Asset allocation in the Athens Stock Exchange:
A variance sensitivity analysis

by

Panayiotis F. Diamandis[1], Anastassios A. Drakos[1]
Georgios P. Kouretas[1,2*] and Leonidas P. Zarangas[3]

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Abstract

This paper provides an analysis of asset allocation using univariate portfolio GARCH models applied on daily data for the period January 1999 to December 2009 on stocks traded in the Athens Stock Exchange, a recently monitored emerging market. Our analysis adopts the variance sensitivity analysis methodology due to Manganelli (2004) and we are able to recover from the univariate approach the multivariate dimension of the portfolio allocation problem. The main results of the analysis are: First, we demonstrate that using a two asset portfolio consisting of blue chips traded in the Greek capital market the estimated variance is a parabolic and convex function of the estimated weights providing evidence that diversification produces significant gains in terms of risk reduction. Second, based on the shape of the first and second derivatives the model misspecification due to the fitting univariate GARCH models is insignificant. Third, we compare the performance of variance sensitivity analysis against that of three popular multivariate GARCH models and it is shown that the adopted methodology provides more efficient results than the competing models. The gains in efficiency get larger as the size of the portfolio increases. Finally, with the application of the Kupiec's test for out-of-sample forecasting performance we demonstrate that the variance sensitivity analysis outperforms all three alternative models at both the 95% and 99% confidence interval independently of the trading position.

Keywords: asset allocation, GARCH models, risk management, sensitivity analysis, Kupiec test

JEL classification: C53, G21; G28

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[1] Department of Business Administration, Athens University of Economics and Business, 76 Patission Street, GR-10434, Athens, Greece.
[2] Centre of International Business and Management, Cambridge Judge Business School, Trumpington Street, Cambridge CB2 1AG, United Kingdom.
[3] Department of Finance and Auditing, Technological Educational Institute of Epirus, 210 Ioanninon Ave., Psathaki, GR-48100, Preveza, Greece.
*corresponding author, email: kouretas@aueb.gr

1. Introduction

The recent financial turmoil of 2007-2009 has brought to the surface once again the need for appropriate estimation of asset allocation given the increase volatility of the prices, derivatives and other financial instruments as well as the implementation and use for evaluating the trading positions, capital adequacy and value-at-risk measures according to the Basle II agreement. During the past two decades, volatility in financial markets and their models and forecasts have attracted growing attention by academic researchers, policy makers and practitioners whereas at the same time several issues has been raised related to the development of theoretical models of asset returns volatility and their applications to real world asset allocation problems.

The Basel Committee of Banking Supervision through the 1988 Basel Accord and the 1996 Amendment of the Basel Accord (or Basel II) which is in force since November 2007 has set the regulation framework for the world financial system. There are three main tools available to regulators for the measurement and control of financial risk, namely minimum risk capital requirements; inspections and reporting requirements and public disclosure and market discipline. Risk management is mainly linked with the minimum risk capital requirements which are imposed by the regulatory body. The Basel Committee currently recommends two types of models for measuring market risk on a daily basis, with VaR being the most popular one.

The enormous growth of trading activity that has been observed during the recent years in both the developed but mainly in the emerging markets has led the financial regulators and supervisory committees to seek well-justified methods to quantify the risk[1]. The importance of financial risk management has significantly increased since the mid-1970s, which saw both the collapse of the fixed exchange rate system and two oil price crises. These major events led to considerable volatility in the capital markets, which together with the emergence of the derivatives market, increased trading volumes and technological advances, led to increasing concerns about the effective measurement and management of financial risk. This need was further reinforced by a number of financial crises that took place in the 1980s; in the 1990s and in the 2000s such as the worldwide stock markets collapse in 1987, the Mexican crisis in 1995, the Asian and Russian financial crises in 1997-1998 as well as the Orange County default, the Barings Bank and Long Term Capital Management bankruptcy cases, the dot.com bubble and certainly the credit and financial crisis of 2007-2009.

The increased financial uncertainty led to a rise of the likelihood for financial institutions to suffer substantial losses as a result of their exposure to unpredictable market changes as it has been made evident once again during the current financial crisis that already led several major financial institutions to bankruptcy in the U.S. and the U.K. These events have made investors become more cautious in their investment decisions, while it has also led to an increased need for more careful study of price volatility in stock markets.

A stylized fact that is well documented in the literature is that stock returns for mature and emerging stock markets behave as martingale processes with leptokurtic distributions and conditionally heteroskedastic errors. Furthermore, these data exhibit volatility across time and the unconditional variance is constant even though the conditional variance during some periods is unusually large. Therefore, it is argued that estimation methods that use conditional variances are more appropriate

[1] For a detailed analysis see the Basel Committee on Banking Supervision’s (1996a, b), Duffie and Pan (1997), Jorion (2000), Alexander (2005, 2008) and Drzik (2005) provide a comprehensive overview of value at risk measures.

for this type of data, as the heteroscedasticity in the disturbances biases the test statistics, leading to incorrect inferences. In the presence of heteroscedasticity, the estimators themselves are no longer efficient and hence for the purpose of forecasting return series, more accurate intervals can be obtained by modeling volatility of returns.

The estimation of multivariate GARCH models, initially proposed by Bollerslev *et al*. (1988), is an extensively used approach for risk management. However, such estimation is very demanding because we are required to estimate a large number of parameters whose number increases exponentially as the number of variables rises.[2] Recently, Manganelli *et al*. (2002) and Manganelli (2004) have developed an approach which provides a solution to the multivariate problem with the GARCH estimation. This approach is based on the estimation of univariate portfolio models and then with the use of certain statistical tools we are able to recover the multivariate dimension which is lost in the estimation of the univariate models. The main reason for using this methodology is that it considers not only the portfolio returns, but also the estimated parameters of the univariate GARCH model as a function of the weights of the assets that form the portfolio. The next step of this approach is to take the first and second derivatives of the variance subject to these weights. This will enable us to deduct important information with respect to the local behaviour (i.e. around the portfolio weights) of the estimated variance.

This paper focuses on the issue of asset allocation in a European emerging market, the Athens Stock Exchange (ASE) a capital market which has been characterized by substantial volatility during the last decade. Although this market has recently grown in size and has been upgraded to the mature market status it still

[2] See for example, Bollerslev *et al*. (1994), Engle and Kroener (1995) and McNeil and Frey (2000) for a detailed analysis.

exhibits features that are found in the emerging markets. The existence of a rational bubble in the period 1998-2000 has dominated its behaviour since then and it has been recently affected substantially by the financial crisis which is still unfolding and has recorded a loss of 75% in capital value during the 2007-2009 financial turmoil. This loss in capitalization has been mainly caused by the capital flight initiated by foreign institutional investors and hedge funds during the current financial, banking and credit crisis which in such an uncertain environment prefer to take positions in safer stock markets and currencies. This negative trend also reflected the recent economic and financial developments in the Greek economy due to its enormous increase in its public debt and the questions raised about its sustainability resulted to a rise of the spread of the Greek 10-year bond spread by 351 base points compared to the German bund in February 2010. It is evident therefore that it is of crucial importance to use appropriate models to obtain portfolio allocation and risk measurement especially for the case of emerging markets and during turbulent economic periods.

We employ daily data of 30 companies listed on the ASE for the period 3 January 2001 to 31 December 2009. We apply the sensitivity analysis proposed by Manganelli *et al*. (2002) and Manganelli (2004) and we provide an evaluation of this approach with the results obtained from the estimation of three alternative models, namely the Dynamic Conditional Correlation model (DCC), the Orthogonal GARCH model (OGARCH) and the Exponentially Weighted Moving Average (EWMA) model. For this application we first estimate the variance sensitivity of a portfolio with two assets traded on the ASE and then we estimate minimum variance portfolios for any given point of time. Finally, we applied the Kupiec (1995) test in order to evaluate the out-of sample performance of the competing models.

There are a number of important findings that stem from our analysis. First, it is shown that the use of sensitivity analysis for asset allocation in this emerging market provides a suitable measure for the diversification opportunities at any given point in time. Second, that in terms of the estimated minimum variance we demonstrated that the VSA model leads to substantial efficiency gains when it is compared to three other GARCH models. Finally, with the application of the Kupiec's test for out-of-sample forecasting performance we demonstrate that the variance sensitivity analysis outperforms all three alternative models at both the 95% and 99% confidence interval independently of the trading position.

The rest of the paper is organized as follows. Section 2 discusses the issues of asset allocation, risk management and the application of alternative GARCH specifications. In section 3 we present the key elements of the variance sensitivity analysis. Section 4 reports the empirical results. In section 5 we present the forecasting performance evaluation of the alternative model specifications with the summary and the concluding remarks provided in a final section.

2. Asset allocation, risk management and GARCH models

The application of multivariate models of conditional volatility has contributed significantly in solving problems which are linked with optimal asset allocation, risk management, derivative pricing and dynamic hedging. However, their empirical application has been surrounded with several difficulties especially in the case of portfolios with a large number of assets due to the fact that the number of parameters that need to be estimated increases exponentially as we move from the univariate framework. The overall evidence from the use of multivariate GARCH with a large number of assets is that they can be tractable only under highly restricted versions of the model. In such circumstances the likelihood of model misspecification can be substantial. When considering the case of risk management within this context it is usually the case that we should be concerned about the behaviour of the predictive density of the asset returns, rather than trying to obtain the model that best approximates the volatility.

Over the last two decades a number of variants of the multivariate GARCH have been developed. These include the conditional constant correlation (CCC) model introduced by Bollerslev (1990), the Riskmetrics specification (1996) proposed by J.P. Morgan and widely used by practitioners, the Orthogonal GARCH developed by Alexander and Chibumba (1995) and Alexander (2000) and the Dynamic Conditional Correlation (DCC) advanced by Engle and Sheppard (2001) and Engle (2002)[3]. As we already mentioned the multivariate volatility models are highly restricted by construction and this could lead to substantial model uncertainty. As a consequence, given standard data limitations and technical considerations it has been long recognized that we are unable to provide rigorous statistical testing and obtain unbiased inference using these multivariate GARCH model specifications. These problems with implementing appropriate testing procedures coupled with the application of model selection techniques may also lead to additional difficulties in the proper design of optimal asset allocation and measuring market risk and these obstacles get higher as the number of assets increases leading and in there is always the case that no single model choice would provide a satisfactory answer to the optimal asset allocation problem.

Pesaran *et al*. (2009) partially addressed the issues linked with the difficulties that we face with the estimation and statistical inference of multivariate volatility

[3] Bauwens *et al*. (2003) and McAleer (2005) provide comprehensive surveys of alternative GARCH specifications.

models. They developed an integrated econometric approach to the asset portfolio optimization under the constraint of VaR in the presence of model uncertainty along with the associated risk monitoring problem. To this end Pesaran *et al*. (2009) adopted model averaging as strategy to risk diversification in order to deal with model uncertainty. They argued that such an approach reduces the model misspecification issues that may arise by the pre-test selection bias which is embodied in the two-step procedure of the AIC and the SIC information criteria. Furthermore, Pesaran *et al*. (2009) argued that the widely used forecast evaluation measures such as the Root Mean Square Error (RMSE) face considerable problems when applied in the case of multivariate volatility models. Therefore, they proposed a forecasting performance criterion that links the VaR performance of their associated portfolios and could be considered as a variant of the Kupiec's (1995) binomial test. The main findings of their analysis were that in most cases the Student *t* Dynamic Conditional Correlation model (TDCC) advanced by Pesaran and Pesaran (2007) managed to pass the VaR diagnostic tests in out-of-sample performance. In addition Pesaran *et al*. (2009) demonstrated that the equal-weighted average model based on the top 25 models of a large set of models it was the one not rejected by the VaR diagnostic tests they implemented. However, a drawback of this approach is that is dependent on the choice of the models under consideration and their corresponding weights and this may be proved a tedious exercise.

3. Econometric methodology

In this section we provide a description of the theoretical consideration of the variance sensitivity analysis within the univariate portfolio GARCH models drawing heavily on Manganelli (2004). The starting point of the analysis is the argument made by Nijman and Sentana (1996) that a linear combination of variables generated by a multivariate GARCH process will only be a weak GARCH process. An implication of this result is that an attempt to fit GARCH processes directly to portfolio returns will generally lead to a model misspecification.

Manganelli *et al*. (2002) and Manganelli (2004) developed an alternative methodology to the direct estimation of any multivariate GARCH specification based on the notion of “quasi-maximum-likelihood” introduced by White (1994), assuming that any GARCH model is only a rough approximation of the true relationship among the observed data. The basic proposition put forward by Manganelli (2004) is that when we change the weights of a portfolio this will lead to a change of the time series of portfolio returns. This change will then lead to an alteration of the information available for the estimation of the univariate GARCH process. A result of this procedure will be that the derived variance can be considered as a function of the portfolio returns via two channels, the first being the vector of portfolio returns and the second being the estimated parameters.[4]

The idea of employing measures of sensitivity to the weights of the portfolio allocation has previously been used in a number of alternative models of estimating VaR. For example Garman (1996) suggested the computation of the derivative of the VaR with respect to the individual elements of a specific portfolio might be used in order to evaluate the potential influence of trading on the VaR of a company. Moreover, Gourieroux *et al*. (2000) have adopted the proposal set forth by Garman (1996) to provide an analysis of its theoretical implications to alternative VaR specifications. Manganelli (2004) argues that the same type of analysis can be used for the variance of a portfolio given the corresponding variance-covariance matrix.

[4] As Manganelli (2004, p. 373) points out the estimated parameters depend on the time series of portfolio returns used in estimation.

Manganelli (2004) makes a significant contribution to the issue of using sensitivity analysis for achieving optimal asset allocation within the context of univariate GARCH models. His approach has a number of significant implications. First, the GARCH sensitivity analysis can be used by portfolio managers and investors to test whether their actual portfolio has minimum variance. According to Manganelli (2004) this test amounts to a value of zero for all derivatives with respect to portfolio weights. Second, this methodology can also be used to examine the effect that any asset has on the variance of the portfolio. This will provide valuable information to portfolio managers enabling them to identify the main sources of market risk and/or to investigate the influence that a specific transaction exercises on the portfolio variance. Finally, Manganelli (2004) develops a simple method for the estimation of the full variance–covariance matrices of portfolio assets.

Let y_t be the return of the portfolio comprised of $n+1$ assets and let $y_{t.i}$ be the ith stock return, for $t=1,.....,T$ and $i=1,.......,n+1$.[5] Denoting the weight of asset i by a_i, the portfolio return at time t is $y_t = \sum_{i=1}^{n+1} a_i y_{t,i}$. Given that the weights a_i must sum to one, we can express one weight as a function of the others, $a_{n+1} = 1 - \sum_{i=1}^{n} a_i$.

Let us assume that y_t follows a zero-mean process with a $\text{GARCH}(p,q)$ conditional variance h_t:

$$y_t = \sqrt{h_t}\varepsilon_t \qquad \varepsilon_t \mid \Omega_t \sim (0,1) \tag{1}$$

$$h_t = z_t'\theta \tag{2}$$

where

$\underset{mx1}{z_t} = (1, y_{t-1}^2,......, y_{t-q}^2, h_{t-1},......., h_{t-p})'$, $\underset{mx1}{\theta} = (\alpha_0, \alpha_1,....., \alpha_q, \beta_1,......, \beta_p)'$, and

[5] For a full analysis of the mathematical and statistical derivations see Manganelli (2004, p. 373-374).

$m = p + q + 1$

$\Omega_t = \{a, [y_{r,1}]_{t=1}^{t-1}, ..., [y_{r,n+1}]_{r=1}^{t-1}\}$, is defined as the information set of the model, where a denotes the n-vector of portfolio weights.[6] It is clear from the definition of the information set that a change in the vector of portfolio weights implies a change in the information set. This is due to the fact that the actual series of the stock returns are included in the information. As, evaluating the potential influence of a transaction on the estimated variance can become extremely complicated necessitating a re-estimation of the complete model, Manganelli (2004) suggests an alternative simpler method.

This method calls for the calculation of the first derivative of the variance with respect to the weights. Thus, a positive derivative would indicate that the change in weights due to trading on a particular asset will result in an increase of the variance of the portfolio while a negative derivative will lead to a reduction in the portfolio's variance. To analyze this point let us define $\hat{h} = \hat{z}_t' \hat{\theta}$ as the estimated variance. The computation of the first derivative of $\hat{h}_t$ is based on the recognition that both the vectors $\hat{z}_t$ and $\hat{\theta}$ (the vector of the estimated coefficients) are functions of the weight a. Then the first derivative is derived as follows:

$$\underset{xx1}{\frac{\partial \hat{h}}{\partial a}} = \underset{nxm}{\frac{\partial \hat{z}_t'}{\partial a}} \underset{nx1}{\hat{\theta}} + \underset{nxm}{\frac{\partial \hat{\theta}'}{\partial a}} \underset{mx1}{\hat{z}}_t \tag{3}$$

[6]As Manganelli (2004, p. 373) notes, that the $(n+1)$ weight equals one minus the sum of the other weights. The respective $(n+1)$ asset is considered to be the benchmark asset against which the sensitivity is conducted. Bollerslev and Wooldridge (1992) also show that the vector of unknown parameters θ can be consistently estimated by maximizing the normal log-likelihood.

In order to analyse carefully the local behaviour of the estimated variance with respect to the portfolio allocation we derive the second derivative, which will allows us to examine its concavity. This is given by:

$$\underset{nxn}{\frac{\partial^2 \hat{h}_t}{\partial a \partial a'}} = \underset{nxm}{\frac{\partial \hat{z}'_t}{\partial a}} \underset{nxm}{\frac{\partial \hat{\theta}}{\partial a}} + \underset{nxm}{\frac{\partial \hat{\theta}'}{\partial a}} \underset{mxn}{\frac{\partial \hat{z}_t}{\partial a'}} + \left(\underset{1xm}{\theta'} \otimes \underset{nxn}{I} \right) \underset{mnxn}{\frac{\partial}{\partial a'}} vec\left(\frac{\partial \hat{z}'_t}{\partial a} \right) + \left(\underset{1xm}{\hat{z}'_t} \otimes \underset{nxn}{I_n} \right) \frac{\partial}{\partial a'} vec\left(\frac{\partial \hat{\theta}'}{\partial a} \right) \quad (4)$$

where $\otimes$ indicates the Kronecker product and I_n is an (nxn) identity matrix.[7] Evaluation of equation (3) and (4) require the computation of the respective derivatives.

Given the above analysis we note that the approach to the optimal asset allocation problem advanced by Manganelli (2004) initially results to the standard solution which is true for all static and dynamic optimization problems that the outcome variance of the portfolio is a function of the weights. The contribution of this approach lies on the specific dynamics of this functional relationship and the goal of minimizing the curvature of the hyper-surface with respect to the weights vector.

The sensitivity analysis approach developed by Manganelli *et al*. (2002) and Manganelli (2004) can now be employed in order to estimate large variance-covariance matrices as well as to analyze the optimal conditional portfolio allocation in a mean-variance framework. The second contribution of this approach is provided by the utilization of the relationship among the estimated variances, covariances and the variance derivatives with respect to the portfolio weights, since it allows the development of a simple method for the estimation of full variance-covariance matrices of large portfolios, which by construction are positive definite.[8] As Manganelli (2004) demonstrated, this problem involves the maximization of a

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[7] Manganelli (2004, p. 374-375 and Appendix B) provides a full mathematical analysis of the evaluation and properties of the first and second derivatives.

[8] Manganelli (2004, p. 374-376) provides the algebraic derivation of the necessary and sufficient conditions of the maximization problem.

function of the conditional mean and the conditional variance with respect to portfolio weights. More explicitly this maximization problem can be solved by maximizing a function of n variables, the portfolio weights, with known their first and second derivatives. Manganelli (2004) notes that univariate portfolio GARCH models are subject to misspecification although the degree of misspecification and its impact on the shape of the function that is maximized is unknown a priori and this can only be revealed on empirical grounds.[9] When we turn to the estimation of large variance-covariance matrix then this involves a three step procedure: (a) Minimization of the portfolio variance with respect to weights; (b) computation of the second derivatives of portfolio variance; and (c) definition of the (n+1)-vector of weights corresponding to each asset entering the portfolio and then compute the variance-covariance matrix by solving the system shown in Manganelli (2004, p. 377).

4. Data and empirical results

In this paper we apply the variance sensitivity analysis using stock returns of companies listed in the Athens Stock Exchange (ASE). We use daily data for the period 3 January 2001 to 31 December 2009. The price data for the ASE is the closing prices quotations of the ASE General Price Index which is a capitalization weighted index. For the present study we chose the top 30 companies by market capitalization, from the banking sector, manufacturing, construction, informatics and telecommunications industries and they are given in the Appendix. The data has been retrieved from DATASTREAM. The sample consists of 2224 observations. The estimation process is conducted for the full sample whereas we use the last 5 years

[9] If the portfolio GARCH models were correctly specified, the function would be quadratic in the weights, and the optimization procedure trivial.

(each year is taken to have 252 trading days) to conduct the out-of-sample forecasting evaluation.

We begin our empirical analysis with the estimation of the first and second derivatives of GARCH variances using a two-asset portfolio composed of the stocks of ALPHA (Banking and financial services) and FOLLI-FOLLIE (A manufacturer of fashion jeweler) traded on the ASE.[10,11] We estimate univariate GARCH (1,1) models for 31 portfolios constructed from these two assets. The weight (a) for ALPHA Bank ranges from -1 to 2, with a step increase of 0.1. For each estimated GARCH model, the first and second derivatives of the estimated variance with respect to the weight (a) have been calculated. The estimated variances on 31 December 2009 for the 31 portfolios with respect to the weight (a) along with the first and second derivatives are illustrated in Figure 1. There are several points to be made regarding these plots. The variance corresponding to $a = 0$ is the variance of FOLLI-FOLLIE whereas the variance corresponding to $a = 1$ is that of ALPHA. Furthermore, those portfolios that have a weight greater than 1 are short of ALPHA and those which have a weight less than zero are short of FOLLI-FOLLIE. The shape of the estimated variance shown in Figure 1 as we have already explained in section 3 is tied to the potential gains from portfolio diversification. Thus given that the estimated variance is considered to be a parabolic and convex function of portfolio weights a there are substantial gains from portfolio diversification measured in terms of risk reduction.

Furthermore, in line with Manganelli (2004) our univariate GARCH estimates produce results close to the theoretical considerations and therefore we may argue that

[10] To check the robustness of our results we have also applied the two asset portfolio case using two other pairs of non-banking sector firms whose stocks exhibit significant trading value and are considered as blue chips. The first pair is Mihaniki (construction) and Inform Lykos (Informatics) and the second is Mitilineos (Mines) and Iatriko (Medical services). We obtained very similar results with respect to the optimization asset allocation problem. To save space these results are available upon request.

[11] All estimations have been run with the MATLAB codes developed by Manganelli.

they are good approximations of the unknown true model. This argument is further strengthened by examining the shape of the first and second derivatives. If the function was a perfect parabola then the first derivative would be a straight line with a positive slope and the second derivative would be a straight line with a slope equal to zero. From Figure 1 we observe that both derivatives are close to the values implied by theory.

Figure 2 shows the plots of the first derivatives of the estimated variance, $\frac{\partial \hat{h}_t(a)}{\partial a}$, for the two degenerated portfolios, i.e. FOLIE-FOLLIE ($a = 0$) and ALPHA $(\alpha = 1)$ .These plots show the magnitude by which the variance would decrease or increase over time, in the situation where an investor moves away from the corner solution of holding either stock. Similar patterns can be derived for any portfolio weight. Therefore, the investor or the portfolio manager has a complete set of information with respect to the effects in terms of risk when the composition of the current portfolio is changed.

Figure 2 also shows that the first derivative of ALPHA is always positive whereas the corresponding one for FOLLI-FOLLIE is mostly negative. This finding implies that during the period under investigation the minimum variance portfolio was formed by a convex combination of these two assets (Manganelli, 2004, p. 379). The evidence that over the last part of the sample both first derivatives were positive indicates that during that period the risk manager needed to take a short position ALPHA in order to obtain the minimum-variance portfolio. In addition, Figure 2 provides useful information with respect to the sources of risk of a particular portfolio. Thus, Manganelli (2004) shows that the greater in absolute value the first derivative is, the greater the risk reduction following a portfolio reallocation will be.

The first derivative of the portfolio which is only composed by the ALPHA stock is much higher on average (in absolute value) than the first derivative of the portfolio including only the FOLLI-FOLLIE stock.[12] Thus, we may conclude that during the 2000s an investor who was active in Athens Stock Exchange could achieve greater variance reduction and therefore could gain more in terms of risk reduction if he/she diversified away from the portfolio with only FOLLI-FOLLIE stock than from the ALPHA portfolio.[13]

Next we consider the information obtained from Figure 3. We report the plots of the second derivative of ALPHA as well as its difference from the average second derivatives computed over all 31 portfolios. Theory suggests that in the case of a model which is correctly specified the second derivative should be a flat line since it should not depend on the portfolio composition. In this case if we take the difference between the average second derivatives and that of ALPHA the second derivative should be zero. Indeed, in Figure 3 we observe that the derived difference is quite smooth around zero and this evidence is related with our analysis of Figure 1 and provides further support in favour of our univariate GARCH model being a good approximation of the true variance.[14]

The second derivative also provides important information to the risk manager with respect to the size of change of the variance sensitivity when a change in the portfolio allocation takes place. This implies that the greater the value of the second derivative, the greater the change in the variance sensitivity will be which in turn will lead to the need for a smaller portfolio reallocation in order to attain a given size of

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[12] The average first derivative for ALPHA is 5.98 and for FOLLI-FOLLIE is -7.61. This means that the variance sensitivity of the portfolio consisting only of FOLLI-FOLLIE was about 27% higher than that of the ALPHA portfolio.

[13] In general, in order to reduce the risk, the risk manager should sell the assets with the highest first derivative and buy those with the lowest one (Manganelli *et al*., 2002; Manganelli, 2004).

[14] The reason that the difference is not exactly equal to zero is due to the misspecification of the univariate GARCH model.

variance reduction. In Figure 3 we observe that in the period 2000-2009 the impact on variance due to portfolio reallocations is much greater compared to that during the 1990s. Specifically, the average value of the second derivative was 7.68 between 1990 and 1999, whereas in the period 2000 to 2009 a substantial increase has been documented. These findings suggest that there has been a significant increase in the concavity of the portfolio variance (as a function of weight, a) for the stocks of ALPHA and FOLLI-FOLLIE over the last five years and risk managers active on the Athens Stock Exchange should take this information into consideration.

The next stage of the present analysis deals with the implementation of the methodology developed by Manganelli (2004) and discussed in section 3, as well. The purpose is to estimate full variance-covariance matrices and to find the allocation that minimizes portfolio variance. We test this approach with the use of different subsamples of the Athens Stock Exchange general index for the period 3 January 2001 to 31 December 2009.[15]

The performance of the variance sensitivity methodology is then evaluated against three alternative specifications of multivariate models. These specifications include the Dynamic Conditional Correlation (DCC), the Orthogonal GARCH (OGARCH) and the Exponentially Weighted Moving Average (EWMA) popularized by Riskemetrics.[16]

For these three alternative specifications we first estimate the variance-covariance matrix on 31 December 2009. Next we compute the weights that lead to

[15] We have calculated the standard summary statistics for the returns of the 30 assets used in the analysis. The typical stylized facts that financial data exhibits is also documented here. Thus, it is observed that the hypothesis of normality is rejected based on the Jarque-Bera test statistic and the data also exhibits high kurtosis. We have also calculate the sample correlations of the returns. The average correlation is 0.29. To save space these results are available upon request.

[16] For daily data the weight λ (the decay coefficient) is usually set to 0.94.

the derivation of the minimum-variance portfolio[17]. We then estimate the univariate GARCH variance associated to this portfolio and present the annualized estimated volatility in Table 1. We conduct this exercise for the DCC, OGARCH and EWMA models and for five alternative portfolios with 2, 5, 10, 20 and 30 stocks.[18]

The estimation of the variance sensitivity analysis (VSA) model is conducted with the direct minimization of the univariate GARCH variance with respect to portfolio weights.[19] We observe that convergence is achieved very quickly and is very robust to the choice of the initial conditions which implies that the objective function behaves appropriately even when we consider the case of problems associated with high dimensions.[20] Following Manganelli (2004) we choose as initial conditions of the variance sensitivity analysis model the optimal weights of exponentially weighted moving average model. Table 1 presents the complete results.

The picture emerging from Table 1 is that the VSA model outperforms the three alternative models in comparison. This result is due to the fact that the VSA model is constructed to estimate the minimum-variance portfolio based on the univariate GARCH model. Furthermore, we observe that the performance of the VSA model relative to the other competing models increases as the number of stocks increase.[21] Thus, we demonstrate that while in the case of the two-asset portfolio the differences in the minimum variances are almost zero, as we move towards larger portfolios these differences get larger for both the DCC and OGARCH models. With five stocks, DCC and OGARCH overestimate the minimum-variance portfolio by

[17] The analytical solution is given in Manganelli (2004, p.382)
[18] A full list of the companies used in the analysis is given in Appendix A. Assets are progressively aggregated in the order reported in this Appendix.
[19] We use the function *fminunc* in Matlab and we insert as inputs the first and second analytical derivatives calculated in section 2.
[20] Convergence for a 30-asset portfolio occurs in less than 20 iterations for randomly chosen initial conditions.
[21] The outperformance of the VSA model is measured by the percentage difference in annualized volatility.

about 2% and 21%, respectively. When we look into the case with ten stocks then the difference rises to 5% and 24%, while for the case of twenty and thirty stocks it ranges from 32% to approximately 143%. These results lead to the conclusion that as the number of stocks in a portfolio rises, the number of restrictions imposed by the typical multivariate GARCH models increases and its computation becomes very complicated.

In contrast the results of the EWMA model, the simplest of all data filters, provide a rather different outcome since its performance does not deteriorate as much and as fast as the DCC and OGARCH models. Manganelli (2004) explains this behaviour of the EWMA model on the grounds of its construction. Since we use the same weight λ for all variance and covariance terms this amounts to the estimation of this model's portfolio variance directly, with coefficient λ. Certainly, this does necessarily imply that the EWMA model provides reasonable estimates of the variance-covariance matrix, (see Manganelli, 2004, p. 384).[22]

The results of the asset allocation analysis derived from an emerging market of the Eurozone confirm the previous evidence obtained by Manganelli (2004) who employed a set of stocks traded in the NYSE and this may further suggest that the degree of misspecification of the univariate GARCH models on which the variance sensitivity analysis is based on is negligible.

5. Forecasting performance evaluation

The final stage of the present analysis involves the evaluation of forecasting performance of the competing models. This task is accomplished with the application

[22] It should be noted that the computation time of the VSA model for a thirty-asset portfolio it takes less than a few minutes to attain optimization .

of the backtesting procedure provided by the Kupiec (1995) test and out-of-sample VaR evaluation criteria.[23]

We test all models with a VaR level of significance, (α), that takes values from 1% and 5% and we then evaluate their performance by calculating the failure rate for the returns series y_t. The failure rate is defined as the number of times returns exceed the forecasted VaR. Following Giot and Laurent (2003) we define a failure rate f_l for the long trading positions, which is equal to the percentage of negative returns smaller than one-step-ahead VaR for long positions. In a similar manner, we define f_s as the failure rate for short positions as the percentage of positive returns larger than the one-step-ahead VaR for short position.[24]

In Kupiec's test, we define f as the ratio of the number of observations exceeding Var(x) to the number of total observation (T) and pre-specified VaR level as a (Tang and Shieh, 2006). The statistic of Kupiec LR test is given by Eq. (5) (Kupiec, 1995). Under the null hypothesis Kupiec (1995) developed a likelihood ratio statistic (LR) distributed as chi-square distribution which is given as follows:

$$LR = 2\left\{\log[f^{x}(1-f)^{T-x}] - \log[\alpha^{x}(1-\alpha)^{T-x}]\right\} \qquad (5)$$

where $f = N/T$ is the failure rate, $\hat{f}$ is the empirical (estimated) failure rate, N is the number of days over a period T that a violation has occurred. Giot and Laurent (2003) suggest that the computation of the empirical failure rate defines a sequence of yes/no, under this testable hypothesis.

[23] For a detailed analysis of selection and evaluation criteria for VaR models see Andersen and Bollerslev (1998) Christoffersen, (1998), Sarma *et al*. (2003) and Bams *et al*. (2005).

[24] When the VaR model is correctly specified then the failure rate should be equal to the pre-specified VaR level.

The VaRs of α quantile for long and short trading position are computed as in Equation 6, 7 and 8 for normal, Student-t and skewed Student-t respectively (Tang and Shieh, 2006).

$$VaR_{long} = \hat{\mu}_t - z_\alpha \hat{\sigma}_t, \qquad VaR_{short} = \hat{\mu}_t + z_\alpha \hat{\sigma}_t \tag{6}$$

$$VaR_{long} = \hat{\mu}_t - st_{\alpha,\nu} \hat{\sigma}_t \qquad VaR_{short} = \hat{\mu}_t + st_{\alpha,\nu} \hat{\sigma}_t \tag{7}$$

$$VaR_{long} = \hat{\mu}_t - skst_{\alpha,\nu,\xi} \hat{\sigma}_t \qquad VaR_{short} = \hat{\mu}_t + skst_{\alpha,\nu,\xi} \hat{\sigma}_t \tag{8}$$

where z_α, $st_{\alpha,\nu}$ and $skst_{\alpha,\nu,\xi}$ are left or right tail quantile at $\alpha\%$ for normal, Student-t and skewed Student-t distributions respectively.

For the out-of-sample forecast evaluation we use 252*5 days forecast sample in order to provide one-step-ahead prediction.[25] Table 2 provides a summary of out-of-sample VaR forecasts for long and short trading positions as well as for all five alternative constructed portfolios. The results of Kupiec out-of-sample forecasting test provide further evidence about the superiority of the VSA model as the appropriate specification for optimal asset allocation. The overall empirical evidence suggests that the VSA model has the best out-of-sample performance against all three competing models for $\alpha = 0.05$ and $a = 0.01$ and for both short and long trading position in most cases. Therefore, we provided further evidence based on the VaR forecasts that variance sensitivity analysis provides an appropriate framework for the asset allocation optimization, which is much needed especially during the current turbulent period in the global financial markets.

[25] Given that the Kupiec test is applicable only for the univariate case we apply it in the case of all competing models by using the estimated residuals from the estimation of each model for the different portfolios produced in the previous section.

6. Summary and concluding remarks

Modelling asset volatility using multivariate GARCH models become cumbersome since they require strong assumptions to make estimations feasible while their dimension increase exponentially as the number of variables increases. A common procedure to avoid the problems raised by the estimation is to fit univariate GARCH models to the time series data of portfolio returns, but this approach has as a shortcoming the loss of the multivariate dimension of the portfolio allocation. Variance sensitivity analysis has been proposed by Manganelli *et al*. (2002) and Manganelli (2004) in order to resolve the problem that arises when trying to model asset volatility using multivariate GARCH models. More, specifically, this approach utilizes the GARCH environment giving at the same time tractable computations and clear-cut conclusions. Manganelli (2004) suggested that the impact of a portfolio reallocation on the estimated variance should be evaluated by calculating the sensitivity of the estimated variance with respect to the weight of the stock involved in the transaction. This task can be accomplished by using as a sensitivity measure the derivative of the estimated variance with respect to portfolio weights. Furthermore, this approach allows us to estimate the full variance-covariance matrix.

In this paper we provided a variance sensitivity analysis using daily data from the Athens Stock Exchange, a closely monitored emerging market. Our sample consists of daily returns of thirty assets traded at the Athens Stock Exchange for the period between 3 January 2001 and 31 December 2009. This is an emerging market which has been closely monitored by portfolio managers as a result of its high returns during the last decade. We conducted our analysis by constructing different portfolios with two, five, ten, twenty and thirty assets. First, we considered a two-asset portfolio consisting of stocks of large capitalization firms traded on the ASE. After the estimation of the variance sensitivity we examined how this sensitivity has been changing over time and emphasized its implications for risk management in this emerging stock market. Furthermore, we calculated the second derivative of the estimated variance for this portfolio with respect to the portfolio weights. The second derivative is a measure that provides an indication of the benefits measured in terms of risks that arise from portfolio diversification between the two assets under examination. Second, following Manganelli (2004) we also computed the minimum variance portfolio at any given point in time for alternative portfolios constructed from the general index of the ASE. Our results were shown to be robust for alternative pairs of stocks. The performance of this methodology was assessed against three popular multivariate GARCH specifications, namely DCC, OGARCH and EWMA models. Finally, with the application of the Kupiec's test for out-of-sample forecasting performance we demonstrate that the variance sensitivity analysis outperforms all three alternative models at both the 95% and 99% confidence interval independently of the trading position.

The overall results of the present analysis leads to the conclusion that the adopted methodology provides more efficient results than the competing multivariate GARCH models. An important point to be made is that the degree of misspecification of the estimated univariate GARCH is insignificant. Finally, our results are in line with those reported by Manganelli (2004) for the NYSE suggesting that this methodology performs well on daily data derived from mature as well as emerging markets and thus can be considered a useful tool for portfolio and risk managers.

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APPENDIX

Table A.1 List of stocks traded in ASE used in the analysis

A/A	NAME
1	ALPHA BANK S.A.
2	ATTICA S.A. HOLDING
3	PIRAEUS BANK
4	COCA-COLA S.A.
5	FOLLI-FOLLIE
6	J&P AVAX S.A.
7	GEKTERNA– CONSTRUCTION
8	ELLAKTOR– CONSTRUCTION
9	DOL-PUBLISHERS
10	BIOHALKO INDUSTRIAL(COPPER AND ALUMINIUM)
11	HELLENIC OIL
12	MITILINEOS S.A.
13	TITAN – CEMENT S.A,
14	BABYLAND
15	HALCOR-METALURGIC
16	ANEK-SHIPPING CO
17	ELVAL
18	NATIONAL BANK OF GREECE
19	SIDENOR-METALURGIC
20	LAVIPHARM-CHEMICAL
21	ALTER-INFORMATICS
22	IATRIKO MEDICAL CENTRE
23	INTRAKOM S.A. INFORMATICS
24	HELLENIC WIRES
25	MAILLES MINES S.A.
26	IMFORM LYKOS-INFORMATICS.
27	METKA S.A.
28	MIHANIKI CONSTRUCTION
29	OTE TELECOMMUNICATIONS S.A.
30	FOURLIS-ELECTRIC

Table 1. Comparison between the VSA methodology and alternative multivariate GARCH models

	Portfolio with 2 assets			Portfolio with 5 assets			Portfolio with 10 assets			Portfolio with 20 assets			Portfolio with 30 assets		
	Min vol	% VSA	Seconds	Min vol	% VSA	Seconds	Min vol	% VSA	Seconds	Min vol	% VSA	Seconds	Min vol	% VSA	Seconds
DCC	31.07%	0.13	16	27.05%	6.31	23	23.52%	13.19	39	18.62%	32.64	86	17.25%	112.23	144
OGARCH	35.44%	14.20	3	29.92%	21.24	5	25.39%	24.47	9	21.67%	40.86	18	19.31%	143.19	27
EWMA	32.44%	4.53	1	24.85%	0.67	1	23.27%	3.88	1	18.83%	22.40	1	19.40%	34.12	1
VSA	31.03%	0	46	24.68%	0	73	20.40%	0	120	13.38%	0	122	11.41%	0	354

Note: DCC is the dynamic conditional correlation; OGARCH is the orthogonal GARCH; EWMA is the exponentially weighted moving average. For each portfolio we report the univariate GARCH annualized volatility associated with the minimum-variance weights implied by the estimated variance-covariance matrix, the percentage difference with respect to VSA and the computation time to estimate the model.

TABLE 2: Out-of- Sample Forecasting Kupiec Test

Portfolio with 2 assets

Out-of- Sample Forecasting 95% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.97238	16.328	0.00538**	0.01322	31.962	0.00791**
OGARCH	0.97178	15.166	0.00284**	0.08712	22.671	0.00028**
EWMA	0.97460	16.441	0.00198**	0.03456	3.545	**0.11441**
VSA	0.96508	2.981	**0.26278**	0.02698	4.289	**0.02879**

Out-of- Sample Forecasting 99% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.99891	14.7891	0.00045**	0.005824	10.567	0.02347*
OGARCH	0.99578	15.4352	0.00123**	0.007935	19.8771	0.03568*
EWMA	0.97368	19.2451	0.01256*	0.008630	17.0156	0.00234**
VSA	0.96257	6.8289	**0.26781**	0.028573	2.3891	**0.34507**

Portfolio with 5 assets

Out-of- Sample Forecasting 95% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.98345	17.873	0.00234**	0.016278	24.961	0.0001**
OGARCH	0.99023	14.254	0.00012**	0.023456	22.559	0.0022**
EWMA	0.97788	18.341	0.00689**	0.028972	20.445	0.0122*
VSA	0.99689	5.2562	**0.24134**	0.017891	2.2952	**0.2416**

Out-of- Sample Forecasting 99% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.99338	22.1529	0.00789**	0.012887	23.9958	0.002347**
OGARCH	0.99167	17.0801	0.00367**	0.015781	11.378	0.01799*
EWMA	0.98679	18.2336	0.01119*	0.028793	28.4572	0.00897**
VSA	0.98389	3.2892	**0.51289**	0.016298	15.277	**0.39695**

Portfolio with 10 assets

Out-of- Sample Forecasting 95% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.99783	19.709	0.00225**	0.01689	24.971	0.0256*
OGARCH	0.98287	15.201	0.00028**	0.02190	15.601	0.0198*
EWMA	0.99234	20.879	0.02379*	0.02877	20.678	0.0023**
VSA	0.99345	8.3356	**0.33768**	0.01198	13.790	**0.2241**

Out-of- Sample Forecasting 99% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.99186	21.539	0.00274**	0.01882	22.9932	0.04347*
OGARCH	0.99283	18.799	0.00448**	0.01952	17.9528	0.00018**
EWMA	0.99091	18.355	0.02567*	0.02009	16.0233	0.00435**
VSA	0.98221	4.6981	**0.28864**	0.01831	1.15173	**0.69695**

Portfolio with 20 assets

Out-of- Sample Forecasting 95% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.97640	20.812	0.00022**	0.01675	32.911	0.00336**
OGARCH	0.97263	16.277	0.01156*	0.01923	22.641	0.00278**
EWMA	0.98678	20.142	0.00336**	0.01908	22.765	0.00035**
VSA	0.96533	1.7128	**0.27897**	0.02234	1.113	**0.41103**

Out-of- Sample Forecasting 99% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.99102	21.129	0.00562**	0.00891	22.678	0.03647
OGARCH	0.99209	18.233	0.00446**	0.00228	23.557	0.00019
EWMA	0.99679	19.221	0.03778*	0.00366	3.0271	**0.21091**
VSA	0.98210	4.224	**0.33989**	0.01224	2.1513	**0.51229**

Portfolio with 30 assets

Out-of- Sample Forecasting 95% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.98622	28.889	0.00345**	0.01255	21.656	0.0156*
OGARCH	0.97334	17.652	0.00019**	0.01368	26.503	0.00015**
EWMA	0.98189	19.989	0.01277*	0.01329	19.255	0.00668**
VSA	0.96679	5.7339	**0.18971**	0.01477	2.8288	**0.31882**

Out-of- Sample Forecasting 99% Confidence Interval						
	VaR for Short Position			**VaR for Long Position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
DCC	0.98556	21.334	0.00335**	0.002981	22.334	0.00561**
OGARCH	0.98673	16.221	0.00167**	0.013245	23.598	0.01664*
EWMA	0.99112	15.902	0.00013**	0.016562	26.033	0.00098**
VSA	0.97299	4.2214	**0.23356**	0.000335	1.4422	**0.44671**

Note: Number of forecasts: 252*5 days and 1 day ahead. (*) and (**) denote 5% and 10% level of significance respectively.

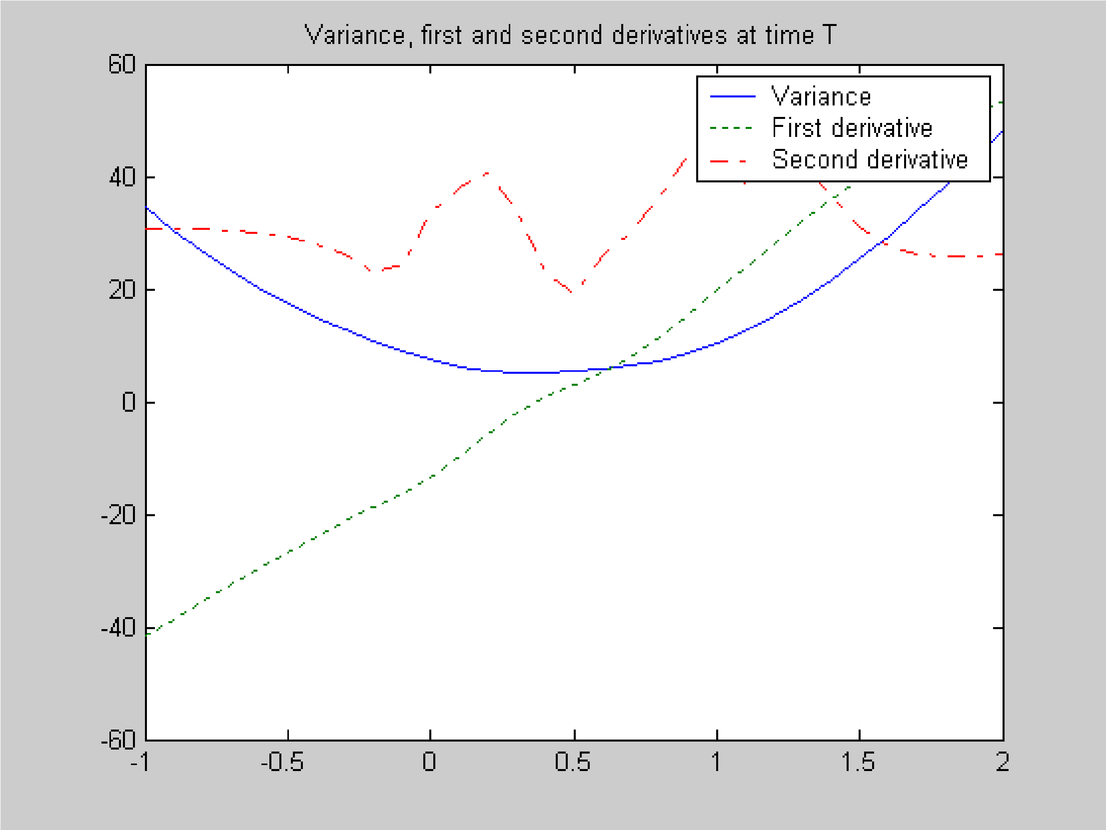

Figure 1. Plot of estimated variance, first and second derivative on December 31, 2009, for 31 portfolios constructed from ALPHA and FOLLIE-FOLLIE. On the horizontal axis is the portfolio weight for GM, which ranges from -1 to 2 with increments of 0.1. The variance is computed by reestimating a GARCH(1,1) model for each of the 31 portfolios. The first and second derivatives are computed analytically, as described in section 2.

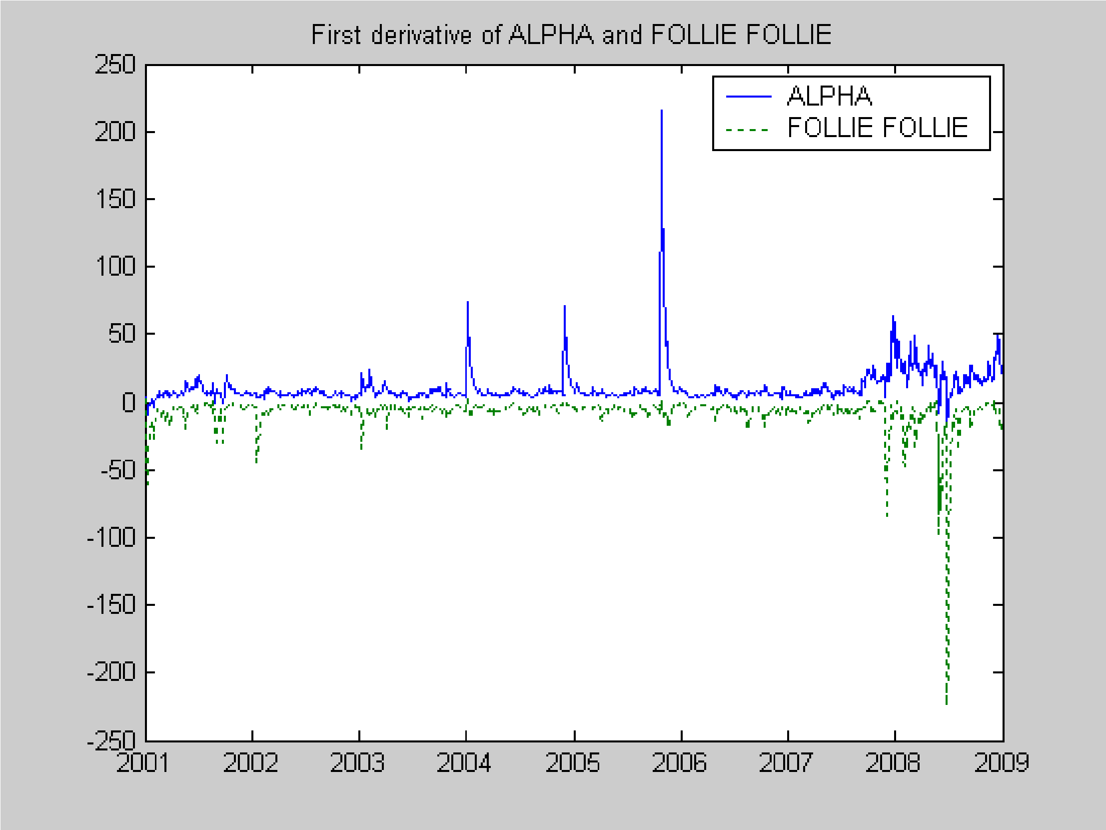

Figure 2. Plot of the first derivative of the estimated variance for the two degenerate portfolios of ALPHA and FOLLIE-FOLLIE.

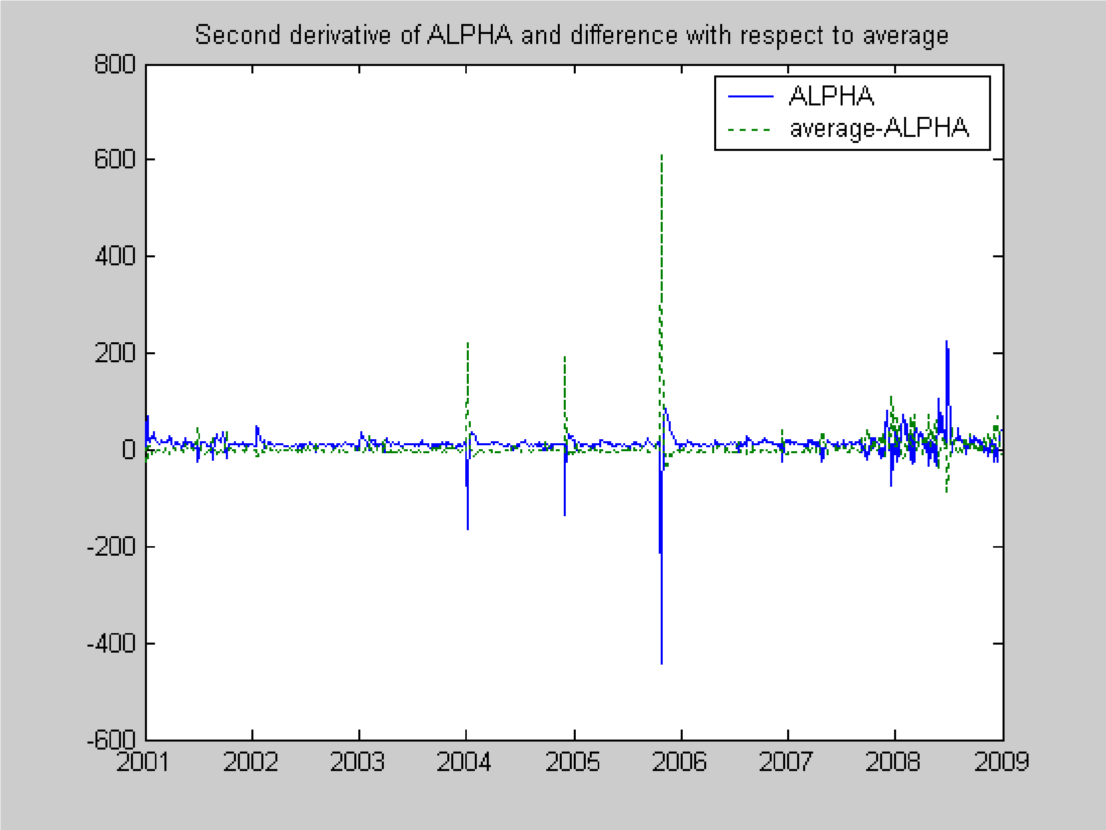

Figure 3. Plot of the estimated second derivatives, computed from the degenerate ALPHA portfolio (upper graph) and its difference w.r.t. the average second derivatives of 31 different portfolios (lower graph).