Predicting Conditional Autoregressive Value at Risk for major stock markets during tranquil and turbulent periods

by

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Abstract

This paper employs the Conditional Autoregressive Value at Risk (CAViaR) methodology developed by Engle and Manganelli (2004) in order to examine market risk in several major equity markets. By interpreting the VaR as the quantile of future portfolio values conditional on current information, Engle and Manganelli (2004) propose this approach to quantile estimation that does not require any of the extreme assumptions of the existing methodologies, mainly normality and i.i.d. returns. The CAViaR model shifts the focus of attention from the distribution of returns directly to the behaviour of the quantile. We provide a comparative evaluation of the predictive performance of four alternative CAViaR specifications, namely *Adaptive*, *Symmetric Absolute Value*, *Asymmetric Slope* and *Indirect GARCH(1,1)* models. The main findings of the present analysis is that we are able to confirm some stylized facts of financial data such as volatility clustering, while the Dynamic Quantile criterion selects different models for different confidence intervals for the case of the five general indices.

Keywords: Non-linear Regression Quantile, Value-at-Risk, Risk Management, Conditional Autoregressive

JEL classification: C53, G21; G28

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1. Introduction

Quantitative risk measure forecasting has become, at least since the market crash in 1987 and then as a consequence of the financial crisis of 1997-1998, as well as the bankruptcy of several financial institutions such as the BCCI and Barrings international banks that led to increased price volatility and financial uncertainty the benchmark for measuring market risk. The global financial crisis of 2007-2009 has called once again into question financial risk management practices, and one key issue is whether risk measures can actually be forecast accurately enough for the purpose. Thus the current financial crisis has indeed highlighted once again the importance of risk management where as our experience over the last twenty years where institutions such as banks and major hedge fund companies are found to inevitably fail from time to time. Such financial uncertainty has increased the likelihood of financial institutions suffering substantial losses as a result of their exposure to unpredictable market changes. These events have made investors become more cautious in their investment decisions, while it has also led to an increased need for more careful study of price volatility in stock markets. Moreover, the recent crisis has shown the necessity of having adequate risk-management protocols in order to achieve greater resilience, and the hence, the need to improve the existing procedures for quantifying the market risk. The present paper is motivated mainly by this concern.

In bank regulation, the effectiveness of capital requirements in preventing funding shortfall rests upon the estimation accuracy of market risk measures. Market risk is one of the four types of risk that financial institutions can expose themselves to. It is considered the most significant one since it represents potential economic loss caused by the reduction in the market value of a portfolio. The existence of market risk and recent financial disasters have raised the need for the development of practical risk management tools for financial institutions. This need has been reinforced by the Basel Committee of Banking Supervision (1996) which called for the use of internal market risk management to capital requirement by financial institutions such as banks and investment firms.[1]

Value-at-Risk has become the standard tool used by financial analysts to measure market risk. VaR is defined as a certain amount lost on a portfolio of financial assets with a given probability over a fixed number of days. The confidence level represents 'extreme market conditions' with a probability that is usually taken to be 99% or 95%. This implies that only 1% (5%) of the cases will lose more than the reported VaR of a specific portfolio. VaR is widely used because of its simplicity. Essentially the VaR provides a single number that represents market risk and therefore it is easily understood.[2]

Although the VaR is conceptually a simple measure of market risk, there exists a controversy with respect to the suitability of the alternative existing techniques employed to estimate the VaR. Indeed, the measurement of VaR is a very interesting statistical problem. Artzner *et al*. (1997, 1999) have derived a set of axioms that specify a *coherent risk* measure. Thus, a risk measure must possesses the following characteristics. First. it should not exceed the maximum possible loss which can occur. Second, the proposed risk measure should be greater than the mean loss, implying capital adequacy to cover losses. Third, in the event that there is a proportional change in the loss, then we require the risk measure to change proportionally as well. Finally, it must satisfy the property of superadditivity,

[1]For a detailed analysis see the Basel Committee on Banking Supervision's (1996), "*Amendment to the Capital Accord to Incorporate Market Risks*". Duffie and Pan (1997), Alexander (2005) and Drzik (2005) provide a comprehensive overview of value at risk measures.
[2] See also Bank for International Settlements (1988, 1999a,b,c, 2001).

implying that the risk measure calculated for two separate losses should be equal to the risk measure calculated on the sum of the two portfolios. As Boyle *et al*. (2005), Alexander *et al*. (2005) and Longin (2001) among others emphasize, the VaR methodology has certain limitations since it does not satisfy the properties of subadditivity and excess of the mean loss. Given these reservations regarding the use of the VaR as a measure for market risk, several researchers have developed alternative risk measures.[3]

Calculating the VaR requires accurate knowledge of the distribution of extreme events. This is a difficult task since the distribution of portfolio returns is not constant over time and, given that VaR is nothing more than a specific quantile of future portfolio values subject to current information, we must find an appropriate model for time varying conditional quantiles. This crucial issue is coupled with the need for providing accurate estimates of the chosen distribution of portfolio returns.

During the last two decades a large number of alternative models have been developed to estimate VaR. These alternative methodologies have mainly focused on modeling the entire distribution of returns and are based on the strict assumptions of normality or i.i.d. returns. However, as Engle and Manganelli (2004) argue, if we do not correctly estimate the underlying market risk then this can lead to an allocation of capital below first-best and that can affect the profitability and/or the financial stability of the corresponding bank or investment firm. Engle and Manganelli (2004) have recently proposed an alternative approach that models not the entire distribution but rather focuses on the regression quantile which does not require the above mentioned strict assumptions. This methodology, which is called Conditional Autoregressive Value at Risk (CAViaR), uses an autoregressive process in order to

[3] CVaR is an alternative risk measure that satisfies the coherency criteria by Artzner *et al*. (1997, 1999). Its advantage over VaR measures is that it focuses on both the frequency and the size of extreme events.

model the evolution of the regression quantile over time. The estimation of the unknown parameters is done with the use of the framework suggested by Koenker and Bassett (1978). Furthermore, Engle and Manganelli (2004) prove that these estimators are asymptotically efficient and consistent. Finally, they develop the Dynamic Quantile test which is used to examine the quality of the CAViaR results.[4]

The main objective of this paper is to investigate the predictive performance of various types of the CAViaR specifications for stock market returns during tranquil and turbulent periods. We estimate and perform an evaluation of the predictive performance of two of the four alternative CAViaR specifications, namely, *Symmetric Absolute Value* and *Asymmetric Slope*. Furthermore, given the recent financial crisis we are further interested in evaluating the predictive performance of the alternative specifications for three out-of-sample evaluation periods (before-crisis, crisis and after-crisis). Within this framework we consider the issue of model stability. It is well documented that the model parameters are estimated from real data, which are often subject to structural changes due to regime shifts or events such as financial crises. Recently, Huang *et al*. (2009) developed an improved CAViaR specification to account for some shortcomings of the original specification. Specifically, they argue that the asymmetric specification of Engle and Manganelli (2004) which is used to investigate the different effects of positive and negative returns on the VaR prediction requires the estimation of four parameters in the optimization procedure. This complicated model may lead to more estimation errors leading to an unstable or unrobust model. Therefore, this extension of the constant-parameter CAViaR allows us to examine whether the parameters of an individual risky asset are driven by the market index return. This extended model allows the tails of a financial series to

[4] Chernozhukov (1999) has derived independently the same dynamic quantile test.

follow different stochastic process and it allows VaR to be influenced by the volatility levele of the driving index.

The remainder of the paper is organized as follows. Section 2 presents some of the most widely used VaR models. In section 3 we discuss the CAViaR methodology and its proposed alternative specifications. In Section 4 we report our empirical results, and finally section 5 provides our concluding remarks.

2. Value-at-Risk-models and methods

2.1. VaR estimation methods

During the 1990s several alternative modeling methodologies for the estimation of the VaR were advanced. The purpose of these models was to provide risk managers with a comprehensive and intuitively easily understood measure of the VaR. The motivation for the development of the VaR models relies on the stylized characteristics of financial data which have were first documented by Mandelbrot (1963) and Fama (1965). To recapitulate, these characteristics imply that the returns of financial assets have leptokurtic distributions, that their distributions are negatively skewed and finally that they exhibit volatility clustering. As Manganelli and Engle (2001) and Engle and Manganelli (2004) point out, these alternative methodologies adopted a common general structure: (a) We mark-to-market the portfolio on a daily basis; (b) Estimation of the distribution of returns; and (c) Computation of the portfolio's VaR. The main difference among the alternative methodologies is linked to the estimation of the appropriate distribution of the portfolio returns. We can briefly discuss the advantages and disadvantages of the alternative VaR models using the following broad classification.[5]

The first class of models is fully parametric and includes applications such as J.P. Morgan's Riskmetrics (1996) and GARCH models. These methodologies combine an econometric model with the assumption of conditional normality for the returns series. Specifically, these models rely on the specification of the variance equation of the portfolio returns and the assumption that the standardized errors are i.i.d. Additionally, when the GARCH methodology is applied we are also required to specify the distribution of the errors, which is usually taken to be the normal one, while it is assumed that the negative returns follow the same process as the rest of portfolio returns (Bams *et al*. 2005; Burns, 2005; Angelidis *et al.* 2004; Alexander *et al*., 2005; Pojarlev and Polasek, Polasek and Pojarlev, 2005; Kuester *et al*., 2006; Haas *et al*., 2006; Guidolin and Timmerman, 2006 and Chen et al., 2013 are among the numerous applications employing alternative GARCH specifications).

The application of the parametric methodologies has been criticized since they tend to give coefficients which underestimate the VaR, mainly due to their failure to take into account the characteristic that the distribution of the portfolio returns have heavy tails. This underestimation of the VaR, as well as possible misspecifications with respect to the variance equation along with the distribution of errors, can be corrected by allowing alternative distributions of the errors such as the Gaussian, *Student's-t* and Generalized Error Distribution. However, it is further shown that the GARCH-type models provide satisfactory estimates of the quantile only when a bad event has already occurred.

[5] Jorion (2000) provides a complete analysis of the VaR methodology and alternative estimation methodologies. Kuester *et al*. (2006) gives an updated survey of alternative VaR methodologies.

The second approach for the estimation of the distribution of profits and losses is the non-parametric historical simulation. This methodology makes no assumption about the distribution of the portfolio returns and is based on the concept of rolling windows. The idea is to select a window which is usually taken to be anywhere between 6 months to 2 years and assume that any portfolio return has the same likelihood to occur. Moreover, a return which falls outside the chosen window has probability equal to zero to occur. This methodology has several deficiencies. It is inappropriate to provide extreme quantiles since we cannot extrapolate beyond past observations. The proposed solution to this problem is the increase of the sample of observations, but this will lead to estimates of the VaR which are biased downwards (or upwards) since we have a mixture of periods with low volatility with periods of high volatility.

Within this group of VaR models falls the hybrid approach developed by Boudoukh *et al*. (1998), which combines the historical simulation and Riskmetrics. This methodology applies weights to the portfolio returns that decline exponentially. Although this approach improves on the previously discussed methodologies, it also has problems since the selection of the parameters as well the calculation of the VaR does not depend on sound statistical theory but is rather ad hoc.

The final group for the estimation of the VaR are the semiparametric models. The first approach in this category is the Extreme Value Theory proposed by Danielsson *et al*. (1998) and Danielsson and de Vries (2000). The advantage of this approach is that it is based on sound statistical theory which offers a parametric form for the tail of a distribution. This approach focuses on the asymptotic form of the tail, rather than modeling the complete distribution of portfolio returns and therefore we are able to obtain more efficient forecasts of the risk associated with a particular market position. Although this methodology is very appealing it does have two shortcomings. First, as Danielson and de Vries (2000) argue, this approach performs well at very low quantiles but fails to provide accurate estimations of the VaR at levels which are not considered very extreme. Second, this methodology is also based on the assumption of i.i.d. standardized errors which is, as we have already discussed, an important limitation. Despite its limitations this approach has found substantial applications recently, (Longin, 2000; McNeil and Saladin, 2000; McNeil and Frey, 2000, Naftci, 2000; Bekiros and Georgoutsos, 2005a,b; Brooks *et al.*, 2005).

Engle and Manganelli (2004) and Manganelli and Engle (2001) proposed an alternative semiparametric method to estimate VaR, namely Conditional Autoregressive Value at Risk (CAViaR). This approach is based on the simple intuition that it is better to model the quantile directly as it evolves through time instead of attempting to model and estimate the entire distribution of portfolio returns. Modelling the quantile instead of the entire distribution has the main advantage that we are not required to adopt the set of extreme assumptions which are invoked by alternative methodologies, among them normality or that returns are i.i.d. In addition, CAViaR takes into consideration volatility clustering of portfolio returns that leads to the understanding that the corresponding distributions are autocorrelated. As a consequence the VaR must also follow a similar pattern since it is directly linked with the standard deviation of the distribution. Therefore, Engle and Manganelli (2004) and Manganelli and Engle (2001) developed CAViaR in order to take into account of the particular characteristic of the VaR. There is only a handful of empirical papers that have applied CAViaR. Kouretas and Zarangas (2005) employ the CAViaR model to measure the market risk of five major equity markets, six blue chip stocks from the Athens Exchange, and six major company stocks listed on the New York Stock

Exchange (NYSE). They compare four alternative CAViaR specifications with mixed results. Bao *et al*. (2006) compare the performance of CAViaR and other VaR measures for five East and Southeast Asian markets before, during and after the financial crisis of 1997-1998. They conclude that CAViaR models are quite satisfactory in a stable period but their performance in less satisfactory during turbulent periods. Kuester et al. (2006) compare CAViaR and alternative approaches for univariate VaR forecasts using daily return data on the NASDAQ Composite Index and alternative evaluation criteria. They conclude that CAViaR specifications performs well overall using unconditional and conditional tests for the predictive performance of VaR, although its forecasting performance is less accurate during periods of crisis and very high volatility. Huang *et al*. (2009) develop an extension of the CAViaR model (a) with a proposition of a new asymmetric CAViaR specification and (b) a mixed data regression model for multi-period VaR prediction. Using market data on WTI daily spot oil prices the improved CAViaR specifications performed well in a battery of evaluation criteria. Huang et al.

2.2. CAViaR

Engle and Manganelli (2004) and Manganelli and Engle (2001) consider a vector of portfolio returns that is observable, defined as $\{y_t\}_{t=1}^{T}$. Let θ be the probability tied to VaR, x_t be a vector of observable variables at time t, and β_θ be a vector of unknown parameters. They also define $f_t(\beta) \equiv f(x_{t-1}, \beta_\theta)$ as the θ-quantile of the distribution of the portfolio returns at time t which has been formed at time $t-1$.[6] Therefore, a general formulation of CAViaR can be written as follows:

[6] For simplicity we have eliminated the subscript θ from the vector of unknown parameters.

$$f_t(\beta) = \gamma_0 + \sum_{i=1}^{q} \gamma_i f_{t-i}(\beta) + \sum_{i=1}^{p} a_i l(x_{t-i}, \varphi) \quad (1)$$

where $\beta' = (a', \gamma', \varphi')$ and l is a function of a finite number of lagged values of observables. Moreover, in order for the quantile to have a smooth transition we use the autoregressive terms $\gamma_i f_{t-i}(\beta), i = 1,.....q$. Finally, they use the term $l(x_{t-i}, \varphi)$ to provide a relationship between the θ-quantile $f_t(\beta)$ and the observable variables which are included in the information set. As Engle and Manganelli (2004) point out, we can consider the lagged portfolio returns as the best choice for x_{t-1}. This implies that as y_{t-1} becomes negative, then one should expect the VaR to increase, while the VaR tends to decline in good days. Therefore, we expect that changes in y_{t-1} will symmetrically affect the VaR.

The purpose is to develop alternative specifications for the function l and then estimate the different models. Engle and Manganelli (2004) propose four alternative CAViaR specifications which we will estimate in our case. The first specification is called *Adaptive* and takes the following formulation:

$$f_t(\beta_1) = f_{t-1}(\beta_1) + \beta_1 \{[1 + \exp(G[y_{t-1} - f_{t-1}(\beta_1)])]^{-1} - \theta\} \quad (2)$$

where G is some positive finite number and we that as $G \to \infty$, the last term of equation (1) converges to $\beta_1[I(y_{t-1} \leq f_{t-1}(\beta_1)) - \theta]$, where $I(.)$ is the indicator function. The intuition behind the *adaptive* specification tells us that in those cases where the VaR has been exceeded, then we should increase its value, whereas in those cases that we do not exceed it, then we should reduce its value by a small magnitude. Such a strategy will lead in a reduction in the probability of observing a sequence of hits while at the same time it is highly unlikely that we will have zero number of hits. Engle and Manganelli (2004) also point out that this CAViaR specification has a unit coefficient on the lagged VaR.

A second specification is called *Symmetric Absolute Value* (SAV) and its mathematical formulation is given by:

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3 \, | \, y_{t-1} \, | \tag{3}$$

This model responds symmetrically to past portfolio returns and it is mean reverting since the coefficient of the lagged VaR is not constrained to equal one. Furthermore, we could properly specify this quantile specification using a GARCH model with the standard deviation (and not the variance), which is considered to follow a symmetric distribution with i.i.d. errors.[7]

The *Asymmetric Slope* (AS) is the third commonly used specification to estimate the l function. It is written as follows:

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3 (y_{t-1})^+ + \beta_4 (y_{t-1})^- \tag{4}$$

The *Asymmetric Slope* model allows for an asymmetric response to positive and negative past portfolio returns.[8] Again this model is mean reverting. As with the SAV model we can correctly specify this specification by fitting a GARCH process with the standard deviation, following this time an asymmetric distribution with i.i.d. errors.

[7] See also Taylor (1986), Schwert (1988) and Engle (2002).

[8] $(x)^+ = \max(x,0), (x)^- = -\min(x,0)$.

The final specification is called *Indirect GARCH(1,1)* which is also mean reverting and as with the SAV specification, it responds symmetrically to past returns. This specification can be correctly modeled under the assumption that the underlying data process follows a true GARCH(1,1) with an i.i.d. error distribution.[9] The algebraic expression of this specification is as follows:

$$f_t(\beta) = (\beta_1 + \beta_2 f_{t-1}^2(\beta) + \beta_3 y_{t-1}^2)^{1/2} \tag{5}$$

The next step to the analysis is the estimation of the parameters of the alternative CAViaR models. They are estimated using linear and non-linear quantile techniques. These techniques were first introduced by Koenker and Basett (1978) who provide a thorough analysis of how to apply the concept of sample quantile to a linear regression model. We consider the following model proposed by Engle and Manganelli (2004):

$$y_t = x_t'\beta^0 + \varepsilon_{\theta t} \qquad\qquad Quant_\theta(\varepsilon_{\theta t} \mid x_t) = 0 \tag{6}$$

where x_t is a p-vector of regressors and $Quant_\theta(\varepsilon_{\theta t} \mid x_t)$ is, as we have already defined, the θ-quantile of $\varepsilon_{\theta t}$ conditional on x_t. White (1994) has shown that if we minimize the regression quantile objective function that was developed by Koenker and Basett (1978) we can obtain consistent estimates under certain assumptions. This minimization can be considered as follows: We define $f_t(\beta) \equiv x_t\beta$. Then any $\hat{\beta}$ that solves the following problem:

[9] It is worth noting that the CAViaR specifications are more general than the fitted GARCH models. They can allow for a wide range of assumptions with respect to the error distribution and they can also handle distributions with non-i.i.d. errors.

$$\min_{\beta} \frac{1}{T}[\theta - I(y_t < f_t(\beta))][y_t - f_t(\beta)] \quad (7)$$

defines the θ^{th} regression quantile.

Within this framework, Engle and Manganelli (2004) show that the only assumption required is the appropriate specification of the quantile process and, more specifically, we do not have to specify the entire distribution of the error terms. Furthermore, even if we erroneously specify the regression quantile process, Engle and Manganelli (2004) argue that we can still obtain a minimization of equation (5) that satisfies the Kullback-Leibler Information Criterion (which measures the deviation between the true specification and the actual model).

Engle and Manganelli (2004) consider the case where $\hat{\beta}$ is a non-linear regression quantile estimator and they prove that this estimator is consistent and asymptotically normal. Furthermore, they show that there is a consistent estimator of the variance-covariance matrix. Engle and Manganelli (2004) then go on to derive the asymptotic distribution of the estimator. This allows us to conduct hypothesis tests of the quantile models.[10]

Engle and Manganelli (2004) also propose a new test for the evaluation of the alternative specifications which has better power properties than other existing tests. This test allows for the inclusion of a variety of alternative specifications. They define:

[10]Following the seminal paper by Koenker and Bassett (1978), a number of alternative regression quantile models have been developed over the last twenty years that take into account alternative assumptions about the errors. Among others, Koenker and Bassett (1982) allow for the case of heteroskedastic errors, whereas Portnoy (1991) considers the case of non-stationary dependent errors. Furthermore, we have extensions that cover the cases of time series models, simultaneous equations and censored regression models and recently we also have extensions that deal with the case of autoregressive quantiles (Koenker and Zhao, 1996). See Engle and Manganelli (2004) for the relevant literature. All these models differ from the Engle and Manganelli (2004) CAViaR models since they are linear in the parameters.

$$Hit_t(\beta^0) \equiv f(y_t < f_t(\beta^0)) - \theta \quad (8)$$

where the function $Hit_t(\beta^0)$ is assumed to take a value $(1-\theta)$ every time y_t falls below the quantile, and it takes the value $-\theta$ in all other cases. Equation (8) implies that the expectation of $Hit_t(\beta^0)$ is zero. Furthermore, based on the definition of the quantile given in equation (1), we also assume that the conditional expectation of $Hit_t(\beta^0)$, given a set of information at period $t-1$, is zero. This implies that $Hit_t(\beta^0)$ must be uncorrelated with its own lagged values as well as with $f_t(\beta^0)$ and its expected value should equal zero. If these assumptions hold for $Hit_t(\beta^0)$ then we are certain that we have no misspecification error introduced, there is no autocorrelation in the *hits*, and we will obtain the correct fraction of exceptions. Based on definition (8), Engle and Manganelli (2004) derive two test statistics. First, they construct an *in-sample Dynamic Quantile test*. This test is a specification test which is used to select among alternative model specifications of a particular CAViaR process. Second, they construct an *out-of-sample Dynamic Quantile test*. This test is useful to the market regulators and/or the risk managers, since they can examine whether the VaR estimates for a particular financial institution satisfy certain properties such as that they are unbiased, they provide independent hits and they give quantile estimates which are independent. Moreover, it is argued that this second test has some nice features since it is simple in its application and it does not depend on the procedure used for the estimation. We obtain the results from this test simply by using a series of VaR values and the respective value of the portfolio.[11]

[11] The complete derivation of the two tests is given in Engle and Manganelli (2004, p. 370-371). Granger *et al*. (1989) and Christoffersen (1998) are among other studies which have developed test statistics for the validity of the forecast model, but as Engle and Manganell; (2004) point out, they have

4. Empirical results

We apply the alternative CAViaR model specifications on daily data for the period March 3, 1995 to August 28, 2013 for the following general stock indices: S&P 500, FTSE 100, NIKKEI225, DAX30, CAC40 and the Athens Exchange General Index . The data was taken from Datastream. Given our earlier discussion that financial returns exhibit certain properties which imply that it is unreasonable to assume that stock return series follow a certain stochastic process. This evidence motivates this study to consider the CAViaR model by accommodating different processes into the prediction horizon. Table 1 reports summary statistics for each stock market daily returns. The significant negative skewness and kurtosis show that the return process deviates from the normal distribution. Figure 1 presents the evolution of prices and returns exhibiting strong evidence of clustering. The daily returns are computed as 100 times the difference of the log of the prices. Finally in order to implement our analysis we construct historical series of portfolio for each case and we choose a specification of the functional form of the quantile.

Our analysis begins with estimation of the four CAViaR specifications described in section 3. For the estimation of the models we used the first 4364 observations and the last 500 to conduct the out-of-sample testing performance. Huang *et al*. (2009) argue that most of the VaR models assume that time series follow a certain stochastic process and the models are fully parametric. The issue here is that the model parameters are estimated from data, which in many cases are subject to structural breaks. To this end, Bao *et al*. (2006) examine the performance of VaR models for a group of Southeast Asian countries under different economic and/or

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low power against misspecification introduced by the presence of serial correlation in the conditional probabilities leading to a quantile measurement error.

political environments by considering three out-of-sample evaluation periods. The main findings of their analysis show that; (a) risk forecasts with VaR would yield poorer results during a period of crisis (1997-1998) than during tranquil periods and (b) none of the VaR models they test is satisfactory in the crisis period, including the CAViaR specifications as well.

We estimated 1% and 5% one day Value-at-Risk.[12] As Engle and Manganelli (2004) proved, all the models are both continuous and continuously differentiable in β.[13] Our results are summarized in Tables 1-2. Each table reports the value of the estimated parameters, the respective standard errors and the one-sided p-values. Furthermore, each table shows the value of the regression objective function given by equation (3) above. Finally, we report the percentage of times the VaR is exceeded and the *in-and out-of-sample p-value* of the Dynamic Quantile test. The computation of the VaR series with the CAViaR models has been done with the initialization of $f_1(\beta)$ to the empirical θ-quantile of the first 300 observations. With respect to the computation of the DQ test we used a constant, the VaR forecast and the first four lagged hits as instrumental variables. In contrast, to avoid the presence of collinearity in the matrix of the first and higher order derivatives, we did not include the constant and the VaR forecast.[14] Following Engle and Manganelli (2004) we compute the standard errors and the variance covariance matrix of the in-sample DQ test and the

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[12] To estimate the Adaptive model we set $G = 10$, in all cases where G entered the definition of the Adaptive model in section 3.

[13] All assumptions which these models need to satisfy are given in Appendix A of Engle and Manganelli (2004); and they refer to asymptotic results but are difficult to verify in finite samples.

[14] As Engle and Manganelli (2004) point out, the lagged hit variables contain the indicator function. Given that the indicator function is Lipschitz continuous it satisfies condition, DQ3 of theorem 4 (Engle and Manganelli, 2004).

calculation of the statistics $\hat{D}_T$ and $\hat{M}_T$ was done with the use of the *k*-nearest neighbour estimators, with $k = 40$ for the 1% VaR and $k = 60$ for the 5% VaR.[15]

The first important observation we make in both tables is that the coefficient β_2 is very significant and this implies that volatility clustering is verified for the stock price returns of the six general indices. More specifically, this carries over to the tails of the distribution. Second, we note the accuracy of the alternative models. This is measured by the percentage of in-sample hits. Consider the results of the 1% VaR. We observe that for either the case of the mature market or the emerging market, or even the case of the general stock indices the Symmetric Absolute Value, the Asymmetric Slope and the Indirect GARCH models provide estimates which are extremely close to the value of 1, which is taken as evidence that they describe the evolution of the tail for most of the cases. Specifically, the results are particularly good for the stock returns of the S&P 500, FTSE100 and DXA30 . In these cases we further observe that the out-of-sample hits are exactly equal to one or about 1.2%. For the stock returns of CAC40, NIKKEI225 and ASE the accuracy of the in-sample hits is fairly good, but the out-of sample hits are substantially below the value of 1. Furthermore, for most cases the Adaptive model provides inferior results either for in- or out-of sample hits. Again we see that the Symmetric Absolute Value, the Asymmetric Slope and the Indirect GARCH models provide accurate in-sample estimations, whereas the Adaptive specification provides estimates well away from the 1% benchmark. Furthermore, the Symmetric Absolute Value, the Asymmetric

[15] The calculation of the two statistics is described in Engle and Manganelli, theorems 3 and 4. Furthermore, for the optimization procedures we adopt the strategy explained in Engle and Manganelli (2004). The computations were made in Matlab 6.1 using the functions *fminsearch* and *fminunc* as the optimization algorithms while the loops to compute the recursive quantile functions were coded in C and have been developed by Manganelli.

Slope and the Indirect GARCH provide in-sample hits near the value of 1, but the out-of sample hits are some distance from this value.

We then turn to the results for the 5% VaR. We first discuss the results for the U.S. companies which are traded on the NYSE. The in-sample hits range from 4.9% to 5.1% for the Symmetric Absolute Value, the Asymmetric Slope and the Indirect GARCH models, while the Adaptive model misses the target in this case as well. The out-of sample forecasts are very accurate for the case of MERK when we apply any of these three models, and also for the case of ALCOA when we apply the Indirect GARCH specification. When we turn to the case of the companies from the emerging markets we note that we obtain estimates which are extremely close to the value of 5%, which is taken as evidence that they describe the evolution of the tail for all cases under consideration. Looking into the out-of-sample forecasts, the performance is similar to the one obtained in the developed market. Finally, a similar pattern emerges for all three specifications as well the adaptive model when we examine the five stock indices. A noticeable exemption is the FTSE20 index. The rejection of all specifications by the Dynamic Quantile-in-sample test may be attributed to the presence of a speculative bubble during the period 1998-2000.

A final comment we make is that in most cases the estimation of the Asymmetric Slope model gives coefficient estimates for the negative lagged returns which are always statistically significant while the estimates associated with the positive returns are not significantly different from zero. Therefore, we may argue there are possible strong asymmetric influences on VaR measures of lagged returns.

The overall results from the present analysis show that the DQ test statistics select different CAViaR specifications for different confidence internals which may lead to the argument that the process guiding the tail behaviour changes over time.

This contradicts the fundamental assumptions of the volatility parametric models that the tails of the distribution follow the same process as the rest of the portfolio returns.

5. Summary and concluding remarks

The recent financial crisis of 2007-2009 has brought in surface the need for more accurate measurement of the downside risk of financial institutions. A significant contribution to this increased volatility has been the substantial rise of capital flows from mature markets towards the emerging markets of South East Asia and the economies of transition. The 1997-1998 financial crisis as well the bankruptcy of several financial institutions during the 1990s have shown quite convincingly how important for the stability of the global financial system is the development and adoption of the appropriate mechanisms for measuring market risk. However, the recent financial turbulence and the reversal of capital flows in an effort of the hedge funds to liquidate their investment in emerging markets has once again led to an increase volatility in the global financial markets.

The *Riskemetrics* methodology by Morgan Stanley (1996) provided the first econometric methodology for the measurement of market risk based on the concept of VaR which was introduced in Banking Supervision's (1996), "*Amendment to the Capital Accord to Incorporate Market Risks*". This led to a fast growing literature on the development of alternative methodologies to measure VaR. The present paper utilized the CAViaR modeling procedure, which has been proposed by Engle and Manganelli (2004). This is a semiparametric method which shifts the analysis of developing a good measure of the VaR from the distribution of the portfolio returns directly to the behavior of the quantile. This methodology is considered to have several advantages over competing VaR methodologies like the GARCH models and the Extreme Value Theory.

We applied this methodology to estimate the VaR using daily observations for the period January 3, 1995 to August 28, 2013. We studied the behaviour of alternative CAViaR specifications for six stock market indices, namely the S&P 500, FTSE 100, DAX30, CAC40, NIKKE225 and the Athens Exchange General Index.

Our overall results led to the conclusion that this methodology provided very accurate measurement of the VaR for all stock price indices. This evidence is based on the Kupiec (1995) in-sample forecasting performance and the Cristoffersen (1998) out-of-sample forecasting performance. These findings may be of interest to portfolio managers, private and institutional investors as well as hedge funds that are active in mature and emerging stock markets.

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TABLE 1: Estimates and Relevant Statistics for the four Conditional Autoregressive Value at Risk Models

1%VaR	**Symmetric Absolute Value**			**Asymmetric slope**			**Indirect GARCH**			**Adaptive**		
	S&P500	**FTSE100**	**DAX30**	**S&P500**	**FTSE100**	**DAX30**	**S&P500**	**FTSE100**	**DAX30**	**S&P500**	**FTSE100**	**DAX30**
Beta 1	**0.0129**	**0.1085**	**0.1364**	0.0310	**0.1378**	**0.0554**	0.1538	0.2204	0.0706	**1.1023**	**0.4687**	**0.3404**
Standard Errors	*0.0053*	*0.0403*	*0.0516*	*0.0242*	*0.0665*	*0.0328*	*0.1029*	*0.1823*	*0.0753*	*0.0943*	*0.1188*	*0.1046*
p values	*0.0076*	*0.0035*	*0.0041*	*0.1004*	*0.0192*	*0.0455*	*0.0674*	*0.1133*	*0.1742*	*0.0000*	*0.0000*	*0.0006*
Beta 2	**0.9923**	**0.9419**	**0.9475**	**0.9653**	**0.8964**	**0.9711**	**0.9731**	**0.9432**	**0.9859**	0	0	0
Standard Errors	*0.0033*	*0.0295*	*0.0162*	*0.0170*	*0.0361*	*0.0161*	*0.0049*	*0.0144*	*0.0044*	*0*	*0*	*0*
p values	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0*	*0*	*0*
Beta 3	**0.0223**	**0.1483**	**0.1059**	-0.0009	**0.1263**	0.0090	0.1081	0.2500	0.0633	0	0	0
Standard Errors	*0.0084*	*0.0729*	*0.0394*	*0.0312*	*0.0713*	*0.0432*	*0.3793*	*0.2190*	*0.3635*	*0*	*0*	*0*
p values	*0.0039*	*0.0209*	*0.0036*	*0.4885*	*0.0382*	*0.4172*	*0.3878*	*0.1268*	*0.4309*	*0*	*0*	*0*
Beta 4	0	0	0	**0.1823**	**0.3119**	**0.0936**	0	0	0	0	0	0
Standard Errors	*0*	*0*	*0*	*0.0993*	*0.1485*	*0.0334*	*0*	*0*	*0*	*0*	*0*	*0*
p values	*0*	*0*	*0*	*0.0332*	*0.0178*	*0.0025*	*0*	*0*	*0*	*0*	*0*	*0*
RQ	195.47	188.14	190.88	190.86	1.88.14	189.86	196.34	190.17	192.91	199.49	190.10	194.03
Hits in-sample(%)	0.9822	0.9822	0.9822	0.9515	0.9822	0.9822	1.0129	1.0129	0.9822	1.0436	1.0743	1.1050
Hits out-of-sample(%)	0.6000	1.0000	1.0000	1.2000	1.4000	1.0000	0.8000	1.0000	1.0000	0.8000	0.4000	1.0000
DQ in-sample (p values)	0.0433	0.0967	0.5054	0.7451	0.7436	0.7340	0.0262	0.0655	0.4657	0.5298	0.0702	0.5975
DQ out-of-sample (p values)	0.9912	0.9430	0.9996	0.8445	0.9044	0.9998	0.9966	0.9809	0.9980	0.8857	0.9133	0.0012
5%VaR	**Symmetric Absolute Value**			**Asymmetric slope**			**Indirect GARCH**			**Adaptive**		
	S&P500	**FTSE100**	**DAX30**	**S&P500**	**FTSE100**	**DAX30**	**S&P500**	**FTSE100**	**DAX30**	**S&P500**	**FTSE100**	**DAX30**
Beta 1	**0.0071**	**0.0387**	**0.0306**	**0.0069**	0.0038	**0.0414**	0.0573	0.0175	0.0043	**0.1501**	**0.2859**	**0.4779**
Standard Errors	*0.0029*	*0.0115*	*0.0167*	*0.0042*	*0.0138*	*0.0214*	*0.0448*	*0.0271*	*0.0241*	*0.0355*	*0.0602*	*0.0675*
p values	*0.0073*	*0.0004*	*0.0339*	*0.0497*	*0.3919*	*0.0267*	*0.1005*	*0.2588*	*0.4299*	*0.0000*	*0.0000*	*0.0000*
Beta 2	**0.9927**	**0.9595**	**0.9652**	**0.9858**	**0.9601**	**0.9512**	**0.9650**	**0.9695**	**0.9667**	0	0	0
Standard Errors	*0.0039*	*0.0068*	*0.0089*	*0.0036*	*0.0086*	*0.0119*	*0.0069*	*0.0045*	*0.0043*	*0*	*0*	*0*
p values	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0*	*0*	*0*
Beta 3	**0.0146**	**0.0792**	**0.0703**	0.0071	**0.0757**	0.0190	0.0677	0.0655	0.0747	0	0	0
Standard Errors	*0.0065*	*0.0142*	*0.0154*	*0.0083*	*0.0232*	*0.0224*	*0.0816*	*0.1061*	*0.2297*	*0*	*0*	*0*
p values	*0.0118*	*0.0000*	*0.0000*	*0.1936*	*0.0006*	*0.1980*	*0.2032*	*0.2687*	*0.3726*	*0*	*0*	*0*
Beta 4	0	0	0	**0.0425**	**0.0785**	**0.1163**	0	0	0	0	0	0
Standard Errors	*0*	*0*	*0*	*0.0079*	*0.0146*	*0.0278*	*0*	*0*	*0*	*0*	*0*	*0*
p values	*0*	*0*	*0*	*0.0000*	*0.0000*	*0.0000*	*0*	*0*	*0*	*0*	*0*	*0*
RQ	677.75	592.47	627.77	674.74	592.88	620.30	677.95	594.59	630.23	681.04	605.12	622.69
Hits in-sample(%)	5.0031	5.0031	4.9724	5.0031	4.9724	4.9724	5.0645	4.9417	5.0645	5.4328	5.1565	4.8496
Hits out-of-sample(%)	4.4000	3.8000	5.4000	4.4000	4.4000	4.8000	5.2000	3.2000	5.0000	3.4000	3.2000	4.4000
DQ in-sample (p values)	0.0140	0.1404	0.0140	0.1782	0.1606	0.7128	0.0858	0.1879	0.0627	0.0528	0.0180	0.3899
DQ out-of-sample (p values)	0.9565	0.8789	0.0189	0.8185	0.8299	0.0001	0.8871	0.5762	0.0932	0.6494	0.2679	0.0001

Note: Significant coefficients at 5% level of significance are given in bold; shaded boxes denote rejection from the DQ test at 1% significance level.

TABLE 2: Estimates and Relevant Statistics for the four Conditional Autoregressive Value at Risk Models

1%VaR	Symmetric Absolute Value			Asymmetric slope			Indirect GARCH			Adaptive		
	CAC40	NIKKEI225	ASE	CAC40	NIKKEI225	ASE	CAC40	NIKKEI225	ASE	CAC40	NIKKEI225	ASE
Beta 1	**0.1531**	**1.4012**	**0.1670**	0.1893	**0.6336**	**0.0669**	1.0975	1.4216	0.1877	**0.3986**	**0.7651**	**0.5317**
Standard Errors	*0.0894*	*0.0463*	*0.0273*	*0.1461*	*0.1865*	*0.0416*	*1.1544*	*0.7568*	*0.1475*	*0.1512*	*0.1216*	*0.0703*
p values	*0.0434*	*0.0003*	*0.0000*	*0.0975*	*0.0002*	*0.0538*	*0.1709*	*0.0302*	*0.1016*	*0.0042*	*0.0000*	*0.0006*
Beta 2	**0.9335**	**0.5229**	**0.8988**	**0.8908**	**0.6457**	**0.8963**	**0.8907**	**0.7763**	**0.8978**	0	0	0
Standard Errors	*0.0341*	*0.1213*	*0.0159*	*0.0159*	*0.0847*	*0.0251*	*0.0622*	*0.0545*	*0.0186*	*0*	*0*	*0*
p values	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0*	*0*	*0*
Beta 3	**0.1428**	**0.6255**	**0.2782**	**0.1347**	**0.3432**	**0.2328**	0.3138	0.8323	0.5366	0	0	0
Standard Errors	*0.0674*	*0.1288*	*0.0392*	*0.0567*	*0.2089*	*0.1054*	*0.2677*	*0.4695*	*0.1362*	*0*	*0*	*0*
p values	*0.0000*	*0.0000*	*0.0000*	*0.0087*	*0.0502*	*0.0133*	*0.1206*	*0.0381*	*0.0000*	*0*	*0*	*0*
Beta 4	0	0	0	**0.2831**	**0.9444**	**0.2960**	0	0	0	0	0	0
Standard Errors	*0*	*0*	*0*	*0.1890*	*0.4007*	*0.0602*	*0*	*0*	*0*	*0*	*0*	*0*
p values	*0*	*0*	*0*	*0.0671*	*0.0092*	*0.0000*	*0*	*0*	*0*	*0*	*0*	*0*
RQ	189.41	177.50	135.52	187.63	173.17	136.14	191.72	176.32	135.89	193.89	185.87	140.26
Hits in-sample(%)	0.9813	1.0120	0.9813	0.9813	0.9813	0.9813	1.0120	1.0120	1.0120	1.0120	1.0426	1.1653
Hits out-of-sample(%)	0.6000	1.0000	1.2000	0.6000	1.2000	1.2000	0.2000	1.2000	1.2000	0	0.6000	0.2000
DQ in-sample (p values)	0.7362	0.5610	0.0429	0.7391	0.6177	0.5510	0.7484	0.5684	0.5809	0.4893	0.0032	0.0807
DQ out-of-sample (p values)	0.9833	0.9966	0.8638	0.9879	0.9860	0.9498	0.7805	0.9943	0.9552	0.9601	0.9073	0.7836

5%VaR	Symmetric Absolute Value			Asymmetric slope			Indirect GARCH			Adaptive		
	CAC40	NIKKEI225	ASE	CAC40	NIKKEI225	ASE	CAC40	NIKKEI225	ASE	CAC40	NIKKEI225	ASE
Beta 1	**0.0946**	**0.1934**	**0.0437**	**0.0662**	**0.1443**	0.0066	**0.1454**	**0.2656**	0.0202	**0.3468**	**0.3201**	**0.205**
Standard Errors	*0.0214*	*0.0515*	*0.0107*	*0.0217*	*0.0323*	*0.0076*	*0.0560*	*0.1431*	*0.0233*	*0.0656*	*0.0366*	*0.042*
p values	*0.0000*	*0.0001*	*0.0000*	*0.0011*	*0.0000*	*0.1948*	*0.0047*	*0.0317*	*0.1928*	*0.0000*	*0.0000*	*0.000*
Beta 2	**0.9344**	**0.8660**	**0.9525**	**0.9319**	**0.8581**	**0.9525**	**0.9368**	**0.8652**	**0.9504**	0	0	
Standard Errors	*0.0131*	*0.0390*	*0.0147*	*0.0151*	*0.0170*	*0.0099*	*0.0081*	*0.0281*	*0.0055*	*0*	*0*	
p values	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0*	*0*	
Beta 3	**0.0893**	**0.1726**	**0.0953**	**0.0648**	**0.0653**	**0.1264**	0.0923	**0.2080**	0.1161	0	0	
Standard Errors	*0.0192*	*0.0525*	*0.0249*	*0.0191*	*0.0306*	*0.0242*	*0.1177*	*0.0587*	*0.1135*	*0*	*0*	
p values	*0.0000*	*0.0005*	*0.0001*	*0.0003*	*0.0163*	*0.0000*	*0.2165*	*0.0002*	*0.1532*	*0*	*0*	
Beta 4	0	0	0	**0.1091**	**0.2681**	**0.0533**	0	0	0	0	0	
Standard Errors	*0*	*0*	*0*	*0.0389*	*0.0277*	*0.0219*	*0*	*0*	*0*	*0*	*0*	
p values	*0*	*0*	*0*	*0.0025*	*0.0000*	*0.0074*	*0*	*0*	*0*	*0*	*0*	
RQ	623.46	579.55	484.60	622.10	568.39	480.85	623.65	579.41	483.60	630.41	579.35	494.3
Hits in-sample(%)	4.9678	4.9678	5.0291	4.9371	5.0291	5.0291	5.0291	5.0598	4.8758	4.9371	5.2131	48.49
Hits out-of-sample(%)	2.4000	3.4000	4.2000	2.4000	3.0000	3.2000	2.4000	3.4000	4.0000	3.6000	4.0000	3.200
DQ in-sample (p values)	0.3575	0.0067	0.5124	0.8367	0.1924	0.3690	0.3200	0.0072	0.3057	0.0460	0.0000	0.099
DQ out-of-sample (p values)	0.1050	0.4659	0.6450	0.1253	0.4333	0.4603	0.1385	0.7249	0.5205	0.3270	0.2391	0.223

Note: Significant coefficients at 5% level of significance are given in bold; shaded boxes denote rejection from the DQ test at 1% significance level.