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# HYBRID OPTIMIZATION METHODOLOGIES FOR TRANSPORTATION LOGISTICS MANAGEMENT – HybOpt

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# Optimization Methodologies for the Family Capacitated Vehicle Routing Problem

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# Outline

- Literature Review
- Problem Formulation
- Real-Life Scenario
- Methodologies
- Computational Results
- Discussion on Components

# The F-CVRP

- Introduced by Bernardino and Paias (2024).
- Extension of the Family Travelling Salesperson Problem.
- Imposes family-based constraints to the classical Capacitated Vehicle Routing Problem.

Families:

- Disjoint sets of customers
- Customers of the same family have the same demand
- Each family has a required number of visits

# Key Characteristics

- Customer Clusters
  - Vehicles are allowed to serve customers from different clusters in any sequence without restriction (soft clustering)
  - Relevant problem: SoftCluVRP *(Defryn and Sorensen 2017)*
- Selective Service
  - Not all customers need to be serviced
  - Relevant problems: OP (*Tsiligirides1984*), TOP (*Butt and Cavalier 1994*)

**Clustering + Selective Service**

- COP (*Angelelli et al. 2014*)
- SOP (*Archetti et al. 2018*)
- STOP (*Nguyen et al. 2025*)

# F-CVRP formulation

- $N = \{1, 2, \ldots n\}$ set of customer nodes
- $K$ identical vehicles
- $Q$ capacity
- $L = \{0, 1, \ldots L\}$ set of families
- $\forall\, l \in L$:
  - $F_l \subseteq N$ set of customer members
  - $d_l$ product demand
  - $v_l$ number of required visits

# F-CVRP formulation (2)

- $G = (V, A)$ directed weighted graph
- $V = \{0\} \cup N$ complete node set
- $A = \{(i, j) \mid i, j \in V, i \neq j\}$ arc set
- Each $(i, j) \in A$ associated with travel cost $c_{ij}$
- $x_{ij}^k = 1$ iff vehicle *k* travels from *i* to *j*
- $y_i = 1$ iff customer *i* is visited

# F-CVRP formulation (3)

**Objective:**

$$\min \sum_{k=1}^{K} \sum_{(i,j)\in A} c_{ij} x_{ij}^{k}$$

**Constraints:**

$$\sum_{k=1}^{K} \sum_{(i,j)\in A} x_{ij}^{k} = y_i, \quad i \in N \tag{2}$$

$$\sum_{k=1}^{K} \sum_{(i,j)\in A} x_{ij}^{k} = y_j, \quad j \in N \tag{3}$$

$$\sum_{j\in N} x_{0j}^{k} = 1, \quad \forall k \in \{1, 2, \ldots, K\} \tag{4}$$

$$\sum_{i\in N} x_{i0}^{k} = 1, \quad \forall k \in \{1, 2, \ldots, K\} \tag{5}$$

$$\sum_{k=1}^{K} \sum_{j\in N} x_{0j}^{k} = K \tag{6}$$

$$\sum_{i,j\in N} d_i x_{ij}^{k} \leq Q, \quad \forall k \in \{1, 2, \ldots, K\} \tag{7}$$

$$\sum_{i\in F_l} y_i = v_l, \quad \forall l \in L \tag{8}$$

$$u_j \geq u_i + 1 - n \cdot (1 - x_{ij}^{k}), \quad \forall i, j \in N, i \neq j, \forall k \in \{1, 2, \ldots, K\} \tag{9}$$

$$x_{ij}^{k} \in \{0, 1\}, \forall (i, j) \in A \tag{10}$$

$$y_i \in \{0, 1\}, \forall i \in N \tag{11}$$

$$1 \leq u_i \leq n, \forall i \in N \tag{12}$$

$$u_0 = 0 \tag{13}$$

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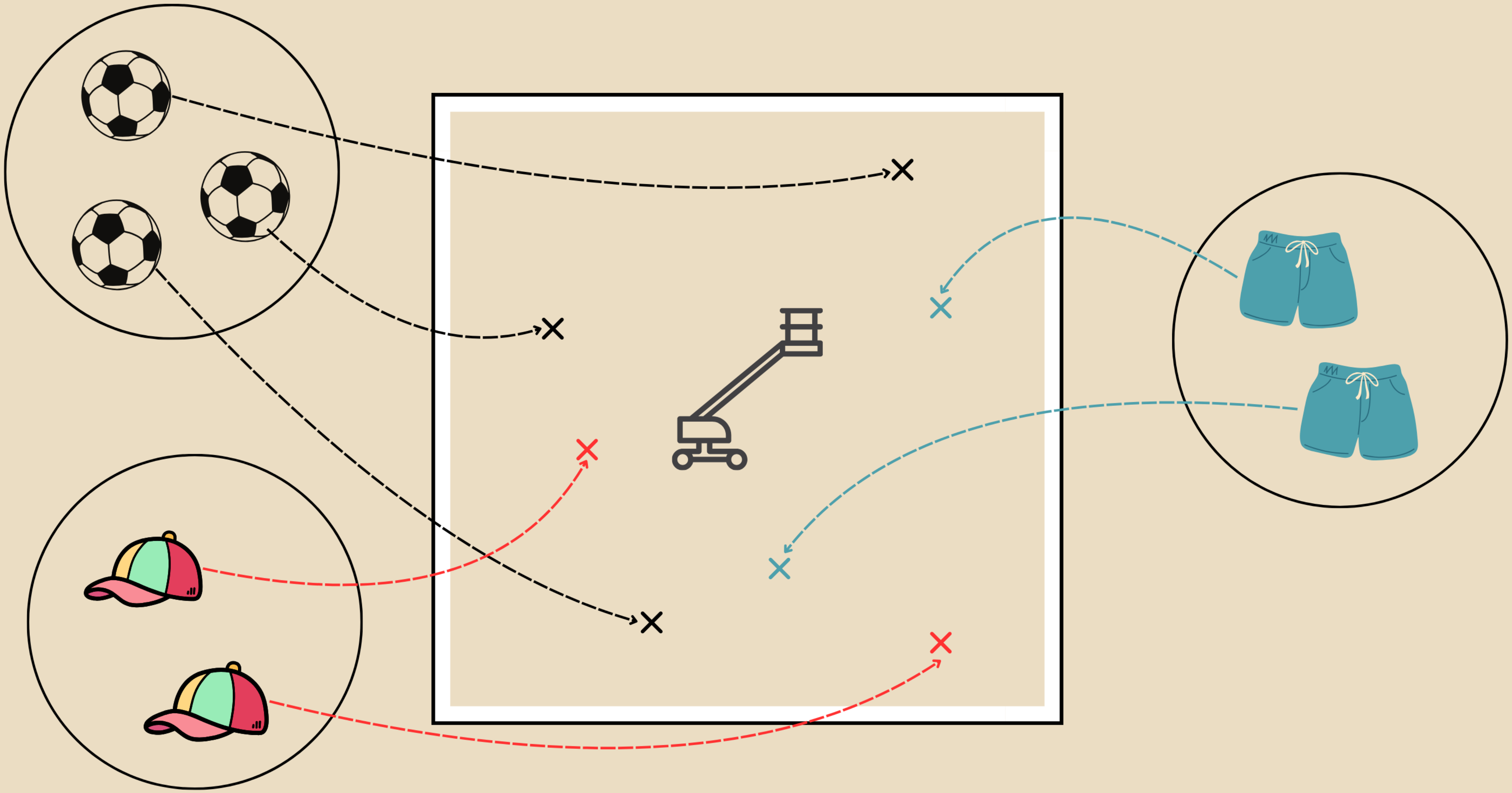
# Warehouse Order Picking



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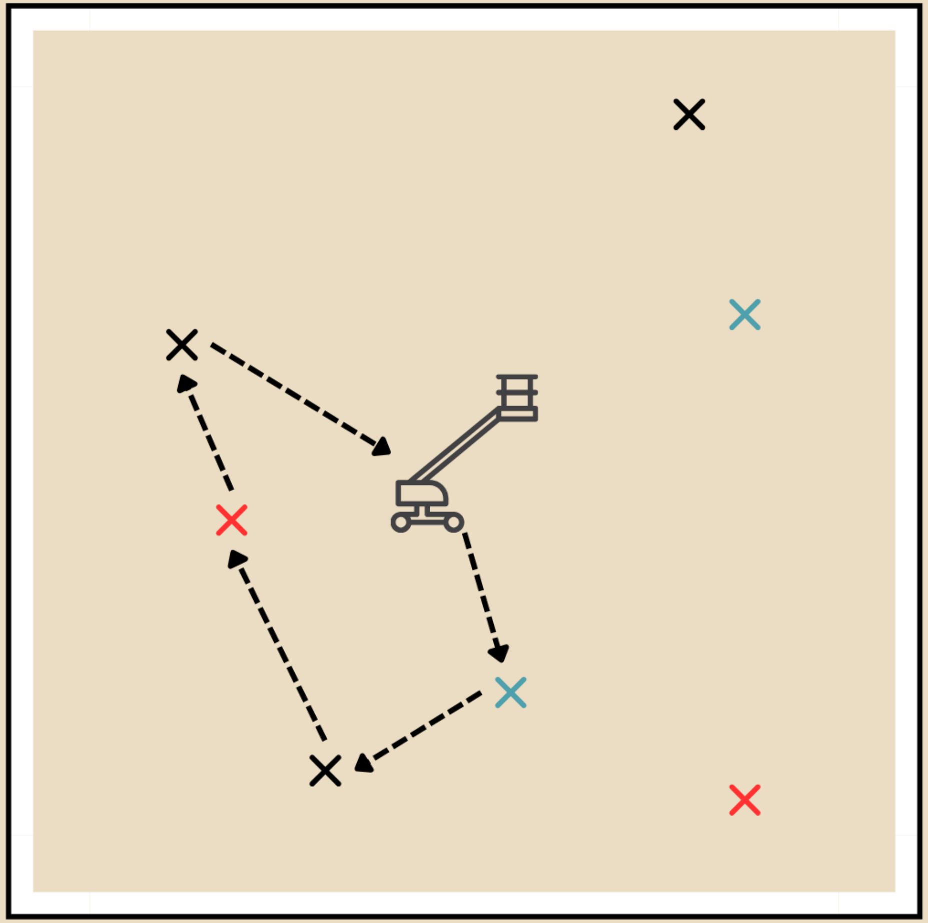
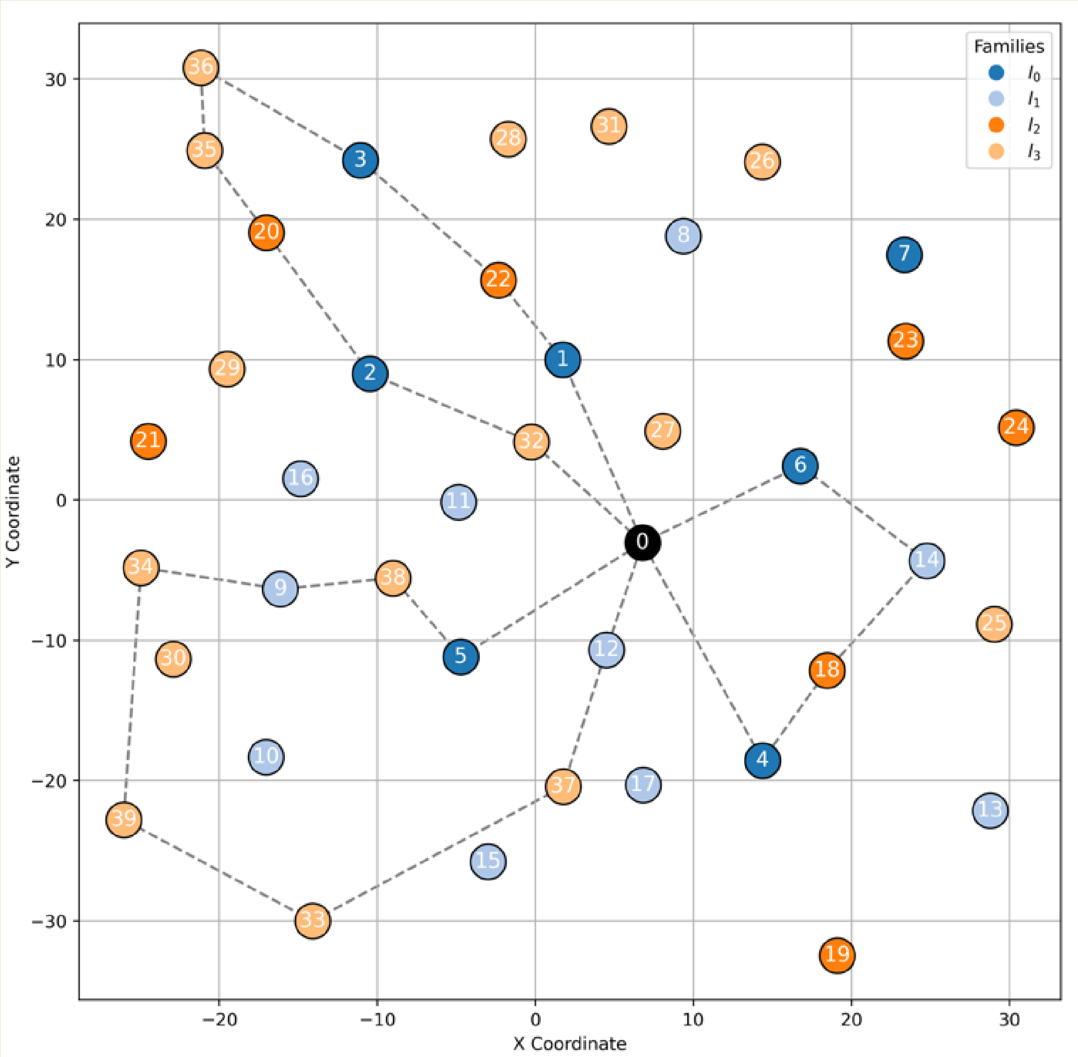
# Warehouse Order Picking



Order

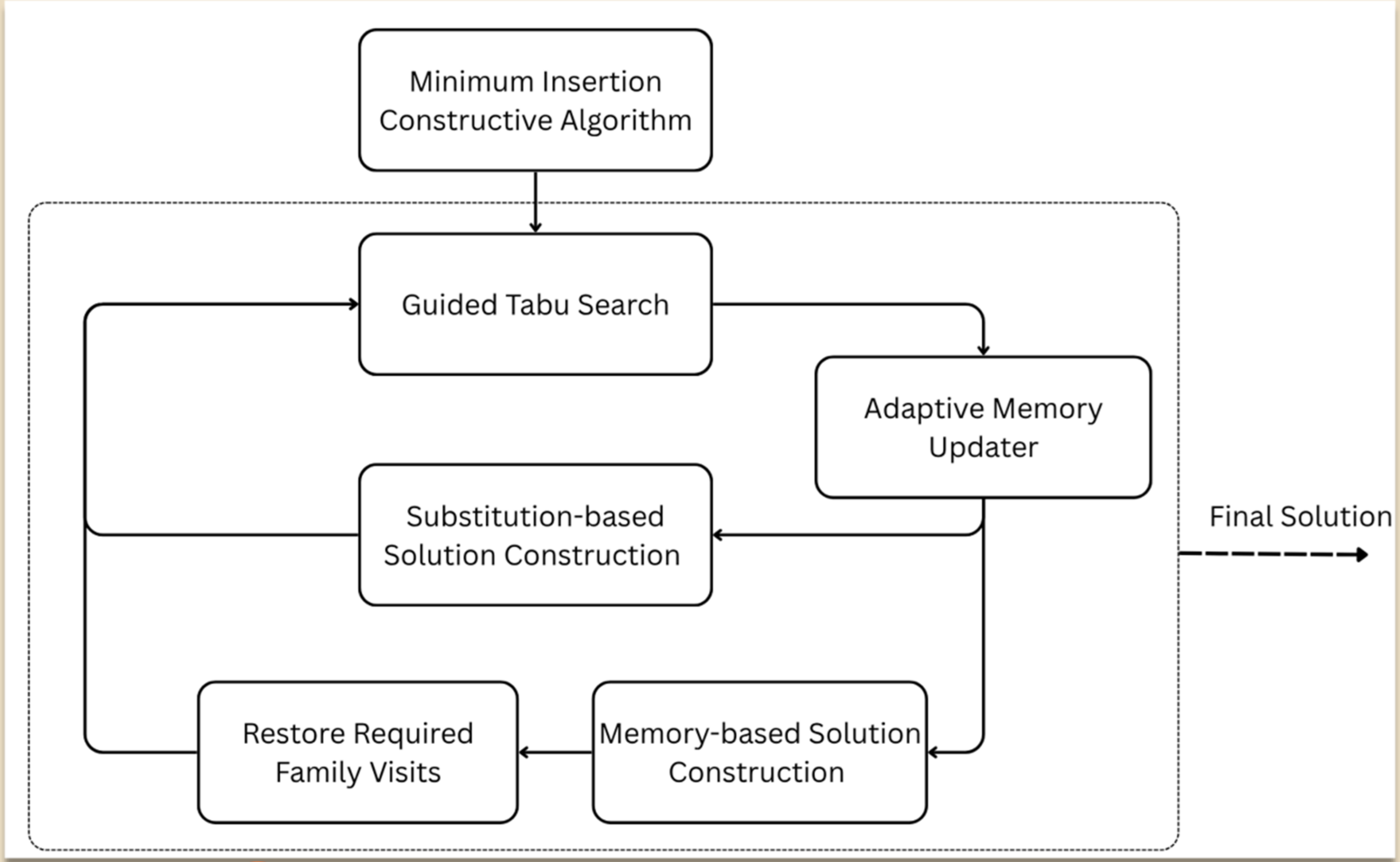
# Sample F-CVRP Solution




| $l$ | $F_l$ | $v_l$ | $d_l$ |
|---|---|---|---|
| 0 | {1,..,7} | 6 | 17 |
| 1 | {8,..,17} | 3 | 16 |
| 2 | {18,..,31} | 3 | 16 |
| 3 | {32,..,39} | 8 | 16 |

$$K = 3$$
$$Q = 140$$

# Algorithmic Components



# Discussion on Components

Initial implementation with Tabu Search

Experimentation with Guided Local Search

GLS converged faster towards global optima in larger instances but was overly restrictive in smaller ones

Solution: create a hybrid GTS to leverage the strengths of both approaches

# Tabu Policy

Tabu list forbids previously deleted edges from being reinserted in future iterations (Glover 1986).

Aspiration Criterion:

- $\forall a \in A_{rem}$, where $A_{rem}$ is the set of arcs removed by a move, assign $p_a \leftarrow Z(S)$
- On subsequent iterations moves that would introduce an arc set $A_{int}$ to form a modified solution $S_{mod}$ are prohibited, unless $Z(S_{mod}) < p_a, \forall a \in A_{int}$
- Periodically reinitialize $p_a \leftarrow +\infty, \forall a \in A$ to avoid an overly restrictive behavior

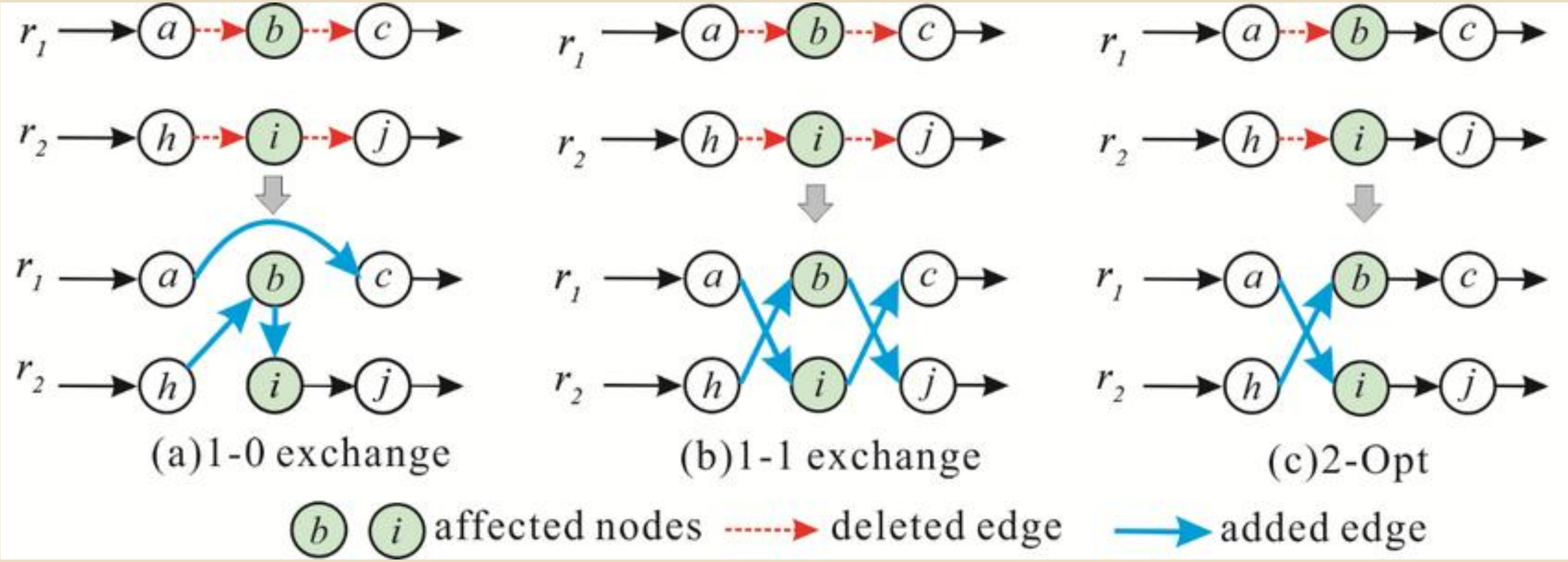
# Guided Tabu Search

- Implements tabu policy and an objective function modification strategy based on Guided Local Search (Voudouris and Tsang 1996).
- Use penalized cost matrix with probability $g$
- Penalize most costly arcs based on the formula:

$$c^*_{i,j} \leftarrow (1 + \lambda \cdot p_{i,j}) \cdot \ c_{i,j}$$

# Local Search Operators

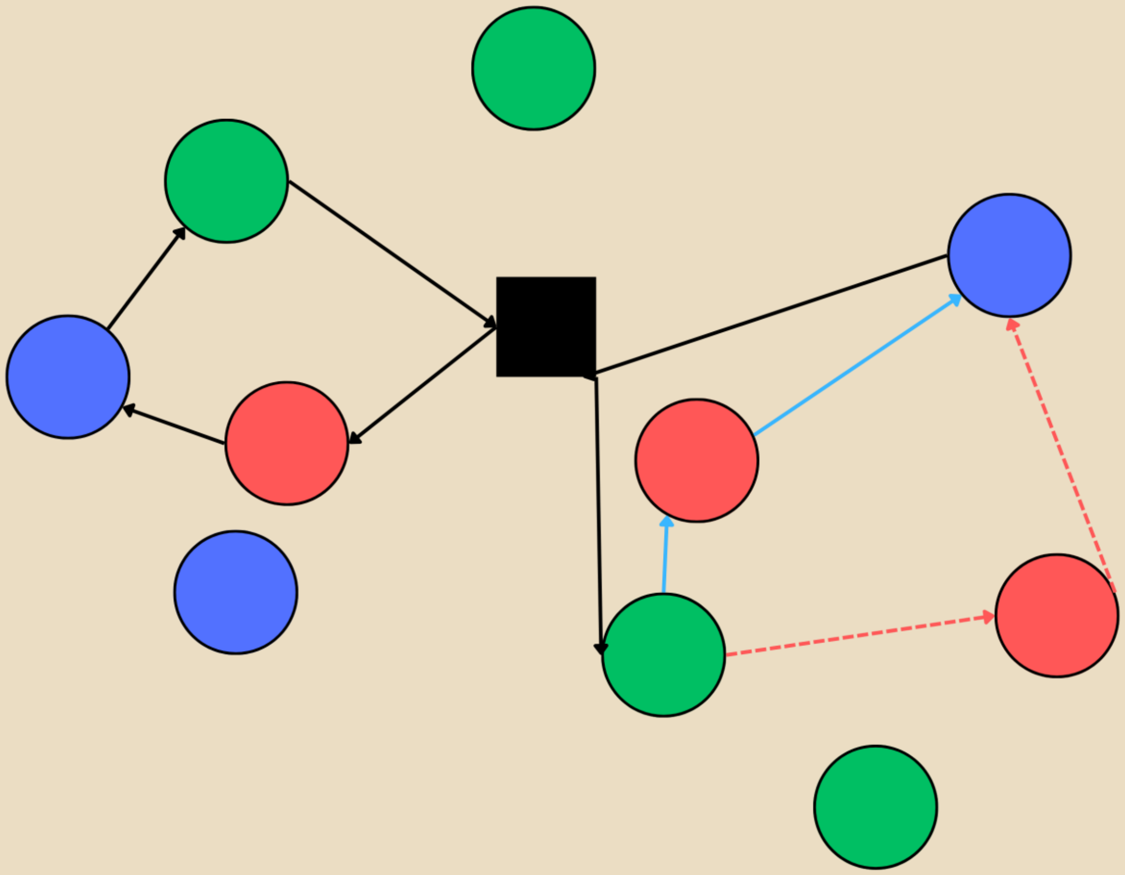
Classical VRP operators:



# Local Search Operators (2)

In-Out Exchange

- Family specific operator
- Substitutes customer in *S* with customer not in *S* belonging to the same family



# Solution Reconstruction Strategies

Memory-based

- Uses routes stored in an Adaptive Memory structure as building blocks for new solution

Substitution-based

- Makes multiple customer substitution operations by solving IP model

A mix of both strategies was found to be the most beneficial

# Memory-based Solution Construction

Adaptive Memory stores routes, with their associated $Z(S)$

Set maximum $poolSize$ for memory

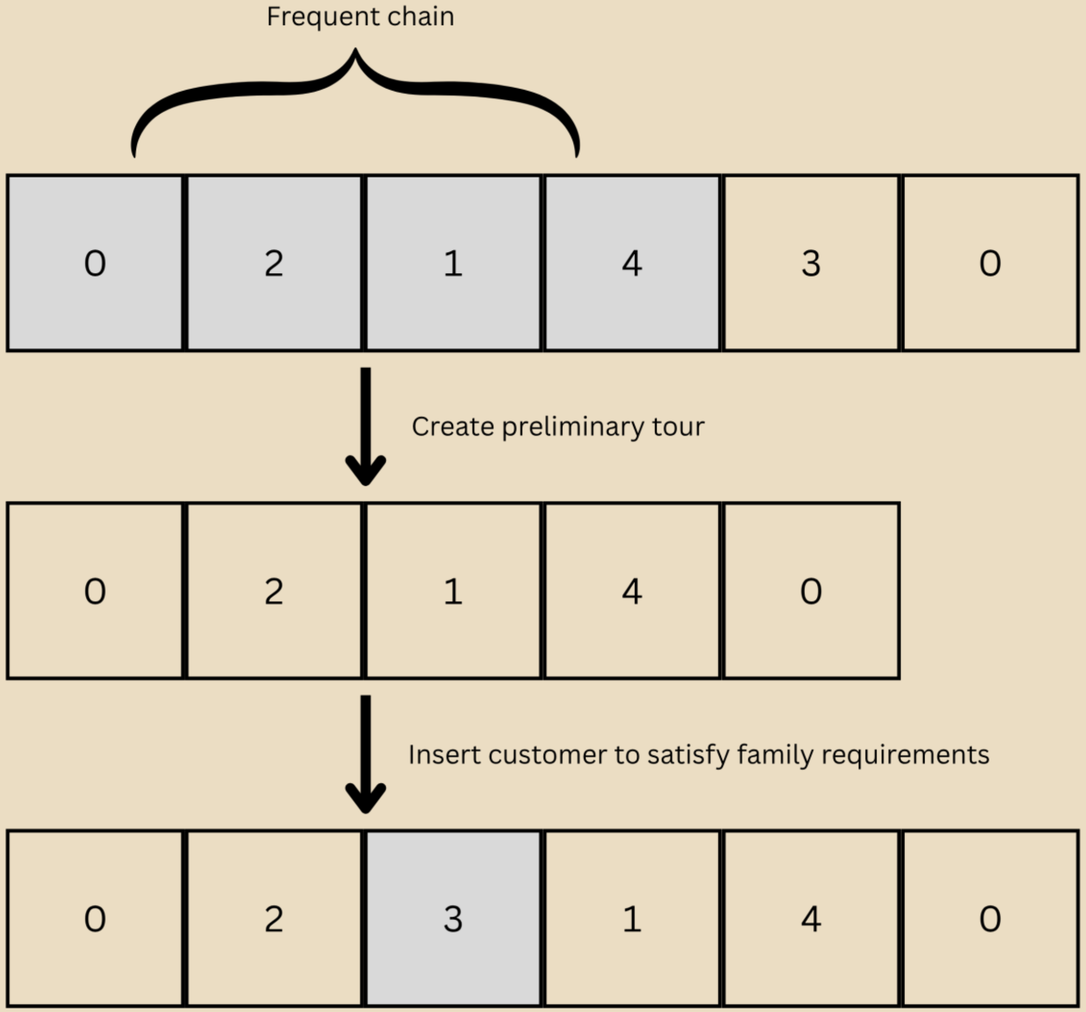
New solutions produced by GTS update the AM contents

Existing routes have their $Z(S)$ updated if necessary

New routes are added

# Memory-based Solution Construction (2)

- Routes are comprised of variable length node sequences or chains
- Chain characteristics:
  - vertex length
  - frequency
  - number of previous selections
- Selection criterion:
  - Length
  - Frequency minus previous selections



# Memory-based Solution Construction (3)

- Restore family requirements by solving an IP model
- Let:
  - $add_{i,k} = 1$ iff node *i* is to be inserted to route *k*
  - $c_{i,k}$: the minimum insertion cost of $i$ into route $k$
  - $req_l$: the number of customers from family *l* that need to be inserted to satisfy its requirements
  - $D_k$: the total load of vehicle *k*
  - $P$: the set of customers currently in the solution

Objective: $$\min \sum_{k=1}^{K} \sum_{i \in P'} c_{i,k} \cdot add_{i,k}$$

Constraints: $$\sum_{k=1}^{K} \sum_{i \in P'} add_{i,k} \leq 1$$

$$D_k + \sum_{i \in P'} d_i \cdot add_{i,k} \leq Q, \forall k \in \{1, 2, ..., K\}$$

$$\sum_{k=1}^{K} \sum_{i \in F_l \cap P'} add_{i,k} = req_l, \forall l \in L$$

# Substitution-based Solution Construction

- Performs multiple simultaneous customer substitutions on incumbent solution $S^*$ by solving IP model
- Let:
  - $s_{i,j} = 1$ iff node $i \in P$ is substituted by node $j \in P'$
  - $subc_{i,j}$: the substitution cost of *i* with *j*
  - $r_i = 1$ iff node $i$ is removed from the solution
  - $a_j = 1$ iff node $j$ is added to the solution
  - $maxSub$: the maximum number of substitutions

# Substitution-based Solution Construction (2)

Objective:

$$min \sum_{i \in P} \sum_{j \in P'} subc_{i,j} \cdot s_{i,j}$$

Constraints:

$$D_k + \sum_{i \in P} \sum_{j \in P'} (d_j - d_i) \cdot s_{i,j} \leq Q, \quad \forall k \in \{1, 2, ..., K\}$$

$$\sum_{i \in F_l} r_i = \sum_{j \in F_l} a_j, \forall l \in L$$

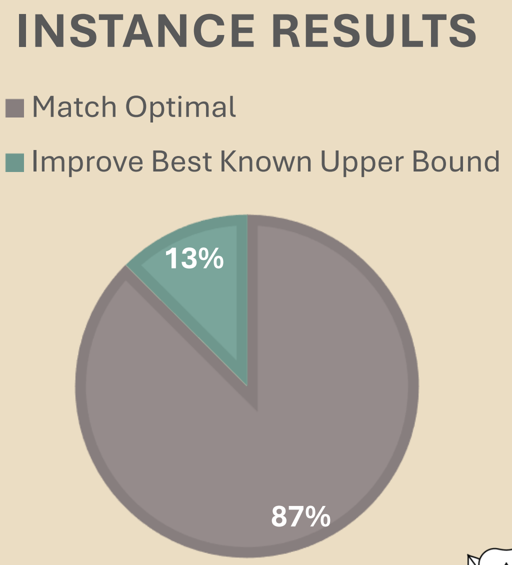
$$\sum_{i \in P} s_{i,j} = a_j, \forall j \in P'$$

$$\sum_{j \in P'} s_{i,j} = r_i, \forall i \in P$$

$$0.75 \cdot maxSub \leq \sum_{i \in P} \sum_{j \in P'} s_{i,j} \leq maxSub$$

# Computational Results

- Methodologies benchmarked against 384 previously published instances
- Small to medium sized (15-100 customers)
- Average % improvement w.r.t. previous best-known results:  0.13%
- New data set was generated with increased number of customers



# Computational Results (2)

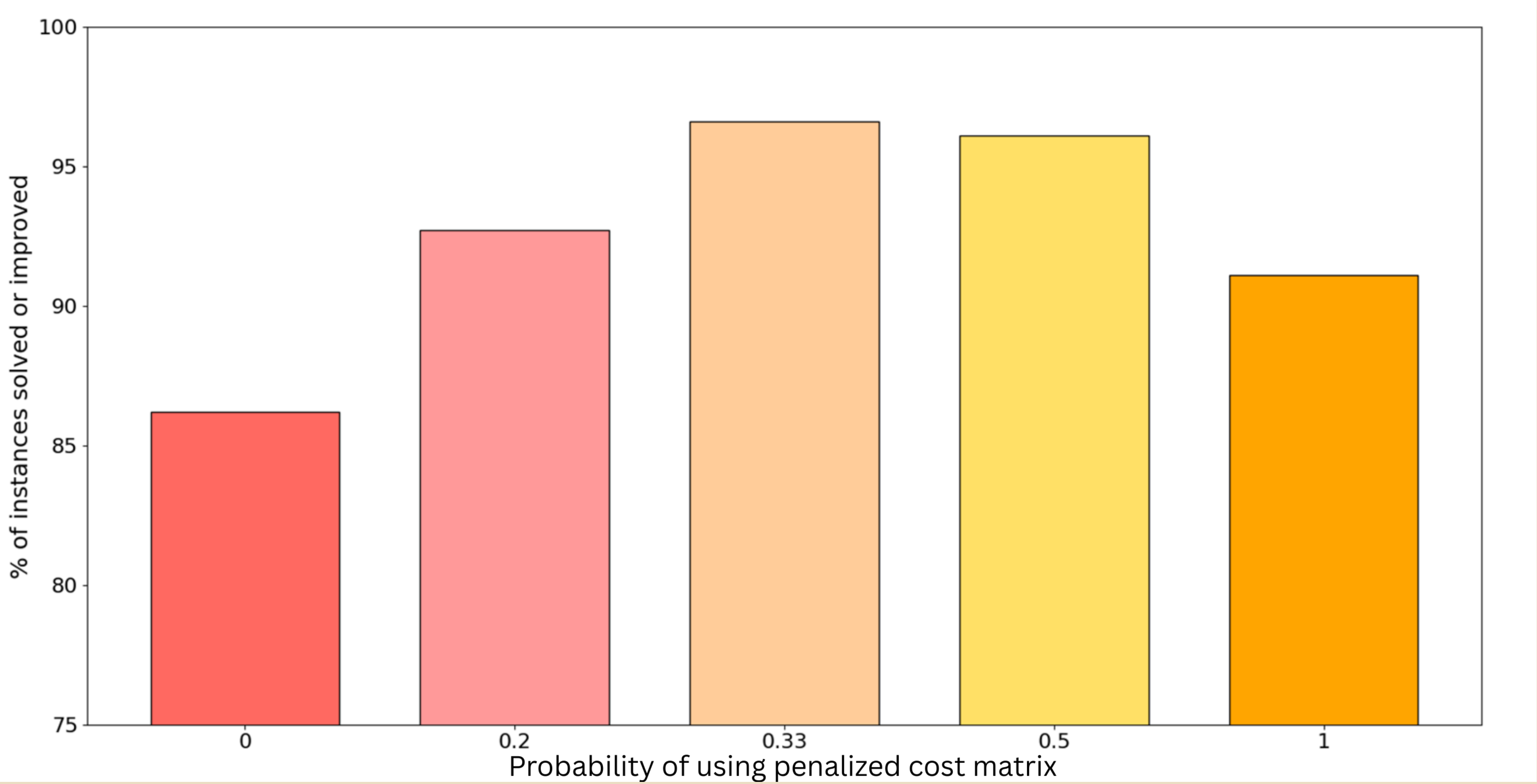
| $\lvert N \rvert$ | $\#Solved$ | $\%Gap_{avg}$ | $t_{avg}(s)$ |
|---|---|---|---|
| 15 | 12/12 | 0.0 | 0 |
| 18 | 12/12 | 0.0 | 0 |
| 19 | 10/10 | 0.0 | 1 |
| 20 | 12/12 | 0.0 | 1 |
| 21 | 24/24 | 0.0 | 1 |
| 22 | 12/12 | 0.0 | 1 |
| 39 | 12/12 | 0.0 | 19 |
| 44 | 12/12 | 0.0 | 28 |
| 49 | 36/36 | 0.0 | 39 |
| 50 | 12/12 | -0.18 | 43 |
| 54 | 48/48 | -0.14 | 71 |
| 59 | 24/24 | -0.36 | 116 |
| 64 | 12/12 | -1.1 | 198 |
| 69 | 12/12 | -0.7 | 261 |
| 75 | 24/24 | -0.26 | 395 |
| 100 | 12/12 | -0.58 | 1440 |

Symmetric instance results

| $\lvert N \rvert$ | $\#Solved$ | $\%Gap_{avg}$ | $t_{avg}(s)$ |
|---|---|---|---|
| 33 | 12/12 | 0.0 | 6 |
| 35 | 12/12 | 0.0 | 10 |
| 38 | 12/12 | 0.0 | 13 |
| 44 | 12/12 | 0.0 | 22 |
| 47 | 12/12 | 0.0 | 35 |
| 55 | 12/12 | 0.0 | 69 |
| 64 | 12/12 | 0.0 | 219 |
| 70 | 12/12 | 0.0 | 243 |

Asymmetric instance results

# Contribution of GLS

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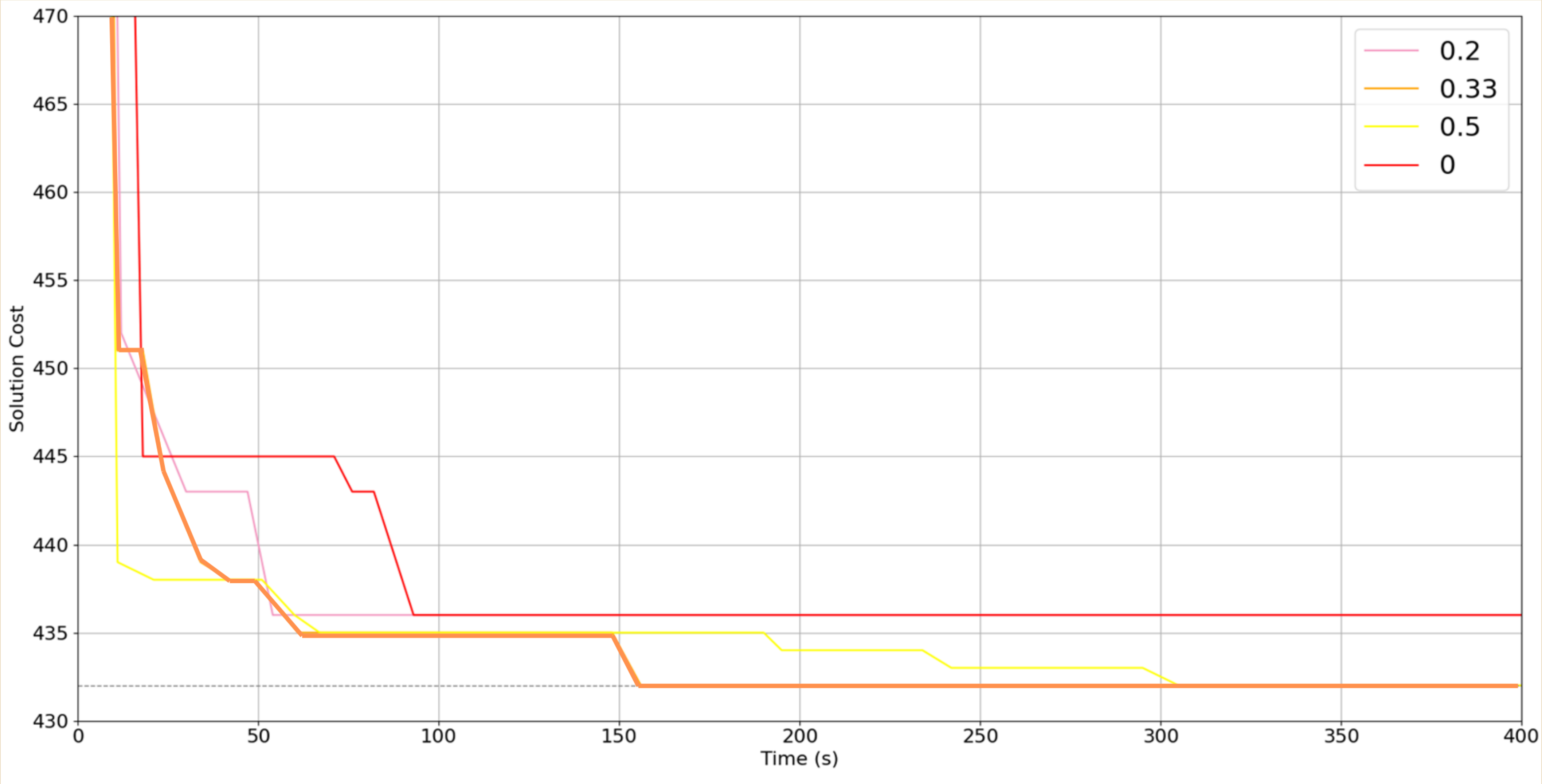
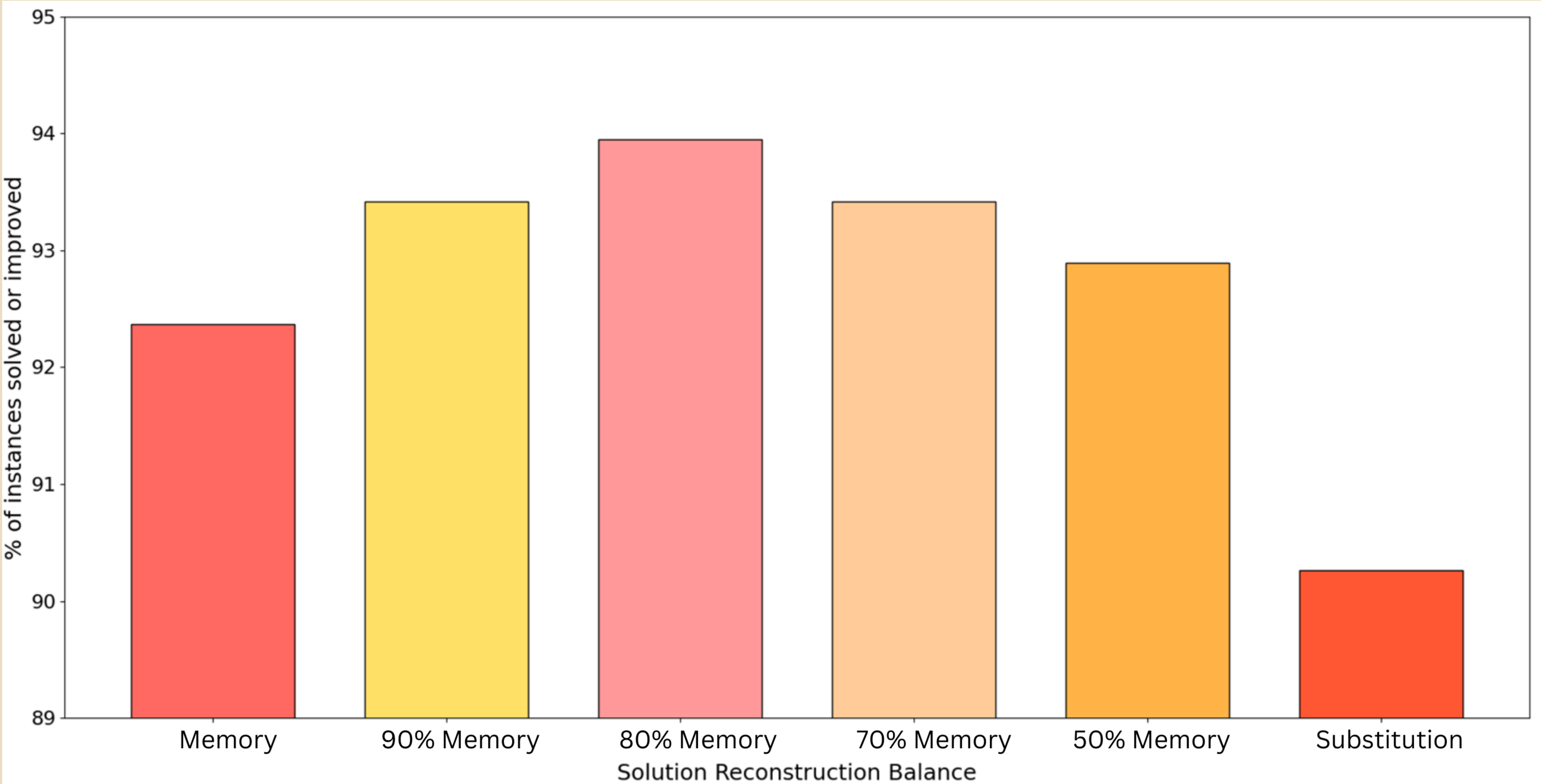
# Contribution of GLS (2)



# Solution Reconstruction Balance



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# Thank you for your attention!

Questions?

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