total effect of Negotiation process	$E\left(\exp\{v+w-u\}\right)$	+12,65%	On initial common-information expected output $E\left(p \middle| I_f \cap I_e\right)$

- **It appears that on average the negotiation process benefits the workers, they do get some of their self-evaluation premium.**

An empirical application

- **REALITY CHECK : Going down to transaction level:**

1993 USD (levels) Conditional mean values	$\hat{\varepsilon} < 0$ $n = 8229$	$\hat{\varepsilon} > 0$ $n = 9690$
$\hat{E}\left(p \middle\| I_f \cap I_e, \varepsilon\right)$	10.37	10.06
Actual Hourly wage	7.39	14.81
%Δ	-28.7%	+47.2%

- **No systematic relation between “typical credentials” and wage outcomes.**

An empirical application

- **Temporal evolution**

A downward trend in the Effect of the Negotiation process

Year	n	$\hat{E}\left(p \mid I_f \cap I_e\right)$ (1993 USD)	$\hat{E}\left(\exp\{w_i - u_i\} \mid \varepsilon_i\right)$ (Gross mark-up)	$\exp\{\hat{\varepsilon}_i\}$ (Gross mark-up)	Actual hourly wage (1993 USD)
1979	454	7.40	1.11	1.19	8.81
1980	615	7.51	1.13	1.21	9.02
1981	745	7.73	1.12	1.19	9.17
1982	964	7.94	1.12	1.21	9.49
1983	1034	8.16	1.10	1.17	9.45
1984	1093	8.57	1.04	1.06	9.03
1985	1248	8.97	1.06	1.09	9.68
1986	1284	9.47	1.08	1.11	10.47
1987	1398	9.93	1.11	1.16	11.42
1988	1443	10.44	1.12	1.16	12.06
1989	1520	10.90	1.11	1.13	12.30
1990	1535	11.42	1.12	1.13	12.92
1991	1570	11.92	1.10	1.10	13.23
1992	1517	12.45	1.08	1.06	13.08
1993	1499	13.11	1.10	1.08	14.08

An empirical application

Percentage of transactions with wage BELOW ABOVE common-information expected output

Year	$\exp\{\varepsilon_i\}<1$	$\exp\{\varepsilon_i\}>1$
1979	40%	$\boxed{60\%}$
1980	38%	62%
1981	39%	61%
1982	39%	61%
1983	42%	58%
1984	52%	48%
1985	49%	51%
1986	47%	53%
1987	44%	56%
1988	43%	57%
1989	44%	56%
1990	46%	54%
1991	48%	52%
1992	51%	49%
1993	52%	$\boxed{48\%}$

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THANK YOU !