# Should investors include commodities in their portfolios after all? New evidence*

Charoula Daskalaki[a] and George Skiadopoulos[b]

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## Abstract

This paper investigates whether an investor is made better off by including commodities in a portfolio that consists of traditional asset classes. To this end, a more general approach is taken compared to the "in-sample, mean-variance" one followed by previous literature. First, the posed question is revisited within an in-sample setting by resorting to a statistical framework. Tests for both mean-variance and non-mean-variance spanning are employed. Then, optimal portfolios are formed by taking into account the higher order moments of the returns distribution and their out-of-sample performance is evaluated. Under the in-sample setting, we find that commodities should be used only by investors whose preferences are described by non mean-variance utility functions. However, these benefits are not preserved out-of-sample. The results challenge the alleged diversification benefits of commodities and are remarkably robust across a number of performance evaluation measures, utility functions, and datasets. They hold even when transaction costs are taken into account and over the recent 2007-2009 crisis period.

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[a] Department of Banking and Financial Management, University of Piraeus, Greece, e-mail: chdask@webmail.unipi.gr

[b] Department of Banking and Financial Management, University of Piraeus, Greece, and Financial Options Research Centre, Warwick Business School, University of Warwick, UK, e-mail: gskiado@unipi.gr

# 1. Introduction

Investments in commodities have grown rapidly over the last years. They take place mainly via commodity futures and commodity index funds. It has been estimated that "*...inflows into commodity investments during 2009 will be a record $60 billion, topping $51 billion from 2006...*" (Wall Street Journal, January 4, 2010) with the prospect being that they will increase even further.[1] Furthermore, Stoll and Whaley (2009) estimate the total commodity index investment in the U.S. to be about $174 billion in 2009. The popularity of investing in commodities has been commonly attributed to the fact that from a theoretical point of view, commodities are expected to form an alternative asset class. Their returns are expected to show small or even negative correlation with the returns of assets that belong to traditional asset classes like stocks and bonds. This is because the value of commodities is driven by factors such as weather and geopolitical conditions, supply constraints in the physical production, and event risk that are distinct from those that determine the value of stocks and bonds (see also Anson, 2002, and Geman, 2005, for a discussion). In fact, a number of empirical studies have confirmed this type of correlation over certain periods of time (see e.g., Bodie and Rosansky, 1980, Erb and Harvey, 2006, Gorton and Rouwenhorst, 2006, Büyükşahin et al., 2010, Chong and Miffre, 2010). Consequently, diversification benefits, i.e. reduction of risk for any given level of expected return, may emerge.[2] However, there is evidence that the growing presence of index funds in commodities markets integrates the commodity markets with the stock and bond ones (Silvennoinen and Thorp, 2010, Tang and Xiong, 2010). This calls the diversification benefits of commodities into question.[3] This paper revisits the common perception on the diversification role of commodities by investigating the benefits of investing in commodities in a more general setting than the one that the previous literature has adopted so far.

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[1] "… according to a survey of more than 300 attendees at Barclays Capital's fifth annual US Commodities Investor Conference…63% of those surveyed indicated they plan to increase their commodity exposure over the next three years." (Barclays press release, December 2009, http://www.barcap.com/About+Barclays+Capital/Press+Office ).

[2] The appeal of investing in commodities is also attributed to their ability to hedge against changes in inflation (see e.g., Bodie and Rosansky, 1980, Bodie, 1983, Gorton and Rouwenhorst, 2006). In addition, there is evidence that profitable trading strategies with commodities can be constructed. A number of studies find that commodity returns can be predicted by a number of variables (see e.g., Hong and Yogo, 2010, and the references therein), and profitable momentum and term structure strategies can be constructed (Erb and Harvey, 2006, Gorton et al., 2007, Miffre and Rallis, 2007, Asness et al., 2009, Fuertes et al., 2010).

[3] The evidence on markets integration is mixed though. Chong and Miffre, (2010) and Büyükşahin et al., (2010) find that commodity and equity markets have become more segmented over the years in contrast to the findings of Tang and Xiong (2010) and Silvennoinen and Thorp (2010).

There is already a number of papers that have examined whether incorporating commodities in the asset menu improves the risk-return profile of investors portfolios. Bodie and Rosansky (1980) and Fortenbery and Hauser (1990) found that by switching from a stock portfolio to a portfolio with stocks and commodities over the periods 1950-1976 and 1976-1985, respectively, can reduce risk without sacrificing the obtained return. Georgiev (2001) performed a similar analysis over the period 1995-2005 and found an increase in the Sharpe ratio. In addition, a number of studies have investigated the role of commodities under the Markowitz (1952) mean-variance (MV) static asset allocation setting and reached similar conclusions. Ankrim and Hensel (1993) studied the diversification benefits of investing in commodities over the period 1972-1990, and concluded that expanding the investable universe with commodities improves the risk/return trade-off of optimal portfolios for any given risk tolerance coefficient. Satyanarayan and Varangis (1996) examined whether the efficient frontier changes when commodity futures are incorporated in international stock portfolios over the period 1970-1992. They found that the inclusion of commodities shifts the efficient frontier upwards. Anson (1999) addressed the same question from another perspective. He formed optimal portfolios by maximising a quadratic expected utility for a range of risk aversion coefficients over the period 1974-1997. He concluded that adding commodities to a portfolio of stocks and bonds increases the Sharpe ratio of optimal portfolios. Jensen et al. (2000) have also found that including commodities in a traditional asset universe improves the risk-return profile of the efficient portfolios over the period 1973-1997. Idzorek (2007) performed a similar empirical analysis over the period 1970-2005 and reached similar conclusions.

Therefore, the above mentioned literature has provided unanimous evidence that the investor is better off by including commodities in her portfolio. However, this conclusion has been reached under a MV setting by comparing the position of the efficient frontiers corresponding to the without-commodities universe and the expanded one that includes commodities, respectively. This approach is subject to three shortcomings though. First, the Markowitz setting may not reflect accurately the gains from investing in commodities since it is founded on two assumptions, i.e. that either the distribution of the asset returns is normal or investor's preferences are described by a quadratic utility function. Neither of these two conditions is expected to hold. In particular, there is ample empirical evidence that asset returns are not distributed normally, especially for relatively short horizons (see e.g., Longin 1996, Peiro, 1999, for equities, and Gorton and Rouwenhorst, 2006, Kat and Oomen, 2007a, for commodity futures). In the case where the non-normality of returns is not taken into account in the optimal portfolio formation process, then there is a utility loss (Jondeau and Rockinger, 2006). Furthermore, a quadratic utility function exhibits negative marginal utility after a certain finite wealth level and increasing absolute risk aversion with respect to wealth (Hanoch and Levy, 1970); both these features are not consistent with rational behavior. The second shortcoming is that the comparison of the position of efficient frontiers should be set within a statistical framework; the previously mentioned commodities papers asses the diversification benefits of investing in commodities by eyeballing the position of efficient frontiers. Third, all previous studies have investigated the benefits of investing in commodities within an in-sample setting. In principle, the portfolio choice should be examined in an out-of-sample setting given that on any given point in time, the investor decides on the portfolio weights and the portfolio returns to be realised over the investment horizon is uncertain.

In light of the previously mentioned shortcomings, this paper takes a more general approach to examine whether commodities should be included in an investors' portfolio. In particular, it considers an investor who allocates funds between equities, bonds, a risk-free asset, and commodities in a standard static asset allocation context and makes the following five contributions to the existing literature. First, it revisits the posed question within an in-sample setting by employing rigorous tests instead of eyeballing the relative position of efficient frontiers based on traditional and traditional augmented with commodities asset universes. To this end, the regression-based spanning techniques are applied to test for spanning when investor preferences are described by utility functions that are consistent with the MV setting, as well as, a more general non-MV one (see e.g., Huberman and Kandel, 1987, DeRoon and Nijman, 2001, for MV spanning, and DeRoon et al., 1996, for generalized non-MV spanning tests).[4]

Second, it examines the question under scrutiny by employing an *out-of-sample* setting. In line with DeMiguel et al. (2009) and Kostakis et al. (2010), static one-period optimal portfolios are formed at any point in time, their corresponding realised returns are calculated and their performance is evaluated under a number of performance measures. Third, optimal portfolios are constructed by taking into account the higher order moments of the returns

[4] Huang and Zhong (2006), Nijman and Swinkels (2008), Scherer and He (2008), and Galvani and Plourde (2010) have also applied spanning techniques to assess the diversification benefits of investing in commodities. However, their analysis is placed under a MV setting.

distributions of the involved assets. To this end, direct utility maximization is performed (e.g., Cremers et al., 2005, Adler and Kritzman, 2007, Sharpe, 2007). The appeal of this approach compared to the MV optimization applied by previous studies is that the optimal portfolios can be derived by maximizing the expected utility of the investor for any assumed type of returns distribution and description of her preferences.

Fourth, the posed question is studied by considering alternative ways of investing in commodities. The popular commodity indexes of S&P GSCI and DJ-UBS CI, as well as individual commodity futures contracts written on different types of commodities are considered. The previous literature on asset allocation with commodities has assumed that the investor can invest only in commodity indexes. In practice, this is not the case; instead investors follow different strategies represented by the available menu of futures written on individual commodities. Most importantly, the use of alternative commodity instruments will serve as a robustness test to the subsequently reported findings. This is because commodities present a significant heterogeneity in terms of their risk-return characteristics that commodity indexes fail to capture (Kat and Oomen, 2007a, and Erb and Harvey, 2006). Finally, an extended dataset that spans the period January 1989 to December 2009 is used. The previous studies rely on data up to 2005 only. Therefore, the effect of the bearish and bullish regimes in commodity prices over the period 2006-2008, the recent 2007-2009 subprime credit crisis, as well as that of the increasing presence of index investors in commodities markets and the potential markets integration has yet been left unexplored within a commodities asset allocation setting.

To check the robustness of the obtained results, a number of tests are conducted. First, various utility/value functions and degrees of risk aversion that describe the preferences of the individual investor are employed. This is because the formation of optimal portfolios is investor specific. In particular, exponential and power utility functions, as well as, the disappointment aversion setting introduced by Gul (1991) are adopted. The latter takes into account behavioural characteristics in investor's preferences. Second, a number of performance measures (Sharpe ratio, opportunity cost, portfolio turnover and risk-adjusted returns net of transaction costs) are used to compare the performance of the optimal portfolio based on traditional and augmented with commodities opportunity sets, respectively. This will enable to take into account the impact of the higher order moments as well as that of transaction costs on performance evaluation. Third, the optimal portfolios are calculated by also maximising the expected utility approximated by its second order (truncated) Taylor series expansion. This serves to check whether the in-sample diversification benefits of commodity investing in a MV framework reported by previous studies still show up in an out-of-sample setting. Finally, the impact of the recent 2007-2009 credit crisis is studied.

The rest of the paper is structured as follows. Section 2 describes the dataset. Section 3 outlines the tests for of spanning and discusses the results. Section 4 sets the asset allocation framework and then compares the out-of-sample performance of optimal portfolios that contain commodities with that of those that do not contain commodities.. Sections 5 and 6 investigate whether the results are robust under a MV setting and over the recent crisis period, respectively. The last section provides a summary of our results.

## 2. The dataset

The dataset is comprised of monthly closing prices of the alternative asset classes used in this study, provided by Bloomberg. In particular, the S&P 500 and Barclays U.S. Aggregate Bond Index are used as representative indexes of the equity and bond markets, respectively. The Libor one-month rate is used for the risk-free rate. To get exposure to the commodity asset class, various well followed commodity futures indexes, as well as individual commodity futures contracts are used. In particular, the S&P Goldman Sachs Commodity Index, Dow Jones-UBS Commodity Index, and five individual futures contracts on Crude oil (NYMEX), Cotton (NYBOT), Copper (COMEX), Gold (COMEX) and Live cattle (CME) are employed. The dataset for all assets spans the period from January 1989 to December 2009, with the exception of DJ-UBS CI that covers the period from January 1991 to December 2009, due to data availability constraints.

Commodity indexes represent passive investment strategies in a number of the shortest expiry commodity futures. To maintain a continuous time series roll-over is performed as every contract approaches expiry. The S&P Goldman Sachs Commodity Index (S&P GSCI) was launched in January 1991. The index currently invests in twenty four commodities classified into five groups: energy, precious metals, industrial metals, agricultural and livestock. The S&P GSCI is heavily concentrated on the energy sector (almost 70% of the total index value) since its portfolio weighting scheme is based on the level of worldwide production for each commodity over the past five years. The Dow Jones-UBS Commodity Index (DJ-UBS CI) was launched in

July 1998 with historical data available from January 1991. The index invests in nineteen commodities from the energy, precious metals, industrial metals, agricultural and livestock sectors. The weights of the individual commodity futures contracts in DJ-UBS CI are primarily based on futures contract liquidity data (the dominant factor), supplemented with commodity world production data. The DJ-UBS CI relies on two important rules to ensure diversification: the minimum and the maximum allowable weight for any single commodity is 2% and 15%, respectively, and the maximum allowable for any sector is 33%.

In the case of individual commodity futures contracts, each one of them has an underlying commodity that belongs to one of the basic five commodity sectors, respectively: energy, industrial metals, precious metals, agriculture, and livestock. Crude oil is the world's most actively traded commodity. Futures contracts on light sweet crude oil (WTI) are traded on NYMEX. They are the world's largest-volume futures contract on a physical commodity. Each futures contract has a 1,000 barrels contract size and its price is quoted in U.S. dollars per barrel. The last trading date for crude oil futures contracts is the third business day prior to the twenty-fifth calendar day of the month preceding the delivery month. Copper is the world's third most widely used metal and is primarily used in the infrastructure and construction industries. Therefore, its price is considered to reflect the current state of the world economy. The contract size is 25,000 pounds and its price is quoted in US cents per pound. The last trading day for copper futures is the third last business day of the delivery month. Next, cotton futures have been traded in New York since 1870. They have been used by the domestic and global cotton industries to price and hedge transactions. The NYBOT cotton futures specifies delivery of 50,000 pounds net weight upon expiry and its price is quoted in terms of U.S. cents per pound. The last trading day is seventeen business days from end of the delivery month. Gold has been a traditional investment vehicle since it serves as a hedge against inflation and a safe haven in periods of market crises (see e.g., Baur and McDermott, 2010). Each gold futures contract (traded on COMEX) has a contract size of 100 troy ounces and its price is quoted in U.S. dollars and cents per troy ounce. The last trading day for gold futures is the third last business day of the delivery month. Finally, the livestock futures market serves mainly commodity merchandisers, producers, and processors. The live cattle futures has 40,000 pounds contract size and its price is quoted in U.S. cents per pound. The last trading day is the last business day of the delivery month.

The Bloomberg generic shortest futures series is used for each one of the five commodity futures. Bloomberg creates continuous time series of future prices by rolling over from the shortest series to the next shortest as the shortest approaches maturity. Roll over takes place on the first day of the month that the futures expires (for a description on generics, see also Chantziara and Skiadopoulos, 2008).

## 2.1. Summary statistics

Table 1 reports the descriptive statistics for the various asset classes and the pairwise correlations (Panels A and B, respectively) over the period from January 1989 to December 2009 (the only exception is DJUBS CI, with data available from January 1991). At this point, few words of caution are in order. Futures contracts are zero-investment instruments i.e. they do not require initial investment, hence their respective returns are considered excess returns (over the risk-free rate). To compare the rate of return on commodity futures with those on stocks and bonds, we approximate the return on a futures position with the sum of the percentage change in the futures prices and the risk-free rate of return (see e.g., Bodie and Rosansky, 1980; Fortenbery and Hauser, 1990). In the case of the commodity indexes (S&P GSCI and DJ-UBS CI), the returns on stocks and bonds are compared with the respective returns on total return indexes.

We can see that the monthly average return on commodity indexes is lower than stocks and bonds and exhibits higher standard deviation. As a result, the annualized Sharpe ratio is considerably higher for bonds and stocks than commodity indexes. The reported evidence is consistent with previous studies that support that the stand-alone performance of commodity indexes is inferior to other asset classes (see e.g., Jensen et al., 2000). In the case of the individual commodity futures, most contracts exhibit greater average annualized return than stock, bond and commodity indexes as well as greater standard deviation. We can see that the performance of stocks and bonds is superior to that of all commodity futures but crude oil and gold in terms of risk-adjusted returns. The Jarque-Bera test rejects the null hypothesis that the commodity asset returns are distributed normally (at a 5% significance level). Panel B of Table 1 shows that the pairwise correlations of commodity futures with the alternative asset classes are low. This indicates the potential diversification benefits of adding commodities to an already diversified portfolio. In addition, the correlation among the individual commodities is low. This is in line with the findings reported by Erb and Harvey (2006) and supports the notion that there is a certain degree of heterogeneity among the various commodities. Hence, the concept of an "average" commodity captured by a single commodity index is hard to be accepted.

## 3. In-sample benefits of commodities: Testing for spanning

The concept of spanning was first introduced by Huberman and Kandel (1987) and was initially restricted to a MV framework. In brief, the literature on MV spanning analyzes the effect that the introduction of additional risky assets (termed test assets) has on the MV frontier of a set of benchmark assets (see DeRoon and Nijman, 2001, for a review). MV spanning occurs when the MV frontier derived from the augmented investment opportunity set (benchmark assets plus the test ones) coincides with the frontier of the benchmark assets. This implies that the MV investors cannot improve their risk/return trade-off by adding the test assets, regardless of their risk aversion level.[5] In this section, we are interested in investigating the economic benefits from investing in various commodity products by means of tests for spanning, without restricting ourselves in an MV framework though. To this end, we follow DeRoon et al., (1996, 2003) and analyse the concept of spanning by means of the stochastic discount factor (SDF) that sets the ground for the ensuing discussion of spanning tests within a non-MV framework.

### 3.1. Definition of spanning: The stochastic discount factor approach

Let an investor who considers a set of *K* risky assets, with $R_{t+1}$ the $(K\times 1)$ vector of the respective gross returns. Asset pricing theory dictates that there exists a SDF (also known as pricing kernel), $M_{t+1}$, such that

$$E\left[M_{t+1}R_{t+1} \mid I_t\right] = \iota_K \tag{1}$$

where $I_t$ denotes the information available at time *t* and $\iota_K$ a *K*-dimensional unit vector. The SDF is derived from the first order conditions of a discrete time intertemporal portfolio selection problem, i.e.

$$M_{t+1} = \rho \frac{U'(C_{t+1})}{U'(C_t)} \tag{2}$$

[5] If the two frontiers have only one point in common, this is known as intersection. In this case, there is only one value of the risk aversion coefficient for which mean-variance investors can not improve their risk/return trade-off by including the test assets in their investment set.

where $\rho$ is the investor's subjective discount factor that reflects her rate of time preference (see e.g., Cochrane, 2005, Pennacchi, 2008). Alternatively, the SDF can also be derived from the first order conditions of a simpler portfolio problem where the investor maximizes the expected utility of her terminal wealth (DeRoon and Nijman, 2001). In this case, the SDF is proportional to the first derivative of the assumed utility function of wealth, given the investor's optimal portfolio choice $w^*$,

$$M_{t+1} = cU'(w^* R_{t+1}) \tag{3}$$

where $c$ is a constant and $w^*$ the $(K\times 1)$ vector of optimal portfolio weights (see also DeRoon, et al., 2003). Equation (3) shows that the SDF varies across investors with different utility functions or the same utility function and different risk aversion coefficients.

The investor has to decide whether or not to incorporate a set of test assets, with gross return $R_{t+1}^{test}$, in the initial $K$-asset universe. Without loss of generality, assume that $R_{t+1}^{test}$ contains only one element. Let $M$ be a set of SDFs that price the $K$ benchmark assets, i.e. for each $M_{t+1}$ that belongs to $M$, equation (1) holds. $M$ includes at least the SDF with the minimum variance for any given level of risk aversion. DeRoon et al. (1996) define the test asset to be $M$-spanned by the $K$ assets if and only if the set $M$ of SDFs that prices each one of the $K$-benchmark assets also prices the test asset, i.e. [6]

$$\forall M_{t+1} \in M : \; E\left[M_{t+1} R_{t+1}^{test} \mid I_t\right] = 1 \tag{4}$$

Equation (4) implies that the investor cannot achieve greater utility by incorporating the test asset in her optimal portfolio of benchmark assets. This may be explained as follows. Equation (3) shows that the SDF is a function of the marginal utility evaluated at the return of the optimal portfolio. Let a given utility function. Then, it follows that all assets are priced by the same pricing kernel (i.e. spanning exists) if and only if the terminal wealth obtained by forming the optimal portfolio of the benchmark assets equals that obtained by the optimal portfolio consisting of the benchmark plus the test asset. DeRoon et al. (1996, Proposition 1, page 6) show that the returns $R_{t+1}^{test}$, of the test asset is $M$-spanned by the returns $R_{t+1}$ of the benchmark assets if and only if

[6] See also Ferson et al. (1993) and Bekaert and Urias (1996) for the equivalent definition of spanning in a MV setting. Notice that in the case of intersection, $M$ is a singleton.

$$\hat{R}_{t+1}^{test} = proj\left(R_{t+1}^{test} \left\| \left[ M \cup \left\{ w'R_{t+1} : w \in W \right\} \right] \right. \right) = w'R_{t+1} \text{ for some } w \in W \tag{5}$$

where $w \in W = \left\{ w \in \mathbb{R}^k : w'\iota_k = 1 \right\}$.

Proposition 1 yields the following testable hypothesis: the new asset is *M*-spanned by the benchmark assets if and only if the return of the new asset can be written as the return of a portfolio of the benchmark assets, and a zero-mean error term $\varepsilon_{t+1}$, i.e

$$H_0 : R_{t+1}^{test} = w'R_{t+1} + \varepsilon_{t+1} \tag{6}$$

where $\varepsilon_{t+1}$ is orthogonal to the set *M* of the pricing kernels under consideration, i.e.

$$E\left[ M_{t+1} \left( R_{t+1}^{test} - w'R_{t+1} \right) | I_t \right] = E\left[ M_{t+1} \varepsilon_{t+1} | I_t \right] = 0 \tag{7}$$

Equation (7) stems from subtracting equation (4) from equation (1) once multiplying both sides of (1) by $w$. It implies that under *M*-spanning, the additional return earned by the new asset has zero price and hence is of no value to the investor. Furthermore, equation (6) implies that the new asset adds only to the variance of the portfolio of benchmark assets.

### 3.2. Mean-Variance spanning tests

First, we test for MV spanning. Hansen and Jagannathan (1991) have shown that the SDFs associated with MV optimizing behavior have the lowest variance among all admissible ones (that price correctly a set of asset returns) and are linear in asset returns, i.e.

$$M_{t+1}(v) = v + \beta'(R_{t+1} - E_t(R_{t+1})), v \in \mathbb{R} \tag{8}$$

with $v = E(M_{t+1})$ and $\beta = Var_t(R_{t+1})^{-1} \left\{ \iota_k - vE_t(R_{t+1}) \right\}$. Hence, equation (5) can be estimated by the following linear regression

$$R_{t+1}^{test} = \alpha + \beta R_{t+1} + \varepsilon_{t+1} \tag{9}$$

The null hypothesis for spanning is

$$H_0 : \alpha = 0 \text{ and } \beta\iota_k = 1 \tag{10}$$

The definition of MV spanning by means of the SDF imposes the same restrictions as those proposed by Huberman and Kandel (1987). The restrictions in (10) are tested by Wald test (see e.g., DeRoon and Nijman, 2001). The standard errors of the estimators are corrected by the Newey and West (1987) method to account for the presence of autocorrelation and heteroskedasticity in the residual term.

In the case that the *K*-benchmark asset universe includes also the risk-free asset, the test for MV spanning is modified. If a risk-free asset exists with $R^f$ being the risk-free rate of return, the spanning tests are formulated in excess returns terms.[7] To fix ideas, define $\alpha_J$ to be the intercept in the regression of the test asset's excess returns on the excess returns of the *K* benchmark assets, i.e.

$$R_{t+1}^{test} - R_t^f = \alpha_J + \beta\left(R_{t+1} - R_t^f i_K\right) + \varepsilon_{t+1} \tag{11}$$

with $E\left(\varepsilon_{t+1}\right) = E\left(\varepsilon_{t+1} R_{t+1}\right) = 0$. In Appendix A, we derive the equivalence between the intercepts of equations (9) and (11), i.e.

$$\alpha_J = \alpha - R_t^f\left(1 - \beta i_K\right) \tag{12}$$

Given the regression model in equation (11), imposing the spanning constraints of equation (10) yields $\alpha_J = 0$, i.e.

$$H_0 : \alpha_J = \alpha - R_t^f\left(1 - \beta i_K\right) = 0 \tag{13}$$

Notice that in the case of the excess returns formulation, the hypothesis of spanning amounts to testing only the intercept term. The slope coefficients of the risky assets do not need to add up to one (see also Huberman and Kandel, 1987, Scherer and He, 2008). The missing allocation is filled by the investment in the risk-free asset.

### 3.3. Non mean-variance spanning tests

Next, we outline the test for spanning in the non–MV case. Let investors' preferences be described by a non-MV utility function *U(·)*, i.e. not a quadratic one. Consequently, the set *M* of pricing kernels under consideration includes also the SDFs of the assumed non-MV utility function that correspond to different risk aversion coefficients. Equation (3) implies that any given value for the risk aversion coefficient imposes a different SDF that should be included in the set *M*. Therefore, when a non-MV utility function is considered, test for spanning should be

[7] In this case, testing for spanning is equivalent to testing for intersection, i.e. whether the two frontiers coincide at a given point. This can be easily perceived by means of the MV efficient frontier. In the case where there is a risk-free asset, two mutual fund separation theorem holds, i.e. the efficient frontier is linear and constructed by combining the risk-free asset with the tangency portfolio. Hence, testing for spanning amounts to testing whether the two linear frontiers, that of the test and benchmark assets and the one that includes only benchmark assets, are the same. This is equivalent to testing whether the tangency portfolios are the same, i.e. testing for intersection.

carried out by examining whether the relative restrictions hold for any value of risk aversion. For the purposes of this study, we employ a wide range of risk aversion coefficient for each non-MV utility function of interest, i.e., *i=1,2,…,n.* Following the approach suggested by DeRoon et al. (1996, 2003), we estimate equation (5) by projecting on the set *M* of SDFs, i.e.:

$$R_{t+1}^{test} = \alpha + \beta R_{t+1} + \sum_{i=1}^{n} \gamma_i U_i' \left( w_i^{*\prime} R_{t+1} \right) + \varepsilon_{t+1} \tag{14}$$

and test jointly for spanning in the MV and non-MV case by evaluating the restrictions

$$H_0 : \beta \iota_k = 1 \text{ } and \text{ } \alpha = \gamma_i = 0 \forall i \tag{15}$$

The restrictions in (15) are again tested by a Wald test, where the standard errors of the estimators are corrected by the Newey and West (1987) method.

In the case that the *K*-benchmark asset universe includes also the risk-free asset, the test for non-MV spanning is modified again by employing excess returns. The equivalent regression equation and spanning restrictions in excess returns are derived for the non-MV case (see Appendix B) and the following linear regression equation is estimated

$$R_{t+1}^{test} - R_t^f = \alpha_J + \beta \left( R_{t+1} - R_t^f i_K \right) + \sum_{i=1}^{n} \gamma_i U_i' \left( w_i^{*\prime} R_{t+1} \right) + \varepsilon_{t+1} \tag{16}$$

Hence, the restrictions that need to hold for the joint existence of MV and non-MV spanning, become[8]

$$H_0 : \alpha_J = 0 \text{ and } \gamma_i = 0 \forall i \tag{17}$$

To perform the regression shown in equation (16), the unobserved regressors (i.e. the marginal utilities) need to be estimated. To this end, an assumption about the utility function needs to be made and the optimal portfolio weights need to be estimated. We consider an investor whose preferences are described by either an exponential utility function or a power utility function, for different levels of risk aversion. The negative exponential utility function is defined as:

$$U(W) = -\boldsymbol{exp}\left\{ -\eta W \right\} / \eta, \quad \eta > 0 \tag{18}$$

where $\eta$ is the coefficient of absolute risk aversion (ARA). The power utility function is defined as

[8] Notice that $\alpha_J$ can be interpreted as Jensen's alpha only under the MV setting.

$$U(W)=\frac{W^{1-\gamma}-1}{1-\gamma},\ \gamma\neq 1 \tag{19}$$

where $\gamma$ is the coefficient of relative risk aversion (RRA).

Regarding the estimation of optimal portfolio weights, this is done by applying the Generalized Method of Moments (GMM, see e.g., Jagannathan et al., 2002, Cochrane, 2005). The moment conditions generated by the SDFs of interest need to be defined. Given the assumed non-MV utility function, equations (1) and (3) imply that the returns on the *K* benchmark assets should satisfy the following conditions:

$$E\left[c_i U_i'\left(w_i^{*\prime} R_{t+1}\right) R_{t+1} \mid I_t\right]=\iota_K \quad \forall i \tag{20}$$

Let the parameter vector $\theta_i=[c_i \;\; w_i^*]$ that corresponds to the *ith* value of risk aversion, *i=1,2,..,n.* Define the errors, $u_{t+1}\left(\theta_i\right)$:

$$u_{t+1}\left(\theta_i\right)=c_i U_i'\left(w_i^{*\prime} R_{t+1}\right) R_{t+1}-\iota_K \tag{21}$$

Then, for a sample of size *T*, the moment conditions $g_T(\theta_i)$ are defined as the sample mean of the errors $u_{t+1}\left(\theta_i\right)$ i.e.

$$g_T(\theta_i)\equiv\frac{1}{T}\sum_{t=1}^{T} u_t\left(\theta_i\right)=E_T\left[u_t\left(\theta_i\right)\right]=E_T\left[c_i U_i'\left(w_i^{*\prime} R_t\right) R_t-\iota_K\right] \tag{22}$$

By definition, the SDF (for each *i*) should price each one of the three benchmark assets. This provides us with three moment conditions in order to estimate $\theta_i$. The GMM estimate of $\theta_i$ is obtained by minimizing a quadratic function

$$J_T(\theta_i)=g_T(\theta_i)' W g_T(\theta_i) \tag{23}$$

where *W* is a positive definite weighting matrix. In our case, *W* is set equal to the identity matrix *I*. This is because the number of the unknowns (optimal weights for the three corresponding benchmark assets) equals the number of moment conditions.

### 3.4. Results and discussion

This section tests the spanning hypothesis when a commodity asset is included in a traditional asset universe, consisting of stocks, bonds, and the risk-free asset. The analysis is conducted by using either a commodity index or a futures written on an individual commodity, as a commodity investment vehicle separately. To this end, the widely used commodity indexes, S&P GSCI and DJ-UBS CI, as well as, individual commodity future contracts written on crude oil, cotton, copper, live cattle and gold are employed. Table 2 reports the Wald test statistics and the respective *p*-values for testing the null hypothesis that there is spanning. The test is conducted for testing only MV spanning, MV and non-MV spanning jointly (MV & exponential, MV & power), as well as non-MV spanning (exponential, power). Risk aversion coefficients for a range of values are used (ARA, RRA=2,4,6,8,10) to conduct the non-MV spanning tests (equation (16) ). We can see that the null hypothesis of MV spanning cannot be rejected at a 5% significance level. This holds for either one for the two commodity indexes and for every individual commodity futures. Therefore, the results suggest that under a MV setting, the performance of traditional portfolios, consisting of stocks, bonds and cash, cannot be significantly improved by investing in commodities. These findings are in line with those reported by DeRoon et al. (1996) and Scherer and He (2008), and in contrast to Galvani and Plourde (2010) who tested the spanning hypothesis for individual energy futures over the period 1990-2008.

On the other hand, in the non-MV case we can see that the spanning hypothesis is rejected for the two commodity indexes and the majority of individual commodity contracts regardless of the assumed non-MV utility function; the only exceptions occur for futures on cotton (for the assumed exponential utility function) and live cattle. Results hold regardless of whether testing is carried out for joint MV and non-MV spanning or for only non-MV spanning. These findings are again in line with DeRoon et al. (1996) who found that commodity futures do not offer any added value to investors with utility functions consistent with the MV setting, while they do in the case where spanning is tested under a non-MV setting.

## 4. Out-of-sample benefits of commodities

Next, we investigate whether the in-sample diversification benefits provided by commodities futures are preserved in an out-of-sample setting, too. To this end, we calculate optimal portfolios separately for an asset universe that includes "traditional" asset classes (stock, bond, risk-free asset) and an "augmented" one that also includes commodities. Their relative performance is evaluated in an out-of-sample setting. The assessment of the out-of-sample performance of the derived portfolios is the ultimate test given that at any given point in time, the investor decides on the portfolio weights; the portfolio returns to be realised over the investment horizon are uncertain. Next, we describe the asset allocation framework, calculate the optimal investment strategies based on the two respective universes, and compare their out-of sample performance.

### 4.1. The asset allocation setting

Let a myopic investor with fixed initial wealth $W_t$ who faces an asset universe of $N$ assets that pay off at time $t$+1. Her utility function $U(W_{t+1})$ is assumed to be continuous, increasing, concave and differentiable. Let $w_i$ be the weight of wealth invested in the risky asset $i$ over the next period. The optimal portfolio at time $t$ is constructed by maximizing the investor's expected utility of wealth at time $t+1$ with respect to the portfolio weights, i.e.

$$\max_{w_i} E[U(W_{t+1})] \tag{24}$$

$$s.t \quad \sum_{i=1}^{N} w_i = 1$$

Let also $r_{i,t+1}$ be the simple rate of return on the individual asset $i$ and $r_{p,t+1}$ the portfolio return. Without loss of generality, we assume that the initial wealth is normalized to one, i.e., $W_t = 1$. The end-of-period wealth is given by:

$$W_{t+1} = W_t(1 + \sum_{i=1}^{N} w_i r_{i,t+1}) = 1 + \sum_{i=1}^{N} w_i r_{i,t+1} = 1 + r_{p,t+1} \tag{25}$$

To solve the expected utility maximization problem, an assumption about the utility function of the investor needs to be made. First, we assume that the preferences of the investors are described by the negative exponential and the power utility functions (equations (18) and (19)), respectively) that are commonly used in the finance literature. To ensure the robustness of our results, various levels of absolute and relative risk aversion (ARA, RRA=2, 4, 6, 8, 10) are used. In addition, we use the disappointment aversion (DA) setting introduced by Gul (1991) to capture behavioral characteristics in investors preferences. In particular, the DA setting is in line with the behavioral finance literature that has documented that the standard utility functions cannot capture the fact that the carriers of value for an investor may be the gains and losses relative to a reference point rather than the terminal wealth (see e.g., Kahneman and Tversky, 1979). This framework has been employed in recent asset allocation studies so as to capture the presence of loss aversion (see e.g., Ang et al., 2005, Driessen and Maenhout, 2007, Kostakis et al., 2010), i.e.

the fact that investors are more sensitive to reductions in their financial wealth than to increases. The advantage of Gul's (1991) DA setting over other behavioral models is that is founded on formal decision theory that retains all assumptions and axioms underlying expected utility theory but the independence axiom that is replaced by a weaker one to accommodate the Allais paradox. In line with Driessen and Maenhout (2007) and Kostakis et al. (2010), a DA value function based on a power utility function is employed, i.e.

$$U(W)=\begin{cases} \dfrac{W_T^{1-\gamma}-1}{1-\gamma} & \text{if } W_T>\mu_W \\ \dfrac{W_T^{1-\gamma}-1}{1-\gamma}-(\dfrac{1}{A}-1)\left[\dfrac{\mu_W{}^{1-\gamma}-1}{1-\gamma}-\dfrac{W_T^{1-\gamma}-1}{1-\gamma}\right] & \text{if } W_T<\mu_W \end{cases} \tag{26}$$

where $\gamma$ denotes the RRA coefficient that controls the loss function in each region, $A \leq 1$ is the coefficient of DA that controls the relative steepness of the value function in the region of gains versus the region of losses and $\mu_W$ is the reference point relative to which gains or losses are measured; the investor gets disappointed in the case where her wealth drops below the reference point. Notice that the loss aversion decreases as $A$ increases; $A$=1 corresponds to the case of the standard power utility function where there is no loss aversion. In accordance with Driessen and Maenhout (2007), two values for $A$=0.6, 0.8 are employed. Furthermore, in line with Barberis et al. (2001) and Kostakis et al. (2010), $\mu_W$ is set equal to the initial wealth invested at the risk-free rate, i.e., $\mu_W = W_t(1+r_f)$. This choice of the reference point implies that the investor uses the risk-free rate as a benchmark to distinguish gains from losses something which is in line with empirical evidence (see Veld and Veld-Merkoulova, 2008).

## 4.2. Calculating the optimal portfolio

The optimization problem in equation (24) can be implemented by performing direct utility maximization defined as the following non-linear optimization problem:

$$\max_{w_i} E[U(W_{t+1})] = \int .....\int U[W_0(1+\sum_{i=1}^{N} w_i r_i)]dF(r_1...r_N) \tag{27}$$

$$s.t \quad \sum_{i=1}^{N} w_i = 1$$

where $F(r_1 ... r_N)$ is the joint cumulative distribution function (CDF) of the *N* returns at time *t*+1. The first order conditions of this problem are given by:

$$\frac{\partial E[U(W_{t+1})]}{\partial w_i} = 0 \quad \forall i = 1,2,...,N \qquad (28)$$

Direct utility maximization provides a more general asset allocation setting compared with the Markowitz MV one since it takes into account the higher order moments of the joint CDF as well, (see Sharpe, 2007, for a discussion). On the other hand, the joint CDF needs to be estimated; this requires assuming either a specific estimator or a parametric form for the CDF leading to an estimation error. To circumvent this, we estimate optimal portfolios by applying the full scale optimization method proposed by Cremers et al. (2005), and Adler and Kritzman (2007). This is a non-parametric technique that is based on a numerical grid search procedure that uses as many asset mixes as necessary to identify the weights that yield the highest expected utility. The method requires no assumptions about the joint CDF of returns or potential estimators. On the other hand, the absence of simplifying assumptions comes at the cost of computational burden. The optimal portfolio weights are restricted to lie within the interval [-2, 1] to allow for short selling.

## 4.3. Out-of-sample performance measures

To ensure the out-of-sample nature of our study, a "rolling-sample" approach is employed. Let the dataset consist of *T* monthly observations for each asset and *K* be the size of the rolling window to be used for the calculation of the portfolio weights, where $K \leq T$. Standing at any given point in time (month) *t*, we use the previous *K* observations to estimate the asset allocation weights that maximize expected utility. The estimated weights at time *t* are then used to compute the out-of-sample realised return over the period *[t,t+1]*. This process is repeated by incorporating the return for the next period and ignoring the earliest one, until the end of the sample is reached. To ensure the robustness of the obtained results, we use alternative rolling windows sizes of *K*=36, 48, 60, 72 monthly observations. This rolling-window approach allows to derive a series of *T-K* monthly out-of-sample optimal portfolio returns, given the preferences of the investor and length of the estimation window. The time series of realised portfolio returns is then used to evaluate the out-of-sample performance of the formed optimal portfolios.

Following DeMiguel et al. (2009) and Kostakis et al. (2010), a number of performance measures are employed, namely the Sharpe ratio (SR), opportunity cost, portfolio turnover, and a measure of the portfolio risk-adjusted returns net of transaction costs are introduced. To fix ideas, let a specific strategy *c*. The SR is defined as the fraction of the sample mean of out-of-sample excess returns $\hat{\mu}_c$, divided by their sample standard deviation $\hat{\sigma}_c$.

$$\widehat{SR}_c = \frac{\hat{\mu}_c}{\hat{\sigma}_c} \tag{29}$$

To test whether the SRs of the two optimal portfolio strategies are statistically different, the statistic proposed by Jobson and Korkie (1981) and corrected by Memmel (2003) is ussed.

However, the SR is suitable to assess the performance of a strategy only in the case where the strategy's returns are normally distributed. Hence, the concept of opportunity cost, introduced by Simaan (1993) is used next to assess the economic significance of the difference in performance of the two optimal portfolios based on the traditional and augmented with commodities asset universes, respectively. Let $r_{wc}$ denote the optimal portfolio realized return obtained by an investor with the expanded investment opportunity set that includes commodities, and $r_{nc}$ the optimal portfolio realized return obtained by the same investor when her investment opportunities are restricted to the traditional asset classes. The opportunity cost $\theta$ is defined as the return that needs to be added to the portfolio return $r_{nc}$ so that the investor becomes indifferent (in utility terms) between the two strategies imposed by the different investment opportunity sets, i.e.

$$E\left[U\left(1+r_{nc}+\theta\right)\right]=E\left[U\left(1+r_{wc}\right)\right] \tag{30}$$

Hence, a positive opportunity cost implies that the investor is better off in case of an investment opportunity set that allows commodity investing. Notice that the opportunity cost takes into account all the characteristics of the utility function and hence it is suitable to evaluate strategies even when the return distribution is not the normal one.

In addition, we use the portfolio turnover metric so as to quantify the amount of trading required to implement each one of the two strategies. The portfolio turnover $PT_c$ for a strategy *c* is defined as the average absolute change in the weights over the *T*-*K* rebalancing points in time and across the *N* available assets i.e.,

$$PT_c = \frac{1}{T-K}\sum_{t=1}^{T-K}\sum_{j=1}^{N}\left(\left| w_{c,j,t+1} - w_{c,j,t+}\right|\right) \tag{31}$$

where $w_{c,j,t}$, $w_{c,j,t+1}$ are the derived optimal weights of asset *j* under strategy *c* at time *t* and *t+1*, respectively; $w_{c,j,t+}$ is the portfolio weight before the rebalancing at time *t+1*; the quantity $\left|w_{c,j,t+1} - w_{c,j,t+}\right|$ shows the magnitude of trade needed for asset *j* at the rebalancing point *t+1*. The *PT* quantity can be interpreted as the average fraction (in percentage terms) of the portfolio value that has to be reallocated over the whole period.

Finally the two investment strategies are also evaluated under the risk-adjusted, net of transaction costs, returns measure proposed by DeMiguel et al. (2009). To fix ideas, let *pc* be the proportional transaction cost and $r_{c,p,t+1}$ the realized portfolio return at *t+1* (before rebalancing). The evolution of the net of transaction costs wealth $NW_c$ for strategy *c,* is given by:

$$NW_{c,t+1} = NW_{c,t}\left(1 + r_{c,p,t+1}\right)\left[1 - pc \times \sum_{j=1}^{N}\left(\left| w_{c,j,t+1} - w_{c,j,t+}\right|\right)\right] \tag{32}$$

Therefore, the return net of transaction costs is defined as

$$RNTC_{c,t+1} = \frac{NW_{c,t+1}}{NW_{c,t}} - 1 \tag{33}$$

The return-loss measure is calculated as the additional return needed for the strategy with the restricted opportunity set to perform as well as the strategy with the expanded opportunity set that includes commodity futures. Let $\mu_{wc}$, $\mu_{nc}$ be the monthly out-of-sample mean of *RNTC* from the strategy with the expanded and the restricted opportunity set, respectively and $\sigma_{wc}$, $\sigma_{nc}$ be the corresponding standard deviations. Then, the return-loss measure is given by:

$$return - loss = \frac{\mu_{wc}}{\sigma_{wc}} \times \sigma_{nc} - \mu_{nc} \tag{34}$$

To calculate $NW_{c,t+1}$, the proportional transaction cost *pc* is assumed to be equal to 50 basis points per transaction for stocks and bonds (see DeMiguel et al., 2009, for a similar choice) and 35 basis points for the commodity indexes and individual commodity futures contracts (based on discussion with practitioners in the commodity markets). *pc* is set equal to zero for the risk-free asset, since in practice no transaction fees are charged in the case where the investor deposits or withdraws an amount from the risk-free savings account.

## 4.4. Direct maximization: Results and discussion

This section discusses the results on the out-of-sample performance of the traditional and augmented with commodities portfolios formed by direct maximization of expected utility. The dataset spans the period from January 1989 to December 2009, with the exception of DJ-UBS CI that covers the period from January 1991 to December 2009, due to data availability constraints.

Tables 3, 4, and 5 show the results for the cases where the preferences of the investor are described by an exponential utility, power utility, and DA value function, respectively. Investors access investment in commodities via the S&P GSCI and DJ-UBS CI commodity indexes. Results are reported for the four performance measures and various levels of (absolute/relative) risk and DA, as well as different sample sizes of the estimation window. To assess the statistical significance of the superiority in SRs, the *p*-values of Memmel's (2003) test are reported within parentheses. The null hypothesis is that the SRs obtained from the traditional investment opportunity set and the investment opportunity set that also includes commodities are equal. We can see that the optimal portfolios formed based on the traditional investment opportunity set yield greater SRs than the corresponding portfolio strategies based on the expanded investment opportunity set. Some exceptions occur where the optimal strategies that include commodities yield greater SRs than the ones that use the traditional opportunity set. However, the *p*-values of Memmel's (2003) test indicate that the differences in SRs are not statistically significant. Interestingly, we can see that for any given level of risk aversion, the SRs decrease as the size of the rolling window increases. This implies that the recently arrived information should be weighted more heavily (see also Kostakis et al., 2010, for a similar finding). An exception to this pattern occurs when the size of the rolling window increases from 60 months to 72 months; this is more pronounced for the S&P GSCI.

Regarding the opportunity cost, we can see that this is negative in most cases. The negative sign indicates that the investor is willing to pay a premium in order to replace the optimal strategy that includes investment in commodities with the optimal one that invests only in the traditional assets. This implies that the investor is better off when the traditional investment opportunity set is considered. These results are in accordance with the ones obtained under the SR despite the fact the distribution of the optimal portfolio returns deviates from normality (evidence is based on unreported results). Interestingly, in most of the cases, the opportunity cost decreases (in absolute terms) as the risk aversion increases. This implies that the investor becomes indifferent in utility terms between including or not commodities in her asset portfolio as she becomes more risk averse.

Furthermore, the portfolios that include only the traditional asset classes induce less portfolio turnover compared with the ones that also include commodities. Interestingly, we can see that in most cases the difference in the portfolio turnovers of the two strategies decreases as the risk aversion increases. This suggests that as the investor becomes more risk averse, she decreases her rebalancing activity since she is willing less to undertake an active bet. Finally, we can see that the return-loss measure that takes into account transaction costs is negative. The negative sign simply confirms the out-of-sample superiority of the portfolios that include only the traditional asset class, even after deducting the incurred transaction costs. In addition, we can see that the return-loss measure decreases (in absolute terms) as the risk aversion increases, just as was the case with the opportunity cost. These findings hold regardless of the commodity index, assumed utility/value function, degree of the investor's relative/absolute risk aversion, degree of DA, and the employed size of the estimation window.

Tables 6 and 7 show the results when investors access investment in commodities via the individual futures contracts and their preferences are described by an exponential / power utility and DA value function, respectively. Due to space limitations, results are reported for ARA, RRA=2,4,6. Results are similar to the ones obtained in the case where commodity indexes were considered, i.e. in most cases, optimal augmented portfolios that include commodity futures do not outperform the ones that do not. In particular, we can see that the optimal portfolios formed based on the traditional investment opportunity set yield greater SRs than the corresponding portfolio strategies based on the expanded with commodity futures opportunity set. Some exceptions occur in the case of crude oil, copper, and gold futures, i.e. greater SRs are delivered for the optimal strategies that include commodities. However, in most cases, the *p*-values of Memmel's (2003) test indicate that the differences in SRs are not statistically significant. These findings hold regardless of the selected commodity future contract, assumed utility/value function, degree of the investor's relative/absolute risk aversion, DA, and the employed size of the estimation window.

Regarding the opportunity cost, we can see that this is negative in almost all cases. Few exceptions occur when gold futures are considered. In most of the cases, the opportunity cost decreases (in absolute terms) as the risk aversion increases. In addition, the portfolios that include only the traditional asset classes induce less portfolio turnover compared with the ones that also include individual commodity futures. Interestingly, we can also see that in most cases the difference in the portfolio turnovers of the two strategies decreases as the risk aversion increases. Finally, regarding the return-loss measure, the results are mixed. This measure is negative in almost all cases across the various levels of risk aversion when crude oil, cotton and live cattle are used as investment vehicle. On the other hand, the measure is positive in the case of metal futures (copper and gold) in more than half of the cases. This implies that even though portfolios based on an investment opportunity set that includes gold /copper futures have greater turnover than the ones based on the traditional opportunity set, the investors can still earn positive risk-adjusted return by investing in these commodities. On the other hand, these results are not robust in the presence of DA, especially in the case that $A$=0.6, where the out-of-sample superiority of the portfolios that include only the traditional asset classes is confirmed by all employed performance measures. Overall, the reported results under the out-of-sample setting are in contrast with the findings within an in-sample non-MV setting (Section 3.4) where it was found that commodities offer diversification benefits to investors with non-MV utility functions.

## 5. Out-of-sample benefits of commodities: Mean-variance analysis

In this section, we perform an additional test to assess the robustness of the results found in the previous section. In particular, the out-of-sample potential benefits of including commodities in an investor's portfolio are examined within a MV setting. This will shed light on whether the previously reported evidence that challenges the diversification role of commodities is due to the inclusion of the higher order moments of the returns distribution.

### 5.1. The setting

We maximize a second order approximation of the expected utility rather than solving the direct maximization problem. Let the mean value of the future wealth, $\overline{W}_{t+1}$, defined by equation (25)

$$\overline{W}_{t+1} = E_t\left(W_{t+1}\right) = 1 + \sum_{i=1}^{N} w_i \mu_{i,t+1} = 1 + \mu_{p,t+1} \qquad (35)$$

where $\mu_{i,t+1}$ denotes the mean rate of return on the individual asset $i$ and $\mu_{p,t+1}$ the mean portfolio return. The expected utility approximated by an infinite Taylor series expansion around $\bar{W}_{t+1}$ is given by:

$$E[U(W_{t+1})] = E\left[\sum_{k=0}^{\infty} U^k\left(\bar{W}_{t+1}\right)\frac{[W_{t+1}-\bar{W}_{t+1}]^k}{k!}\right] \tag{36}$$

Under rather mild conditions of convergence (see e.g., Loistl, 1976, Lhabitant, 1998, Garlappi and Skoulakis, 2008), the expected utility can be expressed in terms of all the central moments of the distribution of the end-of-period wealth and the partial derivatives of the utility function, i.e.

$$E[U(W_{t+1})] = \sum_{k=0}^{\infty}\frac{U^k(\bar{W}_{t+1})}{k!}E\left[\left(W_{t+1}-\bar{W}_{t+1}\right)^k\right] \tag{37}$$

Setting a maximum finite value for $k$ yields

$$E[U(W_{t+1})] \approx \sum_{k=0}^{k_{\max}}\frac{U^k(\bar{W}_{t+1})}{k!}E\left[\left(W_{t+1}-\bar{W}_{t+1}\right)^k\right] \tag{38}$$

We choose $k$=2 that corresponds to the MV optimization proposed by Markowitz (1952). Hence,

$$r_{p,t+1}-\mu_{p,t+1} = \sum_{i=1}^{N} w_i(r_{i,t+1}-\mu_{i,t+1}) \tag{39}$$

Subtracting equation (35) from equation (25) yields

$$W_{t+1}-\bar{W}_{t+1} = r_{p,t+1}-\mu_{p,t+1} = \sum_{i=1}^{N} w_i(r_{i,t+1}-\mu_{i,t+1}) \tag{40}$$

Therefore, the second order Taylor series expansion can be written as:

$$E[U(W_{t+1})] \approx U(\bar{W}_{t+1}) + \frac{U^2(\bar{W}_{t+1})}{2!}E\left[\left(W_{t+1}-\bar{W}_{t+1}\right)^2\right] = U(\bar{W}_{t+1}) + \frac{U^2(\bar{W}_{t+1})}{2!}\sigma^2_{p,t+1} \tag{41}$$

where $\sigma^2_{p,t+1}$ denotes the variance of the portfolio returns. Under the negative exponential and power utility functions, equation (41) is formulated respectively as

$$E\left[U\left(W_{t+1}\right)\right] \approx -\frac{1}{\eta}\exp\left(-\eta\bar{W}_{t+1}\right)\left(1+\frac{\eta^2}{2}\sigma^2_{p,t+1}\right) \tag{42}$$

and

$$E\left[U\left(W_{t+1}\right)\right] \approx \frac{\bar{W}_{t+1}^{1-\gamma}-1}{1-\gamma} - \frac{\gamma}{2}\bar{W}_{t+1}^{-\gamma-1}\sigma^2_{p,t+1} \tag{43}$$

Equations (42) and (43) are maximised with respect to the portfolio weights to obtain the optimal portfolio choice; a grid search over possible values of the assets weights is performed. To implement the maximization, the means and variance-covariance matrix of the asset returns are estimated by their corresponding sample estimators.

## 5.2. Results and discussion

This section discusses the findings on the out-of-sample performance of the traditional and augmented optimal portfolios formed by maximising the second order Taylor series expansion. Tables 8 and 9 show the results for the cases where the preferences of the investor are described by second order Taylor series expansions of exponential and power utility functions, respectively. Investors access investment in commodities via the S&P GSCI and DJ-UBS CI commodity indexes (Panels A and B, respectively). Results are reported for the four performance measures and various levels of absolute and relative risk aversion, as well as different sample sizes of the estimation window. We can see that the MV optimal portfolios formed based on the traditional investment opportunity set yield greater SRs than the corresponding portfolio strategies based on the expanded investment opportunity set. Few exceptions are observed, i.e. higher SRs for the optimal strategies that include commodities. However, the *p*-values of Memmel's (2003) test indicate that the differences in SRs are not statistically significant. Regarding the opportunity cost, we can see that this is negative in almost every case. This implies that the investor is better off when the traditional investment opportunity set is considered. In addition, the portfolios that include only the traditional asset classes induce less portfolio turnover compared with the ones that also include commodities. Finally, we can see that the return-loss measure is negative. These findings hold regardless of the commodity index, assumed utility function, degree of the relative/absolute risk aversion, and the employed size of the estimation window. Therefore, within an out-of-sample framework, the results obtained under the MV setting are qualitatively identical with the ones obtained under the more general direct utility maximisation setting that takes into account the higher order moments, too.

Table 10 reports the results when each one of the five individual futures contracts is included in the traditional asset universe and the preferences of the marginal investor are described by the exponential utility and power utility, respectively. Due to space limitations, results are reported for ARA, RRA=2,4,6. We can see that the optimal portfolios formed based on the traditional investment opportunity set yield greater SRs than the corresponding strategies based on an investment opportunity set that includes commodity futures. Some exceptions are observed when individual contracts on crude oil, copper and gold are considered, i.e. higher SRs for the optimal strategies that include commodity investing. However, in most cases, the *p*-values of Memmel's (2003) test indicate that the differences in SRs are not statistically significant. Regarding the opportunity cost, we can see that this is negative in most cases. Few exceptions are observed when individual contracts on gold are considered. In addition, the portfolios that include only the traditional asset classes induce less portfolio turnover compared with the ones that include also individual commodity futures. Concerning the return-loss metric, results are mixed. In almost every case, when individual contracts on crude oil, cotton and live cattle are considered, the return-loss measure is negative. On the other hand, when individual contracts on gold and copper futures are considered, the return-loss measure is positive. Only a few exceptions are observed.

Overall, the results confirm the conclusions from the non-MV analysis that found that the introduction of commodity instruments in a traditional portfolio is not beneficial for a utility-maximizer investor. In addition, they extend the evidence reported in Section 3 from the spanning tests within an in-sample MV setting where commodities were found to span the returns of stocks, bonds, and the risk-free asset, to an out-of-sample one.

## 6. The effect of the 2007-2009 subprime crisis: A robustness test

In this section, we assess the robustness of our results by examining whether an investor should had included commodities in her portfolio over the recent subprime crisis period. We consider August 2007 as the beginning of the sub-prime debt crisis in line with Gorton (2009) and hence the previous analysis is repeated over the period from August 2007 until December 2009. The motivation for undertaking this analysis stems from the fact that the empirical evidence on the diversification benefits of commodities over periods of market turbulence is mixed. On the one hand, there is a number of empirical papers that examine the pre-2008 era. Their findings imply that the diversification benefits of commodities are more pronounced over turbulent periods (see e.g., Gorton and Rouwenhorst, 2006, Kat and Oomen, 2007b, Chong & Miffre, 2010, Büyükşahin et al., 2010). On the other hand, Silvennoinen and Thorp (2010), Tang and Xiong, (2010), and Buyuksahin et al. (2010) find that the return correlations between commodities and equities have increased substantially during the recent subprime crisis. Our analysis of asset returns' rolling pairwise correlations (unreported results) also uncovers this increasing pattern.

### 6.1. Testing for spanning

This section tests whether commodities (commodity indexes or individual commodity futures) span the standard asset universe, consisting of stocks, bonds and the risk-free asset over the crisis period. We test again the null hypothesis of spanning for investors with preferences described by MV utility functions, as well as, more general ones. The results are the same as the ones obtained when the analysis was conducted over the whole sample period (Section 3.4) ; results are not reported due to space limitations. In particular, we find that the null hypothesis of MV spanning cannot be rejected for either one of the two commodity indexes or for any individual commodity contract at a 5% significance level. On the other hand, in the non-MV case, the spanning hypothesis is rejected for the two commodity indexes and the majority of individual commodity contracts, regardless of the utility function assumed. The only exceptions occur for the individual contracts written on cotton (only for exponential utility function) and live cattle, just as was the case in the whole-sample analysis . Therefore, the results suggest that over the crisis period, investors whose preferences are described by non-MV utility functions become better off when commodities were included in their portfolios. This does not hold for investors with MV utility functions.

### 6.2. Direct maximization

This section discusses the results on the out-of-sample performance of the traditional and augmented with commodities portfolios formed by direct maximization of expected utility over the recent crisis period from August 2007 to December 2009.

Investors access investment in commodities either via the commodity indexes or via the selected individual commodity futures and their preferences are described by an exponential utility, power utility and DA value function. The results are qualitatively similar as the ones obtained when the analysis was conducted over the whole sample period (Section 4.4) ; results are not reported due to space limitations. In particular, the differences in SRs between investment strategies that include commodity investing (on indexes or individual future contracts) and those formed by the traditional asset classes are not statistically significant. The only exception is gold; the investment strategies that include gold futures yield statistically greater SRs. Regarding the opportunity cost, we can see that this is negative in most cases. Few exceptions are observed especially in the case of individual contracts on gold. This implies that the investor is better off when the traditional investment opportunity set is considered. This result may be attributed to the fact that correlations tend to increase over periods with extreme market conditions and hence diversification benefits vanish. Furthermore, the portfolios that include only the traditional asset classes induce less portfolio turnover compared with the ones that also include commodity investing.

Concerning the return-loss measure, the results are mixed for most commodity investment vehicles. The only exceptions occur for crude oil and gold futures where the measure is positive in almost all cases. This implies that even though the portfolios based on an investment opportunity set that includes crude oil/gold futures have greater turnover than the ones based on the traditional opportunity set, the investors can still earn positive risk-adjusted return by investing in these commodities. There results hold regardless of the assumed utility function, degree of risk aversion and size of the estimation window. An exception occurs in the presence of DA for $A$=0.6 where the return-loss measure is negative in almost every case but gold. The findings on the diversification benefits of gold is in accordance with the evidence on its "safe haven" role in periods of crisis (Baur and McDermott, 2010).

## 7. Conclusions

This paper has investigated whether an investor is made better off by including commodities in a portfolio that consists of traditional asset classes, namely stocks, bonds, and cash. To this end, a more general approach than the one followed by the previous literature has been taken. In particular, the previous literature had examined the question under scrutiny only within an in-sample mean-variance (MV) setting.

We have departed from the previous literature in two aspects. First, we have revisited the posed question within an in-sample setting that is consistent with MV as well as non-MV preferences. To this end, the tests for non-MV spanning proposed by DeRoon et al. (1996) have been used. Second, the diversification benefits of commodities have been studied within an out-of-sample static non-MV framework. Optimal portfolios were formed under the traditional and augmented with commodities asset universes, separately, by taking into account the higher order moments of returns distribution. Next, their performance was evaluated. To check the robustness of the obtained results, alternative ways of investing in commodities over the period

1989-2009 have been considered (commodity indexes and individual commodity futures). Various utility/value functions and degrees of risk aversion that describe the preferences of the individual investor have also been employed. Furthermore, a number of performance measures were used to compare the performance of the optimal portfolio based on traditional and augmented with commodities opportunity sets, respectively. The presence of transaction costs has also been considered. Finally, we have investigated whether our findings are robust under the popular MV setting and over the recent 2007-2009 crisis period.

We found that within the in-sample setting, commodities do not have added value for investors with utility function consistent with the MV setting. On the other hand, they do offer diversification benefits to investors with negative exponential and power utility functions. However, these benefits were not preserved in the out-of-sample framework. In most cases, the optimal portfolios that include only the traditional asset classes appear to have superior performance. Given that the out-of-sample setting is the ultimate test for addressing the primary question of this paper, our results challenge the common belief that commodities should be included in investor's portfolios. Most importantly, the results are remarkably robust given that they hold regardless of the performance measure, specification of utility function, and commodity instrument that is used as investment vehicle (gold appears to be the only exception). Furthermore, the superiority of the traditional portfolios has also been confirmed even under the presence of transaction costs. Similar conclusions were reached under a MV setting and over the crisis period. Our findings are consistent with the empirical evidence on the increasing financialization of commodities.

Future research should look at the benefits of commodities within a dynamic asset allocation context (see Brandt, 2009, for a review of the vast literature). Hong and Yogo (2010) find that expected commodity returns have negative conditional correlation with expected stock and bond returns. This implies that commodities may be useful to investors for intertemporal hedging. However, such an exercise should take into account all commodity related factors that affect the dynamics of the investment opportunity set (see Schwartz and Trolle, 2009, and the references therein).[9] This is well beyond the scope of the current paper but deserves to become a topic for future research.

[9] To the best of our knowledge, Dai (2008) is the only study that has studied the intertemporal hedging benefits of investing in commodities. However, his analysis uses a single factor model for the dynamics of commodity prices.

## References


Adler, T., and M. Kritzman, 2007, Mean-variance versus full-scale optimisation: In and out of sample, *Journal of Asset Management* 7, 302-311.

Ang, A., G. Bekaert, and J. Liu, 2005, Why stocks may disappoint, *Journal of Financial Economics* 76, 471-508.

Ankrim, E. M., and C. R. Hensel, 1993, Commodities in asset allocation: A real-asset alternative to real estate?, *Financial Analysts Journal* 49, 20-29.

Anson, M., 1999, Maximizing utility with commodity futures diversification, *Journal of Portfolio Management* 25, 86-94.

Anson, M., 2002, *Handbook of Alternative Assets*, John Wiley & Sons, Inc.

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen, 2009, Value and momentum everywhere, Working paper, New York University.

Barberis, N., M. Huang, and T. Santos, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics* 116, 1-53.

Baur, D.G. and T.K. McDermott, 2010, Is gold a safe haven? International evidence, *Journal of Banking and Finance*, forthcoming.

Bekaert, G., and M. S. Urias, 1996, Diversification, integration and emerging market closed-end funds, *Journal of Finance* 51, 835-869.

Bodie, Z., and V. I. Rosansky, 1980, Risk and return in commodity futures, *Financial Analysts Journal* 36, 27-39.

Bodie, Z., 1983, Commodity futures as a hedge against inflation, *Journal of Portfolio Management* 9, 12–17.

Brandt, M., 2009, Portfolio choice problems, in Y. Ait-Sahalia and L.P. Hansen (ed)., *Handbook of Financial Econometrics*, Elsevier Science: Amsterdam, 269-337.

Büyükşahin, B., M. S. Haigh, and M. A. Robe, 2010, Commodities and equities: Ever a "Market of one"?, *Journal of Alternative Investments* 12, 76-95.

Chantziara, T., and G. Skiadopoulos, 2008, Can the dynamics of the term structure of petroleum futures be forecasted? Evidence from major markets, *Energy Economics* 30, 962-985.

Chong, J., and J. Miffre, 2010, Conditional correlation and volatility in commodity futures and traditional asset markets, *Journal of Alternative Investments* 12, 61-75.

Cochrane, J., 2005, *Asset pricing (Revised Edition)*, Princeton University Press.

Cremers, J. H., M. Kritzman, and S. Page, 2005, Optimal hedge fund allocations, *Journal of Portfolio Management* 31, 70-81.

Dai, R., 2008, Commodities in dynamic asset allocation, Working Paper, Tilburg University.

DeMiguel, V., L. Garlappi, and R. Uppal, 2009, Optimal versus naive diversification: How inefficient is the 1/*n* portfolio strategy?, *Review of Financial Studie*s 22, 1915-1953.

DeRoon, F. A., T. E. Nijman, and B. J. M. Werker, 1996, Testing for spanning with futures contracts and nontraded assets: A general approach, Working paper, Tilburg University.

DeRoon, F. A., and T. E. Nijman, 2001, Testing for mean-variance spanning: A survey, *Journal of Empirical Finance* 8, 111-155.

DeRoon, F. A., T. E. Nijman, and B. J. M. Werker, 2003, Currency hedging for international stock portfolios: The usefulness of mean-variance analysis, *Journal of Banking and Finance* 27, 327-349.

Driessen, J., and P. Maenhout, 2007, An empirical portfolio perspective on option pricing anomalies, *Review of Finance* 11, 561-603.

Erb, C. B., and C. R. Harvey, 2006, The strategic and tactical value of commodity futures, *Financial Analysts Journal* 62, 69-97.

Ferson, W. E., S. R. Foerster, and D. B. Keim, 1993, General tests of latent variable models and mean-variance spanning, *Journal of Finance* 48, 131-156.

Fortenbery, T. R., and R. J. Hauser, 1990, Investment potential of agricultural futures contracts, *American Journal of Agricultural Economics* 72, 721-726.

Fuertes, A. M., J. Miffre and G. Rallis, Tactical allocation in commodity futures markets: Combining momentum and term structure signals, *Journal of Banking and Finance*, forthcoming.

Garlappi, L., and G. Skoulakis, 2008, Taylor series approximations to expected utility and optimal portfolio choice, Working Paper, University of Texas at Austin.

Galvani, V., and A. Plourde, 2010, Portfolio diversification in energy markets, *Energy Economics* 32, 257-268.

Geman, H., 2005, *Commodities and Commodity Derivatives*, John Wiley & Sons, Inc.

Georgiev, G., 2001, Benefits of commodity investment, *Journal of Alternative Investments* 4, 40-48.

Gorton, G. B., 2009, Information, liquidity, and the panic of 2007, *American Economic Review, Papers and Proceedings* 99, 567–572.

Gorton, G. B., F. Hayashi, and G. K. Rouwenhorst, 2007, The fundamentals of commodity futures returns, Working paper, National Bureau of Economic Research.

Gorton, G. B., and G. K. Rouwenhorst, 2006, Facts and fantasies about commodity futures, *Financial Analysts Journal* 62, 47-68.

Gul, F., 1991, A theory of disappointment aversion, *Econometrica* 59, 667-686.

Hanoch, G., and H. Levy, 1970, Efficient portfolio selection with quadratic and cubic utility, *Journal of Business* 43, 181-189.

Hansen L. P., and R. Jagannathan, 1991, Implications of security market data of dynamic economies, *Journal of Political Economy* 99, 225-262.

Hong, H., and M. Yogo, 2010, Commodity market interest and asset return predictability, Working Paper, Princeton University.

Huang, J., and Z. Zhong, 2006, Time-variation in diversification benefits of commodity, REITs, and TIPS, Working Paper, Rutgers, The State University of New Jersey.

Huberman, G., and S. Kandel, 1987, Mean-variance spanning, *Journal of Finance* 42, 873-888.

Idzorek, T. M., 2007, Commodities and strategic asset allocation, in H. Till and J. Eagleeye (ed) *Intelligent Commodity Investing* (London: Risk Books), 113-177.

Jagannathan, R., G. Skoulakis, and Z. Wang, 2002, Generalized method of moments: Applications in finance, *Journal of Business & Economic Statistics* 20, 470-481.

Jensen, G. R., R. R. Johnson, and J. M. Mercer, 2000, Efficient use of commodity futures in diversified portfolios, *Journal of Futures Markets* 20, 489-506.

Jobson, J. D., and B. M. Korkie, 1981, Performance hypothesis testing with the Sharpe and Treynor measures, *Journal of Finance* 36, 889-908.

Jondeau, E., and M. Rockinger, 2006, Optimal portfolio allocation under higher moments, *European Financial Management* 12, 29-55.

Kahneman, D., and A. Tversky, 1979, Prospect theory: An analysis of decision under risk, *Econometrica* 47, 263-291.

Kat, H. M., and R. C. Oomen, 2007a, What every investor needs to know about commodities Part I: Univariate return analysis, *Journal of Investment Management* 5, 4-28.

Kat, H. M., and R. C. Oomen, 2007b, What Every Investor Should Know about Commodities Part II: Multivariate Return Analysis, *Journal of Investment Management* 5, 40-64.

Kostakis, A., N. Panigirtzoglou, and G. Skiadopoulos, 2010, Asset allocation with option-implied distributions: A forward-looking approach, Working Paper, University of Piraeus.

Kroll, Y., H. Levy, and H. M. Markowitz, 1984, Mean-variance versus direct utility maximization, *Journal of Finance* 39, 47-61.

Lhabitant, F. S., 1998, On the (ab) use of Taylor series approximations for portfolio selection, portfolio performance and risk management, Working Paper, University of Lausanne.

Loistl, O., 1976, The erroneous approximation of expected utility by means of a Taylor's series expansion: Analytic and computational results, *American Economic Review* 66, 904-910.

Longin, M., 1996, The asymptotic distribution of extreme stock market returns, *Journal of Business* 69, 383-408.

Markowitz, H., 1952, Portfolio selection, *Journal of Finance* 7, 77-91.

Memmel, C., 2003, Performance hypothesis testing with the Sharpe ratio, *Finance Letters* 1, 21-23.

Miffre, J., and G. Rallis, 2007, Momentum strategies in commodity futures markets, *Journal of Banking and Finance* 31, 1863–1886.

Newey, W. K., and K. D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703-708.

Nijman, T. E., and L. A. P. Swinkels, 2008, Strategic and tactical allocation to commodities for retirement savings schemes, in F. J. Fabozzi, R. Füss, D. G. Kaizer (ed) *The Handbook of Commodity Investing,* John Wiley & Sons, Inc., 522-546.

Peiro , A., 1999, Skewness in financial returns, *Journal of Banking and Finance* 23, 847-862.

Pennacchi, G. G., 2008. *Theory of asset pricing* , Pearson Education.

Satyanarayan, S., and P. Varangis, 1996, Diversification Benefits of Commodity Assets in Global Portfolios, *Journal of Investing* 5, 69-78.

Scherer, B., and L. He, 2008, The diversification benefits of commodity futures indices: A mean variance spanning test, in F. J. Fabozzi, R. Füss, D. G. Kaizer (ed) *The Handbook of Commodity Investing ,* John Wiley & Sons, Inc., 241-265.

Schwartz, E.S., and A.B. Trolle, 2009, Unspanned stochastic volatility and the pricing of commodity derivatives, *Review of Financial Studies* 22, 4423-4461.

Sharpe, W. F., 2007, Expected utility asset allocation, *Financial Analysts Journal* 63, 18-30.

Silvennoinen, A., and S. Thorp, 2010, Financialization, crisis and commodity correlation dynamics, Working paper, Queensland University of Technology.

Simaan, Y., 1993, What is the opportunity cost of mean-variance investment strategies?, *Management Science* 39, 578-587.

Stoll, H. R., and R. E. Whaley, 2009, Commodity index investing and commodity futures prices, Working Paper, Vanderbilt University.

Tang, K., and W. Xiong, 2010, Index investing and the financialization of commodities, Working Paper, Princeton University.

Veld, C., and Y. V. Veld-Merkoulova, 2008, The risk perceptions of individual investors, *Journal of Economic Psychology* 29, 226-252.

## List of Tables

### TABLE 1
### Descriptive Statistics

Entries report the descriptive statistics for the alternative asset classes used in this study. The dataset spans the period from January 1989 to December 2009, with the exception of DJ-UBS CI that covers the period from January 1991 to December 2009. Panel A reports the summary statistics: annualized mean returns, standard deviations and Sharpe Ratios as well as skewness and kurtosis figures. The *p*-values of Jarque-Bera test are also reported. The null hypothesis is that the distribution of returns is normal. Panel B shows the correlation matrix of the assets under consideration.

**PANEL A: Summary Statitics**

| | Average Return | Standard deviation | Sharpe Ratio | Skewness | Kurtosis | Jarque-Bera p-value |
|---|---|---|---|---|---|---|
| S&P 500 Total Return | 10.0% | 14.9% | 0.37 | -0.65 | 4.31 | 0.000 |
| Barclays Aggregate Bond Index | 7.2% | 3.9% | 0.69 | -0.26 | 3.54 | 0.062 |
| S&P GSCI Total Return Index | 8.0% | 21.4% | 0.16 | -0.12 | 5.30 | 0.000 |
| DJUBS Total Return Index | 6.8% | 14.5% | 0.19 | -0.57 | 6.24 | 0.000 |
| Cotton (NYBOT) | 10.1% | 29.4% | 0.19 | -0.10 | 3.68 | 0.092 |
| Crude Oil (NYMEX) | 17.3% | 33.3% | 0.38 | 0.44 | 5.60 | 0.000 |
| Gold (COMEX) | 10.3% | 14.9% | 0.39 | 0.23 | 4.82 | 0.000 |
| Copper (COMEX) | 11.8% | 26.9% | 0.27 | 0.12 | 5.88 | 0.000 |
| Live Cattle (CME) | 6.5% | 16.1% | 0.13 | -0.29 | 5.13 | 0.000 |
| Libor 1-month | 4.5% | 0.7% | | 0.07 | 2.64 | 0.420 |

**PANEL B: Correlation Matrix**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| S&P 500 Total Return | 1.00 | | | | | | | | |
| Barclays Aggregate Bond Index | 0.19 | 1.00 | | | | | | | |
| S&P GSCI Total Return Index | 0.08 | 0.01 | 1.00 | | | | | | |
| DJUBS Total Return Index | 0.23* | 0.07 | 0.90* | 1.00 | | | | | |
| Cotton (NYBOT) | 0.19* | 0.04 | 0.07 | 0.22* | 1.00 | | | | |
| Crude Oil (NYMEX) | -0.02 | -0.06 | 0.88* | 0.73* | 0.00 | 1.00 | | | |
| Gold (CMX) | -0.07 | 0.15** | 0.24* | 0.39* | 0.07 | 0.20* | 1.00 | | |
| Copper (NYMEX) | 0.27* | -0.10 | 0.32* | 0.52* | 0.18* | 0.25* | 0.23* | 1.00 | |
| Live Cattle (CME) | 0.06 | -0.07 | 0.06 | 0.09 | 0.05 | -0.03 | -0.04 | 0.04 | 1.00 |

* Significant at 1%.
** Significant at 5%.

## TABLE 2
## Testing for Spanning: Results

Entries report the Wald test statistics and the respective *p*-values for the null hypothesis that a set of benchmark assets consisting of stocks, bonds and the risk-free asset spans a given test asset from the commodity futures market. The first column reports results for the null hypothesis that there is mean-variance spanning. The next column reports results for the null hypothesis that there is both mean-variance and exponential utility spanning with risk aversion coefficient ranging from 2 to 10. The third column reports results for the null hypothesis that there is spanning only for investors with exponential utility function. The forth column reports results for the null hypothesis that there is both mean-variance and power utility spanning with risk aversion coefficient ranging from 2 to 10. The last column presents the respective results when only power utility function is considered. The initial set of assets is the S&P 500 Total Return Index, Barclays Aggregate Bond Index and Libor 1-month. Results are based on monthly observations from Jan. 1989 –Dec. 2009 for S&P GSCI and Jan. 1991-Dec. 2009 for DJ-UBS CI. All test statistics are based on a Newey-West covariance matrix with five lags.

| **Test Asset** | **Mean - Variance (MV)** | **MV & Exponential** | **Exponential** | **MV & Power** | **Power** |
|---|---|---|---|---|---|
| **S&P GSCI** | 0.23 | 23.41 | 14.39 | 72.58 | 70.95 |
| | *(0.631)* | *(0.001)* | *(0.013)* | *(0.000)* | *(0.000)* |
| **DJ-UBS CI** | 0.06 | 29.94 | 28.67 | 79.63 | 79.62 |
| | *(0.800)* | *(0.000)* | *(0.000)* | *(0.000)* | *(0.000)* |
| **Crude Oil** | 2.67 | 39.80 | 13.72 | 91.50 | 87.62 |
| | *(0.102)* | *(0.000)* | *(0.017)* | *(0.000)* | *(0.000)* |
| **Cotton** | 0.33 | 6.58 | 5.42 | 27.12 | 27.09 |
| | *(0.563)* | *(0.361)* | *(0.367)* | *(0.000)* | *(0.000)* |
| **Copper** | 0.99 | 25.81 | 18.40 | 60.06 | 55.92 |
| | *(0.320)* | *(0.000)* | *(0.003)* | *(0.000)* | *(0.000)* |
| **Gold** | 2.06 | 17.26 | 12.85 | 46.81 | 42.73 |
| | *(0.151)* | *(0.008)* | *(0.025)* | *(0.000)* | *(0.000)* |
| **Live Cattle** | 0.85 | 3.77 | 1.89 | 4.67 | 3.61 |
| | *(0.358)* | *(0.708)* | *(0.864)* | *(0.587)* | *(0.607)* |

# TABLE 3
## Direct Utility Maximization: Commodity Indexes and Exponential Utility

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under an exponential utility. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes commodities. Investors access investment in commodities either via S&P GSCI (Panel A) or via DJ-UBS CI (Panel B). Results are reported for different sizes of the rolling window (K=36,48,60,72 observations) and different degrees of absolute risk aversion (ARA=2,4,6,8,10). Results are based on monthly observations from Jan. 1989 –Dec. 2009 for S&P GSCI and Jan. 1991-Dec. 2009 for DJ-UBS CI.

**Panel A: S&P GSCI (1989-2009)**

| | | ARA=2 | | ARA=4 | | ARA=6 | | ARA=8 | | ARA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.30 | 0.49 | 0.36 | 0.57 | 0.39 | 0.59 | 0.40 | 0.59 | 0.40 | 0.58 |
| | *(p-value)* | *(0.151)* | | *(0.075)* | | *(0.064)* | | *(0.059)* | | *(0.082)* | |
| | Opp. Cost | -6.00% | | -6.24% | | -5.04% | | -4.32% | | -3.24% | |
| | Port.Turnover | 82.75% | 56.28% | 73.42% | 53.09% | 69.46% | 53.78% | 60.45% | 52.32% | 61.28% | 52.91% |
| | Return-Loss | -5.40% | | -5.14% | | -4.24% | | -3.60% | | -2.82% | |
| **K=48** | Sharpe Ratio | 0.21 | 0.34 | 0.31 | 0.44 | 0.35 | 0.47 | 0.34 | 0.44 | 0.31 | 0.42 |
| | *(p-value)* | *(0.239)* | | *(0.180)* | | *(0.176)* | | *(0.176)* | | *(0.160)* | |
| | Opp. Cost | -5.40% | | -4.08% | | -2.76% | | -1.80% | | -1.44% | |
| | Port.Turnover | 71.34% | 44.77% | 57.03% | 40.12% | 49.38% | 37.64% | 50.19% | 42.20% | 48.86% | 40.51% |
| | Return-Loss | -4.32% | | -3.52% | | -2.68% | | -2.08% | | -1.87% | |
| **K=60** | Sharpe Ratio | 0.03 | 0.16 | 0.15 | 0.27 | 0.17 | 0.26 | 0.19 | 0.28 | 0.18 | 0.25 |
| | *(p-value)* | *(0.211)* | | *(0.184)* | | *(0.201)* | | *(0.199)* | | *(0.277)* | |
| | Opp. Cost | -7.44% | | -4.68% | | -3.12% | | -2.52% | | -0.14% | |
| | Port.Turnover | 71.74% | 38.00% | 53.32% | 35.70% | 47.76% | 35.19% | 40.16% | 35.56% | 40.81% | 35.12% |
| | Return-Loss | -5.05% | | -3.58% | | -2.51% | | -1.87% | | -1.30% | |
| **K=72** | Sharpe Ratio | 0.11 | 0.25 | 0.31 | 0.38 | 0.33 | 0.38 | 0.36 | 0.37 | 0.37 | 0.40 |
| | *(p-value)* | *(0.219)* | | *(0.317)* | | *(0.348)* | | *(0.460)* | | *(0.412)* | |
| | Opp. Cost | -8.64% | | -3.96% | | -2.64% | | -1.20% | | -1.44% | |
| | Port.Turnover | 65.49% | 32.74% | 42.48% | 26.24% | 37.89% | 27.05% | 32.59% | 28.45% | 33.31% | 28.67% |
| | Return-Loss | -5.01% | | -2.48% | | -1.62% | | -0.66% | | -0.82% | |

**Panel B: DJ-UBS CI (1991-2009)**

| | | ARA=2 | | ARA=4 | | ARA=6 | | ARA=8 | | ARA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.47 | 0.45 | 0.50 | 0.54 | 0.48 | 0.55 | 0.47 | 0.55 | 0.47 | 0.52 |
| | *(p-value)* | *(0.473)* | | *(0.400)* | | *(0.323)* | | *(0.285)* | | *(0.361)* | |
| | Opp. Cost | -0.12% | | -2.64% | | -2.88% | | -2.88% | | -2.52% | |
| | Port.Turnover | 82.47% | 57.13% | 78.45% | 56.12% | 75.94% | 58.28% | 68.02% | 57.29% | 69.17% | 58.17% |
| | Return-Loss | -0.91% | | -1.97% | | -2.20% | | -2.03% | | -1.37% | |
| **K=48** | Sharpe Ratio | 0.38 | 0.37 | 0.41 | 0.49 | 0.43 | 0.52 | 0.40 | 0.48 | 0.39 | 0.46 |
| | *(p-value)* | *(0.477)* | | *(0.329)* | | *(0.280)* | | *(0.285)* | | *(0.307)* | |
| | Opp. Cost | -0.84% | | -3.60% | | -3.12% | | -2.16% | | -1.80% | |
| | Port.Turnover | 72.89% | 42.64% | 65.43% | 39.67% | 57.82% | 39.34% | 55.96% | 45.56% | 54.50% | 43.76% |
| | Return-Loss | -1.27% | | -2.82% | | -2.65% | | -2.01% | | -1.62% | |
| **K=60** | Sharpe Ratio | -0.07 | 0.08 | 0.04 | 0.21 | 0.06 | 0.21 | 0.10 | 0.23 | 0.08 | 0.21 |
| | *(p-value)* | *(0.230)* | | *(0.147)* | | *(0.149)* | | *(0.187)* | | *(0.187)* | |
| | Opp. Cost | -8.40% | | -7.20% | | -4.56% | | -3.12% | | -2.40% | |
| | Port.Turnover | 69.17% | 36.95% | 61.69% | 36.13% | 54.95% | 36.78% | 39.37% | 34.51% | 42.55% | 35.29% |
| | Return-Loss | -5.24% | | -4.96% | | -3.70% | | -2.43% | | -2.17% | |
| **K=72** | Sharpe Ratio | -0.10 | 0.00 | 0.05 | 0.18 | 0.05 | 0.19 | 0.06 | 0.18 | 0.06 | 0.22 |
| | *(p-value)* | *(0.321)* | | *(0.231)* | | *(0.213)* | | *(0.213)* | | *(0.151)* | |
| | Opp. Cost | -8.88% | | -7.20% | | -5.28% | | -3.60% | | -3.72% | |
| | Port.Turnover | 60.87% | 33.26% | 45.41% | 25.71% | 41.28% | 27.21% | 39.61% | 29.40% | 34.33% | 28.40% |
| | Return-Loss | -3.76% | | -3.85% | | -3.31% | | -2.41% | | -2.37% | |

## TABLE 4

## Direct Utility Maximization: Commodity Indexes and Power Utility

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under power utility. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded opportunity set that includes commodities. Investors access investment in commodities either via S&P GSCI (Panel A) or via DJ-UBS CI (Panel B). Results are reported for different sizes of the rolling window (K=36,48,60,72 observations) and different degrees of relative risk aversion (RRA=2,4,6,8,10). Results are based on monthly observations from Jan. 1989 –Dec. 2009 for S&P GSCI and Jan. 1991-Dec. 2009 for DJ-UBS CI.

**Panel A: S&P GSCI (1989-2009)**

| | | RRA=2 | | RRA=4 | | RRA=6 | | RRA=8 | | RRA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Expanded | Traditional | Expanded | Traditional | Expanded | Traditional | Expanded | Traditional | Expanded | Traditional |
| K=36 | Sharpe Ratio | 0.30 | 0.49 | 0.35 | 0.57 | 0.38 | 0.59 | 0.39 | 0.59 | 0.40 | 0.58 |
| | *(p-value)* | *(0.147)* | | *(0.075)* | | *(0.065)* | | *(0.062)* | | *(0.077)* | |
| | Opp. Cost | -6.12% | | -6.12% | | -4.80% | | -4.08% | | -3.24% | |
| | Port.Turnover | 82.96% | 56.57% | 73.68% | 53.48% | 69.43% | 53.75% | 65.61% | 55.01% | 60.96% | 52.43% |
| | Return-Loss | -5.48% | | -5.15% | | -4.23% | | -3.57% | | -2.89% | |
| K=48 | Sharpe Ratio | 0.20 | 0.34 | 0.30 | 0.44 | 0.35 | 0.47 | 0.33 | 0.44 | 0.31 | 0.42 |
| | *(p-value)* | *(0.229)* | | *(0.180)* | | *(0.178)* | | *(0.156)* | | *(0.138)* | |
| | Opp. Cost | -5.76% | | -3.96% | | -2.40% | | -1.68% | | -1.56% | |
| | Port.Turnover | 71.75% | 44.66% | 57.22% | 39.61% | 49.68% | 37.72% | 48.67% | 38.88% | 48.40% | 40.22% |
| | Return-Loss | -4.48% | | -3.55% | | -2.68% | | -2.27% | | -2.01% | |
| K=60 | Sharpe Ratio | 0.03 | 0.17 | 0.15 | 0.27 | 0.16 | 0.26 | 0.17 | 0.25 | 0.17 | 0.24 |
| | *(p-value)* | *(0.206)* | | *(0.182)* | | *(0.203)* | | *(0.227)* | | *(0.241)* | |
| | Opp. Cost | -7.80% | | -4.68% | | -2.64% | | -1.80% | | -1.68% | |
| | Port.Turnover | 71.47% | 37.58% | 52.99% | 34.89% | 47.52% | 34.75% | 42.64% | 34.47% | 40.30% | 34.75% |
| | Return-Loss | -5.12% | | -3.63% | | -2.53% | | -1.84% | | -1.44% | |
| K=72 | Sharpe Ratio | 0.11 | 0.25 | 0.31 | 0.38 | 0.33 | 0.38 | 0.34 | 0.38 | 0.36 | 0.39 |
| | *(p-value)* | *(0.220)* | | *(0.318)* | | *(0.203)* | | *(0.367)* | | *(0.408)* | |
| | Opp. Cost | -9.84% | | -4.20% | | -2.64% | | -1.80% | | -1.44% | |
| | Port.Turnover | 65.30% | 32.81% | 43.23% | 26.62% | 38.17% | 27.08% | 34.50% | 27.48% | 31.88% | 28.36% |
| | Return-Loss | -4.97% | | -2.50% | | -2.53% | | -1.19% | | -0.81% | |

**Panel B: DJ-UBS CI (1991-2009)**

| | | RRA=2 | | RRA=4 | | RRA=6 | | RRA=8 | | RRA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Expanded | Traditional | Expanded | Traditional | Expanded | Traditional | Expanded | Traditional | Expanded | Traditional |
| K=36 | Sharpe Ratio | 0.47 | 0.46 | 0.49 | 0.53 | 0.48 | 0.55 | 0.47 | 0.54 | 0.47 | 0.52 |
| | *(p-value)* | *(0.481)* | | *(0.403)* | | *(0.331)* | | *(0.306)* | | *(0.359)* | |
| | Opp. Cost | -0.12% | | -2.40% | | -2.52% | | -2.52% | | -2.28% | |
| | Port.Turnover | 82.73% | 57.40% | 78.76% | 56.47% | 76.04% | 58.20% | 72.83% | 60.22% | 68.68% | 57.61% |
| | Return-Loss | -1.01% | | -1.95% | | -2.16% | | -1.89% | | -1.39% | |
| K=48 | Sharpe Ratio | 0.39 | 0.38 | 0.42 | 0.48 | 0.43 | 0.52 | 0.40 | 0.48 | 0.39 | 0.47 |
| | *(p-value)* | *(0.475)* | | *(0.338)* | | *(0.280)* | | *(0.283)* | | *(0.293)* | |
| | Opp. Cost | -0.84% | | -3.48% | | -2.76% | | -1.68% | | -1.80% | |
| | Port.Turnover | 72.50% | 42.46% | 65.30% | 39.00% | 58.01% | 39.38% | 56.80% | 41.88% | 54.82% | 43.37% |
| | Return-Loss | -1.24% | | -2.78% | | -2.67% | | -2.18% | | -1.73% | |
| K=60 | Sharpe Ratio | -0.07 | 0.08 | 0.04 | 0.21 | 0.06 | 0.21 | 0.07 | 0.21 | 0.09 | 0.20 |
| | *(p-value)* | *(0.228)* | | *(0.152)* | | *(0.151)* | | *(0.170)* | | *(0.203)* | |
| | Opp. Cost | -8.64% | | -7.32% | | -4.32% | | -2.76% | | -2.16% | |
| | Port.Turnover | 68.38% | 36.45% | 61.33% | 35.19% | 55.06% | 36.24% | 46.86% | 35.74% | 42.39% | 34.88% |
| | Return-Loss | -5.16% | | -4.92% | | -3.73% | | -2.77% | | -2.07% | |
| K=72 | Sharpe Ratio | -0.09 | 0.00 | 0.04 | 0.17 | 0.04 | 0.19 | 0.05 | 0.20 | 0.07 | 0.21 |
| | *(p-value)* | *(0.320)* | | *(0.235)* | | *(0.185)* | | *(0.171)* | | *(0.176)* | |
| | Opp. Cost | -9.72% | | -7.44% | | -5.04% | | -3.96% | | -3.36% | |
| | Port.Turnover | 61.04% | 33.37% | 45.89% | 26.15% | 41.43% | 27.23% | 37.13% | 28.27% | 33.29% | 28.18% |
| | Return-Loss | -3.77% | | -3.82% | | -3.29% | | -2.69% | | -2.17% | |

# TABLE 5

## Direct Utility Maximization: Commodity Indexes and Disappointment aversion value function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under a disappointment aversion value function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded opportunity set that includes commodities. Results are reported for different sizes of the rolling window (K=36,48,60,72 observations), degrees of relative risk aversion (RRA=2,4,6,8,10) and values of the disappointment aversion parameter (*A*=0.6,0.8). Investors access investment in commodities either via S&P GSCI (Panels A and B) or via DJ-UBS CI (Panels C and D). Results are based on monthly observations from Jan. 1989 –Dec. 2009 for S&P GSCI and Jan. 1991-Dec. 2009 for DJ-UBS CI.

**Panel A: S&P GSCI (A=0.6)**

| | | RRA=2 | | RRA=4 | | RRA=6 | | RRA=8 | | RRA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.39 | 0.59 | 0.34 | 0.50 | 0.38 | 0.49 | 0.38 | 0.46 | 0.38 | 0.44 |
| | *(p-value)* | *(0.071)* | | *(0.100)* | | *(0.196)* | | *(0.253)* | | *(0.310)* | |
| | Opp. Cost | -6.00% | | -4.92% | | -3.36% | | -2.40% | | -2.16% | |
| | Port.Turnover | 72.33% | 52.51% | 63.87% | 50.69% | 53.58% | 46.80% | 50.60% | 41.18% | 43.68% | 37.50% |
| | Return-Loss | -4.34% | | -3.13% | | -1.85% | | -1.46% | | -1.06% | |
| **K=48** | Sharpe Ratio | 0.34 | 0.47 | 0.29 | 0.35 | 0.24 | 0.27 | 0.22 | 0.25 | 0.20 | 0.23 |
| | *(p-value)* | *(0.147)* | | *(0.278)* | | *(0.355)* | | *(0.407)* | | *(0.384)* | |
| | Opp. Cost | -3.84% | | -2.16% | | -1.92% | | -1.56% | | -1.44% | |
| | Port.Turnover | 62.51% | 45.72% | 59.78% | 47.47% | 47.36% | 45.69% | 44.85% | 39.33% | 40.26% | 34.28% |
| | Return-Loss | -3.02% | | -1.53% | | -0.78% | | -0.67% | | -0.69% | |
| **K=60** | Sharpe Ratio | 0.26 | 0.30 | 0.16 | 0.17 | 0.14 | 0.14 | 0.12 | 0.12 | 0.10 | 0.09 |
| | *(p-value)* | *(0.370)* | | *(0.446)* | | *(0.499)* | | *(0.497)* | | *(0.474)* | |
| | Opp. Cost | -2.64% | | -2.04% | | -1.80% | | -1.44% | | -0.96% | |
| | Port.Turnover | 67.82% | 59.96% | 56.55% | 55.50% | 47.48% | 41.79% | 39.33% | 32.05% | 32.69% | 26.44% |
| | Return-Loss | -1.24% | | -0.55% | | -0.57% | | -0.65% | | -0.52% | |
| **K=72** | Sharpe Ratio | 0.42 | 0.39 | 0.28 | 0.28 | 0.26 | 0.27 | 0.26 | 0.27 | 0.26 | 0.26 |
| | *(p-value)* | *(0.390)* | | *(0.489)* | | *(0.471)* | | *(0.459)* | | *(0.491)* | |
| | Opp. Cost | -1.08% | | -1.92% | | -1.56% | | -1.20% | | -0.96% | |
| | Port.Turnover | 53.62% | 66.87% | 51.20% | 53.45% | 42.10% | 38.76% | 32.01% | 28.59% | 25.60% | 23.45% |
| | Return-Loss | 0.66% | | -0.30% | | -0.61% | | -0.68% | | -0.60% | |

**Panel B: S&P GSCI (A=0.8)**

| | | RRA=2 | | RRA=4 | | RRA=6 | | RRA=8 | | RRA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.34 | 0.58 | 0.36 | 0.57 | 0.36 | 0.56 | 0.38 | 0.56 | 0.40 | 0.56 |
| | *(p-value)* | *(0.066)* | | *(0.061)* | | *(0.051)* | | *(0.077)* | | *(0.108)* | |
| | Opp. Cost | -7.32% | | -5.88% | | -5.04% | | -3.72% | | -3.00% | |
| | Port.Turnover | 81.37% | 61.79% | 74.20% | 56.33% | 68.32% | 56.81% | 60.87% | 51.57% | 53.80% | 46.31% |
| | Return-Loss | -5.72% | | -4.47% | | -3.71% | | -2.86% | | -2.18% | |
| **K=48** | Sharpe Ratio | 0.34 | 0.45 | 0.35 | 0.48 | 0.32 | 0.41 | 0.29 | 0.38 | 0.28 | 0.36 |
| | *(p-value)* | *(0.210)* | | *(0.169)* | | *(0.201)* | | *(0.192)* | | *(0.227)* | |
| | Opp. Cost | -1.32% | | -3.24% | | -1.68% | | -1.80% | | -1.44% | |
| | Port.Turnover | 62.51% | 47.07% | 54.95% | 46.49% | 54.58% | 48.50% | 51.63% | 47.68% | 46.33% | 44.28% |
| | Return-Loss | -2.60% | | -2.71% | | -1.80% | | -1.47% | | -1.08% | |
| **K=60** | Sharpe Ratio | 0.14 | 0.32 | 0.19 | 0.30 | 0.18 | 0.26 | 0.17 | 0.23 | 0.16 | 0.21 |
| | *(p-value)* | *(0.119)* | | *(0.190)* | | *(0.238)* | | *(0.277)* | | *(0.319)* | |
| | Opp. Cost | -6.00% | | -3.36% | | -2.28% | | -1.92% | | -1.56% | |
| | Port.Turnover | 56.64% | 33.53% | 53.22% | 40.14% | 47.12% | 41.08% | 42.89% | 40.35% | 40.61% | 37.38% |
| | Return-Loss | -5.07% | | -2.66% | | -1.64% | | -1.15% | | -0.90% | |
| **K=72** | Sharpe Ratio | 0.27 | 0.38 | 0.37 | 0.42 | 0.35 | 0.38 | 0.35 | 0.36 | 0.33 | 0.35 |
| | *(p-value)* | *(0.237)* | | *(0.363)* | | *(0.399)* | | *(0.444)* | | *(0.413)* | |
| | Opp. Cost | -4.68% | | -2.52% | | -1.92% | | -1.56% | | -1.44% | |
| | Port.Turnover | 45.63% | 25.52% | 38.38% | 28.36% | 37.43% | 32.95% | 35.98% | 36.38% | 33.82% | 30.93% |
| | Return-Loss | -3.46% | | -1.54% | | -0.95% | | -0.57% | | -0.72% | |

# TABLE 5 (Cont'd)

## Direct Utility Maximization: Commodity Indexes and Disappointment aversion value function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under a disappointment aversion value function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded opportunity set that includes commodities. Results are reported for different sizes of the rolling window (K=36,48,60,72 observations) and different degrees of relative risk aversion (RRA=2,4,6,8,10). Entries are reported for both values of the disappointment aversion parameter (*A*=0.6,0.8) employed in this study. Investors access investment in commodities either via S&P GSCI (Panels A and B) or via DJ-UBS CI (Panels C and D). Results are based on monthly observations from Jan. 1989 –Dec. 2009 for S&P GSCI and Jan. 1991-Dec. 2009 for DJ-UBS CI.

**Panel C: DJ-UBS CI (A=0.6)**

| | | RRA=2 | | RRA=4 | | RRA=6 | | RRA=8 | | RRA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.48 | 0.56 | 0.43 | 0.48 | 0.44 | 0.45 | 0.43 | 0.41 | 0.41 | 0.39 |
| | *(p-value)* | *(0.301)* | | *(0.381)* | | *(0.474)* | | *(0. 467)* | | *(0.467)* | |
| | Opp. Cost | -4.44% | | -3.48% | | -3.12% | | -2.76% | | -2.28% | |
| | Port.Turnover | 81.75% | 52.83% | 66.82% | 49.72% | 57.04% | 45.80% | 53.27% | 39.70% | 46.75% | 35.60% |
| | Return-Loss | -2.71% | | -1.64% | | -0.98% | | -0.91% | | -0.78% | |
| **K=48** | Sharpe Ratio | 0.38 | 0.52 | 0.35 | 0.40 | 0.31 | 0.34 | 0.31 | 0.33 | 0.28 | 0.31 |
| | *(p-value)* | *(0.177)* | | *(0.358)* | | *(0.398)* | | *(0.454)* | | *(0.404)* | |
| | Opp. Cost | -4.56% | | -2.16% | | -2.52% | | -1.92% | | -1.80% | |
| | Port.Turnover | 72.58% | 48.80% | 60.64% | 49.12% | 53.57% | 45.51% | 48.05% | 37.77% | 42.91% | 31.80% |
| | Return-Loss | -3.58% | | -1.46% | | -1.06% | | -0.86% | | -1.00% | |
| **K=60** | Sharpe Ratio | 0.09 | 0.27 | 0.07 | 0.14 | 0.07 | 0.10 | 0.05 | 0.08 | 0.04 | 0.05 |
| | *(p-value)* | *(0.086)* | | *(0.311)* | | *(0.423)* | | *(0.419)* | | *(0.467)* | |
| | Opp. Cost | -5.04% | | -2.52% | | -2.04% | | -1.92% | | -1.20% | |
| | Port.Turnover | 74.40% | 59.87% | 50.87% | 55.57% | 42.67% | 41.29% | 39.19% | 31.83% | 32.42% | 26.41% |
| | Return-Loss | -4.06% | | -1.10% | | -0.68% | | -0.82% | | -0.60% | |
| **K=72** | Sharpe Ratio | 0.12 | 0.26 | 0.03 | 0.17 | 0.01 | 0.16 | 0.02 | 0.16 | 0.02 | 0.15 |
| | *(p-value)* | *(0.178)* | | *(0.163)* | | *(0.177)* | | *(0.183)* | | *(0.203)* | |
| | Opp. Cost | -4.08% | | -3.36% | | -2.76% | | -2.16% | | -1.68% | |
| | Port.Turnover | 56.06% | 67.19% | 50.07% | 52.87% | 43.28% | 38.32% | 34.03% | 28.19% | 27.90% | 23.27% |
| | Return-Loss | -4.08% | | -1.87% | | -1.59% | | -1.42% | | -1.20% | |

**Panel D: DJ-UBS CI (A=0.8)**

| | | RRA=2 | | RRA=4 | | RRA=6 | | RRA=8 | | RRA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.47 | 0.55 | 0.48 | 0.53 | 0.45 | 0.52 | 0.46 | 0.50 | 0.46 | 0.50 |
| | *(p-value)* | *(0.329)* | | *(0.361)* | | *(0.319)* | | *(0.390)* | | *(0.397)* | |
| | Opp. Cost | -3.72% | | -3.36% | | -3.24% | | -2.76% | | -2.76% | |
| | Port.Turnover | 88.25% | 65.14% | 81.67% | 61.20% | 74.19% | 62.28% | 66.11% | 55.31% | 60.18% | 48.72% |
| | Return-Loss | -2.75% | | -2.04% | | -1.75% | | -1.25% | | -1.20% | |
| **K=48** | Sharpe Ratio | 0.39 | 0.49 | 0.42 | 0.52 | 0.40 | 0.46 | 0.37 | 0.43 | 0.37 | 0.42 |
| | *(p-value)* | *(0.280)* | | *(0.256)* | | *(0.341)* | | *(0.341)* | | *(0.373)* | |
| | Opp. Cost | -4.56% | | -3.60% | | -1.44% | | -2.16% | | -1.68% | |
| | Port.Turnover | 71.88% | 47.63% | 63.99% | 49.55% | 61.77% | 52.65% | 59.83% | 51.40% | 53.13% | 46.68% |
| | Return-Loss | -3.42% | | -2.74% | | -1.56% | | -1.30% | | -0.98% | |
| **K=60** | Sharpe Ratio | 0.01 | 0.27 | 0.06 | 0.26 | 0.07 | 0.22 | 0.08 | 0.20 | 0.07 | 0.17 |
| | *(p-value)* | *(0.070)* | | *(0.083)* | | *(0.132)* | | *(0.199)* | | *(0.240)* | |
| | Opp. Cost | -9.60% | | -5.52% | | -3.12% | | -2.28% | | -1.92% | |
| | Port.Turnover | 68.66% | 33.40% | 60.86% | 41.30% | 50.39% | 40.60% | 43.63% | 39.96% | 40.27% | 36.73% |
| | Return-Loss | -7.45% | | -4.66% | | -2.98% | | -1.88% | | -1.42% | |
| **K=72** | Sharpe Ratio | 0.04 | 0.19 | 0.08 | 0.23 | 0.05 | 0.21 | 0.05 | 0.20 | 0.05 | 0.19 |
| | *(p-value)* | *(0.221)* | | *(0.163)* | | *(0.153)* | | *(0.172)* | | *(0.171)* | |
| | Opp. Cost | -7.56% | | -5.04% | | -3.60% | | -3.12% | | -2.76% | |
| | Port.Turnover | 52.52% | 24.21% | 44.12% | 28.07% | 39.77% | 32.86% | 37.56% | 36.50% | 35.88% | 31.15% |
| | Return-Loss | -4.66% | | -3.57% | | -2.63% | | -1.92% | | -1.75% | |

# TABLE 6

## Direct Utility Maximization: Individual Commodity Futures (Exponential and Power Utility Function)

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under an exponential utility (Panel A) and power utility (Panel B). The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes commodities. Investors access investment in commodities via the selected individual commodity futures contracts. Results are reported for different sizes of the rolling window (K=36,48,60,72 observations) and different degrees of absolute/relative risk aversion (ARA,RRA=2,4,6). Results are based on monthly observations from Jan. 1989 –Dec. 2009.

**Panel A: Individual Commodity Futures Contracts (Exponential Utility)**

| | | ARA=2 | | | | | | ARA=4 | | | | | | ARA=6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** |
| K=36 | Sharpe Ratio | 0.39 | 0.49 | 0.19 | 0.75 | 0.25 | 0.49 | 0.50 | 0.57 | 0.31 | 0.69 | 0.41 | 0.57 | 0.56 | 0.61 | 0.38 | 0.69 | 0.45 | 0.59 |
| | *(p-value)* | *(0.332)* | *(0.499)* | *(0.054)* | *(0.091)* | *(0.049)* | | *(0.349)* | *(0.491)* | *(0.029)* | *(0.279)* | *(0.079)* | | *(0.435)* | *(0.454)* | *(0.033)* | *(0.274)* | *(0.079)* | |
| | Opp. Cost | -4.44% | -0.72% | -10.44% | 8.04% | -6.36% | | -3.36% | -2.40% | -7.68% | 0.48% | -4.20% | | -2.16% | -1.80% | -5.64% | -0.60% | -3.24% | |
| | Port.Turnover | 77.24% | 78.80% | 92.65% | 81.33% | 92.27% | 56.28% | 64.61% | 71.93% | 74.28% | 78.54% | 73.88% | 53.09% | 61.50% | 69.47% | 66.71% | 75.64% | 67.33% | 53.78% |
| | Return-Loss | -3.44% | -0.97% | -8.00% | 5.01% | -6.71% | | -2.12% | -1.14% | -6.04% | 1.12% | -4.07% | | -0.98% | -0.52% | -4.31% | 0.86% | -2.93% | |
| K=48 | Sharpe Ratio | 0.35 | 0.42 | 0.07 | 0.57 | 0.23 | 0.34 | 0.44 | 0.51 | 0.22 | 0.54 | 0.31 | 0.44 | 0.49 | 0.47 | 0.30 | 0.54 | 0.36 | 0.47 |
| | *(p-value)* | *(0.476)* | *(0.377)* | *(0.034)* | *(0.139)* | *(0.210)* | | *(0.497)* | *(0.387)* | *(0.014)* | *(0.310)* | *(0.125)* | | *(0.459)* | *(0.413)* | *(0.017)* | *(0.346)* | *(0.110)* | |
| | Opp. Cost | -2.64% | -0.60% | -8.28% | 5.52% | -3.60% | | -2.16% | -2.04% | -5.64% | -2.52% | -3.60% | | -1.44% | -2.64% | -3.96% | -3.00% | -2.52% | |
| | Port.Turnover | 68.44% | 65.75% | 70.84% | 71.99% | 75.16% | 44.77% | 51.38% | 57.88% | 52.57% | 65.73% | 58.74% | 40.12% | 43.62% | 53.20% | 46.30% | 59.41% | 49.40% | 37.64% |
| | Return-Loss | -1.07% | 0.65% | -7.18% | 3.98% | -3.93% | | -0.79% | 0.14% | -4.91% | 0.75% | -3.41% | | -0.31% | -0.36% | -3.50% | 0.09% | -2.53% | |
| K=60 | Sharpe Ratio | 0.22 | 0.26 | 0.02 | 0.48 | 0.09 | 0.16 | 0.31 | 0.31 | 0.18 | 0.44 | 0.21 | 0.27 | 0.30 | 0.34 | 0.20 | 0.40 | 0.21 | 0.26 |
| | *(p-value)* | *(0.396)* | *(0.353)* | *(0.152)* | *(0.056)* | *(0.317)* | | *(0.421)* | *(0.428)* | *(0.134)* | *(0.194)* | *(0.270)* | | *(0.385)* | *(0.342)* | *(0.157)* | *(0.208)* | *(0.251)* | |
| | Opp. Cost | -6.00% | -5.88% | -5.28% | 6.00% | -3.84% | | -4.08% | -5.28% | -3.00% | -2.76% | -2.64% | | -3.00% | -2.88% | -2.04% | -2.28% | -2.04% | |
| | Port.Turnover | 64.42% | 69.21% | 61.54% | 72.48% | 63.14% | 38.00% | 46.32% | 55.12% | 45.18% | 60.23% | 48.58% | 35.70% | 40.71% | 47.48% | 40.25% | 53.57% | 44.16% | 35.19% |
| | Return-Loss | -0.37% | 0.52% | -4.61% | 5.61% | -3.08% | | -0.30% | -0.51% | -2.54% | 2.04% | -2.04% | | 0.06% | 0.39% | -1.60% | 1.38% | -1.53% | |
| K=72 | Sharpe Ratio | 0.28 | 0.45 | 0.10 | 0.55 | 0.08 | 0.25 | 0.39 | 0.49 | 0.28 | 0.48 | 0.27 | 0.38 | 0.41 | 0.50 | 0.31 | 0.45 | 0.31 | 0.38 |
| | *(p-value)* | *(0.431)* | *(0.201)* | *(0.167)* | *(0.053)* | *(0.093)* | | *(0.484)* | *(0.289)* | *(0.160)* | *(0.287)* | *(0.112)* | | *(0.436)* | *(0.269)* | *(0.175)* | *(0.341)* | *(0.161)* | |
| | Opp. Cost | -6.00% | -1.20% | -4.92% | 6.48% | -5.52% | | -4.80% | -3.12% | -2.88% | -6.12% | -2.88% | | -3.60% | -2.64% | -2.04% | -6.60% | -1.80% | |
| | Port.Turnover | 59.25% | 61.72% | 47.54% | 64.28% | 59.16% | 32.74% | 37.13% | 42.21% | 32.82% | 51.82% | 38.00% | 26.24% | 33.94% | 39.81% | 31.88% | 45.44% | 36.58% | 27.05% |
| | Return-Loss | -0.85% | 3.04% | -4.29% | 5.50% | -5.53% | | -0.91% | 0.90% | -2.44% | 0.43% | -3.03% | | -0.33% | 0.83% | -1.60% | -0.09% | -1.82% | |

**Panel B: Individual Commodity Futures Contracts (Power Utility)**

| | | RRA=2 | | | | | | RRA=4 | | | | | | RRA=6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** |
| K=36 | Sharpe Ratio | 0.38 | 0.49 | 0.20 | 0.75 | 0.26 | 0.49 | 0.50 | 0.56 | 0.31 | 0.69 | 0.41 | 0.57 | 0.56 | 0.61 | 0.38 | 0.69 | 0.45 | 0.59 |
| | *(p-value)* | *(0.320)* | *(0.489)* | *(0.055)* | *(0.096)* | *(0.049)* | | *(0.345)* | *(0.485)* | *(0.029)* | *(0.278)* | *(0.079)* | | *(0.429)* | *(0.459)* | *(0.033)* | *(0.276)* | *(0.080)* | |
| | Opp. Cost | -4.80% | -0.84% | -10.80% | 7.92% | -6.48% | | -3.48% | -2.40% | -7.92% | 0.36% | -4.32% | | -2.28% | -1.56% | -5.64% | -0.60% | -3.24% | |
| | Port.Turnover | 78.26% | 79.35% | 92.61% | 49.30% | 92.32% | 56.57% | 65.67% | 72.75% | 74.62% | 77.41% | 74.46% | 53.48% | 61.58% | 69.86% | 66.47% | 75.59% | 67.59% | 53.75% |
| | Return-Loss | -3.63% | -1.15% | -7.99% | 4.91% | -6.78% | | -2.19% | -1.22% | -6.10% | 1.18% | -4.11% | | -1.03% | -0.58% | -4.33% | 0.85% | -2.98% | |
| K=48 | Sharpe Ratio | 0.35 | 0.42 | 0.07 | 0.58 | 0.23 | 0.34 | 0.43 | 0.50 | 0.21 | 0.54 | 0.31 | 0.44 | 0.48 | 0.51 | 0.29 | 0.54 | 0.36 | 0.47 |
| | *(p-value)* | *(0.486)* | *(0.375)* | *(0.034)* | *(0.139)* | *(0.14)* | | *(0.495)* | *(0.396)* | *(0.014)* | *(0.312)* | *(0.125)* | | *(0.459)* | *(0.419)* | *(0.017)* | *(0.347)* | *(0.115)* | |
| | Opp. Cost | -3.12% | -0.84% | -8.52% | 5.40% | -3.72% | | -2.28% | -2.28% | -5.76% | -3.96% | -3.72% | | -1.44% | -2.64% | -3.96% | -3.60% | -2.52% | |
| | Port.Turnover | 68.82% | 65.53% | 71.15% | 71.92% | 75.03% | 44.66% | 51.05% | 57.92% | 52.43% | 65.25% | 58.48% | 39.61% | 43.85% | 53.05% | 46.78% | 59.50% | 49.69% | 37.72% |
| | Return-Loss | -1.21% | 0.67% | -7.25% | 3.96% | -3.91% | | -0.88% | 0.00% | -4.96% | 0.71% | -3.47% | | -0.31% | -0.41% | -4.33% | 0.08% | -2.51% | |
| K=60 | Sharpe Ratio | 0.22 | 0.27 | 0.02 | 0.48 | 0.09 | 0.17 | 0.30 | 0.31 | 0.17 | 0.44 | 0.21 | 0.27 | 0.30 | 0.33 | 0.19 | 0.40 | 0.21 | 0.26 |
| | *(p-value)* | *(0.394)* | *(0.344)* | *(0.145)* | *(0.057)* | *(0.314)* | | *(0.426)* | *(0.439)* | *(0.130)* | *(0.196)* | *(0.267)* | | *(0.383)* | *(0.350)* | *(0.160)* | *(0.207)* | *(0.261)* | |
| | Opp. Cost | -9.12% | -8.52% | -5.52% | 5.28% | -4.32% | | -4.92% | -6.24% | -3.12% | -4.92% | -2.76% | | -3.36% | -3.12% | -2.04% | -2.76% | -2.04% | |
| | Port.Turnover | 64.20% | 67.66% | 61.76% | 72.10% | 63.23% | 37.58% | 45.28% | 55.05% | 44.73% | 59.93% | 47.61% | 34.89% | 40.31% | 47.24% | 40.17% | 53.68% | 44.22% | 34.75% |
| | Return-Loss | -0.36% | 0.67% | -4.74% | 5.57% | -3.16% | | -0.36% | -0.70% | -2.61% | 1.95% | -2.08% | | 0.07% | 0.30% | -1.61% | 1.37% | -1.51% | |
| K=72 | Sharpe Ratio | 0.28 | 0.45 | 0.10 | 0.55 | 0.08 | 0.25 | 0.39 | 0.49 | 0.28 | 0.48 | 0.26 | 0.38 | 0.41 | 0.49 | 0.31 | 0.45 | 0.31 | 0.38 |
| | *(p-value)* | *(0.434)* | *(0.207)* | *(0.168)* | *(0.056)* | *(0.091)* | | *(0.481)* | *(0.300)* | *(0.161)* | *(0.290)* | *(0.109)* | | *(0.429)* | *(0.275)* | *(0.175)* | *(0.344)* | *(0.165)* | |
| | Opp. Cost | -10.80% | -8.64% | -5.04% | 5.64% | -5.64% | | -6.12% | -4.80% | -2.88% | -10.68% | -2.88% | | -4.32% | -3.36% | -2.04% | -9.24% | -1.68% | |
| | Port.Turnover | 59.38% | 62.50% | 48.00% | 65.28% | 59.21% | 32.81% | 37.95% | 42.90% | 33.09% | 52.80% | 38.66% | 26.62% | 33.79% | 40.30% | 31.96% | 45.98% | 36.14% | 27.08% |
| | Return-Loss | -0.89% | 2.89% | -4.29% | 5.38% | -5.57% | | -0.91% | 0.74% | -2.44% | 0.35% | -3.08% | | -0.28% | 0.75% | -1.61% | -0.14% | -1.80% | |

# TABLE 7

## Direct Utility Maximization: Individual Commodity Futures and Disappointment aversion value function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under a disappointment aversion value function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes commodities. Investors access investment in commodities via the selected individual future contracts. Results are reported for different sizes of the rolling window (K=36,48,60,72 observations), degrees of relative risk aversion (RRA=2,4,6) and values of the disappointment aversion parameter (Panel A for *A*=0.6 and Panel B for *A*=0.8). Results are based on monthly observations from Jan. 1989 –Dec. 2009.

**Panel A: Individual Commodity Futures Contracts (A=0.6)**

| | | RRA=2 | | | | | | RRA=4 | | | | | | RRA=6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** |
| K=36 | Sharpe Ratio | 0.49 | 0.61 | 0.38 | 0.69 | 0.49 | 0.59 | 0.54 | 0.56 | 0.37 | 0.64 | 0.44 | 0.50 | 0.54 | 0.55 | 0.37 | 0.57 | 0.46 | 0.49 |
| | *(p-value)* | *(0.264)* | *(0.472)* | *(0.027)* | *(0.262)* | *(0.138)* | | *(0.385)* | *(0.382)* | *(0.068)* | *(0.186)* | *(0.239)* | | *(0.357)* | *(0.376)* | *(0.082)* | *(0.292)* | *(0.346)* | |
| | Opp. Cost | -4.68% | -3.00% | -5.40% | -0.96% | -2.52% | | -2.28% | -3.24% | -0.84% | -1.80% | -1.92% | | -1.92% | -4.32% | -2.88% | -3.60% | -1.20% | |
| | Port.Turnover | 72.41% | 70.67% | 67.02% | 78.44% | 68.29% | 52.51% | 62.97% | 67.23% | 65.22% | 71.91% | 61.39% | 50.69% | 57.03% | 58.51% | 56.45% | 65.46% | 51.18% | 46.80% |
| | Return-Loss | -2.58% | -0.90% | -4.27% | 0.56% | -2.42% | | -0.24% | -0.33% | -2.66% | 0.80% | -1.41% | | -0.30% | -0.51% | -1.92% | -0.23% | -0.72% | |
| K=48 | Sharpe Ratio | 0.44 | 0.46 | 0.37 | 0.49 | 0.37 | 0.47 | 0.37 | 0.39 | 0.26 | 0.29 | 0.29 | 0.35 | 0.33 | 0.35 | 0.19 | 0.28 | 0.23 | 0.27 |
| | *(p-value)* | *(0.418)* | *(0.530)* | *(0.060)* | *(0.468)* | *(0.109)* | | *(0.423)* | *(0.405)* | *(0.056)* | *(0.376)* | *(0.220)* | | *(0.322)* | *(0.334)* | *(0.078)* | *(0.474)* | *(0.279)* | |
| | Opp. Cost | -3.24% | -5.04% | -2.64% | -4.80% | -2.64% | | -1.92% | -3.60% | -2.04% | -3.84% | -1.68% | | -1.56% | -3.12% | -1.56% | -5.16% | -1.80% | |
| | Port.Turnover | 62.26% | 68.15% | 60.41% | 71.38% | 61.71% | 45.72% | 55.19% | 61.02% | 57.52% | 70.67% | 52.26% | 47.47% | 50.22% | 52.27% | 50.40% | 64.64% | 45.57% | 45.69% |
| | Return-Loss | -1.47% | -1.64% | -2.36% | -1.08% | -2.49% | | -0.27% | -0.47% | -1.64% | -0.54% | -1.24% | | -0.01% | -0.19% | -1.16% | -1.05% | -0.79% | |
| K=60 | Sharpe Ratio | 0.25 | 0.24 | 0.20 | 0.31 | 0.27 | 0.30 | 0.19 | 0.20 | 0.12 | 0.21 | 0.16 | 0.17 | 0.16 | 0.19 | 0.08 | 0.17 | 0.13 | 0.14 |
| | *(p-value)* | *(0.377)* | *(0.395)* | *(0.058)* | *(0.468)* | *(0.363)* | | *(0.434)* | *(0.446)* | *(0.166)* | *(0.409)* | *(0.428)* | | *(0.431)* | *(0.390)* | *(0.130)* | *(0.421)* | *(0.492)* | |
| | Opp. Cost | -5.04% | -8.16% | -2.76% | -6.12% | -1.92% | | -3.24% | -4.44% | -1.44% | -4.80% | -1.80% | | -2.88% | -3.24% | -1.20% | -4.20% | -1.44% | |
| | Port.Turnover | 65.18% | 79.25% | 67.86% | 83.60% | 66.05% | 59.96% | 56.73% | 58.33% | 58.62% | 70.15% | 55.14% | 55.50% | 48.75% | 46.22% | 47.74% | 54.86% | 45.87% | 41.79% |
| | Return-Loss | -1.45% | -2.14% | -2.06% | -0.91% | -0.92% | | -0.23% | -0.42% | -0.88% | -0.52% | -0.38% | | -0.58% | -0.43% | -0.90% | -0.78% | -0.41% | |
| K=72 | Sharpe Ratio | 0.34 | 0.41 | 0.35 | 0.35 | 0.37 | 0.39 | 0.26 | 0.33 | 0.25 | 0.24 | 0.24 | 0.28 | 0.22 | 0.31 | 0.23 | 0.23 | 0.21 | 0.27 |
| | *(p-value)* | *(0.375)* | *(0.458)* | *(0.312)* | *(0.424)* | *(0.399)* | | *(0.452)* | *(0.401)* | *(0.315)* | *(0.406)* | *(0.259)* | | *(0.369)* | *(0.388)* | *(0.224)* | *(0.422)* | *(0.175)* | |
| | Opp. Cost | -4.08% | -4.92% | -1.32% | -7.44% | -1.44% | | -3.12% | -4.08% | -0.96% | -5.64% | -2.16% | | -2.88% | -3.24% | -0.84% | -3.96% | -1.68% | |
| | Port.Turnover | 50.72% | 59.11% | 58.09% | 78.79% | 47.00% | 66.87% | 47.41% | 49.76% | 54.29% | 63.10% | 53.10% | 53.45% | 42.58% | 39.80% | 44.29% | 49.36% | 43.48% | 38.76% |
| | Return-Loss | -0.84% | -0.14% | -0.41% | -1.62% | 0.11% | | -0.53% | -0.16% | -0.40% | -1.44% | -0.75% | | -1.10% | -0.72% | -0.61% | -1.36% | -0.93% | |

**Panel B: Individual Commodity Futures Contracts (A=0.8)**

| | | RRA=2 | | | | | | RRA=4 | | | | | | RRA=6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** | **Crude Oil** | **Copper** | **Cotton** | **Gold** | **Live Cattle** | **Traditional** |
| K=36 | Sharpe Ratio | 0.47 | 0.54 | 0.25 | 0.72 | 0.37 | 0.58 | 0.53 | 0.59 | 0.34 | 0.67 | 0.44 | 0.57 | 0.57 | 0.59 | 0.38 | 0.65 | 0.46 | 0.56 |
| | *(p-value)* | *(0.287)* | *(0.434)* | *(0.015)* | *(0.212)* | *(0.048)* | | *(0.395)* | *(0.470)* | *(0.021)* | *(0.274)* | *(0.094)* | | *(0.493)* | *(0.452)* | *(0.029)* | *(0.283)* | *(0.346)* | |
| | Opp. Cost | -5.16% | -3.12% | -10.20% | 2.28% | -5.52% | | -3.12% | -2.64% | -6.24% | -0.84% | -3.24% | | -2.16% | -2.76% | -4.92% | -2.16% | -1.20% | |
| | Port.Turnover | 70.36% | 79.16% | 83.24% | 89.38% | 85.69% | 61.79% | 64.83% | 71.04% | 70.53% | 84.70% | 76.75% | 56.33% | 61.79% | 66.18% | 61.87% | 76.61% | 51.18% | 56.81% |
| | Return-Loss | -3.05% | -1.82% | -7.67% | 1.63% | -5.16% | | -1.33% | -0.73% | -4.74% | 0.55% | -3.05% | | -0.43% | -0.42% | -3.25% | 0.45% | -0.72% | |
| K=48 | Sharpe Ratio | 0.41 | 0.44 | 0.21 | 0.56 | 0.29 | 0.45 | 0.47 | 0.50 | 0.33 | 0.53 | 0.37 | 0.48 | 0.45 | 0.47 | 0.29 | 0.48 | 0.33 | 0.41 |
| | *(p-value)* | *(0.424)* | *(0.494)* | *(0.012)* | *(0.297)* | *(0.099)* | | *(0.486)* | *(0.459)* | *(0.025)* | *(0.372)* | *(0.169)* | | *(0.400)* | *(0.388)* | *(0.029)* | *(0.329)* | *(0.158)* | |
| | Opp. Cost | -3.84% | -4.44% | -6.60% | -0.96% | -4.68% | | -2.28% | -3.60% | -3.48% | -3.72% | -2.64% | | -1.32% | -2.52% | -2.76% | -2.64% | -1.80% | |
| | Port.Turnover | 57.52% | 64.40% | 58.16% | 70.52% | 63.90% | 47.07% | 46.85% | 58.74% | 52.15% | 65.13% | 56.37% | 46.49% | 48.17% | 55.51% | 50.82% | 62.50% | 55.03% | 48.50% |
| | Return-Loss | -1.70% | -1.26% | -5.54% | 0.91% | -4.12% | | -0.54% | -0.69% | -2.93% | -0.06% | -2.71% | | 0.19% | 0.01% | -2.11% | 0.21% | -1.56% | |
| K=60 | Sharpe Ratio | 0.25 | 0.27 | 0.21 | 0.47 | 0.23 | 0.32 | 0.28 | 0.31 | 0.22 | 0.39 | 0.23 | 0.30 | 0.27 | 0.32 | 0.20 | 0.34 | 0.20 | 0.26 |
| | *(p-value)* | *(0.360)* | *(0.423)* | *(0.095)* | *(0.246)* | *(0.189)* | | *(0.465)* | *(0.466)* | *(0.121)* | *(0.302)* | *(0.190)* | | *(0.458)* | *(0.373)* | *(0.143)* | *(0.320)* | *(0.230)* | |
| | Opp. Cost | -7.56% | -10.44% | -3.60% | -1.80% | -3.12% | | -4.32% | -5.28% | -2.16% | -4.08% | -2.28% | | -3.12% | -3.12% | -1.56% | -3.48% | -2.52% | |
| | Port.Turnover | 49.91% | 57.11% | 45.14% | 63.49% | 48.14% | 33.53% | 43.38% | 52.40% | 44.10% | 58.46% | 47.95% | 40.14% | 41.98% | 47.56% | 44.08% | 52.78% | 46.00% | 41.08% |
| | Return-Loss | -2.76% | -2.67% | -3.12% | 1.57% | -2.80% | | -0.82% | -0.74% | -1.65% | 0.65% | -1.78% | | -0.26% | 0.08% | -1.19% | 0.27% | -1.22% | |
| K=72 | Sharpe Ratio | 0.31 | 0.42 | 0.27 | 0.51 | 0.26 | 0.38 | 0.40 | 0.48 | 0.33 | 0.45 | 0.34 | 0.42 | 0.39 | 0.46 | 0.32 | 0.39 | 0.31 | 0.38 |
| | *(p-value)* | *(0.365)* | *(0.417)* | *(0.122)* | *(0.250)* | *(0.106)* | | *(0.459)* | *(0.362)* | *(0.121)* | *(0.431)* | *(0.150)* | | *(0.481)* | *(0.335)* | *(0.165)* | *(0.480)* | *(0.141)* | |
| | Opp. Cost | -7.68% | -7.08% | -3.24% | -2.52% | -3.36% | | -4.08% | -3.84% | -2.16% | -7.56% | -1.92% | | -3.00% | -3.60% | -1.44% | -6.96% | -2.52% | |
| | Port.Turnover | 45.09% | 46.86% | 34.71% | 56.49% | 43.52% | 25.52% | 34.48% | 39.92% | 34.10% | 46.72% | 36.20% | 28.36% | 34.70% | 38.20% | 35.67% | 44.65% | 36.65% | 32.95% |
| | Return-Loss | -2.79% | -0.72% | -2.91% | 0.95% | -3.36% | | -0.98% | 0.01% | -1.78% | -0.81% | -1.74% | | -0.43% | 0.14% | -1.11% | -0.93% | -1.34% | |

# TABLE 8

## Mean-Variance Optimization: Commodity Indexes and Taylor series expansion of Exponential utility function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under a second order Taylor series expansion of exponential utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes commodities. Investors access investment in commodities either via S&P GSCI (Panel A) or via DJ-UBS CI (Panel B). Results are reported for different sizes of the rolling window (K=36,48,60,72 observations) and different degrees of absolute risk aversion (ARA=2,4,6,8,10). Results are based on monthly observations from Jan. 1989 – Dec. 2009 for S&P GSCI and Jan. 1991-Dec. 2009 for DJ-UBS CI.

**Panel A: S&P GSCI (1989-2009)**

| | | ARA=2 | | ARA=4 | | ARA=6 | | ARA=8 | | ARA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.31 | 0.49 | 0.38 | 0.58 | 0.39 | 0.60 | 0.40 | 0.60 | 0.42 | 0.59 |
| | *(p-value)* | *(0.155)* | | *(0.089)* | | *(0.063)* | | *(0.065)* | | *(0.066)* | |
| | Opp. Cost | -5.88% | | -6.24% | | -5.52% | | -3.96% | | -3.84% | |
| | Port.Turnover | 80.96% | 54.85% | 71.65% | 52.94% | 69.12% | 52.81% | 60.09% | 52.81% | 61.65% | 53.45% |
| | Return-Loss | -5.34% | | -4.92% | | -4.43% | | -3.76% | | -3.06% | |
| **K=48** | Sharpe Ratio | 0.21 | 0.33 | 0.31 | 0.45 | 0.38 | 0.49 | 0.38 | 0.47 | 0.32 | 0.42 |
| | *(p-value)* | *(0.254)* | | *(0.169)* | | *(0.189)* | | *(0.197)* | | *(0.154)* | |
| | Opp. Cost | -5.16% | | -4.56% | | -3.24% | | -2.40% | | -2.04% | |
| | Port.Turnover | 70.91% | 44.89% | 56.77% | 40.02% | 48.50% | 37.41% | 46.99% | 39.86% | 49.53% | 41.90% |
| | Return-Loss | -4.12% | | -3.67% | | -2.55% | | -1.87% | | -1.87% | |
| **K=60** | Sharpe Ratio | 0.03 | 0.16 | 0.16 | 0.28 | 0.19 | 0.28 | 0.24 | 0.29 | 0.19 | 0.25 |
| | *(p-value)* | *(0.222)* | | *(0.176)* | | *(0.191)* | | *(0.278)* | | *(0.280)* | |
| | Opp. Cost | -7.08% | | -4.92% | | -3.60% | | -2.16% | | -2.04% | |
| | Port.Turnover | 71.33% | 37.95% | 52.10% | 33.62% | 48.67% | 36.49% | 41.34% | 38.39% | 40.82% | 35.61% |
| | Return-Loss | -4.90% | | -3.72% | | -2.55% | | -1.39% | | -1.23% | |
| **K=72** | Sharpe Ratio | 0.11 | 0.24 | 0.30 | 0.38 | 0.34 | 0.39 | 0.33 | 0.36 | 0.35 | 0.39 |
| | *(p-value)* | *(0.218)* | | *(0.278)* | | *(0.331)* | | *(0.392)* | | *(0.379)* | |
| | Opp. Cost | -7.92% | | -3.96% | | -2.76% | | -2.04% | | -1.80% | |
| | Port.Turnover | 65.85% | 32.47% | 42.60% | 25.49% | 36.36% | 26.58% | 37.68% | 33.80% | 32.24% | 29.21% |
| | Return-Loss | -5.08% | | -2.84% | | -1.69% | | -0.94% | | -0.87% | |

**Panel B: DJ-UBS CI (1991-2009)**

| | | ARA=2 | | ARA=4 | | ARA=6 | | ARA=8 | | ARA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.47 | 0.45 | 0.51 | 0.55 | 0.48 | 0.56 | 0.46 | 0.55 | 0.49 | 0.55 |
| | *(p-value)* | *(0.475)* | | *(0.413)* | | *(0.312)* | | *(0.280)* | | *(0.317)* | |
| | Opp. Cost | 0.00% | | -2.52% | | -3.60% | | -3.24% | | -2.76% | |
| | Port.Turnover | 81.95% | 55.62% | 76.13% | 56.13% | 76.03% | 57.21% | 70.78% | 59.01% | 68.24% | 58.75% |
| | Return-Loss | -0.96% | | -1.77% | | -2.35% | | -2.08% | | -1.55% | |
| **K=48** | Sharpe Ratio | 0.38 | 0.37 | 0.42 | 0.49 | 0.46 | 0.53 | 0.43 | 0.50 | 0.41 | 0.46 |
| | *(p-value)* | *(0.482)* | | *(0.324)* | | *(0.299)* | | *(0.292)* | | *(0.351)* | |
| | Opp. Cost | -0.84% | | -5.16% | | -3.36% | | -2.88% | | -2.28% | |
| | Port.Turnover | 73.62% | 42.91% | 64.08% | 39.83% | 57.24% | 39.25% | 53.87% | 41.11% | 55.30% | 45.63% |
| | Return-Loss | -1.33% | | -2.82% | | -2.49% | | -2.07% | | -1.36% | |
| **K=60** | Sharpe Ratio | -0.07 | 0.07 | 0.04 | 0.22 | 0.09 | 0.24 | 0.12 | 0.23 | 0.12 | 0.21 |
| | *(p-value)* | *(0.225)* | | *(0.120)* | | *(0.144)* | | *(0.183)* | | *(0.233)* | |
| | Opp. Cost | -8.04% | | -7.20% | | -4.92% | | -3.24% | | -2.52% | |
| | Port.Turnover | 69.28% | 36.96% | 60.31% | 33.80% | 54.72% | 38.37% | 23.32% | 36.43% | 43.06% | 36.02% |
| | Return-Loss | -5.20% | | -5.52% | | -3.68% | | -2.61% | | -1.80% | |
| **K=72** | Sharpe Ratio | -0.10 | 0.00 | 0.05 | 0.19 | 0.08 | 0.21 | 0.08 | 0.21 | 0.10 | 0.22 |
| | *(p-value)* | *(0.321)* | | *(0.206)* | | *(0.178)* | | *(0.173)* | | *(0.209)* | |
| | Opp. Cost | -8.28% | | -6.84% | | -5.04% | | -4.44% | | -3.00% | |
| | Port.Turnover | 60.87% | 32.94% | 45.13% | 24.83% | 40.11% | 26.55% | 37.97% | 28.54% | 33.17% | 29.48% |
| | Return-Loss | -3.76% | | -4.23% | | -3.31% | | -2.65% | | -1.85% | |

## TABLE 9

## Mean-Variance Optimization: Commodity Indexes and Taylor series expansion of Power utility function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under a second order Taylor series expansion of power utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes commodities. Investors access investment in commodities either via S&P GSCI (Panel A) or via DJ-UBS CI (Panel B). Results are reported for different sizes of the rolling window (K=36,48,60,72 observations) and different degrees of relative risk aversion (ARA=2,4,6,8,10). Results are based on monthly observations from Jan. 1989 –Dec. 2009 for S&P GSCI and Jan. 1991-Dec. 2009 for DJ-UBS CI.

**Panel A: S&P GSCI (1989-2009)**

| | | RRA=2 | | RRA=4 | | RRA=6 | | RRA=8 | | RRA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.31 | 0.49 | 0.38 | 0.59 | 0.39 | 0.60 | 0.41 | 0.60 | 0.42 | 0.59 |
| | *(p-value)* | *(0.155)* | | *(0.084)* | | *(0.063)* | | *(0.067)* | | *(0.066)* | |
| | Opp. Cost | -5.88% | | -6.36% | | -5.76% | | -3.84% | | -4.08% | |
| | Port.Turnover | 81.25% | 55.01% | 71.89% | 52.42% | 68.55% | 52.99% | 65.97% | | 61.80% | 53.24% |
| | Return-Loss | -5.31% | | -5.13% | | -4.47% | | -3.95% | | -3.14% | |
| **K=48** | Sharpe Ratio | 0.21 | 0.33 | 0.30 | 0.44 | 0.38 | 0.49 | 0.38 | 0.48 | 0.33 | 0.42 |
| | *(p-value)* | *(0.258)* | | *(0.166)* | | *(0.184)* | | *(0.174)* | | *(0.171)* | |
| | Opp. Cost | -5.16% | | -4.68% | | -3.36% | | -2.64% | | -2.16% | |
| | Port.Turnover | 70.84% | 44.62% | 57.92% | 40.37% | 49.15% | 37.86% | 47.11% | 37.93% | 48.79% | 41.34% |
| | Return-Loss | -4.11% | | -3.80% | | -2.66% | | -2.14% | | -1.76% | |
| **K=60** | Sharpe Ratio | 0.02 | 0.16 | 0.16 | 0.28 | 0.19 | 0.28 | 0.21 | 0.28 | 0.20 | 0.25 |
| | *(p-value)* | *(0.221)* | | *(0.177)* | | *(0.201)* | | *(0.238)* | | *(0.283)* | |
| | Opp. Cost | -7.32% | | -5.04% | | -3.60% | | -2.64% | | -2.04% | |
| | Port.Turnover | 72.53% | 38.18% | 52.35% | 33.15% | 49.33% | 37.42% | 44.11% | 35.21% | 41.16% | 35.79% |
| | Return-Loss | -5.00% | | -3.81% | | -2.49% | | -1.77% | | -1.20% | |
| **K=72** | Sharpe Ratio | 0.10 | 0.24 | 0.29 | 0.37 | 0.34 | 0.40 | 0.33 | 0.38 | 0.35 | 0.39 |
| | *(p-value)* | *(0.221)* | | *(0.274)* | | *(0.339)* | | *(0.345)* | | *(0.155)* | |
| | Opp. Cost | -8.16% | | -4.08% | | -2.88% | | -2.28% | | -1.80% | |
| | Port.Turnover | 66.71% | 32.79% | 43.27% | 25.31% | 36.51% | 26.59% | 35.26% | 27.73% | 32.73% | 28.64% |
| | Return-Loss | -5.09% | | -2.95% | | -1.68% | | -1.27% | | -0.95% | |

**Panel B: DJ-UBS CI (1991-2009)**

| | | RRA=2 | | RRA=4 | | RRA=6 | | RRA=8 | | RRA=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** | **Expanded** | **Traditional** |
| **K=36** | Sharpe Ratio | 0.47 | 0.45 | 0.51 | 0.55 | 0.49 | 0.56 | 0.48 | 0.55 | 0.48 | 0.55 |
| | *(p-value)* | *(0.473)* | | *(0.404)* | | *(0.318)* | | *(0.298)* | | *(0.308)* | |
| | Opp. Cost | 0.00% | | -2.52% | | -3.60% | | -3.24% | | -3.00% | |
| | Port.Turnover | 82.40% | 55.71% | 75.74% | 55.43% | 75.96% | 57.45% | 71.60% | 59.06% | 68.57% | 58.61% |
| | Return-Loss | -0.96% | | -1.88% | | -2.32% | | -1.99% | | -1.63% | |
| **K=48** | Sharpe Ratio | 0.38 | 0.37 | 0.41 | 0.49 | 0.46 | 0.54 | 0.45 | 0.52 | 0.41 | 0.47 |
| | *(p-value)* | *(0.475)* | | *(0.331)* | | *(0.301)* | | *(0.300)* | | *(0.342)* | |
| | Opp. Cost | -0.72% | | -3.48% | | -3.48% | | -2.88% | | -2.28% | |
| | Port.Turnover | 73.54% | 42.50% | 64.18% | 40.06% | 57.64% | 39.67% | 53.26% | 40.87% | 54.58% | 44.96% |
| | Return-Loss | -1.27% | | -2.77% | | -2.49% | | -2.03% | | -1.42% | |
| **K=60** | Sharpe Ratio | -0.07 | 0.07 | 0.03 | 0.22 | 0.10 | 0.24 | 0.11 | 0.24 | 0.11 | 0.22 |
| | *(p-value)* | *(0.233)* | | *(0.119)* | | *(0.149)* | | *(0.162)* | | *(0.199)* | |
| | Opp. Cost | -8.04% | | -7.44% | | -4.92% | | -3.60% | | -2.40% | |
| | Port.Turnover | 69.57% | 37.14% | 60.44% | 33.23% | 55.56% | 39.36% | 48.50% | 37.13% | 43.34% | 36.28% |
| | Return-Loss | -5.11% | | -5.64% | | -3.65% | | -2.82% | | -2.05% | |
| **K=72** | Sharpe Ratio | -0.10 | -0.01 | 0.04 | 0.18 | 0.08 | 0.22 | 0.07 | 0.21 | 0.09 | 0.22 |
| | *(p-value)* | *(0.324)* | | *(0.209)* | | *(0.183)* | | *(0.165)* | | *(0.170)* | |
| | Opp. Cost | -8.28% | | -7.08% | | -5.16% | | -4.20% | | -3.36% | |
| | Port.Turnover | 61.46% | 33.24% | 45.85% | 24.55% | 39.79% | 26.45% | 37.99% | 28.65% | 33.80% | |
| | Return-Loss | -3.73% | | -4.30% | | -3.30% | | -2.75% | | -2.22% | |

# TABLE 10

## Mean-Variance Optimization: Individual Commodity Futures and Taylor series expansions of Exponential and Power utility functions

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under a second order Taylor series expansion of exponential (Panel A) and power utility (Panel B). The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes commodities. Investors access investment in commodities via the selected individual commodity futures contracts. Results are reported for different sizes of the rolling window (K=36,48,60,72 observations) and different degrees of absolute/relative risk aversion (ARA,RRA=2,4,6). Results are based on monthly observations from Jan. 1989 to Dec. 2009.

**Panel A: Individual Commodity Futures Contracts (Taylor series expansion of Exponential Utility)**

**ARA=2**

| K | | Crude Oil | Copper | Cotton | Gold | Live Cattle | Traditional |
|---|---|---|---|---|---|---|---|
| K=36 | Sharpe Ratio | 0.40 | 0.49 | 0.19 | 0.75 | 0.26 | 0.49 |
| | *(p-value)* | *(0.350)* | *(0.497)* | *(0.051)* | *(0.092)* | *(0.047)* | |
| | Opp. Cost | -3.84% | -1.32% | -10.08% | 7.92% | -6.12% | |
| | Port.Turnover | 74.95% | 78.71% | 90.60% | 82.28% | 91.49% | 54.85% |
| | Return-Loss | -3.13% | -1.00% | -8.06% | 4.90% | -6.69% | |
| K=48 | Sharpe Ratio | 0.36 | 0.42 | 0.07 | 0.57 | 0.22 | 0.33 |
| | *(p-value)* | *(0.463)* | *(0.377)* | *(0.034)* | *(0.143)* | *(0.204)* | |
| | Opp. Cost | -2.28% | -0.84% | -8.16% | 5.52% | -3.60% | |
| | Port.Turnover | 68.48% | 66.38% | 70.96% | 71.86% | 75.36% | 44.89% |
| | Return-Loss | -0.89% | 0.64% | -7.16% | 3.87% | -3.97% | |
| K=60 | Sharpe Ratio | 0.21 | 0.25 | 0.02 | 0.48 | 0.09 | 0.16 |
| | *(p-value)* | *(0.411)* | *(0.354)* | *(0.162)* | *(0.055)* | *(0.319)* | |
| | Opp. Cost | -4.68% | -4.56% | -5.04% | 6.60% | -3.48% | |
| | Port.Turnover | 66.15% | 69.54% | 61.30% | 72.83% | 63.04% | 37.95% |
| | Return-Loss | -0.58% | 0.49% | -4.45% | 5.67% | -3.03% | |
| K=72 | Sharpe Ratio | 0.28 | 0.46 | 0.10 | 0.55 | 0.08 | 0.24 |
| | *(p-value)* | *(0.427)* | *(0.188)* | *(0.164)* | *(0.05)* | *(0.099)* | |
| | Opp. Cost | -3.84% | 1.32% | -4.80% | 7.20% | -5.28% | |
| | Port.Turnover | 59.10% | 60.24% | 47.11% | 62.63% | 59.05% | 32.47% |
| | Return-Loss | -0.80% | 3.30% | -4.35% | 5.67% | -5.42% | |

**ARA=4**

| K | | Crude Oil | Copper | Cotton | Gold | Live Cattle | Traditional |
|---|---|---|---|---|---|---|---|
| K=36 | Sharpe Ratio | 0.53 | 0.61 | 0.33 | 0.70 | 0.43 | 0.58 |
| | *(p-value)* | *(0.390)* | *(0.454)* | *(0.029)* | *(0.270)* | *(0.075)* | |
| | Opp. Cost | -2.76% | -3.60% | -7.20% | 0.84% | -3.96% | |
| | Port.Turnover | 63.34% | 70.37% | 74.19% | 80.00% | 73.38% | 52.94% |
| | Return-Loss | -1.73% | -0.50% | -5.97% | 1.18% | -4.02% | |
| K=48 | Sharpe Ratio | 0.45 | 0.54 | 0.23 | 0.55 | 0.32 | 0.45 |
| | *(p-value)* | *(0.487)* | *(0.339)* | *(0.013)* | *(0.296)* | *(0.114)* | |
| | Opp. Cost | -2.04% | -2.16% | -5.52% | -0.72% | -3.48% | |
| | Port.Turnover | 52.20% | 57.39% | 53.60% | 64.95% | 58.83% | 40.02% |
| | Return-Loss | -0.73% | 0.77% | -4.98% | 0.97% | -3.48% | |
| K=60 | Sharpe Ratio | 0.31 | 0.33 | 0.19 | 0.44 | 0.22 | 0.28 |
| | *(p-value)* | *(0.415)* | *(0.395)* | *(0.141)* | *(0.203)* | *(0.265)* | |
| | Opp. Cost | -3.12% | -3.96% | -2.88% | -0.60% | -2.40% | |
| | Port.Turnover | 45.72% | 53.66% | 44.30% | 61.44% | 47.56% | 33.62% |
| | Return-Loss | -0.34% | -0.16% | -2.56% | 1.90% | -2.11% | |
| K=72 | Sharpe Ratio | 0.37 | 0.51 | 0.28 | 0.48 | 0.27 | 0.38 |
| | *(p-value)* | *(0.480)* | *(0.250)* | *(0.154)* | *(0.282)* | *(0.111)* | |
| | Opp. Cost | -3.48% | -0.96% | -2.88% | -2.16% | -2.88% | |
| | Port.Turnover | 36.65% | 40.68% | 32.58% | 51.74% | 38.04% | 25.49% |
| | Return-Loss | -1.24% | 1.46% | -2.56% | 0.56% | -3.07% | |

**ARA=6**

| K | | Crude Oil | Copper | Cotton | Gold | Live Cattle | Traditional |
|---|---|---|---|---|---|---|---|
| K=36 | Sharpe Ratio | 0.57 | 0.65 | 0.40 | 0.70 | 0.47 | 0.60 |
| | *(p-value)* | *(0.428)* | *(0.395)* | *(0.038)* | *(0.272)* | *(0.079)* | |
| | Opp. Cost | -2.16% | -3.24% | -5.04% | -0.48% | -3.00% | |
| | Port.Turnover | 58.57% | 69.31% | 66.41% | 77.73% | 67.07% | 52.81% |
| | Return-Loss | -0.99% | -0.02% | -4.12% | 0.75% | -2.90% | |
| K=48 | Sharpe Ratio | 0.50 | 0.56 | 0.32 | 0.56 | 0.38 | 0.49 |
| | *(p-value)* | *(0.468)* | *(0.352)* | *(0.016)* | *(0.341)* | *(0.105)* | |
| | Opp. Cost | -1.68% | -2.52% | -3.96% | -1.80% | -2.64% | |
| | Port.Turnover | 44.20% | 52.39% | 46.28% | 59.32% | 48.68% | 37.41% |
| | Return-Loss | -0.42% | 0.26% | -3.48% | 0.14% | -2.49% | |
| K=60 | Sharpe Ratio | 0.31 | 0.37 | 0.22 | 0.40 | 0.23 | 0.28 |
| | *(p-value)* | *(0.416)* | *(0.314)* | *(0.160)* | *(0.237)* | *(0.247)* | |
| | Opp. Cost | -2.76% | -2.28% | -1.57% | -1.44% | -1.92% | |
| | Port.Turnover | 43.78% | 48.85% | 41.64% | 57.33% | 46.21% | 36.49% |
| | Return-Loss | -0.18% | 0.71% | -1.57% | 1.13% | -1.54% | |
| K=72 | Sharpe Ratio | 0.40 | 0.52 | 0.32 | 0.46 | 0.32 | 0.39 |
| | *(p-value)* | *(0.482)* | *(0.233)* | *(0.163)* | *(0.337)* | *(0.171)* | |
| | Opp. Cost | -2.40% | -1.08% | -2.04% | -2.76% | -1.80% | |
| | Port.Turnover | 34.14% | 39.10% | 31.76% | 44.67% | 34.93% | 26.58% |
| | Return-Loss | -0.61% | 1.24% | -1.71% | 0.00% | -1.75% | |

**Panel B: Individual Commodity Futures Contracts (Taylor series expansion of Power Utility)**

**RRA=2**

| K | | Crude Oil | Copper | Cotton | Gold | Live Cattle | Traditional |
|---|---|---|---|---|---|---|---|
| K=36 | Sharpe Ratio | 0.39 | 0.49 | 0.19 | 0.75 | 0.25 | 0.49 |
| | *(p-value)* | *(0.336)* | *(0.500)* | *(0.050)* | *(0.092)* | *(0.045)* | |
| | Opp. Cost | -4.32% | -1.32% | -10.44% | 8.04% | -6.24% | |
| | Port.Turnover | 76.64% | 78.82% | 91.35% | 82.84% | 92.27% | 55.01% |
| | Return-Loss | -3.44% | -1.04% | -8.20% | 4.91% | -6.85% | |
| K=48 | Sharpe Ratio | 0.36 | 0.42 | 0.06 | 0.57 | 0.22 | 0.33 |
| | *(p-value)* | *(0.458)* | *(0.378)* | *(0.033)* | *(0.136)* | *(0.208)* | |
| | Opp. Cost | -2.40% | -0.84% | -8.40% | 5.76% | -3.60% | |
| | Port.Turnover | 69.18% | 66.54% | 71.65% | 72.83% | 75.65% | 44.62% |
| | Return-Loss | -0.86% | 0.61% | -7.33% | 4.03% | -3.98% | |
| K=60 | Sharpe Ratio | 0.20 | 0.25 | 0.02 | 0.48 | 0.09 | 0.16 |
| | *(p-value)* | *(0.429)* | *(0.356)* | *(0.159)* | *(0.054)* | *(0.317)* | |
| | Opp. Cost | -5.52% | -4.80% | -5.16% | 6.72% | -3.60% | |
| | Port.Turnover | 69.26% | 70.52% | 62.26% | 73.37% | 64.11% | 38.18% |
| | Return-Loss | -0.84% | 0.44% | -4.58% | 5.71% | -3.12% | |
| K=72 | Sharpe Ratio | 0.28 | 0.44 | 0.09 | 0.55 | 0.07 | 0.24 |
| | *(p-value)* | *(0.415)* | *(0.198)* | *(0.161)* | *(0.049)* | *(0.100)* | |
| | Opp. Cost | -3.84% | 0.84% | -4.92% | 7.32% | -5.40% | |
| | Port.Turnover | 59.92% | 63.04% | 48.11% | 62.81% | 32.79% | 32.79% |
| | Return-Loss | -0.65% | 3.10% | -4.45% | 5.72% | -5.45% | |

**RRA=4**

| K | | Crude Oil | Copper | Cotton | Gold | Live Cattle | Traditional |
|---|---|---|---|---|---|---|---|
| K=36 | Sharpe Ratio | 0.53 | 0.60 | 0.33 | 0.70 | 0.43 | 0.59 |
| | *(p-value)* | *(0.384)* | *(0.468)* | *(0.026)* | *(0.273)* | *(0.071)* | |
| | Opp. Cost | -3.00% | -3.84% | -7.68% | 0.84% | -4.08% | |
| | Port.Turnover | 63.88% | 70.47% | 74.67% | 80.16% | 73.62% | 52.42% |
| | Return-Loss | -1.87% | -0.68% | -6.25% | 1.12% | -4.18% | |
| K=48 | Sharpe Ratio | 0.44 | 0.53 | 0.22 | 0.55 | 0.31 | 0.44 |
| | *(p-value)* | *(0.492)* | *(0.358)* | *(0.013)* | *(0.291)* | *(0.113)* | |
| | Opp. Cost | -2.28% | -2.76% | -5.76% | -0.60% | -3.60% | |
| | Port.Turnover | 53.30% | 58.24% | 54.27% | 65.46% | 59.63% | 40.37% |
| | Return-Loss | -0.81% | 0.54% | -5.10% | 1.07% | -3.57% | |
| K=60 | Sharpe Ratio | 0.31 | 0.32 | 0.18 | 0.44 | 0.22 | 0.28 |
| | *(p-value)* | *(0.425)* | *(0.414)* | *(0.137)* | *(0.203)* | *(0.263)* | |
| | Opp. Cost | -3.36% | -4.68% | -3.00% | -0.60% | -2.52% | |
| | Port.Turnover | 45.79% | 53.61% | 44.11% | 61.81% | 47.23% | 33.15% |
| | Return-Loss | -0.44% | -0.39% | -2.66% | 1.93% | -2.16% | |
| K=72 | Sharpe Ratio | 0.36 | 0.50 | 0.27 | 0.49 | 0.26 | 0.37 |
| | *(p-value)* | *(0.466)* | *(0.257)* | *(0.154)* | *(0.268)* | *(0.107)* | |
| | Opp. Cost | -3.84% | -1.08% | -2.88% | -1.92% | -3.00% | |
| | Port.Turnover | 37.49% | 40.94% | 32.62% | 52.36% | 38.66% | 25.31% |
| | Return-Loss | -1.42% | 1.39% | -2.61% | 0.70% | -3.20% | |

**RRA=6**

| K | | Crude Oil | Copper | Cotton | Gold | Live Cattle | Traditional |
|---|---|---|---|---|---|---|---|
| K=36 | Sharpe Ratio | 0.56 | 0.64 | 0.40 | 0.70 | 0.46 | 0.60 |
| | *(p-value)* | *(0.417)* | *(0.412)* | *(0.035)* | *(0.092)* | *(0.076)* | |
| | Opp. Cost | -2.28% | -4.08% | -5.28% | 8.04% | -3.12% | |
| | Port.Turnover | 58.77% | 70.57% | 66.61% | 78.48% | 67.20% | 52.99% |
| | Return-Loss | -1.09% | -0.20% | -4.24% | 4.91% | -2.97% | |
| K=48 | Sharpe Ratio | 0.49 | 0.56 | 0.32 | 0.56 | 0.38 | 0.49 |
| | *(p-value)* | *(0.483)* | *(0.368)* | *(0.015)* | *(0.346)* | *(0.010)* | |
| | Opp. Cost | -1.80% | -2.88% | -4.08% | -1.92% | -2.76% | |
| | Port.Turnover | 44.84% | 53.45% | 46.92% | 59.55% | 50.48% | 37.86% |
| | Return-Loss | -0.54% | 0.12% | -3.58% | 0.13% | -2.62% | |
| K=60 | Sharpe Ratio | 0.31 | 0.36 | 0.22 | 0.41 | 0.23 | 0.28 |
| | *(p-value)* | *(0.414)* | *(0.324)* | *(0.158)* | *(0.234)* | *(0.265)* | |
| | Opp. Cost | -2.88% | -2.52% | -2.04% | -1.56% | -1.92% | |
| | Port.Turnover | 44.71% | 50.42% | 42.94% | 58.15% | 46.52% | 37.42% |
| | Return-Loss | -0.15% | 0.62% | -1.63% | 1.20% | -1.45% | |
| K=72 | Sharpe Ratio | 0.40 | 0.52 | 0.32 | 0.46 | 0.33 | 0.40 |
| | *(p-value)* | *(0.492)* | *(0.237)* | *(0.162)* | *(0.340)* | *(0.173)* | |
| | Opp. Cost | -2.52% | -1.20% | -2.04% | -2.88% | -1.80% | |
| | Port.Turnover | 34.02% | 38.46% | 31.78% | 44.24% | 34.96% | 26.59% |
| | Return-Loss | -0.69% | 1.27% | -1.74% | 0.00% | -1.76% | |

## Appendix A: Mean-Variance Spanning Tests in Excess Returns

Under the MV setting, when the test is formulated in gross returns, equation (9) needs to be estimated and the constraints under the null hypothesis for spanning are given by equation (10). When the initial $K$-benchmark asset universe includes also the risk-free asset, the test for MV spanning is modified and is formulated in excess returns terms. Subtracting the risk-free rate from both sides of (9), yields

$$R_{t+1}^{test} - R_t^f = \alpha + \beta R_{t+1} - R_t^f + \varepsilon_{t+1} \Rightarrow R_{t+1}^{test} - R_t^f = \alpha + \beta R_{t+1} - \left(R_t^f \left(1 - \beta\iota_K\right) + R_t^f \beta\iota_K\right) + \varepsilon_{t+1} \Rightarrow$$

$$R_{t+1}^{test} - R_t^f = \left[\alpha - R_t^f \left(1 - \beta\iota_K\right)\right] + \beta\left(R_{t+1} - R_t^f \iota_K\right) + \varepsilon_{t+1} \tag{44}$$

Let $\alpha_J$ be the intercept in the regression of the test asset's excess returns on the excess returns of the $K$ benchmark assets [see equation (11)]. Equation (44) establishes the equivalence between the intercepts of equations (9) and (11), i.e. $\alpha_J = \alpha - R_t^f \left(1 - \beta i_K\right)$ . Since the restrictions when the test is formulated in gross returns are $\alpha = 0$ and $\beta i_K = 1$, the equivalent restriction in excess returns is that $\alpha_J = 0$.

Hence, when the $K$-benchmark asset universe includes also the risk-free asset, equation (11) need to be estimated and the constraints under the null hypothesis of spanning refer only to the intercept term (equation (13)). The slope coefficients multiply only the excess returns of the *(K-1)* risky assets and therefore they do not need to add up to one.

## Appendix B: Non Mean-Variance Spanning Tests in Excess Returns

Under the non-MV setting, when the test is formulated in gross returns, equation (14) needs to be estimated and the null hypotheses for spanning to be tested [equation (15)]. When the initial $K$-benchmark asset universe includes also the risk-free asset, the test for non- MV spanning is also formulated in terms of excess returns. Subtracting the risk-free rate from both sides of (14) yields

$$R_{t+1}^{test} - R_f = \alpha + \beta R_{t+1} - R_f + \sum_{i=1}^{n} \gamma_i U_i' \left(w_i^{*\prime} R_{t+1}\right) + \varepsilon_{t+1} \Rightarrow$$

$$R_{t+1}^{test} - R_f = \alpha + \beta R_{t+1} - \left(R_f \left(1 - \beta\iota_K\right) + R_f \beta\iota_K\right) + \sum_{i=1}^{n} \gamma_i U_i' \left(w_i^{*\prime} R_{t+1}\right) + \varepsilon_{t+1} \Rightarrow$$

$$R_{t+1}^{test} - R_f = \alpha + \beta\left(R_{t+1} - R_f \iota_K\right) - R_f \left(1 - \beta\iota_K\right) + \sum_{i=1}^{n} \gamma_i U_i' \left(w_i^{*\prime} R_{t+1}\right) + \varepsilon_{t+1} \Rightarrow$$

$$R_{t+1}^{test} - R_f = \left[\alpha - R_f\left(1 - \beta\iota_K\right)\right] + \beta\left(R_{t+1} - R_f\iota_K\right) + \sum_{i=1}^{n}\gamma_i U_i'\left(w_i^{*\prime} R_{t+1}\right) + \varepsilon_{t+1} \qquad (45)$$

Let $\alpha_J$ again be the intercept in the regression (45), i.e. $\alpha_J = \alpha - R_f\left(1 - \beta\iota_K\right)$. Since the restrictions when the test is formulated in gross returns are $\alpha = \gamma_i = 0\ \forall i$ and $\beta i_K = 1$, the equivalent restriction in excess returns is that $\alpha_J = \gamma_i = 0\ \forall i$ .

Hence, when the test is formulated in excess returns, equation (16) need to be estimated and the constraints under the null hypothesis of spanning refer only to the intercept term and the coefficients of the SDFs (equation (17)). Again, the slope coefficients multiply only the excess returns of the *(K-1)* risky assets and therefore they do not need to add up to one.